Global Normalization at Base 2 (GN(2)) $\Rightarrow$ FLT: A Coq-Verified Conditional Route

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Abstract. We present an explicit-base formulation of Dedenko's idea in which the sole hypothesis is the global normalization at base 2 (GN(2)): for any putative natural solution of Fermat's equation $x^n + y^n = z^n$ with $n > 2$, one must have the equality $2^{\,n} = 2 \cdot n$. Together with the elementary growth fact $2^{\,n} > 2 \cdot n$ for all $n \geq 3$, this yields an immediate contradiction and hence Fermat's Last Theorem (FLT). The result is stated as the conditional implication Arithmetic + GN(2) $\Rightarrow$ FLT and is fully formalized in Coq. The core formal proof is purely over the naturals; a convenient real "coverage" predicate $\mathrm{pow}\,2\,n = 2 \cdot \mathrm{INR}\,n$ is linked to $2^{\,n} = 2 \cdot n$ by a bridge lemma. Standard parametrization $(z, x) = (m^n + p^n,\, m^n - p^n)$ and parity identities are included as motivation/consistency checks only and play no role in the final step. This GN(2) presentation replaces the earlier global-normalizer $o > 1$ formulation.

Keywords: Fermat's Last Theorem · GN(2) · normalization · Coq · formal verification

1 Introduction

We consider Fermat's equation

$$x^n + y^n = z^n, \qquad x, y, z \in \mathbb{N},\ n \in \mathbb{N}. \tag{1}$$

In the present reading the analysis is collapsed by a single, explicit-base hypothesis:

Definition 1 (Global normalization at base 2 (GN(2))). *For every $n > 2$ and all $x, y, z \in \mathbb{N}$,*

$$x^n + y^n = z^n \implies 2^{\,n} = 2 \cdot n.$$

Combining GN(2) with the elementary growth inequality $2^{\,n} > 2 \cdot n$ for $n \geq 3$ immediately yields FLT.

2 Motivation: algebraic setup and parity (not used in the core)

Following the standard trick, set $z := m^n + p^n$ and $x := m^n - p^n$ (initially over $\mathbb{R}$ so that ring equalities are straightforward). Then

$$y^n = z^n - x^n = (m^n + p^n)^n - (m^n - p^n)^n$$

is the odd–binomial sum. Passing to $\mathbb{Z}$, one obtains that $z \pm x$ are even; in Coq this is captured by lemmas `sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, and `parity_condition_Z`. These parity facts are logically independent from the final step and are included for completeness only.

3 Coq formalization: naturals core and a real wrapper

The development proves the elementary growth comparisons $2^n > 2n$ for $n \geq 3$ (and $3^n > 2n$ for $n \geq 1$) and packages them as a lemma that $2^{\,n} = 2 \cdot n$ forces $n \in \{1, 2\}$ (`pow_eq_linear_positive`).

The GN(2) hypothesis is encoded directly over $\mathbb{N}$:

GN(2) (Coq).

```
Definition GN2 : Prop :=
  forall (n x y z : nat),
    2 < n ->
    Nat.pow x n + Nat.pow y n = Nat.pow z n ->
    2 ^ n = 2 * n.
```

From GN(2) the contradiction for $n > 2$ is immediate:

FLT from GN(2) (Coq).

```
Lemma FLT_from_GN2 :
  GN2 ->
  forall n x y z,
    2 < n ->
    Nat.pow x n + Nat.pow y n = Nat.pow z n -> False.
```

For convenience, a real "coverage" predicate is also used:

$$\mathrm{pow}\, 2\, n \;=\; 2 \cdot \mathrm{INR}\, n,$$

which is linked back to $2^n = 2 \cdot n$ via bridge lemmas `covers_two_nat`, `INR_two_mul_nat` and the implication `GN2_R_implies_GN2`. This yields the corollary `fermat_last_theorem_from_GN2_R`.

4 What is *not* assumed

The reading here does *not* rely on any unconditional congruence like $(m^n + p^n)^n - (m^n - p^n)^n \equiv 0 \pmod{2n}$ (which is false in general). The only extra assumption is GN(2); algebraic parametrization and parity serve as motivation/consistency checks and are not used in the final step.

5 Article–Coq correspondence

Article (item)	**Coq formalization (lemma/theorem)**
Algebraic parametrization over $\mathbb{R}$; integer parity facts	`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`.
GN(2) hypothesis over $\mathbb{N}$	`GN2` (definition of the Prop).
Growth vs. linear comparison; $2^n = 2 \cdot n \Rightarrow n \in \{1, 2\}$	`pow2_gt_linear`, `pow3_gt_linear`, `pow_eq_linear_positive`.
Real wrapper and bridge back to $\mathbb{N}$	`covers_two_nat`, `INR_two_mul_nat`, `GN2_R`, `GN2_R_implies_GN2`.
FLT from GN(2) (direct) / via real wrapper	`FLT_from_GN2` / `fermat_last_theorem_from_GN2_R`.

Table 1. Mapping between the paper's steps and the Coq development.

6 Conclusion

Under the single GN(2) assumption attached to any hypothetical counterexample, the Coq file derives FLT for all $n > 2$ using only elementary growth lemmas. Parity constraints from the parametrization are checked separately. This GN(2) presentation replaces the earlier global-normalizer ($o > 1$) ansatz.

Appendix: selected Coq declarations (names)

`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`, `pow2_gt_linear`, `pow3_gt_linear`, `pow_eq_linear_positive`, `GN2`, `GN2_R`, `covers_two_nat`, `INR_two_mul_nat`, `GN2_R_implies_GN2`, `FLT_from_GN2`, `fermat_last_theorem_from_GN2_R`.

References

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