Gene Siriviboon
Phys 2030
Prof. Brad Marston

Phys 2030 Research Proposal

Project Title: Simulating Chaos in the Forced Astrojax Pendulum with Neural Network.

Introduction:

Astrojax is a system of two masses connected with a single string. The system resembles a double pendulum except that there is an extra degree of freedom from the mass sliding along the string, which could give the system a richer possible phenomenon. However, due to its high degree of freedom, the equation of motion of the system is extremely complex. Even though the formulation of Lagrangian with constrain could simplify the system, the solution to the Lagrange multiplier still need numerical algorithms such as the Newton-Raphson method (Toit, 2005), making it non-parallelizable and computationally expensive. This is limiting especially for phase space analysis which needs an extensive search on the parameter space. This project aims to tackle both the methodology and the nature of the Astrojax system itself with the following questions: does the stable state in the Astrojax system exists? If it is, what are the initial conditions required and how can we search for such states efficiently?

We would explore the possibility of using the neural network as a function approximator for the Lagrange multiplier in the constraints, which could accelerate the phase space exploration due to its parallelizable nature. The stability of the state at each configuration would be calculated by the Lyapunov exponent, the evolution of the ensemble of nearly identical would be visualized, and the bifurcation diagram of the system would be constructed similarly to the literature (Stachowiak & Okada, 2006).

Objective:

1. To use a neural network to simulate the force of constraint without using labels from the a priori simulated data and compare the result with the one obtained with the traditional Newton-Raphson method.
2. To map the Lyapunov exponent of the system as a function of the initial coordinate and momentum of the system.
3. To visualize the evolution of an ensemble of the system in phase space and determine the ergodicity of the system.
4. To construct a bifurcation diagram of the Astrojax pendulum.

Computational Algorithm:

1. Neural Network would be used trained on a different configuration of the Astrojax system with the constrain equation as the loss function.
2. The Newton-Raphson method will be used as a baseline for calculating the Lagrange multiplier.
3. The Runge-Kutta method will be used to integrate the equation of motion.

Bibliography:

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Toit, P. D. (2005). The Astrojax Pendulum and the N-Body Problem on the Sphere: A study in reduction, variational integration, and pattern evocation. Retrieved October 29, 2020, from https://www.semanticscholar.org/paper/The-Astrojax-Pendulum-and-the-N-Body-Problem-on-the-Toit/533cf17f06055a76a5ffb2001557712f4f8c3707