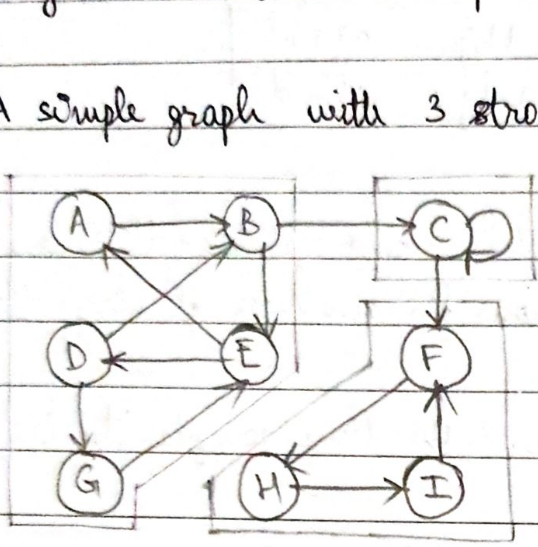
08.04.2020

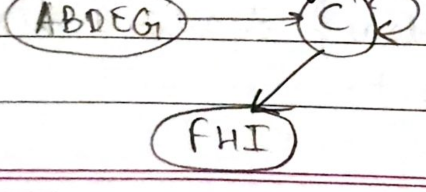
Strongly Connected Components (SCC)

- A strongly connected component C, of a graph G, is the largest subset of vertices $C \subset V$ such that for every pair of vertices $v, w \in C$, we have a path from v to w and a path from w to v.

eg: A simple graph with 3 strongly connected vertices.

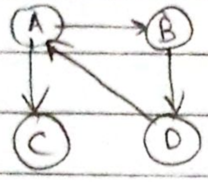


- Once the strongly connected components have been identified, we can show a simplified view of the graph by combining all the vertices in one strongly connected component into a single larger vertex:

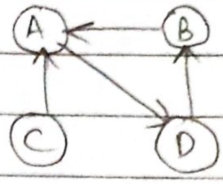


- We can create a powerful & efficient algorithm using DFS.
- The <u>transposition</u> of a graph G is defined as the graph G^T where all the edges in the graph G have been reversed, ie, if there's a directed edge from node A to node B in G, then G^T will contain an edge from node B to node A.

eg:



G



G'

Notice that G and G' have the same 2 strongly connected components.

- <u>SCC algorithm:</u>
  1. Call DFS(G) to compute the finish times for each vertex.
  2. Compute G^T.
  3. Call DFS(G^T), but in the main loop of DFS, explore each vertex <u>in decreasing order of finish time.</u>
  4. Each tree in the forest computed in step 3 is a SCC. Output the vertex ids for each vertex in each tree in the forest to identify the component.