Easy way to remember Strassen's Matrix Equation

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Strassen's matrix is a Divide and Conquer method that helps us to multiply two matrices(of size n X n).

You can refer to the link, for having the knowledge about Strassen's Matrix first :
Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)

But this method needs to cram few equations, so I'll tell you the simplest way to remember those :

$$p1 = a(f - h) \qquad p2 = (a + b)h$$
$$p3 = (c + d)e \qquad p4 = d(g - e)$$
$$p5 = (a + d)(e + h) \qquad p6 = (b - d)(g + h)$$
$$p7 = (a - c)(e + f)$$

The A x B can be calculated using above seven multiplications.
Following are values of four sub-matrices of result C

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{X} \times \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}}_{C}$$

X , Y and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2
p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2

You just need to remember 4 Rules :

- AHED (Learn it as 'Ahead')
- Diagonal
- Last CR
- First CR

Also, consider X as (Row +) and Y as (Column -) matrix

Follow the Steps :

- Write P1 = A; P2 = H; P3 = E; P4 = D
- For P5 we will use Diagonal Rule i.e.
  (Sum the Diagonal Elements Of Matrix X) * (Sum the Diagonal Elements Of Matrix Y), we get
  P5 = (A + D)* (E + H)

$$X = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad Y = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

-AHED
-Diagonals
-Last CR
-First CR

P1 = A
P2= H
P3= E
P4= D
P5= (A + D) * (E + H)

- For P6 we will use Last CR Rule i.e. Last Column of X and Last Row of Y and remember that Row+ and Column- so i.e. (B – D) * (G + H), we get
  P6 = (B – D) * (G + H)
- For P7 we will use First CR Rule i.e. First Column of X and First Row of Y and remember that Row+ and Column- so i.e. (A – C) * (E + F), we get
  P6 = (A – C) * (E + F)

Check for Row (+)

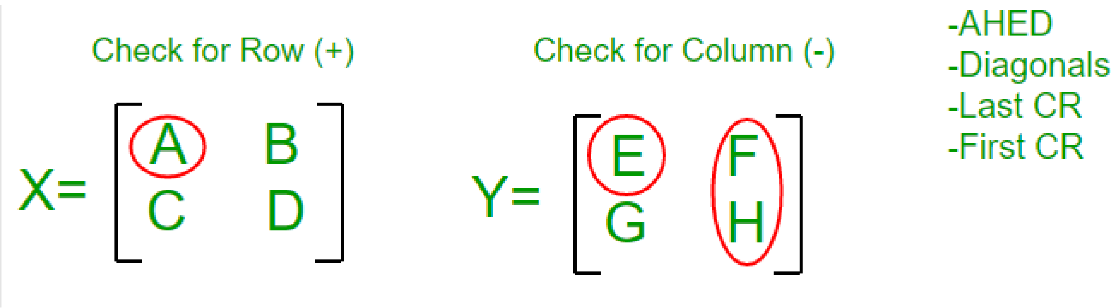
Check for Column (-)

$$X = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad Y = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

-AHED
-Diagonals
-Last CR
-First CR

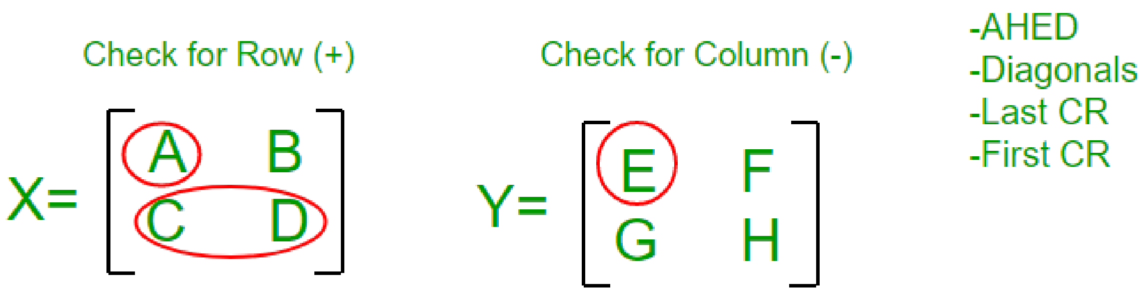
P1 = A
P2= H
P3= E
P4= D
P5= (A + D) * (E + H)
P6= (B – D) * (G + H)
P7= (A – C) * (E + F)

- Come Back to P1 : we have A there and it's adjacent element in Y Matrix is E, since Y is Column Matrix so we select a column in Y such that E won't come, we find F H Column, so multiply A with (F – H)
  So, finally P1 = A * (F – H)



P1 = A * (F – H)
P2= H
P3= E
P4= D
P5= (A + D) * (E + H)
P6= (B – D) * (G + H)
P7= (A – C) * (E + F)

- Come Back to P2 : we have H there and it's adjacent element in X Matrix is D, since X is Row Matrix so we select a Row in X such that D won't come, we find A B Column, so multiply H with (A + B)
  So, finally P2 = H * (A + B)
- Come Back to P3 : we have E there and it's adjacent element in X Matrix is A, since X is Row Matrix so we select a Row in X such that A won't come, we find C D Column, so multiply E with (C + D)
  So, finally P3 = E * (C + D)



P1= A * (F – H)
P2= H * (A + B)
P3= E * (C + D)
P4= D

P5= (A + D) * (E + H)
P6= (B – D) * (G + H)
P7= (A – C) * (E + F)

- Come Back to P4 : we have D there and it's adjacent element in Y Matrix is H, since Y is Column Matrix so we select a column in Y such that H won't come, we find G E Column, so multiply D with (G – E)
  So, finally P4 = D * (G – E)
  We are done with P1 – P7 equations, so now we move to C1 – C4 equations in Final Matrix C :

- Remember Counting : Write P1 + P2 at C2
- Write P3 + P4 at its diagonal Position i.e. at C3
- Write P4 + P5 + P6 at 1st position and subtract P2 i.e. C1 = P4 + P5 + P6 – P2
- Write odd values at last Position with alternating – and + sign i.e. P1 P3 P5 P7 becomes
  C4 = P1 – P3 + P5 – P7

$$XY= \begin{bmatrix} P6 + P5 + P4 - P2 & P1 + P2 \\ P3+ P4 & P1 + P5 - P3 - P7 \end{bmatrix}$$

Improved By : BhavayAnand