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Logic Symbols: ≥ ≤ ≠ ¬ ∧ ∨ ⊕ ≡ → ↔ ∃ ∀

Set Symbols: ∈ ∉ ⊆ ⊂ ⊇ ⊃ ∅ ∪ ∩ ×

1

Use proof by contradiction to show that If a and b are rational numbers with b ≠ 0 and x is an irrational number, then a + bx is irrational.

P=a and b are rational numbers (b!=0) and x is an irrational number

Q=a + bx is irrational

P→Q

Assume that this isn’t the case. By contradiction:

P∧¬Q

P= a and b are rational numbers (b!=0) and x is an irrational number

¬Q=a + bx is rational

Say, a=w/v and b=y/z where b ≠ 0. Say s = a+bx.

Isolate x:

s=a+bx

s-a=bx

((s-a)/b)=x

a=w/v, b=y/z

((s-(w/v))/(y/z))=x

((s-(w/v)) and (y/z) are both integers since s, w, v, y, and z are all integers and since sums, differences, products, and division of integers are integers. Thus, by definition of a rational, x is rational which contradicts the supposition. Hence, the supposition is false and the original statement is proven true by contradiction.

2

Suppose a ∈  Z. If a^22 is even, then a is even. Use proof by contradiction.

P=a^2 is even

Q=a is even

P→Q

Assume this is not the case. By contradiction:

P∧¬Q

P= a^2 is even

¬Q=a is odd

Suppose x is a particular but arbitrarily chosen element of Z.

x=2k

So, (2k)^2 = 4k^2 which is even for all integers.

Thus, the supposition is false. Hence, by contradiction the original statement is true.

3

Let  A ={x  ∈  Z | x = 5a + 2  for some integer a } and

B ={y  ∈  Z | y = 10b − 3 for some integer b}.

Prove  or disprove that  B  ⊆ A

Suppose that m is a particular but arbitrarily chosen element of B. We need to show that this element m is in A. Thus by the definition of B, we can show that m=10b-3. By the definition of A, we can show that m=5a+2. Say we use integer a, such that

10b-3=5a+2. If we solve for a:

10b-3=5a+2

10b-5=5a

2b-1=a = integer

Then check the definitions for A and B:

5a+2=

=5(2b-1)+2

=10b-5+2

=10b-3

Thus m ∈ A and B  ⊆ A is true.

4

True or false : { φ } = φ

False. One is a set and the other is an element.

5

True or false : Z^- ∩   Z^+  = Z

False, no negative numbers intersect positive numbers

6

True or false: {8} ∈ { 6, 8, 10}

False. Only element 8 not set 8.

7

True or false: {2, 4} ⊆ {x ∈ N | x is even}

True

8

Find the power set of  A, where A = { x ∈  Z |  -4 < x < 2} .

P(x) = {∅,{-3}, {-2}, {-1}, {0}, {1}, {2}, {-3, -2, -1, 0, 1, 2}}

9

Let,  A={3n  ∈  Z | -1≤ n ≤ 2, n  ∈  Z } ,

B={1, {2}}  and

C = {{1,2}}.

Find A X ( B X C) .

A = -3, 0, 3, 6

(BxC) = {(1, {1, 2}), ({2}, {1, 2})}

Ax(BxC) ={(-3, (1, {1, 2})), (-3, ({2}, {1, 2})), (0, (1, {1, 2})), (0, ({2}, {1, 2})), (3, (1, {1, 2})), (3, ({2}, {1, 2})), (6, (1, {1, 2})), (6, ({2}, {1, 2}))