Devin Gendron
Set 5.9 - 6, 10, 13.b, 16

6. Define a set S recursively as follows:
   I. BASE: $a \in S$
   II. RECURSION: If $s \in S$, then,
   a. $sa \in S$ b. $sb \in S$
   III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

   Use structural induction to prove that every string in S begins with an a.

(1)By I, a ∈ S
(2)By (1) and II(a), sa ∈ S. But sa is the concatenation of a, which equals a. So a ∈ S.
(3)By (2) and II(b), ab ∈ S and every string in S begins with an a.

H 10. Define a set S recursively as follows:
   I. BASE: $0 \in S$, $5 \in S$
   II. RECURSION: If $s \in S$ and $t \in S$ then
   a. $s + t \in S$ b. $s - t \in S$
   III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

   Use structural induction to prove that every integer in S is divisible by 5.

Assume that s and t are both divisible by 5. The integers p and q can represent this divisibility such that s=5*p and t=5*q. Thus:

(1) s+t= 5*p + 5*q = 5(p+q)
(2) s-t = 5*p – 5*q = 5(p-q)
(3) so, both s+t and s-t are divisible by 5.

13. Consider the set P of parenthesis structures defined in Example 5.9.4. Give derivations showing that each of the following is in P.

a. ()(()) b. (())(())

b.

(1) By I, () is in P.
(2) By (1) and II(a), (()) is in P.
(3) By (2), (2), and II(b), (())(()) is in P.

16. Give a recursive definition for the set of all strings of 0's and 1's for which all the 0's precede all the 1's.

Let S be the set of all strings of 0's and 1's where all the 0's precede all the 1's.

I. Basic Clause: $0 \in S$, $1 \in S$

II. Recursion: If $s \in S$ then:

a. $s0 \in S$ b. $s1 \in S$

III. Restriction: Nothing is in S other than objects defined in I and II above.