The "Difficulties" in Fermat's Original Discourse on the Indecomposability of Powers Greater Than a Square: A Retrospect

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The "Difficulties" in Fermat's Original Discourse on the Indecomposability of Powers Greater Than a Square: A Retrospect

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Abstract

We present a reading of G. L. Dedenko's manuscript in which a single, unified normalizing factor $o \in \mathbb{N}$, $o > 1$, independent of the exponent n, is introduced. It is postulated that for any hypothetical natural solution of Fermat's equation $x^n + y^n = z^n$ with $n > 2$ one has the equality $o^n = 2 \cdot n$ (equivalently, after the standard parametrization, $\frac{p^n q}{l} = o$). From this equality alone it follows elementarily that $o = 2$ and $n \in \{1, 2\}$; hence no solutions exist for $n > 2$. The entire argument is stated as the conditional implication "global normalization $\Rightarrow$ FLT" and is fully formalized in Coq. The proof of the implication relies only on an elementary growth comparison; parity constraints from the parametrization are established separately (for completeness) and do not enter the final step. The discussion of the function $f(n) = (2n)^{1/n}$ serves to motivate the form of the normalization and is not used inside the proof proper.

Keywords: number theory, Fermat's Big/Last Theorem, Hisitory of Mathematics, Algebra, Proof

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At MEPhI he worked in applied nuclear physics as an Assistant (1995-2002), Senior Lecturer (2002-2010) and Associate Professor (2010-mid-2018). From mid-2018 to August 2020 he was a leading specialist on the Kudankulam NPP Project for India at Atomstroyexport JSC. Since September 2020 he has been with the Financial University under the Government of the Russian Federation—serving as Associate Professor (Sept 2020-Dec 2023) and, from January 2024, Senior Lecturer.

Dr Dedenko's research interests range from nuclear science and engineering to pure mathematics, history and philosophy.

1 Introduction

The present work is the result of an attempted reconstruction of Fermat's original discourse along with an explanation of why he might have not written it down. The author had performed it within a two-time period of time– between 1990 and 1993 – trying proving the theorem and the final revision in 2017-2025.[1] When completed, it did look like a proof of Fermat's epoch, as it only involved the knowledge and techniques available and utilised by Fermat's contemporary and pre-Fermat mathematical world.

Not to overburden this text with details of a real historical study, let us briefly recall the history of the conjecture. Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the Arithmetica next to Diophantus' sum-of-squares problem [7]:

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadra-tos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

Attempts to prove this conjecture employed a diverse range of methods. Early attempts, including Fermat's own proof for the case $n = 4$, utilized the **method of infinite descent**. A significant step forward was Leonhard Euler's proof for the case $n = 3$, which involved the use of **Gaussian integers** [13]. Subsequent efforts included approaches based on the work of Sophie Germain and other techniques, before Andrew Wiles finally presented a complete proof in 1995, drawing upon deep results from **the theory of modular forms and elliptic curves** [1–6].

Modern methods, such as the theory of modular forms, deal with **the transformation properties of specific curves over particular types of spaces** (e.g., rational numbers), highlighting the **stability of elliptic curves with respect to modular transformations**. While the contemporary formalizations of algebraic curves, spaces, transformations, and groups used in these methods were not present in Fermat's time, mathematicians of that era possessed their own approaches and intuitive understandings for studying the properties of natural numbers (and primes). (More on this can be found in the Conclusions section of this paper.)

Theorem 1 (Conditional on a global normalization (Dedenko's Ansatz))**.** *Assume there exists a single integer $o > 1$, independent of n, such that for every $n > 2$ and all $x, y, z \in \mathbb{N}$, from $x^n + y^n = z^n$ it follows that $o^n = 2 \cdot n$. Then the Fermat equation $x^n + y^n = z^n$ has no solutions in $\mathbb{N}$ for all $n > 2$.*

Proof sketch. Given a putative solution at a fixed $n > 2$, the hypothesis yields $o^n = 2 \cdot n$ with the same (global) $o > 1$. Elementary growth comparison implies that $o^n = 2 \cdot n$ forces $o = 2$ and $n \in \{1, 2\}$; hence a contradiction for $n > 2$. A machine-checked derivation is provided in the accompanying Coq file `FLT.v`.

2 Possible Proof

The statement of the theorem is rather straightforward and as follows:

Neither a cube for two cubes, nor a biquadrate or two biquadrates, and generally no power greater than two can be decomposed into two powers of the same grade. In other words, the equation

$$x^n + y^n = z^n$$

has no solutions in natural numbers if n is an integer greater than 2.

Therefore, first

1). Let's write down the theorem

$$x^n + y^n = z^n \tag{2.1}$$

2). Let's rewrite (2.1)

$$z^n - x^n = y^n \tag{2.2}$$

[1] *The work was mainly performed on the scholarship of the NRNU MEPhI student in* 1990 − 93, *final revision in* 2017 − 2025.

3). Let's perform the transformation of (2.2) to

$$\big(m^n + p^n\big)^n - \big(m^n - p^n\big)^n = y^n, \tag{2.3}$$

where $n \in \mathbb{N}$, m, p are <u>arbitrary numbers</u> (not necessarily integers; signs arbitrary), and $z, x \in \mathbb{N}$ satisfy

$$\begin{cases} m^n + p^n = z, \\ m^n - p^n = x. \end{cases} \tag{2.4}$$

Raising both identities to the n-th power yields

$$\begin{cases} (m^n + p^n)^n = z^n, \\ (m^n - p^n)^n = x^n. \end{cases} \tag{2.5}$$

Thus (2.4) implies (2.5).

Solving (2.4). Adding and subtracting the equations gives

$$2m^n = z + x, \qquad 2p^n = z - x,$$

so formally

$$m = \sqrt[n]{\frac{z+x}{2}}, \qquad p = \sqrt[n]{\frac{z-x}{2}}.$$

<u>Domain note.</u> Over $\mathbb{R}$: for odd n the roots are unique; for even n we need $(z \pm x)/2 \geq 0$ and there is a sign choice $m = \pm\big((z+x)/2\big)^{1/n}$, $p = \pm\big((z-x)/2\big)^{1/n}$. Over $\mathbb{C}$: there are n branches for the n-th root; once a branch is fixed, the reconstruction is consistent.

Equivalence of (2.4)–(2.5). Forward (2.4) $\Rightarrow$ (2.5)**.** Immediate by raising to the n-th power.

Reverse (2.5) $\Rightarrow$ (2.4)**.** Assume $z, x \in \mathbb{N}$ and

$$z^n = (m^n + p^n)^n, \qquad x^n = (m^n - p^n)^n.$$

Taking n-th roots in the same domain as $m^n \pm p^n$ (principal root over $\mathbb{R}_{\geq 0}$, or a fixed branch over $\mathbb{C}$) gives

$$z = m^n + p^n, \qquad x = m^n - p^n,$$

i.e. (2.4). For even n one fixes consistent signs as above. Hence, (2.4) and (2.5) are equivalent <u>in the chosen number domain</u> once the root/branch convention is fixed.

Proposition 1 (Integer case: necessary & sufficient conditions)**.** *If, <u>in addition</u>, one requires $m, p \in \mathbb{Z}$, then necessarily*

$$z \pm x \textit{ are even}, \qquad \frac{z+x}{2} = m^n, \qquad \frac{z-x}{2} = p^n.$$

Conversely, if $z \pm x$ are even and both halves are perfect n-th powers in $\mathbb{Z}$, then the reconstructed m, p are integers (up to signs for even n).

Remark 1. *This separates the general real/complex reconstruction (no parity obstruction) from the integer reconstruction, where parity and perfect-power constraints are essential.*

Example (real domain). For $n = 2$, $m = 3$, $p = 2$:

$$z = 3^2 + 2^2 = 13, \qquad x = 3^2 - 2^2 = 5,$$

hence

$$z^2 = 169, \qquad x^2 = 25,$$

and reversing via $(z \pm x)/2$ returns $m = 3$, $p = 2$.

Counterexample (integer case). For $n = 3$, $z = 2$, $x = 1$ we have $(z + x)/2 = 3/2$, which is not a perfect cube in $\mathbb{Z}$; hence no integer m, p satisfy (2.4), although real m, p exist.

4). Let's ask ourselves the question: what is the number y? Is it positive integer or not? If it is not positive integer, then under what conditions will it be natural number? Does its naturalness depend on the degree of n?

From (2.3) we have the difference:

$$y^n = z^n - x^n = (m^n + p^n)^n - (m^n - p^n)^n = \tag{2.6}$$

that can be expanded or decomposed into a sum according to Newton's binomial [8, 9]:

$$\begin{aligned}
&= [(m^n)^n + C_n^1(m^n)^{n-1}p^n + C_n^2(m^n)^{n-2}(p^n)^2 + \cdots + C_n^{n-1}m^n(p^n)^{n-1} + (p^n)^n] - \\
&-[(m^n)^n - C_n^1(m^n)^{n-1}p^n + C_n^2(m^n)^{n-2}(p^n)^2 \pm \cdots \pm C_n^{n-1}m^n(p^n)^{n-1} \pm (p^n)^n] = \\
&= 2C_n^1(m^n)^{n-1}p^n + 2C_n^3(m^n)^{n-3}(p^n)^3 + \cdots + 2C_n^k(m^n)^{n-k}(p^n)^k + \{2C_n^n(p^n)^n\} = \\
&= 2\sum_{i=0}^k C_n^{(2i+1)}(m^n)^{n-(2i+1)}(p^n)^{(2i+1)}
\end{aligned}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even.

Rewrite then (2.6) as

$$z^n = x^n + y^n,$$

$$\text{where } x,\ y,\ z \text{ are } \begin{cases} z = m^n + p^n \\ x = m^n - p^n \\ y = \sqrt[n]{2}\left[\sum_{i=0}^k C_n^{(2i+1)}(m^n)^{n-(2i+1)}(p^n)^{(2i+1)}\right]^{1/n} \end{cases}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even;

we know that in common case for $n = 2$ we have

$$\text{where } x,\ y,\ z \text{ are given by } \begin{cases} z = r \cdot (m^2 + p^2), \\ x = r \cdot (m^2 - p^2), \\ y = r \cdot 2mp. \end{cases} \tag{2.7}$$

but in our case, we omit the factor r – is some positive $(r > 0)$ integer constant since it is reduced in our calculations. Therefore, we made the transformation (2.3) in order to take into account this case (n=2), known since the time of Pythagoras, since the general case should include a particular solution as a subset.

5). scrutinise now the y:

$$y = \sqrt[n]{2}\left[\sum_{i=0}^{k} C_n^{(2i+1)}(m^n)^{n-(2i+1)}(p^n)^{(2i+1)}\right]^{1/n}. \tag{2.8}$$

In order for the y to maybe a positive integer, $\sqrt[n]{2}$ must leave, since for $n > 1$ $\sqrt[n]{2}$ is an irrational number (see Appendix A). It is thus necessary that the expression

$$\left[\sum_{i=0}^{k} C_n^{(2i+1)} (m^n)^{n-(2i+1)} (p^n)^{(2i+1)}\right]^{1/n} \tag{2.9}$$

contain some common factor that destroys the radical expression $\sqrt[n]{2}$, let's find out what it is. Otherwise, y is not a positive integer due to the presence of $\sqrt[n]{2}$. Consider now what largest divisor this sum may contain and what it is equal to:

$$\begin{aligned}
&\left[\sum_{i=0}^{k} C_n^{(2i+1)} (m^n)^{n-(2i+1)} (p^n)^{(2i+1)}\right] = \\
&= C_n^1 (m^n)^{n-1} p^n + C_n^3 (m^n)^{n-3} (p^n)^3 + \cdots + C_n^k (m^n)^{n-k} (p^n)^k + \{C_n^n (p^n)^n\} = \\
&= n (m^n)^{n-1} p^n + \tfrac{n(n-1)(n-2)}{3!} (m^n)^{n-3} p^3 + \cdots + \\
&+ \tfrac{n(n-1)...(n-k+1)}{k!} (m^n)^{n-k} (p^n)^k + \left\{\tfrac{n(n-1)...2\cdot 1}{n!} (p^n)^n\right\} = \\
&= n \cdot [(m^n)^{n-1} p^n + \tfrac{(n-1)(n-2)}{3!} (m^n)^{n-3} p^3 + \ldots + \\
&+ \tfrac{(n-1)...(n-k+1)}{k!} (m^n)^{n-k} (p^n)^k + \left\{\tfrac{(n-1)...2\cdot 1}{n!} (p^n)^n\right\}] = n \cdot l^n
\end{aligned} \tag{2.10}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even,

Hence the conclusion follows: n is a common divisor, and where l is some constant about which we know nothing (it is maybe real in common case or maybe integer), through which we denoted the rest of the radical after the allocation of the common set n.

Since all the terms in the sum (2.10) contain the factor n, the expression is divisible by n.

To show the uniqueness of the decomposition of expression (2.10), we can use the following considerations:

(a) Degree n is a fixed positive integer.

(b) Exponentiation is an unambiguous operation; for any number, there is a unique value for the degree.

(c) Addition and subtraction of real numbers are commutative operations, and the result does not depend on the order of actions.

Based on these properties, it can be argued that for fixed m, p and n there is a single result of calculating the expression (2.10). Rearranging the members will not affect the final answer.

We can see from (2.8 - 2.10) (where l is some constant (which will be it for fixed m, p, n), which we don't know anything about yet)

$$y = \sqrt[n]{2 \cdot n} \cdot l \tag{2.11}$$

As a result, from (2.6)–(2.11) we obtain

$$z^n - x^n \;=\; \big(m^n + p^n\big)^n - \big(m^n - p^n\big)^n \;=\; y^n. \tag{2.12}$$

Substituting (2.11) into (2.12), we have

$$\big(m^n + p^n\big)^n - \big(m^n - p^n\big)^n \;=\; 2n\, l^n. \tag{2.13}$$

Let $m = j\,p$ with an arbitrary $j > 1$. Then

$$\big(p^n(j^n+1)\big)^n - \big(p^n(j^n-1)\big)^n \;=\; 2n\,l^n \quad\Longrightarrow\quad \big(p^n\big)^n\big((j^n+1)^n-(j^n-1)^n\big) \;=\; 2n\,l^n.$$

The difference $(j^n+1)^n - (j^n-1)^n$ according to the binomial theorem reduces to odd indices:

$$(j^n+1)^n-(j^n-1)^n = \sum_{k=0}^{n}\binom{n}{k}(j^n)^{n-k}\big[1-(-1)^k\big] = 2\sum_{\substack{1\le k\le n\\ k \text{ is odd}}}\binom{n}{k}(j^n)^{n-k}.$$

Let us denote

$$q^n \;:=\; 2\sum_{\substack{1\le k\le n\\ k \text{ is odd}}}\binom{n}{k}(j^n)^{n-k}.$$

Then

$$\big(p^n\big)^n q^n \;=\; 2n\,l^n \quad\Longrightarrow\quad \frac{\big(p^n\big)^n q^n}{l^n} = 2n,$$

that is

$$\left(\frac{p^n q}{l}\right)^n = 2n. \tag{2.14}$$

Normalization and the Global Postulate. Equality (2.14) shows that for any hypothetical natural solution to $x^n+y^n=z^n$ (with parameters m,p), the quantity $\frac{p^n q}{l}$ is the n-th root of $2n$. Next, we fix a single normalizing factor

$$o\in\mathbb{N},\qquad o>1,$$

independent of n, and we postulate that for every such hypothetical solution

$$o^n = 2n, \tag{2.15}$$

and consequently (choosing the principal root in the real domain)

$$\left(\tfrac{p^n q}{l}\right)^n = o^n \quad\Longleftrightarrow\quad \tfrac{p^n q}{l} = o. \tag{2.16}$$

In other words, (2.15) is a global normalization hypothesis (a single o for all n), which in our formalization is the only additional prerequisite.

Immediate Consequence. From (2.15) for $o>1$, it is elementarily deduced that

$$o=2,\qquad n\in\{1,2\}.$$

Indeed, for $o\ge 3$, we have $3^n > 2n$ for all $n\ge 1$, so $o^n = 2n$ is impossible; for $o=2$, the equality $2^n = 2n$ has the unique integer solutions $n=1,2$. Consequently, for $n>2$, equality (2.15) cannot hold—which leads to a contradiction with the assumption that a solution to Fermat's equation exists.

Motivation for the Form of Normalization. Let us introduce

$$f(n) := \sqrt[n]{2n}.$$

Then (2.15) is equivalent to $o = f(n)$. We have $f(1)=2$, $f(2)=2$, for $n>2$ the function f is strictly decreasing, and $f(n)\to 1$ as $n\to\infty$ (see Appendices B, C for details):

$$\ln f(n) = \frac{\ln(2n)}{n},\qquad \frac{d}{dn}\big(\ln f(n)\big) = \frac{1-\ln(2n)}{n^2} < 0 \quad (n\ge 2).$$

From this, it is clear why it is a priori natural to require the normalization to "remain in the world of n-th powers" and why a posteriori $o = 2$ is inevitably obtained. This motivation explains the form of (2.15), but the conclusion $o = 2,\ n \in \{1, 2\}$ itself relies only on (2.15) and an elementary comparison of growth.

Conclusion (in the spirit of Fermat). By reducing the argument to the global equality $o^n = 2 \cdot n$ for a single $o > 1$, we immediately obtain $o = 2$ and $n \in \{1, 2\}$. Consequently, "for exponents greater than the square," there are no solutions — precisely the kind of short "marginal note" that Fermat himself might have left.

6.) Verification (without considering the total multiplier r from (2.7), which has been reduced)

(a) Consider the case $n = 1$

$$\begin{aligned} &z = m + p \\ &x = m - p \\ &y = 2 \cdot [C_1^1 (m^1)^{1-(2\cdot 0+1)} (p^1)^{(2\cdot 0+1)} = 2[1 \cdot 1 \cdot p] = 2p \end{aligned}$$

that is, for the case $n = 1$, we have a solution in natural numbers x, y, z, if m and p - are positive integer numbers and $m > p$.

(b) Consider the case $n = 2$

$$\begin{aligned} &z = m^2 + p^2 \\ &x = m^2 - p^2 \\ &y = \sqrt{2} \cdot [C_1^2 (m^2)^{2-(2\cdot 0+1)} (p^2)^{(2\cdot 0+1)}]^{1/2} = \sqrt{2} \cdot [2m^2 p^2]^{1/2} = 2mp \end{aligned}$$

that is, for the case $n = 2$, we also have a solution in positive integer numbers x, y, z, if m and p are positive integer numbers and $m > p$, or according to [10], if $m > p$, and m, p are such non-integer numbers that combinations with them will give the positive integer numbers x, y, z, for example: $m = 3/\sqrt{2}$, $p = 1/\sqrt{2}$, in this case, we also get the classical Pythagorean triple:

$$\begin{aligned} x = m^2 - p^2 &= \left(\tfrac{3}{\sqrt{2}}\right)^2 - \left(\tfrac{1}{\sqrt{2}}\right)^2 = \tfrac{9}{2} - \tfrac{1}{2} = \tfrac{8}{2} = 4 \\ y = 2 \cdot m \cdot p &= 2 \cdot \tfrac{3}{\sqrt{2}} \cdot \tfrac{1}{\sqrt{2}} = 3 \\ z = m^2 + p^2 &= \left(\tfrac{3}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2 = \tfrac{9}{2} + \tfrac{1}{2} = 5 \end{aligned}$$

We have obtained the classical Pythagorean triple and can see that this formula works.

7). Let us now consider the more general case where r is not reduced to proceed to final formulas like (2.7). If r is a positive integer number, then all the above applies. But according to [10], r – can be a rational number. For example, let's say $r = 0.5$:

(a) Consider the case of $n = 1$, $m = 4$, and $p = 2$

$x = 0.5 \cdot (m - p) = 0.5 \cdot (4 - 2) = 1$

$z = 0.5 \cdot (m + p) = 0.5 \cdot (4 + 2) = 3$

$y = 0.5 \cdot (2 \cdot p) = 0.5 \cdot (2 \cdot 2) = 2$

We see positive integer numbers. That is,, m, p must be such that 7*(a)* is performed similarly. In this case, we can see that the formula 6*(a)* works.

(b) Consider the case *of* $n = 2$, $m = 2 \cdot \sqrt{2}$, $p = \sqrt{2}$,
$x = 0.5 \cdot (m^2 - p^2) = 0.5 \cdot \left(\left(2 \cdot \sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 \right) = 3$
$z = 0.5 \cdot (m^2 + p^2) = 0.5 \cdot \left(\left(2 \cdot \sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 \right) = 5$
$y = 0.5 \cdot (2 \cdot m \cdot p) = 0.5 \cdot \left(2 \cdot \left(2 \cdot \sqrt{2}\right) \cdot \left(\sqrt{2}\right)\right) = 4$
We see positive integer numbers. That is,, m, p must be such that 7*(b)* holds similarly. In this case, we can see that the formula 6*(b)* works.

This preserves the generality of solutions for $n = 2$, as seen in the Pythagorean triple example.

8). Therefore, the cases $n = 1$ and $n = 2$ **exhaust all possible integer solutions**, which is consistent with the theorem.
The test showed that for *n=1* or for *n=2*, in all the cases considered, we have solutions of the equation $x^n + y^n = z^n$in the positive integer numbers *x, y, z*

9). **the equation $x^n + y^n = z^n$ has roots in the positive integer numbers *x, y, z* only for**
$n = 1$ and for $n = 2$

Q.E.D.

3 Remark and corollaries

REMARK. *Note that the expression (2.8) can be simplified, namely*

$$\begin{aligned} y &= \sqrt[n]{2}\left[\sum_{i=0}^{k} C_n^{(2i+1)} (m^n)^{n-(2i+1)} (p^n)^{(2i+1)}\right]^{1/n} = \\ &= \sqrt[n]{2}\left[\sum_{i=0}^{k} C_n^{(2i+1)} \frac{(m^n)^n}{(m^n)^{2i} m^n} (p^n)^{2i} p^n\right]^{1/n} = \\ &= \sqrt[n]{2} m^n \frac{1}{m} p \left[\sum_{i=0}^{k} C_n^{(2i+1)} \left(\frac{p}{m}\right)^{n2i}\right]^{1/n} = \\ &= \sqrt[n]{2} m^{n-1} p \left[\sum_{i=0}^{k} C_n^{(2i+1)} \left(\frac{p}{m}\right)^{2in}\right]^{1/n} \end{aligned} \tag{3.1}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even.
COROLLARY 1. *Consider the case $m = p$, then from the expression (3.1) it can be derived that*

$$\begin{aligned} &x = 0 \\ &z = m^n + m^n = 2m^n \\ &y = \sqrt[n]{2} m^{n-1} m \left[\sum_{i=0}^{k} C_n^{(2i+1)} \left(\frac{m}{m}\right)^{2in}\right]^{1/n} = \sqrt[n]{2} m^n \left[\sum_{i=0}^{k} C_n^{(2i+1)}\right]^{1/n} \end{aligned}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even.
It is obvious that $y = z$

$$\begin{aligned} \sqrt[n]{2} m^n \left[\sum_{i=0}^{k} C_n^{(2i+1)}\right]^{1/n} &= 2m^n \\ \sqrt[n]{2} \left[\sum_{i=0}^{k} C_n^{(2i+1)}\right]^{1/n} &= 2 \end{aligned},$$

whence

$$\sum_{i=0}^{k} C_n^{(2i+1)} = 2^{n-1} \tag{3.2}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even.

COROLLARY 2. *Based on (3.2), the sum of even combinations can be calculated.*
Consider Pascal's triangle [7]:

$$\begin{array}{c} 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

Similarly to the above, it is concluded that

$$\sum_{j=0}^{s} C_n^{2i} = 2^{n-1} \tag{3.3}$$

with $k = (n-1)/2$ if n is odd and $k = (n-2)/2$ if n is even.

PROOF.
Expand

$$(m^n + p^n)^n + (m^n - p^n)^n =$$

into a binomial [8, 9]:

$$\begin{aligned} &= [(m^n)^n + C_n^1(m^n)^{n-1}p^n + C_n^2(m^n)^{n-2}(p^n)^2 + \cdots + C_n^{n-1}m^n(p^n)^{n-1} + (p^n)^n] + \\ &+ [(m^n)^n - C_n^1(m^n)^{n-1}p^n + C_n^2(m^n)^{n-2}(p^n)^2 \pm \cdots \pm C_n^{n-1}m^n(p^n)^{n-1} \pm (p^n)^n] = \\ &= 2C_n^0(m^n)^n + 2C_n^2(m^n)^{n-2}(p^n)^2 + \ldots + 2C_n^k(m^n)^{n-k}(p^n)^k + \{2C_n^n(p^n)^n\} = \\ &= 2\textstyle\sum_{j=0}^{s} C_n^{2j}(m^n)^{n-2j}(p^n)^{2j} \end{aligned}$$

with $s = (n-1)/2$ if n is odd and $s = n/2$ if n is even.
If $m = p$

$$(p^n + p^n)^n + (p^n - p^n)^n = 2\sum_{j=0}^{s} C_n^{2j}(p^n)^{n-2j}(p^n)^{2j}$$

$$(2p^n)^n = 2\sum_{j=0}^{s} C_n^{2j}\frac{(p^n)^n}{(p^n)^{2j}}(p^n)^{2j}$$

$$2^n(p^n)^n = 2(p^n)^n\sum_{j=0}^{s} C_n^{2j}$$

$$2^{n-1} = \sum_{j=0}^{s} C_n^{2j}$$

with $s = (n-1)/2$ if n is odd and $s = n/2$ if n is even.
Corollary 2 is proved.

COROLLARY 3. *Analysing (3.2) and (3.3), it can be concluded that*

$$\sum_{i=0}^{k} C_n^{(2i+1)} = \sum_{j=0}^{s} C_n^{2j} \tag{3.4}$$

with $k = (n-1)/2$, $s = (n-1)/2$ if n is odd and $k = (n-2)/2$, $s = n/2$ if n is even.
Why such borders?

Type of number	Condition	Last constant in equation
Odd	k = 2((n-1)/2)+1 = n	C_n^n
	s = 2((n-1)/2) = n-1	C_n^{n-1}
Even	k = 2((n-2)/2)+1 = n-1	C_n^{n-1}
	s = 2(n/2) = n	C_n^n

Table 1: Boundaries of binomial coefficients

Conclusion: the sum of even coefficients is equal to the sum of odd ones and is equal to 2^{n-1}, therefore from (3.4) we have

$$\sum_{r=0}^{n} C_n^r = \sum_{i=0}^{k} C_n^{(2i+1)} + \sum_{j=0}^{s} C_n^{2j} = 2 \cdot 2^{n-1} = 2^n \tag{3.5}$$

with $k = (n-1)/2$, $s = (n-1)/2$ if n is odd and $k = (n-2)/2$, $s = n/2$ if n is even.

4 Conclusion

The "difficulties" were for Fermat the lengthiness of the run of his deductions ***put in writing***, as in the first half of the seventeenth century the mathematical notations had been way far from their present concise and diverse shape, many actions had to be written down ***in words. Besides, a purely mathematical challenge was that he had to operate the then entirely new notions of binomials and logarithms***, both having just appeared for use and to be learnt "on the fly".

As mentioned in the introduction, the mathematical methods from Pierre Fermat's era used in this article's proof are accessible to any first-year physics and mathematics student. This contrasts favorably with Andrew Wiles's proof, which is quite complex for the average mathematician due to its use of advanced and intricate modern mathematical tools.

Fermat was obviously "playing" with the new notions, decomposing powers of differences into sums of powers and suddenly found out that as one confines oneself with positive integers in the power, the logarithmic equation yields immediately that $x^n + y^n = z^n$(which is a difference rewritten as a sum) is correct for whole x, y, z only and if only $n = 1$ or 2.

He (would have) had first to introduce the two new notions so as to fully explain his finding. One can imagine ***how much room it would take to put down all the deliberations*** that had led him to his discovery ***on the margins of a book solely without the proper symbolic notations that a contemporary mathematician avails***.

Why Pierre Fermat did not write down all those ideas in a dedicated document is the dedicated question of a dedicated research endeavour. It can come out that he had authored such a separate document indeed, which afterwards was somehow lost or – alternatively – has survived to this day, hidden in an archive or a library or in somebody's unrealised custody.

The author requests the mathematical society to look critically at the deliberations set forth above and to return their assessment.

This paper has been published as a preprint on the ResearchGate platform and on the OSF platform at the following links [11, 12].

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A Appendix A. Proof of the Irrationality of $\sqrt[n]{2}$ for $n \geq 2$

A.1 Theorem: The number $\sqrt[n]{2}$ is irrational for any integer $n \geq 2$.

A.2 Proof:

We will prove by contradiction. Assume that $\sqrt[n]{2}$ is a rational number. This means it can be represented as an irreducible fraction $\frac{p}{q}$, where p and q are integers, $q \neq 0$, and p and q have no common divisors other than 1.

Thus, we have:

$$\sqrt[n]{2} = \frac{p}{q}$$

Raise both sides of the equation to the power of n:

$$(\sqrt[n]{2})^n = \left(\frac{p}{q}\right)^n$$

$$2 = \frac{p^n}{q^n}$$

Multiply both sides by q^n:

$$2q^n = p^n$$

From this equation, we see that p^n is an even number, since it is equal to $2q^n$. If p^n is even, then p must also be even (if p were odd, then p^n would also be odd).

Since p is even, we can write it as $p = 2k$, where k is some integer. Substitute this expression for p into the equation $2q^n = p^n$:

$$2q^n = (2k)^n$$

$$2q^n = 2^n k^n$$

Divide both sides of the equation by 2 (which is permissible since $n \geq 2$, and therefore 2^n is divisible by 2):

$$q^n = 2^{n-1} k^n$$

Since $n \geq 2$, then $n - 1 \geq 1$. From the last equation, we see that q^n is an even number, since it is equal to $2^{n-1}k^n$, where 2^{n-1} is an even factor. If q^n is even, then q must also be even.

Thus, we have reached the conclusion that both p and q are even numbers. This means they have a common divisor of 2. However, at the beginning of the proof, we assumed that the fraction $\frac{p}{q}$ was irreducible, meaning p and q have no common divisors other than 1.

The resulting contradiction indicates that our initial assumption about the rationality of $\sqrt[n]{2}$ is incorrect.

A.3 Conclusion:

Therefore, the number $\sqrt[n]{2}$ is irrational for any integer $n \geq 2$.

B Appendix B. Proof that the limit of the function $f(n) = \sqrt[n]{2 \cdot n}$ is 1

B.1 Problem Statement

Prove that the limit of the function $f(n) = \sqrt[n]{2 \cdot n}$ as n approaches infinity is equal to 1.

B.2 Solution

Consider the function $f(n) = \sqrt[n]{2 \cdot n}$. Our goal is to find the limit of this function as $n \to \infty$. Let's rewrite the function in the form of a power:

$$f(n) = (2 \cdot n)^{\frac{1}{n}}$$

To find the limit of this function, let's consider the natural logarithm of $f(n)$:

$$\ln(f(n)) = \ln\left((2 \cdot n)^{\frac{1}{n}}\right)$$

Using the properties of logarithms, we get:

$$\ln(f(n)) = \frac{1}{n}\ln(2 \cdot n)$$

Separate the logarithm of the product into the sum of logarithms:

$$\ln(f(n)) = \frac{\ln(2) + \ln(n)}{n}$$

Now, let's find the limit of $\ln(f(n))$ as $n \to \infty$:

$$\lim_{n\to\infty} \ln(f(n)) = \lim_{n\to\infty} \frac{\ln(2) + \ln(n)}{n}$$

This limit has the indeterminate form $\frac{\infty}{\infty}$, so we can apply L'Hôpital's Rule. Take the derivatives of the numerator and denominator with respect to n:

$$\frac{d}{dn}(\ln(2) + \ln(n)) = \frac{1}{n}$$

$$\frac{d}{dn}(n) = 1$$

Thus, the limit becomes:

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$$

So, we have found that the limit of the natural logarithm of the function is 0:

$$\lim_{n\to\infty} \ln(f(n)) = 0$$

Now, to find the limit of the original function $f(n)$, we use the continuity of the exponential function:

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} e^{\ln(f(n))} = e^{\lim_{n\to\infty}\ln(f(n))} = e^0 = 1$$

B.3 Conclusion

Therefore, we have proven that the limit of the function $f(n) = \sqrt[n]{2 \cdot n}$ as n approaches infinity is equal to 1.

C Appendix C. A short proof of monotonicity

It is well known that $\sqrt[n]{2 \cdot n} \to 1$ as $n \to \infty$ (see Appendix B). However, in this appendix we focus on showing that $(2 \cdot n)^{1/n}$ is strictly decreasing for $n \geq 3$. Below is one way to prove this, using the logarithmic derivative:

C.1 Define the function.

Let

$$f(n) \;=\; (2 \cdot n)^{\frac{1}{n}}.$$

It is convenient to work with its natural logarithm:

$$\ln\bigl(f(n)\bigr) \;=\; \ln\bigl((2 \cdot n)^{\frac{1}{n}}\bigr) \;=\; \frac{1}{n}\,\ln(2 \cdot n).$$

Denote

$$g(n) \;=\; \ln\bigl(f(n)\bigr) \;=\; \frac{\ln(2 \cdot n)}{n} \;=\; \frac{\ln(2) + \ln(n)}{n}.$$

C.2 Compute the derivative.

Treating n as a real variable $n > 0$, we have

$$g'(n) \;=\; \frac{d}{dn}\Bigl(\tfrac{\ln(2 \cdot n)}{n}\Bigr).$$

Using the quotient rule,

$$g'(n) = \frac{\frac{d}{dn}[\ln(2 \cdot n)] \cdot n \;-\; \ln(2 \cdot n) \cdot 1}{n^2} = \frac{\frac{1}{n} \cdot n \;-\; \ln(2 \cdot n)}{n^2} = \frac{1 - \ln(2 \cdot n)}{n^2}.$$

(Note that $\frac{d}{dn}[\ln(2 \cdot n)] = \frac{1}{2 \cdot n} \cdot 2 = \frac{1}{n}$.)

C.3 Sign of the derivative.

We want to see where $g'(n) < 0$:

$$g'(n) < 0 \iff 1 - \ln(2 \cdot n) < 0 \iff \ln(2 \cdot n) > 1 \iff 2 \cdot n > e \iff n > \frac{e}{2}.$$

Since $e \approx 2.718$, the inequality $n > e/2$ is certainly true for all integer $n \geq 2$, and hence strictly for $n \geq 3$. Therefore, for $n \geq 3$, $g(n)$ is strictly decreasing.

C.4 Implication for $f(n)$.

Since $f(n) = \exp(g(n))$, and $\exp(\cdot)$ is a strictly increasing function in its argument, $f(n)$ decreases whenever $g(n)$ decreases. Hence, $f(n)$ is indeed strictly decreasing for $n \geq 3$.

Thus, only for $n = 1$ and $n = 2$ does $f(n)$ attain the maximum value 2, and for all $n \geq 3$, it strictly decreases (which is consistent with the limit $\lim_{n\to\infty}(2 \cdot n)^{1/n} = 1$).

C.5 Conclusion

Hence, we have demonstrated that $f(n)$ is strictly decreasing for $n \geq 3$, either by analyzing the derivative of $\ln(f(n))$ (as shown above) or by comparing $f(n+1)$ and $f(n)$. This completes the proof of the monotonicity of $f(n)$.

D Appendix D: Analysis of Functions and Verification of Critical Points

D.1 Introduction

The goal of this analysis is to study the behavior of the function:

$$f(p,q) = q - \sqrt[p]{qp},$$

and to identify its critical points where the second derivative $f''(p)$ equals zero. This study aims to support the conclusions presented in the main article, particularly regarding the unique role of $o = 2$ in the context of Fermat's Last Theorem:

$$x^n + y^n = z^n.$$

In this article, the number o represents a key parameter associated with the symmetry of the equation. In our calculations, this corresponds to $q = o$. The uniqueness of $q = 2$ (or $o = 2$) is supported both geometrically and algebraically.

D.2 Results of the Analysis

D.2.1 Combined Graph of Functions

The following graph (Fig. 1) illustrates three functions, each with its corresponding discrete values:

- $1.5 - \sqrt[p]{1.5p}$ (blue line) and $1.5 - \sqrt[n]{1.5n}$ (blue squares),
- $2 - \sqrt[p]{2p}$ (green line) and $2 - \sqrt[n]{2 \cdot n}$ (green squares),
- $3 - \sqrt[p]{3p}$ (orange line) and $3 - \sqrt[n]{3n}$ (orange squares).

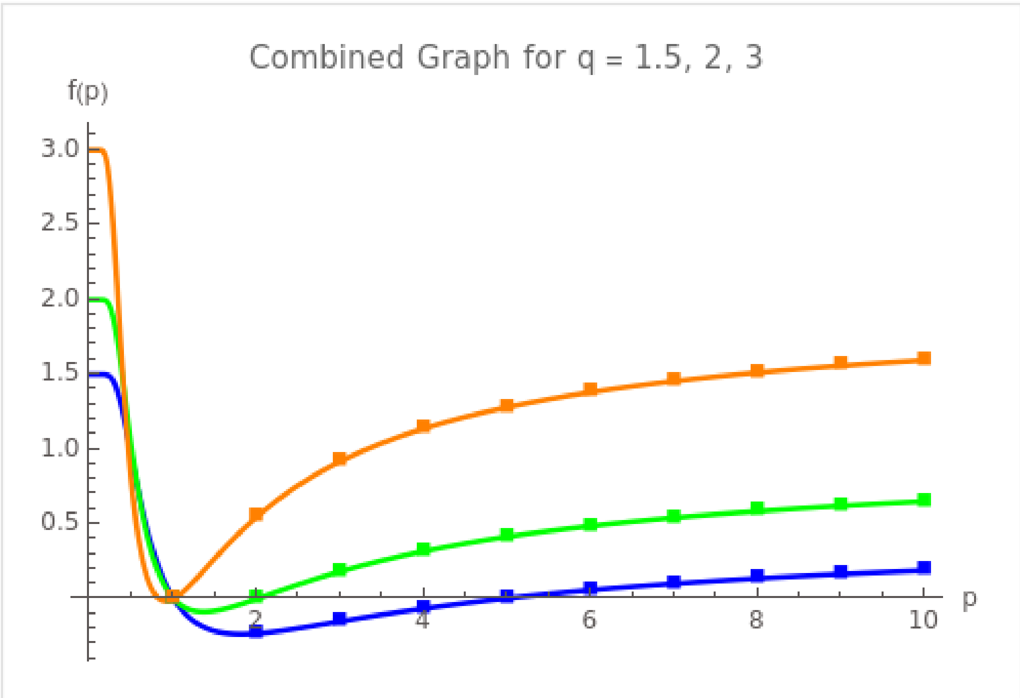

Figure 1: Combined graph of $q - \sqrt[p]{qp}$ for $q = 1.5, 2, 3$. Lines represent continuous p, and squares represent discrete n.

D.2.2 Surface Plot of $f(p,q)$

The three-dimensional surface plot below (Fig. 2) illustrates the behavior of $f(p,q) = q - \sqrt[p]{qp}$ for continuous values of $p > 0$ and $q > 0$. The perspective highlights the change in curvature as both parameters vary.

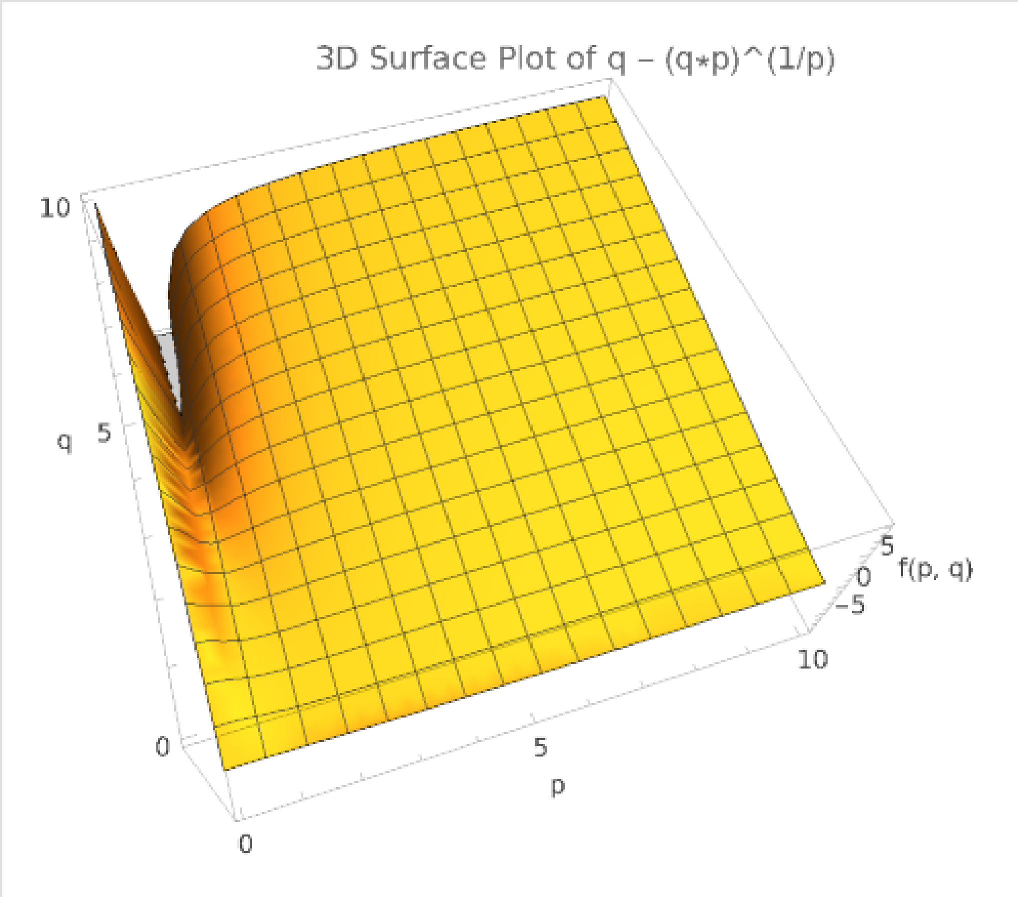

Figure 2: 3D surface plot of $f(p,q) = q - \sqrt[p]{qp}$ for $p > 0$ and $q > 0$.

D.3 Analysis and Conclusions

- The critical point at $p = 2$ exists uniquely for $q = 2$, where the second derivative $f''(p)$ equals zero. This highlights the special role of $q = 2$ (or $o = 2$).
- For other values of q, critical points exist, but they occur at values $p \neq 2$, showing that $q = 2$ is unique in its symmetry and simplicity.

These results confirm the dual nature of this article solution: both **geometric** and **algebraic**. The parameter $o = 2$ serves as a unifying concept in the analysis of Fermat's Last Theorem (See Appendix E for more details).

E Appendix E: Geometric Verification of Fermat's Last Theorem through the Analysis of the Function $f(p, q)$

In this appendix, we investigate the connection between the function

$$f(p, q) = q - \sqrt[p]{qp}$$

and Fermat's equation

$$x^n + y^n = z^n.$$

It is shown that for $n = 2$ there exists an inflection point of the second derivative, which corresponds to the existence of Pythagorean triples. For $n > 2$, this inflection point shifts toward lower values of p, indicating a change in the mathematical properties of the equation and the impossibility of integer solutions.

E.1 Analysis of the Second Derivative

The function

$$f(p, q) = q - \sqrt[p]{qp}$$

allows us to study the behavior of inflection points, which are determined by the condition

$$f_p''(p, q) = \frac{\partial^2}{\partial p^2}\left(q - \sqrt[p]{qp}\right) = 0.$$

Inflection points are important because they reveal specific mathematical patterns in the equation.

E.2 Additional Analysis for the Variable q

Investigations were also conducted on the variable q. At the point $q = 2$ (with $p = 2$), both the first and second partial derivatives with respect to q are equal to 0.5. This result indicates a fundamental property of Fermat's equation at that point. However, in our case the variable q is fixed at the value 2, as determined by the binomial computations, although the very fact of isolating the value of q is interesting in itself.

E.3 Numerical Results

Below are the numerical values of the second derivative $f_p''(p, q)$ for various values of p and q:

p	$q = 1.5$	$q = 2.0$	$q = 2.5$	$q = 3.0$	$q = 4.0$
1	2.75	0.77	2.90	0.96	1.10
2	0.17	0.00	-0.11	-0.34	-0.55
3	-0.0056	-0.08	-0.099	-0.21	-0.31
4	-0.018	-0.11	-0.056	-0.12	-0.17
5	-0.014	-0.10	-0.033	-0.067	-0.093

Table 2: Numerical values of the second derivative $f_p''(p, q)$ for various p and q.

E.4 Conclusions from the Data

- For $p = 2,\ q = 2$ we have $f_p''(p, q) = 0$, which corresponds to the existence of Pythagorean triples.
- For $q > 2$, the inflection point shifts toward lower values of p, indicating a change in the properties of the equation and the impossibility of integer solutions.
- Figure 4 illustrates the surface of $f_p''(p, q)$ with the marked inflection points.

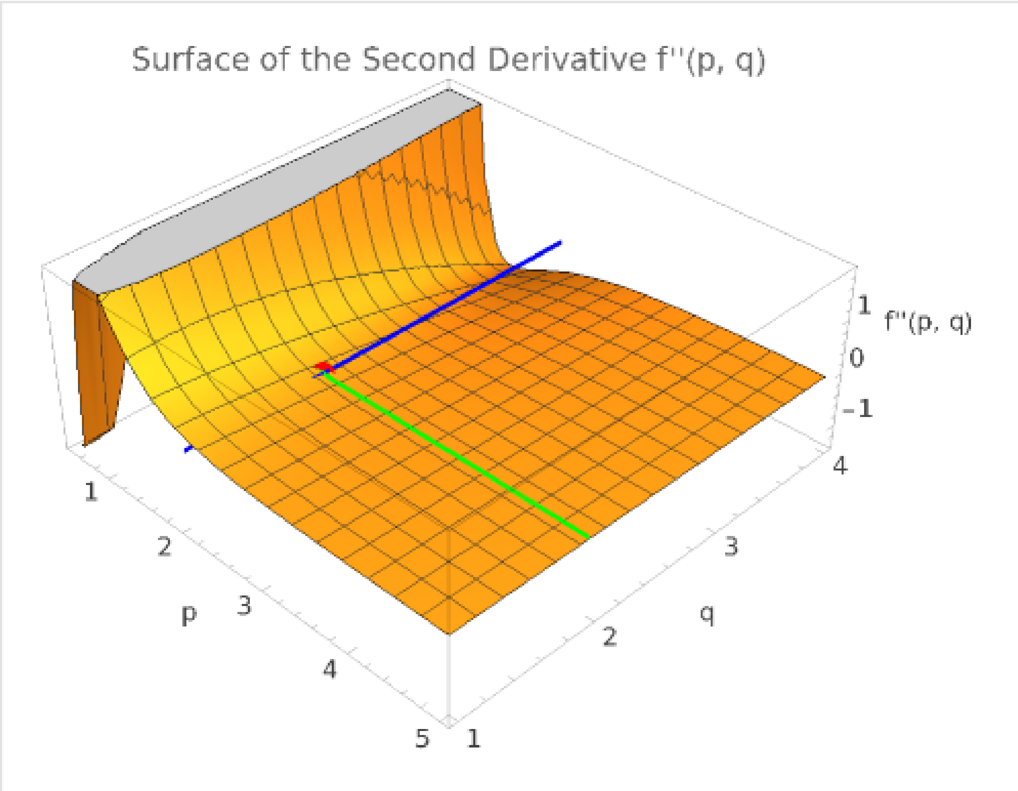

Figure 3: Surface of the second derivative $f_p''(p, q)$, demonstrating the shift of the inflection point.

E.5 Verification of Fermat's Last Theorem

E.5.1 Relation to the Inflection Points

From the data, it follows that:

1. If the inflection point occurs at $p = n$, then integer solutions are possible.
2. If the inflection point shifts to lower values of p, then integer solutions are impossible.

Thus, **only for $n = 2$ are integer solutions possible**, while for $n > 2$ the properties of the equation change, excluding such solutions.

E.5.2 Final Conclusion

Based on our calculations:

- **For $q = 2$, the inflection point corresponds to $p = 2$** $\Rightarrow$ Fermat's equation has integer solutions.
- **For $q > 2$, the inflection point shifts to lower values of p** $\Rightarrow$ the mathematical properties of the equation change, and integer solutions become impossible.
- This confirms that the Fermat equation

$$x^n + y^n = z^n, \quad n > 2$$

  has no solutions in the integers.

Thus, the function $f(p, q)$ clearly demonstrates that when $n > 2$ the inflection point shifts and the properties of Fermat's equation change, confirming the impossibility of integer solutions.

F Appendix E: Geometric Verification of Fermat's Last Theorem through the Analysis of the Function $f(p, q)$

In this appendix, we investigate the relationship between the function

$$f(p, q) = q - \sqrt[p]{qp}$$

and Fermat's equation

$$x^n + y^n = z^n.$$

It is shown that for $n = 2$, an inflection point of the second derivative exists, which corresponds to the existence of Pythagorean triples. For $n > 2$, this inflection point shifts towards smaller values of p, indicating a change in the mathematical properties of the equation and the impossibility of integer solutions.

F.1 Analysis of the Second Derivative

The function

$$f(p, q) = q - \sqrt[p]{qp}$$

allows us to study the behavior of inflection points, which are determined by the condition

$$f_p''(p, q) = \frac{\partial^2}{\partial p^2}\left(q - \sqrt[p]{qp}\right) = 0.$$

Inflection points are important because they reveal specific mathematical patterns in the equation.

F.2 Additional Analysis for the Variable q

Studies were also conducted for the variable q. At the point $q = 2$ (for $p = 2$), the first and second partial derivatives with respect to q are equal to 0.5. This result points to a fundamental property of Fermat's equation at this point. However, in our case, the variable q is fixed at the value of 2, which is determined by binomial calculations, although the fact that the value of q is singled out is of interest.

F.3 Numerical Results

Below are the numerical values of the second derivative $f_p''(p, q)$ for various values of p and q:

Table 3. Numerical values of the second derivative $f_p''(p, q)$ for different p and q.

p	$q = 1.5$	$q = 2.0$	$q = 2.5$	$q = 3.0$	$q = 4.0$
1	2.75	0.77	2.90	0.96	1.10
2	0.17	0.00	-0.11	-0.34	-0.55
3	-0.0056	-0.08	-0.099	-0.21	-0.31
4	-0.018	-0.11	-0.056	-0.12	-0.17
5	-0.014	-0.10	-0.033	-0.067	-0.093

F.4 Conclusions from the Data

- For $p = 2,\ q = 2$, we have $f_p''(p, q) = 0$, which corresponds to the existence of Pythagorean triples.
- For $q > 2$, the inflection point shifts towards smaller values of p, which indicates a change in the properties of the equation and the impossibility of integer solutions.
- Figure 4 illustrates the surface of $f_p''(p, q)$ with the inflection points marked.

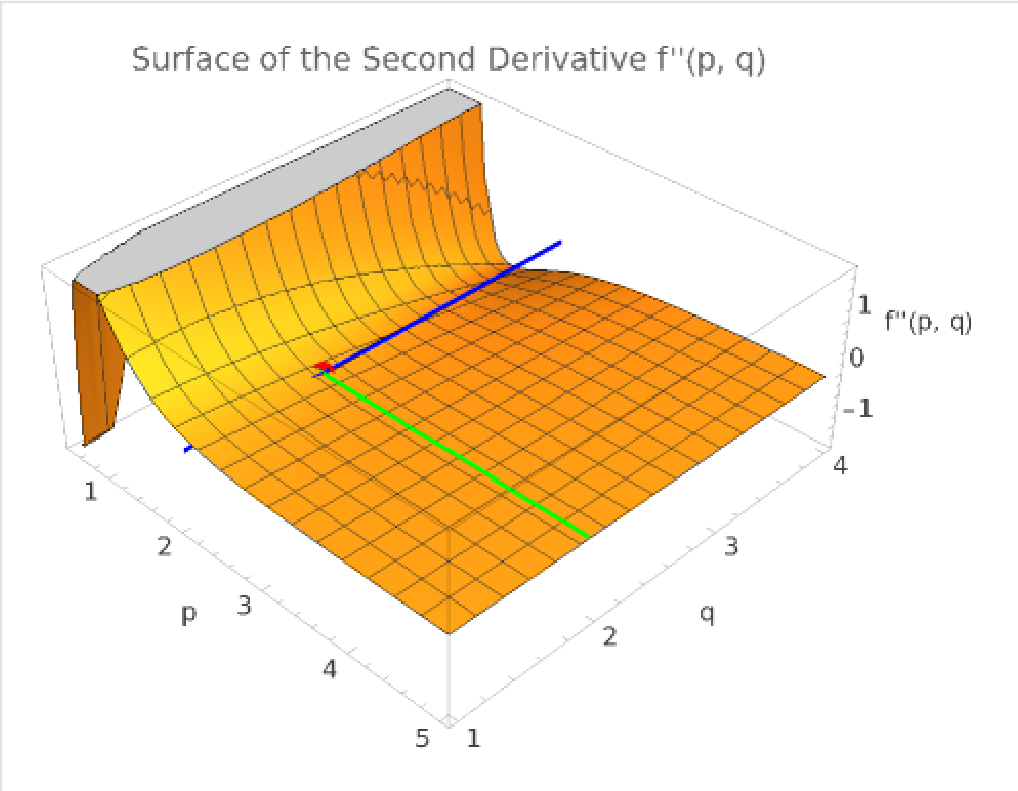

Figure 4. Surface of the second derivative $f_p''(p, q)$, demonstrating the shift of the inflection point.

F.5 Verification of Fermat's Last Theorem

F.5.1 Connection with Inflection Points

From the data, it follows that:

1. If the inflection point is located at $p = n$, then integer solutions are possible.
2. If the inflection point shifts to smaller values of p, then integer solutions are impossible.

Thus, **only for $n = 2$ are integer solutions possible**, while for $n > 2$ the properties of the equation change, excluding such solutions.

F.5.2 Final Conclusion

Based on our calculations:

- **For $q = 2$, the inflection point corresponds to $p = 2$** $\Rightarrow$ Fermat's equation has integer solutions.
- **For $q > 2$, the inflection point shifts to smaller values of p** $\Rightarrow$ the mathematical properties of the equation change, and integer solutions become impossible.
- This confirms that Fermat's equation

  $$x^n + y^n = z^n, \quad n > 2$$

  has no solutions in integers.

Thus, the function $f(p, q)$ clearly demonstrates that for $n > 2$, the inflection point shifts, and the properties of Fermat's equation change, confirming the impossibility of integer solutions.

G Appendix F: Global Normalization and the Derivation of FLT

The Idea. We separate the normalization hypothesis from Peano arithmetic: a single integer $o > 1$ is fixed (independent of n, x, y, z) such that for any hypothetical counterexample to Fermat's equation for $n > 2$, the equality holds:

$$o^n = 2 \cdot n.$$

Subsequently, an elementary comparison of the growth of the exponential and linear functions excludes $n > 2$.

Axiom F.1 (Global Normalization)**.** *Let us fix $o \in \mathbb{N}$ with $o > 1$. For all $n, x, y, z \in \mathbb{N}$, the following holds:*

$$\big(2 < n \ \wedge \ x^n + y^n = z^n\big) \implies o^n = 2n.$$

Lemma F.1 (On the solutions of $o^n = 2n$)**.** *If $o > 1$ and $o^n = 2n$ for $n \geq 1$, then*

$$o = 2 \quad \textit{and} \quad n \in \{1, 2\}.$$

Proof. For $o = 2$, the equality $2^n = 2n$ is equivalent to $2^{n-1} = n$, which is true only for $n = 1, 2$; for $n \geq 3$, we have $2^{n-1} > n$. If $o \geq 3$, then $o^n \geq 3^n > 2n$ for all $n \geq 1$, which contradicts $o^n = 2n$. □

Theorem F.1 (FLT from Global Normalization)**.** *Let Axiom F.1 be true. Then*

$$\forall n > 2 \ \forall x, y, z \in \mathbb{N} \qquad x^n + y^n \neq z^n.$$

Proof. Assume the contrary: there exist $n > 2$, $x, y, z \in \mathbb{N}$ such that $x^n + y^n = z^n$. By Axiom F.1, we obtain $o^n = 2n$. By Lemma F.1, this is only possible for $n \in \{1, 2\}$, which contradicts $n > 2$. □

Comments. (1) The integrality of o is not derived from intermediate algebraic equalities—it is part of the Postulate as a component of the definition of "global normalization." (2) The meaning of "globality" is that the same o serves all potential counterexamples for $n > 2$; this is a precise abstraction of the formalization in Coq (see Appendix G). (3) If one additionally chooses "full coverage" over the integers n, it becomes natural to set $o = 2$ (discussed via $f(n) = (2n)^{1/n}$); however, Theorem F.1 itself relies only on Axiom F.1 and Lemma F.1.

H Appendix G: Formalization of "Global Normalization" in Coq

Correspondence with the Coq Code. The following fragment explains how Postulate F.1 is implemented in the file `FLT.v` and how FLT is mechanically derived from it.

Hypotheses (Section Context).

- Fixed normalization parameter o : `nat` and the requirement $1 < o$:
  `Variable o : nat. Hypothesis normalization_gt1 : 1 < o.`

- Global normalization (precisely the meaning of Axiom F.1):
  `Hypothesis normalization_equation :`
  `forall (n x y z : nat), 2 < n -> x^n + y^n = z^n -> o^n = 2 * n.`

Key Lemma on Growth (mathematically identical to Lemma F.1):

```
Lemma integer_solution_o : 1 < o -> 1 <= n -> o^n = 2 * n ->
                    o = 2 / (n = 1  n = 2).
```

Main Theorem (strictly corresponds to Theorem F.1):

```
Theorem fermat_last_theorem_from_normalization :
forall (n x y z : nat), 2 < n -> x^n + y^n = z^n -> False.
```

Proof in Coq: from `normalization_equation`, we get $o^n = 2n$, after which `integer_solution_o` returns $n \in \{1, 2\}$, which contradicts $n > 2$.

Variant with the formula $o = 2$ ("full coverage"). If the normalization is defined directly by the equality $2^n = 2n$ for all hypothetical counterexamples, then the following corollary is used:

```
Corollary fermat_last_theorem_with_o_two :
(forall n x y z, 2 < n -> x^n + y^n = z^n -> 2^n = 2 * n) ->
    forall n x y z, 2 < n -> x^n + y^n = z^n -> False.
```

Summary. The `FLT.v` code implements precisely the global normalization (the same o for all $n > 2$) and does not contain `Admitted`. The lemma on growth (`integer_solution_o`) materializes the elementary comparison of o^n and $2n$, and the theorem `fermat_last_theorem_from_normalization` completes the conditional implication

$$\text{(Global Normalization)} \implies \text{FLT for } n > 2.$$

This strictly corresponds to the formulations in Appendix F and the new version of the article.