CS261 Data Structures

Hash Tables

Open Address Hashing

Goals

- Open Address Hashing
- Complexity Analysis

Hash Tables: Resolving Collisions

There are two general approaches to resolving collisions:

1. Open address hashing: if a spot is full, probe for next empty spot

2. Chaining (or buckets): keep a collection at each table entry

Open Address Hashing

- All values are stored in an array
- Hash value is used to find initial index to try
- If that position is filled, the next position is examined, then the next, and so on until an empty position is found
- The process of looking for an empty position is termed *probing*, specifically linear probing when we look to the next element

Open Address Hashing: Example

Eight element table using Amy's hash function(alphabet position of the 3rd letter of the name):

Already added: Amina, Andy, Alessia, Alfred, and Aspen

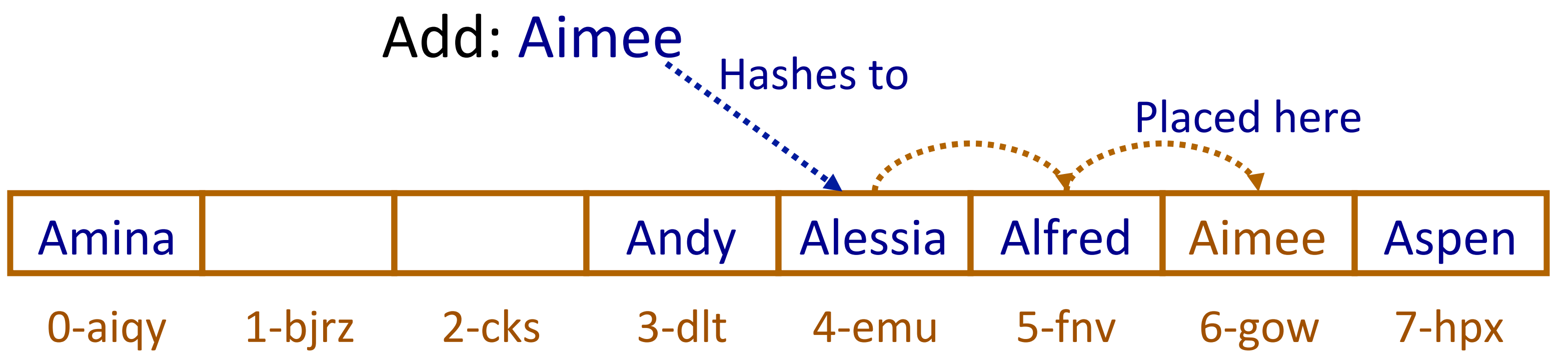
Amina			Andy	Alessia	Alfred		Aspen
0-aiqy	1-bjrz	2-cks	3-dlt	4-emu	5-fnv	6-gow	7-hpx

Note: We've shown where each letter of the alphabet maps to for simplicity here (given a table size of 8) …so you don't have to calculate it!

e.g. Y is the 25th letter (we use 0 index, so the integer value is 24) and 24 mod 8 is 0

Open Address Hashing: Adding

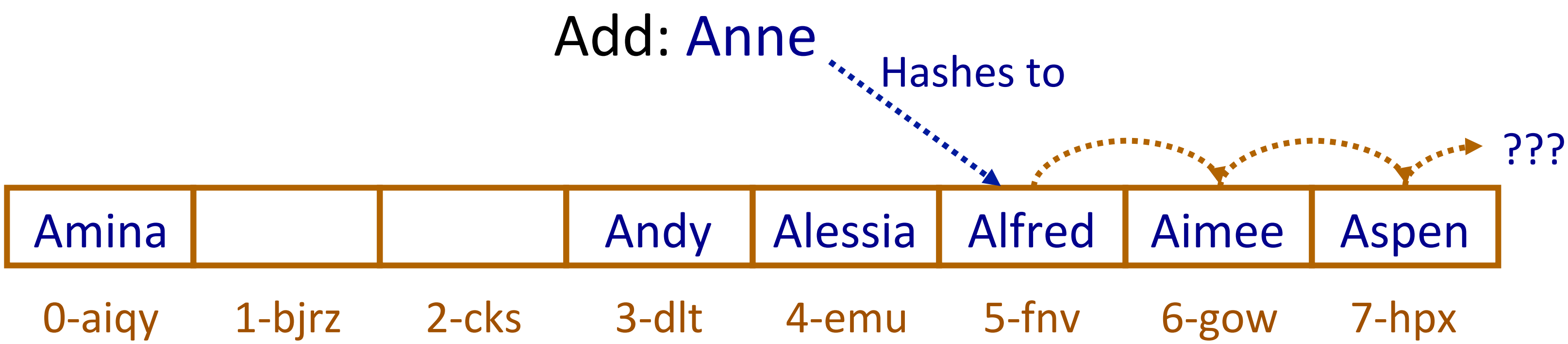
Now we need to add: Aimee



The hashed index position (4) is filled by Alessia: so we ***probe*** to find next free location

Open Address Hashing: Adding (cont.)

Suppose Anne wants to join:

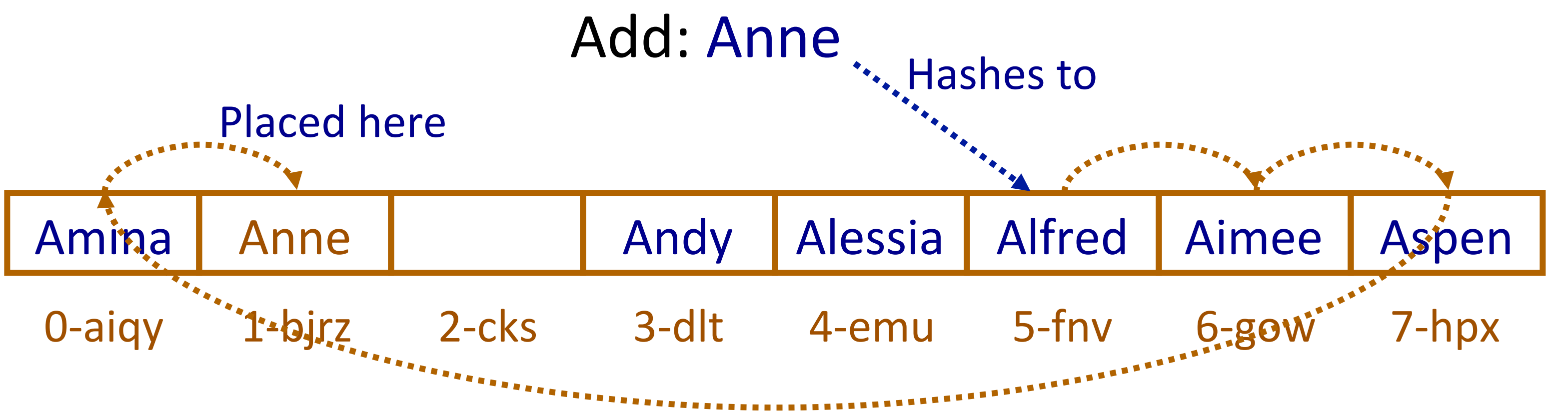


The hashed index position (5) is filled by Alfred:

– Probe to find next free location → what happens when we reach the end of the array

Open Address Hashing: Adding (cont.)

Suppose Anne wants to join:



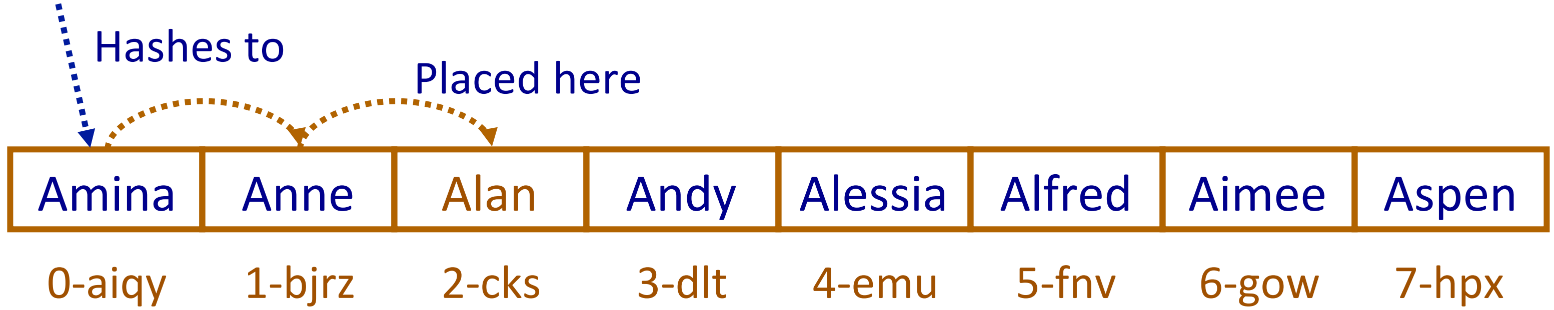
The hashed index position (5) is filled by Alfred:

- Probe to find next free location
- When we get to end of array, wrap around to the beginning
- Eventually, find position at index 1 open

Open Address Hashing: Adding (cont.)

Finally, Alan wants to join:

Add: Alan



The hashed index position (0) is filled by Amina:

- Probing finds last free position (index 2)
- Collection is now completely filled
- What should we do if someone else wants to join? (More on this later)

Open Address Hashing: Contains

- Hash to find initial index
- probe forward until
  - value is found, or (return 1)
  - *empty location is found* (return 0)

Amina			Andy	Alessia	Alfred	Aimee	Aspen
0-aiqy	1-bjrz	2-cks	3-dlt	4-emu	5-fnv	6-gow	7-hpx

- Notice that search time is not uniform

Open Address Hashing: Remove

- Remove is tricky
- What happens if we delete Anne, then search for Alan?

Remove: Anne

Amina	~~Anne~~	Alan	Andy	Alessia	Alfred	Aimee	Aspen
0-aiqy	1-bjrz	2-cks	3-dlt	4-emu	5-fnv	6-gow	7-hpx

Find: Alan

Hashes to

Probing finds null entry → Alan not found

Amina		Alan	Andy	Alessia	Alfred	Aimee	Aspen
0-aiqy	1-bjrz	2-cks	3-dlt	4-emu	5-fnv	6-gow	7-hpx

Open Address Hashing: Handling Remove

- Simple solution: Don’t allow removal (e.g. words don’t get removed from a spell checker!)
- Better solution: replace removed item with “tombstone”
  - Special value that marks deleted entry
  - Can be replaced when adding new entry
  - But doesn’t halt search during contains or remove

Find: Alan

Hashes to

Probing skips tombstone → Alan found

Amina	_TS_	Alan	Andy	Alessia	Alfred	Aimee	Aspen
0-aiqy	1-bjrz	2-cks	3-dlt	4-emu	5-fnv	6-gow	7-hpx

Hash Table Size: Load Factor

Load factor:

# of elements

Load factor $\lambda = n / m$ Size of table

- represents the portion of the buckets that are filled
- For open address hashing, load factor is between 0 and 1 (often somewhere between 0.5 and 0.75)

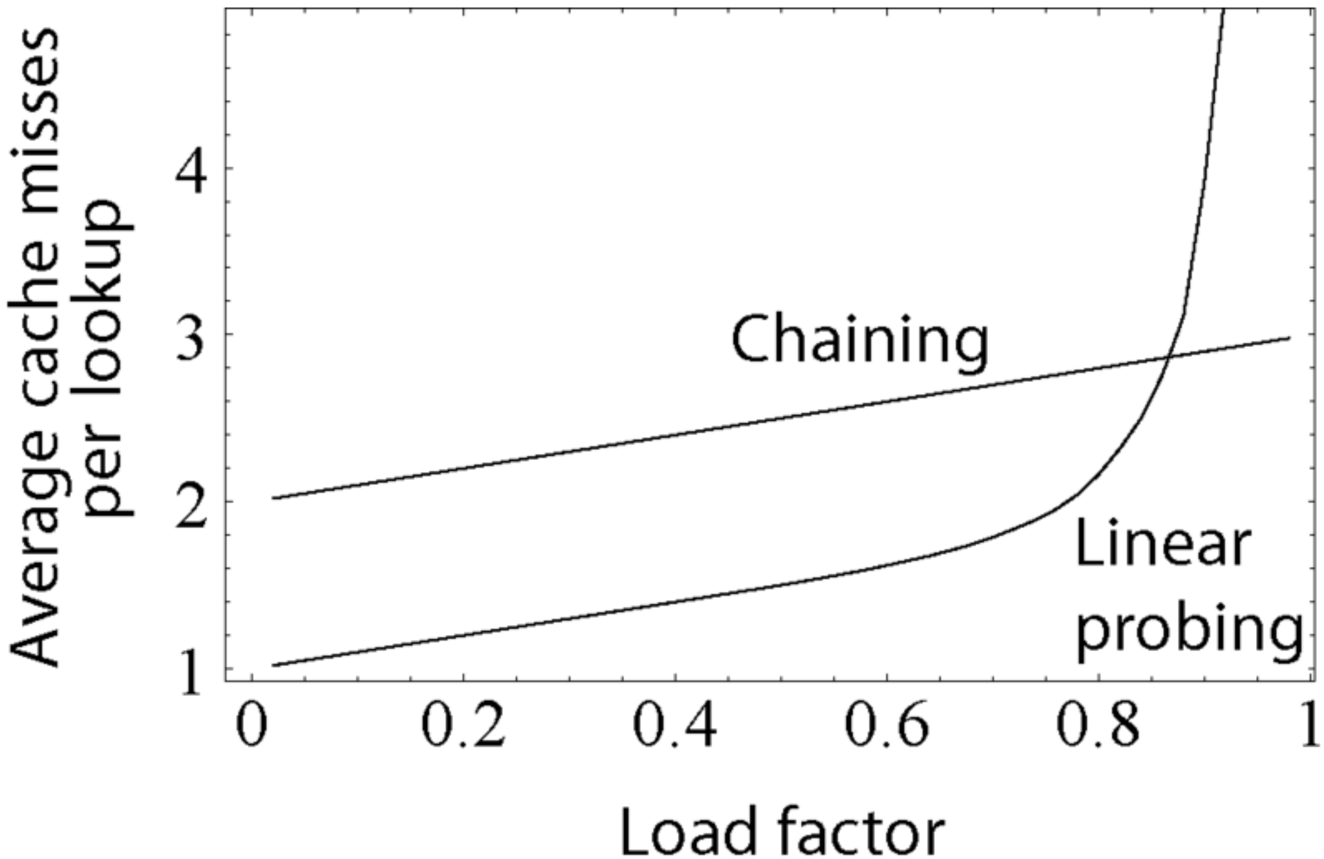
Want the load factor to remain small in order to avoid collisions

Clustering

- Assuming uniform distribution of hash values what's the probability that the next value will end up in index 1, 2, 6 ?

- As load factor gets larger, the tendency to cluster increases, resulting in longer search times upon collision
- Also affected by removals with _TS_

Performance vs. Load Factor



http://en.wikipedia.org/wiki/Hash_table

Double Hashing

- Rather than use a linear probe (ie. looking at successive locations)…
  - Use a second hash function to determine the probe step
- Helps to reduce clustering

Large Load Factor: What to do?

- Common solution: When load factor becomes too large (say, bigger than 0.75) → Reorganize
- Create new table with twice the number of positions
- Copy each element, ***rehashing*** using the new table size, placing elements in new table
- Delete the old table

Hash Tables: Algorithmic Complexity

- Assumptions:
  - Time to compute hash function is constant
  - Worst case analysis → All values hash to same position
  - Best case analysis → Hash function uniformly distributes the values
- Find element operation:
  - Worst case for open addressing → $O(n)$
  - Best case for open addressing → $O(1)$

Hash Tables: Average Case

- What about average case?
- Turns out, it's $1 / (1 - \lambda)$
- So keeping load factor small is very important

λ	$1 / (1 - \lambda)$
0.25	1.3
0.5	2.0
0.6	2.5
0.75	4.0
0.85	6.6
0.95	19.0

Hashing in Practice

- Need to find good hash function → uniformly distributes keys to all indices
- Open address hashing:
  - Need to tell if a position is empty or not
  - One solution → store only pointers & check for null (== 0)

Your Turn

- Complete Worksheet #37: Open Address Hashing