MATHS FORMULAE

module 3

(i) $$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)}$$

$$\begin{cases} a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx \\ a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx \\ b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx \end{cases}$$

if dirichlets conditions are satisfied.
(i) periodic
(ii) finite discontinuities

(ii) Fourier Integral:

$$\boxed{f(x) = \int_{\omega=0}^{\infty} \frac{1}{\pi}\int_{t=-\infty}^{\infty} f(t)\cdot\cos(\omega(t-x))\,dt\,d\omega}$$

OR

$$\boxed{f(x) = \int_0^{\infty}[A(\omega)\cos\omega x + B(\omega)\sin\omega x]\,d\omega}$$

$$\begin{cases} A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt \\ B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt \end{cases}$$

(iii) Fourier sine and Cosine Integral:

FOURIER SINE INTEGRAL } $f(x) = \int_0^{\infty} B(\omega)\sin\omega x\,d\omega$; $B(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t)\sin\omega t\,dt$; $\boxed{f(t) \rightarrow \text{odd fn.}}$

FOURIER COSINE INTEGRAL } $f(x) = \int_0^{\infty} A(\omega)\cos\omega x\,d\omega$; $A(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t)\cos\omega t\,dt$; $\boxed{f(t) \rightarrow \text{even fn.}}$

(iv) Fourier Transform:

- COMPLEX FORM OF FOURIER INTEGRAL ~~TRANSFORM~~ } $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\cdot e^{i\omega(x-t)}\,dt\,d\omega$

FOURIER TRANSFORM OF $f(x)$ } $f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$

INVERSE FOURIER TRANSFOM OF $f(x)$ } $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\omega)\,e^{-i\omega x}\,d\omega$

(v) Fourier Sine & Cosine Transform:

FOURIER SINE TRANSFORM } $f_s(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\sin\omega t\,dt$; FOURIER INV. SINE TRANSFORM } $f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f_s(\omega)\sin\omega x\,d\omega$

FOURIER COSINE TRANSFORM } $f_c(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos\omega t\,dt$; FOURIER INV. COSINE TRANSFORM } $f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f_c(\omega)\cos\omega x\,d\omega$

(vi) Properties of Fourier Transform:

(a) Linearity property : $F\{af(x) + bg(x)\} = aF\{f(x)\} + bF\{f(x)\}$

(b) Change of Scale : $F\{f(ax)\} = \frac{1}{a}\hat{F}(\omega/a)$

(c) Shifting Property : $F\{f(x-a)\} = e^{-i\omega a}\hat{f}(\omega)$

(d) Frequency shift property : $F\{e^{-iax}f(x)\} = \hat{f}(\omega - a)$ & $\boxed{F^{-1}\{\hat{f}(\omega-a)\} = e^{iax}f(x)}$

fourier transform for derivatives :

$$F\{f'(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f'(t)\cdot e^{-i\omega t}\,dt \quad \ldots \quad \boxed{F\{f^{n}(x)\} = (i\omega)^n \hat{f}(\omega)}$$

UNIT STEP FUNCTION

$$u(t-a) = u_a(t) = \begin{cases} 0, & 0<t<a \\ 1, & t\geq a \end{cases}$$

$\to$ FOURIER TRANSFORM $\}$ $\dfrac{-ie^{-i\omega a}}{\sqrt{2\pi}}\left(\dfrac{1}{\omega}\right)$

IMPULSE FUNCTION

$$\delta_\epsilon(t) = \begin{cases} \frac{1}{\epsilon}, & a\leq t\leq a+\epsilon \\ 0, & \text{otherwise} \end{cases} \qquad [\epsilon\to 0]$$

$\to$ FOURIER TRANSFORM $\}$ $= \dfrac{e^{-i\omega a}}{\sqrt{2\pi}}$

CONVOLUTION

$f(x),\ g(x) \longrightarrow$

$$(f*g)(x) = \int_{-\infty}^{\infty} f(u)\cdot g(x-u)\,du$$

$\downarrow$

$$F\{(f*g)(x)\} = \sqrt{2\pi}\; F\{f(x)\}\cdot F\{g(x)\}$$

$\downarrow$

$$F^{-1}\{\hat{f}(\omega)\,\hat{g}(\omega)\} = (f*g)(x)$$

Module 4

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$ ⇒ Laplace Transform

$L^{-1}\{F(s)\} = f(t)$ ⇒ Inverse Laplace Transform

→ $\boxed{\text{Laplace transform operator}}$

→ exists if : ① piecewise continuous $\{[0,\infty)\}$

② exponential order $\{\lim_{t\to\infty} e^{-st} f(t) \text{ exists}\}$

STANDARD LAPLACE TRANSFORMS

1. $L\{1\} = \frac{1}{s}$; $s>0$
2. $L\{e^{at}\} = \frac{1}{s-a}$; ~~s>a~~ $s>a$
3. $L\{\sin at\} = \frac{a}{s^2+a^2}$;
4. $L\{\cos at\} = \frac{s}{s^2+a^2}$
5. $L\{\sinh at\} = \frac{a}{s^2-a^2}$
6. $L\{\cosh at\} = \frac{s}{s^2-a^2}$
7. $L\{t^n\} = \frac{n!}{s^{n+1}}$

STD. INV. LAPLACE TRANSFORMS

1. $L^{-1}\{1/s\} = 1$
2. $L^{-1}\{1/(s-a)\} = e^{at}$
3. $L^{-1}\{1/(s^2+a^2)\} = \frac{1}{a}\sin at$
4. $L^{-1}\{s/(s^2+a^2)\} = \cos at$
5. $L^{-1}\{1/(s^2-a^2)\} = \frac{1}{a}\sinh at$
6. $L^{-1}\{s/(s^2-a^2)\} = \cosh at$
7. $L^{-1}\{\frac{1}{s^n}\} = \frac{t^{n-1}}{(n-1)!}$

--- by shifting property -----

8. $L^{-1}\{1/((s-b)^2+a^2)\} = \frac{e^{bt}}{a}\sin at$
9. $L^{-1}\{(s-b)/((s-b)^2-a^2)\} = e^{bt}\cosh at$
10. $L^{-1}\{1/(s+a)^n\} = e^{-at}\cdot\frac{t^{n-1}}{(n-1)!}$

and so on...

Properties

(i) Linearity property :

$L\{a\,f(t) + b\,g(t)\} = a\cdot L\{f(t)\} + b\cdot L\{g(t)\}$

(ii) First shifting property :

$L\{e^{at} f(t)\} = F(s-a)$

(iii) Change of scale property :

$L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

Multiplication by t^n ⇒

$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(s))$

Division by t ⇒

$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$

for derivatives ⇒

$L\{f^{n}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{n-1}(0)$

for integrals : $L\{\int_0^t f(u)\}s = \frac{F(s)}{s}$

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inverses } $L^{-1}\{F(s-a)\} = e^{at} \cdot f(t)$

$$L^{-1}\left\{\frac{d}{ds}(F(s))\right\} = -t f(t)$$

$$L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)\,dt$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \;;$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{2l}\int_{-l}^{l} f(t)\,dt + \sum_{n=1}^{\infty}\frac{1}{l}\int_{-l}^{l} f(t)\left\{\cos\left(\frac{n\pi t}{l}\right)\cos\left(\frac{n\pi x}{l}\right) + \sin\left(\frac{n\pi t}{l}\right)\cdot\sin\left(\frac{n\pi x}{l}\right)\right\}dt$$

$$= 0 + \sum_{n=1}^{\infty}\frac{1}{l}\int_{-l}^{l} f(t)\cos\left(\frac{n\pi(t-x)}{l}\right)dt$$

$$\delta\omega = \omega_{n+1} - \omega_n = \frac{(n+1)\pi}{l} - \frac{n\pi}{l} = \frac{\pi}{l} \Rightarrow l = \frac{\pi}{\delta\omega}$$

$$n\cdot\delta\omega = \omega$$

$$= \sum_{n=1}^{\infty}\frac{\delta\omega}{\pi}\int_{-l}^{l} f(t)\cos(\omega(t-x))\,dt$$

$$f(x) = \int_{\omega=0}^{\infty}\frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos(\omega(t-x))\,dt$$

---

on further simplification: ← fourier' Integral

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cdot[\cos\omega t\cos\omega x + \sin\omega t\sin\omega x]\,dt\,d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty}\left(\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt\right)\cos\omega x\,d\omega + \frac{1}{\pi}\int_0^{\infty}\left(\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt\right)\sin\omega x\,d\omega$$

Let $A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt$ & $B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt$

$$\to f(x) = \int_0^{\infty} A(\omega)\cos\omega x\,d\omega + \int_0^{\infty} B(\omega)\sin\omega x\,d\omega$$

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fourier sine & cosine integral:

COSINE INTEGRAL REPRESENTATION $\left.\right\}$ $f(x) = \int_0^{\infty} A(\omega)\cos\omega x\,d\omega \;:\; A(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t)\cos\omega t\,dt$ — $f(t) \to$ even fn.

SINE INTEGRAL REPRESENTATION $\left.\right\}$ $f(x) = \int_0^{\infty} B(\omega)\sin\omega x\,d\omega \;;\; B(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t)\sin\omega t\,dt$ — $f(t) \to$ odd fn.

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fourier transform

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos(\omega(t-x))\,dt\,d\omega$$

OR

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\cos(\omega(x-t))\,dt\,d\omega \quad (a)$$

mehl ↳ even fn.

Imagine $0 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\sin(\omega(x-t))\,dt\,d\omega \quad (b)$

mehl ↳ odd fn.

(a) + i(b) ⇒ COMPLEX FORM OF FOURIER INTGR:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)[\cos\omega(x-t) + i\sin\omega(x-t)]\,dt\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\,e^{i\omega(x-t)}\,dt\,d\omega$$

$$\therefore f(x) = \frac{1}{2\pi}\int_{-\infty}\int_{-\infty} ...$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt\right) e^{i\omega x} d\omega = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\omega)\, e^{i\omega x}\, d\omega$$

$\therefore$ fourier transform of $f(x)$ $\Big\}$ $f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty} f(t)\, e^{-i\omega t}\, dt$

inverse transform of $f(x)$ $\Big\}$ $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\omega)\, e^{i\omega x}\, d\omega$

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fourier sine & cosine tranform:

As we know,

fourier sine integral $\Big\}$ $f(x) = \int_0^{\infty} B(\omega) \sin \omega x \, d\omega$

- $B(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t) \sin \omega t \, dt$

sub. $\Rightarrow$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\int_0^{\infty} f(t) \sin\omega t \sin \omega x \, dt\, d\omega$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty}\left[\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\sin\omega t\, dt\right] \sin \omega x\, d\omega$$

$\downarrow$

fourier sine transform $\Big\}$ $f_s(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t) \sin\omega t\, dt$

inverse fourier sine transform $\Big\}$ $f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f_s(\omega) \sin \omega x\, d\omega$

fourier cosine integral $\Big\}$ $f(x) = \int_0^{\infty} A(\omega) \cos \omega x \, d\omega$

- $A(\omega) = \frac{2}{\pi}\int_0^{\infty} f(t)\, \cancel{\sin \omega t} \cos \omega t\, dt$

sub. $\Rightarrow$

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\int_0^{\infty} f(t)\, \cancel{\sin\omega t\, dt} \cos\omega t \cos \omega x\, dt\, d\omega$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty}\left(\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t) \cos \omega t\, dt\right) \cos\omega x\, d\omega$$

$\downarrow$

fourier ~~sine~~ cosine transform $\Big\}$ $f_c(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t) \cos \omega t\, dt$

inverse fourier cosine transform $\Big\}$ $f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f_c(\omega) \cos \omega x\, d\omega$

Properties

① Linearity property : $F\{af(x) + b\, g(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(af(t) + bg(t))\cdot e^{-i\omega t}\, dt$

$$= \frac{1}{\sqrt{2\pi}}\, a \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt + \frac{1}{\sqrt{2\pi}}\, b \int_{-\infty}^{\infty} g(t)\, e^{-i\omega t}\, dt$$

$$= a\hat{f}(\omega) + b\,\hat{g}(\omega)$$

② Change of scale property : $F\{f(ax)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(at)\, e^{-i\omega t}\, dt$

- $u = at$
- $dt = \frac{1}{a} du$

$$F\{f(ax)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\cdot e^{-\frac{i\omega u}{a}}\cdot \frac{du}{a}$$

$$= \frac{1}{a}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\cdot e^{-\frac{i\omega u}{a}}\, du\right)$$

$$= \frac{1}{a}\hat{f}(\omega/a)$$

(3) Shifting property :

$$F\{f(x-a)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t-a)\, e^{-i\omega t}\, dt$$

$$\left.\begin{array}{l} u = t-a \\ \rightarrow dt = du \end{array}\right\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\cdot e^{-i\omega(u+a)}\, du = e^{-i\omega a}\cdot\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\cdot e^{-i\omega u}\, du$$

$$= e^{-i\omega a}\,\hat{f}(\omega)$$

(4) frequency shifting property :

$$F\{e^{+iax}\cdot f(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{+iat}\cdot f(t)\cdot e^{-i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{-i(\omega-a)t}\, dt$$

$$\therefore \quad = \hat{f}(\omega - a)$$

~~derivatives of fourie~~

fourier transform for derivatives:

$$F\{f'(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f'(t)\, e^{-i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\left[\cancel{e^{-i\omega t} f(t)\Big|_{-\infty}^{\infty}}_{\,0} - \int_{-\infty}^{\infty}(-i\omega)\, e^{-i\omega t}\cdot f(t)\, dt\right]$$

$$= i\omega\,\hat{f}(\omega)$$

$$F\{f''(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f''(t)\, e^{-i\omega t}\, dt$$

$$\cancel{= \frac{1}{\sqrt{2\pi}}\left[e^{-i\omega t} f(t) + i\omega e^{-i\omega t} f(t)\right.}$$

$$= \frac{1}{\sqrt{2\pi}}\left[\cancel{e^{-i\omega t} f'(t)\Big|_{-\infty}^{\infty}}_{\,0} - \int_{-\infty}^{\infty}(-i\omega)\, e^{-i\omega t} f'(t)\, dt\right]$$

$$= i\omega\cdot F\{\hat{f}(\omega)\}$$

$$= (i\omega)^2\,\hat{f}(\omega)$$

$\therefore$ generally, $F\{f^{n}(x)\} = (i\omega)^n\,\hat{f}(\omega)$; $\hat{f}(\omega) = F\{f(x)\}$

MA

$$L\{1\} = \int_0^\infty 1 \cdot e^{-st}\, dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{s}$$

$$L\{e^{at}\} = \int_0^\infty e^{at} \cdot e^{-st}\, dt = \int_0^\infty e^{-(s-a)t}\, dt = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = \frac{1}{s-a}$$

$$L\{\sin at\} = \int_0^\infty \sin at \cdot e^{-st}\, dt = \left\{\frac{e^{-st}}{s^2+a^2}\left[-s\sin at - a\cos at\right]\right\}_0^\infty = \frac{a}{s^2+a^2}$$

$$L\{\cos at\} = \int_0 \cos at \cdot e^{-st}\, dt = \left\{\frac{e^{-st}}{s^2+a^2}\left[-s\cos at + a\sin at\right]\right\}_0^\infty = \frac{s}{s^2+a^2}$$

$$L\{\sinh at\} = \int_0^\infty \left(\frac{e^{at}-e^{-at}}{2}\right) e^{-st}\, dt = \frac{1}{2}\int_0^\infty \left[e^{-(s-a)t} - e^{-(s+a)t}\right] dt = \frac{1}{2}\left\{\frac{1}{s-a} - \frac{1}{s+a}\right\}$$

$$= \frac{a}{s^2-a^2}$$

$$L\{\cosh at\} = \int_0^\infty \left(\frac{e^{at}+e^{-at}}{2}\right) e^{-st}\, dt = \frac{1}{2}\int_0^\infty \left[e^{-(s-a)t} + e^{-(s+a)t}\right] dt = \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\}$$

$$= \left\{\frac{s}{s^2-a^2}\right\}$$

$$L\{t^n\} = \int_0^\infty t^n \cdot e^{-st}\, dt \Rightarrow \cdot\ st = u \to t = \frac{u}{s} \to dt = \frac{du}{s} \Rightarrow$$

$$= \int_0^\infty \frac{u^n}{s^n} \cdot e^{-u} \frac{du}{s} = \frac{1}{s^{n+1}} \int_0^\infty u^n e^{-u}\, du = \frac{n!}{s^{n+1}}$$

$$\left\{ \because \Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1}\, dx \quad \Gamma(n+1) = n! = n\Gamma(n) \right\}$$

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properties

① $L\{a f(t) + b g(t)\} = a \cdot L\{f(t)\} + b \cdot L\{g(t)\}$

$$= a\int_0^\infty f(t) e^{-st}\, dt + b\int_0^\infty g(t) e^{-st}\, dt$$

$$= a F(t) + b G(t)$$

② $L\{e^{at} f(t)\} = \int_0^\infty e^{at} \cdot f(t) \cdot e^{-st}\, dt$

$$= \int_0^\infty f(t) \cdot e^{-(s-a)t}\, dt$$

$$= F(s-a)$$

③ $L\{f(at)\} = \int_0^\infty f(at) \cdot e^{-st}\, dt \Rightarrow \cdot\ u = at \to t = u/a \to dt = du/a \Rightarrow$

$$= \int_0^\infty f(u) \cdot e^{-\frac{su}{a}} \cdot \frac{du}{a} \Rightarrow \frac{1}{a}\int_0^\infty f(u) \cdot e^{-(s/a)u}\, du = \frac{1}{a} f(s/a)$$

$L\{t\, f(t)\} = -\frac{d}{ds}(F(s))$ proof by induction?

$L\{t^2 f(t)\} = \frac{d^2}{ds^2}(F(s))$

$\vdots$

$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(s))$

$\div t$

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