Saathi

Date 21 / 02 / 2019 Tutorial - I

Q1. If the probability distribution of a discrete R.V ~~is~~ X is given as $P(X=x) = \begin{cases} \frac{x}{15}, & 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{cases}$

Find

1) $P(1 \text{ or } 2)$

2) $P(\frac{1}{2} < x < \frac{5}{2} \mid x > 1)$

Q2. Consider a lot of 10 items containing 3 defectives from which a sample of 4 items is drawn at random. ~~Find~~ Let X denote no. of defective item in a sample. Find:

(i) PMF of X.

(ii) $P(X < 1)$

(iii) distribution fn.

Q3. Find the μ & σ^2 of discrete uniform P.D given by $f(x) = \frac{1}{n}$ for $x = 1, 2, 3, \ldots, n$.

Q4. If 1, 3, 5, 6, 7 and 8 are six uniformly selected observations, then obtain the μ & σ^2.

Q5. The average no. of phone calls per minute coming into a switch board b/w 2 and 4 pm is 2.5, Determine the probability that during 1 particular min. there will be:

(i) 4 or fewer (ii) more than 6 telephone calls

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Ans 1: 1) $P(1 \text{ or } 2) = P(x=1) + P(x=2) = \frac{1}{15} + \frac{2}{15} = \frac{3}{15} = \frac{1}{5}$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

2) $P\left(\left(\frac{1}{2} < x < \frac{5}{2}\right) / x > 1\right) = \frac{P\left(\left(\frac{1}{2} < x < \frac{5}{2}\right) \cap P(x>1)\right)}{P(x>1)}$

$= \frac{P\left(\left(\frac{1}{2} < x < \frac{5}{2}\right) \cap P(x>1)\right)}{P(x>1)} = \frac{\left(\frac{2}{15}\right)}{\frac{14}{15}} = \frac{1}{7}$

Ans 2:

x_i	0	1	2	3
(i) $P(X=x_i)$	$\frac{{}^7C_4}{{}^{10}C_4} = \frac{7 \times 6 \times 5 \times 4}{10 \times 9 \times 8 \times 7} = \frac{1}{6}$	$\frac{{}^7C_3 \times {}^3C_1}{{}^{10}C_4} = \frac{1}{2}$	$\frac{{}^7C_2 \times {}^3C_2}{{}^{10}C_4} = \frac{3}{10}$	$\frac{{}^7C_1 \times {}^3C_3}{{}^{10}C_4} = \frac{7 \times 1 \times 24}{10 \times 9 \times 8 \times 7} = 1/30$

X: no. of defective items.

(ii) $P(X<1) = P(X=0) = \frac{1}{6}$

(iii) x_i	0	1	2	3
$P(X \le x_i)$	1/6	2/3	29/30	1

Ans 3:

x_i	1	2	3	4	. . .	n
$f(x_i)$	$1/n$	$1/n$	$1/n$	$1/n$		$1/n$

$\mu = \sum x_i f(x_i) = \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \frac{4}{n} + \dots + \frac{n}{n}$

$\Rightarrow \mu = \frac{1}{n}(1 + 2 + \dots + n) = \frac{1}{n} \times \frac{(n)(n+1)}{2} = \frac{(n+1)}{2}$

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x_i	1	2	3	. . .	n
x_i^2	1	4	9	. . .	n^2
$f(x)$	$1/n$	$1/n$	$1/n$		$1/n$

$$\sigma^2 = E[x^2] - (E[x])^2 = \frac{1}{n}\left[1+4+9+\dots+n^2\right] - \frac{(n+1)^2}{4}$$

$$= \frac{1}{\cancel{n}} \times \left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right] - \frac{(n+1)^2}{4}$$

$$= \frac{(2n^2+3n+1)}{6} - \frac{(n^2+2n+1)}{4}$$

$$= \frac{2n^2-2}{24} = \frac{n^2-1}{12}$$

Ans:4

x_i	1	3	5	6	7	8
$P(X=x_i)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$
x_i^2	1	9	25	36	49	64

$$\mu = \sum x_i P(x_i) = \cancel{\frac{1 \times 8 \times 9}{6 \quad 2}} = \cancel{6} \quad \frac{1}{6}\left[1+3+5+6+7+8\right]$$

$$= \frac{30}{6} = 5$$

$$\sigma^2 = E[x^2] - (E[x])^2$$

$$= \sum x_i^2 p(x_i) - 25$$

$$= \frac{1}{6}\left[1+9+25+36+49+64\right] - 25$$

$$= \frac{1}{6}\left[10+100+74\right] - 25 = \frac{184}{6} - 25 = \frac{184-150}{6}$$

$$= \frac{34}{6} = 5.67$$

Ans:5

$X \rightarrow$ no. of phone calls b/w 2pm and 4pm per minute.

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$\mu = 2.5$

x	0	1	2	.	.	.
$P(x)$	.					

$$P(X=0) = \frac{e^{-\lambda} \times \lambda^{x}}{x!} = \frac{e^{-2.5} \times \lambda^{0}}{0!} = \underline{0.0821}$$

$$P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= e^{-\lambda}\left[1 + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!}\right]$$

$$= e^{-2.5}\left[1 + 2.5 + \frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!}\right]$$

$$= \underline{0.891}$$

$$P(X > 6) = 1 - P(X \leq 6)$$

$$= 1 - (P[X=1] + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6))$$

$$= 1 - \left(0.891 + e^{-2.5}\left[\frac{2.5^5}{5!} + \frac{2.5^6}{6!}\right]\right)$$

$$= 1 - (0.891 + 0.0946)$$

$$= \underline{0.0144}$$

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