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Assignment 3.2

pgs 336-348. Set 6.1 - 3, 7, 13, 18, 33, 34

3. Let sets R, S, and T be defined as follows:

$$R = \{x \in \mathbf{Z} \mid x \text{ is divisible by } 2\}$$
$$S = \{y \in \mathbf{Z} \mid y \text{ is divisible by } 3\}$$
$$T = \{z \in \mathbf{Z} \mid z \text{ is divisible by } 6\}$$

a. Is $R \subseteq T$? Explain.
b. Is $T \subseteq R$? Explain.
c. Is $T \subseteq S$? Explain.

a) No, R is not in T because there are parts of R that are not in T. Simply, 2 is in R, but 2 is not in T because 2 cannot be divided by 6.

b) Yes, T is in R because there are parts of T that are in R. Simply, 6 is in T and is divisible by 2, thus T is a subset of R.

c) Yes, T is in S because there are parts of T that are in S. Simply, 6 is in T and is divisible by 3, thus T is a subset of S.

7. Let $A = \{x \in \mathbf{Z} \mid x = 6a + 4 \text{ for some integer } a\}$,
$B = \{y \in \mathbf{Z} \mid y = 18b - 2 \text{ for some integer } b\}$, and
$C = \{z \in \mathbf{Z} \mid z = 18c + 16 \text{ for some integer } c\}$.
Prove or disprove each of the following statements.
a. $A \subseteq B$ b. $B \subseteq A$ c. $B = C$

a) Suppose that x is a particular but arbitrarily chosen element of A
If x is an element in A and A is a subset of B, then x is an element of B as long as x is an integer.

$6a + 4 = 18a – 2$
$6a + 6 = 18a$
$a + 1 = 3a$
$1 = 2a$
$a = ½$

Thus $A \not\subset B$ because a is not an integer.

b) Suppose that x is a particular but arbitrarily chosen element of B.
If x is an element in B and B is a subset of A, then x is an element of B as long as x is an integer.

Let $x = a = 3a – 1$
$6a + 4 = 18a – 2$
$6(3a – 1) + 4 = 18a – 2$
$18a – 2 = 18a – 2$

Thus x is an element of B and $B \subseteq A$

c) Suppose that B = C

$18b – 2 = 18c + 16$ we must test each way.

$B \subseteq C$

If $b = 1c + 1$
$18b – 2 = 18c + 16$
$18(1c + 1) – 2 = 18c + 16$
$18c + 16 = 18c + 16$

So $18b – 2 = 18(1c + 1) – 2 = 18c + 16$, then by definition of B, b is an element of B and a subset of C

For, C⊆B
If c = 1b – 1
18b – 2 = 18c + 16
18b – 2 = 18(1b – 1) + 16
18b – 2 = 18b -2

So 18c + 16 = 18(1b – 1) + 16 = 18b – 2, then by definition of C, c is an element of C and a subset of B

Since B⊆C is true and C⊆B is true, then B = C is true.

13. Indicate which of the following relationships are true and which are false:

a. $\mathbf{Z}^+ \subseteq \mathbf{Q}$
b. $\mathbf{R}^- \subseteq \mathbf{Q}$
c. $\mathbf{Q} \subseteq \mathbf{Z}$
d. $\mathbf{Z}^- \cup \mathbf{Z}^+ = \mathbf{Z}$
e. $\mathbf{Z}^- \cap \mathbf{Z}^+ = \emptyset$
f. $\mathbf{Q} \cap \mathbf{R} = \mathbf{Q}$
g. $\mathbf{Q} \cup \mathbf{Z} = \mathbf{Q}$
h. $\mathbf{Z}^+ \cap \mathbf{R} = \mathbf{Z}^+$
i. $\mathbf{Z} \cup \mathbf{Q} = \mathbf{Z}$

a) True
b) False
c) False
d) False
e) False
f) True
g) True
h) True
i) False

Figure 6.1.3

18. a. Is the number 0 in $\emptyset$? Why?
b. Is $\emptyset = \{\emptyset\}$? Why?
c. Is $\emptyset \in \{\emptyset\}$? Why?
d. Is $\emptyset \in \emptyset$? Why?

a) There are no elements that exists within ∅ so 0 is not there.
b) No ∅ = { ∅ } because the element ∅ and the set { ∅ } are two different things.
c) Yes, ∅ is an element in the set { ∅ } and can be mapped to ∅.
d) No because they are both elements not given a set.

33. a. Find $\mathscr{P}(\emptyset)$.
b. Find $\mathscr{P}(\mathscr{P}(\emptyset))$.
c. Find $\mathscr{P}(\mathscr{P}(\mathscr{P}(\emptyset)))$.

a) P ∅ = { ∅}

b) P(P(∅)) = P({∅}) = { ∅, {∅}}
c) P (P(P(∅))) = P { ∅, {∅}} = { ∅, {∅}, {∅}}

34. Let $A_1 = \{1, 2, 3\}$, $A_2 = \{u, v\}$, and $A_3 = \{m, n\}$. Find each of the following sets:

a. $A_1 \times (A_2 \times A_3)$ b. $(A_1 \times A_2) \times A_3$
c. $A_1 \times A_2 \times A_3$

a) A1 * (A2 * A3) = {(1(u, m)), (1(u, n)), (1(v, m)), (1(v, n)), (2(u, m)), (2(u, n)), (2(v, m)), (2(v, n)), (3(u, m)), (3(u, n)), (3(v, m)), (3(v, n))

b) (A1 * A2) * A3 = {((1, u)m), ((1, v)m), ((1, u)n), ((1, v)n), ((2, u)m), ((2, v)m), ((2, u)n), ((2, v)n), ((3, u)m), ((3, v)m), ((3, u)n), ((3, v)n),

c) A1 * A2 * A3 = {(1, u, m), (1, v, m), (1, u, n), (1, v, n), (2, u, m), (2, v, m), (2, u, n), (2, v, n), (3, u, m), (3, v, m), (3, u, n), (3, v, n)