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CS225-402 Midterm
11/12/17 – 6:50PM EST Exam Time

Logic Symbols: ≥ ≤ ≠ ¬ ⋀⋁ ⊕ ≡ → ↔ ∃∀
Set Symbols: ∈ ∉ ⊆ ⊂ ⊇⊃ ∅ ⋃ ∩ ×

1.

a)
P=to log on to the server
Q=to have a valid password
"If we want to log on to the server, then a valid password is necessary"
The contrapositive = ~Q→~P, so:
~P=to not log on to the server
~Q=to not have a valid password

Thus, the contrapositive is: "If there is not a valid password, then you cannot log on to the server"

b)
P=x is nonnegative
Q=x is positive OR x is zero
The contrapositive = ~Q→~P, so:
~P=x is not nonnegative
~Q=x is not positive and x is not zero

Thus, the contrapositive is: "If x is not positive and x is not zero, then x is not nonnegative."

1.2.

a)
P=you pay the bill
Q=you lose the company's services
~(P⋁ Q) =(~P⋀ ~Q)

Thus the negation is: "You do not pay the bill and you do not lose the company's services."

b)
P=A positive integer is prime
~Q=it has no divisors other than 1 and itself
~(P→~Q) = P∩Q   Thus the negation is:
"A positive integer is prime and it does have divisors other than 1 or itself."

2.

$P(x)$: x is a mathematics problem

$Q(x)$: x is time consuming

$R(x)$: x is easy

$S(x)$: x is solvable

a) For all problems in science, such that if it is a mathematics problem, then it is solvable or it is not easy.

b) Not all problems are solvable if they are easy. -Or- There exists a problem that is solvable and is not easy.

c) There exists a problem in science such that it is a mathematics problem and it is time consuming.

d) There does not exist a problem in science such that it is time consuming and it is solvable.

3.

a) "For all people, if they are young, then they are not good athletes."
$\forall x \mid S(x) \rightarrow \neg W(x)$

b) "There exists a person, such that, they are female and not good athletes, but they are young"
$\exists x \mid B(x) \rightarrow \neg W(x) \wedge S(x)$

c) Not all young people are good athletes.
$\neg \forall x \mid S(x) \rightarrow W(x)$

d) There exists a person who is not a good athlete and is not a female.
$\exists x \mid \neg W(x) \wedge \neg B(x)$

4.

P	Q	R	~R	$(Q \vee R)$	$(P \wedge \neg R)$	$(P \rightarrow (Q \vee R))$	$((P \wedge \neg R) \rightarrow Q)$
T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	F	T	F	T	T
F	F	F	T	F	F	T	T

Since $(P \rightarrow (Q \vee R))$ and $((P \wedge \neg R) \rightarrow Q)$ have the same truth table values, they are both logically equivalent.

5.
Given two rational numbers r and s with $r< s$, we must find if there is another rational number between r and s. By definition of a rational number, $r=a/b$ and $s=c/d$ where a, b, c, and d are integers and b and d $!=0$. To find if there is a number between r and s that is rational, we add r and s together $(r+s)$ and then divide it by two $((r+s)/2)$. We then plug in our integers a, b, c, d:

$(((a/b) + (c/d))/2)$ -we then solve to get:
$=(ad + cb)/2bd$

Since a, b, c, and d are integers, by the definition of summation, and by the definition of a product, $(ad + cb)/2bd$ is rational and the original statement is proven. An easy way to view this is: $r < ((r+s)/2) < s$

6.
Show that if k is an integer and $5k + 4$ is odd then k is odd. Suppose not. Suppose that $5k+4$ is odd and k is even. By contradiction, P=k is an integer and $5k+4$ is odd, Q=k is odd, then P^~Q= “k is an integer and $5k+4$ is odd and k is even.

So, if k is even, then by definition of an even number, $k=2m$ where m is an arbitrary integer that is even. If we plug in k, we get $5(2m)+4$ which is even and not odd. Thus by contradiction we prove that the supposition is false and the original statement is true.

7.

By the definition of a set, B $\subseteq$ A. For some arbitrary integer x, x is an element in B and x is an element in A by the previously defined set B $\subseteq$ A. To determine if B $\subseteq$ A, then 18b-2 and 6(random integer)+4 will produce a random integer a that is in the set B.

18b-2=6a+4	solve for a
18b-6=6a	divide by 6
3b-1=a	

3b-1=a is an integer by definition of a difference. So if we plug it back in to 6a+4 we get:

6(3b-1)+4 =
=18b-6+4
=18b-2 which is in the set A.

Thus B $\subseteq$ A is proven.

8.

$(B^c \cup (B^c - A))^c = B$

$(B^c \cup (B^c - A))^c = (B^c \cup (B^c \cap A^c))^c$	by the set difference law
$= ((B^c)^c \cap ((B^c)^c \cup (A^c)^c)$	by De Morgan's Laws for all sets A and B
$= B \cap (B \cup A)$	by the double complement law
$= B$	by the absorption laws

9.

9.1:

∑ i=1 →7: 4i - ∑ i=1 →2: 4i + ∑ i=0 →7: (-1)^i - ∑ i=0 →2: (-1)^i =

=(7(7+1)/2) – (2(2+1)/2) + ((-1^(7+1)-1)/-2) - ((-1^(2+1)-1)/-2)
=28 – 3 + 0 + (-1) = 24(ans)

9.2:

∑ i=0 →4: $(4^{j+3}+6)$ = 64*∑ i=0 →4: (4^{j}) + ∑ i=0 →4: 6 =

64*$((4^{4+1}$ – 1)/4-1) + 6*4 = (ans)

64*$((4^{5}$ – 1)/3) + 24 = (ans)

10.

P(n) = 3^0 + 3^1 + ……..+3^n = ½(3^(n+1) -1)

Base Case P(0):
LHS: $3^0 = 1$
RHS: ½(3^{0+1}-1) = 1
P(0) is true.

Inductive Case: If P(k) is true, then show that P(k+1) is also true.

P(k) = 3^0 + 3^1 + ……..+3^k = ½(3^(k+1) -1)

Then:

P(k + 1) = 3^0 + 3^1 + ……..+3^(k+1) = ½(3^(k+2) -1)

Thus to prove that P(k+1) is true solve:

$\frac{1}{2}(3^{k+1}-1) + 3^{k+1}$

$= \frac{1}{2}(3^{k+1}-1) + (2/2)*3^{k+1}$

$=\frac{1}{2}(3^{k+1}-1 + 2*3^{k+1})$

$=\frac{1}{2}(1^1* 3^{k+1}-1 + 2*3^{k+1})$

$=\frac{1}{2}(3^{k+2}-1)$

Thus by mathematical induction, P(k+1) is proven to be true.