Problem Set 7 – ACSE-2 – November 2020

1) **Material derivatives:** Given material moving through a temperature field, where the motion of a particle of the continuum can be described as:
$x_1' = \xi_1 + kt\xi_2, x_2' = (1+kt)\xi_2, x_3' = \xi_3$
Here ξ_i is the initial Lagrangian position of the particle, and x'_i the position reached at time t. k is a constant.
The temperature field in spatial coordinates x_i is given by:
$$T(\mathbf{x}, t) = \alpha(x_1 + x_2)$$
   (a) Find the material description of temperature
   (b) Obtain the velocity of a material particle in material and spatial description
   (c) Determine the material time derivative of the temperature in both material and spatial descriptions
   (d) Plot how an initially square 1x1 box in x_1, x_2 plane deforms over two time steps, from t=0 to t=1 and t=2. Take k=1.

2) **Infinitesimal strain:** With reference to a rectangular Cartesian coordinate system, the state of strain at a point is given by:
$$\mathbf{E} = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times 10^{-4}$$
   (a) What is the unit elongation in the direction of $2\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3$?
   (b) What is the change in angle between two perpendicular lines (in undeformed state) emanating from the point and in the directions of $2\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3$ and $3\hat{\mathbf{e}}_1 - 6\hat{\mathbf{e}}_3$?

3) **Maximum strain:** For the displacement field $u_1 = k\xi_1^2, u_2 = k\xi_2\xi_3, u_3 = k(2\xi_1\xi_3 + \xi_1^2)$, $k=10^{-6}$, find the maximum unit elongation for an element that is initially at (1,0,0).

4) **Finite and infinitesimal strain**:
Given the displacement components: $u_1 = k\xi_2^2$, $u_2=u_3=0$, $k=10^{-4}$.
   (a) Sketch the deformation of a unit square OABC, with as corner points: O=(0,0,0), A=(1,0,0), B=(1,1,0), C(0,1,0).
   (b) Find the deformed vectors $d\mathbf{x}'^{(1)}$ and $d\mathbf{x}'^{(2)}$ of the material elements $\mathbf{d\xi}^{(1)} = d\xi_1\hat{\mathbf{e}}_1$ and $\mathbf{d\xi}^{(2)} = d\xi_2\hat{\mathbf{e}}_2$, which were originally at point C(0,1,0).
   (c) Determine the difference between the deformed and undeformed lengths of the two elements
   (d) Determine the change in angle between the two elements.
   (e) Obtain the infinitesimal strain tensor **E**.
   (f) Estimate unit elongation for material elements $\mathbf{d\xi}^{(1)} = d\xi_1\hat{\mathbf{e}}_1, \mathbf{d\xi}^{(2)} = d\xi_2\hat{\mathbf{e}}_2$ using the infinitesimal strain tensor **E**.
   (g) And also find the decrease in angle between these two elements from the strain tensor **E**
   (h) Compare the answers from (f), (g) and (c), (d).

5) **Conservation of mass:** Given the following velocity field, find the density of a material particle as a function of time, i.e. derive $\rho(t)$.
$$v_i = \frac{kx_i}{1+kt}$$