$$\nabla \times \hat{v} = 0$$

$$\hat{v} = -k \nabla \phi = -k \begin{pmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial z} \end{pmatrix}$$

$$\nabla \times v = \begin{pmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{pmatrix}$$

$$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = \frac{\partial}{\partial x}\left(-k \frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-k \frac{\partial \phi}{\partial x}\right)$$

$$= -k \frac{\partial^2 \phi}{\partial x \partial y} + k \frac{\partial^2 \phi}{\partial y \partial x}$$

but $\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x}$

$$= 0$$

$$\nabla \times \left(\rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \nabla \vec{u}\right) = \nabla \times \left(-\nabla P + \mu \nabla^2 \vec{u} + \rho \vec{g}\right)$$

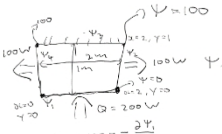
$$\rho \nabla \times \frac{\partial \vec{u}}{\partial t} + \rho \nabla \times (\vec{u} \cdot \nabla \vec{u})$$

$$= -\cancel{\nabla \times (\nabla P)} + \nabla \times (\mu \nabla^2 \vec{u}) + \rho \nabla \times \vec{g}$$

$$\rho \frac{\partial \nabla \times \vec{u}}{\partial t} + \rho\left((\vec{u} \cdot \nabla)\vec{\omega} + (\overset{0}{\cancel{\nabla \cdot \vec{u}}})\omega - (\vec{\omega} \cdot \nabla)\vec{u}\right) = \mu \nabla^2 (\nabla \times \vec{u}) + \rho \nabla \times \vec{g}$$

$$\rho \frac{\partial \vec{\omega}}{\partial t} + \rho (\vec{u} \cdot \nabla)\vec{\omega} = \rho(\vec{\omega} \cdot \nabla)\vec{u} + \mu \nabla^2 \vec{\omega} + \rho \nabla \times \vec{g}$$

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u} + \frac{\mu}{\rho} \nabla^2 \vec{\omega} + \nabla \times \vec{g}$$



$\Psi_2 = 100y$

$F_y = 100 = -\frac{\partial \Psi_1}{\partial x}$

$\frac{d\Psi_1}{dx} = -100$

$d\Psi_1 = -\int 100\,dx$

$\Psi_1 = -100x + A$

at $x = 2$ $\Psi_1 = 0$

$100 \times 2 = A$

$A = 200$

$\Psi_1 = 200 - 100x$

Boundary 2: $F_x = \frac{100}{1} = \frac{\partial \Psi_2}{\partial y}$

$d\Psi_2 = \int 100\,dy$

$\Psi_2 = 100y + B$

at $y=0$ $\Psi_2 = 0$

$\therefore B = 0$

$\Psi_2 = 100y$

Boundary 3: $F_y = 0 = -\frac{\partial \Psi_3}{\partial x}$

$\Psi_3 = C$

$\Psi_3 = 100$

Boundary 4: $F_x = -100 = \frac{\partial \Psi_4}{\partial y}$

$\int d\Psi_4 = \int -100\,dy$

$\Psi_4 = -100y + D$

at $y = 1$ $\Psi_4 = 100$

$100 = -100 + D$

$D = 200$

$\Psi_4 = 200 - 100y$

$$a_n = \frac{2}{L}\int_0^L (mx+c)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$X = \frac{n\pi x}{L} \qquad x = \frac{L}{n\pi}X$$

$$dx = \frac{L}{n\pi}dX$$

$$a_n = \frac{2}{n\pi}\int_0^{n\pi}\left(\frac{mL}{n\pi}X + c\right)\sin(X)\,dX$$

$$a_n = \frac{2}{n\pi}\left(\int_0^{n\pi}\frac{mL}{n\pi}X\sin X\,dx + \int_0^{n\pi} c\sin X\,dX\right)$$

$$a_n = \frac{2}{n\pi}\left(\left[\frac{mL}{n\pi}\left(-X\cos X + \sin X\right)\right]_0^{n\pi} + \left[-c\cos X\right]_0^{n\pi}\right)$$

$$a_n = \left(\left(-n\pi\cos(n\pi)\right) - c\cos(n\pi)\right)\frac{mL}{n\pi} + c\Big)\frac{2}{n\pi}$$

if n is even

$\cos(n\pi) = 1$

if n is odd

$\cos(n\pi) = -1$

if n is even

$$a_n = -\frac{2mL}{n\pi}$$

if n is odd

$$a_n = \frac{2mL}{n\pi} + \frac{4L}{n\pi}$$

$W \Leftrightarrow H$

$y \Rightarrow x$

$x \Rightarrow H - y$

$$\phi_2 = \sum_{n=1}^{\infty} \gamma_n \sin\left(\frac{n\pi x}{W}\right)\left(\cosh\left(\frac{n\pi(H-y)}{W}\right) - \coth\left(\frac{n\pi H}{W}\right)\sinh\left(\frac{n\pi(H-y)}{W}\right)\right)$$

$$\gamma_n = \frac{2}{W}\int_0^W \psi_b \sin\left(\frac{n\pi x}{W}\right)dx$$