

Mathematical Proof for Project Euler Problem 1: Multiples of 3 or 5

Problem Statement

Find the sum of all natural numbers below 1000 that are divisible by 3 or 5.

Mathematical Solution

Step 1: Identify All Relevant Numbers

A number n is included if it is a multiple of either 3 or 5. To avoid double counting, we apply the principle of inclusion-exclusion.

Step 2: Principle of Inclusion-Exclusion

Let:

$$A = \{n \mid n < 1000, n \text{ is a multiple of } 3\}$$

$$B = \{n \mid n < 1000, n \text{ is a multiple of } 5\}$$

Using the inclusion-exclusion principle:

$$\text{Sum}(A \cup B) = \text{Sum}(A) + \text{Sum}(B) - \text{Sum}(A \cap B)$$

Step 3: Calculate Individual Sums

Multiples of 3 below 1000:

$$(3, 6, 9, \dots, 999)$$

Number of terms:

$$n = \frac{999 - 3}{3} + 1 = 333$$

Sum:

$$\text{Sum}(A) = \frac{333}{2}(3 + 999) = 166833$$

Multiples of 5 below 1000:

$$(5, 10, 15, \dots, 995)$$

Number of terms:

$$n = \frac{995 - 5}{5} + 1 = 199$$

Sum:

$$\text{Sum}(B) = \frac{199}{2}(5 + 995) = 99500$$

Multiples of 15 below 1000:

$$(15, 30, 45, \dots, 990)$$

Number of terms:

$$n = \frac{990 - 15}{15} + 1 = 66$$

Sum:

$$\text{Sum}(A \cap B) = \frac{66}{2}(15 + 990) = 33165$$

Step 4: Apply Inclusion-Exclusion

$$\text{Sum}(A \cup B) = 166833 + 99500 - 33165 = 233168$$

Conclusion

The sum of all natural numbers below 1000 that are multiples of 3 or 5 is:

$$\boxed{233168}$$