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Quiz over Week 6 and 7

1
$d(0) = 2$

$dk = 2d_{k-1}+4$, for all integers $k>=1$

$d(0) = 2$

$d(1) = 2(d(0)) + 4 = 2(2) + 4 = 2^2 + 4$

$d(2) = 2(d(1)) + 4 = 2(2^2 + 4) + 4 = 2^3 + 2*4 + 4$

$d(3) = 2(d(2)) + 4 = 2(2^3 + 2*4 + 4) + 4 = 2^4 + 2^2*4 + 2*4 + 4$

$d(4) = 2(d(3)) + 4 = 2(2^4 + 2^2*4 + 2*4 + 4) + 4 = 2^5 + 2^3*4 + 2^2*4 + 2*4 + 4$

$d(5) = 2(d(4)) + 4 = 2(2^5 + 2^3*4 + 2^2*4 + 2*4 + 4) + 4 =$

$\mathbf{2^6 + 2^4*4 + 2^3*4 + 2^2*4 + 2*4 + 4}$

Guess:

$d(n) = 2^{n+1} + 2^{n-1}*4 + 2^{n-2}*4 + \ldots\ldots\ldots + 2^2*4 + 2*4 + 4$

$= 2^{n+1} + 4\sum(i=0, n-1)\ 2^i$

$= 2^{n+1} + 4 * (2^n-1)/(2-1)$

$=2^{n+1} + 4 * (2^n-1)$

$=2^{n+1} + 4*2^n – 4$

$=2*2^n + 4*2^n – 4$

$= 6*2^n – 4$

2

a

Basis Step: $6 \in S$

Recursive Step: If $x, y \in S$, then $x+y \in S$

b

Basis Step: $\lambda \in S$

Recursive Step: If $c \in S$, then $00c \in S$, $01c \in S$, $10c \in S$ and $11c \in S$.

3

We must prove that every possible integer that exists in S will be divisible by 5.

So our base case is,

Base Case: $0 \in S$, $5 \in S$. When we test this, both 0 and 5 are divisible by 5, so our base case is true.

Now we must prove or Recursive step using structural induction,

Inductive Case: We must assume that both s and $t \in S$. Thus s and t are both divisible by 5 by the base case. To prove this case, we can define s and t as s=5a and t=5b. To prove (a), we set 5a+5b. This is equal to 5(a+b) which is divisible by 5. Thus, $s + t \in S$ and is proven to be true. To prove (b), we set 5a-5b. This is equal to 5(a-b) which is divisible by 5. Thus, $s - t \in S$ and is proven to be true.

Therefore all integers that are in S are divisible by 5.

4

a

We use the pigeonhole principle to solve this. Logically, if there are a greater number of people than days in the year, then there will be those who share birthdays. Given that our group is 2000 people, we must determine if there are at least 5 that have the same birthday. By applying the pigeonhole principle, k < 2000/365, we get k = 5.48. So k+1 or more objects or in this case people will be equal to 6. Thus, there are at least 6 people from the group of 2000 that share the same birthday and therefore proves that at least 5 also have the same birthday.

b

Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

{5, 6}, {4, 7}, {3, 8}, {2, 9}, {1, 10}

There are 5 sets of pairs whose sum is 11. If we are to choose 7 integers from our set S, then we can use the pigeonhole principle to determine if at least 2 pairs of them will have a sum of 11. So for 7 integers or pigeons, we have to fit them into 5 sets that sum to 11 or pigeonholes. Thus, at least two of the 7 integers must have a sum that is 11.

5

a

Lets check how many integers are multiples of 2 from 1 through 100 first. So every second integer is a multiple of 2 so each can be represented as 2k for some integer k from 1 to 50. So there are 50 multiples of 2 in this range.

Now lets check how many integers are multiples of 9 from 9 to 99. So every 9^{th} integer can be represented in the form 9m for some integer 1 to 11. Thus there are 11 multiples of 9.

Before we calculate the final answer we must consider numbers that are both a multiple of 2 and 9. So when 9 is also a multiple of 2 between 9 and 99 exactly 5 times.

So if there are 50 multiples of 2 in the range of 1 through 100 and 11 multiples of 9 in the range of 1 through 100 minus the overlap of 9 also being a multiple of 2 we get:

50 + 11 – 5 = 56

We want the numbers between 1 and 100 that are not multiples of 2 or 9 so:

100-56 = 44. Thus, 44 integers from 1 through 100 are neither multiples of 2 or 9.

b

Since the string is of length 4, we must create 4 difference cases to determine the answer.

Case 1: One s – C(4,1) * 25^3

Case 2: Two s' – C(4,2) * 25^2

Case 3: Three s' - C(4,3) * 25^1

Case 4: Four s' – The only case is that the string is 'ssss'

Thus we can combine the product and sum rule to get:

C(4,1) * 25^3 + C(4,2) * 25^2 + C(4,3) * 25^1 + 1

= 4 * 15625 + 6 * 625 + 4 * 25 + 1 = 66351 strings of four lowercase letters that have s in them.