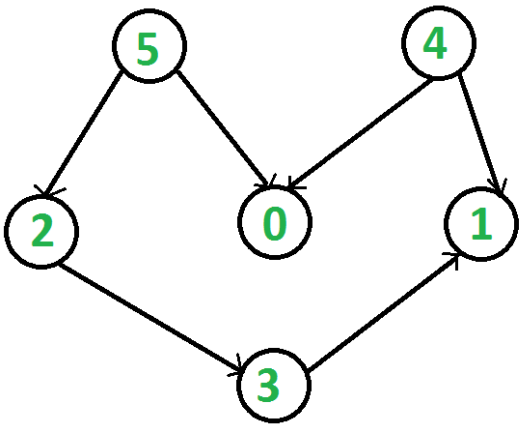
Topological Sorting

Topological sorting for Directed Acyclic Graph (DAG) is a linear ordering of vertices such that for every directed edge uv, vertex u comes before v in the ordering. Topological Sorting for a graph is not possible if the graph is not a DAG.

For example, a topological sorting of the following graph is “5 4 2 3 1 0”. There can be more than one topological sorting for a graph. For example, another topological sorting of the following graph is “4 5 2 3 1 0”. The first vertex in topological sorting is always a vertex with in-degree as 0 (a vertex with no incoming edges).

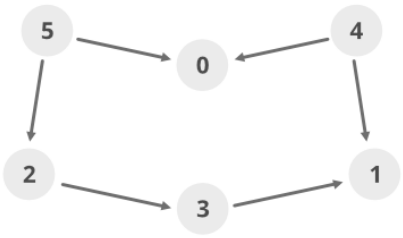


Topological Sorting vs Depth First Traversal (DFS):
In DFS, we print a vertex and then recursively call DFS for its adjacent vertices. In topological sorting, we need to print a vertex before its adjacent vertices. For example, in the given graph, the vertex ‘5’ should be printed before vertex ‘0’, but unlike DFS, the vertex ‘4’ should also be printed before vertex ‘0’. So Topological sorting is different from DFS. For example, a DFS of the shown graph is “5 2 3 1 0 4”, but it is not a topological sorting

Recommended: Please solve it on “*PRACTICE*” first, before moving on to the solution.

Algorithm to find Topological Sorting:
We recommend to first see implementation of DFS here. We can modify DFS to find Topological Sorting of a graph. In DFS, we start from a vertex, we first print it and then recursively call DFS for its adjacent vertices. In topological sorting, we use a temporary stack. We don’t print the vertex immediately, we first recursively call topological sorting for all its adjacent vertices, then push it to a stack. Finally, print contents of stack. Note that a vertex is pushed to stack only when all of its adjacent vertices (and their adjacent vertices and so on) are already in stack.

Below image is an illustration of the above approach:



Adja cent list (G) 0 →
1 →
2 → 3
3 → 1
4 → 0, 1
5 → 2, 0

visited

0	1	2	3	4	5
false	false	false	false	false	false

Stack(empty)

Step 1: Topological Sort(0), visited[0] = true

↓

List is empty. No more recursion call.

Stack | 0 |

Step 2: Topological Sort(1), visited[1] = true

↓

List is empty. No more recursion call.

Stack | 0 | 1 |

Step 3: Topological Sort(2), visited[2] = true

↓

Topological Sort(3), visited[3] = true

↓

'1' is already visited. No more recurrsion call

Stack | 0 | 1 | 3 | 2 |

Step 4: Topological Sort(4), visited[4] = true

↓

'0' , '1' are already visited. No more recurrsion call

Stack | 0 | 1 | 3 | 2 | 4 |

Step 5: Topological Sort(5), visited[5] = true

↓

'2' , '0' are already visited. No more recurrsion call

Stack | 0 | 1 | 3 | 2 | 4 | 5 |

Step 6: Print all elements of stack from top to bottom

Following are the implementations of topological sorting. Please see the code for Depth First Traversal for a disconnected Graph and note the differences between the second code given there and the below code.

C++

```
// A C++ program to print topological sorting of a DAG
#include<iostream>
#include <list>
#include <stack>
using namespace std;

// Class to represent a graph
class Graph
{
    int V;    // No. of vertices'

    // Pointer to an array containing adjacency listsList
    list<int> *adj;

    // A function used by topologicalSort
    void topologicalSortUtil(int v, bool visited[], stack<int> &Stack);
public:
    Graph(int V);   // Constructor

     // function to add an edge to graph
    void addEdge(int v, int w);

    // prints a Topological Sort of the complete graph
    void topologicalSort();
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w); // Add w to v’s list.
}

// A recursive function used by topologicalSort
void Graph::topologicalSortUtil(int v, bool visited[],
                                stack<int> &Stack)
{
    // Mark the current node as visited.
    visited[v] = true;

    // Recur for all the vertices adjacent to this vertex
    list<int>::iterator i;
    for (i = adj[v].begin(); i != adj[v].end(); ++i)
        if (!visited[*i])
            topologicalSortUtil(*i, visited, Stack);

    // Push current vertex to stack which stores result
    Stack.push(v);
}

// The function to do Topological Sort. It uses recursive
// topologicalSortUtil()
```

```
void Graph::topologicalSort()
{
    stack<int> Stack;

    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for (int i = 0; i < V; i++)
        visited[i] = false;

    // Call the recursive helper function to store Topological
    // Sort starting from all vertices one by one
    for (int i = 0; i < V; i++)
      if (visited[i] == false)
        topologicalSortUtil(i, visited, Stack);

    // Print contents of stack
    while (Stack.empty() == false)
    {
        cout << Stack.top() << " ";
        Stack.pop();
    }
}

// Driver program to test above functions
int main()
{
    // Create a graph given in the above diagram
    Graph g(6);
    g.addEdge(5, 2);
    g.addEdge(5, 0);
    g.addEdge(4, 0);
    g.addEdge(4, 1);
    g.addEdge(2, 3);
    g.addEdge(3, 1);

    cout << "Following is a Topological Sort of the given graph \n";
    g.topologicalSort();

    return 0;
}
```

Java

```
// A Java program to print topological sorting of a DAG
import java.io.*;
import java.util.*;

// This class represents a directed graph using adjacency
// list representation
class Graph
{
    private int V;   // No. of vertices
    private LinkedList<Integer> adj[]; // Adjacency List

    //Constructor
    Graph(int v)
    {
```

```
        V = v;
        adj = new LinkedList[v];
        for (int i=0; i<v; ++i)
            adj[i] = new LinkedList();
    }

    // Function to add an edge into the graph
    void addEdge(int v,int w) { adj[v].add(w); }

    // A recursive function used by topologicalSort
    void topologicalSortUtil(int v, boolean visited[],
                             Stack stack)
    {
        // Mark the current node as visited.
        visited[v] = true;
        Integer i;

        // Recur for all the vertices adjacent to this
        // vertex
        Iterator<Integer> it = adj[v].iterator();
        while (it.hasNext())
        {
            i = it.next();
            if (!visited[i])
                topologicalSortUtil(i, visited, stack);
        }

        // Push current vertex to stack which stores result
        stack.push(new Integer(v));
    }

    // The function to do Topological Sort. It uses
    // recursive topologicalSortUtil()
    void topologicalSort()
    {
        Stack stack = new Stack();

        // Mark all the vertices as not visited
        boolean visited[] = new boolean[V];
        for (int i = 0; i < V; i++)
            visited[i] = false;

        // Call the recursive helper function to store
        // Topological Sort starting from all vertices
        // one by one
        for (int i = 0; i < V; i++)
            if (visited[i] == false)
                topologicalSortUtil(i, visited, stack);

        // Print contents of stack
        while (stack.empty()==false)
            System.out.print(stack.pop() + " ");
    }

    // Driver method
    public static void main(String args[])
    {
        // Create a graph given in the above diagram
        Graph g = new Graph(6);
        g.addEdge(5, 2);
```

```
        g.addEdge(5, 0);
        g.addEdge(4, 0);
        g.addEdge(4, 1);
        g.addEdge(2, 3);
        g.addEdge(3, 1);

        System.out.println("Following is a Topological " +
                           "sort of the given graph");
        g.topologicalSort();
    }
}
// This code is contributed by Aakash Hasija
```

Python

```
#Python program to print topological sorting of a DAG
from collections import defaultdict

#Class to represent a graph
class Graph:
    def __init__(self,vertices):
        self.graph = defaultdict(list) #dictionary containing adjacency List
        self.V = vertices #No. of vertices

    # function to add an edge to graph
    def addEdge(self,u,v):
        self.graph[u].append(v)

    # A recursive function used by topologicalSort
    def topologicalSortUtil(self,v,visited,stack):

        # Mark the current node as visited.
        visited[v] = True

        # Recur for all the vertices adjacent to this vertex
        for i in self.graph[v]:
            if visited[i] == False:
                self.topologicalSortUtil(i,visited,stack)

        # Push current vertex to stack which stores result
        stack.insert(0,v)

    # The function to do Topological Sort. It uses recursive
    # topologicalSortUtil()
    def topologicalSort(self):
        # Mark all the vertices as not visited
        visited = [False]*self.V
        stack =[]

        # Call the recursive helper function to store Topological
        # Sort starting from all vertices one by one
        for i in range(self.V):
            if visited[i] == False:
                self.topologicalSortUtil(i,visited,stack)

        # Print contents of the stack
        print stack
```

```
g= Graph(6)
g.addEdge(5, 2);
g.addEdge(5, 0);
g.addEdge(4, 0);
g.addEdge(4, 1);
g.addEdge(2, 3);
g.addEdge(3, 1);

print "Following is a Topological Sort of the given graph"
g.topologicalSort()
#This code is contributed by Neelam Yadav
```

Output:

```
Following is a Topological Sort of the given graph
5 4 2 3 1 0
```

Time Complexity: The above algorithm is simply DFS with an extra stack. So time complexity is the same as DFS which is O(V+E).

Note : Here, we can also use vector instead of stack. If the vector is used then print the elements in reverse order to get the topological sorting.

Applications:
Topological Sorting is mainly used for scheduling jobs from the given dependencies among jobs. In computer science, applications of this type arise in instruction scheduling, ordering of formula cell evaluation when recomputing formula values in spreadsheets, logic synthesis, determining the order of compilation tasks to perform in makefiles, data serialization, and resolving symbol dependencies in linkers [2].

Related Articles:
Kahn's algorithm for Topological Sorting : Another O(V + E) algorithm.
All Topological Sorts of a Directed Acyclic Graph

References:
http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/GraphAlgor/topoSort.htm
http://en.wikipedia.org/wiki/Topological_sorting

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