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Logic Symbols: $\geq \leq \neq \neg \wedge \vee \oplus \equiv \rightarrow \leftrightarrow \exists \forall$ Set Symbols: $\in \notin \subseteq \subset \supseteq \supset \emptyset \cup \cap \times$
Quiz 4

1

Let $A = \{x \mid -2 < x \leq 4\}$, $B = \{x \mid -9 \leq x \leq 1\}$ and $C = \{x \mid 2 \leq x < 4\}$, where x represents an integer number. Determine the sets $(A - C) \cup A$, and $B \cap C^c$.

$$(A - C) \cup A = (A - C) = \{x \in \mathbb{R} \mid -2 < x < 2\} \text{ then } (A - C) \cup A = \{x \in \mathbb{R} \mid -2 < x \leq 4\}$$

$$(A \cap B) - C = (A \cap B) = \{x \in \mathbb{R} \mid -2 < x \leq 1\} \text{ then } (A \cap B) - C = \{x \in \mathbb{R} \mid -2 < x \leq 1\}$$

$$B \cap C^c = \{x \in \mathbb{R} \mid -9 \leq x \leq 1\}$$

2

Use an element argument to prove the statement:

For all sets A and B , $(A \cup B)^c \subseteq A^c \cap B^c$

Suppose $x \in (A \cup B)^c$. By definition of complement, $x \notin A$ and $x \notin B$. This states that x is not an element of either A or B . Therefore, x is not an element in $(A \cup B)^c$ and follows that x is not in $A^c \cap B^c$. Hence we can conclude that $(A \cup B)^c \subseteq A^c \cap B^c$.

3

For all sets A and B , simplify the given expression,

$$A - (A \cap B)$$

Cite a property from Theorem 6.2.2 for every step of the proof. [Theorem 6.2.2.png](#)

$$\begin{aligned} A - (A \cap B) &= A \cap (A \cap B)^c \text{ by the set difference law} \\ &= A \cap (A^c \cup B^c) \text{ by DeMorgan's Law} \\ &= (A \cap A^c) \cup (A \cap B^c) \text{ by associative laws} \\ &= (\emptyset) \cup (A \cap B^c) \text{ by the complement laws} \\ &= (A \cap B^c) \cup (\emptyset) \text{ by the commutative law} \\ &= (A \cap B^c) \text{ by the identity law} \\ &= A - B \text{ by the set difference law} \end{aligned}$$

4

What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$, where a_n equals:

1) $a_n = (-1)^{n+2} * n^2$, For all $n \geq 0$

2) $a_n = 5$

1 : $a_0 = (-1)^{0+2} * 0^2 = 1$
 $a_1 = (-1)^{1+2} * 1^2 = -1$
 $a_2 = (-1)^{2+2} * 2^2 = 4$
 $a_3 = (-1)^{3+2} * 3^2 = -9$

2 : $a_0 = 5$
 $a_1 = 5$
 $a_2 = 5$
 $a_3 = 5$

5

Given that, $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. Use this identity to find a simple expression for

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{(k-1)} - \frac{1}{k}\right) + \left(\frac{1}{k} - \frac{1}{(k+1)}\right)$$

$$= 1 - \frac{1}{(k+1)}$$

Compute the following summations. (Instructions: Showing your work is necessary and you must make use of the formula provided in the attached document [summation formula.pdf](#) . An intermediate form will be acceptable, so you don't need to calculate the final result.)

a) $\sum 5i = 3(7i+6)$

b) $\sum 3k = 0(3^{k+3})$

c) $\sum 5j = 2(2^{*}(-2)^j)$

a

$$\begin{aligned}\sum 5i = 3(7i+6) &= \sum 5i = 3(7i+6) - \sum_{i=1}^7 (7i+6) = ((7*(3(3+1)))/2 + 6) - ((7*(2(2+1)))/2 + 6) \\ &= ((7*(3(4)))/2 + 6) - ((7*(2(3)))/2 + 6)\end{aligned}$$

b

$$\sum 3k = 0(3^{k+3}) = \sum (3^k 3^3) = 9 \sum (3^k) = (9 * 3^{(3+1)} - 1) / (3 - 1) = \frac{9 * (3^4 - 1)}{2}$$

c

$$\begin{aligned}\sum 5j = 2(2^{*}(-2)^j) &= 2 \sum (-2)^j - 2 \sum (-2)^j = 2 * (((-2)^{(5+1)} - 1) / (2 - 1)) - 2 * (((-2)^{(1+1)} - 1) / (2 - 1)) \\ &= 2 * (((-2)^{(6)} - 1) / (1)) - 2 * (((-2)^{(2)} - 1) / (1))\end{aligned}$$