IMPERIAL COLLEGE LONDON

MSc ASSESSMENT 2020/21

For internal students of Imperial College London

Taken by students of MSc Applied Computational Science and Engineering

ACSE-2 Assessment - Week 3

Friday 20 November 2020

This assessment comprises THREE questions. Please answer ALL questions.

The marks to be obtained from answering questions 1 to 3 are as follows:
Question 1: 35 marks
Question 2: 30 marks
Question 3: 35 marks

The assessments are run as open-book assessments, and as such we have worked hard to create a coursework that assesses synthesis of knowledge rather than factual recall. Be aware that access to the internet, notes or other sources of factual information in the time provided may not be too helpful and may well limit your time to successfully synthesise the answers required. The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use may also constitute an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you. The College will investigate all instances where an assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all assessments, we will analyse performance against performance on the rest of the course and against data from previous years and use an evidence-based approach to maintain a fair and robust assessment. As with all assessments, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Please write clearly, annotate any graphs or sketches and explain your answers. Only partial marks will be given for correct answers without an explanation or derivation. Make sure you put a page number on each page of your answer script.

Question 1 (35 marks)

It has been found that a slurry can be approximated as a Bingham plastic with a yield stress, τ_0, and a limiting viscosity, μ_∞. The fluid has a density, ρ. We wish to predict the velocity, v, at which spheres of mass, M, and radius, r, will fall through this fluid under the influence of gravity (acceleration, g). These parameters have the following units:

Symbol	**Description**	**Units**
τ_0	Yield stress of the fluid	*Pa*
μ_∞	Limiting viscosity	*Pa.s*
ρ	Fluid density	*kg/m^3*
v	Velocity of the sphere	*m/s*
M	Mass of the sphere	*kg*
r	Radius of the sphere	*m*
g	Acceleration of gravity	*m/s^2*

(i) How many dimensionless groups are required to describe this system? Show working (note that some units can be broken into more fundamental units). (5)

(ii) Derive an appropriate set of dimensionless groups to investigate the behaviour of the spheres. Show all working. You may use Python to calculate the inverse and do the matrix multiplications as required, but write out the input matrix, the inverse and any associated calculations. (20)

(iii) There is more than one set of dimensionless groups that could be used to describe this problem and, while there is no "correct" set, some sets are more appropriate than others. What do the dimensionless groups in the set that you have derived represent physically and why do you feel that this is an appropriate set? (10)

Question 2 (30 marks)

A block of rock is being heated from below and cooled on the left, right and top sides. We will assume that the rock is long enough in the other (z) direction that the problem can be considered a 2D one. The block is 2m wide and 1m high.

The total heat flow into the bottom of the rock is 100 W per length in the z direction.

We can assumed that the edges are being evenly cooled (i.e. the flux through each of the cooled boundaries has the same magnitude and is constant w.r.t. position along the boundary).

The bottom boundary is unevenly heated with a parabolic heat flux that is at a maximum in the middle of the block, going to zero at both edges of the block.

We wish to use a stream function formulation to describe this problem.

(i) Calculate the heat fluxes as a function of position on each boundary. (10)

(ii) Use the calculated fluxes to calculate the value of the stream functions around the boundaries of the domain. (20)

Question 3 (35 marks)

In order to measure the rheology of a mixture of two liquid phases a researcher decided to use a parallel plate rheometer in which the upper plate is moved at a constant velocity, v_{top}, and the lower plate is kept stationary. The shear stress at the top plate, τ_{top}, is then measured in order that the apparent viscosity of the mixture can be calculated. The separation between the plates is L.

Unfortunately the researcher left the mixture too long before carrying out the measurement and the two liquid phases separated to form two layers of liquid. In your calculations you may assume that the fraction of the denser fluid in the mixture is ϕ (which will also be the fraction of the gap between the plates that this fluid occupies). Each of the fluids can be assumed to have a Newtonian rheology, but with a different viscosity (μ_{upper} and μ_{lower}).

The flow can be assumed to be steady, with no instabilities and the plates can assumed to be large enough that end/edge effects can be ignored. Assume that the top plate is moving in the *x* direction and the separation between the plates is in the *y* direction. You can also assume that body forces can be ignored and there is no pressure gradient in the direction of flow.

(i) Simplify the following Navier-Stokes equation to an equation appropriate to this problem.

Navier-Stokes equation: $\rho \frac{D\mathbf{u}}{Dt} = -\nabla P + \nabla \cdot \boldsymbol{\tau} + \rho\, \mathbf{g}$ (5)

Clue: There is only one non-zero shear stress component in this problem (τ_{yx}).

(ii) Calculate the shear stress as a function of vertical distance using the shear stress at the top wall as a boundary condition. Note that the shear stress will be continuous over the interface between the two fluids. (5)

(iii) Calculate the velocity profile between the plates. Note that in a Newtonian rheology:

$$\boldsymbol{\tau} = 2\mu\, \mathbf{S} \quad \text{where } \mathbf{S} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$$

The velocity profile can be calculated by noting that there should be no slip at either of the two boundaries and that the two fluids must have the same velocity at the interface between them. (15)

Clue: The only non-zero velocity component is in the *x* direction and it only varies in the *y* direction.

(iv) As a function of μ_{upper}, μ_{lower} and ϕ, what will be the apparent viscosity that the researcher will think his mixture has? (10)

Clue: This is the same as asking what viscosity would a single fluid have to have in order for it to generate the same shear stress at the wall for a given wall velocity.