$-2.92\sigma + 30 = 0 \Rightarrow \sigma = \frac{30}{2.92} = 10.27$

$\mu = 58.16$

~~P(60< P(X<60) - P(X<60) - P(X<~~

~~P(45<~~ $P(45 \leq X \leq 60) = P(X \leq 60) - P(X \leq 45)$

$= P(z \leq 0.179) - P(z \leq -1.28)$

$= 0.57142 - 0.10027$

$= 0.47115 \approx 47.12\%$

$P(60 < X < 75) = P(X < 75) - P(X < 60)$

$= P(z < 1.64) - P(z < 0.179)$

$= 0.94950 - 0.57142$

$= 0.37808 \approx 38\%$

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(7) The income distributⁿ of workers in a certain factory was found to be normally distributed with mean Rs 1000 & S.D Rs. 100. There were 180 persons getting above 1200. How many persons were there in all?

ans $X \rightarrow$ ~~No. of persons getting above 1200~~ income of workers

$\mu = 1000$

$\sigma = 100$

$P(X > 1200) = 1 - P(X \leq 1200)$

$= 1 - P(z \leq 2)$

$= 1 - 0.97725 = 0.02275$

$P(X > 1200) \times 100 =$ ~~22.75%~~ 2.275%

~~$\frac{180 \times 100}{T} = 22.75 \Rightarrow T = \frac{18000}{22.75}$~~ $\frac{180 \times 100}{T} = 2.275$

$\Rightarrow T = \frac{18000}{2.275} \approx 7912$

(8) If X is a normally distributed R-V with mean 12 & σ^2 16, find the value of a such that $P(X > a) = 0.24$

ans $P(X > a) = 0.24$ $\mu = 12$ $\sigma = 4$

$P(X>a) = 1 - P(X \le a) = 0.24 \Rightarrow P(X \le a) = 0.76$

$\cancel{\Rightarrow \frac{a-12}{4} = 0.76} \Rightarrow P\left(\frac{X-12}{4} \le \frac{a}{4}\right) = 0.76$

$\cancel{P(X\le} \Rightarrow P(X \le 4a+12) = 0.76$

$\cancel{4a+12 = 0.71 \Rightarrow 4a =}\ \frac{a-12}{4} = 0.71 \Rightarrow a = 14.84$

(9) For a normally distributed population, 7% of the items have their values less than 35, 89% have their values less than 63. Find the mean & S.D of the distributn.

ans. $X \to$ Value of items

$P(X<35) = 0.07 \Rightarrow P\left(Z < \frac{35-\mu}{\sigma}\right) = 0.07 = P(Z<-1.47)$

$\Rightarrow \frac{35-\mu}{\sigma} = -1.47 \Rightarrow \mu - 1.47\sigma - 35 = 0$

$P(X<63) = 0.89 \Rightarrow P\left(Z < \frac{63-\mu}{\sigma}\right) = 0.89 = P(Z<1.23)$

$\Rightarrow \frac{63-\mu}{\sigma} = 1.23 \Rightarrow \mu + 1.23\sigma - 63 = 0$

$\Rightarrow -2.7\sigma + 28 = 0 \Rightarrow \sigma = \frac{28}{2.7} = 10.37$

$\mu = 35 + 15.24 = 50.24$

(10) The time required to assemble a piece of machinery is a random variable having approximately a normal distributn with $\mu = 12.9$ & $\sigma = 2$ mins. What are the probabilities that:

Assembling of a piece of machinery of this kind will take:

(i) At least 11.5 mins

(ii) Anywhere from 11.6 to 14.8 mins

ans. $X \to$ Time taken

$\mu = 12.9$, $\sigma = 2$

(i) $P(X \ge 11.5\text{mins}) = 1 - P(X \le 11.5)$

$= 1 - P(Z \le -0.7)$

$= 0.75804$

(ii) $P(11.6 \le X \le 14.8) = P(X \le 14.8) - P(X \le 11.6)$

$= P(X \le 0.95) - P(X \le -0.65)$

$= 0.57109$

(11) A salestax officer has reported that the avg. sales of the 500 business that he has to deal with during a year is Rs 36000 with $\sigma = 10000$. Assuming normal dist, find

(i) No. of business whose sale > Rs 40,000

(ii) The % of business whose sales are likely to range btw 30,000 & 40,000

ans. $X \rightarrow$ Sale in Rs

$\sigma = 10,000$, $\mu = 36,000$

(i) $P(X > 40000) = 1 - P(X \le 40000)$

$= 1 - P(Z \le 0.4)$

$= 0.34458$

$\frac{N}{500} = \frac{35}{100} \Rightarrow N = 175$

(ii) $P(30000 \le X \le 40000) = P(X \le 40000) - P(X \le 30000)$

$= P(Z \le 0.4) - P(Z \le -0.6)$

$= 0.65542 - 0.27425$

$= 0.38117$

$\frac{N}{500} = \frac{38}{100} \Rightarrow N = 190$

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(12) A R.V has a normal distributn with $\mu = 62.4$. Find it's σ if the probability is 0.20 that it will take on a value greater than 79.2

ans. $\mu = 62.4$

$P(X > 79.2) = 0.20$

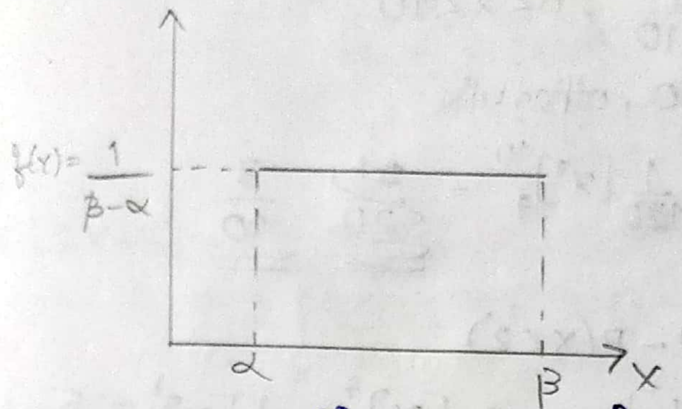
$1 - P(X \le 79.2) = 0.20 \Rightarrow P(X \le 79.2) = 0.80$

$\Rightarrow P(Z \le 0.85) = 0.80$

$\Rightarrow \frac{79.2 - 62.4}{\sigma} = 0.85 \Rightarrow \sigma = \frac{16.8}{0.85} = 19.765$

Uniform Distribut^n [Rectangular Distribut^n]

P.D.F $f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha < X < \beta \\ 0, & \text{otherwise} \end{cases}$



$$\text{Mean} = E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{\alpha}^{\beta} \frac{x}{\beta-\alpha}\,dx = \left[\frac{x^2}{2(\beta-\alpha)}\right]_{\alpha}^{\beta}$$

$$= \frac{1}{2(\beta-\alpha)}[\beta^2-\alpha^2] = \frac{\beta+\alpha}{2}$$

$$\text{Variance} = E[X^2] - (E[X])^2$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{\alpha}^{\beta} \frac{x^2}{\beta-\alpha}\,dx = \frac{1}{3(\beta-\alpha)}[x^3]_{\alpha}^{\beta}$$

$$= \frac{1}{3(\beta-\alpha)}[\beta^3-\alpha^3] = \frac{\beta^2+\alpha\beta+\alpha^2}{3}$$

$$\text{Variance} = \frac{\beta^2+\alpha\beta+\alpha^2}{3} - \frac{(\beta^2+\alpha^2+2\alpha\beta)}{4}$$

$$= \frac{4\beta^2+4\alpha\beta+4\alpha^2-3\beta^2-3\alpha^2-6\alpha\beta}{12}$$

$$= \frac{\beta^2-2\alpha\beta+\alpha^2}{12} = \frac{(\beta-\alpha)^2}{12}$$

Q) A bus arrives every 10 mins @ a bus stop. Assuming waiting time X for bus is uniformly distributed, find the probability that a person has to wait for the bus

(i) More than 7 mins

(ii) Btw 2 & 7 mins

ans. $X \to$ Waiting time

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha < X < \beta \\ 0, & \text{otherwise} \end{cases}$$

$$\beta - \alpha = 10 \Rightarrow f(x) = \begin{cases} \frac{1}{10}, & 0 < X < 10 \\ 0, & \text{otherwise} \end{cases}$$

(i) $P(X>7) = \int_7^{10} \frac{\cancel{x}}{10} dx = \frac{1}{1\cancel{2}0}[x^{\cancel{2}}]_7^{10} = \cancel{\frac{51}{20}} \quad \frac{3}{10}$

(ii) $P(2<X<7) = P(X<7) - P(X<2)$

$$= \int_0^7 \frac{dx}{10} - \int_0^2 \frac{dx}{10} = \frac{1}{10}[x]_0^7 - \frac{1}{10}[x]_0^2 = \frac{5}{10} = \frac{1}{2}$$

(2) If a conference room cannot be reserved for more than 4 hrs, find probability that a given conference lasts more than 3 hrs.

ans. $X \to$ Time for reservation in hrs

$$f(x) = \begin{cases} 1/4, & 0 \le X \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$P(X>3) = \int_3^4 \frac{dx}{4} = 1/4$$

(3) A string of length 10m is divided into 2 parts. If the part of random length X is uniformly distributed, then find

(i) $E[3X+2]$

(ii) $Var[2X+3]$

(iii) $E[X(10-X)]$

ans. $X \to$ Length of ~~random~~ part

$$f(x) = \begin{cases} \frac{1}{10}, & 0 \le X \le 10 \\ 0, & \text{otherwise} \end{cases}$$

(i) $E[3X+2] = 3E[X]+2 = 3\int_0^{10} \frac{x\,dx}{10} + 2 = \frac{3}{20}[x^2]_0^{10} + 2 = 15+2 = 17$

(ii) $Var[2X+3] = E[4X^2+12X+9] - (E[2X+3])^2$

$E[4X^2+12X+9] = 4E[X^2]+12E[X]+9$

$= 4\int_0^{10} \frac{x^2\,dx}{10} + 12\int_0^{10}\frac{x\,dx}{10} + 9$

$= \frac{4}{30}\left[x^3\right]_0^{10} + \frac{12}{20}(x^2)_0^{10} + 9$

$= \frac{4}{3\cancel{0}} \times 100\cancel{0} + 60 + 9$

$= \frac{400+207}{3} = \frac{607}{3} = \underline{202.33}$

$E[2X+3] = 2E[X]+3$

$= 2\int_0^{10}\frac{x\,dx}{10} + 3 = \frac{2}{20}[x^2]_0^{10} + 3 = 10+3 = \underline{\underline{13}}$

$Var[2X+3] = 202.33 - 169 = \underline{\underline{33.33}}$

(iii) $E[X(10-X)] = E(10X - X^2) = 10E[X] - E[X^2]$

$= 10\times5 - \frac{100}{3} = \frac{150-100}{3} = \frac{50}{3} = \underline{\underline{16.67}}$

④ If X follows uniform distributⁿ in $(-\alpha,\alpha)$. find α such that $P(X<1) = 1/3$

ans. $P(X<1) = \frac{1}{3} = \int_{-\alpha}^{1} \cancel{\frac{dx}{3}} \frac{dx}{2\alpha}$ $\qquad f(x) = \begin{cases} 1/2\alpha, & -\alpha<x<\alpha \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow \frac{1}{3} = \frac{1}{2\alpha}[x]_{-\alpha}^{1} \Rightarrow \frac{1}{3} = \frac{1+\alpha}{2\alpha} \Rightarrow 2\alpha = 3+3\alpha \Rightarrow \underline{\underline{\alpha=-3}}$

⑤ If X is uniformly distributed in $(-2,2)$
Find
(i) $P(X<1)$
(ii) $P(|X-1| \geq 1/2)$ $\qquad x-1<-1/2$ & $x-1>1/2$

ans. $\qquad x<1/2$ & $x>3/2$ $= P(X<1/2)+P(X>3/2)$

(ii) $f(x) = \begin{cases} 1/4, & -2<x<2 \\ 0, & \text{otherwise} \end{cases}$

(i) $P(X<1) = \int_{-2}^{1}\frac{dx}{4} = \frac{1}{4}[x]_{-2}^{1} = \underline{\underline{3/4}}$

(ii) $P(|X-1| \geq 1/2) \Rightarrow x-1<-1/2$ & $x-1>1/2$

$\Rightarrow x<1/2$ & $x>3/2 \Rightarrow P(X<1/2)+P(X>3/2)$

$$= \int_{-2}^{1/2} \frac{dx}{4} + \int_{3/2}^{2} \frac{dx}{4} = \frac{1}{4}[x]_{-2}^{1/2} + \frac{1}{4}[x]_{3/2}^{2}$$

$$= \frac{1}{4}\left[\frac{1}{2}+2\right] + \frac{1}{4}\left[2-\frac{3}{2}\right] = \frac{5}{8} + \frac{1}{8} = \underline{\underline{3/4}}$$

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Exponential Distributⁿ

P.D.F $f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & \beta > 0,\ x > 0 \\ 0, & \text{otherwise} \end{cases}$

Exponential distributⁿ is commonly used to model the time b/w independent events that occur at a const avg rate
eg. The inter-arrival time @ a service facility i.e. the waiting time b/w successive arrivals has exponential distributⁿ.

$$\text{Mean} = E[X] = \int_{-\infty}^{\infty} \frac{x e^{-x/\beta}}{\beta} dx = \int_{-\infty}^{0} x f(x) dx + \int_{0}^{\infty} x f(x) dx$$

$$= \int_{0}^{\infty} x \cdot \frac{1}{\beta} e^{-x/\beta} dx$$

$$= \frac{1}{\beta}\left[x \int e^{-x/\beta} dx - \int\int e^{-x/\beta} dx\right]_0^{\infty}$$

$$= \frac{1}{\beta}\left[x \cdot \frac{e^{-x/\beta}}{-1/\beta} - \int \frac{e^{-x/\beta}}{-1/\beta} dx\right]_0^{\infty}$$

$$= \frac{1}{\beta}\left[-x\beta e^{-x/\beta} + \int \beta e^{-x/\beta} dx\right]_0^{\infty}$$

$$= \frac{1}{\beta}\left[-\beta x e^{-x/\beta} - \beta^2 e^{-x/\beta}\right]_0^{\infty}$$

$$= \frac{1}{\beta}\left[0 - 0 - 0 + \beta^2\right] = \underline{\beta}$$

Variance = $Var[X] = E[X^2] - (E[X])^2$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_0^{\infty} \frac{x^2}{\beta} e^{-x/\beta} dx$$

$$= \cancel{\frac{1}{\beta}\left[x^2 \int e^{-x/\beta} dx - 2x\right.} = \frac{1}{\beta}\left[x^2\left(\frac{e^{-x/\beta}}{-1/\beta}\right) - (2x)\left(\frac{e^{-x/\beta}}{1/\beta^2}\right) + 2\left(\frac{e^{-x/\beta}}{-1/\beta^3}\right)\right.$$