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CS6A
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Strassen's Matrix Multiplication

When multiplying two matrices we can use the divide and conquer strategy until we are left with 2X2 matrices.

~~For example, when multiplying.~~

We can consider these 2X2 matrices as:

$$\overset{A}{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}} \times \overset{B}{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}} = \overset{C}{\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}}$$

Where $C_{11} = A_{11} \times B_{11} + A_{12} \times B_{21}$

$C_{12} = A_{11} \times B_{12} + A_{12} \times B_{22}$

$C_{21} = A_{21} \times B_{11} + A_{22} \times B_{21}$

$C_{22} = A_{21} \times B_{12} + A_{22} \times B_{22}$

The regular multiplication method does 8 multiplications,

$\therefore \quad T(n) = 8\,w\left(\frac{n}{2}\right) + O(n^2)$

Complexity is $O(n^{\log_2 8}) = \underline{O(n^3)}$

Strassen's algorithm reduces the number of multiplications to 7, reducing the complexity to $O(n^{\log_2 7}) = O(n^{2.81})$

It calculates the values of C's:

$$C_{11} = P + S - T + V$$

$$C_{12} = R + T$$

$$C_{21} = Q + S$$

$$C_{22} = P + R - Q + U$$

Where P, Q, R, S, T, U, V are:

$$P = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$Q = B_{11}(A_{21} + A_{22})$$

$$R = A_{11}(B_{12} - B_{22})$$

$$S = A_{22}(B_{21} - B_{11})$$

$$T = B_{22}(A_{11} + A_{12})$$

$$U = (B_{11} + B_{12})(A_{21} - A_{11})$$

$$V = (A_{12} - A_{22})(B_{21} + B_{22})$$