$$\frac{d^2C^*}{dx^{*2}} = \frac{d}{dx^*}\left(\frac{dC^*}{dx^*}\right) = Pe\,\frac{dC^*}{dx^*}$$

$$\int \frac{d\left(\frac{dC^*}{dx^*}\right)}{\frac{dC^*}{dx^*}} = \int Pe\, dx^*$$

$$\ln\left(\frac{dC^*}{dx^*}\right) = Pe\,x^* + A$$

$$\frac{dC^*}{dx^*} = e^{Pe\,x^* + A}$$

$$\frac{dC^*}{dx^*} = e^{A} e^{Pe\,x^*}$$

$$\int dC^* = \int e^{A} e^{Pe\,x^*} dx^*$$

$$C^* = \underbrace{\frac{e^{A}}{Pe}}_{k} e^{Pe\,x^*} + B$$

$$C^* = k e^{Pe\,x^*} + B$$

at $x^* = 0$ $C^* = 1$

$$1 = k + B$$

at $x^* = 1$ $C^* = 0$

$$k e^{Pe} + B = 0$$

$$B = -k e^{Pe}$$

$$1 = k - k e^{Pe}$$

$$k = \frac{1}{1 - e^{Pe}}$$

$$C^* = \frac{1}{1-e^{Pe}} e^{Pe\,x^*} - \frac{1}{1-e^{Pe}} e^{Pe}$$

$$C^* = \frac{e^{Pe\,x^*} - e^{Pe}}{1 - e^{Pe}}$$

d_n d_d γ g ρ

L L $\gamma \to \frac{F}{L}$ $\frac{L}{T^2}$ $\frac{M}{L^3}$

$$\frac{F}{L}$$

$$\frac{M\frac{L}{T^2}}{L}$$

$$\frac{M}{T^2}$$

$$\begin{array}{c|ccccc} & d_n & \gamma & \rho & g & d_d \\ \hline M & 0 & 1 & 1 & 0 & 0 \\ L & 1 & 0 & -3 & 1 & 1 \\ T & 0 & -2 & 0 & -2 & 0 \end{array}$$

$$\begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$N_1 = \frac{d_n^2 \rho g}{\gamma} \qquad N_2 = \frac{d_d}{d_n}$$

$$
\begin{array}{c} \\ M \\ L \\ T \end{array}
\begin{array}{c}
\begin{array}{cccccc} \rho_u & \mu & g & \rho_l & \Delta v & h \end{array} \\
\left(\begin{array}{ccc|ccc}
1 & 1 & 0 & 1 & 0 & 0 \\
-3 & -1 & 1 & -3 & 1 & 1 \\
0 & -1 & -2 & 0 & -1 & 0
\end{array}\right)
\end{array}
$$

$\mu_s \quad \frac{F\,T}{L^2} \qquad F \to M\frac{L}{T^2}$

$\frac{M}{LT}$

$$
\begin{array}{c} \rho_u \\ \mu \\ g \\ \rho_l \\ \Delta v \\ h \end{array}
\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
\qquad
\begin{pmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ 0 \\ 1 \\ 0 \end{pmatrix}
\qquad
\begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

$N_1 = \frac{\rho_l}{\rho_u} \qquad N_2 = \frac{\Delta v\, \rho_u^{\frac{1}{3}}}{\mu^{\frac{1}{3}}\, g^{\frac{1}{3}}} \qquad N_3 = \frac{h\, \rho_u^{\frac{2}{3}}}{\mu^{\frac{2}{3}}\, g^{\frac{1}{3}}}$