Devin Gendron

Set 2.1- 22, 42, 45

Determine whether the statement forms in 16–24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

22. p∧(q ∨r) and (p∧q)∨(p∧r)

p	q	r	(q ∨r)	(p∧q)	(p∧r)	**p∧(q ∨r)**	**(p∧q)∨(p∧r)**
T	T	T	T	T	T	**T**	**T**
T	T	F	T	T	F	**F***	**T***
T	F	T	T	F	T	**T**	**T**
T	F	F	F	F	F	**F**	**F**
F	T	T	T	F	F	**F**	**F**
F	T	F	T	F	F	**F**	**F**
F	F	T	T	F	F	**F**	**F**
F	F	F	F	F	F	**F**	**F**

- p∧(q ∨r) and (p∧q)∨(p∧r) have different truth table values, so they are not logically equivalent. If the values indicated by the '*' were the same, then p∧(q ∨r) and (p∧q)∨(p∧r) would be logically equivalent.

Use truth tables to establish which of the statement forms in 40–43 are tautologies and which are contradictions.

42. ((~p∧q)∧(q ∧r))∧~ q

p	q	r	~p	~ q	(~p∧q)	(q ∧r)	**((~p∧q)∧(q ∧r))∧~ q**
T	T	T	F	F	F	T	**F**
T	T	F	F	F	F	F	**F**
T	F	T	F	T	F	T	**F**
T	F	F	F	T	F	F	**F**
F	T	T	T	F	T	T	**F**
F	T	F	T	F	T	F	**F**
F	F	T	T	T	F	F	**F**
F	F	F	T	T	F	F	**F**

-All F's so ((~p∧q)∧(q ∧r))∧~ q is a contradiction.

In 44 and 45, determine whether the statements in (a) and (b) are logically equivalent.

45.

a. Bob is a double math and computer science major and Ann is a math major, but Ann is not a double math and computer science major.

b. It is not the case that both Bob and Ann are double math and computer science majors, but it is the case that Ann is a math major and Bob is a double math and computer science major.

p = Bob is a double math major and computer science major

q = Ann is a math major

r = Ann is a double math major and computer science major

a) $\mathbf{p \wedge q \wedge \sim r}$

p	q	r	$\sim r$	$\mathbf{p \wedge (q \wedge \sim r)}$
T	T	T	F	**F**
T	T	F	T	**T**
T	F	T	F	**F**
T	F	F	T	**F**
F	T	T	F	**F**
F	T	F	T	**F**
F	F	T	F	**F**
F	F	F	T	**F**

b) $\mathbf{\sim(p \wedge r) \wedge q \wedge p}$

p	q	r	$(p \wedge r)$	$\sim(p \wedge r)$	$\mathbf{\sim(p \wedge r) \wedge q \wedge p}$
T	T	T	T	F	**F**
T	T	F	F	T	**T**
T	F	T	T	F	**F**
T	F	F	F	T	**F**
F	T	T	F	T	**F**
F	T	F	F	T	**F**
F	F	T	F	T	**F**
F	F	F	F	T	**F**

-Statements (a) and (b) are logically equivalent.

Set 2.2 - 11, 13.b , 15, 20, 38, 41, 43, 45

Construct truth tables for the statement forms in 5–11.

11. $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

p	**q**	**r**	**$(q \rightarrow r)$**	**$(p \wedge q)$**	**$(p \rightarrow (q \rightarrow r))$**	**$((p \wedge q) \rightarrow r)$**	**$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$**
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**F**	**F**	**F**
T	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**T**
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**T**	**T**

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.

b. $\sim(p \rightarrow q) \equiv p \wedge \sim q$.

p	**q**	**$\sim q$**	**$(p \rightarrow q)$**	**$\sim(p \rightarrow q)$**	**$p \wedge \sim q$**
T	**T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**

$\sim(p \rightarrow q)$ and $p \wedge \sim q$ have the same truth table values, so they are logically equivalent.

15. Determine whether the following statement forms are logically equivalent:

$p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$

p	q	r	$(q \rightarrow r)$	$(p \rightarrow q)$	**$p \rightarrow (q \rightarrow r)$**	**$(p \rightarrow q) \rightarrow r$**
T	T	T	T	T	**T**	**T**
T	T	F	F	T	**F**	**F**
T	F	T	T	F	**T**	**T**
T	F	F	T	F	**T**	**T**
F	T	T	T	T	**T**	**T**
F	T	F	F	T	**T**	**F**
F	F	T	T	T	**T**	**T**
F	F	F	T	T	**T**	**F**

- $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$ are not logically equivalent because their truth table values are not equivalent.

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

**The negation of an if-then statement does not start with the word if!*
$\sim(p \rightarrow q) \equiv p \wedge \sim q$

a. If P is a square, then P is a rectangle.

Ans: P is a square and P is not a rectangle.

b. If today is New Year's Eve, then tomorrow is January.

Ans: Today is New Year's Eve and tomorrow is not January.

c. If the decimal expansion of r is terminating, then r is rational.

Ans: The decimal expansion of r is terminating and r is not rational.

d. If n is prime, then n is odd or n is 2.

Ans: n is prime and n is neither odd or 2.

e. If x is nonnegative, then x is positive or x is 0.

Ans: X is nonnegative and x is neither positive or 0.

f. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

Ans: Tom is Ann's father and Jim is not her uncle and Sue is not her aunt.

g. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Ans: n is divisible by 6 and n is neither divisible by 2 or 3.

In 37–39, rewrite the statements in if-then form.

38. Ann will go unless it rains.

Ans: If it does not rain, then Ann will go.

Using: *If r and s are statements,* ***r unless s*** *means if ~s then r.*

Rewrite the statements in 40 and 41 in if-then form.

41. Having two 45◦ angles is a sufficient condition for this triangle to be a right triangle.

Ans: If two 45◦ angles are present, then it is a right triangle.

Use the contrapositive to rewrite the statements in 42 and 43 in if-then form in two ways.

43. Doing homework regularly is a necessary condition for Jim to pass the course.

Ans1: If Jim does not pass the course, then he didn't do the homework regularly.

Ans2: If Jim does his homework regularly, then he will pass the course.

> Using: *If p and q are statements,*
>
> *p* ***only if*** *q means "if not q then not p,"*
>
> *or, equivalently, "if p then q."*

Note that "a sufficient condition for s is r" means r is a sufficient condition for s and that "a necessary condition for s is r" means r is a necessary condition for s. Rewrite the statements in 44 and 45 in if-then form.

45. A necessary condition for this computer program to be correct is that it not produce error messages during translation.

Ans: If the computer program does not produce error messages during translation, then the program is correct.

> *Using: "A necessary condition for s is r = r is a necessary condition for s". Thus, "if r, then s".*