The G-dynamics for Hilbert-Pólya conjecture

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Abstract

With the non-universal validity of the QPB that has been replaced by the complete QCPB theory. In order to be self-consistent with QCPB theory, it is necessary to extend the Hermitian operator to the non-Hermitian operator generally. In such universal case, the G-dynamics contained in the QCHS theory has played a critical role in the whole QCPB theory. It legitimately proves the Hilbert-Pólya conjecture and then the Riemann hypothesis as a byproduct becomes a direct inference, the G-dynamics as a bridge successfully connects quantum mechanics with general relativity which directly leads to the quantum gravity.

Keywords: QCPB theory; QCHS theory; G-dynamics; Hilbert-Pólya conjecture; Riemann hypothesis; quantum gravity.

1 Introduction

1.1 Riemann zeta function and RH

In 1737s, Euler discovered a formula relating the zeta function, which involves summing an infinite sequence of terms containing the positive integers, and an infinite product that involves every prime number:

Theorem 1 (Euler product formula). *[1] For the real number $\sigma > 1$,*

$$\sum_{n=1}^{\infty} \frac{1}{n^{\sigma}} = \prod_{p} \left(1 - \frac{1}{p^{\sigma}}\right)^{-1}$$

Where p is prime number.

The Euler product formula is also called the analytic form of the fundamental theorem of arithmetic. It demonstrates how the Riemann zeta function encodes information on the prime factorization of integers and the distribution of primes. Riemann

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"G-dynamics is magic actuator for the Universe or Multiverse"

extended the study of the zeta function to include the complex numbers. Riemann's zeta function, depending on the complex variable z, is defined in more compact notation as

$$\zeta(z) = \sum_{n} \phi_n(z) = \prod_{p} (1 - \phi_p(z))^{-1}$$

where $\phi_n(z) = \phi_n(x, y) = n^{-x} n^{-\sqrt{-1}y} = n^{-z}$ is denoted. It can also be defined by the integral

$$\zeta(z) = \frac{1}{\Gamma(z)} \int_0^\infty \frac{1}{e^x - 1} x^z \frac{\mathrm{d}x}{x}$$

where $\Gamma(z) = \displaystyle\int_0^\infty e^{-x} x^z \frac{\mathrm{d}x}{x}$ is the gamma function. The Riemann zeta function can be given by

$$\frac{4\xi(z)}{z(z-1)} = 2\pi^{-\frac{z}{2}} \Gamma\left(\frac{z}{2}\right) \zeta(z) = \int_0^\infty (\theta(\sqrt{-1}t) - 1) t^{\frac{z}{2}-1}\, \mathrm{d}t,$$

in terms of Jacobi's theta function $\theta(\tau) = \displaystyle\sum_{n=-\infty}^{\infty} e^{\sqrt{-1}\pi n^2 \tau}$. Riemann also found a symmetric version of the functional equation applying to the xi-function:

$$\xi(z) = \frac{1}{2} \pi^{-\frac{z}{2}} z(z-1) \Gamma\left(\frac{z}{2}\right) \zeta(z),$$

However, this integral only converges if the real part of z is greater than 1, but it can be regularized. This gives the following expression for the zeta function, which is well defined for all z except 0 and 1:

$$\pi^{-z/2} \Gamma(z/2) \zeta(z) = \frac{1}{z-1} - \frac{1}{z} + \frac{1}{2} \int_0^1 \left(\theta\left(\sqrt{-1}t\right) - t^{-1/2}\right) t^{z/2-1} dt$$
$$+ \frac{1}{2} \int_1^\infty \left(\theta\left(\sqrt{-1}t\right) - 1\right) t^{z/2-1} dt$$

The Riemann hypothesis states that in fact all these zeros $z_n = 1/2 + \sqrt{-1}\gamma_n = \left(\frac{1}{2},\ \gamma_n\right)$ lie on the critical line $\operatorname{Re} z = 1/2$. Actually, it represents that all coordinates are shown as $z_n = \left(\frac{1}{2},\ \gamma_n\right) \in \mathbb{C}$ in the complex plane $\mathbb{C}$ with a fixed abscissa $\operatorname{Re} z_n = 1/2$. The numbers γ_n are defined by

$$\zeta\left(1/2 + \sqrt{-1}\gamma_n\right) = 0, \quad \gamma_n \neq 0$$

If the Riemann hypothesis is true, all the (infinitely many) γ_n are real, and are the heights of the zeros above the real z axis.

In a recent paper [2], we have given a deeply geometric resolution of the RH. Firstly, we propose the non-Hermitian operator that $\hat{a} \neq \hat{a}^\dagger$ always holds in terms of the quantum operator $\hat{a}$ for QM and GR, and we prove that there always exists a NG operator $\hat{a}^{(g)} \neq 0$ satisfying $\hat{a}^{(g)} = -\hat{a}^{(g)\dagger}$ such that any non-Hermitian operator $\hat{a}$ can be formally shown as $\hat{a} = \hat{a}^\dagger + \hat{a}^{(g)}$, then the Hermitian symmetry breaking is caused by NG operator $\hat{a}^{(g)} \neq 0$ as a skew Hermitian operator that represents terms related to gravity. The skew Hermitian operator automatically replies to Hilbert-Pólya conjecture and exactly answers RH in analytic number theory, actually, RH as a byproduct to be solved is correspondingly done. Since the n-manifolds admits an Einstein metric, by considering two-manifolds with Einstein metric, it is then geometrically proved that the non-trivial zeros of Riemann ζ function actually exist as a quantified set for QG such that [1]

$$\zeta\left(1/2 + w^{(g)}/2\right) = 0$$

where NG operator $\hat{a}^{(g)}$ in terms of the frequency w is the geometric frequency $w^{(g)} = \pm 2\sqrt{-1}R$ that relates to the discrete energy spectrum $\rho = 1/2 + \sqrt{-1}E^{(QG)}/\hbar$. The scalar curvature R forming a discrete set only relies on the Einstein tensor $G_{ij} = 0$ for a vacuum field equation, and by using the Planck length l_P that is given for a coupling constant $q_G = \pi {l_P}^2/c \sim 10^{-78}$ of QG along with $\sqrt{q_G} \sim 10^{-39}$ that coincidentally fits the strength of intensity 10^{-39} of gravity in four interactions.

Unsolved problem: RH the Riemann Hypothesis (RH), has never been proved or disproved, and is probably the most important unsolved problem in mathematics.

Unsolved problem: QG How can the theory of quantum mechanics be merged with the theory of general relativity / gravitational force and remain correct at microscopic length scales? What verifiable predictions does any theory of quantum gravity make?

As we have proven, there is a deep connection for both unsolved problems, it states as follows.

Theorem 2 (RH $\sim$ QG)**.** *[2] The RH holds for QG such that $G_{ij}(R) = \zeta(\rho) = 0$, where non-trivial zeros $\rho = 1/2 + \sqrt{-1}R$.*

This partially answers the unsolved problems above together, this step has greatly opened up a new perspective for understanding the quantum behaviors of gravity. In particular, it helps us better perceive the relation between the analytic number theory and Riemannian geometry. It is widely hoped that a theory of QG would allow us to understand problems of very high energy and very small dimensions of space, such as the behavior of black holes, and the origin of the universe.

This paper aims at the description of how RH and QG connects in both number theory and Riemann geometry. We surely believe that the Riemann zeros are related to the eigenvalues of a vibration frequencies.

2 General relativity

A pseudo-Riemannian manifold (M, g) is a differentiable manifold M equipped with an everywhere non-degenerate, smooth, symmetric metric tensor g. Such a metric is called a pseudo-Riemannian metric. Applied to a vector field, the resulting scalar field value at any point of the manifold can be positive, negative or zero. Some basic theorems of Riemannian geometry can be generalized to the pseudo-Riemannian case. In particular, the fundamental theorem of Riemannian geometry is true of pseudo-Riemannian manifolds as well. This allows one to speak of the Levi-Civita connection on a pseudo-Riemannian manifold along with the associated curvature tensor.

[1] See [2, 4] for more details.

2.1 Einstein field equation and Einstein tensor

As understood, the metric tensor is a central object in general relativity that describes the local geometry of spacetime as a result of solving the Einstein field equation. The contracted second Bianchi identity is given by [3]

$$\nabla^j G_{ij} = \nabla^j \left(R_{ij} - \frac{1}{2} R g_{ij} \right) = 0 \tag{1}$$

to interpret the contracted second Bianchi identity $\nabla^j R_{ij} = \frac{1}{2}\partial_i R$, where Einstein tensor is $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ which is a symmetric second-rank tensor that is a function of the metric, as described, identity $\nabla^j R_{ij} = \frac{1}{2}\partial_i R \neq 0$ holds. Actually, $G_{ij} = 0$ is the zero solution of the (1). Einstein tensor $G_i^j = R_i^j - \frac{1}{2}\delta_i^j R$ must satisfy the Bianchi identity that can be written in the form

$$\left(R_i^j - \frac{1}{2}\delta_i^j R \right)_{;j} = \nabla_j \left(R_i^j - \frac{1}{2}\delta_i^j R \right) = 0$$

Furthermore, it gets $R_{;l} = R_{,l} = 2g^{ij}R_{il;j} = 2R^j_{l;j}$ by using the fact that the metric tensor is covariantly constant. In general, under the covariant conservation laws and general relativity is consistent with the local conservation of energy and momentum expressed as $T^j_{l;j} = 0$, and field equation needs, $R^j_{l;j} \neq 0$ is strictly asked to be kept, but it has successfully found $G^j_{l;j} = 0$ to fit the covariant conservation laws of matter field proven by the foundation of general relativity. On two dimensional manifolds, the Einstein tensor $G_i^j = R_i^j - \frac{1}{2}\delta_i^j R$ automatically becomes zero.

Because the material tensor satisfies the covariant conservation law

$$\nabla^i T_{ij} = 0 \tag{2}$$

and the Ricci tensor itself does not satisfy the covariant conservation law $\nabla^i R_{ij} \neq 0$, the system of equations

$$R_{ij} = \kappa T_{ij} \tag{3}$$

is incompatible, this incorrect equation proposed by Einstein earlier, it differs from the correct equation by a term proportional to the curvature scalar: $-\frac{1}{2}Rg_{ij}$. This raises the question that Einstein has not yet obtained the correct field equation. At the same time, the equations (3) they wrote did not succeed in explaining physical phenomena. Although the equation is beautiful and satisfies many things, Einstein still can't explain the deviation between Mercury's perihelion precession and Newton's equation, so he knows that the equation (3) is still unsuccessful. In addition, the correct gravitational field equation together with the contracted Bianchi identity ensures the covariant conservation of energy and momentum, which, depending on the specific material system, contains all or part of the information of the equation of motion of matter, which is now well known.

The energy momentum tensor T_{ij} of matter field satisfies the law of covariant conservation (2), Einstein tensor also satisfies the law of covariant conservation $\nabla^i G_{ij} = 0$ which fulfills $R_{;l} = R_{,l} = 2R^j_{l;j} \neq 0$ to unite two covariant conservation leads to the Einstein field equation

$$G_{ij} = \frac{8\pi G}{c^4} T_{ij} \tag{4}$$

it's conclusively compatible with covariant conservation (2) of matter field. It follows vacuum Einstein field equation that is $G_{ij} = 0$, with the support of the law of covariant conservation for both energy momentum tensor and Einstein tensor, it indicates that R in the vacuum field equation $R_{ij} = \frac{1}{2}Rg_{ij}$ satisfies $R_{;l} = R_{,l} \neq 0$, which means in two dimensional R is a variable.

In conclusions, the basic facts are clearly listed as follows

1. covariant conservation of matter field (2) $\nabla^i T_{ij} = 0$ which expresses the local conservation of stress-energy. This conservation law is a physical requirement. With field equations, Einstein ensured that general relativity is consistent with this conservation condition.
2. contracted second Bianchi identity $\nabla^i G_{ij} = 0, \quad G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$.
3. According to the 1 and 2, it has identity $\nabla^i R_{ij} = \frac{1}{2}\nabla_j R = \frac{1}{2}\partial_j R \neq 0$ held.

The Einstein tensor is a rank 2 tensor defined over Riemannian manifolds. The Ricci tensor depends only on the metric tensor, so the Einstein tensor can be defined directly with just the metric tensor.

The Bianchi identities can also be easily expressed with the aid of the Einstein tensor: $\nabla_i G^{ij} = 0$. The Bianchi identities automatically ensure the conservation of the stress-energy tensor in curved spacetimes: (2). The geometric significance of the Einstein tensor is highlighted by this identity.

The Einstein-Hilbert action is the action that yields the Einstein's field equations through the principle of least action. Einstein's field equation (4) for describing the gravity is in a specific expression given by

$$R_{ij} - \frac{1}{2}g_{ij}R = \kappa T_{ij} \tag{5}$$

where $\kappa = \frac{8\pi G}{c^4}$ has been chosen such that the non-relativistic limit yields the usual form of Newton's gravity law. The expression on the left represents the curvature of spacetime as determined by the metric; the expression on the right represents the matter or energy content of spacetime. The Einstein field equation can then be interpreted as a set of equations dictating how matter or energy determines the curvature of spacetime. These equations, together with the geodesic equation, which dictates how freely-falling matter moves through spacetime, form the core of the mathematical formulation of general relativity.

Einstein modified his original field equations to include a cosmological term Λ proportional to the metric

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij} \tag{6}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar or scalar curvature, and the metric tensor g describe the structure of spacetime, the stress-energy tensor T describes the energy and momentum density and flux of the matter in that point in spacetime. When Λ is zero, this reduces to the field equation of general relativity. The constant Λ is the cosmological constant. Since Λ is constant, the energy conservation law is unaffected. Despite Einstein's misguided motivation for introducing the cosmological constant term, there is nothing inconsistent with the presence of such a term in the equations. Indeed, recent improved astronomical techniques have found that a positive value of Λ is needed to explain the accelerating universe. Einstein thought of the cosmological constant as an independent parameter, but its term in the field equation can also be moved algebraically to the other side, written as part of the stress-energy tensor. The existence of a cosmological constant is thus equivalent to the existence of a non-zero vacuum energy.

2.2 Einstein-Hilbert action

The Einstein-Hilbert action in general relativity is the action that yields the Einstein field equations through the principle of least action. With the $(-,+,+,+)$ metric signature, the gravitational part of the action is given as [3]

$$I = \frac{1}{2\kappa} \int R\sqrt{-g}\, \mathrm{d}^4 x + I_m \tag{7}$$

where $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor matrix, R is the Ricci scalar, and $\kappa = 8\pi G c^{-4}$ is Einstein's constant, G is the gravitational constant and c is the speed of light in vacuum). If it converges, the integral is taken over the whole spacetime. If it does not converge, I is no longer well-defined, but a modified definition where one integrates over arbitrarily large, relatively compact domains, still yields the Einstein equation as the Euler-Lagrange equation of the Einstein-Hilbert action. Note that $\sqrt{-g}d^4x$ is a form invariant 4-dimensional volume element.

Now that we have all the necessary variations at our disposal, we can insert them into the equation of motion for the metric field to obtain. The action principle then tells us that the variation of this action $\delta I = 0$ with respect to the inverse metric is zero, yielding (5).

Taking the trace with respect to the metric of both sides deduces the result $R = -\kappa T$ at $\delta^\mu_\mu = 4$, and (5) is written as

$$R_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)$$

If $\delta^\mu_\mu = 2$ holds, then $T = 0$ is obtained.

When a cosmological constant Λ is included in the Lagrangian, Euler-Lagrange equation is of the form $L_{gra} = \frac{1}{2\kappa}(R - 2\Lambda)\sqrt{-g}$, where $[\Lambda] \sim \left[l^{-2}\right]$, the action

$$I = \frac{1}{2\kappa} \int\limits_M (R - 2\Lambda)\sqrt{-g} d^4 x + I_m$$

yields the field equations and the cosmological constant Λ appears in Einstein's field equation (6). Accordingly, (6) is rewritten by

$$R_{ij} - \Lambda g_{ij} = \kappa\left(T_{ij} - \frac{1}{2} T g_{ij}\right)$$

with $R - 4\Lambda + \kappa T = 0$. If $T_{ij} = \frac{1}{2} T g_{ij}$ holds, then $R_{ij} = \Lambda g_{ij}$ or $R = 4\Lambda$.

2.3 Equivalent formulations

It can be proved that in Riemannian geometry, there are g_{ij} and its first and second derivatives, and for the second tensor with linear combination of second derivatives, it is possible to take the following form $c_1 R_{ij} + c_2 R g_{ij} + c_3 g_{ij}$, where c_1, c_2, c_3 all are constants. Therefore, the most common possible form of the field equation should be

$$R_{ij} + c_2 R g_{ij} + \Lambda g_{ij} = \kappa T_{ij}$$

where κ is uniquely determined by the gravitational constant. c_2, Λ are undetermined constants. In general relativity, the law of conservation of energy and momentum is expressed as a covariant form $\nabla_j T^{ij} = T^{ij}{}_{;j} = 0$, as a result, then

$$\nabla_j \left(R^{ij} + c_2 R g^{ij}\right) = \left(R^{ij} + c_2 R g^{ij}\right)_{;j} = 0$$

Comparing this with Einstein tensor $\nabla_j G^{ij} = G^{ij}{}_{;j} = 0$, namely, $\nabla_j G^{ij} = \nabla_j T^{ij} = 0$, then $c_2 = -1/2$, and (6) then follows.

Taking the trace with respect to the metric of both sides of the EFE (6) one gets

$$R - \frac{n}{2} R + n\Lambda = \frac{8\pi G}{c^4} T \tag{8}$$

where n is the spacetime dimension. In vacuum case, namely, condition $T = 0$ holds, it has form given by $(n-2) R = 2n\Lambda$ Obviously, if $n = 2$ is taken in vacuum case, then $\Lambda = 0$, it reveals that Λ has different properties in terms of R at dimension two. If $T \neq 0$ is assumed, then $\Lambda = \frac{4\pi G}{c^4} T$ holds on two dimensional manifolds.

The expression (8) can be rewritten as

$$-R + \frac{n\Lambda}{\frac{n}{2} - 1} = \frac{8\pi G}{c^4} \frac{T}{\frac{n}{2} - 1}, \quad n \neq 2$$

If one adds $-\frac{1}{2} g_{ij}$ times this to the EFE, one gets the following equivalent trace reversed form

$$R_{ij} - \frac{\Lambda g_{ij}}{\frac{n}{2} - 1} = \frac{8\pi G}{c^4} \left(T_{ij} - \frac{1}{n-2} T g_{ij}\right).$$

The stress-energy tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in spacetime, generalizing the stress tensor of Newtonian physics. It is an attribute of matter, radiation, and non-gravitational force fields. The stress-energy tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass is the source of such a field in Newtonian gravity.

3 Non-Hermitian operators

3.1 Hermitian symmetry

Hermitian symmetry exists in most quantum theories, but it is not necessary. If we give up this symmetry, the system will have wonderful physical phenomena and interesting applications. Schrödinger equation is the core of quantum mechanics. It consists of two parts: Hamiltonian operator and wave function. The Hamiltonian depends on the system environment, and the wave function contains all the useful information about the quantum state. For a particular system, we write the system properties into Hamiltonian operators, and Schrödinger's equation tells us what the wave function looks like in the system.

Usually, physicists will ask the Hamiltonian operator to satisfy some basic mathematical conditions to satisfy symmetry in nature, for example, the energy of quantum states is real number, particle number and energy conservation. These basic mathematical conditions are called Hermitian symmetry, and almost all basic quantum theories are based on the premise that the Hamiltonian operator is Hermitian symmetry.

3.2 Symmetry breaking of Hermiticity and NG operator

The complex plane or z-plane is a geometric representation of the complex numbers established by the real axis and the perpendicular imaginary axis. It can be thought of as a modified Cartesian plane, with the real part of a complex number represented by a displacement along the x-axis, and the imaginary part by a displacement along the y-axis.

A complex number takes the form $z = x + \sqrt{-1}y$ where x and y are real, and $\sqrt{-1}$ is an imaginary number. The complex conjugate of $z = x + \sqrt{-1}y$ is defined by $\overline{z} = x - \sqrt{-1}y$[2], and it is obtained by a reflection across the real axis in the plane. In fact a complex number z is real if and only if $z = \overline{z}$, and it is purely imaginary if and only if $z = -\overline{z}$. The reader should have no difficulty checking that

$$\mathrm{Re}\,(z) = \frac{z+\overline{z}}{2} = x, \quad \mathrm{Im}\,(z) = \frac{z-\overline{z}}{2\sqrt{-1}} = -\sqrt{-1}\frac{(z-\overline{z})}{2} = y \tag{9}$$

We call x and y the real part and the imaginary part of z, respectively, the imaginary part of z is $\sqrt{-1}y = \frac{z-\overline{z}}{2}$. A complex number with zero real part is said to be purely imaginary. Throughout our presentation, the set of all complex numbers is denoted by $\mathbb{C}$. The complex numbers can be visualized as the usual Euclidean plane by the following simple identification: the complex number $z = x + \sqrt{-1}y \in \mathbb{C}$ is identified with the point $(x, y) \in \mathbb{R}^2$. Naturally, the x and y axis of $\mathbb{R}^2$ are called the real axis and imaginary axis, because they correspond to the real and purely imaginary numbers, respectively. A complex number z and its complex conjugate $\overline{z}$ can be expressed as

$$z = x + \sqrt{-1}y = (x, y)\,, \quad \overline{z} = x - \sqrt{-1}y = (x, -y)$$

on the complex plane. Furthermore, modulus (or absolute value) is given by $r = |z| = \sqrt{z\overline{z}}$, as an application, such as all non-trivial zeros of $\zeta(z)$ on the complex plane can be shown in the form

$$\rho_n = 1/2 + \sqrt{-1}\gamma_n = (1/2, \gamma_n)$$

Hence, its complex conjugate of non-trivial zeros is $\overline{\rho_n} = 1/2 - \sqrt{-1}\gamma_n = (1/2, -\gamma_n)$, subsequently, based on (9), $\rho_n + \overline{\rho_n} = 1$, $\frac{\rho_n+\overline{\rho_n}}{2} = 1/2$, And we then have

$$\sqrt{-1}\gamma_n = \frac{\rho_n - \overline{\rho_n}}{2}$$

More precisely, it easily yields $\rho_n \neq \overline{\rho_n}$ and $\rho_n - \overline{\rho_n} = 2\sqrt{-1}\gamma_n \neq 0$. Above conclusions hold absolutely. To gather all similar mode the complex function or complex situation

[2]The complex conjugate of a complex number z is written as $\overline{z}$ or z^*. The first notation, a vinculum, avoids confusion with the notation for the conjugate transpose of a matrix, which can be thought of as a generalization of the complex conjugate. The second is preferred in physics, where dagger (†) is used for the conjugate transpose, while the bar-notation is more common in pure mathematics.

leads to a common feature

$$z^{(g)} = z - \overline{z} = 2\sqrt{-1}y$$
$$\rho_n{}^{(g)} = \rho_n - \overline{\rho_n} = 2\sqrt{-1}\gamma_n$$
$$\hat{a}^{(g)} = \hat{a} - \hat{a}^{\dagger} = 2\sqrt{-1}\hat{a}^{(hi)}$$

Subsequently, we obtain

$$\sqrt{-1}\gamma_n = \frac{\rho_n - \overline{\rho_n}}{2}, \quad \sqrt{-1}y = \frac{z - \overline{z}}{2}, \quad \sqrt{-1}\hat{a}^{(hi)} = \frac{\hat{a} - \hat{a}^{\dagger}}{2}$$

Accordingly, it gets

$$\rho_n = 1/2 + \sqrt{-1}\gamma_n = 1/2 + (\rho_n - \overline{\rho_n})/2 = \frac{1}{2}(1 + \rho_n - \overline{\rho_n})$$

In other words, it's also expressed as $\rho_n = \frac{1}{2}\left(1 + 2\sqrt{-1}\gamma_n\right)$. Thusly, the RH means $\zeta(\rho_n) = \zeta\left(1/2 + \rho_n^{(g)}/2\right) = 0$.

Definition 1 (Symmetry breaking of Hermiticity). *[2] Non-Hermitian operator*

$$\hat{a} = \hat{a}^{\dagger} + \hat{a}^{(g)} \in NHer$$

holds, where $\hat{a}^{(g)} = 2\hat{a}^{(sh)} \neq 0 \in SHer$ *is called NG operator.*

The mathematics of almost all eigenvalue problems encountered in wave physics is essentially the same, but the richest source of such problems is quantum mechanics, where the eigenvalues are the energies of stationary states (levels), rather than frequencies as in acoustics or optics, and the operator is the hamiltonian. Reflecting this catholicity of context, we will refer to the γ_n as frequencies.

The basis of the Riemann quantum analogy, which is an identification of the periodic orbits in the conjectured dynamics underlying the Riemann zeros, made by comparing formulae for the counting functions of the γ_n and of asymptotic quantum eigenvalues. The significance of the long periodic orbits in giving rise to universal behaviour in classical and semiclassical mechanics and, by analogy, the Riemann zeros, with a physical interpretation in terms of resurgence of long periodic orbits that implies new interpretations of the periodic orbit sum for quantum spectra.

In conclusion comprehensively, we can say that this spectrum given by the nontrivial zeros of the zeta function is definitely frequency for unknown system.

As a result, it implies an analogous attempt.

Corollary 1. *There exists a NG operator* $\hat{w}^{(g)} \neq 0$ *such that*

$$\zeta\left(1/2 + w^{(g)}/2\right) = 0$$

holds for its eigenvalue $w^{(g)}$ *in quantum mechanics.*

Since a complex number $z = x + \sqrt{-1}y$ is uniquely determined by a pair of ordered real numbers (x, y), for a given rectangular coordinate system on a plane, all the complex numbers correspond to all the points on the plane one by one, so the complex number $z = x + \sqrt{-1}y$ can be represented by the points whose coordinates are (x, y) on the plane.

The same pure imaginary modes appear in complex number theory

$$\begin{cases} \sqrt{-1}y = \frac{z-\overline{z}}{2} \\ \sqrt{-1}\gamma_n = \frac{\rho_n - \overline{\rho_n}}{2} \\ \sqrt{-1}\hat{a}^{(hi)} = \frac{\hat{a}-\hat{a}^\dagger}{2} \end{cases} \Rightarrow y = \gamma_n = a_n{}^{(hi)} \in \mathbb{R}$$

Therefore, we formally assume that above identity holds in quantum mechanics. It strongly implies that $\gamma_n = a_n{}^{(hi)}$ correspond to the eigenvalues of Hermitian operator $\hat{a}^{(hi)}$ in CQM, it arises a question, what is the corresponding quantum system for the non-trivial zeros of zeta function.

$$\hat{a}^{(hi)} = -\sqrt{-1}\hat{a}^{(sh)} = -\frac{\sqrt{-1}}{2}\hat{a}^{(g)} \in Her$$

We naturally speculate Hermitian operator $\hat{a}^{(hi)}$ is a hidden discrete operator with unique properties. To speculate on this question is a gain.

3.3 Non-Hermitian Hamiltonian operator

Theorem 3. *For given non-Hermitian Hamiltonian operator $\hat{H} \in NHer$, then*

$$\hat{H} = \hat{H}^{(\mathrm{re})} + \hat{H}^{(sh)} \in NHer, \quad \hat{H}^{(sh)} \in SHer \tag{10}$$

where

$$\hat{H}^{(\mathrm{re})} = \frac{1}{2}\left(\hat{H} + \hat{H}^\dagger\right) \in Her$$

$$\hat{H}^{(\mathrm{im})} = -\sqrt{-1}\hat{H}^{(sh)} = \frac{1}{2\sqrt{-1}}\left(\hat{H} - \hat{H}^\dagger\right) \in Her$$

so that its eigenvalues are $\lambda = \lambda^{(\mathrm{re})} + \sqrt{-1}\lambda^{(\mathrm{im})}$; $\lambda^{(\mathrm{re})}, \lambda^{(\mathrm{im})} \in C^\infty(\mathbb{R})$.

Obviously, $\hat{H}^{(\mathrm{re})}$ is a symmetric operator while $\hat{H}^{(sh)}$ is a antisymmetric operator. As usually, $\hat{H}^{(\mathrm{re})}$ is taken as the form $\hat{H}^{(\mathrm{re})} = \hat{H}^{(cl)} = \sqrt{-1}\hbar\partial_t \in Her$. It turns out that $\hat{H}^{(sh)} = \sqrt{-1}\hat{H}^{(\mathrm{im})} \in SHer$ is a skew Hermitian operators given by non-Hermitian Hamiltonian operator. And then NG operator in terms of Hamiltonian operator emerges basically

$$\hat{H}^{(g)} = \hat{H} - \hat{H}^\dagger = 2\hat{H}^{(sh)} = 2\sqrt{-1}\hat{H}^{(im)} \in SHer$$

Hermitian operators are systems of $\hat{H}^{(\mathrm{re})} = \hat{H}^{(\mathrm{re})\dagger}$, which correspond to eigenvalues of real numbers. Hermitian system is the system whose physical quantities are Hermitian operators, that is, the eigenvalues of physical quantities are real numbers.

For a non-Hermitian system, the eigenvalues of physical quantities contain imaginary numbers, such as the time-dependent Hamiltonian. Notice that non-Hermitian operator is comprised by Hermitian operator $\hat{H}^{(\mathrm{re})}$ and skew Hermitian operator $\hat{H}^{(sh)} = \sqrt{-1}\hat{H}^{(\mathrm{im})} \in SHer$, it well explains that all system which get systems involved is always a non-Hermitian operator, correspondingly. In other words, skew Hermitian operator $\hat{H}^{(sh)} = \sqrt{-1}\hat{H}^{(\mathrm{im})} \in SHer$ is the part missed by the quantum mechanics, it fits the explanations why the general relativity can't reconcile the quantum mechanics, obviously, the QG is a non-Hermitian system described by skew Hermitian operator.

4 Covariant dynamics, generalized Heisenberg equation, G-dynamics

Quantum geometric bracket defined by geomutator is such a beautiful thing ever existed in the universe.

In this section, we will briefly review the entire theoretical framework of quantum covariant Hamiltonian system defined by the quantum covariant Poisson bracket totally based on the paper [5]. More precisely, the time covariant evolution of any observable $\hat{f}$ in the covariant dynamics is given by both the generalized Heisenberg equation of motion and G-dynamics.

Theorem 4 (The covariant dynamics). *[5] The covariant dynamics, the generalized Heisenberg equation, G-dynamics can be formally formulated as*

The covariant dynamics: $\frac{\mathcal{D}\hat{f}}{dt} = \frac{1}{\sqrt{-1}\hbar}\left[\hat{f}, \hat{H}\right]$

The generalized Heisenberg equation: $\frac{d\hat{f}}{dt} = \frac{1}{\sqrt{-1}\hbar}\left[\hat{f}, \hat{H}\right]_{QPB} - \frac{1}{\sqrt{-1}\hbar}\hat{H}\left[s, \hat{f}\right]_{QPB}$

G-dynamics: $\hat{w} = \frac{1}{\sqrt{-1}\hbar}\left[s, \hat{H}\right]_{QPB}$.

respectively, where $\frac{\mathcal{D}}{dt} = \frac{d}{dt} + \hat{w}$ *is covariant time operator, and* $\hat{H}$ *is the Hamiltonian and* $[\cdot, \cdot]$ *denotes the GGC of two operators.*

With the non-universal validity of the QPB that has been replaced by the complete QCPB theory. Definitely, the QCPB theory leads us to a complete picture of the quantum mechanics and its deep secrets. We can definitely employ the QCPB approach to discover more hidden laws of the quantum mechanics. More importantly, the QCPB theory can naturally reconcile the quantum mechanics and the general relativity.

This is a picture of the future as painted by the QCPB theory-a picture that will surely evolve over time as we dig for more clues to how our story will unfold. Much of the science is very recent and new puzzle pieces are still waiting to be found.

4.1 Imaginary geomenergy

In this section, the imaginary geomenergy is defined based on the G-dynamics, by using this new concept, we can better give a presentation for the covariant dynamics, ect.

Definition 2. *[5] The imaginary geomenergy is defined by* $E^{(\mathrm{Im})}(\hat{w}) = \sqrt{-1}\hbar\hat{w}$*, where* $\hat{w}$ *means G-dynamics.*

Thus, a dynamical variable $\hat{f}$ with the geomenergy defined above, then covariant dynamics is rewritten in the form

$$\sqrt{-1}\hbar\frac{\mathcal{D}}{dt}\hat{f} = \left[\hat{f}, \hat{H}\right] = \sqrt{-1}\hbar\frac{d}{dt}\hat{f} + \hat{f}E^{(\mathrm{Im})}(\hat{w})$$

As a consequence of the imaginary geomenergy, we can say that imaginary geomenergy is a new kind of Hamiltonian operator. Obviously, if the covariant equilibrium equation $\left[\hat{f}, \hat{H}\right] = 0$ holds, then it yields

$$\frac{d}{dt}\hat{f} + \hat{f}\hat{w} = 0$$

or this covariant equilibrium formula in the form shows $\frac{d}{dt}\hat{f} = -\hat{f}\hat{w}$, accordingly, $\hat{f}$ is said to be a quantum covariant conserved quantity.

5 G-dynamics for the Riemann hypothesis

Since the G-dynamics has been discovered as an independent dynamics to picture the quantum mechanics, we have studied its precise properties [see [4, 5] for more details], throughout this process, we surprisedly find a proper Hermitian operator for solving the RH.

5.1 Hilbert-Pólya theorem

More precisely, the G-dynamics as a new dynamical form appears in the quantum mechanics, we mainly focus on the G-dynamics

$$\hat{w} = \frac{1}{\sqrt{-1}\hbar}\left[s, \hat{H}\right]_{QPB}$$

that'll take us into the physical future. With the S-dynamics in the GCHS, clearly, we can say that the G-dynamics describes rotating quantum physics, as a result of this, we can naturally give a conclusion, the G-dynamics is in charge of the non-inertial system, hence, we expect to use it to accomplish the compatible combination of the quantum mechanics and general relativity. As [4, 5] already stated, the G-dynamics is only induced by the structure function s, and meanwhile, it's independent to the other observables. There is a clear fact of the G-dynamics taken as a different form by choosing different Hamiltonian operators, in one word, various Hamiltonian operators correspond to the multiple G-dynamics. Therefore, for a given wave function ψ, the eigenvalue equation of operator is accordingly given by $\widehat{w}\psi = w\psi$, where w is the geometric frequency eigenvalue. By using G-dynamics, it gets corresponding imaginary geomenergy

$$E^{(\mathrm{Im})}(\hat{w}) = \sqrt{-1}\hbar\hat{w} = \left[s, \hat{H}\right]_{QPB}$$

Thusly, it then has energy spectrum given by $E^{(\mathrm{Im})}\psi = \sqrt{-1}E^{(g)}\psi$, where $E^{(g)} = \hbar w$. Besides, $E^{(\mathrm{Im})}/\hbar = \sqrt{-1}\widehat{w}$, in general. Obviously, we can see that the G-dynamics is completely determined by the structure function s, it implies that the G-dynamics represents the properties of the spatial manifolds, in other words, the spatial manifolds has abundant activities. As a result of this point, we need to seek more clues to unlock this quantum characters.

The most general form is the time-dependent Schrödinger equation, which gives a description of a system evolving with time [6]

$$\sqrt{-1}\hbar\frac{\partial\psi}{\partial t} = \widehat{H}^{(cl)}\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi \tag{11}$$

where $\partial/\partial t$ symbolizes a partial derivative with respect to time t. By plugging the classical Hamiltonian operator $\hat{H}^{(cl)} \in Her$ to the G-dynamics, it generates the such

Hermitian operator $\hat{w}^{(cl)} \in Her$ to positively correspond to the reasonable answer for Hilbert-Pólya conjecture, meanwhile, we have skew Hermitian operator

$$E^{(\mathrm{Im})}\left(\widehat{w}^{(cl)}\right)/\hbar = \sqrt{-1}\widehat{w}^{(cl)} \in SHer$$

the NG-operator that is made

$$\widehat{w}^{(g)} = 2E^{(\mathrm{Im})}\left(\widehat{w}^{(cl)}\right)/\hbar = 2\sqrt{-1}\widehat{w}^{(cl)} \in SHer$$

as well, or $\widehat{w}^{(g)} = -2\sqrt{-1}\widehat{w}^{(cl)} \in SHer$. The G-dynamics is mainly for describing the quantum rotation in a geometric frequency. Such Hermitian operator $\hat{w}^{(cl)}$ is rightly a description of frequency of a proper quantum system, let's confirm this system and describe it.

Theorem 5 (Hilbert-Pólya theorem)**.** *[2] The nontrivial zeros of Riemann ζ function correspond to the eigenvalues of a Hermitian operator $\hat{w}^{(cl)}$.*

This theorem certainly ensures the Hilbert-Pólya conjecture that such Hermitian operator $\hat{w}^{(cl)}$ really exists, it answers the deep connection between the analytic algebra and the differential geometry, the structure function s as such bridge appears to get it done well.

Theorem 6. *[2, 4] The heat equation for the structure function s based on the Schrödinger equation* (11) *is given by*

$$\partial_t s = \sqrt{-1}a_c\left(\Delta s/2 + \nabla s \cdot \nabla \ln \psi\right)$$

for a given wave function ψ.

Note that the heat equation above can be rewritten in a simple form by considering its one-dimensional case, $\hat{w}^{(cl)} = -\gamma_m\left(2u\frac{\partial}{\partial x} + \frac{\partial u}{\partial x}\right)$, where $u = \frac{\partial s}{\partial x}$, $\gamma_m = \frac{\sqrt{-1}\hbar}{2m}$ and $\hat{w}^{(cl)}\psi = w^{(q)}\psi$ is its eigenvalues equation, its solution of above eigenvalue equation has been figured out, meanwhile, we obtain simplified version of the geometric wave function

$$\Gamma\left(u, w^{(q)}\right) = u^{-1/2}\exp\left(\sqrt{-1}a_c^{-1}\int\frac{w^{(q)}}{u}dx\right) \tag{12}$$

where $a_c = \hbar/m$. As shown, this geometric wave function is composed of two parts, $\left|\Gamma\left(u, w^{(q)}\right)\right| = \left|\Gamma\left(u\right)\right| = u^{-1/2}$ and geometric phase factor $\exp\left(\sqrt{-1}a_c^{-1}\int\frac{w^{(q)}}{u}dx\right)$, geometric phase is $a_c^{-1}\int\frac{w^{(q)}}{u}dx$, obviously, the former describes the geometric amplitude while the latter depicts the geometric wave. In particular, the geometric amplitude $\left|\Gamma\left(u, w^{(q)}\right)\right| = u^{-1/2}$ has its discrete expression expressed as $\left|\Gamma\left(n\right)\right| = n^{-1/2}$ which highlights quantum characteristics. As we can easily see, the geometric wave function manifests the basic characteristics of geometry, in fact, it can be treated as a geometric function along with the wave. Essentially, it can nicely describe a complete quantum mechanics duo to the QCPB theory is complete, observe that this is a natural way to introduce the differential geometry into the quantum mechanics, furthermore, the general relativity can be compatibly introduced to quantum mechanics for the sake of the achievement of the quantum gravity.

Notice that sub-unit $\phi_n(z) = n^{-x-\sqrt{-1}y}$ for Riemann zeta function is comprised of two parts, $|\phi_n(z)| = |\phi_n(x)| = n^{-x}$ as a discrete pattern and $n^{-\sqrt{-1}y}$ also holds in a discretization form as well, the latter undoubtedly describes some kinds of the wave. As a result, we can see a natural corresponding pattern between the (12) and sub-unit $\phi_n(z) = n^{-x-\sqrt{-1}y}$ for Riemann zeta function.

$$\begin{aligned} |\Gamma(u)| &= u^{-1/2} \to |\phi_n(x)| = n^{-x} \\ \exp\left(\sqrt{-1}a_c^{-1}\int \frac{w^{(q)}}{u}dx\right) &\to \exp\left(-\sqrt{-1}yT_n\right) \end{aligned} \tag{13}$$

where $T_n = \ln n$. That is to say, $\Gamma\left(u, w^{(q)}\right) \to \phi_n(z)$, it's clear to see that $|\Gamma(n)| = |\phi_n(1/2)| = n^{-1/2}$ always holds. In order to better analyze the connection (13), we have proposed the following concepts to simplify the connection (13), based on this, it points out that

Theorem 7. *[2] The heat equation is $w^{(q)} = c_0 a_c u^2$ given for geometric wave-particle duality*

$$p^{(s)} = \hbar u, \quad E^{(q)} = \hbar w^{(q)}, \quad v^{(s)} = c_0 a_c u$$

such that $E^{(q)} = v^{(s)}p^{(s)}$, the $v^{(s)} = c_0 a_c u$ is the quantum geometric velocity of particles in a mass m, c_0 is a dimensionless constant.

Note that geometric wave-particle duality is completely induced by the structure function s with respect to the spacetime, respectively. Especially, quantum geometric velocity $v^{(s)} = c_0 a_c u$ of particles seems to be mainly determined by the geometric variable u induced by s in terms of the derivative of the space. This feature embodies a clear trajectory of the movement of the particle, it implies the quantum path might be there for a unique existence.

Actually, the quantum geometric velocity $v^{(s)} = dx/dt$ represents a classical feature corresponding to the quantum movements. With the help of the geometric wave-particle duality, let's simplify the geometric phase factor $\exp\left(\sqrt{-1}a_c^{-1}\int \frac{w^{(q)}}{u}dx\right)$, in fact, by a directly computation, it gets $v^{(s)}/u = dx/(udt) = c_0 a_c$, and then the geometric phase factor simply becomes $\exp\left(\sqrt{-1}c_0\int w^{(q)}dt\right)$, where c_0 is in charge of the conjugation, here we take $c_0 = -1$ for a better understanding, due to the stability of the geometric frequency $w^{(q)}$, so that $\partial_t w^{(q)} = 0$ always holds for $\exp\left(-\sqrt{-1}w^{(q)}T\right)$, where T corresponds to the period of the geometric frequency $w^{(q)}$. Therefore, the geometric wave function (12) becomes simply

$$\Gamma\left(u, w^{(q)}\right) = u^{-1/2}\exp\left(-\sqrt{-1}w^{(q)}T\right)$$

Its conjugate form is given by $\Gamma^*\left(u, w^{(q)}\right) = u^{-1/2}\exp\left(\sqrt{-1}w^{(q)}T\right)$, essentially, this is a real face of the geometric wave function (12). Then as for the geometric variable u, it has $\Gamma\left(n, w^{(q)}\right) = |\Gamma(n)|\exp\left(-\sqrt{-1}w^{(q)}T\right)$ follows, thusly,

$$\sum_n \Gamma\left(n, w^{(q)}\right) = \sum_n n^{-1/2}\exp\left(-\sqrt{-1}w^{(q)}T\right)$$

As a result, let's reconsider the corresponding relation (13), and using the discrete expression $|\Gamma(n)| = n^{-1/2}$, obviously, we can obtain a clear relation for both cases,

$$\begin{aligned} &|\Gamma(n)| = |\phi_n(1/2)| = n^{-1/2} \\ &\exp\left(-\sqrt{-1}w^{(q)}T\right) \to \exp\left(-\sqrt{-1}yT_n\right) \end{aligned}$$

Thusly, then $e^{-\sqrt{-1}w^{(q)}T} = e^{-\sqrt{-1}yT_n}$ holds for all n if and only if $T \equiv T_n = \ln n,\ y = w^{(q)}$, it means the period T is discrete while all $y = w^{(q)}$ lie on the line $x = 1/2$, then $\phi_n\left(1/2, w^{(q)}\right) = n^{-1/2-\sqrt{-1}w^{(q)}}$ perfectly embodies the core feature of the non-trivial zeros of the Riemann zeta function. As a consequence, the non-trivial zeros pattern for sub-unit $\phi_n(z)$ for Riemann zeta function appears accordingly

$$\Gamma\left(n, w^{(q)}\right) = |\Gamma(n)| \exp\left(-\sqrt{-1}w^{(q)}T_n\right) = n^{-1/2-\sqrt{-1}w^{(q)}}$$

By making use of the discrete procedure

$$\begin{aligned} &u^{-1/2} \to n^{-1/2} \\ &T \to T_n \end{aligned}$$

It obviously leads to an equality $\Gamma\left(n, w^{(q)}\right) = \phi_n\left(1/2, w^{(q)}\right)$, it reveals that all non-trivial zeros are on the critical line 1/2, in other words, all geometric frequency $w^{(q)}$ that relates to energy spectrum lie on the critical line 1/2.

Theorem 8 (Riemann theorem)**.** *All non-trivial zeros of $\zeta(z)$ lie on the critical line* $\operatorname{Re} z = 1/2$.

Most importantly, the discrete period implies the discrete geometric frequency ${w_n}^{(q)}$ to form the non-trivial zeros $\rho = 1/2+\sqrt{-1}w^{(q)}$ or $\rho = 1/2+\sqrt{-1}E^{(q)}/\hbar$ which satisfies $\zeta(\rho) = 0$. Therefore, we prove the Hilbert-Pólya conjecture that is right, and thus, then the Riemann hypothesis as a byproduct becomes a direct inference.

Consequently, due to the irreducible of the prime number, hence we have ${w_n}^{(q)} \to {w_p}^{(q)}$, and $\phi_n(z) \to \phi_p(z)$. The complete eigenvalue equation rewrites

$$\hat{w}^{(cl)}\phi_p\left(1/2, w^{(q)}\right) = {w_p}^{(q)}\phi_p\left(1/2, w^{(q)}\right)$$

As the Hilbert-Pólya theorem 5 described, the discrete geometric frequency ${w_p}^{(q)}$ form a frequency spectrum that relates to the energy spectrum ${E_p}^{(q)} = \hbar w_p^{(q)}$, thusly, the non-trivial zeros can be shown as

$$\rho_p = 1/2 + \sqrt{-1}E_p^{(q)}/\hbar$$

All discrete geometric wave function reaches to a steady state $\sum\limits_n \phi_n\left(1/2, w^{(q)}\right) = 0$ in terms of the geometric frequency $w^{(q)}$, the geometric frequency ${w_n}^{(q)}$ is only associated with partial derivative with respect to time of structure function s.

We always choose structure function s on the Riemann manifolds to be $s = \ln\sqrt{g}$ as a scalar function, as a consequence, geometric frequency can be explicitly verified, $w^{(q)} = -\partial_t \ln\sqrt{g} = R$, where we have used the Ricci flow, and R is the scalar curvature, it leads to a certain formula for Riemann zeta function, globally, that is, $\zeta\left(1/2+\sqrt{-1}R\right)=0$ in general, or definitely,

$$\zeta\left(1/2+\sqrt{-1}R_p\right)=0$$

this implies the deep relation between the prime numbers and the quantum gravity, in short, the quantum gravity labelled by the prime numbers is a unique quantum feature, and the secrets of the prime numbers p link to the quantum gravity. Conclusively, the non-trivial zeros of Riemann zeta function are in a form $\rho = 1/2+\sqrt{-1}R$ or $\rho_p = 1/2+\sqrt{-1}R_p$, as it shows, the non-trivial form of the zeros is so profoundly done.

Technologically, by using the NG operator as a skew Hermitian operator, we can actually go a step further for analyzing the essence of the scalar curvature R in the formula $\zeta(\rho)=0$, it turns out that the following compactly result can be proven to hold that is closely associated with the vacuum field equation $G_{ij}=0$ or the Einstein manifold in two dimensional. Our discussion begins with a brief summary of the progress in the paper [2]. Then comes the beautiful part where the amazing result begins to show the way it once has been,

Theorem 9. *[2] The non-trivial zeros $\rho = 1/2\pm\sqrt{-1}R$ of the zeta function holds for the Einstein tensor such that $G_{ij}=\zeta(\rho)=0$.*

Note that theorem 9 has strengthened the conclusion $\zeta(\rho)=0$ and it has revealed a more deeper relation on the basis of the $\zeta\left(1/2+\sqrt{-1}R\right)=0$ generally. It connects the vacuum field equation to the non-trivial zeros $\rho = 1/2+\sqrt{-1}R$ of the Riemann zeta function. Meanwhile, the theorem 9 has profoundly announced the relation for both Riemann manifold and algebra, the left $G_{ij}=0$ states the two Einstein manifolds or the vacuum field equation while the latter $\zeta(\rho)=0$ nicely gives the elaborate discrete data to be a response to the vacuum field equation. By the way, the contracted second Bianchi identity (1) leads to $\nabla^j R_{ij}=\frac{1}{2}\partial_i R$ which can be interpreted as a physical equilibrium. This identity profoundly brings together the most important aspects of the two deepest things in the quantum gravity: gravitational geometry and data of quantization. Realizing this wonder in our universe is truly existed, this incredible connection is just there to have been discovered.

As Hilbert-Pólya theorem 5 given, the eigenvalues of the Hermitian operator $\hat{w}^{(cl)}$ as a right candidate to prove the Riemann hypothesis beautifully works, it can be seen that the Riemann hypothesis is exactly as a byproduct to becomes a direct inference within the framework of the entire QCPB theory. We hope something unexpectedly good happens to quantum mechanics again in this entirely new QCPB theory.

5.2 Discussions of Hermitian operator $\hat{w}^{(cl)}$

As for the Hermitian operator $\hat{w}^{(cl)}$ that describes the geometric frequency of a spin or the rotation of the geometric object represented by the structure function s, so varied, mysterious, grandiose, affecting, it definitely writes in a form

$$\widehat{w}^{(cl)}/\gamma_m = -2u\frac{\partial}{\partial x}-\frac{\partial u}{\partial x} \tag{14}$$

We can go deep further researching about it such as $\widehat{w}^{(cl)}x=-\gamma_m\left(2u+x\frac{\partial u}{\partial x}\right)$, and $\widehat{w}^{(cl)}u=-\frac{3}{2}\gamma_m\frac{\partial u^2}{\partial x}$, then $\hat{w}^{(cl)}u=-\sqrt{-1}\frac{3}{4}\frac{\partial w^{(q)}}{\partial x}\neq 0$, or the case $\hat{w}^{(cl)}s/\gamma_m=-2u^2-$

$s\frac{\partial u}{\partial x}$, etc. As we can see, the G-dynamics $\hat{w}^{(cl)}$ as a Hermitian operator can act onto different mathematical object, then it implies different meanings. For convenient discussions, consider $u > 0$, as an example, we have $\widehat{w}^{(cl)}x = -4u\gamma_m \left(x\frac{\partial \ln u^{1/4}}{\partial x} + 1/2\right)$. As we can observe, the right side of the above equation is very similar to the Berry-Keating's Hamiltonian $\widehat{H}^{(\mathrm{bk})}$ in the form. Above formula can be simplified as $\widehat{w}^{(cl)}x = \sqrt{-1}v^{(s)}\left(1 + x\partial \ln u^{1/2}/\partial x\right)$, where we have used $-4u\gamma_m = 2\sqrt{-1}v^{(s)}$, and $c_0 = -1$ is taken. For the (14), if we make a transformation like $u \to ux$, then we can obtain

$$-\hat{w}_1^{(cl)}/\gamma_m = 2ux\frac{\partial}{\partial x} + u + x\frac{\partial u}{\partial x} = 2u\left(x\frac{\partial}{\partial x} + 1/2\right) + x\frac{\partial u}{\partial x}$$

Notice that the term $x\frac{\partial}{\partial x} + 1/2$ of right side rightly reflects the Berry-Keating's Hamiltonian $\widehat{H}^{(\mathrm{bk})}$ in the form, hence, it's equal to $-\hat{w}_1^{(cl)}/\gamma_m - x\frac{\partial u}{\partial x} = 2u\left(x\frac{\partial}{\partial x} + 1/2\right)$. Meanwhile, we can get an interesting relation given by

$$x\hat{w}^{(cl)}/\gamma_m - \hat{w}_1^{(cl)}/\gamma_m = u$$

or in a simple form $x\hat{w}^{(cl)} - \hat{w}_1^{(cl)} = \gamma_m u$, then $x\hat{w}^{(cl)} = \hat{w}_1^{(cl)} + u\gamma_m$, and it yields $x\hat{w}^{(cl)} = \hat{w}_1^{(cl)} - \sqrt{-1}v^{(s)}/2$.

As for the geometric wave function given by (12), its conjugate geometric wave function is $\Gamma^*\left(u, w^{(q)}\right) = u^{-1/2}\exp\left(-\sqrt{-1}a_c^{-1}\int \frac{w^{(q)}}{u}dx\right)$ with the module $|\Gamma(u)| = |\Gamma^*(u)| = u^{-1/2}$, accordingly, the discrete formula follows $|\Gamma(n)| = |\Gamma^*(n)| = n^{-1/2}$. More precisely, the distribution of the zeros of zeta function can be precisely organized as follows

$$\begin{aligned}
&\operatorname{Re} z < 0, \quad \zeta(z) = 0, \quad z = -2k, \quad k \in Z^+ = \{1, 2, 3, \cdots\} \\
&\operatorname{Re} \rho = 1/2, \quad \zeta(\rho) = 0, \quad \rho = 1/2 + \sqrt{-1}R \in \mathbb{C}
\end{aligned}$$

As a result of this conclusion, we certify the Riemann theorem 8 that always holds for all discrete scalar curvature R lying on the critical line $1/2$.

6 Conclusions

The QCPB theory as a covariant extension of the classical QPB theory has been built for a complete quantum mechanics, then it naturally leads to the QCHS that is associated with G-dynamics as a critical component. With the help of the G-dynamics based on the Schrödinger equation, we found such Hermitian operator $\hat{w}^{(cl)}$ to prove the Hilbert-Pólya conjecture, in particular, the geometric wave function in discreteness is rightly corresponding to the sub-unit of the Riemann zeta function, then RH is accordingly proven in this way. On the one hand, we prove the all non-trivial zeros lie on the critical line 1/2; on the other, we certainly verify precise form of the imaginary part of the non-trivial zeros that are the scalar curvature R translated as the geometric frequency, most importantly, RH connects the QG, the formula is $G_{ij} = \zeta(\rho) = 0$. The scalar curvature R in the vacuum field equation forms a discrete set which asserts the frequency spectrum, it constructs the energy spectrum of the QG, the gap theorem in terms of the scalar curvature R has thusly substantiated.

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