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Q. When a train is moving at 25 m/s, steam is shut off & brakes are applied. The speed v of the train in m/s after t sec is given in the following table. Find the distance moved by the train before coming to a halt using Simpson's rule & trapezoidal rule.

t	0	10	20	30	40	50	60
v	25	18.1	12.3	7.6	4	1.6	0

04.05.2019

* Numerical Solution of First order D.E :

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0 \rightarrow \text{initial cond}^n.$$

$x \rightarrow$ independent

$y \rightarrow$ dependent

$y = y(x)$ $\qquad$ $x_1 = x_0 + h$

$y_0 = y(x_0)$ $\qquad$ $x_2 = x_1 + h$

$y_1 = y(x_1)$ $\qquad$ $\vdots$

$y_2 = y(x_2)$

$\vdots$

Euler method

Improved Euler method

Runge - Kutta method (4th order)

Q. To solve $\frac{dy}{dx} = f(x, y)$ $y(x_0) = y_0$ numerically, we will be finding the values of y at $x_1, x_2, \ldots$

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where $y_1 = y(x_1)$, $y_2 = y(x_2)$ where $x_1 = x_0 + h$
$x_2 = x_1 + h$.

I. Euler method: $y_{n+1} = y_n + hf(x_n, y_n)$; $(n = 0, 1, 2, 3 \ldots$

Q. Solve: $y' + 0.2y = 0$; $y(0) = 5$; $h = 0.2$. Do 10 steps by Euler formula.

$y_{n+1} = y_n + hf(x_n, y_n)$

Here $f(x, y) = -0.2y$; $x_0 = 0$, $y_0 = 5$, $h = 0.2$.

$n = 0$: $y_1 = y_0 + hf(x_0, y_0) = 5 + 0.2(-1) = \underline{4.8}$

$n = 1$: $y_2 = y_1 + hf(x_1, y_1) = 4.8 + 0.2(-0.2 \times 4.8)$

$= 4.8[1 - 0.04] = 4.8 \times 0.96$

$= \underline{4.608}$.

$$\begin{array}{r} 96 \times \\ 48 \\ \hline 768 + \\ 3840 \\ \hline 4.608 \end{array}$$

$n = 2$: $y_3 = y_2 + hf(x_2, y_2) = 4.608 + (0.2)(-0.2 \times 4.608)$

$= \underline{4.42368}$.

$n = 3 = y_4 = y_3 + h(-0.2y_3) = y_3[1 - 0.04] = y_3 \times 0.96$

$= 4.2467328$

$n = 4 \Rightarrow y_5 = 0.96 \times y_4 = 4.076863488$

$n = 5 \rightarrow y_6 = 3.913788948$

$n = 6 \Rightarrow y_7 = 3.757237391$

$n = 7 \Rightarrow y_8 = 3.606947895$

$n = 8 \Rightarrow y_9 = 3.462669979$

$n = 9 \Rightarrow y_{10} = 3.32416318$

Q. Solve: $y' = (x + y)^2$; $y(0) = 0$; $h = 0.1$

$f(x, y) = (x + y)^2 \Rightarrow x_0 = 0$, $y_0 = 0$; $h = 0.1$

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$\therefore n = 0$: $y_1 = y_0 + hf(x_0, y_0) = \underline{0}$; $y_2 = 0 + 0.1(\ldots$

x	y	$f(x, y)$	$hf(x, y)$	$f(x,y) + hf(x,y)$
$x_0 = 0$	$y_0 = 0$	0	0	
$x_1 = 0.1$	$y_1 = 0$	0.01	0.001	0
$x_2 = 0.2$	$y_2 = 0.001$	4.004001	0.4004001	0.001
$x_3 = 0.3$	$y_3 = 0.4004001$	0.49056	0.049056	5.0401×10^{-3}
$x_4 = 0.4$	$y_4 = 0.049056$	0.793097	0.0793097	0.014345
$x_5 = 0.5$	$y_5 = 0.0793097$			0.0315132
$x_6 = 0.6$	$y_6 = 0.059764$			0.059764
$x_7 = 0.7$	$y_7 = 0.103293$			0.103293
$x_8 = 0.8$	$y_8 = 0.167821$			0.167821
$x_9 = 0.9$	$y_9 = 0.261488$		0.396394	0.261488

$y_5 = 0.0315132$

$y_{10} = \underline{\underline{0.396394}}$

Q. Given $y' = x^2 + y$; $y(0) = 1$; determine the value of y when $x = 0.1$

$h = 0.1 \Rightarrow \frac{1}{2}$ mark

$h = 0.05 \Rightarrow 2$ steps

$h = 0.025 \Rightarrow 4$ steps

Let $h = 0.05$; $x_0 = 0$, $y_0 = 1$; $f(x, y) = x^2 + y$.

x	y	$f(x, y)$	$hf(x, y)$	(y_{n+1})
0	1	1	0.05	(1.05)
0.05	1.05	1.1525	~~0.057450~~ ~~2.625×10^{-3}~~ 0.052625	(1.102625)
0.1	~~0.057450~~ ~~2.625×10^{-3}~~ 1.102625	0.06775 ~~0.012625~~	~~6.3125×10^{-4}~~ 0.05563125	1.15825625

$\Rightarrow$ $y(0.1) = \underline{1.15825625}$

Adv. of paging: (i) avoids ext. fragmentation (ii) avoid need for compaction, whereas segment. does not (iii) solves considerable prob. of fitting mem. chunks of varying sizes onto the backing store.

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* ~~How~~ Paging:

(i) Phy. address space of a process can be non-contiguous.

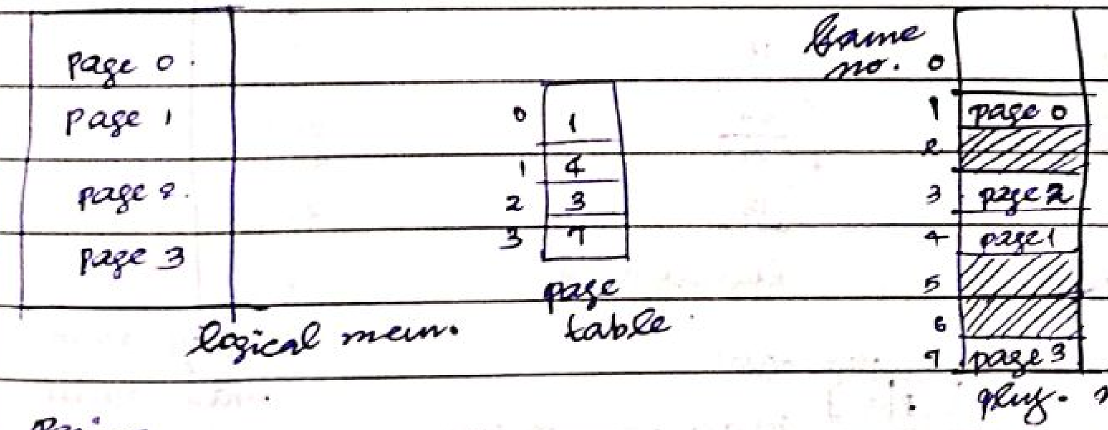
(ii) Process is divided into fixed-size blocks, each of which may reside in a diff. part of phy. mem.

(iii) Divide phy. mem. into fixed-sized blocks called frames.
- size of a frame is 2^x where $512 \text{ bytes} < 2^x \leq 16\text{M bytes}$

(iv) Divide logical mem. into blocks of same size as frames called pages.

(v) Backing store, where the pgm. is permanently residing, is also split into ~~blocks~~ storage units - called blocks.

$$(\text{Block size})_{\text{phy. mem}} = (\text{Block size})_{\text{logical mem.}}$$



(vi) Paging
- avoids external fragmentation ∵ page size is same.
- still have internal " . eg: page size = 50 ⇒ 155 bytes process requires 4 pages though last page has just 5 bytes.

segment size may differ

1. Keep track of all free frames.

2. To run a pgm. of size N pages, need to find N free frames to load pgm. from backing store.

3. Set up a page table to translate logical to phy. address

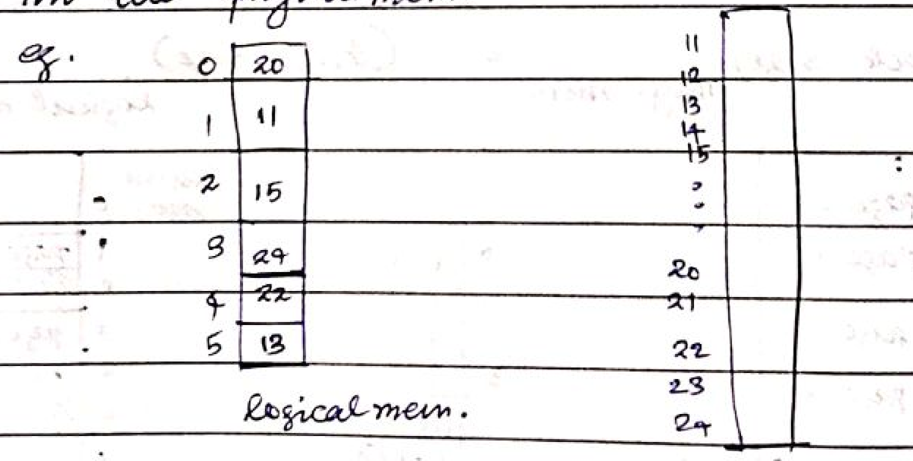
4. Page table is kept in memory:

- page-table BR (PTBR) points to the page table
- " " length reg. (PTLR) indicates size of the page table.

No. of entries in logical mem. = n (phy. mem.)

NOTE logical mem. entries contain the page frame in the phy. ~~a~~ mem.

eg.



page no. page offset

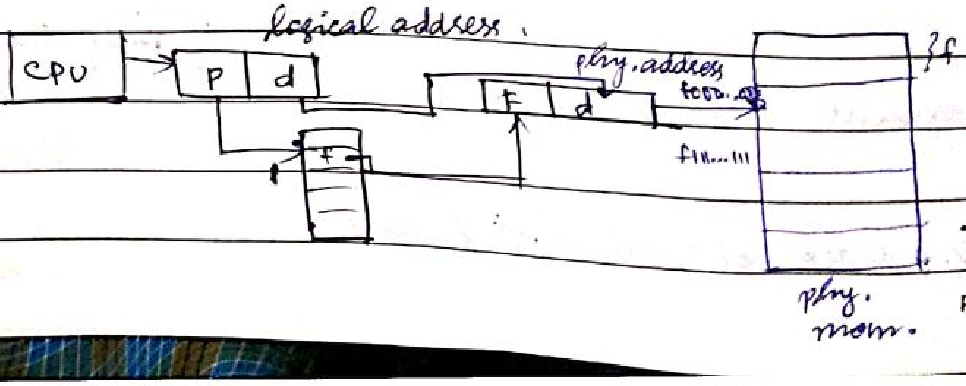
p	d
$m-n$	n

(2)100 ≡ 15 100

where 2^n = page size.

logical address space = 2^m

* Paging H/w:



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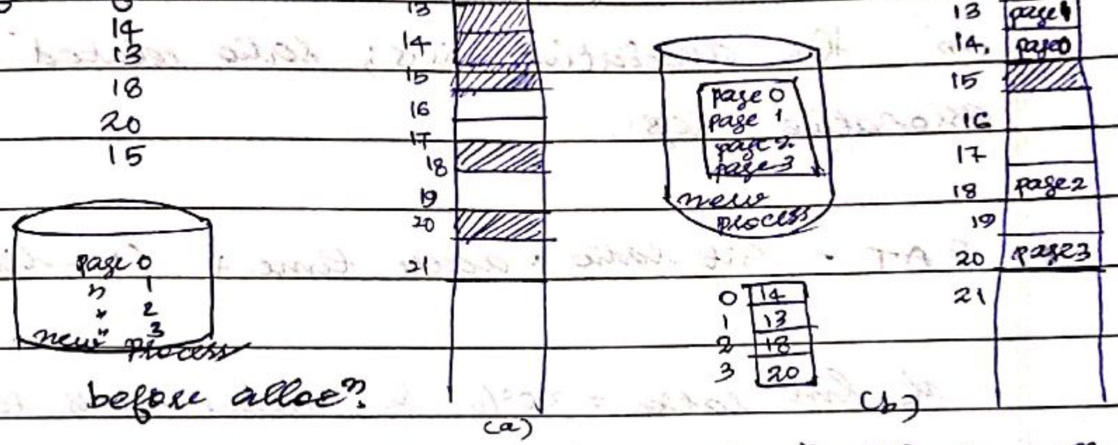
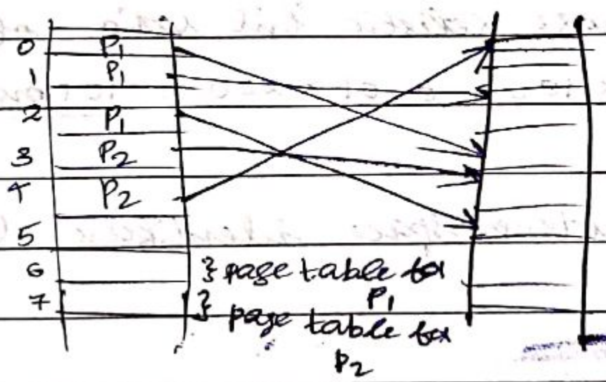
- Address generated by CPU has 2 parts — p and d.

free-frame list



free frames (a) before alloca^n. & (b) after alloc^n.

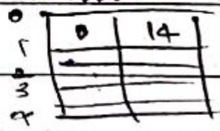


$P_1 = \underline{0}\,\underline{210}$
p d
$P_2 = 0210$

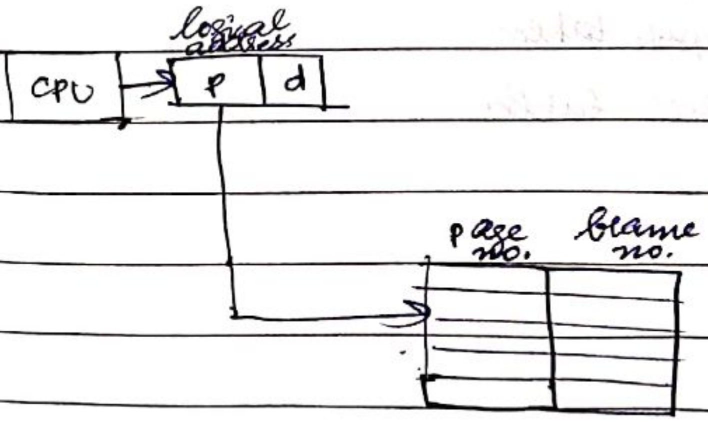
mem. access takes more time.

★ TLB — associative mem.

associative mem. / translation look-aside buffers.



paging h/w with TLB :



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* Eff. access time (EAT):
- Hit ratio: % of times that a page no. is found in the associative regs; ratio related to no. of associative regs.

• EAT = hit ratio * access time + (1 - hit ratio) * 2 * access time

if hit ratio = 80% & mem. access takes 100ns:

$EAT = 0.80 \times 100 + 0.20 \times 200 = 120 ns$

Consider more relistic hit ratio of 99%:

$EAT = 0.99 \times 100 + 0.01 \times 200 = \underline{101 ns}$ → EAT (access time)

TLB issues: address-space identifiers (ASIDs)

• Memory protec^n:
  valid/invalid bit.

* Page table for large address space:
Hierarchical paging
Hashed page tables.
Inverted page tables.