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Reading: 244-256 and 258-263
Assignment: Set 5.2 – 9, 27, 35 & Set 5.3 – 10, 18, 23.b

9. Prove the statement using mathematical deduction. Do not derive them from Theorem 5.2.2 or Theorem 5.2.3.

9. For all integers $n \geq 3$,

$$4^3 + 4^4 + 4^5 + \cdots + 4^n = \frac{4(4^n - 16)}{3}.$$

1. S3 = 4^3 = (4(4^(3) – 16))/3
   = (4(48))/3
   4^3 = 64 = 4^3 = true

2. Assume S(k) is true: 4^3 + 4^4 + 4^5 + … + 4^k = (4(4^(k) – 16))/3

3. S(k+1): 4^3 + 4^4 + 4^5 + … + 4^(k+1) = (4(4^(k+1) – 16))/3

4. S(k): 4^3 + 4^4 + 4^5 + … +4^(k) + 4^(k+1) = ((4(4^(k) – 16))/3) + 4^(k+1)

5. Simplify right side:

$$= \left(\frac{4(4^k - 16)}{3}\right) + 4^{k+1}$$

$$= \frac{\left(4(4^k - 16)\right) + 3*4^{k+1}}{3}$$

$$= \frac{\left((4^{k+1} - 64)\right) + 3*4^{k+1}}{3}$$

$$= \frac{4^{k+1} + 3*4^{k+1} - 64}{3}$$

$$= \frac{1^1 * 4^{k+1} + 3^1 * 4^{k+1} - 64}{3}$$

$$= \frac{4^1 * 4^{k+1} - 64}{3}$$

$= \frac{4^{k+2} - 64}{3}$ which is equal to (4(4^(k+1) – 16))/3. As was to be shown and proven by mathematical induction.

27. Use the formula for the sum of the first *n* integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 20–29 or to write them in closed form.

27. $5^3 + 5^4 + 5^5 + \cdots + 5^k$, where k is any integer with $k \geq 3$.

Geometric Sequence: Formula = $\frac{r^{n+1}-1}{r-1}$

$=\frac{5^{k+1}-1}{5-1}$

$=\frac{5^{k+1+1}-1}{4}$

$=\frac{5^{k+2}-1}{4}$ is this true?

$=\left(\frac{5^{k+1}-1}{5-1}\right) + r^{k+1}$ $=\left(\frac{5^{k+1}-1}{5-1}\right) + 5^{k+1}$

$=\left(\frac{5^{k+1}-1}{4}\right) + (4 * 5^{k+1})/4$

$=\left(\frac{5^{k+1}-1+4*5^{k+1}}{4}\right)$ $=\left(\frac{5^{k+1}+4*5^{k+1}-1}{4}\right)$ $=\left(\frac{1^1*5^{k+1}+4^1*5^{k+1}-1}{4}\right)$ $=\left(\frac{5^1*5^{k+1}-1}{4}\right)$

$=\left(\frac{5^{k+2}-1}{4}\right)$ as was to be shown.

35. p254 Find the mistakes in the proof fragments.

H **35. Theorem:** For any integer $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

"Proof (by mathematical induction): Let the property $P(n)$ be $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$.

Show that P(1) is true: When $n = 1$

$$\sum_{i=1}^{1} i(i!) = (1+1)! - 1$$

So $\qquad 1(1!) = 2! - 1$

and $\qquad 1 = 1$

Thus $P(1)$ is true."

They began from a statement to deduce that the conclusion was true. This is incorrect and does not always prove that the statement is true. They should transform the left-hand side and the right-hand side independently until they are seen to be equal or transform one side of the equation until it is seen to be the same as the other side of the equation.

10. $n^3 - 7n + 3$ is divisible by 3, for each integer $n \geq 0$.

Let the property P(n) be the sentence "n^3 – 7n + 3 is divisible by 3".

Show that P(0) is true:
To establish P(0), we must show that

(0)^3 – 7(0) + 3 is divisible by 3.

But

(0)^3 – 7(0) + 3 = 3

and 3 is divisible by 3 because 3=3*1. Hence, P(0) is true.

Show that for all integers k >= 0, if P(k) is true then P(k+1) is also true:
Suppose that P(k) is true for a particular but arbitrarily chosen integer k>=0. That is: Let k be any integer with k>=0, and suppose that

k^3 – 7k + 3 is divisible by 3.

By definition of divisibility, this means that

k^3 – 7k + 3 = 3r for some integer r

We must show that P(k+1) is true. That is, we must show that:

(k+1)^3 – 7(k+1) + 3 is divisible by 3

Solving: (k+1)^3 – 7(k+1) + 3 = (k+1)(k+1)(k+1) – 7k – 7 + 3
= (k+1)(k+1)(k+1) – 7k – 7 +3
= k^3 + 3k^2 + 3k +1 -7k -7 +3
= $(\mathrm{k}^3 - 7k + 3) + (3k^2 + 3k - 6)$
= $(\mathrm{k}^3 - 7k + 3) + 3(k^2 + k - 2)$

The first part of the equation ($(\mathrm{k}^3 - 7k + 3)$ is proven divisible by 3 in our basis. The second part of the equation $3(k^2 + k - 2)$ is equal to 3*r for some integer r. So by divisibility, the equation $(\mathrm{k}^3 - 7k + 3) + 3(k^2 + k - 2)$ is divisible by 3.

18. $5^n + 9 < 6^n$, for all integers $n \geq 2$.

Let the property P(n) be the sentence “5^n + 9 < 6^n”.

Show that P(2) is true:
To establish P(2), we must show that

5^(2) + 9 < 6^(2).

But

5^(2) + 9 < 6^(2) = 34 < 36

Hence, P(2) is true.

Show that for all integers k >= 2, if P(k) is true then P(k+1) is also true:
Suppose that P(k) is true for a particular but arbitrarily chosen integer k>=2. That is: Let k be any integer with k>=2, and suppose that

5^(k) + 9 < 6^(k).

Thus the conclusion to be shown is

5^(k+1) + 9 < 6^(k+1).

Solving:

6(5^(k) + 9) < 6(6^(k))
=(5 + 1)(5^(k) + 9) < 6^(k+1)
=5*5^k + 45 + 5^k +9 < 6^(k+1)
=5^(k+1) + 9 + (5^k +45) < 6^(k+1)
=5^(k+1) + 9 < 6^(k+1) as was to be shown.

23. a. $n^3 > 2n + 1$, for all integers $n \geq 2$.
b. $n! > n^2$, for all integers $n \geq 4$.

For any given statement, let the property P(n) be the inequality n! > n^2

Show that P(2) is true:

4! > 4^2 = 4*3*2*1 > 4*4 = 24>16 which is true.

Show that for all integers k>=4, if p(k) is true, then p(k+1) is true.

P(k) = k! > k^2 which is true by our basis

P(k+1) = (k+1)! > (k+1)^2
=(k+1)! - (k+1)^2 > 0
=(k+1)k! – (k+1)^2 >0
=(k+1)(k! – (k+1)) >0

>(k+1)(k^2 – (k-1))
>(k+1)(K^2 –k + (1/4) – (1/4) -1)
>(k+1)((k-(1/2))^2-5/4)

=(k+1)(k! – (k+1)) >(k+1)((k-(1/2))^2-5/4)>0
=(k+1)! > (k+1)^2
as was to be shown.