Fermat + Coq: FLT from Global Normalization via a Coverage Parameter

We present a *global-normalization* reading of G. L. Dedenko's manuscript. A single real factor $o > 1$ is postulated to serve all putative counterexamples to Fermat's equation

$$x^n + y^n = z^n \qquad (n > 2),$$

and is linked to the exponent n by the *coverage identity*

$$\mathrm{pow}(o, n) \;=\; 2 \cdot \mathrm{INR}(n).$$

Under the *maximum-coverage* principle (the same o covers exactly $n \in \{1, 2\}$ and no other exponents), the development shows that $o = 2$, and that $\mathrm{pow}(2, n) = 2 \cdot \mathrm{INR}(n)$ can hold only for $n \in \{1, 2\}$. Hence any putative solution with $n > 2$ yields a contradiction, and Fermat's Last Theorem (FLT) follows.

What is formalized in Coq.

- *Coverage predicate:*

  ```
  Definition covers_with (o:R) (n:nat) := pow o n = 2 * INR n.
  ```

- *Global normalization hypotheses* (as section assumptions):
  - `normalization_gt1`: $1 < o$.
  - `maximum_coverage`: covers_with$(o, 1)$ and covers_with$(o, 2)$, and $\forall n,$ covers_with$(o, n) \Rightarrow n \in \{1, 2\}$.
  - `normalization_equation`: if $n > 2$ and $x^n + y^n = z^n$ in $\mathbb{N}$, then covers_with(o, n).

- *Consequences in Coq:*
  - `normalization_parameter_is_two`: $o = 2$ (from $\mathrm{pow}(o, 1) = 2$).
  - `normalization_forces_small_exponent`: no solutions for any $n > 2$.

- *Concrete realisation $o = 2$:* `covers_two_one`, `covers_two_two`, and `covers_two_only_small` prove that $o = 2$ indeed satisfies maximum coverage.

- *Bridges & growth:* `covers_two_nat`, `INR_two_mul_nat`, and the lemmas `pow2_gt_linear`, `pow3_gt_linear`, `pow_eq_linear_positive` establish that $2^n = 2n$ forces $n \in \{1, 2\}$.

- *Parity & parametrization (motivation only):* over $\mathbb{R}$ and $\mathbb{Z}$, `sum_diff_from_parameters_R/Z`, `parity_condition_Z` show evenness of $z \pm x$ for $z := m^n + p^n,\ x := m^n - p^n$; these are not used in the final contradiction.

Motivation vs. proof. The coverage-parameter viewpoint subsumes the earlier explicit-base phrasing. While the behaviour of $f(n) = (2n)^{1/n}$ motivates why $o = 2$ is the "full-coverage" choice, the *proof* relies only on the formal hypotheses above and elementary growth lemmas, not on an a priori GN(2) postulate.

Repository (code and PDFs): github.com/Gendalf71/FLT-Coq

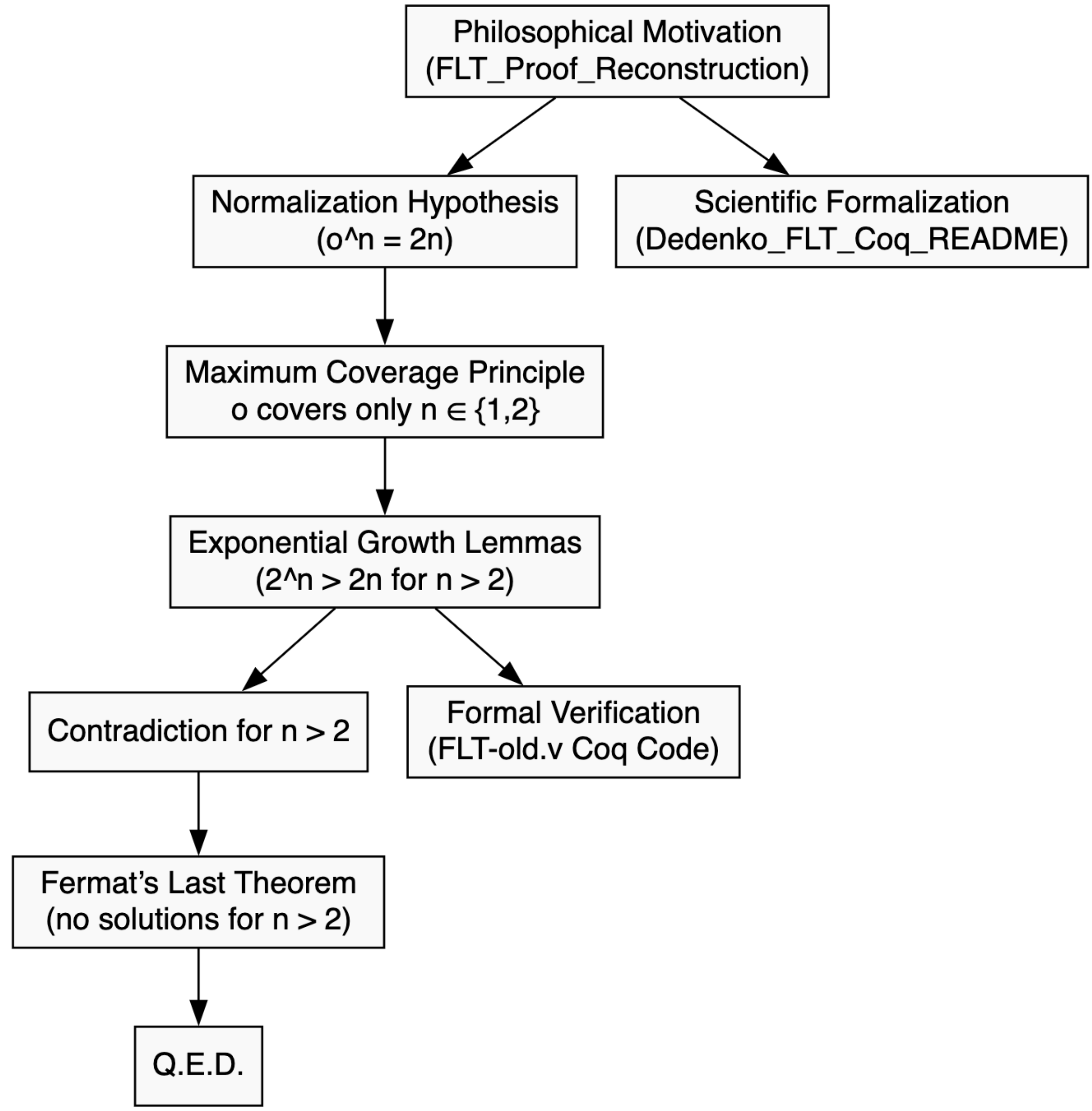

Figure 1: Formal pipeline: global normalization ($o > 1$) $\Rightarrow$ FLT (Coq).

The package includes:

- `FLT.v`: Coq development (no `Admitted`); proofs compile.
- A reasoning flowchart (figure above).
- Explanatory PDFs (EN/RU), updated to the coverage-parameter reading.

Further reading:

- Reconstruction of Fermat's Proof (ResearchGate) — RU
- Formalization & discussion — EN