Global Normalization via a Coverage Parameter ($o > 1$) $\Rightarrow$ FLT: A Coq-Verified Conditional Route

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Abstract. We develop a *global-normalization* reading of Dedenko's idea. A single real factor $o > 1$ is postulated to serve all putative counterexamples to Fermat's equation $x^n + y^n = z^n$ with $n > 2$, yielding the *coverage* identity $\mathrm{pow}(o, n) = 2 \cdot \mathrm{INR}(n)$. Under the *maximum-coverage* condition (the same o covers precisely the exponents $n \in \{1, 2\}$ and no others), the Coq development proves $o = 2$ and shows that $\mathrm{pow}(2, n) = 2 \cdot \mathrm{INR}(n)$ can hold only for $n \in \{1, 2\}$. Hence any putative solution with $n > 2$ yields a contradiction, and Fermat's Last Theorem follows. The normalization premise is kept as an explicit hypothesis over $\mathbb{N}$; bridge lemmas connect the real equation $\mathrm{pow}(o, n) = 2 \cdot \mathrm{INR}(n)$ with the integer comparison 2^n vs. $2n$. The classical parametrization $(z, x) = (m^n + p^n, m^n - p^n)$ and parity identities appear only as motivation and are not used in the final contradiction. This coverage-parameter presentation replaces the earlier fixed-base GN(2) wording.

Keywords: Fermat's Last Theorem · global normalization · coverage · Coq · formal verification

1 Introduction

We consider Fermat's equation

$$x^n + y^n = z^n, \qquad x, y, z \in \mathbb{N},\ n \in \mathbb{N}. \tag{1}$$

In the present reading the analysis is collapsed by a single global normalizer $o > 1$ and a coverage predicate:

Definition 1 (Coverage predicate). *For a real $o > 1$ and $n \in \mathbb{N}$, set*

$$\mathrm{covers\_with}(o, n) \ :\Longleftrightarrow\ \mathrm{pow}(o, n) = 2 \cdot \mathrm{INR}(n).$$

Definition 2 (Global normalization hypothesis). *There exists a fixed $o > 1$ such that for every integer $n > 2$ and all $x, y, z \in \mathbb{N}$,*

$$x^n + y^n = z^n \implies \mathrm{covers\_with}(o, n).$$

Definition 3 (Maximum coverage). *The same o satisfies* $\mathrm{covers\_with}(o, 1)$ *and* $\mathrm{covers\_with}(o, 2)$*, and for all n, if* $\mathrm{covers\_with}(o, n)$ *then* $n \in \{1, 2\}$.

Theorem 1 (Conditional on global normalization (coverage parameter)). *Assume there exists $o > 1$ satisfying Definitions 2 and 3. Then the Fermat equation $x^n + y^n = z^n$ has no solutions in $\mathbb{N}$ for any $n > 2$.*

2 Motivation: algebraic setup and parity (not used in the core)

Following the standard trick, set $z := m^n + p^n$ and $x := m^n - p^n$ (initially over $\mathbb{R}$ so that ring equalities are straightforward). Then

$$y^n = z^n - x^n = (m^n + p^n)^n - (m^n - p^n)^n$$

is the odd–binomial sum. Passing to $\mathbb{Z}$, one obtains that $z \pm x$ are even; in Coq this is captured by lemmas `sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, and `parity_condition_Z`. These parity facts are logically independent from the final step and are included for completeness only.

3 Coq formalization: naturals core with a real wrapper

The development proves the elementary growth comparisons $2^n > 2n$ for $n \geq 3$ (and $3^n > 2n$ for $n \geq 1$), and packages them into the lemma `pow_eq_linear_positive` showing that $2^n = 2 \cdot n$ forces $n \in \{1, 2\}$.

Coverage predicate (Coq).

```
Definition covers_with (o : R) (n : nat) := pow o n = 2 * INR n.
```

Global normalization and maximum coverage (Coq, section hypotheses).

```
Variable o : R.
Hypothesis normalization_gt1 : 1 < o.
Hypothesis maximum_coverage :
  covers_with o 1%nat /\
  covers_with o 2%nat /\
  (forall n, covers_with o n -> n = 1%nat \/ n = 2%nat).
Hypothesis normalization_equation :
  forall (n x y z : nat),
    2 < n ->
    Nat.pow x n + Nat.pow y n = Nat.pow z n ->
    covers_with o n.
```

From these, the Coq file derives:

- $o = 2$ (lemma `normalization_parameter_is_two` via $\mathrm{pow}(o, 1) = 2$),
- the contradiction for all $n > 2$ (lemma `normalization_forces_small_exponent`).

For convenience, the explicit choice $o = 2$ is verified to realise maximum coverage:

```
Lemma covers_two_one : covers_with 2 1%nat.  (* pow(2,1) = 2 · INR(1) *)
Lemma covers_two_two : covers_with 2 2%nat.  (* pow(2,2) = 2 · INR(2) *)
Lemma covers_two_only_small (n : nat) :
  covers_with 2 n -> n = 1%nat \/ n = 2%nat.
```

Combining the global normalization with this explicit choice yields `fermat_last_theorem_via_maximum_coverage`.

4 What is *not* assumed

The reading here does *not* rely on any unconditional congruence like $(m^n + p^n)^n - (m^n - p^n)^n \equiv 0 \pmod{2n}$ (which is false in general), nor on a fixed-base hypothesis GN(2). Algebraic parametrization and parity serve as motivation/consistency checks and are not used in the final step.

5 Article–Coq correspondence

6 Conclusion

Under the single global-normalization premise encoded by a coverage parameter $o > 1$, the Coq file derives FLT for all $n > 2$ using only elementary growth lemmas and the maximum-coverage principle that isolates $n \in \{1, 2\}$. Parity constraints from the parametrization are checked separately. This coverage-parameter presentation replaces the earlier fixed-base GN(2) wording.

Article (item)	**Coq formalization (lemma/theorem)**
Algebraic parametrization over $\mathbb{R}$; integer parity facts	`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`.
Coverage predicate and bridge to naturals	`covers_with`, `covers_two_nat`, `INR_two_mul_nat`.
Growth vs. linear comparison; $2^n = 2 \cdot n \Rightarrow n \in \{1, 2\}$	`pow2_gt_linear`, `pow3_gt_linear`, `pow_eq_linear_positive`.
Global normalization and maximum coverage (hypotheses/section)	`normalization_gt1`, `maximum_coverage`, `normalization_equation`.
Consequences: $o = 2$ and contradiction for $n > 2$	`normalization_parameter_is_two`, `normalization_forces_small_exponent`.
Explicit realisation with $o = 2$; final FLT corollaries	`covers_two_one`, `covers_two_two`, `covers_two_only_small`, `fermat_last_theorem_from_global_normalization`, `fermat_last_theorem_via_maximum_coverage`.

Table 1. Mapping between the paper's steps and the Coq development.

Appendix: selected Coq declarations (names)

`sum_diff_from_parameters_R`, `sum_diff_from_parameters_Z`, `parity_condition_Z`, `no_parameters_if_parity_violation`, `no_parameters_if_odd`, `pow2_gt_linear`, `pow3_gt_linear`, `pow_eq_linear_positive`, `covers_two_nat`, `INR_two_mul_nat`, `covers_with`, `normalization_gt1`, `maximum_coverage`, `normalization_equation`, `normalization_parameter_is_two`, `normalization_forces_small_exponent`, `covers_two_one`, `covers_two_two`, `covers_two_only_small`, `fermat_last_theorem_from_global_normalization`, `fermat_last_theorem_via_maximum_coverage`.

References

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