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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

FOURTH SEMESTER B.TECH DEGREE EXAMINATION MAY 2017

**MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND UMERICAL METHODS**

Max. Marks: 100 Duration: 3 Hours

***Normal distribution table is allowed in the examination hall.***

**PART A (MODULES I AND II)**

***Answer two full questions.***

1. a. Given that $f(x)=\frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x=0,1,2,3$ and 4, find k. Find the cumulative distribution function. (7)

   b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch

   i) none of the new buildings that violate the building code

   ii) one of the new buildings that violate the building code

   iii) at least two of the new buildings violate the building code (8)

2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np=\lambda$ a constant. (7)

   b. Find the value of k for the probability density $f(x)$ given below and hence find its mean and variance where

$$f(x)=\begin{cases} kx^3 & 0<x<1 \\ 0 & otherwise \end{cases}$$

(8)

3. a. A random variable has normal distribution with $\mu=62.4$. Find it's standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)

   b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will

   i) have to be reset in less than 20 days

   ii) not have to be reset in at least 60 days. (8)

**PART B (MODULES III AND IV)**

***Answer two full questions.***

4. a. Use Fourier integral to show that $\int_0^\infty \frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases} 0 & if\ x<0 \\ & if\ x=0 \\ \pi e^{-x} & if\ x>0 \end{cases}$ (7)

   b. Represent $f(x)=\begin{cases} x^2 & 0<x<1 \\ 0 & x>1 \end{cases}$ as a Fourier cosine integral. (8)



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5. a. Find the Fourier transform of $f(x) = \begin{cases} 1 & if\ |x| < 1 \\ 0 & otherwise \end{cases}$ (7)

b. Find the Laplace transforms of the following

i) $\cos t - t \sin t$ ii) $4t\, e^{-2t}$ (8)

6. a. Find the inverse Laplace transform of the following

i) $\dfrac{2s+1}{s^2+2s+5}$ ii) $\dfrac{(2s-10)}{s^3} e^{-5s}$ (8)

b. Solve $y'' + 2y' + 5y = 25t \quad y(0) = -2 \quad y'(0) = -2$ using Laplace transforms (7)

## PART C (MODULES V AND VI)

***Answer two full questions.***

7. a. Solve $f(x) = x - 0.5\cos x = 0$ near $x = 0$ by fixed point iteration method. (7)

b. Solve $f(x) = 2x - \cos x = 0$ by Newton Raphson's method (7)

c. Find $f(9.2)$ from the values given below by Lagrange's interpolation formula

| $x$ | 8 | 9 | 9.5 | 11 |
|---|---|---|---|---|
| $f(x)$ | 2.197225 | 2.251292 | 2.397895 | 2.079442 |

(6)

8. a. Given $(x_j, f(x_j)) = (0.2, 0.9980)$ $(0.4, 0.9686)$, $(0.6, 0.8443)$, $(0.8, 0.5358)$, $(1,0)$, find $f(0.7)$ based on 0.2 0.4 and 0.6rtusin g Newton 's interpolation formula. (10)

b. Solve $10x_1 + x_2 + x_3 = 6,\ x_1 + 10x_2 + x_3 = 6,\ x_1 + x_2 + 10x_3 = 6$ by Gauss-Seidel iteration method starting at $x_1 = 1 \quad x_2 = 1$ and $x_3 = 1$ correct to 4 digits. (10)

9. a. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution. (7)

b. Solve $y' = y,\ y(0) = 1$ by Euler method to find $y(1)$ with $h = 0.2$ (7)

c. Solve $y' = 1 + y^2 \quad y(0) = 0$ by fourth order Runge-Kutta method with $h = 0.1$, 5 steps. (6)

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1) a) Given that $f(x) = k/2^x$, is a probability distribution of a random variable that can take on the values $x = 0, 1, 2, 3$ and 4, find $k$. Find the cumulative distribution function.

$$f(x) = \frac{k}{2^x} \; ; \; x = 0, 1, 2, 3, 4$$

| X: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f(x): | k | k/2 | k/4 | k/8 | k/16 |

$\Sigma f(x) = 1$ ie, $k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} + \frac{k}{16} = 1$

$$\frac{k(16+8+4+2+1)}{16} = 1$$

$$31k = 16$$

$$k = 16/31$$

$\therefore$ Pdf is,

| X : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f(x) : | 16/31 | 8/31 | 4/31 | 2/31 | 1/31 |

Corresponding cdf is,

| X : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| F(x) : | 16/31 | 24/31 | 28/31 | 30/31 | 1 |

b) If 6 of the 18 new buildings in a city violate the buiding code, what is the probability that a building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch
i) none of the new buildings that violate the building code
ii) one of the new buildings that violate the building code.
iii) at least two of the new buildings violate the building code.

→ Let X denote no. of buildings violating the building code
P: probability of building violating the building code

$p = \frac{6}{18} = 1/3$ , $\therefore q = 2/3$ , $n = 4$

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$f(x) = {}^nC_x p^x q^{n-x}$

$f(x) = {}^4C_x (1/3)^x (2/3)^{4-x}$

i, $P(x=0) = {}^4C_0 (1/3)^0 (2/3)^4$

$= (2/3)^4$

$= 16/81$

ii, $P(x=1) = {}^4C_1 (1/3)^1 (2/3)^3$

$= \frac{4!}{1!3!} \times \frac{1}{3} \times \frac{8}{27}$

$= 32/81$

iii, $P(\text{atleast } 2) = P(x \geq 2)$

$= 1 - P(x < 2)$

$= 1 - [P(0) + P(1)]$

$= 1 - \left[\frac{16}{81} + \frac{32}{81}\right]$

$= 1 - \frac{48}{81}$

$= \frac{81-48}{81}$

$= \frac{33}{81}$

2, a) Prove that binomial distribution with parameters $n$ and $p$ can be approximated to Poisson distribution when $n$ is large and $p$ is small with $np = \lambda$ a constant.

We know,

$b(x,n,p) = {}^nC_x p^x q^{n-x}$

$= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$= \frac{n(n-1)(n-2)\cdots(n-x+1)\cancel{(n-x)!}}{x!\cancel{(n-x)!}} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x}$

$= \frac{n^x}{x!}\left[(1-1/n)(1-2/n)\cdots\left(1-\frac{(x-1)}{n}\right)\right] \frac{\lambda^x}{n^x} \frac{[1-\lambda/n]^n}{[1-\lambda/n]^x}$

as $n \to \infty$ and $p \to 0$ we get

$b(x,n,p) = \frac{1}{x!} \times 1 \times \lambda^x \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n$

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$$= \frac{1^x e^{-1}}{x!}$$

$$= f(x,1)$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right) = e^{-1}$$

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b) Find the value of k for the probability f(x) given below and hence find its mean and variance where

$$f(x) = \begin{cases} kx^3, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

we know, $\int_{-\infty}^{\infty} f(x)\,dx = 1$, $\because$ it is pdf

$$\int_0^1 kx^3\,dx = 1$$

$$\left[k\frac{x^4}{4}\right]_0^1 = 1$$

$$\frac{k}{4} = 1$$

$$k = 4$$

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)$$

$$= \int_0^1 x k x^3\,dx$$

$$= \int_0^1 4x^4\,dx$$

$$= \left[\frac{4x^5}{5}\right]_0^1$$

$$= 4/5$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)$$

$$= \int_0^1 x^2 \cdot 4x^3\,dx$$

$$= \int_0^1 4x^5\,dx$$

$$= \left[\frac{4x^6}{6}\right]_0^1$$

$$= 4/6 = 2/3.$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

$$= 2/3 - (4/5)^2$$

$$= 2/3 - 16/25$$

$$= \frac{50-48}{75}$$

$$= 2/75$$

Mean = $4/5$

Variance = $2/75$

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3) a) A random variable has normal distribution with $\mu = 62.4$. find its standard deviation if the probability is 0.2 that will take on a value greater than 79.2

$$\mu = 62.4$$

$$P(X > 79.2) = 0.2$$

$$P\left(Z > \frac{79.2-\mu}{\sigma}\right) = 0.2$$

$$P\left(Z > \frac{79.2-62.4}{\sigma}\right) = 0.2$$

$$P\left(Z > \frac{16.8}{\sigma}\right) = 0.2$$

$$0.5 - P\left(0 < Z < \frac{16.8}{\sigma}\right) = 0.2$$

$$P\left(0 < Z < \frac{16.8}{\sigma}\right) = 0.3$$

$$\frac{16.8}{\sigma} = 0.84$$

$$\underline{\underline{\sigma = 20}}$$

b) The amount of time that a survilance camera will see without having to be reset is a random variable having exponential distribution with parameter 50 days. Find the probability that such a camera will

i) have to be reset in less than 20 days

ii) not have to be reset in atleast 60 days

$$\rightarrow F(x) = \frac{1}{\beta} e^{-x/\beta} \quad ; \beta = 50 \text{ days}$$

$$F(x) = \frac{1}{50} e^{-x/50}$$

$$P(X < 20) = \int_{-\infty}^{20} \frac{1}{50} e^{-x/50} dx$$

$$= \left[\frac{1}{50} \frac{e^{-x/50}}{-1/50}\right]_{-\infty}^{20}$$

$$= \left[\frac{e^{-x/50}}{-1}\right]_{-\infty}^{20}$$

$$= -e^{-20/50} - -e^{-\infty/50}$$

$$\underline{\underline{P(X < 20) = 1 - e^{-2/5}}}$$

$$P(X > 60) = \int_{60}^{\infty} \frac{1}{50} e^{-x/50} dx$$

$$= \frac{1}{50}\left[\frac{e^{-x/50}}{-1/50}\right]_{60}^{\infty}$$

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$$= \frac{1}{50} \times \frac{-50}{1} \left[ e^{-\infty/50} - e^{-60/50} \right]$$

$$= -0 + e^{-6/5}$$

$$= \underline{\underline{e^{-6/5}}}$$

4) a) Use fourier integral to show that,

$$\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0, & x<0 \\ \pi/2, & x=0 \\ \pi e^{-x}, & x>0 \end{cases}$$

Let $f(x) = \begin{cases} 0, & x<0 \\ \pi/2, & x=0 \\ \pi e^{-x}, & x>0 \end{cases}$

let the fourier integral representation of $f(x)$ is,

$$f(x) = \int_0^\infty [A(\omega) \cos \omega x + B(\omega) \sin \omega x] d\omega$$

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos \omega x \, dx$$

$$= \frac{1}{\pi} \int_0^\infty \pi e^{-x} \cos \omega x \, dx$$

$$= \left[ \frac{e^{-x}}{1+\omega^2} (-\cos \omega x + \omega \sin \omega x) \right]_0^\infty$$

$$= 0 - \frac{1}{1+\omega^2} (-1)$$

$$A(\omega) = \underline{\underline{\frac{1}{1+\omega^2}}}$$

$$B(\omega) = \frac{1}{\pi} \int_0^\infty \pi e^{-x} \sin \omega x \, dx$$

$$= \left[ \frac{e^{-x}}{1+\omega^2} (-\sin \omega x - \omega \cos \omega x) \right]_0^\infty$$

$$= 0 - \frac{1}{1+\omega^2} (-\omega)$$

$$B(\omega) = \underline{\underline{\frac{\omega}{1+\omega^2}}}$$

$\therefore$ fourier integral representation is,

$$f(x) = \int_0^\infty \left[ \frac{1}{1+\omega^2} \cos \omega x + \frac{\omega}{1+\omega^2} \sin \omega x \right] d\omega$$

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when $x=0$

$$f(0) = \int_0^\infty \frac{1}{1+\omega^2}\, d\omega$$

$$= [\tan^{-1}\omega]_0^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}0$$

$$= \pi/2 - 0$$

$$= \underline{\underline{\pi/2}}$$ hence verified

b) Represent $f(x) = \begin{cases} x^2, & 0<x<1 \\ 0, & x>1 \end{cases}$ as a Fourier cosine integral.

→ Fourier cosine integral representation is,

$$f(x) = \int_0^\infty A(\omega)\cos\omega x\, d\omega$$

$$A(\omega) = \frac{2}{\pi}\int_0^1 x^2 \cos\omega x\, dx$$

$$= \frac{2}{\pi}\left[x^2\left(\frac{\sin\omega x}{\omega}\right) - 2x\left(-\frac{\cos\omega x}{\omega^2}\right) + 2\left(-\frac{\sin\omega x}{\omega^3}\right)\right]_0^1$$

$$= \frac{2}{\pi}\left[\frac{\sin\omega}{\omega} + \frac{2\cos\omega}{\omega^2} - \frac{2\sin\omega}{\omega^3} - 0\right]$$

$$A(\omega) = \underline{\underline{\frac{2}{\pi\omega^3}\left[\omega^2\sin\omega + 2\omega\cos\omega - 2\sin\omega\right]}}$$

$$\therefore f(x) = \underline{\underline{\int_0^\infty \frac{2}{\pi\omega^3}\left[\omega^2\sin\omega + 2\omega\cos\omega - 2\sin\omega\right]\cos\omega x\, d\omega}}.$$

5. a) Find the fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x|<1 \\ 0 & \text{otherwise} \end{cases}$

→ Fourier transform of $f(x)$ is,

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-i\omega x}}{-i\omega}\right]_{-1}^{1}$$

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$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{e^{-i\omega} - e^{i\omega}}{-i\omega} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{\cos\omega - i\sin\omega - \cos\omega - i\sin\omega}{-i\omega} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{-2i\sin\omega}{-i\omega} \right]$$

$$= \frac{2\sin\omega}{\sqrt{2\pi}\cdot\omega}$$

$$= \sqrt{\frac{2}{\pi}}\,\frac{\sin\omega}{\omega}.$$

b. Find the Laplace transforms of the following.

i, $\cos t - t\sin t$ ii, $4te^{-2t}$

i, $L\{\cos t - t\sin t\} = L\{\cos t\} - L\{t\sin t\}$

$$= \frac{s}{s^2+1} - (-1)\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= \frac{s}{s^2+1} + \frac{-1}{(s^2+1)^2}\cdot 2s$$

$$= \frac{s}{s^2+1} - \frac{2s}{(s^2+1)^2}$$

ii, $L\{4te^{-2t}\} = 4\,L\{te^{-2t}\}$

$$= 4\cdot\frac{1}{(s+2)^2}$$

$$= \frac{4}{(s+2)^2}.$$

6. a) Find the Inverse Laplace transform of the following

i, $\dfrac{2s+1}{s^2+2s+5}$ ii, $\dfrac{(2s-10)}{s^3}\cdot e^{-5s}$

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i) $\dfrac{2s+1}{s^2+2s+5} = \dfrac{2s+1}{s^2+2s+1+4}$

$= \dfrac{2s+1}{(s+1)^2+4}$

$= \dfrac{2s+1+1-1}{(s+1)^2+2^2}$

$= \dfrac{2s+2}{(s+1)^2+2^2} - \dfrac{1}{(s+1)^2+2^2}$

$= \dfrac{2\cdot(s+1)}{(s+1)^2+2^2} - \dfrac{1}{(s+1)^2+2^2}$

$L^{-1}\left\{\dfrac{2s+1}{s^2+2s+5}\right\} = 2L^{-1}\left\{\dfrac{s+1}{(s+1)^2+2^2}\right\} - L^{-1}\left\{\dfrac{1}{(s+1)^2+2^2}\right\}$

$= 2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t$

$= 2e^{-t}\left[\cos 2t - \frac{1}{4}\sin 2t\right]$

ii) $\dfrac{(2s-10)}{s^3}e^{-5s}$

$L^{-1}\left\{e^{-5s}\cdot\dfrac{(2s-10)}{s^3}\right\} = f(t-5)\,u(t-5)$

$L^{-1}\left\{\dfrac{2s-10}{s^3}\right\} = L^{-1}\left\{\dfrac{2s}{s^3} - \dfrac{10}{s^3}\right\}$

$= L^{-1}\left\{\dfrac{2}{s^2} - \dfrac{10}{s^3}\right\}$

$= 2t - \frac{10}{2}t^2$

$= 2t - 5t^2$

$\therefore L^{-1}\left\{e^{-5s}\dfrac{(2s-10)}{s^3}\right\} = \left[2(t-5) - 5(t-5)^2\right]\begin{cases}0, & t<2\\ 1, & t>2\end{cases}$

$= \left[2t-10-5t^2+50t-125\right]\begin{cases}0, & t<2\\ 1, & t>2\end{cases}$

$= 52t - 5t^2 - 135, \quad t>2$

b) Solve $y'' + 2y' + 5y = 25t$, $y(0) = -2$, $y'(0) = -2$ using Laplace transforms.

$\rightarrow y'' + 2y' + 5y = 25t$

Taking laplace transform on both sides,

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$$L\{y''\} + 2L\{y'\} + 5L\{y\} = 25\,L\{t\}$$

$$s^2 L\{y\} - s\,y(0) - y'(0) + 2[sL\{y\} - y(0)] + 5L\{y\} = \frac{25}{s^2}$$

$$L\{y\}\,[s^2+2s+5] - s\,y(0) - y'(0) - 2\,y(0) = \frac{25}{s^2}$$

$$(s^2+2s+5)\,L\{y\} + 2s + 2 + 4 = \frac{25}{s^2}$$

$$[s^2+2s+5]\,L\{y\} = \frac{25}{s^2} - 2s - 6$$

$$L\{y\} = \frac{25}{s^2(s^2+2s+5)} - \left[\frac{6+2s}{s^2+2s+5}\right]$$

$$\frac{25-6s^2-2s^3}{s^2(s^2+2s+5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+2s+5}$$

$$25-6s^2-2s^3 = As(s^2+2s+5) + B(s^2+2s+5) + [Cs+D]\,s^2$$

if $s=0$, $25 = 5B$

$\Rightarrow \underline{\underline{B=5}}$

equating coefficient of $s^3 \Rightarrow -2 = A + C$

$s^2 \Rightarrow 6 = 2A + B + D \Rightarrow 2A + D = -11$

$s \Rightarrow 0 = 5A + 2B \Rightarrow 5A = -10$

$\underline{\underline{A=-2}}$

$\therefore \underline{\underline{C=0}}$

$\therefore D = -11+4 = \underline{\underline{-7}}$

$$\therefore L\{y\} = \frac{-2}{s} + \frac{5}{s^2} - \frac{7}{s^2+2s+5}$$

$$L\{y\} = \frac{-2}{s} + \frac{5}{s^2} - \frac{7}{(s+1)^2+4}$$

$$y = -2L^{-1}\left[\frac{1}{s}\right] + 5L^{-1}\left\{\frac{1}{s^2}\right\} - 7L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}$$

$$y = -2 + 5t - 7e^{-t}\cdot\frac{1}{2}\sin 2t$$

$$\therefore \underline{\underline{y = -2 + 5t - \frac{7}{2}e^{-t}\sin 2t}}$$

7 a) Solve $f(x) = x - 0.5\cos x = 0$ near $x=0$ by fixed point iteration method.

→ Let $f(x) = x - 0.5\cos x$

$f(0) < 0$, $f(1) > 0$ $\therefore$ one root lies btw 0 & 1

$\therefore I = (0,1)$

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$x - 0.5\cos x = 0$

$x = 0.5\cos x$

$g(x) = 0.5\cos x$

$g'(x) = -0.5\sin x$

$|g'(0)| = 0 < 1$

$|g'(1)| = 0.42 < 1$

hence $|g'(x)| < 1 \ \forall x \in I$

Apply, $x_{n+1} = 0.5\cos x_n$

Choose, $x_0 = 0.5$

$x_1 = 0.5\cos(0.5) = 0.43879$

$x_2 = 0.5\cos(0.43879) = 0.45263$

$x_3 = 0.5\cos(0.45263) = 0.44965$

$x_4 = 0.5\cos(0.44965) = 0.45029$

$x_5 = 0.5\cos(0.45029) = 0.45016$

$x_6 = 0.5\cos(0.45016) = 0.45018$

$x_7 = 0.5\cos(0.45018) = 0.45018$

$\therefore$ One of the root is $\approx 0.45018$.

b) Solve $f(x) = 2x - \cos x = 0$ by Newton Raphson's method

→ Let $f(x) = 2x - \cos x$

$f'(x) = 2 + \sin x$

$f(0) < 0$ & $f(1) > 0$

$\therefore$ one root lies btw $(0,1)$

$\therefore I = (0,1)$

Now, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$x_{n+1} = x_n - \frac{[2x_n - \cos x_n]}{[2 + \sin x_n]} = \frac{2x_n + x_n\sin x_n - 2x_n + \cos x_n}{2 + \sin x_n}$$

$$x_{n+1} = \frac{x_n\sin x_n + \cos x_n}{2 + \sin x_n}$$

Choose, $x_0 = 0.5$

$$x_1 = \frac{0.5\sin(0.5) + \cos(0.5)}{2 + \sin(0.5)} = 0.4506$$

$x_2 = 0.4502$

$x_3 = 0.4502$

$\therefore$ one root is $\approx 0.4502$

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c) Find $f(9.2)$ from the values given below by Lagrange's interpolation for formula.

| $x$ | 8 | 9 | 9.5 | 11 |
|---|---|---|---|---|
| $f(x)$ | 2.197225 | 2.251292 | 2.397895 | 2.079442 |

→ By Lagrange's interpolation

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

to find → $f(9.2)$

$\therefore x = 9.2,\ x_0 = 8,\ x_1 = 9,\ x_2 = 9.5,\ x_3 = 11$

$y_0 = 2.197225,\ y_1 = 2.251292,\ y_3 = 2.397895,\ y_4 = 2.079442.$

$$y = f(9.2) = \frac{(9.2-9)(9.2-9.5)(9.2-11)}{(8-9)(8-9.5)(8-11)} \times 2.197225 + \frac{(9.2-8)(9.2-9.5)(9.2-11)}{(9-8)(9-9.5)(9-11)} \times 2.251292$$

$$+ \frac{(9.2-8)(9.2-9)(9.2-11)}{(9.5-8)(9.5-9)(9.5-11)} \times 2.397895 + \frac{(9.2-8)(9.2-9)(9.2-9.5)}{(11-8)(11-9)(11-9.5)} \times 2.079442$$

$f(9.2) = -0.0527334 + 1.458837216 + 0.92079168 - 0.016635536$

$= \underline{\underline{2.3103}}$

$\therefore f(9.2) = \underline{\underline{2.3103}}$

8. a) Given $(x_j, f(x_j)) = (0.2, 0.9980), (0.4, 0.9686), (0.6, 0.8443)$ $(0.8, 0.5358), (1, 0)$. Find $f(0.7)$ based on $0.2, 0.4$ & $0.6$ using Newton's interpolation formula.

| $x_i$ | $f(x_i)$ | $\nabla f_i$ | $\nabla^2 f_i$ | $\nabla^3 f_i$ |
|---|---|---|---|---|
| 0.2 | 0.9980 | | | |
| | | -0.0294 | | |
| 0.4 | 0.9686 | | -0.0949 | |
| | | -0.1243 | | -0.0893 |
| 0.6 | 0.8443 | | -0.1842 | |
| | | -0.3085 | | |
| 0.8 | 0.5358 | | | |

$$r = \frac{x - x_n}{h}$$

$$r = \frac{0.7 - 0.8}{0.2}$$

$$r = \underline{\underline{-0.5}}$$

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$r = -0.5$

Backward difference formula.

$$f(x) = f_n + r\nabla f_n + \frac{r(r+1)}{2!}\nabla^2 f_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 f_n$$

$$= 0.5358 + (-0.5)[-0.3085] + \frac{(-0.5)(-0.5+1)}{2!}[-0.1842] + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}[-0.0893]$$

$$= 0.5358 + 0.15425 + 0.023025 + 5.58125 \times 10^{-3}$$

$\therefore f(0.7) = 0.71866$

b) Solve $10x_1 + x_2 + x_3 = 6$, $x_1 + 10x_2 + x_3 = 6$, $x_1 + x_2 + 10x_3 = 6$ by Gauss-Seidal iteration method starting at $x_1 = 1$, $x_2 = 1$ and $x_3 = 1$ correct to 4 digits.

→ The system is diagonally dominant.

$$\therefore x_1 = \frac{6 - x_3 - x_2}{10}$$

$$x_2 = \frac{6 - x_3 - x_1}{10}$$

$$x_3 = \frac{6 - x_1 - x_2}{10}$$

Choose $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

| Iteration | $x_1 = \frac{6 - x_3 - x_2}{10}$ | $x_2 = \frac{6 - x_3 - x_1}{10}$ | $x_3 = \frac{6 - x_1 - x_2}{10}$ |
|---|---|---|---|
| 1 | 0.4 | 0.46 | 0.514 |
| 2 | 0.5026 | 0.4983 | 0.4999 |
| 3 | 0.5002 | 0.4999 | 0.5 |
| 4 | 0.5 | 0.5 | 0.5 |

$\therefore$ The soln. is $x_1 = 0.5$, $x_2 = 0.5$, $x_3 = 0.5$

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9 a) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution.

→ According to Simpsons rule,

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+f(b)+2\{f(x_2)+f(x_4)+\dots\}+4\{f(x_1)+f(x_3)+\dots\}\right]$$

$h = \frac{b-a}{n}$ , here $b=1,\ a=0,\ n=4$

$h = \frac{1-0}{4} = \underline{\underline{0.25}}$

| $x$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.9412 | 0.8 | 0.64 | 0.5 |

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{3}\left[f(0)+f(1)+2\{f(0.5)\}+4\{f(0.25)+f(0.75)\}\right]$$

$$= \frac{0.25}{3}\left[1+0.5+2(0.8)+4(0.9412+0.64)\right]$$

$$= \frac{0.25}{3}\left[1.5+1.6+6.3248\right]$$

$$= \underline{\underline{0.7854}}$$

→ $$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^1$$

$$= \tan^{-1}(1-0)$$

$$= \underline{\underline{0.7854}}$$

b) Solve $y' = y$, $y^{(0)} = 1$ by Euler method to find $y^{(1)}$ with $h = 0.2$

→ $y' = y$ $\quad \therefore f(x,y) = y$

$y(0) = 1$

$x_0 = 0$

$y_0 = 1$

$h = 0.2$

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$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + 0.2[1]$$

$$y_1 = 1.2 \qquad \therefore y(0.2) = 1.2$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.2 + 0.2 f(0.2, 1.2)$$

$$= 1.2 + 0.2[1.2]$$

$$y_2 = 1.44 \qquad \therefore y(0.4) = 1.44$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$= 1.44 + 0.2 f(0.4, 1.44)$$

$$= 1.44 + 0.2[1.44]$$

$$y_3 = 1.728 \qquad \therefore y(0.6) = 1.728$$

$$y_4 = y_3 + hf(x_3, y_3)$$

$$= 1.728 + 0.2 f(0.6, 1.728)$$

$$= 1.728 + 0.2(1.728)$$

$$y_4 = 2.0736 \qquad \therefore y(0.8) = 2.0736$$

$$y_5 = y_4 + hf(x_4, y_4)$$

$$= 2.0736 + 0.2 f(0.8, 2.0736)$$

$$= 2.0736 + 0.2[2.0736]$$

$$y_5 = 2.4883 \qquad \therefore y(1) = 2.4883$$

$$\Rightarrow y(1) = 2.4883$$

c) Solve $y' = 1 + y^2$, $y(0) = 0$ by fourth order Runge-Kutta method with $h = 0.1$, 5 steps.

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c) Solve $y' = 1 + y^2$, $y(0) = 0$ by forth order Runge Kutta method with $h = 0.1$ in 2 steps.

$\rightarrow$ $y' = 1 + y^2$ ; $y(0) = 0$

$x_0 = 0$, $y_0 = 0$, $f(x, y) = 1 + y^2$, $h = 0.1$

$k_1 = h f(x_0, y_0) = h f(0, 0)$

$k_1 = 0.1[1 + 0]$

$\underline{\underline{k_1 = 0.1}}$

$k_2 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$

$= 0.1 f\left(\frac{0.1}{2}, \frac{0.1}{2}\right)$

$= 0.1\left[1 + \left(\frac{0.1}{2}\right)^2\right]$

$\therefore \underline{\underline{k_2 = 0.10025}}$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1 f\left(\frac{0.1}{2}, \frac{0.10025}{2}\right)$

$= 0.1\left[1 + \left(\frac{0.10025}{2}\right)^2\right]$

$\therefore \underline{\underline{k_3 = 0.10025}}$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.1 f(0.1, 0.10025)$

$= 0.1[1 + (0.10025)^2]$

$\therefore \underline{\underline{k_4 = 0.1010}}$

$y_1 = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$

$= 0 + \frac{1}{6}[0.1 + 2(0.10025 + 0.10025) + 0.1010]$

$\underline{\underline{y_1 = y(x_1) = 0.10033}}$.

Now; $x_1 = 0.1$, $y_1 = 0.10033$, $h = 0.1$

$k_1 = h f(x_1, y_1) = 0.1 f(0.1, 0.10033)$

$= 0.1[1 + (0.10033)^2]$

$\therefore \underline{k_1 = 0.10101}$

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$$k_2 = h f\left(x_1 + \frac{h}{2},\ y_1 + \frac{k_1}{2}\right)$$

$$= 0.1 f(0.15, 0.1508)$$

$$= 0.1\left[1 + (0.1508)^2\right]$$

$$\therefore k_2 = 0.10228$$

$$k_3 = h f\left[x_1 + \frac{h}{2},\ y_1 + \frac{k_2}{2}\right]$$

$$= 0.1 f(0.15, 0.15147)$$

$$= 0.1\left[1 + (0.15147)^2\right]$$

$$\therefore k_3 = 0.10229$$

$$k_4 = h f(x_1 + h,\ y_1 + k_3)$$

$$= 0.1 f(0.2, 0.20262)$$

$$= 0.1\left[1 + (0.20262)^2\right]$$

$$\therefore k_4 = 0.1041$$

$$y_2 = y_1 + \frac{1}{6}\left[k_1 + 2(k_2 + k_3) + k_4\right]$$

$$= 0.10033 + \frac{1}{6}\left[0.10101 + 2(0.10228 + 0.10229) + 0.1041\right]$$

$$\therefore y_2 = 0.20271$$

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