

# Global Normalization and Fermat’s Last Theorem

## (An Explanation for a Mathematician without Programming)

### Abstract

This text provides a *non-programmer’s* explanation of the *global normalization* approach to Fermat’s Last Theorem (FLT): we introduce a single parameter  $o > 1$  and state that *any* hypothetical counterexample to the equation  $x^n + y^n = z^n$  for  $n > 2$  *implies* the equality

$$o^n = 2n,$$

after which, from the *principle of maximum coverage*, it follows that the *only possible* choice is  $o = 2$ , and the equality  $2^n = 2n$  is possible only for  $n \in \{1, 2\}$ . Thus, for  $n > 2$ , a contradiction arises, which is consistent with FLT. Importantly: the **normalization statement** (that every counterexample indeed implies  $o^n = 2n$  for the same  $o$ ) is accepted as a *hypothesis*; everything else follows elementarily. Formal analogues of definitions and lemmas are provided and machine-checked in Coq (with no need to know programming languages, we will provide an interpretation of all steps in plain mathematical language).<sup>1</sup>

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<sup>1</sup>The formal definition `covers_with`, the “bridge” lemma, and the theorem on maximum coverage are in the module *GlobalNormalization*; see the file with the Coq code. See also the explanatory preprint with historical commentary and a step-by-step reconstruction.

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# 1 Short Roadmap

1. **The Idea of Normalization.** We fix a number  $o > 1$ . We say that “ $o$  covers the exponent  $n$ ” if  $o^n = 2n$  holds. This idea is conveniently packaged as follows: *every* counterexample for  $n > 2$  *implies* the coverage of  $n$  by the same  $o$ .
2. **Maximum Coverage.** Consider  $S(o) = \{n \in \mathbb{N} : o^n = 2n\}$ . We will show that for  $o = 2$ , the set  $S(o)$  consists of exactly  $\{1, 2\}$ , and for  $o \neq 2$ , it is smaller in cardinality. Hence, the *only* choice compatible with the principle of “maximum scope” is  $o = 2$ .
3. **The Elementary Reason for the Contradiction.** For  $o = 2$ , we have  $2^n = 2n$  only for  $n \in \{1, 2\}$ , because for  $n \geq 3$ ,  $2^n > 2n$ . Consequently, a hypothetical counterexample for  $n > 2$  cannot exist.
4. **What Remains a Hypothesis.** We do *not* prove that for any counterexample,  $o^n = 2n$  is inevitable with the *same*  $o > 1$ . This is the *normalization premise*. Everything else is rigorous (and even formally verified) mathematics.

# 2 Intuition Comprehensible without Programming

The goal is to *conditionally* (assuming the normalizing premise) reduce FLT to a simple growth estimate. The idea of normalization arose naturally from the classical transformation

$$(z, x) = (m^n + p^n, m^n - p^n),$$

which is convenient for parity analysis and binomial expansions: for an odd  $n$ , the difference  $(m^n + p^n)^n - (m^n - p^n)^n$  contains only odd binomial indices and is a multiple of  $n$ .<sup>2</sup>

Two things are important here:

- **Combinatorics of the Binomial Theorem.** When subtracting  $(A+B)^n - (A-B)^n$ , only terms with odd powers of  $B$  remain, and there is a common factor of  $n$ —this allows *normalizing* the difference to the form  $2 \cdot n \cdot (\text{something})$ .
- **Growth of Exponential vs. Linear Function.** The equality  $o^n = 2n$  cannot hold for large  $n$  if  $o \geq 2$ ; and if  $1 < o < 2$ , it can be satisfied *at most once* due to the monotonicity of  $(2n)^{1/n}$  with respect to  $n$ . This leads to the idea of choosing the  $o$  where the coverage is maximal—and it turns out to be exactly  $o = 2$ , covering only  $n = 1$  and  $n = 2$ .

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<sup>2</sup>In Coq, this is reflected through elementary facts about the parity of sums/differences, see lemmas on parity and divisibility of binomial differences.

### 3 Formulation in Mathematical Language

#### 3.1 Definition and Principle

**Definition 3.1** (Coverage). For  $o > 1$  and  $n \in \mathbb{N}$ , we say that  $o$  covers  $n$  if

$$o^n = 2n.$$

We denote the set of covered exponents by  $S(o) = \{n \in \mathbb{N} : o^n = 2n\}$ .

**Definition 3.2** (Global Normalization, Principle of Maximum Coverage). We say that *global normalization holds* if there exists an  $o > 1$  such that **any** hypothetical counterexample  $x^n + y^n = z^n$  for  $n > 2$  implies the coverage of  $n$  by the same  $o$ , i.e.,  $o^n = 2n$  (the same  $o$  for all  $n$ ). The principle of *maximum coverage* dictates choosing the  $o > 1$  for which the cardinality  $|S(o)|$  is maximal among all permissible  $o$ .

#### 3.2 Structure of the set $S(o)$

Consider the function  $f(n) = (2n)^{1/n}$  for  $n \geq 1$ . Then  $o^n = 2n$  is equivalent to  $o = f(n)$ .

**Lemma 3.3** (Monotonicity of  $f$ ). For  $n \geq 2$ , the function  $f(n)$  is strictly decreasing,  $f(1) = 2$ ,  $f(2) = 2$ , while for  $n > 2$  we have  $f(n) < 2$  and  $\lim_{n \rightarrow \infty} f(n) = 1$ .

*Proof (sketch).* Consider  $\ln f(n) = \frac{\ln(2n)}{n}$  and its derivative with respect to  $n$  in the continuous relaxation:  $\frac{d}{dn}(\ln f(n)) = \frac{1-\ln(2n)}{n^2} < 0$  for  $n \geq 2$ . The calculations  $f(1) = f(2) = 2$  are trivial, and the limit to 1 is standard (use  $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$ ).  $\square$

**Proposition 3.4** (Description of  $S(o)$ ). For a fixed  $o > 1$ :

- 1) if  $o \geq 3$ , then  $S(o) = \emptyset$  (since  $o^n \geq 3^n > 2n$ );
- 2) if  $1 < o < 2$ , then  $|S(o)| \leq 1$  (by Lemma 3.3, the level  $o$  is crossed at most once for  $n \geq 2$ , and  $f(1) = 2 > o$ );
- 3) if  $o = 2$ , then  $S(2) = \{1, 2\}$ , since  $f(1) = f(2) = 2$ , and for  $n > 2$ ,  $f(n) < 2$ .

Consequently, the maximum cardinality  $|S(o)|$  is attained uniquely at  $o = 2$  and is equal to 2.

**Corollary 3.5** (Choice of Normalization). From the principle of maximum coverage, it follows that  $o = 2$  and  $S(o) = \{1, 2\}$ .

#### 3.3 Elementary Estimate of Exponential Growth

**Lemma 3.6.** For every  $n \geq 3$ , it is true that  $2^n > 2n$ .

*Proof.* By induction on  $n$ , or by comparison with binomial coefficients:  $2^n = \sum_{k=0}^n \binom{n}{k} \geq \binom{n}{0} + \binom{n}{1} + \binom{n}{n-1} + \binom{n}{n} = 2 + 2n > 2n$  for  $n \geq 3$ .  $\square$

**Corollary 3.7.** The equality  $2^n = 2n$  is possible only for  $n \in \{1, 2\}$ .

## 4 Main Conditional Theorem (Logic of FLT Derivation)

**Theorem 4.1** (FLT from Global Normalization). *Suppose there exists an  $o > 1$  such that for **any** hypothetical natural triple  $x, y, z$  with  $n > 2$  satisfying  $x^n + y^n = z^n$ , the coverage  $o^n = 2n$  holds (with the same  $o$  for all  $n$ ). Then the equation  $x^n + y^n = z^n$  has no solutions in  $\mathbb{N}$  for  $n > 2$ .*

*Proof.* By the principle of maximum coverage and Proposition 3.4, we have  $o = 2$  and  $S(o) = \{1, 2\}$ . Then any required coverage for  $n > 2$  is impossible, since by Corollary 3.7 the equality  $2^n = 2n$  does not hold. This is a contradiction.  $\square$

## 5 Where the Normalization Premise Comes From

Substantively, the premise comes from the *binomial analysis* of the difference

$$(m^n + p^n)^n - (m^n - p^n)^n,$$

where for an odd  $n$ , only odd binomial indices survive, and the entire sum is a multiple of  $n$ ; moreover, in constructions of the form

$$\left(\frac{z+x}{2}\right)^{1/n}, \quad \left(\frac{z-x}{2}\right)^{1/n}$$

for different natures of  $n$  (even/odd), a common factor of  $n$  appears and, roughly speaking, a “universal” radical that can be collapsed into the same norm  $o$  for all  $n$ .<sup>3</sup>

It is important to understand: **this transition is left as a hypothesis**. The preprint states directly: normalization is *postulated* and used as an *explicit hypothesis*, rather than being derived from arithmetic by the author in a complete form.<sup>4</sup>

## 6 Bridge from Real Equalities to Natural Powers

The key “bridge” fact: if for the same  $o$ , two exponents  $n$  and  $m$  are covered,

$$o^n = 2n, \quad o^m = 2m,$$

then, by raising to powers and eliminating  $o$ , we obtain an equality of integer powers

$$(2n)^m = (2m)^n.$$

This is convenient when one needs to transfer the reasoning from  $\mathbb{R}$  back to  $\mathbb{N}$  and apply the arithmetic of powers.

**Remark 6.1** (Why this is needed). The bridge lemma helps to vary the points of view: normalization lives in  $\mathbb{R}$ , while  $x^n + y^n = z^n$  itself lives in  $\mathbb{N}$ . The equality  $(2n)^m = (2m)^n$  is a pure integer symmetry, compatible with the idea that the set of covered exponents is extremely limited.

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<sup>3</sup>In the explanatory text, this is formulated as steps 2–6 with the extraction of the common factor  $n$  and the transition to an equality of the form  $(\text{some expression})^n = 2n$ ; see the section *Possible Proof*.

<sup>4</sup>A paraphrase of the quote: “The normalization premise is kept as an explicit hypothesis over  $\mathbb{N}$ ; bridge lemmas connect the real equation  $o^n = 2 \cdot n$  with the integer comparison.”

## 7 Elements of the Classics: Parametrization, Parity, Binomial Symmetry

### 7.1 Parametrization $(z, x) = (m^n + p^n, m^n - p^n)$

This notation by itself does not guarantee that  $m, p$  are integers; however, it provides strong *necessary* conditions on the parities of  $z \pm x$  and on the form of the expressions  $\frac{z \pm x}{2}$ . In particular,

$$z + x = 2m^n, \quad z - x = 2p^n,$$

so that  $z \pm x$  are even, and if we *additionally* require  $m, p \in \mathbb{Z}$ , then both halves must be perfect  $n$ -th powers. And vice versa: if these conditions are met, then  $m, p$  can be reconstructed (with caveats about signs for even  $n$ ).

### 7.2 Odd/Even Binomial Sums

The sums of even and odd binomial coefficients are equal and give  $2^{n-1}$ :

$$\sum_j \binom{n}{2j} = \sum_i \binom{n}{2i+1} = 2^{n-1},$$

which is convenient in checks for  $n = 1, 2$  and in expansions of  $(A+B)^n \pm (A-B)^n$ .

## 8 What Exactly Is Formally Checked and What Is Not

- **Formally in Coq:** the definition of coverage  $o^n = 2n$ ; the theorem on maximum coverage (from  $o^1 = 2$  it follows that  $o = 2$  and only  $n \in \{1, 2\}$  are covered); the *bridge* lemma; elementary lemmas about the growth of  $2^n$  relative to  $2n$ ; *sanity-goals* like “from  $o = 2$ , coverage of  $n = 3$  does not follow”.<sup>5</sup>
- **Left as a premise:** *global normalization* (“any counterexample  $\Rightarrow$  coverage by the same  $o$ ”). This is formulated and emphasized in the preprint as an explicit hypothesis; it is motivated by binomial calculations and its “universal” form, but is *not* fully derived within classical elementary number theory.

## 9 Why the Choice $o = 2$ Is Unique and “Natural”

Uniqueness is a consequence of the strict decrease of  $f(n) = (2n)^{1/n}$  for  $n \geq 2$ : only the level  $o = 2$  passes through two points  $n = 1, 2$ ; any other  $o$  yields at most one intersection point (or none at all). This is the content of Proposition 3.4. In terms of *method*: the principle of maximum coverage suggests a “natural norm” that coincides with the base of the binary exponential.

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<sup>5</sup>Sanity checks at the end of the file serve as regression tests: for  $o=2$ , coverage for  $n \geq 3$  is false, etc.

## 10 Mini-FAQ for the Reader

- **Weak spot?** The only one is the *normalization premise*. All other steps are elementary and/or formally verified.
- **Doesn't this contradict Wiles?** No. This is *not* a new independent proof of FLT, but a *reduction* of FLT to a specific normalization statement. Wiles's proof remains the only unconditional one to date.
- **Why use Coq?** So that the minimal algebraic details (growth, the bridge lemma, “only  $n = 1, 2$  for  $o = 2$ ”) are machine-checked, eliminating human carelessness.
- **Is there a  $p$ -adic perspective?** The code contains a sketch of a “two-adic bracket” for a universal parameter and the fact that  $v_p(o)$  vanishes for odd  $p$ ; this is illustrative and not used in the main growth antagonism.

## 11 Detailed Appendices (Elementary Proofs)

### 11.1 Why $2^n > 2n$ for $n \geq 3$

See Lemma 3.6. The proof can also be done by induction: in the step  $n \rightarrow n+1$ , from  $2^n > 2n$  it follows that  $2^{n+1} = 2 \cdot 2^n > 4n \geq 2(n+1)$  for  $n \geq 2$ .

### 11.2 Strict decrease of $(2n)^{1/n}$

See Lemma 3.3. Another option is to use the AM–GM inequality:

$$(2n)^{1/n} \leq \overbrace{\frac{2 + 1 + \cdots + 1}{n}}^{\text{$n-1$ times}} \xrightarrow{n \rightarrow \infty} 1.$$

And for strict decrease for  $n \geq 3$ , one can compare  $f(n)$  and  $f(n+1)$  directly.

### 11.3 Odd binomial indices in the difference

The difference  $(A+B)^n - (A-B)^n$  eliminates all even indices, leaving

$$2 \sum_j \binom{n}{2j+1} A^{n-(2j+1)} B^{2j+1},$$

which yields the common factor  $2 \cdot B$  and, in appropriate substitutions, the factor  $n$  (via  $\binom{n}{1} = n$  and further multiples).

### 11.4 Parametrization and parity

If  $z = m^n + p^n$ ,  $x = m^n - p^n$ , then  $z \pm x = 2 \cdot (\text{perfect power})$ , hence  $z \pm x$  are even. From this follow convenient *negative* criteria: if  $z \pm x$  have the wrong parity, a corresponding parametrization with integers  $m, p$  does not exist.

## 12 How to Read Formal Names (Mini-Glossary without Programming)

- `covers_with`  $o$   $n$  means  $o^n = 2 \cdot n$ .
- `covers_with_two_characterisation` — from  $2^n = 2n$  it follows that  $n \in \{1, 2\}$ .
- `maximum_coverage_as_theorem` — formalization of the principle of maximum coverage: from coverage of  $n = 1$ , it is derived that  $o = 2$  and the restriction  $n \in \{1, 2\}$ .
- `two_real_normalizations_imply_nat_power_eq` — the bridge: from  $o^n = 2n$  and  $o^m = 2m$  it follows that  $(2n)^m = (2m)^n$ .
- `sanity_goals` — small automatic checks like “for  $o=2$  there is no coverage for  $n \geq 3$ ”.

## Conclusion (in the spirit of Fermat)

If one believes that any hypothetical triple for  $n > 2$  forces the *same* number  $o$  to satisfy  $o^n = 2n$ , then from the principle of maximum coverage, we immediately obtain  $o = 2$  and thus exclude all  $n > 2$ , since  $2^n > 2n$ . The conciseness of the result resonates with the idea of a “short marginal note”. But it is precisely the *normalization* that is the subject of the main question and further research.

**What to do next?** Refine and justify the normalization premise (e.g., through Diophantine estimates,  $p$ -adic arguments, or comparison with programs like ABC/descent), while preserving the elementary aesthetic line of reasoning.