Practice Exam 2
11/12/17

1.
a)

p= I wear glasses
q=I can read the chalkboard

p^~q = ~p^q = I do not wear glasses and I can read the chalkboard.

b)

p=if I have a lottery ticket
q= then I can win that lottery

I have a lottery ticket and I cannot win that lottery.

1.2.

a) John is rich if and only if John is wealthy and he is not healthy.
b) John is not rich or it is not the case that he is healthy and wealthy.

2

p	q	r	~q	P or q	~q or r	(p or q) & (~q or r)	Q or r	(p or q) & (~q or r) → (Q or r)
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	F	F
F	T	T	F	T	T	T	T	T
F	T	F	F	T	F	F	T	T
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	F	T

Since the truth table values are not all true, there is no tautology.

3

a) There exists a student such that they are a math major and are a computer science major

$\exists x \mid M(x) \wedge C(x)$

b) Not all students such that if they are math majors, then they are required to take cs225

$\sim\forall x \mid M(x) \rightarrow A(x)$

c)For all students such that if they are comp sci students, then they are not required to take cs 225

$\forall x \mid C(x) \rightarrow \sim A(x)$

d)There exists a student such that they are a mth major and a computer major but is not required to take cs225.

$\exists x \mid M(x) \wedge C(x) \wedge \sim A(x)$

4

Let A, B, and C be sets with A is a subset of B. Proof: A U C is a subset of B intersects C.

By the definition of a intersection, x is an element in A and x is an element in C. Since A is a subset of B, and x is an element of A and C, then C is an element in B. On the right hand side, we see that B intersects C by the definition of intersection. This states that x is an element in B and x is an element in C. Since A is a subset of B, and x is an element in C that is intersected by A, it can be concluded that B intersects with C. As was to be shown as B intersects C.

5

Proof: That B is a subset of A. For y is an element in B, 18b-16 for some integers b. For x is an element in A, 18(some integers) – 2. We must find if they are equivalent and solve for a, so:

$18b+16 = 18a - 2$
$18b + 18 = 18a$
$b + 1 = a$

Thus a is an integer so we can check it

$18(b + 1) - 2$
$= 18b + 18 - 2$
$= 18b + 16$

Thus, B is shown to be a subset of A.

6

By definition of a rational number, r=a/b where b cannot equal 0. So 3r^2 – 2r + 4 = 3(a/b)^2 - 2(a/b) + 4. So,

3(a^2/b^2) – 2(a/b) + 4 where a and b are integers is a rational number by the definition of a sum, difference, and product, a and b are also integers.

7

"If n^3 + 5 is even, then n is odd for all natural numbers"

p= n^3 +5 is even
q= n is odd for all natural numbers

~p= n^3 + 5 is odd
~q= n is even for all natural numbers

So by contraposition:

> "if n is even for all natural numbers, then n^3 + 5 is odd"

So n can be written as 2k for any natural number k. Thus when tested to be odd in n^3 + 5, (2k)^3 + 5 is 8k^3 +5 where it is in fact odd. Thus our original statement is proved by contraposition.

8

"If a nonzero rational and an irrational number are multiplied, then their product is irrational"

If p=a nonzero rational
q= an irrational number
r=their product is irrational

By contradiction, "If a nonzero rational and an irrational number are multiplied, then their product is rational"

So if m is rational and n is irrational, then their product s is rational.
So m=a/b, n is irrational, and s=c/d.

So, (a/b)(n)=(c/d)
= (bc/ad) = n proving that n is in fact rational and that the original statement is false by contradiction.

9

P(n) = any postage of n cents (n>=18) can be formed using only 3cent and 10cent stamps)

Base Case: P(18)= six 3cent stamps, P(19)= three 3 cent stamps and one 10 cent stamp.

Inductive case:
Since the base case is true, P(k) of any postage of k cents (k>=18) can be formed using only 3cent and 10cent stamps). Assume that we can form (k+1)cents worth of postage from only 3cent and 10cent stamps we can take k-2 and add a 3cent stamp.

10

1: (j-(-1)^j) = j – j- (-1)^j – (-1)^j = (6(6+1)/2 – 1) – (6^(6+1)-1)/-2 – (6^(1+1) -1)/-2 = ans

2: 3^i*3^3 = 27 * ((3^i) – (3^i)) = 27((3^(5+1)-1/3-1) – (3^(2+1)-1)/3-1) = ans