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% of students failed = $\frac{10}{100}$; % of students who got distinction = 5.

$P(x < 45) = 0.10 \Rightarrow P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.10$

$\Rightarrow P\left(z < \frac{45-\mu}{\sigma}\right) = 0.10 \qquad \Rightarrow \frac{45-\mu}{\sigma} = -1.28 \quad -(1)$

$\Rightarrow P(x > 75) = 0.05 \Rightarrow P\left(\frac{x-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right) = 0.05$

$\Rightarrow P\left(z > \frac{75-\mu}{\sigma}\right) = 0.05 \Rightarrow 1 - P\left(z \le \frac{75-\mu}{\sigma}\right) = 0.05$

$\Rightarrow P\left(z \le \frac{75-\mu}{\sigma}\right) = 0.95 \Rightarrow \frac{75-\mu}{\sigma} = 1.65 \quad -(2)$

$(1) \Rightarrow 1.28\sigma - \mu + 45 = 0$

$(2) \Rightarrow 1.65\sigma + \mu - 75 = 0$

$\Rightarrow \sigma = \frac{75-45}{1.28+1.65} = \frac{30}{2.93} = \underline{10.2389}$

$\Rightarrow \mu = \frac{75 \times 1.28 + 45 \times 1.65}{1.28 + 1.65} = \frac{170.25}{2.93} = \underline{58.1058}$

$\Rightarrow P(45 < x < 60) = P\left(\frac{45-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{60-\mu}{\sigma}\right)$

$= P(-1.28 < z < 0.185) = P(z < 0.185) - P(z < -1.28)$

$= 0.57142 - 0.10027 = \underline{0.47115}$

$\therefore$ ~~47.11~~ approximately 47% of students secured 2nd class

$\Rightarrow$ remaining ie $85 - 47 = 38\%$ of students secured 1st class $(0.95 - 0.57142 = 0.37858)$

9.02.2019

Q. The income distribution of workers in a certain factory was found to be N.D with $\mu = ₹1000$ and $\sigma = ₹100$. There were 180 persons getting above ₹1200, how many persons were there in all?

$X \rightarrow$ income of workers.

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$\mu = 1000$, $\sigma = 100$.

$P(X > 1200) = \frac{180}{x}$

$x \to$ total no. of workers.

$\Rightarrow 1 - P(X \le 1200) = \frac{180}{x} \Rightarrow P(X \le 1200) = 1 - \frac{180}{x}$.

$\Rightarrow P\left(\frac{X-\mu}{\sigma} \le \frac{1200 - 1000}{100}\right) = 1 - \frac{180}{x}$.

$\Rightarrow P(z \le 2) = 1 - \frac{180}{x} \Rightarrow 0.97725 = 1 - \frac{180}{x}$.

$\Rightarrow x - 0.97725x \mp 180 = 0$

$\Rightarrow 0.02275x = 180 \Rightarrow x = \underline{7912}$.

Q. If X is a N.D random variable with $\mu = 12$ & $\sigma^2 = 16$

Find the value of a s.t. $P(X > a) = 0.24$.

$\Rightarrow 1 - P(X \le a) = 0.24 \Rightarrow P(X \le a) = 0.76$

$\Rightarrow P\left(z \le \frac{a-12}{4}\right) = 0.76$

$\Rightarrow \frac{a-12}{4} = 0.71 \Rightarrow a = 2.84 + 12 \Rightarrow a = 14.84$.

Q. For a N.D population 7% of the items have ~~the~~ their values less than 35 & 89% of the items have their values less than 63. Find μ & σ.

$x \to$ values.

$P(X < 35) = 0.07 \Rightarrow P\left(z < \frac{35-\mu}{\sigma}\right) = 0.07$

$P(X < 63) = 0.89 \Rightarrow P\left(z < \frac{63-\mu}{\sigma}\right) = 0.89$

$\Rightarrow \frac{35-\mu}{\sigma} = -1.47 \Rightarrow 1.47\sigma - \mu + 35 = 0$

$\Rightarrow \frac{63-\mu}{\sigma} = 1.23 \Rightarrow 1.23\sigma + \mu - 63 = 0$.

$\Rightarrow \sigma = \frac{63-35}{1.47+1.23} = \frac{28}{2.70} = \underline{10.37}$

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$\Rightarrow \mu = 1.49 \times 10.37 + 35 = \underline{50.24}$

Q. The time required to assemble a piece of machinery is a random variable having $\mu = 12.9$ min, $\sigma = 2$ min. What are the probabilities that assembling a piece of machinery of this kind will take:

(i) atleast 11.5 min

(ii) anywhere from 11.6 to 14.8 min.

X - time required to assemble a piece of machinery.

(i) $P(X \geq 11.5) = 1 - P\left(z \leq \frac{11.5 - 12.9}{2}\right) = 1 - P(z \leq -0.7)$

$= 0.75804$

(ii) $P(11.6 \leq X \leq 14.8) = P\left(\frac{11.6 - 12.9}{2} \leq z \leq \frac{14.8 - 12.9}{2}\right)$

$= P\left(\frac{-1.3}{2} \leq z \leq \frac{1.9}{2}\right) = P(-0.65 \leq z \leq 0.95)$

$= P(z \leq 0.95) - P(z \leq -0.65)$

$= 0.82894 - 0.25785 = 0.\underline{57109}$

HW Q. A saletax officer has reported that the average sales of the 500 business that he has to deal with in a year is ₹ 36000 with $\sigma = 10{,}000$. Assuming N.D, find:

(i) the no. of business whose sales is above ₹ 40000

(ii) the % of " " " are likely to range b/w ₹ 30,000 & ₹ 40,000

(i) X - sales per year.

$P(X > 40000) = P\left(\frac{X-\mu}{\sigma} > \frac{40000-36000}{10000}\right) = P(z > 0.4)$

$= 1 - P(z \le 0.4) = 1 - 0.65542 = 0.34458$

$\Rightarrow \frac{x}{500} = 0.34458 \Rightarrow x = 172.29 \approx 172$ sales.

(ii) $P(30000 < X < 40000) = P\left(\frac{-6000}{10000} < \frac{X-\mu}{\sigma} < 0.4\right)$

$= P(-0.6 < z < 0.4) = P(z < 0.4) - P(z < -0.6)$

$= \overset{0.65542}{\cancel{0.34458}} - 0.27425 = \overset{0.38117}{\cancel{0.07033}} = \cancel{7.033\%}$ 38.117%

26-02-2019

Q. A random variable has a N.D with $\mu = 62.4$, find its σ if the probability is 0.20, that it will take on a value > 79.2

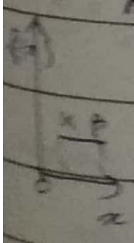
$\mu = 62.4$, $\sigma = ?$, $P = 0.2$, $x > 79.2$.

$\Rightarrow P(x > 79.2) = 0.2 \Rightarrow P\left(z > \frac{79.2 - 62.4}{\sigma}\right) = 0.2$

$\Rightarrow P\left(z > \frac{16.8}{\sigma}\right) = 0.2 \Rightarrow 1 - P\left(z \le \frac{16.8}{\sigma}\right) = 0.2$

$\Rightarrow P\left(z \le \frac{16.8}{\sigma}\right) = 0.8 \Rightarrow z \le 0.85$

$\Rightarrow \frac{16.8}{\sigma} = 0.85 \Rightarrow \sigma = \frac{16.8}{0.85} = 19.76$



* Uniform distribution: / Rectangular distribution

$$\text{pdf} \rightarrow f(x) = \begin{cases} \frac{1}{\beta - \alpha} & ; \alpha < x < \beta \\ 0 & ; \text{otherwise} \end{cases}$$

$\cancel{\sum_{x=\alpha}^{\beta} x_i f_i}$ Mean $= E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \frac{(\beta^2 - \alpha^2)}{2(\beta - \alpha)} = \frac{\alpha + \beta}{2}$

Variance $= E[x^2] - (E[X]^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \frac{(\alpha+\beta)^2}{4}$

$= \int_{\alpha}^{\beta} \frac{x^2}{(\beta - \alpha)}\,dx - \frac{(\alpha+\beta)^2}{4} = \frac{(\beta^3 - \alpha^3)}{3(\beta - \alpha)} - \frac{(\alpha+\beta)^2}{4}$

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$$= \frac{\alpha^2 + \alpha\beta + \beta^2}{3} - \frac{(\alpha^2 + \beta^2 + 2\alpha\beta)}{4}.$$

$$= \frac{\alpha^2 - 2\alpha\beta + \beta^2}{12} = \frac{(\alpha - \beta)^2}{12} := \frac{(\beta - \alpha)^2}{12}.$$

Q. A bus arrives every 10 min. at a bus stop. Assuming the waiting time X is U.D. Find the probability that the person has to wait for the bus:

(i) > 7 min (ii) 2 < X < 7 min.

X : time person has to ~~to~~ wait / waiting time.

(i) $P(X>7)$

$$f(x) = \begin{cases} \frac{1}{10} & ; \; \overset{0}{\cancel{\alpha}} < x < \overset{10}{\cancel{\alpha+10}} \\ 0 & ; \text{ otherwise} \end{cases}$$

$$\Rightarrow P(X>7) = \int_{7}^{\infty} f(x)\,dx = \int_{7}^{10} f(x)\,dx + \int_{10}^{\infty} f(x)\,dx.$$

$$= \frac{(10-7)}{10} = \frac{3}{10} = 0.3$$

(ii) $$P(2<X<7) = \int_{2}^{7} f(x)\,dx = \int_{2}^{7} \frac{1}{10}\,dx = \frac{1}{2} = 0.5$$

Q. If a conference room can't be reserved for more than 4 hrs. Find the probability that a given conference last more than 3 hrs.

$$f(x) = \begin{cases} \frac{1}{4} & , \; 0 < x < 4 \\ 0 & ; \text{ otherwise.} \end{cases}$$

* : conference time.

$P(X=3) = \int_3^\infty f(x)\,dx = \int_3^4 \frac{1}{4}\,dx = \frac{1}{4} = 0.25$.

Q. A string of length 10m is divided into 2 parts. If the part of random length exist U.D, then find:

(i) $E[3X+2]$ (ii) $\sigma^2(2X+3)$ (iii) $E[X(10-X)]$

$X \to$ part of random length.

$$f(x) = \begin{cases} \frac{1}{10}, & 0 < x < 10 \\ 0, & \text{otherwise.} \end{cases}$$

(i) $\Rightarrow E[3X+2] = 3E[X] + 2 = 3\times\left[\frac{10+0}{2}\right] + 2 = 15+2$

$= \underline{17}$

$2\times5+3 = 13$.

(ii) $\sigma^2(2X+3) = E([2X+3])^2 - (E[2X+3])^2$

$= E[4X^2+12X+9] - \cancel{289} \left(\frac{10+0}{2}\right)^2$

$= 4E[X^2] + 12E[X] + 9 - \cancel{289}\ 169$

$\begin{array}{r} 17 \\ 17 \\ \hline 49 \\ 14 \\ 1 \\ \hline 289 \end{array}$

$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{10} \frac{x^2}{10}\,dx = \frac{10^3}{30} = \frac{100}{3}$.

$\Rightarrow \sigma^2(2X+3) = \frac{4\times100}{3} + 60 + 9 - \cancel{289}\ 169$.

$= \frac{400}{3} + 60 + 9 - \cancel{289}\ 169 = \cancel{\frac{400}{3} + 44} = \cancel{\frac{400+132}{3}}$

$\cancel{= \frac{532}{3}} = \cancel{177.33} \cancel{86.69}\ \frac{100}{3} = \underline{33.33}$

(iii) $E[X(10-X)] = 10E[X] - E[X^2] = 50 - \frac{100}{3} = \frac{50}{3} = \underline{16.67}$

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Q. If x follows U.P in $(-\alpha, \alpha)$, find α s.t. $P(x<1) = 1/3$

$$f(x) = \begin{cases} \frac{1}{2\alpha}, & -\alpha < x < \alpha \\ 0, & \text{otherwise} \end{cases}$$

$P(x<1) = 1/3$

$-\infty \quad \alpha \quad 1 \quad 0 \quad \infty$

Case I: when $-\alpha < \alpha < 1$:

$$P(x<1) = \int_{-\infty}^{-\alpha} f(x)\,dx + \int_{-\alpha}^{1} f(x)\,dx$$

$$= \int_{-\alpha}^{1} \frac{1}{2\alpha}\,dx = \frac{1}{2\alpha}(1+\alpha) = 1/3$$

$\Rightarrow 3 + 3\alpha = 2\alpha \Rightarrow \alpha = -3$

$\Rightarrow -\alpha = 3$

$-3 < x < 3$

Q. If x is U.D in $(-2, 2)$, find (i) $P(x<1)$
(ii) $P(|x-1| \geq 1/2)$

$$f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) $P(x<1) = \int_{-\infty}^{1} f(x)\,dx = \int_{-\infty}^{-2} 0\,dx + \int_{-2}^{1} \frac{1}{4}\,dx$

$$= \frac{1}{4}(3) = \frac{3}{4} = 0.75$$

(ii) $|x-1| \geq \frac{1}{2} \Rightarrow (x-1) \geq \frac{1}{2}$ and ~~(1-x)~~ $x-1 < -1/2$

$\Rightarrow x \geq \frac{3}{2}$ and $x < 1/2$

When $x>1$: $|x-1|>0$ $\Rightarrow$ $(x-1)\geq 1/2 \Rightarrow x \geq 3/2$

When $x<1$: $\cancel{(x-1)<0}$ $\Rightarrow$ $(1-x)\geq 1/2 \Rightarrow x \leq 1/2$

$$P(x \geq 3/2) + P(x < 1/2)$$

$$= \int_{3/2}^{2} \frac{1}{4}\,dx + \int_{-2}^{1/2} \frac{1}{4}\,dx = \frac{1}{4}\left(\frac{1}{2}\right) + \frac{1}{4}\left(\frac{5}{2}\right)$$

$$= \frac{6}{8} = \frac{3}{4}$$

25.02.2019

* Exponential distribution:

pdf $f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & \beta>0,\ x>0 \\ 0, & \text{otherwise.} \end{cases}$

Exponential distrib^n is a commonly used to model the time b/w independent events that occur at a constant average rate.

eg: The inter-arrival time at a service facility ie the waiting time b/w successive arrivals has exp. distribu^n

- Mean :

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{0} \frac{x e (0)}{\beta}\,dx + \int_{0}^{\infty} \frac{x}{\beta} e^{-x/\beta}\,dx$$

$$= 0 + \frac{1}{\cancel{\beta}} \frac{\cancel{\beta}\, x\, e^{-x/\beta}}{-1} + \int \frac{1}{\cancel{\beta}} \frac{e^{-x/\beta}}{+1} \times \cancel{\beta}\,dx$$

$$= \left[-x e^{-x/\beta} - \beta e^{-x/\beta}\right]_0^{\infty} = \beta$$