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Logic Symbols: ≥ ≤ ≠ ¬ ⋀ ⋁ ⊕ ≡ → ↔ ∃ ∀ Set Symbols: ∈ ∉ ⊆ ⊂ ⊇ ⊃ ∅ ∪ ∩

Question 6: Give the converse, the contrapositive, and the inverse of the following statements-

1) If the number is 64, then it is both even and a power of 4.

Converse: If it is both even and a power of 4, then it is the number 64.

Contrapositive: If it is neither even and a power of 4, the number is not 64.

Inverse: If the number is not 64, then it is neither even or a power of 4.

2) Having a microscope is a necessary condition for (our) seeing viruses.

Converse: Seeing viruses is a necessary condition for (our) having a microscope.

Contrapositive: Not seeing viruses is a necessary condition for (our) not having a microscope.

Inverse: Not having a microscope is necessary for (our) not seeing viruses.

3) John will break the world's record for the mile run only if he runs the mile in under four minutes.

Converse: If he runs the mile in under four minutes, John will break the world's record for the mile run.

Contrapositive: If he does not run the mile in under four minutes, John will not break the world's record.

Inverse: John will not break the world's record for the mile run if he does not run the mile in under four minutes.

Question 7: Negate the following sentences. Be sure to justify your work :

i) John is a bachelor is sufficient to know that he is a male.
ii) A necessary condition for Jim to pass the course is that he does the home works regularly.
iii) it is neither raining nor sleeting.
iv)If the security code is not entered then the door will not open.
v) Only if Marc studies will he pass the test.

Answers:

i)John is a bachelor = r , he is a male = s.

Thus it is in “if r then s” form. = If John is a bachelor, then he is a male.

We know $\neg (r \rightarrow s) = r \wedge \neg s$….. So,

John is a bachelor and he is not a male.

ii) “r if and only if, s” = “For Jim will pass the course it is a necessary that he does the home work regularly”

“if he does the home works regularly, then jim will pass the course”

so,

He does the home works regularly and Jim will not pass the course.

iii) negating conditionals = $r \wedge \neg s$ so,

It is raining and it is not sleeting

iv) the security code is not enter = $\neg r$, the door will not open = $\neg s$

Thus it is in “if r then s” form. =

We know $\neg (\neg r \rightarrow \neg s) = r \wedge \neg s$….. So,

The security code is entered and the door will not open.

v) p = Marc Studies, q = he will pass the test

We know $\neg (p \rightarrow q) = p \wedge \neg q$ so,

Marc studies and he will not pass the test.

Question 8:

1)

P = 1 + 1 = 3 (F)

Q = 3 + 4 = 9 (F)

False if and only if False (True since they have the same values)

2)

P = 1 + 1 = 2 (T)

Q = 3 + 4 = 9 (F)

True only if False (True)

Question 9:

P = alice is smart

Q = honest

"Alice is smart, or she is smart but honest." = $P \vee (P \wedge Q)$ which is equivalent to P via absorption

Question 10:

a) $P \rightarrow R$

b) $Q \leftrightarrow (R \vee \neg P)$

c) $\neg(Q \vee R)$

Question 11: Decide whether or not the following pairs of statements are logically equivalent (using truth table method) .

P --> (Q -->R)   and   (~P ∨ ~Q) --> R

P	Q	R	~P	~Q	(Q -->R)	(~P ∨ ~Q)	P --> (Q -->R)	(~P ∨ ~Q) --> R
T	T	T	F	F	T	F	T	T
T	T	F	F	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	F

Not logically equivalent due to different truth table values.

Question 12: Simplify the following equation (use the tables attached herewith [Tables.png]())

~(~(P ∨ ~ Q) ∨ (~P ∧ ~Q))

~(~(P ∨ ~ Q) ∨ (~P ∧ ~Q)) ≡ ~(~P ∧ ~~ Q) ∨ (~P ∧ ~Q))   by DeMorgans Law

≡ ~(~P ∧ Q) ∨ (~P ∧ ~Q))   by double negation

≡ ~(~P ∧ (Q ∨~Q))   by distribution

≡ ~(~P ∧ t)   by negation law

≡ ~(~P)   by identity law

≡ ~(~P)   by double negative law

≡ P