24.04.2020

24. Fractional Knapsack problem.

- We have a knapsack that has a weight limit W.
- There are items $i_1, i_2, \ldots, i_n$ each having weight $w_1, w_2, \ldots, w_n$ and some benefit (value or profit) associated with it $v_1, v_2, \ldots, v_n$.
- Our objective is to maximize the profit such that the total weight inside the knapsack is at most W.
- We are allowed to take fractional quantities of an item.

Consider 5 items $I = \langle I_1, I_2, I_3, I_4, I_5 \rangle$ along with their weights $W = \langle 5, 10, 20, 30, 40 \rangle$ and values $V = \langle 30, 20, 100, 90, 160 \rangle$.

The capacity of the knapsack $M = 60$.

(i) calculate value per weight (V/W) for each item (we can call this value density).

Item	Weight W	Value V	Density V/W
I_1	5	30	6
I_2	10	20	2
I_3	20	100	5
I_4	30	90	3
I_5	40	160	4

(ii) Sort the items as per the value density in decreasing order

Item(I)	Weight (W)	Value (V)	Density (V/W)
I_1	5	30	6
I_3	20	100	5
I_5	40	160	4
I_4	30	90	3
I_2	10	20	2

(iii) Take as much item as possible not already in knapsack. (limit wt M = 60).

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Item(I)	weight (W)	value (V)	Total wt	Benefit
I_1	5	30	5	30
I_3	20	100	25	130
I_5	35	$\frac{7}{8} \times 160 = 140$	60	270
I_4	0	0	60	270
I_2	0	0	60	270

check I_1: is $W(I_1) + tot\_wt \leq M$
$5 + 0 \leq 60$ ✓ ⇒ take whole item

check I_3: is $W(I_3) + tot\_wt \leq M$
$20 + 5 \leq 60$ ✓ ⇒ take whole

check I_5: is $W(I_5) + tot\_wt \leq M$?
$40 + 25 \leq 60$ ✗ → take 35kg.
value = (35/40) = 7/8th.
∴ value = $\frac{7}{8} \times 160^{20} = 140$

Bag/Knapsack full ⇒ no more space for other items.