Week 2 – Timed Assessment – EXAMPLE
All questions are mandatory.

You will have 2 hours to complete this assessment. Total marks for the assessment = 100 (35 for Q1, 25 for Q2, 40 for Q3). Please write clearly, annotate any graphs or sketches and explain your answers. Only partial marks will be given for correct answers without an explanation or derivation. Make sure you put a page number on each page of your answer script. GOOD LUCK.

(1) Consider a plane of reflection that passes through the origin. Let $\hat{\mathbf{n}}$ be the unit normal vector to the plane and let $\mathbf{r}$ be the position vector for a point in space. [*35 marks*]

(a) Show that the reflected vector for $\mathbf{r}$ is given by $\mathbf{T} \cdot \mathbf{r} = \mathbf{r} - 2(\mathbf{r} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$, where $\mathbf{T}$ is the transformation that corresponds to the reflection.

(b) Let $\hat{\mathbf{n}} = \frac{1}{\sqrt{5}}(\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2)$. Find the matrix of $\mathbf{T}$.

(2) The position $\mathbf{x}$ at time t of a particle initially at $\boldsymbol{\xi}=(\xi_1, \xi_2, \xi_3)$ is given by: $x_1 = \xi_1 + {\xi_2}^2 t^2$, $x_2 = \xi_2 + \xi_2 t$, $x_3 = \xi_3$. Find the initial position and acceleration of a particle that is at $\mathbf{x}=(2,4,3)$ at $t=1$. [*25 marks*]

(3) Given an isotropic elastic material such that $\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$, where λ and μ are the Lamé constants. At a point in the material it is under the following state of stress:

$$\boldsymbol{\sigma} = \begin{bmatrix} 5\lambda + 4\mu & 2\mu & 0 \\ 2\mu & 5\lambda + 4\mu & 8\mu \\ 0 & 8\mu & 5\lambda + 2\mu \end{bmatrix} 10^{-4} \text{ Pa.}$$

[*40 marks*]

(a) What is the traction on a plane with a normal in direction x_3 direction?
(b) What is the corresponding strain tensor and the unit elongation of an element originally oriented in the direction of this plane's normal?
(c) Sketch the components of this stress tensor that represent tractions in x_2 direction and label them with their magnitudes (in terms of λ and μ)