Week 3 – Timed Assessment – EXAMPLE
All questions are mandatory.
You will have 2 hours to complete this assessment. Total marks for the assessment = 100 (35 for Q1, 30 for Q2, 35 for Q3). Please write clearly, annotate any graphs or sketches and explain your answers. Only partial marks will be given for correct answers without an explanation or derivation.
Make sure you put a page number on each page of your answer script.

Question 1

A foam is being tested in a rheometer. The average shear stress, τ, is measured as a function of the strain rate, S. We know that the rheological properties of the foam are likely to depend upon the surface tension at the liquid gas interfaces, γ, the viscosity of the fluid, μ, and the diameter of the bubbles, d_b. It will also depend upon the liquid content of the foam, ϕ.

Symbol	**Description**	**Units**
τ	Shear stress	*Pa*
S	Strain rate	s^{-1}
γ	Surface tension	*N/m*
μ	Fluid viscosity	*Pa.s*
d_b	Bubble diameter	*m*
ϕ	Liquid content of foam	-

(i) How many dimensionless groups are required to describe this system? Show working (Clue: What are the base dimensions?) (5)

(ii) Derive a suitable set of dimensionless groups to investigate the rheological behaviour of the foam. Show all working. You may use Python to calculate the inverse of matrices as required, but write out the input matrix, the inverse and any associated calculations. (20)

(iii) There are multiple potential sets of dimensionless groups that you could have chosen and there is no “correct” set. Why do you think that the set that you have chosen is appropriate and what ratio of effects or forces does each of your dimensionless groups represent? (10)

Question 2

A packed bed of activated carbon is being use to remove impurities from water. The bed is 2m wide (*x* direction) and 1m high (*y* direction). It is also 1m deep (*z* direction), but you can assume that flows do not vary in this direction making this a 2D problem.

The fluid flows in through the bottom of the bed and out of the top, with no flow through the left and right hand sides. The total flow through the bed is 0.5 m^3/s.

There is a constant flux out of the top of the bed, but at the inlet the flux follows a sinusoidal profile over half a period (a single peak), going to zero at both the left and right hand sides of the domain.

We wish to use a stream function formulation to describe this problem.

(i) Calculate the fluxes as a function of position on each boundary. (10)

(ii) Use the calculated fluxes to calculate the value of the stream functions around the boundaries of the domain. (20)

Question 3

A fluid is flowing down an inclined plane that makes an angle θ with the horizontal. We will assume that the plane is large enough that end/edge effects can be ignored and that the fluid is of constant depth, H, over the plane. In these calculations it is useful to align your axes with the plane (x being the distance down the plane, y being the distance vertically away from the plane). We can also assume that the flow is steady:

(i) Simplify the Navier-Stokes equation to obtain the appropriate momentum balance for this problem.

Navier-Stokes equation: $\rho \frac{D\mathbf{u}}{Dt} = -\nabla P + \nabla \cdot \boldsymbol{\tau} + \rho\, \mathbf{g}$

Solve this momentum balance to obtain the viscous shear stress, τ_{yx}, as a function of the distance from the plane, y (use an appropriate boundary condition).

(15)

(ii) Calculate the velocity profile in the fluid if it is assumed that it exhibits a power law rheology:

$$\boldsymbol{\tau} = 2k|2\mathbf{S}|^{n-1}\mathbf{S}$$

Where $\boldsymbol{\tau}$ is the viscous stress tensor, $\boldsymbol{S}$ is the strain rate tensor ($\boldsymbol{S} = \frac{1}{2}(\nabla\mathbf{v} + \nabla\mathbf{v}^T)$) and $\mathbf{v}$ is the velocity vector. Note that for this system the only non-zero shear stress component is the τ_{yx} component and the only non-zero differential in the strain rate is $\frac{\partial v_x}{\partial y}$.

(20)