Calculating the metric tensor of an egg.

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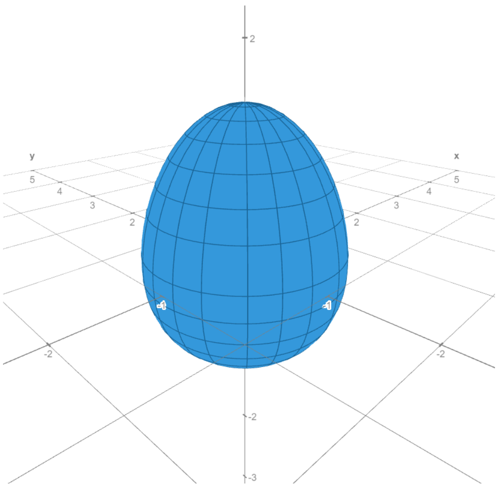
1 Introduction.

The surface is described by these equations:

$$X = \cos v \sin u$$

$$Y = \sin v \sin u$$

$$Z = \left(\frac{\cos u + 1}{4} + 1\right) \cos u$$



The metric tensor:

$$g_{ij} = \begin{bmatrix} \frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta u} & \frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta v} \\ \frac{\delta\vec{R}}{\delta v} \cdot \frac{\delta\vec{R}}{\delta u} & \frac{\delta\vec{R}}{\delta v} \cdot \frac{\delta\vec{R}}{\delta v} \end{bmatrix}$$

2 Calculating.

$$g_{11} = \frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta u}$$

$$\frac{\delta\vec{R}}{\delta u} = \frac{\delta c^i}{\delta u}\frac{\delta\vec{R}}{\delta c^i} = \frac{\delta X}{\delta u}\frac{\delta\vec{R}}{\delta X} + \frac{\delta Y}{\delta u}\frac{\delta\vec{R}}{\delta Y} + \frac{\delta Z}{\delta u}\frac{\delta\vec{R}}{\delta Z}$$

$$\frac{\delta X}{\delta u}\frac{\delta\vec{R}}{\delta X} = \cos v \cos u \frac{\delta\vec{R}}{\delta X}$$

$$\frac{\delta Y}{\delta u}\frac{\delta\vec{R}}{\delta Y} = \sin v \cos u \frac{\delta\vec{R}}{\delta Y}$$

$$\frac{\delta Z}{\delta u}\frac{\delta\vec{R}}{\delta Z} = -\frac{1}{4}(\sin u)(2\cos u + 5)\frac{\delta\vec{R}}{\delta Z}$$

$$\frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta u} = \frac{\delta c^i}{\delta u}\frac{\delta\vec{R}}{\delta c^i} \times \frac{\delta c^j}{\delta u}\frac{\delta\vec{R}}{\delta c^j}$$

When $i \neq j$ the multiplication will be just equal to zero so we can skip it.

$$\frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta u} = \cos v \cos u \frac{\delta\vec{R}}{\delta X} \times \cos v \cos u \frac{\delta\vec{R}}{\delta X} + \sin v \cos u \frac{\delta\vec{R}}{\delta Y} \times \sin v \cos u \frac{\delta\vec{R}}{\delta Y} +$$

$$+(-\frac{1}{4}(\sin u)(2\cos u + 5)\frac{\delta\vec{R}}{\delta Z}) \times (-\frac{1}{4}(\sin u)(2\cos u + 5)\frac{\delta\vec{R}}{\delta Z}) =$$

$$= (\cos v \cos u)^2 + (\sin v \cos u)^2 + \frac{1}{16}(\sin u)^2(2\cos u + 5)^2 =$$

$$= (\cos v)^2(\cos u)^2 + (\sin v)^2(\cos u)^2 + \frac{1}{16}(\sin u)^2(2\cos u + 5)^2 =$$

$$= (\cos u)^2((\cos v)^2 + (\sin v)^2) + \frac{1}{16}(\sin u)^2(2\cos u + 5)^2 =$$

$$= (\cos u)^2 + \frac{1}{16}(\sin u)^2(2\cos u + 5)^2$$

Now let's calculate g_{22}:

$$g_{22} = \frac{\delta\vec{R}}{\delta v} \cdot \frac{\delta\vec{R}}{\delta v}$$

$$\frac{\delta\vec{R}}{\delta v} = \frac{\delta c^i}{\delta v}\frac{\delta\vec{R}}{\delta c^i} = \frac{\delta X}{\delta v}\frac{\delta\vec{R}}{\delta X} + \frac{\delta Y}{\delta v}\frac{\delta\vec{R}}{\delta Y} + \frac{\delta Z}{\delta v}\frac{\delta\vec{R}}{\delta Z}$$

$$\frac{\delta X}{\delta v}\frac{\delta\vec{R}}{\delta X} = -\sin v \sin u \frac{\delta\vec{R}}{\delta X}$$

$$\frac{\delta Y}{\delta v}\frac{\delta\vec{R}}{\delta Y} = \cos v \sin u \frac{\delta\vec{R}}{\delta Y}$$

$$\frac{\delta Z}{\delta v}\frac{\delta\vec{R}}{\delta Z} = 0 \texttt{ Because Z is not dependent on v}$$

$$\frac{\delta\vec{R}}{\delta v}\cdot\frac{\delta\vec{R}}{\delta v} = -\sin v \sin u \frac{\delta\vec{R}}{\delta X} \times -\sin v \sin u \frac{\delta\vec{R}}{\delta X} + \cos v \sin u \frac{\delta\vec{R}}{\delta Y} \times \cos v \sin u \frac{\delta\vec{R}}{\delta Y} =$$

$$= (\sin v \sin u)^2 + (\cos v \sin u)^2 = (\sin v)^2(\sin u)^2 + (\cos v)^2(\sin u)^2 =$$

$$= (\sin u)^2((\cos v)^2 + (\sin v)^2) = (\sin u)^2$$

$$\frac{\delta\vec{R}}{\delta u} \cdot \frac{\delta\vec{R}}{\delta v} = \frac{\delta\vec{R}}{\delta v} \cdot \frac{\delta\vec{R}}{\delta u} = g_{21} = g_{12} = 0$$

3 Result.

$$g_{ij} = \begin{bmatrix} (\cos u)^2 + \frac{1}{16}(\sin u)^2(2\cos u + 5)^2 & 0 \\ 0 & (\sin u)^2 \end{bmatrix}$$

:D (Wait for the next part: "Finding the geodesic curves on an egg")