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Quiz 2

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1.

In the domain of all students, we define predicates
M(x) : x is a math major
C(x) : x is a computer science major
A(x) : x is required to take CS 225 .

Express each of the following English sentences in terms of M(x), C(x), A(x), quantifiers, and logical connectives.

(a) Some math majors are not required to take CS 225.
(b) Not all math majors are computer science majors.
(c) All students who are both computer science and math majors are not required to take CS 225.
(d) There is a student who is both math and computer major is not required to take CS 225.

a. $\exists x\ M(x) \rightarrow \neg A(x)$
b. $\neg \forall x\ M(x) \rightarrow C(x)$
c. $\forall x\ C(x) \land M(x) \rightarrow \neg A(x)$
d. $\exists x\ M(x) \land C(x) \land \neg A(x)$

2.

Let B(x), S(x), and A(x) be the predicates
B(x) : x is a good basketball player
S(x) : x is a good soccer player
A(x) : x is a good athlete

Translate each of the following quantified logic expressions (provided in the file) into English considering the domain to consist of all people .

1. If an athlete is good, then they are a good soccer or basketball player.
2. Not every athlete is a good basketball player.
3. Some people are good soccer players and bad basketball players or bad athletes.
4. Some people are bad soccer players or good basketball players.

3.

Negate each of the following statements:

1) Everything in that store is either overpriced or poorly made.

2) Some suspicions were substantiated.

3) No exercises have answers.

1) P=Overpriced
Q= poorly made

$\forall x\ P(x) \vee Q(x)$ so the negation is: $\neg(\forall x\ P(x) \vee Q(x))$

$= \exists x\ \neg P(x) \wedge \neg Q(x)$

=”Some things in that store are underpriced and nicely made”

2) P= suspicions were substantiated

$\exists xP(x)$ so the negation is: $\neg(\exists xP(x))$

$= \forall x \neg P(x)$

=”All suspicions were unsubstantiated”

3) P=have answers

$\forall x\ \neg P(x)$ so the negation is: $\neg(\forall x\ \neg P(x))$

$= \exists xP(x)$

="Some exercises have answers."

4.

Prove or disprove that $\forall x\, (P(x) \to Q(x))$ and $\sim \exists x \sim(\sim Q(x) \to \sim P(x))$ are logically equivalent. (Hint: Start with the double negation rule $\sim(\sim(\forall x\, (P(x) \to Q(x)))$

Is $\forall x\, (P(x) \to Q(x)) \equiv \sim \exists x \sim(\sim Q(x) \to \sim P(x))$?

$\sim \exists x \sim(\sim Q(x) \to \sim P(x))$ is equivalent to $\sim \exists x \sim(\sim(\sim Q(x)) \vee \sim P(x))$

$\sim \exists x \sim(\sim(\sim Q(x)) \vee \sim P(x)) = \sim \exists x \sim(Q(x)) \vee \sim P(x))$ by double negation

$= \sim \exists x\, (\sim Q(x)) \wedge P(x))$ by DeMorgan's

$= \forall x\, (Q(x) \vee \sim(P(x)))$ by DeMorgan's for quantifiers

$= \forall x\, ((\sim(P(x))) \vee Q(x))$ by commutative

$= \forall x\, (P(x) \to Q(x))$ by implication

Thus, $\forall x\, (P(x) \to Q(x)) \equiv \sim \exists x \sim(\sim Q(x) \to \sim P(x))$

5.

True or false: For the set of all negative integers, $\exists x\, (x + 1 < - x)$

true

6.

Use direct method to prove that given any two rational numbers p and q with p < q, there is another rational number between p and q.

Suppose that p and q are any given rational: p=a/b , q=c/d where a/b<c/d

So if there is a rational number between p and q, then that number = x where

x= (p+q)/2. So, p < x < q. X is equivalent to ((a/b) + (c/d))/2 = (ad + cb)/2db which is rational. Therefore, x=(p+q)/2 is rational and is between p and q.

7.

Prove by contraposition that for all integers m and n, if m + n is even then m and n are both even or m and n are both odd.

P= m + n is even

Q= m and n are both even or m and n are both odd

$P \rightarrow Q$

Assume contraposition. $\neg Q \rightarrow \neg P$

$\neg Q$ = m and n are not both odd or even

$\neg P$ = m + n is odd

"If m and n are not both odd or even, then m + n is odd"

Assume $\neg Q$

m or n are odd and even: $m=2k+1$ and $n=2k$

So, $(2k+1) + (2k) = (4k+1)$ = odd

Since $(4k+1)$ = odd, $\neg Q \rightarrow \neg P$ is true. Thus $P \rightarrow Q$ is true by contraposition.