Global Normalization for Fermat's Equation: From $o^n = 2 \cdot n$ to FLT, with Coq Verification

Grigoriy Dedenko

Abstract. We present a reading of G. L. Dedenko's manuscript in which a single, unified normalizing factor $o \in \mathbb{N}$, $o > 1$, independent of the exponent n, is introduced. It is postulated that for any hypothetical natural solution of Fermat's equation $x^n + y^n = z^n$ with $n > 2$ one has the equality $o^n = 2 \cdot n$ (equivalently, after the standard parametrization, $\frac{p^n q}{l} = o$). From this equality alone it follows elementarily that $o = 2$ and $n \in \{1, 2\}$; hence no solutions exist for $n > 2$. The entire argument is stated as the conditional implication "global normalization $\Rightarrow$ FLT" and is fully formalized in Coq. The proof of the implication relies only on an elementary growth comparison; parity constraints from the parametrization are established separately (for completeness) and do not enter the final step. The discussion of the function $f(n) = (2n)^{1/n}$ serves to motivate the *form* of the normalization and is not used inside the proof proper.

Keywords: Fermat's Last Theorem · Dedenko · normalization · Ansatz · Coq · formal verification

1 Introduction

We consider Fermat's equation

$$x^n + y^n = z^n, \qquad x, y, z \in \mathbb{N},\ n \in \mathbb{N}. \tag{1}$$

In the reading advocated here, following Dedenko's manuscript, one introduces a *single global normalizing factor* $o \in \mathbb{N}$, $o > 1$, *independent of* n, and postulates that for any putative solution of (1) with $n > 2$ one has

$$o^n = 2 \cdot n. \tag{2}$$

This normalization collapses the analysis of all exponents at once. A simple growth comparison then yields that (2) forces $(o, n) = (2, 1)$ or $(2, 2)$ only; thus no solution exists for $n > 2$.

We formalize the above conditional implication in Coq. The algebraic parametrization is recorded over $\mathbb{R}$ for convenience, while parity constraints are proved over $\mathbb{Z}$. The global normalization assumption is represented by a single parameter o together with a universal condition (2) attached to any hypothetical counterexample.

2 Algebraic setup and parity

Following the standard trick, set $z := m^n + p^n$ and $x := m^n - p^n$ (initially over $\mathbb{R}$ so that ring equalities are straightforward). Then

$$y^n = z^n - x^n = (m^n + p^n)^n - (m^n - p^n)^n$$

is the odd–binomial sum. Passing to $\mathbb{Z}$, the specialization implies that $z \pm x$ are even; in Coq this is captured by a lemma (*parity_condition_Z*). These parity facts are logically independent from the final growth step and are included for completeness.

3 Global normalization (Ansatz)

We fix *one* $o \in \mathbb{N}$, $o > 1$, and assume:

Definition 1 (Global normalization principle). *For every $n, x, y, z \in \mathbb{N}$ with $n > 2$, if $x^n + y^n = z^n$ holds, then*

$$o^n = 2 \cdot n. \tag{3}$$

Equivalently, after the standard parametrization and notations of the manuscript, (3) reads $\left(\frac{p^n q}{l}\right)^n = 2 \cdot n$ or $\frac{p^n q}{l} = o$. The analysis of $f(n) = (2n)^{1/n}$ explains why choosing the normalizer in the *n-th power form* is natural, but those analytic properties are not used in the final implication.

4 Coq formalization: growth and the main theorem

The Coq development proves the elementary growth comparisons $2^n > 2n$ for $n \geq 3$ and $3^n > 2n$ for $n \geq 1$, and packages them into:

Lemma 1. *If $o > 1$ and $o^n = 2 \cdot n$ with $n \geq 1$, then $(o, n) = (2, 1)$ or $(2, 2)$.*

In the Coq file, this is `integer_solution_o`. With the global normalization principle as hypothesis, we obtain:

Theorem 1 (FLT from global normalization). *Assume Definition 1. Then for every $n > 2$ there are no solutions to (1) in $\mathbb{N}$. In Coq: `fermat_last_theorem_from_normalization`.*

Proof (Idea). Given $n > 2$ and a putative solution, (3) yields $o^n = 2 \cdot n$; by Lemma 1, this forces $n \in \{1, 2\}$, a contradiction.

For completeness, the development also includes a corollary where one *chooses* $o = 2$ ("full-coverage normalization"), obtaining $2^n = 2 \cdot n$ and the same contradiction; see `fermat_last_theorem_with_o_two`.

5 What is *not* assumed

The reading here *does not* rely on any unconditional congruence like $(m^n + p^n)^n - (m^n - p^n)^n \equiv 0 \pmod{2n}$ (which is false in general). Instead, the only extra assumption is the single global normalizer $o > 1$ satisfying (2) for any hypothetical counterexample.

6 Article–Coq correspondence

Article (item)	**Coq formalization (lemma/theorem)**
Algebraic parametrization over $\mathbb{R}$; integer parity facts	`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`.
Global normalization principle (fixed $o > 1$, independent of n)	Section `Normalization_Parameter`: `Variable o`, `normalization_gt1`, `normalization_equation`.
Growth vs. linear comparison	`pow2_gt_linear`, `pow3_gt_linear`.
Only $(o, n) = (2, 1), (2, 2)$ solve $o^n = 2n$	`integer_solution_o`.
FLT from the normalization principle	`fermat_last_theorem_from_normalization`.
Optional "$o = 2$" corollary	`fermat_last_theorem_with_o_two`.

Table 1. Mapping between the paper's steps and the Coq development.

7 Conclusion

Under the single global normalization assumption $o^n = 2 \cdot n$ attached to any hypothetical counterexample, the Coq file derives FLT for all $n > 2$ using only elementary growth lemmas. Parity constraints from the parametrization are checked separately. The analytic discussion of $f(n) = (2n)^{1/n}$ motivates the n-th power *shape* of the normalizer but is not used in the final implication.

Appendix: selected Coq declarations (names)

`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`, `pow2_gt_linear`, `pow3_gt_linear`, `integer_solution_o`, `Normalization_Parameter` (section), `fermat_last_theorem_from_normalization`, `fermat_last_theorem_with_o_two`.

References

1. A. Wiles. Modular elliptic curves and Fermat's Last Theorem. *Annals of Mathematics* 141 (1995), 443–551.
2. G. L. Dedenko. The "Difficulties" in Fermat's Original Discourse on the Indecomposability of Powers Greater Than a Square: A Retrospect. Preprint, 2025. DOI: 10.13140/RG.2.2.24342.32321.
3. The Coq Development Team. The Coq Proof Assistant. `https://coq.inria.fr`.