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Reading: 268–274
Assignment: Set 5.4 – 2, 10

2. Suppose $b_1, b_2, b_3, \ldots$ is a sequence defined as follows:

$$b_1 = 4, \ b_2 = 12$$
$$b_k = b_{k-2} + b_{k-1} \quad \text{for all integers } k \geq 3.$$

Prove that b_n is divisible by 4 for all integers $n \geq 1$.

Proof:

Let b_1, b_2, b_3,… be the sequence defined by specifying that b_1 = 4, b_2 = 12, and b_k = b_{k-2} + b_{k-1} for all integers k>=1 and is divisible by 4. 4/4 is true and 4/12 is true. So b_1 and b_2 are both true.

Since b_{k-2} is divisible by 4 and b_{k-1} is divisible by 4 and it is the sum of two numbers that are divisible by 4, b_k is also divisible by four. Thus b_n is divisible by 4 for all integers n>=1 as was to be shown.

***H* 10.** The problem that was used to introduce ordinary mathematical induction in Section 5.2 can also be solved using strong mathematical induction. Let $P(n)$ be "any collection of n coins can be obtained using a combination of 3¢ and 5¢ coins." Use strong mathematical induction to prove that $P(n)$ is true for all integers $n \geq 14$.

For the inductive step, note that $k+1=$

$[(k+1)-3]+3$, and if $k \geq 16$, then $(k+1)-3 \geq 14$.

Proof (by strong mathematical induction): Let the property P(n) be the sentence:

"any collection of n coins can be obtained using a combination of 3cent and 5 cent coins."

Show that P(14), P(15), and P(16) is true:

P(14) = 3 cent + 3 cent + 3 cent + 5 cent = 14 cents = true
P(15) = 5 cent + 5 cent + 5 cent = 15 cents = true
P(16) = 3 cent + 3 cent + 5 cent + 5 cent = 16 cents = true

Thus, P(14), P(15), and P(16) are all true.

Show that for any integer k>= 14, if P(i) is true for all integers i with 14 <= i <= k, then P(k+1) is true:

Suppose that P(k) is true for a particular but arbitrarily chosen integer k>=14. That is, suppose that k is any integer with k>=14 such that:

Kcents can be obtained using 3cent and 5cent coins

We must show that:

(k+1)cents can be obtained using 3cent and 5 cent coins.

Case 1: There is a 5cent coin among those used to make up the kcent:

In this case replace the 5 cent by two 3cent coins, the result will be (k+1)cents.

Case 2: There is not a 5cent coin among those used to make up the kcent:

In this case, because k>=14, at least five 3cent coins must have been used. So remove two 3cent coins and replace them by one 5cent coin, the result will be (k+1)cents.

Thus in either case (k+1)cents can be obtained using 3cent and 5cent coins – as was to be shown.