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HW 7.1

Required Reading : pp. 525- 529, 540-549

Homework Assignment - Set 9.2 - 11.c, 14.c,e, 17, Set 9.3 - 5, 24, 33-e,f

11. **a.** A bit string is a finite sequence of 0's and 1's. How many bit strings have length 8?
b. How many bit strings of length 8 begin with three 0's?
c. How many bit strings of length 8 begin and end with a 1?

c

Since the first and last bits are known to be both 1, so two bits of the 8 bit length are used. So, $2^6=64$ is the number of bit strings of length 8 that begin and end with a 1.

14. Suppose that in a certain state, all automobile license plates have four letters followed by three digits.
a. How many different license plates are possible?
b. How many license plates could begin with A and end in 0?
c. How many license plates could begin with *TGIF*?
d. How many license plates are possible in which all the letters and digits are distinct?
e. How many license plates could begin with *AB* and have all letters and digits distinct?

c

Since the first four slots take up *TGIF*, we only need to find the remaining digits which would be $1*10*10*10 = 1000$

e

The first two letters slots are taken by A and B, and all the remaining slots are to be distinct, so: $1 * 24 * 23 * 10 * 9 * 8 = 397440$

17. a. How many integers are there from 1000 through 9999?
 b. How many odd integers are there from 1000 through 9999?
 c. How many integers from 1000 through 9999 have distinct digits?
 d. How many odd integers from 1000 through 9999 have distinct digits?
 e. What is the probability that a randomly chosen four-digit integer has distinct digits and is odd?

a

There are 9000 since we include 1000 when counting.

b

We can use $2k+1$, where $2k=1000$ and $k=500$. We will eventually reach $2*4999 + 1$ after starting from $2*500 + 1$ which will give us the answer 4500 because $4999-500 + 1 = 4500$.

c

Each digit has a number of possibilities that it could be. The first of a 4 digit number can't be 0, so it will have 9. Since the first cannot use 0, but is in the 10 count spot, 9 still follows. The third cannot have an equal amount so it is 8 and the last digit 7:

$$9 * 9 * 8 * 7 = 4536$$

d

Each digit has a number of possibilities and must be odd. So each digit's range is odd numbers (0-9).

$$5*4*3*2 = 120$$

e

What is the probability that a randomly chosen four digit integer has distinct digits?

We must take the distinct/total, so:

$$\begin{aligned} & 9 * 9 * 8 * 7 / 9 * 9 * 9 * 9 \\ & = 4536 / 9000 = 0.504 \end{aligned}$$

If it has distinct digits and is odd, then:

$$120 / 9000 = 0.013$$

5. a. How many five-digit integers (integers from 10,000 through 99,999) are divisible by 5?
 b. What is the probability that a five-digit integer chosen at random is divisible by 5?

a

$$((99995-10000)/5) + 1 = 18000 \text{ five digit integers that are divisible by 5.}$$

b

$$18000/99999 = 0.1800$$

24. a. How many integers from 1 through 1,000 are multiples of 2 or multiples of 9?
 b. Suppose an integer from 1 through 1,000 is chosen at random. Use the result of part (a) to find the probability that the integer is a multiple of 2 or a multiple of 9.
 c. How many integers from 1 through 1,000 are neither multiples of 2 nor multiples of 9?

a

If A = integers from 1 through 1,000 are multiples of 2 and B = integers from 1 through 1,000 are multiples of 9. Thus A=500 and B=111. The union of A and B (multiples of 2 and multiples of 9), $A \cup B$ is the number of multiples of both, but we need to account for the multiples that intersect so we must subtract $A \cap B$ which is 55 ($2 \cdot 9 = 18$ through $990 = 55$). Thus:

$$A \cup B = A + B - A \cap B = 500 + 111 - 55 = 556$$

b

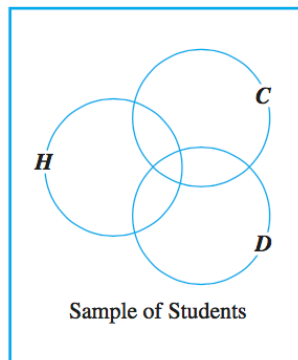
The probability that the integer is a multiple of 2 or a multiple of 9:
 $556/1000 = 0.556$

c

The number of integers from 1 through 1000 are neither multiples of 2 nor multiples of 9 :

$$1000-556 = 444$$

33. A college conducted a survey to explore the academic interests and achievements of its students. It asked students to place checks beside the numbers of all the statements that were true of them. Statement #1 was "I was on the honor roll last term," statement #2 was "I belong to an academic club, such as the math club or the Spanish club," and statement #3 was "I am majoring in at least two subjects." Out of a sample of 100 students, 28 checked #1, 26 checked #2, and 14 checked #3, 8 checked both #1 and #2, 4 checked both #1 and #3, 3 checked both #2 and #3, and 2 checked all three statements.
- How many students checked at least one of the statements?
 - How many students checked none of the statements?
 - Let H be the set of students who checked #1, C the set of students who checked #2, and D the set of students who checked #3. Fill in the numbers for all eight regions of the diagram below.



- How many students checked #1 and #2 but not #3?
- How many students checked #2 and #3 but not #1?
- How many students checked #2 but neither of the other two?

$$\#1=28, \#2=26, \#3=14$$

$$\#1\&\#2=8, \#1\&\#3=4, \#2\&\#3=3, \#1\&\#2\&\#3=2$$

e

$$\#2\&\#3 - \#1\&\#2\&\#3 = 3-2=1$$

f

$$\#2 - \#1\&\#2 - \#2\&\#3 - \#1\&\#2\&\#3 = 26 - 8 - 3 - 2=13$$