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HW 4.1 **pgs 341-343, 352-364, 367-372**, **Set 6.1 - 12, 16,**, **Set 6.2 - 4, 10, 14**, **Set 6.3 - 12, 37, 42**

12. Let the universal set be the set $\mathbf{R}$ of all real numbers and let $A = \{x \in \mathbf{R} \mid -3 \le x \le 0\}$, $B = \{x \in \mathbf{R} \mid -1 < x < 2\}$, and $C = \{x \in \mathbf{R} \mid 6 < x \le 8\}$. Find each of the following:
    a. $A \cup B$ b. $A \cap B$ c. A^c d. $A \cup C$
    e. $A \cap C$ f. B^c g. $A^c \cap B^c$
    h. $A^c \cup B^c$ i. $(A \cap B)^c$ j. $(A \cup B)^c$

a. $A \cup B$ = {x∈R | x ∈ [-3, 2)} = [-3, 2)
b. $A \cap B$ = {x∈R | x ∈ (-1, 0] = (-1, 0]
c. A^c = {x∈R | x < -3 or x > 0}
d. $A \cup C$ = {x∈R |-3 <= x <= 0 or 6 < x <=8}
e. $A \cap C = \varnothing$
f. B^c = {x∈R | x <= -1 or x >= 2}
g. $A^c \cap B^c$ = {x∈R | x <= -1 or x > 0}
h. $A^c \cup B^c$ = {x∈R | x < -3 or x >= 2}
i. $(A \cap B)^c$ = $A^c \cup B^c$ = {x∈R | x < -3 or x >= 2}
j. $(A \cup B)^c$ = $A^c \cap B^c$ = {x∈R | x <= -1 or x > 0}

16. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.
    a. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?
    b. Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?
    c. Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

a. $A \cup (B \cap C) = \{a, b, c\}$
$(A \cup B) \cap C = \{b, c\}$
$(A \cup B) \cap (A \cup C) = \{a, b, c\}$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. $A \cap (B \cup C) = \{b, c\}$
$(A \cap B) \cup C = \{b, c, e\}$
$(A \cap B) \cup (A \cap C) = \{b, c\}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

c. $(A-B)-C = (A \cap B^c) \cap C^c) = \{a\}$
$A-(B-C) = (A \cap (B \cap C^c)) = \{a, b, c\}$
No, these sets are not equivalent.

4. The following is a proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks.

Proof: Suppose A and B are any sets and $A \subseteq B$. *[We must show that* $\underline{\text{(a)}}$*.]* Let $x \in \underline{\text{(b)}}$. *[We must show that* $\underline{\text{(c)}}$*.]* By definition of union, $x \in \underline{\text{(d)}}$ $\underline{\text{(e)}}$ $x \in \underline{\text{(f)}}$. In case $x \in \underline{\text{(g)}}$, then since $A \subseteq B$, $x \in \underline{\text{(h)}}$. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in \underline{\text{(i)}}$ *[as was to be shown]*.

a = A∪B⊆B

b = A⊆B

c = x∈A

d= A∪

e =B

f = A

g = A

h = A

I = A∪B⊆B

10. For all sets A, B, and C,

$$(A - B) \cap (C - B) = (A \cap C) - B.$$

Proof: We use membership tables to prove this identity. One can also prove by showing each side is a subset of the other side.

A	B	C	(A-B)	(C-B)	(A-B)∩(C-B)	(A∩C)	(A∩C) - B
1	1	1	0	0	0	1	0
1	1	0	0	0	0	0	0
1	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0

Thus they are equivalent.

14. For all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

Proof: Suppose A, B, and C are sets and $A \subseteq B$. Let $x \in A \cup C$. By definition of union, $x \in A$ and $x \in C$. But since $A \subseteq B$ and $x \in A$, then $x \in B$. Hence $x \in B$ and $x \in C$, and so, by definition of union, $x \in B \cup C$.
Thus $A \cup C \subseteq B \cup C$ by definition of subset.

H **12.** For all sets A, B, and C,

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

Proof: For all sets A, B, and C. If x∈ A∩(B-C), then x∈A and x∈B and x!∈C.
If x∈ (A∩B)-(A∩C), then x∈A and x∈B and since (A∩C) is subtracted from the set, x!∈C. So A∩(B-C) and (A∩B)-(A∩C) are equivalent and the statement is true.

37. For all sets A and B, $(B^c \cup (B^c - A))^c = B$.

Proof: If x∈ $(B^c \cup (B^c - A))^c$, then:

$(B^c \cup (B^c - A))^c$d

$=(B^c \cup (B^c \cap A^c))^c$ by set difference law

$=((B^c \cup B^c) \cap (B^c \cup A^c))^c$ by associative law

$=((B^c) \cap (B^c \cup A^c))^c$ by idempotent law

$=((B) \cup (B \cap A))$ by DeMorgan's law

=B by absorption law

thus $(B^c \cup (B^c - A))^c$=B and the statement is proven true.

42. $(A - (A \cap B)) \cap (B - (A \cap B))$

$(A-(A\cap B)) \cap (B-(A\cap B))$

$=(A\cap (A\cap B)^c) \cap (B\cap (A\cap B)^c)$ by the set difference law

$=(A\cap (A^c\cup B^c)) \cap (B\cap (A^c \cup B^c))$ by DeMorgan's law

$=((A\cap A^c) \cup (A\cap B^c) \cap ((B\cap A^c)\cup (B\cap B^c))$ by the associative law

$=(\varnothing) \cup (A\cap B^c) \cap ((B\cap A^c)\cup (\varnothing))$ by the complement law

$=(A\cap B^c) \cap ((B\cap A^c)$by identity laws

$=(A-B)\cap(B-A)$ by set difference law

$= \varnothing$ by complement laws