IMPERIAL COLLEGE LONDON

MSc ASSESSMENT 2020/21

For internal students of Imperial College London

Taken by students of MSc Applied Computational Science and Engineering

ACSE-2 Assessment – Week 1

09:00 – 11:00 Friday 6 November 2020

This assessment comprises THREE questions. Please answer **ALL** questions.

The marks to be obtained from answering questions 1 to 3 are as follows:

Question 1: 30 marks
Question 2: 30 marks
Question 3: 40 marks

The assessments are run as open-book assessments, and as such we have worked hard to create a coursework that assesses synthesis of knowledge rather than factual recall. Be aware that access to the internet, notes or other sources of factual information in the time provided may not be too helpful and may well limit your time to successfully synthesise the answers required. The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use may also constitute an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you. The College will investigate all instances where an assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all assessments, we will analyse performance against performance on the rest of the course and against data from previous years and use an evidence-based approach to maintain a fair and robust assessment. As with all assessments, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

1. In lecture 1, as part of our discussion of Taylor series we derived an expression for the linear approximation to a function f of one variable. We introduced the linearisation

$$y(x) = c + mx$$

where m is the gradient (or slope) of the line and c is the intercept.

We had two unknowns, m and c, and we used two pieces of information to fix these – that the linear function agrees with f at an expansion point x_0: $y(x_0) = f(x_0)$; and that the derivative also agree at the expansion point: $y'(x_0) = f'(x_0)$.

(i) In the lecture we noted that $y'(x_0) = f'(x_0)$ implies immediately that $m = f'(x_0)$ and then we jumped straight to the final form of the expansion without explicitly deriving c:

$$y(x) = f(x_0) + (x - x_0)f'(x_0).$$

Derive c from the knowledge that $m = f'(x_0)$ and using $y(x_0) = f(x_0)$, and confirm that you end up with the same expression for $y(x)$ as given just above.

(ii) Extend the linearisation derivation above to derive the next term in the Taylor series, i.e. start from the quadratic expansion

$$y(x) = c + mx + nx^2$$

where c, m and n are three unknowns, and use the three pieces of information we want to be true: $y(x_0) = f(x_0)$, $y'(x_0) = f'(x_0)$ and $y''(x_0) = f''(x_0)$. Does your result agree with the expression for Taylor series we wrote down in class?

Note that the c and the m you derive here will not be the same as in part (i).

(iii) Now consider the functions $f(x) = \sin(x)$ and $f(x) = \cos(x)$. For each, derive Taylor series expansions about the point $x_0 = 0$. Based on this what are very simple approximations to each of these functions which are reasonably accurate for very small values of x?

(30 marks)

2. (i) Consider the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$$

What is its range, rank and null-space. Suppose this matrix appears in a linear system of the form $A\boldsymbol{x} = \boldsymbol{b}$, give examples of RHS vectors $\boldsymbol{b}$ where this system has a non-trivial (i.e. non-zero) solution and where it has no solutions. If you have a solution, how can the presence of a null-space allow you to obtain other valid solutions (give an example for this A)?

(ii) Consider the matrix

$$A = \begin{pmatrix} 2 & 4 \\ 4 & -4 \end{pmatrix}$$

By first computing its eigenvalues and eigenvectors, demonstrate how you can diagonalise this matrix.

(30 marks)

3. Give a brief (less than ~250 words) explanation of the Method of Manufactured Solution (MMS). Include in your explanation a short description of what it can be used for, and why it is such a valuable technique. You may use mathematics and diagrams to help support your answer.

(40 marks)