

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SIXTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019

Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

- 1 Define the terms Best case, Worst case and Average case time complexities. (3)
- 2 What is the smallest value of n such that an algorithm whose running times is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine? (3)
- 3 State Master Theorem. (3)
- 4 Explain the UNION and FIND-SET operations in the linked-list representation of disjoint sets. Discuss the complexity. (3)

PART B

Answer any two full questions, each carries 9 marks.

- 5 a) Determine the time complexities of the following two functions fun1() and fun2(): (2)

```
int fun1(int n)
{
    if (n <= 1) return n;
    return 2*fun1(n-1);
}
```

```
int fun2(int n)
{
    if (n <= 1) return n;
    return fun2(n-1) + fun2(n-1);
}
```

- b) Find the solution to the recurrence equation using iteration method: (3)
- $$T(2^k) = 3 T(2^{k-1}) + 1,$$
- $$T(1) = 1$$
- c) Solve the recurrence using recursion tree method: (4)
- $$T(1) = 1$$
- $$T(n) = 3T(n/4) + cn^2$$
- 6 a) Determine the best case and worst-case time complexity of the following function: (3)

```
void fun(int n, int arr[])
{
    int i = 0, j = 0;
    for(; i < n; ++i)
        while(j < n && arr[i] < arr[j])
            j++;
}
```

- }
 b) Explain the advantages of using height Balanced Trees? Explain AVL Rotations. (4)
 c) Find the minimum and maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0. (2)
- 7 a) List the Properties of B-Trees. (2)
 b) A 2-3-4 tree is defined as a B-Tree with minimum degree $t=2$. Create a 2-3-4 tree by successively inserting the elements (in the given order) 42, 56, 24, 89, 1, 5, 87, 8, 61, 6, 78, 7, 12, 34. (4)
 c) Delete the elements 89, 78, 12 and 8 from the above resultant tree. (3)

PART C

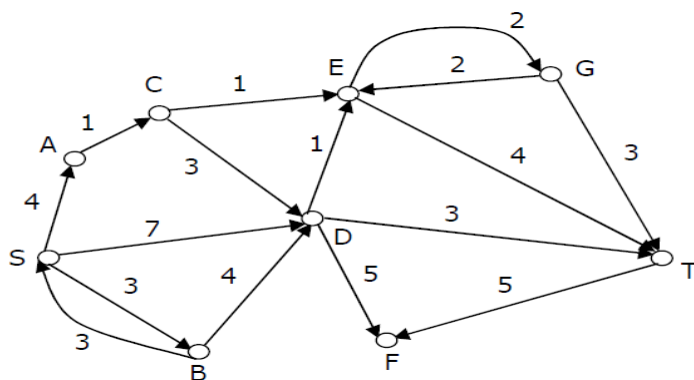
Answer all questions, each carries 3 marks.

- 8 In a weighted graph, assume that the shortest path from a source 's' to a destination 't' is correctly calculated using a shortest path algorithm. Is the following statement true? If we increase weight of every edge by 1, the shortest path always remains same. Justify your answer with proper example. (3)
- 9 Define Strongly Connected Components of a graph. (3)
 Write the algorithm to find Strongly Connected Components in a graph.
- 10 Write an algorithm to merge two sorted arrays and analyse the complexity. (3)
- 11 Write notes on Dynamic Programming Approach. List the sequence of steps to be followed in Dynamic Programming. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) State Shortest Path Problem and Optimal substructure of Shortest Path. (2)
 b) Write Dijkstra's Single Source Shortest path algorithm. Analyse the complexity. (4)
 c) Find the shortest path from s to all other vertices in the following graph using Dijkstra's Algorithm. (3)



- 13 a) Write the algorithm for DFS and analyse its complexity. (4)
 b) Multiply the following two matrices using Strassen's Matrix Multiplication Algorithm. (5)

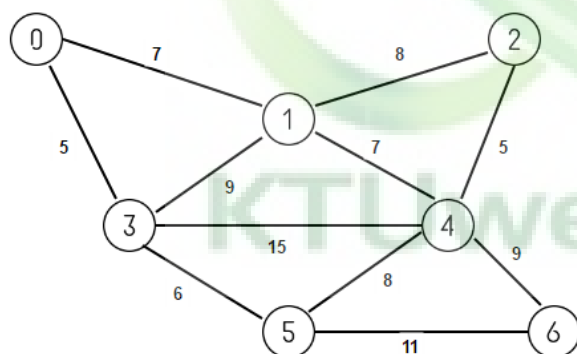
$$A = \begin{bmatrix} 6 & 8 \\ 9 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

- 14 a) State Matrix Chain Multiplication Problem. Write Dynamic Programming Algorithm for Matrix Chain Multiplication Problem. (4)
- b) Using Dynamic Programming, find the fully parenthesized matrix product for multiplying the chain of matrices $\langle A_1 A_2 A_3 A_4 A_5 A_6 \rangle$ whose dimensions are $\langle 30 \times 35 \rangle$, $\langle 35 \times 15 \rangle$, $\langle 15 \times 5 \rangle$, $\langle 5 \times 10 \rangle$, $\langle 10 \times 20 \rangle$ and $\langle 20 \times 25 \rangle$ respectively. (5)

PART E

Answer any four full questions, each carries 10 marks.

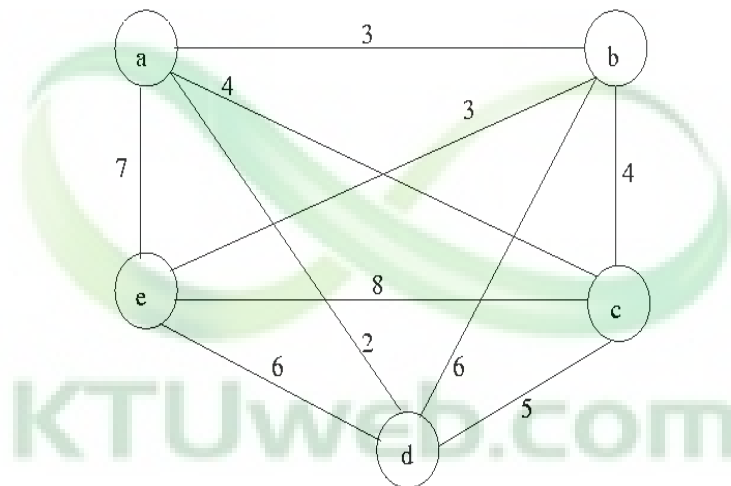
- 15 a) Explain Greedy Approach. Write the general greedy algorithm. (3)
- b) Formulate Fractional Knapsack Problem. Write Greedy Algorithm for fractional Knapsack Problem. (4)
- c) Find the optimal solution for the following fractional Knapsack problem. (3)
 $n=4$, $m = 60$, $W = \{40, 10, 20, 24\}$ and $P = \{280, 100, 120, 120\}$
- 16 a) Write the Kruskal's algorithm for Minimum Spanning Tree. Analyse its complexity. (6)
- b) Compute the Minimum Spanning Tree and its cost for the following graph using Kruskal's Algorithm. Indicate each step clearly. (4)



- 17 a) An undirected graph $G=(V, E)$ contains n ($n > 2$) nodes named v_1, v_2, \dots, v_n . Two vertices v_i, v_j are connected if and only if $0 < |i - j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i + j$. What will be the cost of the minimum spanning tree (as a function of n) of such a graph with n nodes? (4)
- b) Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$. Entry w_{ij} in the matrix W below is the weight of the edge $\{i, j\}$. What is the Cost of the Minimum Spanning Tree T using Prim's Algorithm in this graph such that vertex 0 is a leaf node in the tree T ? (6)

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

- 18 a) State and Explain N Queens Problem. Write the backtracking algorithm for solving N Queens problem. (5)
- b) Show the state space tree for 4 Queens problem. Show the steps in solving 4 Queens problem using backtracking method to print all the solutions. (5)
- 19 a) Explain Branch and Bound method for solving Travelling Salesman Problem. (5)
- b) Solve Travelling Salesman problem for the following graph using Branch and Bound Technique. (5)



- 20 a) Define NP- Hard and NP – Complete Problems. (2)
- b) What are the steps used to show a given problem is NP-Complete? (4)
- c) Write notes on polynomial time reducibility. Give Examples. (4)
