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Practice Exam
11/5/17
Logic Symbols: ≥ ≤ ≠ ¬ ⋀ ⋁ ⊕ ≡ → ↔ ∃ ∀
Set Symbols: ∈ ∉ ⊆ ⊂ ⊇ ⊃ ∅ ∪ ∩ ×

1

a) if it snows, then they do not drive the car.

P =it snows
Q=they drive the car

P→¬Q = ¬P⋁¬Q. When negated is: P⋀Q

If it snows and they drive the car.

b) Only if Susan studies well she will pass the test.
P=she will pass the test.
Q= Susan studies well

Rewritten as: "If susan passes the test, then she studies well."

P→Q = ¬P⋁Q. When negated: P⋀¬Q

Susan passes the test, but she doesn't study well.

c) My car is in the repair shop and I can't get to class.

P=My car is in the repair shop
Q=I can get to class

P⋀¬Q when negated is ¬P⋁Q :

My car is not in the repair shop or I can get to class.

d) He swims if and only if the water is warm.

P=He swims
Q=if the water is warm

P↔Q = (P→Q) ⋀ (Q→P). When negated: (P⋀¬Q) ⋁ (Q⋀¬P)

He swims and the water is warm or the water is warm if he does not swim.

2

1) If the sun is shining and it is Friday, then the basketball game will not take place.

2) If the basketball game will take place, then the sun is shining or it is Friday.

3) It is not the case that the sun is shining or it is Friday.

4) The basketball game will not take place if and only if the sun is not shining.

3

P	Q	R	$(Q \to R)$	$(P \land Q)$	$(P \to (Q \to R))$	$((P \land Q) \to R)$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	F	T	T
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

They share the same true table values so they are logically equivalent.

4

a) Not all female are good athletes.

$\exists x\ B(x) \land \neg W(x)$

b) Some female are not good athletes but young.

$\exists x\ B(x) \land \neg S(x) \land \neg(W(x)$

c) If someone is a female then she is young or a good athlete.

$\forall x\ B(x) \to S(x) \lor (W(x)$

d) There is someone who is a good athlete and female.

$\exists x\ W(x)\ \land B(x)$

5

If two integers have opposite parity, then their product is even.

Let the two integers be x and y. If they have opposite parity, then x is odd and y is even. Let x = 2k+1 and y=2j. K and J are both integers. We must find if there product is even so X*Y = (2k+1)*(2j). Then 2(2kj+j). Since both k and j are integers, there product is even.

6

Use a proof by contraposition to show that if m+n is a irrational number then either m is irrational or n is irrational.

Let the statement "if m+n is a irrational number then either m is irrational or n is irrational"have p q and r. Where P=m+n is a irrational number, Q=m is irrational, R= n is irrational. Contraposition is ¬Q→¬P, so in this case, ¬(Q∨R)→¬P which is equal to: ¬Q∧¬R→¬P. This means, "If M is not irrational and R is not irrational, then m+n is a rational number". So if m is rational and r is rational, then m+n is rational. This can be proved by setting m=a/b and n=c/d where a, b, c, and d are all integers. Then m+n = a/b + c/d = (ad+bc)/bc. Thus it is proven to be rational.

7

Show by contradiction that Suppose a ∈ Z. If a^2 is even, then a is even

For the sake of contradiction, suppose that a^2 is even and a is not even. Since a is odd then a=2k+1. Then $a^2 = (2k+1)^2 = (4k^2 + 4k + 1)$, thus a^2 is odd. Since the contradiction is proven to be false, the original statement is proven false by contradiction.

8

Let A, B and C be sets.

Prove that (A-B) - C ⊆ A – (B–C)

Proof: Prove that (A-B) - C ⊆ A – (B–C) . By the definition of difference we know that x ∈A and x not∈B. Again by the definition of difference we know that x ∈A and x not ∈C. On the right hand side, by the definition of difference x ∈B and x not ∈C. And again by the definition of difference, x ∈A and x not ∈B. Since x ∈A on the left and right side, then we can conclude that (A-B) - C ⊆ A – (B–C).

9

a) ((1/j)-(1/j+1)) = ((1/1)-(1/1+1)) + ((1/2)-(1/2+1)) + ((1/3)-(1/3+1)) + ((1/4)-(1/4+1)) + ((1/j)-(1/5+1)) = 1- (1/(5+1)) = 5/6

b) $4(2^i) - 4(2^i)$ *now use geometric formula* = $4((2^{i+1}-1)/(2-1)) - 4((2^{1+1}-1)/(2-1))$ =

$4((2^5-1)/(2-1)) - 4((2^2-1)/(2-1)) = 4(63 – 3) = 4(60)= 240$

10

Using weak induction prove that For all n є N , $n(n^2 + 5)$ is a multiple of 6.

Proof: For all n є N , $n(n^2 + 5)$ is a multiple of 6

Basis: 0(0+5)=6*0

Induction:

Inductive hypothesis: $n(n^2+5)$ is a multiple of 6.

Inductive steps:

Show that $(n+1)((n+1)^2+5)$ is a multiple of 6.

This equals: $(n+1)((n+1)(n+1) + 5) = (n+1)(n^2+2n+1+5)$

$=(n+1)(n^2+5) + (n+1)(2n+1)$

$=n(n^2+5) + (n^2+ 5) + (n+1)(2n+1)$

$= n(n^2+5) + (n^2+ 5) + (2n^2+3n+1)$

$= n(n^2+5) + (3n^2+3n+6)$

$= n(n^2+5) + 3(n^2+n) +6$

Using our inductive hypothesis, we know that $n(n^2+5)$ is a multiple of 6. To prove that $3(n^2+n)$ is also a multiple of 6, suppose that n=2. This equals 6. An odd number like n=3 is equal to 12. So both odd and even numbers are multiples of 6. Thus the proof is proven.