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List of definite integrals

In mathematics, the definite integral:

$$\int_a^b f(x)\,dx$$

is the area of the region in the *xy*-plane bounded by the graph of *f*, the *x*-axis, and the lines *x* = *a* and *x* = *b*, such that area above the *x*-axis adds to the total, and that below the *x*-axis subtracts from the total.

The fundamental theorem of calculus establishes the relationship between indefinite and definite integrals and introduces a technique for evaluating definite integrals.

If the interval is infinite the definite integral is called an *improper integral* and defined by using appropriate limiting procedures. for example:

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\left[\int_a^b f(x)\,dx\right]$$

A constant, such pi, that may be defined by the integral of an algebraic function over an algebraic domain is known as a period.

The following is a list of the most common definite Integrals. For a list of indefinite integrals see *List of indefinite integrals*

Contents

Definite integrals involving rational or irrational expressions

$$\int_0^\infty \frac{dx}{x^2+a^2} = \frac{\pi}{2a}$$

$$\int_0^\infty \frac{x^m\,dx}{x^n+a^n} = \frac{\pi a^{m-n+1}}{n\sin\left(\dfrac{m+1}{n}\pi\right)} \quad \text{for } 0<m+1<n$$

$$\int_0^\infty \frac{x^{p-1}\,dx}{1+x} = \frac{\pi}{\sin(p\pi)} \quad \text{for } 0<p<1$$

$$\int_0^\infty \frac{x^m\,dx}{1+2x\cos\beta+x^2} = \frac{\pi}{\sin(m\pi)}\cdot\frac{\sin(m\beta)}{\sin(\beta)}$$

$$\int_0^a \frac{dx}{\sqrt{a^2-x^2}} = \frac{\pi}{2}$$

$$\int_0^a \sqrt{a^2-x^2}\,dx = \frac{\pi a^2}{4}$$

$$\int_0^a x^m(a^n-x^n)^p\,dx = \frac{a^{m+1+np}\Gamma\left(\dfrac{m+1}{n}\right)\Gamma(p+1)}{n\Gamma\left(\dfrac{m+1}{n}+p+1\right)}$$

$$\int_0^\infty \frac{x^m\,dx}{(x^n+a^n)^r} = \frac{(-1)^{r-1}\pi a^{m+1-nr}\Gamma\left(\frac{m+1}{n}\right)}{n\sin\left(\frac{m+1}{n}\pi\right)(r-1)!\,\Gamma\left(\frac{m+1}{n}-r+1\right)} \quad \text{for } n(r-2) < m+1 < nr$$

Definite integrals involving trigonometric functions

$$\int_0^\pi \sin(mx)\sin(nx)dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad \text{for } m, n \text{ positive integers}$$

$$\int_0^\pi \cos(mx)\cos(nx)dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad \text{for } m, n \text{ positive integers}$$

$$\int_0^\pi \sin(mx)\cos(nx)dx = \begin{cases} 0 & \text{if } m+n \text{ even} \\ \frac{2m}{m^2-n^2} & \text{if } m+n \text{ odd} \end{cases} \quad \text{for } m, n \text{ integers.}$$

$$\int_0^{\frac{\pi}{2}} \sin^2(x)dx = \int_0^{\frac{\pi}{2}} \cos^2(x)dx = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \sin^{2m}(x)dx = \int_0^{\frac{\pi}{2}} \cos^{2m}(x)dx = \frac{1\times 3\times 5\times\cdots\times(2m-1)}{2\times 4\times 6\times\cdots\times 2m}\cdot\frac{\pi}{2} \quad \text{for } m = 1, 2, 3\ldots$$

$$\int_0^{\frac{\pi}{2}} \sin^{2m+1}(x)dx = \int_0^{\frac{\pi}{2}} \cos^{2m+1}(x)dx = \frac{2\times 4\times 6\times\cdots\times 2m}{1\times 3\times 5\times\cdots\times(2m+1)} \quad \text{for } m = 1, 2, 3\ldots$$

$$\int_0^{\frac{\pi}{2}} \sin^{2p-1}(x)\cos^{2q-1}(x)dx = \frac{\Gamma(p)\Gamma(q)}{2\Gamma(p+q)} = \frac{1}{2}\mathrm{B}(p,q)$$

$$\int_0^\infty \frac{\sin(px)}{x}dx = \begin{cases} \frac{\pi}{2} & \text{if } p > 0 \\ 0 & \text{if } p = 0 \\ -\frac{\pi}{2} & \text{if } p < 0 \end{cases}$$ (see Dirichlet integral)

$$\int_0^\infty \frac{\sin px\cos qx}{x}\,dx = \begin{cases} 0 & \text{if } q > p > 0 \\ \frac{\pi}{2} & \text{if } 0 < q < p \\ \frac{\pi}{4} & \text{if } p = q > 0 \end{cases}$$

$$\int_0^\infty \frac{\sin px\sin qx}{x^2}\,dx = \begin{cases} \frac{\pi p}{2} & \text{if } 0 < p \leq q \\ \frac{\pi q}{2} & \text{if } 0 < q \leq p \end{cases}$$

$$\int_0^\infty \frac{\sin^2 px}{x^2}\,dx = \frac{\pi p}{2}$$

$$\int_0^\infty \frac{1-\cos px}{x^2}\,dx = \frac{\pi p}{2}$$

$$\int_0^\infty \frac{\cos px - \cos qx}{x}\,dx = \ln\frac{q}{p}$$

$$\int_0^\infty \frac{\cos px - \cos qx}{x^2}\, dx = \frac{\pi(q-p)}{2}$$

$$\int_0^\infty \frac{\cos mx}{x^2+a^2}\, dx = \frac{\pi}{2a} e^{-ma}$$

$$\int_0^\infty \frac{x \sin mx}{x^2+a^2}\, dx = \frac{\pi}{2} e^{-ma}$$

$$\int_0^\infty \frac{\sin mx}{x(x^2+a^2)}\, dx = \frac{\pi}{2a^2}\left(1-e^{-ma}\right)$$

$$\int_0^{2\pi} \frac{dx}{a+b\sin x} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

$$\int_0^{2\pi} \frac{dx}{a+b\cos x} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{a+b\cos x} = \frac{\cos^{-1}\left(\dfrac{b}{a}\right)}{\sqrt{a^2-b^2}}$$

$$\int_0^{2\pi} \frac{dx}{(a+b\sin x)^2} = \int_0^{2\pi} \frac{dx}{(a+b\cos x)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}}$$

$$\int_0^{2\pi} \frac{dx}{1-2a\cos x + a^2} = \frac{2\pi}{1-a^2} \quad \text{for } 0 < a < 1$$

$$\int_0^{\pi} \frac{x \sin x\, dx}{1-2a\cos x + a^2} = \begin{cases} \dfrac{\pi}{a}\ln|1+a| & \text{if } |a|<1 \\ \dfrac{\pi}{a}\ln\left|1+\dfrac{1}{a}\right| & \text{if } |a|>1 \end{cases}$$

$$\int_0^{\pi} \frac{\cos mx\, dx}{1-2a\cos x + a^2} = \frac{\pi a^m}{1-a^2} \quad \text{for } a^2 < 1\,,\ m = 0,1,2,\dots$$

$$\int_0^\infty \sin ax^2\, dx = \int_0^\infty \cos ax^2 = \frac{1}{2}\sqrt{\frac{\pi}{2a}}$$

$$\int_0^\infty \sin ax^n = \frac{1}{na^{1/n}}\Gamma\left(\frac{1}{n}\right)\sin\frac{\pi}{2n} \quad \text{for } n > 1$$

$$\int_0^\infty \cos ax^n = \frac{1}{na^{1/n}}\Gamma\left(\frac{1}{n}\right)\cos\frac{\pi}{2n} \quad \text{for } n > 1$$

$$\int_0^\infty \frac{\sin x}{\sqrt{x}}\, dx = \int_0^\infty \frac{\cos x}{\sqrt{x}}\, dx = \sqrt{\frac{\pi}{2}}$$

$$\int_0^\infty \frac{\sin x}{x^p}\, dx = \frac{\pi}{2\Gamma(p)\sin\left(\dfrac{p\pi}{2}\right)} \quad \text{for } 0 < p < 1$$

$$\int_0^\infty \frac{\cos x}{x^p}\, dx = \frac{\pi}{2\Gamma(p)\cos\left(\dfrac{p\pi}{2}\right)} \quad \text{for } 0 < p < 1$$

$$\int_0^\infty \sin ax^2 \cos 2bx\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}\left(\cos\frac{b^2}{a} - \sin\frac{b^2}{a}\right)$$

$$\int_0^\infty \cos ax^2 \cos 2bx\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}\left(\cos\frac{b^2}{a} + \sin\frac{b^2}{a}\right)$$

Definite integrals involving exponential functions

$$\int_0^\infty \sqrt{x}\,e^{-x}\,dx = \frac{1}{2}\sqrt{\pi}$$ (see also Gamma function)

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$$

$$\int_0^\infty \frac{e^{-ax}\sin bx}{x}\,dx = \tan^{-1}\frac{b}{a}$$

$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad \text{for } a>0$$ (the Gaussian integral)

$$\int_0^\infty e^{-ax^2}\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{\left(\frac{-b^2}{4a}\right)}$$

$$\int_0^\infty e^{-(ax^2+bx+c)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{\left(\frac{b^2-4ac}{4a}\right)}\cdot \operatorname{erfc}\frac{b}{2\sqrt{a}},\ \text{where } \operatorname{erfc}(p) = \frac{2}{\sqrt{\pi}}\int_p^\infty e^{-x^2}\,dx$$

$$\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}e^{\left(\frac{b^2-4ac}{4a}\right)}$$

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

$$\int_0^\infty x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}} \quad \text{for } a>0$$

$$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{2n-1}{2a}\int_0^\infty x^{2(n-1)}e^{-ax^2}\,dx = \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{2n+1}}} = \frac{(2n)!}{n!2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}} \quad \text{for } a>0\,,\ n=1,2,3\ldots$$
(where !! is the double factorial)

$$\int_0^\infty x^3 e^{-ax^2}\,dx = \frac{1}{2a^2} \quad \text{for } a>0$$

$$\int_0^\infty x^{2n+1} e^{-ax^2}\,dx = \frac{n}{a}\int_0^\infty x^{2n-1}e^{-ax^2}\,dx = \frac{n!}{2a^{n+1}} \quad \text{for } a>0\,,\ n=0,1,2\ldots$$

$$\int_0^\infty x^m e^{-ax^2}\,dx = \frac{\Gamma\left(\frac{m+1}{2}\right)}{2a^{\left(\frac{m+1}{2}\right)}}$$

$$\int_0^\infty e^{\left(-ax^2-\frac{b}{x^2}\right)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{-2\sqrt{ab}}$$

$$\int_0^\infty \frac{x}{e^x-1}\,dx = \zeta(2) = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n)\zeta(n)$$

$$\int_0^\infty \frac{x}{e^x+1}\,dx = \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots = \frac{\pi^2}{12}$$

$$\int_0^\infty \frac{\sin mx}{e^{2\pi x}-1}\,dx = \frac{1}{4}\coth\frac{m}{2} - \frac{1}{2m}$$

$$\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x} = \gamma$$ (where γ is Euler–Mascheroni constant)

$$\int_0^\infty \frac{e^{-x^2} - e^{-x}}{x}\,dx = \frac{\gamma}{2}$$

$$\int_0^\infty \left(\frac{1}{e^x - 1} - \frac{e^{-x}}{x}\right)dx = \gamma$$

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x\sec px}\,dx = \frac{1}{2}\ln\frac{b^2+p^2}{a^2+p^2}$$

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x\csc px}\,dx = \tan^{-1}\frac{b}{p} - \tan^{-1}\frac{a}{p}$$

$$\int_0^\infty \frac{e^{-ax}(1-\cos x)}{x^2}\,dx = \cot^{-1}a - \frac{a}{2}\ln\left|\frac{a^2+1}{a^2}\right|$$

$$\int_{-\infty}^\infty e^{-x^2}\,dx = \sqrt{\pi}$$

$$\int_{-\infty}^\infty x^{2(n+1)}e^{-\frac{1}{2}x^2}\,dx = \frac{(2n+1)!}{2^n n!}\sqrt{2\pi} \quad \text{for } n = 0, 1, 2, \ldots$$

Definite integrals involving logarithmic functions

$$\int_0^1 x^m(\ln x)^n\,dx = \frac{(-1)^n n!}{(m+1)^{n+1}} \quad \text{for } m > -1, n = 0, 1, 2, \ldots$$

$$\int_0^1 \frac{\ln x}{1+x}\,dx = -\frac{\pi^2}{12}$$

$$\int_0^1 \frac{\ln x}{1-x}\,dx = -\frac{\pi^2}{6}$$

$$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$

$$\int_0^1 \frac{\ln(1-x)}{x}\,dx = -\frac{\pi^2}{6}$$

$$\int_0^\infty \frac{\ln(a^2+x^2)}{b^2+x^2}\,dx = \frac{\pi}{b}\ln(a+b) \quad \text{for } a, b > 0$$

$$\int_0^\infty \frac{\ln x}{x^2+a^2}\,dx = \frac{\pi\ln a}{2a} \quad \text{for } a > 0$$

Definite integrals involving hyperbolic functions

$$\int_0^\infty \frac{\sin ax}{\sinh bx}\,dx = \frac{\pi}{2b}\tanh\frac{a\pi}{2b}$$

$$\int_0^\infty \frac{\cos ax}{\cosh bx}\,dx = \frac{\pi}{2b}\cdot\frac{1}{\cosh\frac{a\pi}{2b}}$$

$$\int_0^\infty \frac{x}{\sinh ax}\,dx = \frac{\pi^2}{4a^2}$$

$$\int_{-\infty}^\infty \frac{1}{\cosh x}\,dx = \pi$$

Frullani integrals

$\int_0^\infty \frac{f(ax) - f(bx)}{x}\, dx = \left(\lim_{x\to 0} f(x) - \lim_{x\to\infty} f(x)\right) \ln\left(\frac{b}{a}\right)$ holds if the integral exists and $f'(x)$ is continuous.

See also

- List of integrals
- Indefinite sum
- Gamma function
- List of limits

References

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