Ficha Extra nº 1

1.

a) A = matriz quadrada (3×3) triangular superior
B = matriz quadrada (3×3) triangular inferior
C = matriz diagonal (3×3)
D = matriz escalar (3×3)
E = matriz identidade (4×4)
F = matriz simétrica (3×3)

b) $A^T = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & -1 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$C^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$; $D^T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$E^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; $F^T = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$

2. $\sqrt{2} \times A = \sqrt{2} \times \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ 1 & 1/2 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \\ 4\sqrt{2} & 5\sqrt{2} \\ \sqrt{2} & \sqrt{2}/2 \end{bmatrix}$

3.

a) $2A - B + C = 2 \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 6 \end{bmatrix} - \begin{bmatrix} 5 & 1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} -3 & 1 & 2 \\ 5 & 3 & 6 \end{bmatrix}$

b) $2(A-B+C) = 2\left(\begin{bmatrix}1&1&1\\2&1&3\end{bmatrix} - \begin{bmatrix}5&1&0\\0&2&4\end{bmatrix} + \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}\right)$

$= 2\begin{bmatrix}-4&0&1\\3&2&3\end{bmatrix} = \begin{bmatrix}-8&0&2\\6&4&6\end{bmatrix}$

c) $3A + B - C = 3\begin{bmatrix}1&1&1\\2&1&3\end{bmatrix} + \begin{bmatrix}5&1&0\\0&2&4\end{bmatrix} - \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}$

$= \begin{bmatrix}3&3&3\\6&3&9\end{bmatrix} + \begin{bmatrix}5&1&0\\0&2&4\end{bmatrix} - \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}$

$= \begin{bmatrix}8&4&3\\5&2&9\end{bmatrix}$

d) $3\left(A - \frac{1}{2}B\right) + C = 3\left(\begin{bmatrix}1&1&1\\2&1&3\end{bmatrix} - \frac{1}{2}\begin{bmatrix}5&1&0\\0&2&4\end{bmatrix}\right) + \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}$

$= 3\left(\begin{bmatrix}1&1&1\\2&1&3\end{bmatrix} - \begin{bmatrix}5/2&1/2&0\\0&1&2\end{bmatrix}\right) + \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}$

$= 3\left(\begin{bmatrix}-3/2&1/2&1\\2&0&1\end{bmatrix}\right) + \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix}$

$= \begin{bmatrix}-9/2&3/2&1\\6&0&3\end{bmatrix} + \begin{bmatrix}0&0&0\\1&3&4\end{bmatrix} = \begin{bmatrix}-9/2&3/2&1\\7&3&7\end{bmatrix}$

(4) a) $A^2 = \begin{bmatrix}2&1\\-1&6\end{bmatrix} \times \begin{bmatrix}2&1\\-1&6\end{bmatrix} = \begin{bmatrix}2\times2+1\times(-1)&2\times1+1\times6\\-1\times2+6\times(-1)&-1\times1+6\times6\end{bmatrix}$

$= \begin{bmatrix}3&8\\-8&35\end{bmatrix}$

$$A^3 = A^2 \times A = \begin{bmatrix} 3 & 8 \\ -8 & 35 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times2+8\times(-1) & 3\times1+8\times6 \\ -8\times2+35\times(-1) & -8\times1+35\times6 \end{bmatrix} = \begin{bmatrix} -2 & 51 \\ -51 & 202 \end{bmatrix}$$

b) $A^2 \times B = \begin{bmatrix} 3 & 8 \\ -8 & 35 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix} =$

$$= \begin{bmatrix} 3\times1+8\times2 & 3\times5+8\times(-3) \\ -8\times1+35\times2 & -8\times5+35\times(-3) \end{bmatrix} = \begin{bmatrix} 19 & -9 \\ 62 & -145 \end{bmatrix}$$

5. $X \times Y = \begin{bmatrix} 0 & 0 & 4 \\ 5 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -2 & 1 & 2 & 0 \\ 2 & 3 & 8 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0\times(-2)+0\times2+4\times1 & 0\times1+0\times3+4\times1 & 0\times2+0\times8+4\times1 & 0\times0+0\times0+4\times0 \\ 5\times(-2)+1\times2+0\times1 & 5\times1+1\times3+0\times1 & 5\times2+1\times8+0\times1 & 5\times0+1\times0+0\times0 \\ 2\times(-2)+1\times2+0\times1 & 2\times1+1\times3+0\times1 & 2\times2+1\times8+0\times1 & 2\times0+1\times0+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & 0 \\ -8 & 8 & 18 & 0 \\ -2 & 5 & 12 & 0 \end{bmatrix}$$

6.

a) $A \cdot B = \begin{bmatrix} 1 & 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} 1\times2+0\times0+(-1)\times(-1)+2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \end{bmatrix}$$

b) $B \cdot A = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1 & 2\times0 & 2\times(-1) & 2\times2 \\ 0\times1 & 0\times0 & 0\times(-1) & 0\times2 \\ -1\times1 & -1\times0 & -1\times(-1) & -1\times2 \\ 3\times1 & 3\times0 & 3\times(-1) & 3\times2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -2 \\ 3 & 0 & -3 & 6 \end{bmatrix}$$

c) Não. Porque $AB \neq BA$.