MODULE 1

DISCRETE PROBABILITY DISTRIBUTION

Random Experiment (RE):

An experiment that can result in different outcomes eventhough repeated under same conditions every time is called random experiment.

In random experiment outcomes are known before the experiment but which outcome is going to happen is not known.

Sample Space:-

Set of all outcomes in a random experiment is called sample space and is denoted by 'S'

eg: In tossing a coins then $S = \{HH, HT, TH, TT\}$

Throwing 2 dices then $S = \{(1,1), (1,2), (1,3) \ldots (1,6)$
$(2,1), (2,2), (2,3) \ldots (2,6)$
$\vdots$
$(6,1), (6,2), (6,3) \ldots (6,6)\}$

Random Variable (RV):-

corresponding to each outcomes in a random exper. it can define a real number which is called a random variable.

It can be defined as a function from 'S' to 'R' [set of all real numbers] such that $X(s) = x$.

Domain of RV fun. is 'S' and Range is a subset of 'R'

eg:-

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eg: ① consider tossing of two coins

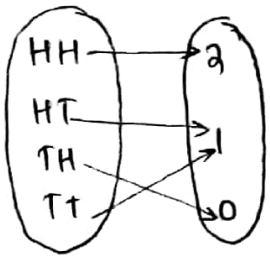
$$S = \{HH, HT, TH, TT\}$$

* RV are denoted by X, Y .... etc and the values taken by it are taken as x, y .... etc

(above) $\Rightarrow$ X = no. of heads

then X can take values $\{0, 1, 2\}$

$X(HH) = 2$, $X(HT) = 1$, $X(TH) = 1$, $X(TT) = 0$



② Throwing 2 dies

X = sum of values of the die

then $X = \{2, 3, \ldots 12\}$

$X(5,2) = 7$, $X(6,6) = 12$ .. etc

③ consider the random experiment of tossing a coin until a head occurs

X = no. of throws

then it takes values $= \{1, 2, 3 \ldots$ etc$\}$

④ X denotes the height of students in the class then X takes all possible values in the interval $X = \{150, 180\}$

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⇒ Discrete and continuous random variables :-

If the random variables takes only discrete no. of values then the R.V is said to be discrete RV [D.R.V]

If the R.V takes all possible values in an interval (a,b). Then it is said to be continuous CR random variable (C.R.V)

ie $X = a \leq x \leq b$

① The life expectancy of a ckt in a computer is CRV

② The no. of accidents in a day ⇒ DRV

⇒ Discrete probability distribution or probability mass fn (PMF) :-

PMF :-

A fn $f(x)$ = probability of $f(x) = x$ of a DRV is said to be a PMF if it satisfies the following condition.

1. $f(x) \geq 0$
2. $\sum f(x) = 1$

PMF is expressed as an eqn. or as a table which shows the values of random variable on the associated probabilities

eg:- In tossing a coins

X = no. of heads

The PMF is

X	$P(x=0) = P(TT) = \frac{1}{4}$
$f(x)$	$P(x=1) = P(HT \cup TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
	$P(x=2) = P(HH) = \frac{1}{4}$

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X	0	1	2
f(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

2) Random experiment of throwing a die

X = no. of in the die

$X = \{1, 2, 3, 4, 5, 6\}$

$P(X=1) = P(X=2) \ldots\ldots P(X=6) = \frac{1}{6}$

$\therefore$ PMF is $f(x) = \frac{1}{6}$ for $x = 1, 2, \ldots 6$

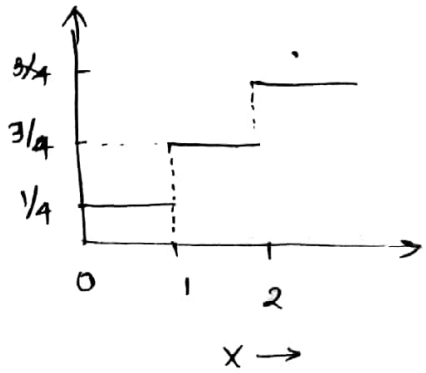
⇒ <u>cumulative distribution function or distribution function (CDF) :-</u>

If x is a ~~distri~~ discrete random variable then the CDF is defined as

$f(x) = P(X \leq x) = \sum_{-\infty}^{x} f(x)$

In example ①

X	0	1	2
f(x)	1/4	1/2	1/4
f(x)	1/4	3/4	1



In example ②

X	1	2	3	4	5	6
f(x)	1/6	1/6	1/6	1/6	1/6	1/6
f(x)	1/6	2/6	3/6	4/6	5/6	1

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⇒ Properties of CDF :-

① $f(\infty) = 1$

② $P(a < x \leq b) = F(b) = f(a)$

③ $F(x)$ is a non decreasing fun. (or increasing funcs)

④ $f(x)$ is a step function

Qn. Check whether the following function can be the PMF of a random variable X.

i) $f(x) = x - 3$ for $x = 1, 2, 3, 4, 5$

x	1	2	3	4	5
$f(x)$	-1	-1	1	1	0

since $f(x=1)$ and $f(x=2)$ are -ve so it is not PMF

ii) $f(x) = \frac{x^2}{30}$ for $x = 0, 1, 2, 3, 4$

⇒

x	0	1	2	3	4
$f(x)$	0	1/30	4/30	9/30	16/30
$f(x)$	0	1/30	5/30	14/30	1

since $\Sigma f(x) = 1$ & $f(x) > 1$ it is a PMF

(iii) $f(x) = 0.26$

$f(1) = 0.26$ $f(2) = 0.26$ $f(3) = 0.26$ $f(4) = 0.26$

⇒ since its sum is not 1 is not a PMF

iv) The PMF of a random variable X is given by

X	0	1	2	3	4	5	6
$f(x)$	k	3k	5k	7k	9k	11k	13k

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ⓐ find the value of k?

ⓑ $P(x<4)$

ⓒ $P(x \geq 5)$

ⓓ Find $F(x)$

ⓐ since it is a PMF $\Sigma f(x) = 1$

so $k+3k+5k+7k+9k+11k+13k=1$

$\Rightarrow 49k = 1$

$k = 1/49$

ⓑ $P(x<4)$

$\Rightarrow k+3k+5k+7k$

$\Rightarrow 16k$ so $\frac{16}{49} = k$

ⓒ $P(x \geq 5)$

$\Rightarrow 11k+13k$

$\Rightarrow 24k$

$\Rightarrow \frac{24}{49} = k$

Qn. Given that $f(x) = \frac{k}{2^x}$ for $x = 0, 1, 2, 3, 4$ is the PMF of the random variable x

i) Find the value of k

ii) Find the $P(0<x\leq 2)$

iii) Find the distribution fn or CDF

ans:-

x	0	1	2	3	4
f(x)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$	$\frac{k}{16}$

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since $f(x)$ is the PMF $\Sigma f(x) = 1$, so

$$k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} + \frac{k}{16} = 1$$

$$k\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = 1$$

$$k\left[\frac{16+8+4+2+1}{16}\right] = 1$$

$$k = \frac{16}{31}$$

Hence the PMF is

X	0	1	2	3	4
$f(x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$
$F(x)$	$\frac{16}{31}$	$\frac{24}{31}$	$\frac{28}{31}$	$\frac{30}{31}$	1

ii) $P(0 < x \leq 2)$

$P(x = 1, 2)$

$P(x=1) + P(x=2)$

$\Rightarrow \frac{8}{31} + \frac{4}{31} = \frac{12}{31}$

Qn. If a random variable X has a PMF $f(x) = k\left(\frac{2}{3}\right)^x$ for $x = 1, 2, 3 \ldots$ Then find k.

ans:- $\Sigma f(x) = 1$ since PMF

$$f(x) = \sum_{x=1}^{\infty} k\left(\frac{2}{3}\right)^x = 1$$

$$\Rightarrow k\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots\right] = 1$$

For geometric

If $r < 1$

$S = \frac{a}{1-r}$.

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$$\Rightarrow \quad k\left[\frac{\frac{2}{3}}{1-\frac{2}{3}}\right] = 1$$

$$k\left[\frac{2/3}{1/3}\right] = 1$$

$$\underline{\underline{k = 1/2}}$$

Qn. Consider a lot of 10 items containing 3 defectives from which a sample of 4 items is drawn, at random. Let the random variable X denotes the no. of defective items in the sample.

i) The PMF of X

ii) Find the DF

iii) Find $P(X<1)$

ans: $X \Rightarrow$ no. of defective items in the sample

$X = 0, 1, 2, 3$

probability of $X=0$ ∴ie $P(X=0)$

$$\frac{{}^7C_4}{{}^{10}C_4} = \frac{\cancel{35}\,7}{\cancel{210}\,30} \Rightarrow 7/30 = 1/6$$

$$P(X=1) = \frac{{}^7C_3 \times {}^3C_1}{{}^{10}C_4} = \frac{1}{2}$$

$$P(X=2) = \frac{{}^7C_2 \times {}^3C_2}{{}^{10}C_4} = 0.3$$

$$P(X=3) = \frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_4} = 1/30$$

ii)

X	0	1	2	3
f(x)	1/6	1/2	3/10	1/30
F(x)	1/6	4/6	29/30	1

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$P(x<1)$

$= \frac{1}{6}$

Qn A random variable X has the following PMF

X	1	2	3	4	5	6	7
f(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find the following

i) Find the value of k

ii) $P(0<x<5)$

iii) $P(x>5)$

i) $\Sigma f(x) = 1$

$80\ k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$10k^2+9k=1$

$10k^2+9k-1=0$

$10k^2+10k-k-1=0$

$10k(k+1)-1(k+1)=0$

$(10k-1)(k+1)=0$

$k=\frac{1}{10}$ $k=-1$

since f(x) is +ve the suitable value is $k=\frac{1}{10}$

ii) $P(0<x<5)$

$k+2k+2k+3k$

$\Rightarrow 3k \Rightarrow \frac{8}{10}$

iii) $P(x>5)$

$2k^2+7k^2+k$

$9k^2+k = \frac{9}{100}+\frac{1}{10} = \frac{19}{100}$

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Qn. In an experiment of tossing 3 coins, find the PMF and the DF of the following random variables

i) X = no. of heads

ii) Y = Head runs

iii) Z = no. of heads in the head runs.

iv) $x+y$

i)

S	HHH	HHT	HTH	THH	THT	HTT	TTH	TTT
X	3	2	2	2	1	1	1	0
Y	1	1	0	1	0	0	0	0
Z	3	2	0	2	0	0	0	0
$x+y$	4	3	2	3	1	1	1	0

PMF

X	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
F(x)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

Y	0	1
f(y)	5/8	3/8
F(Y)	5/8	1

Z	0	2	3
f(z)	5/8	2/8	1/8
F(z)	5/8	7/8	1

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$x+y$	0	1	2	3	4
$f(x+y)$	1/8	3/8	1/8	2/8	1/8
$F(x+y)$	1/8	4/8	5/8	7/8	1

H.W problems at 1.1 Qn 1, 4, 5, 6, 7 8

V mean and variance of a discrete R.V

Mean:-

If x is a discrete R.V which takes values $x_1, x_2, x_3 \ldots$ with PMF $f(x)$; then the mean or the expectation of R.V. x is defined as $E(x) = \sum_{i=1} x_i f(x)$ which is also denoted by μ ie

$$\boxed{E(x) = \mu = \Sigma x f(x)}$$

Properties

I) $E(a) = \Sigma a f(x) = a \Sigma f(x) = 1$

II) $E(ax) = \Sigma a x f(x) = a \Sigma x f(x) \Rightarrow aE(x)$

III) $E(ax+b) = aE(x) + b$

IV) $E(g(x)) = \Sigma g(x) f(x)$

Qn. Find the expectation of the no. on the die

PMF

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(x) = \Sigma x f(x)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + 1$$

$$= \frac{21}{6} = \frac{7}{3}$$

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$E(x^2) = \frac{1}{6}[1+4+9+16+25+36]$

$= \frac{91}{6}$

Variance of a DRV

If X is D.R.V then the variance of X is defined as

$$\boxed{\sigma^2(X) = E[(X-\mu)^2]}$$

where μ is the mean or expectation of X

Expector

$E[(X-\mu)^2] = E[(X^2 - 2X\mu + \mu^2)]$

$= E(X^2) - E(2X\mu) + E(\mu^2)$

$= E(X^2) - 2\mu E(X) + \mu^2$

$= E(X^2) - 2[E(x)]^2 + [E(x)]^2$ $[\mu = E(x)]$

$$\boxed{\sigma^2(x) = E(X^2) - [E(X)]^2}$$

where $E(X^2) = \Sigma x^2 f(x)$

$E(X) = \Sigma x f(x)$

Properties

1) $\sigma^2(k) = E(k^2) - [E(k)]^2$

$= k^2 - k^2 = 0$

variance of a constant is 0

2) $\sigma^2(kx) = E(k^2x^2) - [E(kx)]^2$

$= k^2[E(x^2)] - [kE(x)]^2 = k^2E(x^2) - k^2[E(x)]^2$

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$= k^2[E(X^2) - [E(X)]^2]$

$= k^2\sigma^2(X)$

3) $\sigma^2(aX+b) = a^2\sigma^2(X)$

4) square root of variance is called the standard deviation (S.D)

$$S.D = \sqrt{E(X^2) - [E(X)]^2} = \sigma(X)$$

Qn. Find the mean and variance of the sum of no.s thrown when 2 unbiased dis are thrown.

X = sum denotes the sum of no. on the dice

X take values from 2 to 12

X	2	3	4	5	6	7	8	9	10	11	12
f(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
F(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$

$$\mu = \Sigma x f(x) = \frac{2\times 1}{36} + \frac{3\times 2}{36} + 4$$

$$= \frac{1}{36}[(2\times1)+(3\times2)+(4\times3)+(5\times4)+(6\times5)+(7\times6)+(8\times5)+(4\times9)$$
$$+(10\times3)+(11\times2)+(12\times1)]$$

$$= \frac{1}{36}[2+6+12+20+30+42+40+36+30+22+12]$$

$$= \frac{252}{36}$$

$$E(X^2) = \frac{4}{36} + \frac{1}{36^2}[4+36+144+400+9000+42^2+40^2+36^2+30^2$$
$$+22^2+12^2]$$

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$$= \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36} = \frac{1974}{36}$$

$$\sigma^2(x) = E(x^2) - [E(x)]^2 = \frac{1974}{36} - 49 = \frac{210}{36} = \underline{5.83}$$

Qn. A random variable x follows a PMF as given below

X	0	1	2	3
f(x)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k+1}{3}$	$\frac{2k+1}{6}$

1) Find the value of k

2) Find the mean and S.D of random variable x

1) $\frac{k}{2} + \frac{k}{3} + \frac{k+1}{3} + \frac{2k+1}{6} = 1$

$3k + 2k + 2k + 2 + 2k + 1 = 6$

$9k + 3 = 6$

$9k = 3$

$k = 1/3$

2)

X	0	1	2	3
f(x)	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{5}{18}$

$\mu = \Sigma x f(x)$

$= \frac{1}{9} + \frac{8}{9} + \frac{15}{18}$

$= 1 + \frac{3}{6} \quad \frac{2 + 16 + 15}{18}$

$= \frac{6+3}{6} = \frac{9}{6} = \frac{3}{2} \qquad 1 + \frac{5}{6} = \frac{11}{6}$

$\begin{array}{r} 18 \\ 15 \\ 3 \end{array}$

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$$E(X^2) = \frac{1}{9} + \frac{16}{9} + \frac{9 \times 5}{18}$$

$$= \frac{1}{9} + \frac{16}{9} + \frac{45}{18}$$

$$= \frac{2 + 32 + 45}{18}$$

$$= \frac{79}{18}$$

$$S.D = \sqrt{\frac{79}{18} - \left(\frac{11}{6}\right)^2}$$

$$= 1.013$$

Q. X is a random variable for which mean = 10 and variance = 25. Find the +ve values α and β such that $Y = \alpha X - \beta$ has expectation 0 and variance 1

ans: Given

$E(X) = 10$

$\sigma^2(X) = 25$

and $E(Y) = 0$

$\sigma^2(Y) = 1$

$$E(Y) = E(\alpha X - \beta)$$

$$= E(\alpha X) - \beta$$

$$= \alpha E(X) - \beta$$

$$10\alpha - \beta = 0$$

$$\sigma^2(Y) = \sigma^2(\alpha X - \beta)$$

$$= \sigma^2(\alpha X) = \alpha^2 \sigma^2(X) = \alpha^2 25 = 25\alpha^2$$

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$25\alpha^2 = 1$

$25\alpha^2 = \frac{1}{25}$

$\alpha = \frac{1}{5}$

$10 \times \frac{1}{5} - \beta = 0$

$2 = \beta$

Qn. Let x be random variable with PMF $f(x) = \frac{x}{6}$ & $x = 1, 2, 3$ or equal to zero and Find $E[(X+2)^2]$

ans. The PMF is

X	1	2	3
f(x)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

$E(X) = \Sigma x f(x)$

$= \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{2}{6}\right) + \left(3 \times \frac{3}{6}\right)$

$= \frac{1}{6} + \frac{4}{6} + \frac{9}{6}$

$= \frac{14}{6} = \frac{7}{3}$

$E(X^2) = \Sigma x^2 f(x)$

$= \left(1 \times \frac{1}{6}\right) + \left(4 \times \frac{2}{6}\right) + \left(\frac{9 \times 3}{6}\right)$

$= \frac{1}{6} + \frac{8}{6} + \frac{27}{6} = \frac{36}{6} = 6$

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$$E[(X+2)^2] = E[X^2 + 4X + 4]$$

$$= E(X^2) + E(4X) + E(4)$$

$$= \frac{1}{6} + 4 \times \frac{7}{3} + 4$$

$$= \frac{6}{6} + \frac{28}{3} + 4 \quad = 10 + \frac{28}{3}$$

$$= \frac{1}{6} + \frac{56}{6} + \frac{24}{6} \quad \frac{88}{6} \quad = \frac{30+28}{3}$$

$$= \frac{81}{6} \quad = \frac{58}{3}$$

$$=$$

Qn. With equal probability, from the observation 5, 10, 8, 2, 7 show the no. of defective units found during 5 inspections in a laboratory. Find mean and variance.

X	5	10	8	2	7
f(x)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$$E(X) = \frac{1}{5}[5+10+8+2+7] = \frac{32}{5} = 6.4$$

$$E(X^2) = \frac{1}{5}(25+100+64+4+49)$$

$$= \frac{200}{5} = 40 \quad \frac{242}{5} = 48.4$$

$$\sigma^2(X) = 48.4 - 40.96 = 7.44$$

Qn. 100 fair dies are thrown find the expectation of the sum of nos thrown

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Let X denotes the sum of numbers when 100 dies are thrown.

X_i → denotes no. on i^{th} die $\quad i = 1, 2, \ldots 100.$

$$X = X_1 + X_2 + X_3 + \cdots X_{100}$$

$$= \sum_{i=1}^{100} X_i$$

$$E(X) = E(X_1 + X_2 + \cdots X_{100})$$

$$= E(X_1) + E(X_2) + \cdots E(X_{100})$$

The PMF of X_1 is

X_1	1	2	3	4	5	6
$f(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X_1) = \Sigma x f(x)$$

$$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6]$$

$$= \frac{21}{6}$$

$$= \frac{7}{2} = \underline{\underline{3.5}}$$

Similiarly $E(X_2) = \frac{7}{2}$

$\vdots$

$E(X_{100}) = \frac{7}{2}$

$$E(X) = 100 \times \frac{7}{2}$$

$$= \underline{\underline{350}}.$$

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Qn. A player is to toss 3 coins. He wins Rs 10 if 3 heads appears, Rs 5 if 2 heads appears, Rs 1 if one head appear. He will loss Rs 12 if no head appears. What is the expected amount he gains.

X: denotes no. of heads. The amount he gains

X	10	5	1	-12
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(X) = \Sigma x f(x)$$

$$= \left(10 \times \frac{1}{8}\right) + \left(5 \times \frac{3}{8}\right) + \left(1 \times \frac{3}{8}\right) + \left(-12 \times \frac{1}{8}\right)$$

$$= \frac{10}{8} + \frac{15}{8} + \frac{3}{8} - \frac{12}{8}$$

$$= \frac{16}{8}$$

$$= 2$$

Problem at 1.2 Qn: 2, 4, 5, 7

<u>Problem 1.1</u>

1. A random variable X takes p values 1, 2, 3, 4 such that

$2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$

Find probability distribution of X

X	1	2	3	4
$P(X)$	$k/2$	$k/3$	k	$k/5$
$F(X)$	$15/61$	$10/61$	$30/61$	$6/61$

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$\frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$

$k = \frac{30}{61}$

4. Check whether fcll can be P.MF. Explain your ans.

I) $f(x) = \frac{x}{15}$, $x = 0, 1, 2, 3, 4, 5$

X	0	1	2	3	4	5
f(x)	0	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

$\frac{1+2+3+4+5}{15} = 1$

since $f(x) \geq 1$ & $\sum_{0}^{5} f(x) = 1$ $\therefore$ it can be PMF

II) $f(x) = \frac{1}{4}$, $x = -2, 2, 4, 6$

X	-2	2	4	6
f(x)	1/4	1/4	1/4	1/4

$\sum f(x) = 4 \times \frac{1}{4} = 1$

since $f(x) \geq 1$, & $\sum f(x) = 1$ it can be PMF

III) $f(x) = \frac{5-x^2}{6}$ $x = 2, 3, 4$

X	2	3	4
f(x)	$\frac{1}{6}$	$\frac{-4}{6}$	$\frac{-4}{6}$

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since $f(x) < 0$, it can not be PMF

5.

X	-2	-1	0	1	2	3
f(x)	0.1	k	0.2	2k	0.3 k	4k

Find k

$$\Sigma f(x) = 0.1 + k + 0.2 + 2k + 0.3k + 0.6 + 4k = 1$$

$$4k = 0.4$$

$$k = 1/10$$

6. Two bad eggs are mixed accidentally with 10 good ones. Find probability distribution of the number of bad eggs in 3, drawn at random without replacement, from this lot

X	0	1	2
f(x)	$\frac{12}{22}$	$\frac{9}{22}$	$\frac{1}{22}$

$$P(X=0) = \frac{{}^{10}C_3}{{}^{12}C_3} = \frac{120}{220} = \frac{12}{22}$$

$$P(X=1) = \frac{{}^{10}C_2 \times {}^{2}C_1}{{}^{12}C_3} = \frac{90}{220} = \frac{9}{22}$$

$$P(X=2) = \frac{{}^{10}C_1 \times {}^{2}C_2}{{}^{12}C_3} = \frac{10}{220} = \frac{1}{22}$$

8. Obtain distribution function of total no. of heads occurring in 3 tosses of an unbiased coin.

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S = HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

X	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
F(x)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

Problem 1.2

2. The foll table gives the probability distribution of X

X	1	2	3	4	5
f(x)	0.1	0.1	0.3	0.3	0.2

i) Computes mean ii) Find E(2X−1) iii) Find variance.

$\mu = E(X) = (1 \times 0.1) + (2 \times 0.1) + (3 \times 0.3) + (4 \times 0.3) + (5 \times 0.2) = 3.4$

$E(2X-1) = 2E(X) - E(1) = 6.8 - 1 = \underline{\underline{5.8}}$

$\sigma^2(X) = E(X^2) - [E(X)]^2$

$E(X^2) = 0.1 + 0.4 + 2.7 + 4.8 + 5 = 13$

$\sigma^2(X) = 13 - (3.4)^2 = \underline{\underline{1.44}}$

4. The foll table gives prob distribution of random variable X

X	0	1	2	3	4	5	6
f(x)	0.17	0.29	0.24	0.16	0.01	0.03	0.01

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Find: mean of prob distribution
standard deviation

$\mu = xf(x) = 0.29 + 0.54 + 0.48 + 0.28 + 0.15 + 0.06 = \underline{1.8}$

$E(x^2) = 0.29 + 1.08 + 1.44 + 1.12 + 0.75 + 0.36 = \underline{5.04}$

$\sigma(x) = \sqrt{5.04 - (1.8)^2} = \sqrt{1.8} = \underline{1.34}$

6. A R.V x has fell probability distribution where x is the no. of messages sent per hour over a computer network

X	10	11	12	13	14	15
f(x)	0.08	0.15	0.3	0.2	0.2	0.07

Determine the mean & S.D of the no. of messages per hour.

$E(X) = 0.8 + 1.65 + 3.6 + 2.6 + 2.8 + 1.05 = 12.5$

$E(X^2) = 8 + 18.15 + 43.2 + 33.8 + 39.2 + 15.45$

$= 158.1$

$\sigma^2(x) = E(X^2) - [E(x)]^2 \quad -185$

$\sigma(x) = 1.36$

7. If the mean & variance of the random variable X are

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3 & 2, respectively; then find the mean & variance of the random variable $Y = 4X + 9$

$E(X) = 3 \quad \sigma^2(X) = 2$

$E(Y) = 4E(X) + 9 = 12 + 9 = 21$

$\sigma^2(Y) = 4^2\sigma^2(X) + \sigma^2(9) = 4^2\sigma^2(X) = 16 \times 2 = \underline{\underline{32}}$

Topic 6 BINOMIAL DISTRIBUTION (BI)

The discrete random variable x is said to follow BI if it PMF ~~f(x)~~ $\boxed{f(x) = P(X=x) = nC_x p^x q^{n-x}}$ and $p+q=1$ and is denoted by $X \sim b(x, n, p)$. ~ follow.

$f(0) = b(0, n, p) = q^n$

$f(1) = b(1, n, p) = npq^{n-1}$

$\vdots$

$f(n) = b(n, n, p) = p^n$

These are the different terms in the binomial expansion of $(q+p)^n$. Hence the name binomial expansion $[(q+p)^n = q^n + nq^{n-1}p + \cdots p^n]$

CDF, $F(x) = P(P(X \le x))$

$= \sum_{k=0}^{x} nC_k p^k q^{n-k}$

$= f(0) + f(1) + f(2) + \cdots f(x)$

$= B(x, n, p)$.

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eg:- $B(3, n, p) = f(0) + f(1) + f(2) + f(3)$

$B(0, n, p) = 1$ because fro

$f(3) = b(3, n, p) = F(3) - F(2)$

$= B(3, n, p) - B(2, n, p)$

Note!

1. Binomial distribution (BD) is a discrete distribution which takes values $0, 1, 2, 3, \ldots n$.

2. p is called the probability of success.

3. $q = 1 - p$ is called probability of failure.

4. If x follows binomial distribution then there are only two events success and failure.

5. $f(x) = P(X = x) = P(x$ success out of n trials$)$

eg:- 3 coins are tossed ~~5 times~~ then find the probability of getting ~~2~~ exactly 2 heads.

X denotes the no. of heads occured.

X takes values 0, 1, 2, 3

$p =$ probability getting a head $= \frac{1}{2}$

$q =$ prob. of getting a tail $= \frac{1}{2}$

$P(X = x)$

$P(x \text{ heads}) = {}^3C_x \, p^x q^{3-x}$ $\quad x = 0, 1, 2, 3$

$f(2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{3}{8}$

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eg: 4 dis are thrown. Find the prob. of getting two 6's

$X \Rightarrow$ ~~f~~ denotes 6 occurs.

$x = 0, 1, 2, 3, 4$

$p =$ probability of getting a 6 $= \frac{1}{6}$

$q = \frac{5}{6}$

$f(x) = \cancel{4Cx p^{1/6}}$ $4Cx \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{4-x}$

probab. of getting 2 6 $= f(2)$

$f(2) = 4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= \frac{25}{216}$

Note

If an exp. containing n trials ~~done~~ repeated N times. Then in all N sets, the no. of sets with x success $= Nf(x)$

$= N \times nC_x p^x q^{n-x}$ $\quad x = 0, 1, \ldots n$

eg:- 8 fair ~~dis~~ coins are thrown 256 times. In how many throws do you expect atleast 1 head

$X =$ no. of heads.

$x = 0, 1, 2, 3, 4, 5, 6, 7, 8$

$n = 8$

$N = 256$

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$p =$ probability of head $= \frac{1}{2}$

$q = \frac{1}{2}$

$f(x) = {}^8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$

$= {}^8C_x \left(\frac{1}{2}\right)^8$

prob of getting atleast 1 head $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - f(0)$

$= \frac{255}{256}$

$f(0) = {}^8C_0 \left(\frac{1}{2}\right)^8$

$= \frac{255}{256}$ $\frac{1}{256}$.

no. of throws in which atleast 1 head occurs $= 256 \times \frac{255}{256} = \underline{255}$

eg: out of 2000 families with 4 children how many would you expect h have.

1) atleast 1 boy.

3) 1/2 1 or 2 girls

Let X denotes getting a boy.

X takes values 0, 1, 2, 3, 4

$n = 4$

$N = 2000$

$p = q = \frac{1}{2}$

$f(x) = {}^4C_x p^x q^{n-x}$

$= {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

$= {}^4C_x \left(\frac{1}{2}\right)^4$

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$P(x \geq 1)$

$= 1 - P(x > 1)$

$= 1 - f(0)$

$= 1 - \left(\frac{1}{2}\right)^4$

$= \frac{15}{16}$

no. of values with atleast 1 boy $= 2000 \times \frac{15}{16} = \underline{\underline{1875}}$

ii) probability of getting 1 or 2 girls = prob of getting 3 or 2 ~~girls~~ boys

$f(3) = 4C_3 \left(\frac{1}{2}\right)^4$

$f(2) = 4C_2 \left(\frac{1}{2}\right)^4$

$= 4C_3 \left(\frac{1}{2}\right)^4 + 4C_2 \left(\frac{1}{2}\right)^4$

$= \underline{\underline{\frac{10}{16}}}$

no. of families in 2000 $\therefore$

$= \frac{10 \times 2000}{16}$

$= \underline{\underline{1250}}$.

Q

Derive the mean and variance of the binomial distribution

$f(x) = nC_x p^x q^{n-x}$. $x = 0, 1, 2, 3 \ldots x$.

$$\text{mean} = E(x) = \sum_{x=0}^{n} x f(x)$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n(n-1)!}{(x-1)!((n-1)-(x-1))!} p \cdot p^{x-1} q^{((n-1)-(x-1))}$$

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$$= np \sum_{x=1}^{n} (n-1)C_{x-1}\, p^{x-1} q^{(n-1-(x-1))}$$

$x=1$

$$= \cancel{np \sum_{n=1}^{n}}$$

$$= np\,(n-1)C_0\, p^{0} q^{n-1}$$

$$= \cancel{np(n-1)}$$

$$= np\, q^{n-1}$$

$x=2$

$$= np\,(n-1)C_1\, p^{1} q^{(n-1-1)}$$

$$= np(n-1)\, q^{n-2}$$

$$E(x) = np\left[q^{n-1} + (n-1)q^{n-2} + \cdots p^{n-1}\right]$$

$$= np\,(q+p)^{n-1}$$

$$\boxed{E(X) = np}$$

$$Var(X) = \sigma^2(X) = E(X^2) - (E(X))^2 \longrightarrow (1)$$

$$E(X^2) = \sum_{0}^{n} x^2 f(x) = \sum_{x=0}^{n} [x(x-1)+x]\, f(x)$$

$$= \sum_{x=0}^{n} x(x-1) f(x) + \sum x f(x)$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!}\, p^x q^{n-x} + np$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{[(x-2)!\,(n-2-(x-2))]!} + np$$

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$$= n(n-1)p^2 \sum_{x=2}^{n} (n-2)C_{x-2}\, p^{x-2}\, q^{(n-2-(x-2))} + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np$$

$$= n(n-1)p^2 + np$$

Substituting in ①

$$Var(X) = [n(n-1)p^2 + np] - (np)^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np - np^2$$

$$\boxed{\sigma^2(X) = np - np^2}$$

$$\boxed{\sigma^2(X) = np(1-p)} \Rightarrow \sigma^2(X) = npq$$

S.D $\sigma(X) = \sqrt{np(1-p)}$

$$\boxed{\sigma(X) = \sqrt{npq}}$$

Qn. The mean and variance of binomial distribution is 6 and 2. Find the probability of $P(5 \le X \le 7)$

Ans Given

mean $= np = 6$

variance $= npq = 2$

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$q = \frac{2}{6} = \frac{1}{3}$

Hence $p = 1 - q = \frac{2}{3}$

$6 = \frac{2}{3} n$

$\frac{18}{2} = n$

$\underline{\underline{9 = n}}$

$f(x) = {}^{9}C_{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{9-x}$ $\quad x = 0$ to 9

Pro5

The given prob. $= f(5) + f(6) + f(7)$

$= {}^{9}C_{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{4} + {}^{9}C_{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{3} + {}^{9}C_{7}\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{2}$

$= 126 \times \left(\frac{2}{3}\right)^{5} \times \left(\frac{1}{3}\right)^{4} + 84 \times \left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{3} + 36\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{2}.$

$= \frac{126 \times 32}{19683} + \frac{5376}{19683} + \frac{4608}{19683}$

$= \frac{14016}{19683} = \underline{\underline{0.712}}.$

Dec 2016

Qn. During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating. Find the prob. that among 15 chips

1) atleast 12 will have thick enough coating

2) atmost 6 will have thick enough coating.

3) exactly 10 will have thick enough coating.

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X denotes the chips with thick enough coating

$X = 0, 1, 2, 3 \ldots 15$

$p = 0.7$

$q = 0.3$

$f(x) = 15C_x (0.7)^x (0.3)^{15-x}$ $\quad x = 0, \text{to } 15$

1) $P(X \geq 12) = 1 - P(X < 12)$ Pg no: 286 – 293

$= 1 - B(11, 15, .7)$ Pg no 294 – 298

$= 1 - 0.7031$ Pg no. 301 – 304

$= \underline{\underline{0.2969}}$.

2) $P(X \leq 6) = B(6, 15, .7)$

$= \underline{\underline{0.0152}}$

3) $P(X = 10) = f(10) = B(10, 15, .7) - B(9, 15, .7)$

$= 15C_{10} (0.7)^{10} (0.3)^5$

$= \underline{\underline{0.206}}$

2012

Hw An insurance company agent accepts policies of 5 men all of identical age and good health. Probability that a man of this age will be alive 30 yrs is $\frac{2}{3}$. Find the prob. that in 30 yrs

1) All of 5 men will be alive

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2) Atleast one man will be alive

3) Atmost 3 will be alive.

X denotes no. of persons to be alive

x takes value 0, 1, 2, 3, 4, 5

$p = \frac{2}{3}$

$q = \frac{1}{3}$

$f(x) = 5C_x \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{5-x}$ $\quad x = 0 \text{ to } 5$

1) $f(5) = 5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{5-5}$

$= \left(\frac{2}{3}\right)^5 = \frac{32}{243}$

2) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - f(0)$

$= 1 - \left(\frac{1}{3}\right)^5$

$= \frac{242}{243}$

3) $P(X \leq 3) = 1 - [f(4) + f(5)]$

$= 1 - 5C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right) + \frac{32}{243}$

$= \frac{131}{243}$

4)

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POISSON DISTRIBUTION

A D.R.V, X is said to follow poisson distribution if its PMF

$$\boxed{f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad x=0,1,\dots n}$$

, λ is called the parameter.

The C.D.F of the P.D;

$$F(x,\lambda) = P(X \leq x) = \sum_{k=0}^{x} f(k)$$

which is given in table II

Poisson distribution is the limiting case of binomial distribution when n is very large ($n \to \infty$) and p is very small ($p \to 0$) & np is finite ($np = \lambda$). Then binomial distribution tends to Poisson. Poisson distribution can be used to explain the behaviour of D.R.V, where the probability of occurence of event is very small and the total no. of possibilities very large is rare events follow poisson's distribution. eg:

eg:- 1) no. of accidents in a day
2) no. of defective materials produced in a company &
3) no. of wrong telephone calls.

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Imp.

P.T binomial distribution tends to Poisson distr. as $n \to \infty$; $p \to 0$ & $np =$

$$e^{x} = \underset{n\to\infty}{Lt}\left(\left[1+\frac{x}{n}\right]^{n}\right)$$

$$e^{-x} = \underset{n\to\infty}{Lt}\left(\left[1-\frac{x}{n}\right]^{n}\right)$$

Since X follows binomial distribution

$$f(x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x} \qquad x = 0, 1, 2, \ldots n$$

$$= \frac{n!}{x! \cdot (n-x)!}\, p^{x} (1-p)^{n-x}$$

$$= \frac{n(n-1)\ldots n-(x-1)}{x!}\, p^{x} (1-p)^{n-x}$$

Taking n outside from each term

$$= \frac{(np)^{x}}{x!}\left(1-\frac{1}{n}\right)\ldots\left[1-\frac{(x-1)}{n}\right](1-p)^{n}(1-p)^{-x}$$

As $n \to \infty$, then $1-\frac{1}{n} \to 1$ ..

$\vdots$

$1-\left(x-\frac{1}{n}\right) \to 1 \longrightarrow (1)$.

$n \to \infty$, $p \to 0$; $np = \lambda \longrightarrow (1)$

$$f(x) = \frac{\lambda^{x}}{x!}\left[\underset{n\to\infty}{Lt}\left(1-\frac{\lambda}{n}\right)^{n}\right]\left[\underset{n\to\infty}{Lt}\left(1-\frac{\lambda}{n}\right)^{-x}\right]$$

$$= \frac{\lambda^{x}}{x!}\, e^{-\lambda} \cdot \times 1$$

$$= \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Qn. Find the mean and variance of poisson distribution.

Ans: $f(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots n$

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$$\text{mean} = E(x) = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \lambda \cdot \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \qquad 0! = 1$$

$$= e^{-\lambda} \cdot \lambda \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$= \underline{\underline{\lambda}}$$

$$\boxed{E(x) = \lambda}$$

Variance

$$\sigma^2(x) = E(x^2) - [E(x)]^2 \rightarrow (1)$$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 f(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1) + x] f(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1) f(x)] + \sum x f(x)$$

$$= \sum_{x=0}^{\infty} (x-1) x f(x) + \sum x f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^{x}}{x!} + \lambda$$

$$= \lambda^2 e^{-\lambda} \sum_{2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= \lambda^2 e^{-\lambda} \left[\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \right] + \lambda \right]$$

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$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$

$= \lambda^2 + \lambda$

Substituting in ①

$\sigma^2(X) = \lambda^2 + \lambda - \lambda^2$

$= \lambda$

Table values: binomial distr

* $n = 10$

$p = .9$

$P(X \leq 5) = 0.0016$

* $P(X > 4) = 1 - P(X \leq 4)$

$1 - B(4, 10, .9)$

$= 1 - 0.0001$

$= 0.9999$

* Using table find $P(4 \leq x \leq 10)$, $n = 10$, $p = .35$

$P(4 \leq x \leq 10)$

$= B(10, 10, .35) - B(3, 10, .35)$

$= 1 - 0.5138$

$= .4862$

Qn. In a certain city, the probability of an accident on any given day is 0.005 and accidents are independent of each other. i) what is the probability that in any given period of 400 days, there will be an accident

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on one day (i) what will be the probability that there are atmost 3 days for an accident.

* X :- denotes the no. of days X follows.

$n = 0 \to 400$

$P = 0.005$

X follows Poisson's distribution and $\lambda = np$

$\lambda = np = 400 \times 0.005 = 2$

$$f(x) = \frac{e^{-2} 2^x}{x!} \quad x = 0, 1, 2 \ldots 400$$

(i) $f(1) = \frac{e^{-2} 2}{1!}$

$= \underline{\underline{.2706}} = .271$

(ii) ~~f(x)~~

(ii) $P(X \leq 3) = F(3, 2)$

$= \underline{\underline{.857}}$

Qn if X is a poisson variable such that

$3P(X=4) = \frac{1}{2} P(X=2) + P(X=0)$

Find

(i) The mean of X

(ii) $P(X \leq 2)$ (iii) $P(X > 3)$

ans. Since X is a poisson variable.

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$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(X=4) = f(4) = \frac{e^{-\lambda}\lambda^{4}}{4!}$$

$$P(X \leq 2) = P(X=2) = f(2) = \frac{e^{-\lambda}\lambda^{2}}{2!}$$

$$P(X=0) = f(0) = \underline{\underline{e^{-\lambda}}}$$

substitute the value in given eqn

~~fcx~~

$$\frac{3e^{-\lambda}\lambda^{4}}{4!} = \frac{e^{-\lambda}\lambda^{2}}{2!} + e^{-\lambda}$$

$$\frac{3\lambda^{4}}{4!} = \frac{\lambda^{2}}{2!} + 1$$

$$\frac{3\lambda^{4}}{4!} - \frac{\lambda^{2}}{2!} = 1$$

$$\frac{\lambda^{4}}{8} - \frac{\lambda^{2}}{2} = 1$$

$4 \times 3 \times 2 \times 1$

$\underline{24}$

$$\frac{3\lambda^{4}}{24} - \frac{\lambda^{2}}{2} = 1$$

$$3\lambda^{4} - 12\lambda^{2} = 24$$

$$\lambda^{2}(3\lambda^{2} - 12) = 24$$

$$\lambda^{4} - 4\lambda^{2} = 8$$

$$u^{2} - 4u^{2} = 8$$

$\lambda^{2} = u$

$$3u^{2} - 12u = 24$$

$$u = +12 \pm \sqrt{144 -}$$

$$u - \lambda^{2} = 4$$

$$\lambda = \pm 2$$

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i) mean $= \lambda = 2$

ii) $P(X \leq 2) = F(2, 2), = \underline{\underline{0.677}}$.

iii) $P(X > 3) = 1 - P(X \leq 3)$

$= \cancel{1 - .677 \quad .323 \quad .875}$

$= \underline{\underline{.323}} \quad \cancel{.677}$

$= \cancel{1 - .875}$

$= 1 - .857$

$= \underline{\underline{.143}}$

Qn The no. of γ rays emitted / sec by a certain radioactive substance with mean = 5.8. If a recording instrument becomes inoperative when there are more than 12 rays / sec, what is the probability that this instrument becomes inoperative during any given sec.

$$f(x) = \frac{e^{-5.8} \, 5.8^{x}}{x!} \qquad x = 0, 1, 2, \ldots \infty$$

$P(X > 12) = 1 - P(X \leq 12)$

$= 1 - .993$

$= \underline{\underline{.007}}$

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Ans. Using binomial table $\sum_{k=4}^{8} b(k, 10, 0.35)$

$B(10, 10, 0.35) - B(3, 10, .35)$

$= 1 - .5135$

$= 0.4862$

✓ $B(8, 10, 0.35) - B(2, 10, .35)$

$= .9995 - .5135$

$= \underline{\underline{.4857}}$

Qn. The incidents of occupational disease in an industry is such that the workers have 20% chance of suffering from it. what is the probability that out of 6 workers choosen at random 4/more will suffer from the disease

ans. X = no. of persons suffering the disease

$n = 6$

$p = .2$

$q = .8$

$\therefore f(x) =$ ~~$6C_x 1$~~ ~~$6C_x 6$~~

$f(x) = 6C_x \cdot 2^x \cdot .8^{n-x} \quad x = 0, 1, \cdots 6$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - B(3, 6, .2)$

$= 1 - .9880$

$= 0.017$

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Dec 2019

Qn. In a given city 6% of all drivers get atleast 1 parking ticket per year use poisson distribution to determine the probability that among 80 drivers

① ~~atleas~~ 4 will get atleast 1 ~~party~~ parking ticket en any given year.

② atleast 3 will get atleast 1 parking ticket

③ 3 to 6 inclusive will get atleast 1 parking ticket.

X = no. of drivers who will get atleast 1 parking ticket

$n = 80$

$p = .06$

$\lambda = 80 \times .06$

$= \underline{\underline{4.8}}$

$\therefore \quad f(x) = \dfrac{e^{-4.8} 4.8^{x}}{x!} \qquad x = 0, 1, \ldots 80$

① $f(4) = \dfrac{e^{-4.8} 4.8^{4}}{4!}$

$= F(4, 4.8) - F(3, 4.8)$

$= 0.476 - .294$

$= \underline{\underline{.182}}$

② atleast $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - F(2, 4.8)$

$= 1 - .143 = \underline{.857}$

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(iii) $P(3 \le X \le 6) = F(6, 4.8) - F(3, 4.8)$

$= .791 - .143$

$= \underline{\underline{.648}}$

HW Problems set 1.3 Qn :- 3, 4, 5, 9, 12.. Pg (27)

Pg 37: problem set 1.4 Qn :- 5, 7, 14.

Problem set 1.3

3. In 100 sets of 10 tosses of an unbiased coin, in how many cases do you except to get i) 7 heads and tails ii) atleast 7 heads.

ii) $n = 10$

$p = 1/2$

$x = 0, 1, 2, 3..$

X denotes no. of heads

$P(X \ge 7) = 1 - P(X < 6)$

$= 1 - B(6, 10, 1/2)$

$= 1 - .8281$

$= .1719$

P (atleast 7 heads) $= .1719 \times 100 = \underline{\underline{17}}$

i) X : denotes no. of heads

$n = 10$

$p = 1/2$

$q = 1/2$

$x = 0, 1, 2, 3$

$P(X=7) \cdot f(x) = 10C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$

$= 10C_7 \left(\frac{1}{2}\right)^{10}$

$= \frac{15}{128}$

P(7 heads and tails) $= 100 \times \frac{15}{128} = 11.71 = \underline{\underline{12}}$

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5. A discrete RV X has its mean and var

6. Probability that a batman scores a century in a cricket match is 1/3. Find the probability that out of 5 matches he may score century in i) exactly two matches ii) no matches iii) atleast in one match.

18. The mean of binomial distribution is 3 and the variance is 9/4. Find i) value of n ii) the PMF, iii) $P(X>7)$, iv) $P(1 \le X < 6)$

$np = 3 = E(X)$

$\sigma^2(X) = np(1-p) = 9/4$

$np = 3 \quad p = 3/n$

$np - np^2 = 9/4$

$3 - np^2 = 9/4$

$3 - \frac{n9}{n^2} = 9/4$

$3 - \frac{9}{n} = 9/4$

$3n - 9 = 9/4\, n$

$3n - \frac{9}{4}n = 9$

$\cancel{n = \frac{12n - 9n}{4}}$

$=$

$12n - 9n = 36$

$3n = 36$

$n = 12$

$12p = 3$

$p = 3/12 = \underline{\underline{1/4}}$

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ii) PMF

$f(x) = {}^{12}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{12-x}$

$x = 0, 1, 2, 3 \ldots\ldots 12$

iii) $P(X > 7) = 1 - P(X \leq 7)$

$= 1 - B(7, 12, 1/4)$

$= 1 - .9972$

$= 0.0028$

$= \underline{\underline{0.003}}$

$p = .09 \cdot 9$

$P(X \geq 16)$

$1 - P(X \leq 15)$

$1 - B(15, 18, .9)$

v) $P(1 \leq X \leq 6)$; $n = 12$, $p = 1/4$

$= \cancel{B(5, 12, .25) - B(0, 12, .25)}$

$= \cancel{.9857 - .0317}$

$= B(5, 12, .25) - B(0, 12, .25)$

$= .9456 - .0317$

$= \underline{\underline{0.9139}}$

9. An agricultural operative claims that 90% of the watermelons shipped out are ripe and ready to eat. Find the probability that among 18 watermelons shipped out @ atleast 16 are ripe and ready to eat (b) at most 14 are ripe and ready to eat.

$\cancel{f(x) = 1 - .2662}$

D) $P(X \geq 16)$

$= 1 - P(X \leq 15)$

$= 1 - B(15, 18, .9) = 1 - .2662 = .7338$

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ii) $P(X \leq 14)$

$= B(14, 18, .9)$

$= .0982$

Problem set 1.4

5. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which some demand is refused.

$n = 2$

$\lambda = 1.5$

i) $\frac{e^{-\lambda} \lambda^x}{x!}$ $x = 0, 1, 2$.

$x = 0$, $\frac{e^{-\lambda} \lambda^0}{0!} = e^{1.5} = 0.2231$

ii) P(some demand is refused) = P(demand is more than 2 cars per days)

$= P(X > 2)$

$= 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - e^{-1.5}\left[1 + 1.5 + \frac{2.25}{2}\right]$

$= 0.1912$

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6 coins are tossed 6400 times. using P.D, what is the app. probability of getting 6 heads in x time?

Let the coin be unbiased

$\therefore$ The probability of getting 6 heads with six coins

$$= \left(\frac{1}{2}\right)^6 = \frac{1}{64} = p$$

$$\therefore np = 6400 \times \frac{1}{64} = 100 = \lambda$$

$$f(x) = P(X = x) = \frac{e^{-100}(100)^x}{x!}$$

customer arrive in a counter at an avg of 1.5 per minute. Find the probability that

i) at most 4 will arrive in any given time

ii) at least 3 will arrive ~~down~~ during an interval of 2 min

$\lambda = 1.5$

i) $P(X \leq 4) = F(4, 1.5) = \underline{\underline{.981}} = .981$

ii) $P(X \geq 3) = 1 - P(X \leq 3) = 1 - F(2, 3) = \cancel{1-.647}\ 1 - .423$

$$= \underline{\underline{.577}}$$

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TUTORIAL

Qn. Given the PMF of D.R.V is

x	-3	-2	-1	0	1	2	3
$f(x)$	$1/6$	$1/6$	$1/6$	$1/12$	$1/12$	$1/6$	$1/6$

Find i) $P(|x|<1)$

ii) $P(|x|>2)$

iii) $P(2x+3\leq 5)$

iv) $F(x)$

i) $P(|x|<1) = P(-1<x<1)$

$= f(0)$

$= \underline{\underline{1/12}}$

ii) $P(|x|>2) = P(x=-3) + P(x=3)$

$= 1/6 + 1/6$

$= \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$

iii) $P(2x+3\leq 5) = P(2x\leq 2)$

$= P(x\leq 1)$

$= 1-(P(x=2)+P(x=3))$

$= \frac{1}{1} - \left(\frac{1}{6}+\frac{1}{6}\right)$ $\quad \frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}$ 1

$= \frac{1}{1} - \frac{2}{6} = \cancel{\frac{1}{12}}\ \cancel{\frac{4}{3}}\ \cancel{\frac{2}{3}}$ $\quad 1-\frac{2}{6} = 1-\frac{1}{3} = \underline{\frac{2}{3}}$

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Qn

X	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$
$F(X)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{10}{12}$	1

$\frac{3}{6} + \frac{1}{12}$

B -

$\frac{8}{12} + \frac{2}{6}$

$\frac{10}{12} \times 2$

Qn. Find the mean and ~~sta~~ S.D of

X	8	12	16	20	24
$f(x)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

$E(X) = \Sigma x f(x)$

$= 8 \times \frac{1}{8} + 12 \times \frac{1}{6} + 16 \times \frac{3}{8} + 20 \times \frac{1}{4} + 24 \times \frac{1}{12}$

$= 1 + 2 + 6 + 5 + 2$

$= 16$

$\sigma^2(X) = \cancel{\Sigma x^2 f(x)} \quad E(X^2) - [E(X)]^2$

$E(X^2) = \Sigma x^2 f(x)$

$= 8^2 \times \frac{1}{8} + 12^2 \times \frac{1}{6} + 16^2 \times \frac{3}{8} + 20^2 \times \frac{1}{4} + 24^2 \times \frac{1}{12}$

$= \frac{64}{8} + \frac{144}{6} + \frac{768}{8} + \frac{400}{4} + \frac{576}{12}$

$= 276$

$\sigma^2(X) = 276 - 256$

$= 20$

$S.D = \sqrt{20}$

$= \underline{4.47}$

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Qn. 10 coins are ~~sin~~ thrown simultaneously. Find

i) probability of getting atleast 7 heads

2) probability of getting exactly 7 heads

3) probability of getting atmost 7 heads.

$n = 10$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

X : denotes no. of heads

$f(x) = 10C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x}$

$= 10C_x \left(\frac{1}{2}\right)^{10}$ $\quad x = 0$ to 10

i) $P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - B(6, 10, .5)$

$= 1 - .8281$

$= \underline{\underline{.1719}}$

ii) $P(X = 7) = 10C_7 \left(\frac{1}{2}\right)^{10} = f(7)$ or $B(7, 10, .5) - B(6, 10, .5)$

$= \underline{\underline{0.1171}}$

iii) $P(X \leq 7) = B(7, 10, .5)$

$= \underline{\underline{.9453}}$

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Qn. The mean and variance of a binomial variant X are 16 and 8. Find

i) $P(X=1)$

ii) $P(X \geq 2)$

Given

$np = 16$

$np(1-p) = 8$

$\frac{1}{1-p} = 2$

$1 = 2-2p$

$2p = 1$

$p = 1/2$

$\frac{n}{2} = 16$

$n = 32$

$f(x) = {}^{32}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}$

$= {}^{32}C_x \left(\frac{1}{2}\right)^{16}$

i) $P(X=1) = {}^{32}C_1 \left(\frac{1}{2}\right)^{32} = 32\,C_1 \left(\frac{1}{2}\right)^{32}$

$=$

ii) $P(X \geq 2) = 1 - P(X \leq 1)$ or $1 - [f(0) + f(1)]$

$= 1 - B(1, 32, 0.5)$ $\quad = 1 - [[1+32]\left(\frac{1}{2}\right)^{32}]$

$= 1 - 32C_1 \left(\frac{1}{2}\right)^{32}$ $\quad = 1 - 33 \times \left(\frac{1}{2}\right)^{32}$

$= .99$ $\quad = .99$

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Qn. A coin is tossed until a head appears

1) Find PMF of no. of tosses

2) Find expectation of the no. of tosses.

x = no. of tosses until a head appears

x takes value $1, 2, 3, \ldots$

i) PMF is

x	1	2	3	...
$f(x)$	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	...

ii) mean $= \Sigma x f(x)$

$$= \sum_{x=1}^{\infty} x f(x)$$

$$= 1 \times \frac{1}{2} + 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{2}\right)^3 + \ldots$$

$$= \frac{1}{2}\left[1 + 2 \times \left(\frac{1}{2}\right) + 3 \times \left(\frac{1}{2}\right)^2 + \ldots\right]$$ which is an arithmetico geometric series

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}} + \frac{1 \times \frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}\right]$$

$a = 1$

$d = 1$

$r = \frac{1}{2}$

$$= \frac{1}{2}\left[2 + \frac{1/2}{(1/2)^2}\right]$$

$$= \frac{1}{2}[2 + 2]$$

$$= \underline{\underline{2}}$$

if $r < 1$

$$\left[S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}\right]$$

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Qn. A die is thrown until the face 6 occurs. Find PMF of the no. of ~~rolls~~ throws.

Ans

X	1	2	3	4	----
f(x)	$\frac{1}{6}$	$\left[\frac{5}{6}\times\frac{1}{6}\right]$	$\left[\left(\frac{5}{6}\right)^2\times\frac{1}{6}\right]$	$\left[\left(\frac{5}{6}\right)^3\times\frac{1}{6}\right]$	

Qn. In a certain factory making razor blades, there is small chance $\frac{1}{500}$ for any blade to be defective. The blades are in a packet of 10 use Poisson distribution to calculate appr probability

1) no defective
2) 1 defective
3) 2 defective

respectively in consignment of 10,000 packets.

$n = 10$

$p = \frac{1}{500}$

$\lambda = np$

$= 10 \times \frac{1}{500} = \frac{1}{50} = 0.02$

$$f(x) = \frac{e^{-.2}(.2)^x}{x!} \qquad x = 0, 1, \ldots 10$$

1) $P(X=0) = \frac{e^{-.2}(.2)^0}{0!}$

$= e^{-.2}$

$= \underline{.98}$

no. of packets in which no defective $= 10{,}000 \times .98$

$= \underline{\underline{9800}}$

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ii) $P(X=1) = \frac{e^{-.02}(.02)^1}{1!}$

$= .0196$

no of packets with 1 defective = 196

ii) $P(X=2) = \frac{e^{-.02}(.02)^2}{2!}$

$= 1.960 \times 10^{-4}$

no of packets with 2 defective = 1.96

= 2

Qn. X is poisson variant $P(X=2) = \frac{2}{3} P(X=1)$. Find $P(X=3)$ and $P(X \leq 3)$

ans since x follows poisson dist. $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given $f(2) = \frac{2}{3} f(1)$

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1}$$

$$e^{-\lambda}\lambda = \frac{4}{3} e^{-\lambda}$$

$$\lambda = \frac{4}{3} = 1.33$$

$$f(x) = \frac{e^{-1.33}\, 1.33^x}{x!}$$

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i) $P(X=3)$

$$f(3) = \frac{e^{-1.33}(1.33)^3}{3!}$$

$$= \underline{\underline{0.103}}$$

ii) $P(X \leq 3) = F(3, 1.33)$

$$= \underline{\underline{0.957}} \cancel{\times 0.948}$$

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