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CS225 Final Exam
Logic Symbols: ≥ ≤ ≠ ¬ ⋀ ⋁ ⊕ ≡ → ↔ ∃ ∀ Set Symbols: ∈ ∉ ⊆ ⊂ ⊇ ⊃ ∅ ∪ ∩ ×

Q1:

P=a is rational and ab is irrational
Q=b is irrational

By contradiction:

P= a is rational and ab is irrational
¬Q=b is rational

Thus: “If a is rational and ab is irrational, then b is rational”.

So by definition of a rational integer, a=m/n and b=p/q. We can then test this:

$ab=(m/n)(p/q)$

$=mp/nq$

As we can see, mp/nq is rational and proves by contradiction that the original statement, “If a is rational and ab is irrational, then b is irrational”, is true.

Q2:

P(n) : For every integer n greater than 1, $n! < n^n$

Base Case: Test P(n) with an n greater than 1. So,

P(2): $2! = 2*1 = 2$
$2^2=4$

This proves the base case P(2).

Inductive Case: If our base case is true, then P(k) is also true. If P(k) is true, then we can check to see if P(k+1) is true.

$(k+1) < (k+1)^{(k+1)}$. As can be seen, any integer (k+1) will be less than $(k+1)^{(k+1)}$ for any integer n>1. For example, P(2):

$(2+1) < (2+1)^{(2+1)} = 3<3^3 = 3<27$

Thus, proven by mathematical induction.

Q3:
Let A and B be sets with $A \subseteq B$. Let x be an element in $A \cap B$. By the definition of an intersection, x is an element in A and x is an element in B. Since A intersects with B in $A \cap B$, then x is also an element in A on the right-hand side. Thus, proving that $A \cap B = A$.

Q4:

$d_k = 3d_{k-1} + 2$, for all integers $k \geq 2$

d1 = 3
d2 = 3(d1) + 2 = 3(3) + 2 = $3^2 + 2$ =11
d3 = 3(d2) + 2 = $3(3^2 + 2) + 2 = 3^3 + 3*2 + 2 = 35$
d4 = 3(d3) + 2 = $3(3^3 + 3*2 + 2) + 2 = 3^4 + 3^2*2 + 3*2 + 2$= 107

Guess:

$3^4 + 3^2*2 + 3*2 + 2$

$= 3^n + 3^{n-2}*2 + 3^{n-3}*2 + 2$

$= 3^n +(\sum k=0, n-2)\ 3^i$

$= 3^n + ((3^{n-2+1} - 1)/(3-1))$

$=3^n + ((3^{n-1} - 1)/2)$

$=((2*3^n)/2) + ((3^{n-1} - 1)/2)$

Q5:
Base step: According to I, a ∈ S. By II, s ∈ S so sa and sb will be elements of S.

Inductive step: According to (I), a ∈ S, so by recursion, (II.a) sa will begin the string, followed by (II.b) sb. Since both sa and sb are elements of S, and a is an element of s, every string will begin with a and then followed by b. This is within the means of the restrictions, thus proving that every string S begins with an a.

Q6:

a. There are 900 integers between 100 and 999. To determine how many 3-digit integers between 100 and 999 are divisible by 5, we check:

[ ] [ ] [ ] = box 1 = 1-9 since it cannot be a zero = 9!
Box 2 = 0-10 = 10!
Box 3 = 0-2 since only two integers are divisible by 5 = 2!

Thus: 9! * 10! * 2! = the number of 3-digit integers divisible by 5.

b. {3, 12}, {4, 11}, {5, 10}, {6, 9}, {7, 8} = 5 sets of integers that could equal 15.
By the pigeon theorem, we can check if there must be two integers whose sum is 15.
So, there are 6 objects (pigeons) and 5 sets equaling 15 (holes). Thus for at least one hole, there will be 2 pigeons, so there must be two integers whose sum is 15.

Q7: "MANUSCRIPT" = 10 letters. If the letters US must be next to each other, then their new ordering will be: "MAN[US]CRIPT" = 9 letters.

9!=ans.

Q8:

7 types
40 batteries
including 4 batteries of a7b
40-4=36

C(n + r -1, n)

C(36 + 7 -1, 36) = C(42, 36) = 42!/36!(42-36)! = ans

Q9:
There cannot be a simple graph that have differing degrees. For example, if n=4, and the degree of each vertex is: 1, 2, 3, and 4, then the additive of them is 10e. When divided by 2, this equals

5 which is odd thus proving that a simple graph that has n vertices all of different degrees cannot be.

Q10:
This graph is proven to have a Euler Circuit. No edges are repeated, but vertices are repeated. This is a closed circuit. One method of the circuit can be completed as:

F->G->A->B->C->E->F->A->D->C->F

This shows that we begin and end at the same vertex without repeating any edges, only vertices, and is thus a Euler Circuit.