Questions 1

To implement yourself in Python.

Question 2

The momentum balance in the z direction is as follows:

$$-\frac{dP}{dz}+\left[\frac{1}{r}\frac{d}{dr}(r\,\tau_{rz})\right]=0$$

$$\frac{dP}{dz}=-\frac{\Delta P}{L}$$

Integrating:

$$\tau_{rz}=-\frac{\Delta P}{2L}r+\frac{A}{r}$$

The pipe is symmetric around the centre, with a zero stress at this point: $A=0$

$$\tau_{rz}=-\frac{\Delta P}{2L}r$$

$$\tau_{rz}=k\frac{du_z}{dr}\left|\frac{du_z}{dr}\right|^{n-1}$$

As the velocity is +ve, at goes to zero at the outer boundary, the gradient of the velocity w.r.t. radius in the pipe is -ve:

$$\left|\frac{du_z}{dr}\right|=-\frac{du_z}{dr}$$

$$\frac{du_z}{dr}\left(-\frac{du_z}{dr}\right)^{n-1}=-\frac{\Delta P}{2Lk}r$$

$$\frac{du_z}{dr}=-\left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}}r^{\frac{1}{n}}$$

$$u_z=-\frac{n}{1+n}\left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}}r^{\frac{1+n}{n}}+A$$

Boundary condition:

$u_z=0$ at $r=R$:

$$A=\frac{n}{1+n}\left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}}R^{\frac{1+n}{n}}$$

$$u_z=\frac{n}{1+n}\left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}}\left(R^{\frac{1+n}{n}}-r^{\frac{1+n}{n}}\right)$$

Question 3

Let *x* be the down the slope direction and *y* the distance from the plane.

The momentum balance down the plane is thus as follows:

$$\frac{d\tau_{yx}}{dy} + \rho g_x = 0$$

$$g_x = g\sin(30°) = \frac{g}{2}$$

Integrate:

$$\tau_{yx} = -\rho\frac{g}{2}y + A$$

At y=H $\tau_{yx} = 0$ (no stress at free surface):

$$\tau_{yx} = \rho\frac{g}{2}(H - y)$$

$$if\ |\tau_{yx}| > \tau_0: \quad \tau_{yx} = sign\left(\frac{du_x}{dy}\right)\left(\mu_\infty\left|\frac{du_x}{dy}\right| + \tau_0\right)$$

$$if\ |\tau_{yx}| < \tau_0: \quad \frac{du_x}{dy} = 0$$

The shear stress is positive (and the flow is in the +ve direction and increases with distance from the plane):

$\frac{du_x}{dy} \geq 0$ and $\tau_{yx} \geq 0$:

$$if\ \tau_{yx} > \tau_0: \quad \tau_{yx} = \left(\mu_\infty\frac{du_x}{dy} + \tau_0\right)$$

$$if\ \tau_{yx} < \tau_0: \quad \frac{du_x}{dy} = 0$$

Let us first find the critical distance ($\tau_0 = \tau_{yx}$):

$$\frac{2\tau_0}{\rho g} = (H - y_{crit})$$

We also know that τ_0 is 80% of the value where flow just stops:

$$\tau_0 = 0.4\,\rho g H$$

Therefore:

$$y_{crit} = 0.2H$$

Lets consider the flow in the region that is sheared ($y < y_{crit}$ and $\tau_{yx} > \tau_0$):

$$\frac{du_x}{dy} = \frac{\rho g}{2\mu_\infty}(0.2H - y)$$

Integrate and note that $u_x = 0$ at $y = 0$:

$$u_x = \frac{\rho g}{2\mu_\infty}(0.2Hy - 0.5y^2)$$

The maximum velocity is when $y = y_{crit} = 0.2H$. This is also the constant velocity in the region where $y > y_{crit}$:

$$u_{x\,max} = 0.01\frac{\rho g H^2}{\mu_\infty}$$

Plugging in the other known and specified values:

$$u_{x\,max} \approx 2.45\ m/s$$