Devin Gendron
Set 4.6 - pgs 198-204, 207-211 - #'s 12, 16, 28

Prove Each Statement By Contradiction

12. If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then $a + br$ is irrational.

P = a and b are rational numbers
Q = r is an irrational number
R = a + br is irrational

Assume for the sake of contradiction:

If $P \wedge Q \to R$ is negated to: $P \vee Q \wedge \neg R$ = "a and b are rational, r is irrational, and a+br is rational.

a = (x/y) , b = (w/z) , r = √t thus,

a + br is == (x/y) + (w/z)*(√t)

= (x/y) + (w(√t)/z)
= (xz + wy√t)/yz

= thus a+br is irrational and the original statement "if a and b are rational numbers, b≠0, and r is an irrational number, then a+br is irrational" is true and proven by contradiction.

□

H ✶16. For all odd integers a, b, and c, if z is a solution of $ax^2 + bx + c = 0$ then z is irrational. (In the proof, use the properties of even and odd integers that are listed in Example 4.2.3.)

P= z is a solution of ax^2 + bx + c = 0
Q= z is irrational

P→Q

Assume for the sake of contradiction. So, the contradiction of If P then Q is:

¬(P→Q) = P∧¬Q so the negations are:

P = z is a solution of ax^2 + bx + c = 0
Q = z is rational

So if z is rational, z=d/e. If by substituting in z we get:

a(d/e)^2 + b(d/e) + c = 0

= (ad^2)/(e^2) + (bd/e) + c = 0
=a * d^2 + b * d * e + c * e^2 = 0

There can be multiple cases for d and e.

Case 1: For d is odd and e is even: a * d^2 is an odd portion of the equation, b * d * e is an even portion of the equation, and c * e^2 is an even portion of the equation. Thus, the equation is odd and is a contradiction.

Case 2: For d is even and e is odd: a * d^2 is an even portion of the equation, b * d * e is an odd portion of the equation, and c * e^2 is an odd portion of the equation. Thus, the equation is odd and is a contradiction.

Case 3: For d and e are both odd: a * d^2 is an odd portion of the equation, b * d * e is an odd portion of the equation, and c * e^2 is an odd portion of the equation. Thus, the equation is odd and is a contradiction.

□

Prove Each Statement by (a) contraposition and (b) contradiction

28. For all integers m and n, if mn is even then m is even or n is even.

a) contraposition:

P = mn is even
Q = m is even or n is even

$P \rightarrow Q$

Suppose that $P \rightarrow Q$ is negated. So, the contrapositive of If P then Q is:

$\neg Q \rightarrow \neg P$ so there negations are:

$\neg Q$ = m is odd and n is odd , thus $m = (2k + 1)$ and $n = (2k + 1)$
$\neg P$ = mn is odd

so, $mn = (2k + 1)(2h + 1) = 4kh + 2h + 2k + 1 = 2(2kh + h + k) + 1$

Hence, mn is odd and $\neg Q \rightarrow \neg P$ is proven. Thus, $P \rightarrow Q$ is proven by contraposition.

b) contradiction:

P = mn is even
Q = m is even or n is even

$P \rightarrow Q$

Assume for the sake of contradiction. So, the contradiction of If P then Q is:

$\neg(P \rightarrow Q) \quad = \quad P \wedge \neg Q$ so the negations are:

P = mn is even
$\neg Q$ = m is odd and n is odd , thus $m = (2k+1)$, $n = (2h+1)$, $mn = 2k$

so, $mn = (2k + 1)(2h + 1) = 4kh + 2h + 2k + 1 = 2(2kh + h + k) + 1$, which is odd.

Thus, $mn = 2(2kj + h + k) + 1$. This contradicts shows that our supposition is false, hence, $P \rightarrow Q$ is true.