URBAN EDGE 5/2/20

(ii) If the row sums remain const. $T(n) \in \Theta(g(n)\log n)$

(iii) If the row sums form a decreasing geometric series, then $T(n) \in \Theta(g(n))$ which is proportional to the non-recursive cost of the root.

** Imp — Master Theorem

Sure Qn

The soln of the recurrence equation $T(n) = bT(n/c) + g(n)$

where $E = \frac{\log b}{\log c}$; critical exponent

(i) If $g(n) \in O(n^{E-\varepsilon})$ for any +ve ε, then $T(n) \in \Theta(n^E)$, which is proportional to the no. of leaves in the recursion tree.

(ii) If $g(n) \in \Theta(n^E)$, then $T(n) \in \Theta(g(n)\log n)$ as all node depths contribute equally.

(iii) If $g(n) \in \Omega(n^{E+\varepsilon})$ for any +ve ε, ~~then~~ & $g(n) \in O(n^{E+\delta})$ for some $\delta \geq \varepsilon$, then $T(n) \in \Theta(g(n))$, which is proportional to the non-recursive cost of the root of the recursion tree.

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① Solve the recurrence $T(n) = T(3n/4) + 1$ using Master theorem.

ans: $T(n) = T(3n/4) + 1$

$b = 1,\ c = 4/3,\ g(n) = 1$

$E = \frac{\log b}{\log c} = \frac{\log 1}{\log \frac{4}{3}} = 0$

$\therefore n^E = n^0 = 1$

Case 1:- $g(n) \in O(n^{E-\varepsilon})$

$\Rightarrow 1 \in O(n^{0-\varepsilon})$

$\Rightarrow n^0 \in O(n^{0-\varepsilon}) \Rightarrow$ Case 1 failed.

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We cannot find a +ve ε which satisfies the condit^n

Case 2:- $f(n) \in \Theta(n^E)$

$\Rightarrow 1 \in \Theta(n^0)$

$\Rightarrow n^0 \in \Theta(n^0)$

$\Rightarrow 1 \in \Theta(1) \Rightarrow$ Case 2 passed

$\therefore T(n) \in \Theta(1 \cdot \log n)$

② Solve the recurrence $T(n) = 4T(n/2) + n^3$

ans. $b=4, c=2, f(n)=n^3$

$E = \dfrac{\log b}{\log c} = \dfrac{\log 4}{\log 2} = 2$

$\therefore n^E = n^2$

Case 1:- $f(n) \in O(n^{E-\varepsilon})$

$\Rightarrow n^3 \in O(n^{2-\varepsilon}) \Rightarrow$ Case 1 failed

We cannot find ~~an~~ +ve ε that satisfies the condit^n

Case 2:- $f(n) \in \Theta(n^E)$

$\Rightarrow n^3 \in \Theta(n^2) \rightarrow$ case 2 failed.

Case 3:- $f(n) \in \Omega(n^{E+\varepsilon})$

$\Rightarrow n^3 \in \Omega(n^{2+\varepsilon}) \Rightarrow$ ~~Case 3 passed~~ $\Rightarrow$ Accepted $\Rightarrow \varepsilon = 1$

$f(n) \in O(n^{E+\delta})$

$\Rightarrow n^3 \in O(n^{2+\delta})$, $\delta \geq \varepsilon \Rightarrow$ Case 3 passed $\Rightarrow \delta = 1$

$\therefore T(n) \in \Theta(n^3)$

③ Solve $T(n) = 4T(n/2) + n$

ans $b=4, c=2, f(n)=n$ $\quad E = \log b / \log c = \log 4 / \log 2 = 2$

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Case 1:- $f(n) \in O(n^{c-\epsilon})$

$\Rightarrow n \in O(n^{2-\epsilon})$

$\epsilon = 1 \Rightarrow n \in O(n^{1}) \Rightarrow$ Case 1 accepted

$\therefore T(n) \in \Theta(n^2)$

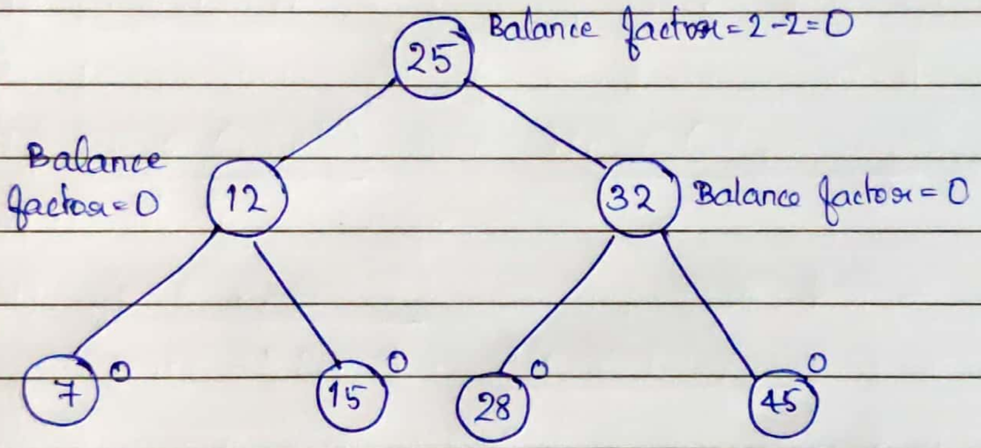
**Imp

Score qn.

AVL Trees

Height-balanced tree

Balance factor = | Height of left subtree − Height of right subtree |



If balance factor = 0 or 1 then the ~~scenario~~ tree is an AVL tree

If a single node has balance factor other than 0 or 1, then it is not an AVL tree