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19. Find the optimal parenthesization of a matrix chain product of the matrices A1, A2, A3, A4 using dynamic programming.

The order of matrices are: A1 – 5×4, A2 – 4×6, A3 – 6×2, A4 – 2×7.

A1	A2	A3	A4
5×4	4×6	6×2	2×7

m	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

s	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

- $m[i,i] = 0 \quad \forall\ i \in \{1,2,3,4\}$
- $m[1,2] = 5\times4\times6 = 120$ ($A1 \cdot A2$, $5\times4\ \ 4\times6$) $\quad s[1,2]=1$
- $m[2,3] = \text{cost}(A2 \cdot A3) = 4\times6\times2 = 48$ ($4\times6 \cdot 6\times2$) $\quad s[2,3]=2$
- $m[3,4]$ cost$(A3 \cdot A4) = 6\times2\times7 = 84$ ($6\times2\ \ 2\times7$) $\quad s[3,4]=3$
- $m[1,3]$

$$\min\{\ A1\cdot(A2\cdot A3)\ ,\ (A1\cdot A2)\cdot A3\ \}$$
$$5\times4\ \ 4\times6\ \ 6\times2 \qquad\qquad 5\times4\ \ 4\times6\ \ 6\times2$$

$$= \min\{\ m[1,1] + m[2,3] + 5\times4\times2\ ,\ m[1,2] + m[3,3] + 5\times6\times2\ \}$$
$$= \min\{\ 88,\ 180\ \} = 88$$

split at A_1 $\therefore s[1,3]=1$

• m[2,4]

min { $A_2 \cdot (A_3 A_4)$, $(A_2 A_3) \cdot A_4$ }
(4×6, 6×2, 2×7)

= min { m[2,2] + m[3,4] + 4×6×7 , m[2,3] + m[4,4] + 4×2×7 }
(0, 84, 168; 48, 0, 56)

= min { 252, 104 } = 104

∴ s[2,4] = 3.

• m[1,4]

A1 A2 A3 A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4

min { $A_1 \cdot (A_2 A_3 A_4)$, $(A_1 A_2) \cdot (A_3 A_4)$, $(A_1 A_2 A_3) \cdot A_4$ }

= min { m[1,1] + m[2,4] + 5×4×7 , → 0 + 104 + 80 = 284
m[1,2] + m[3,4] + 5×6×7 , → 120 + 84 + 210 = 414
m[1,3] + m[4,4] + 5×2×7 } → 88 + 0 + 70 = 158

= 158. ∴ s[1,4] = 3.

Finding the optimal parenthesization using the split matrix s.

s[1,4] = 3 ⇒ (A1 A2 A3)(A4) (fn(s, 1, 3), 4, 4)

s[1,3] = 1 ⇒ (A1)·(A2·A3)·(A4)