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Basic Techniques in Computer Graphics

Assignment 2

Date Published: October 22nd 2019, Date Due: October 29th 2019

- All assignments (programming and text) have to be completed in teams of 3–4 students. Teams with fewer than 3 or more than 4 students will receive no points.
- Hand in **one solution per team per assignment**.
- Every team must work independently. Teams with identical solutions will receive no points.
- Solutions are due 14:30 on October 29th 2019. Late submissions will receive zero points. No exceptions!
- Instructions for **programming assignments**:
  - Download the solution template (a zip archive) through the Moodle course room.
  - Complete the solution.
  - Prepare a new zip archive containing your solution. It must contain exactly those files that you changed. **Only change those files you are explicitly asked to change in the task description.** The directory layout must be the same as in the archive you downloaded.
  - Upload your zip archive through Moodle before the deadline. Use the Moodle group submission feature. Only in the first week (when Moodle groups have not been created yet), list all members of your group in the file `assignmentXX/MEMBERS.txt`. Remember, only one submission per group.
  - Your solution must compile and run correctly **on our lab computers** using the exact same `Makefile` provided to you. Do not include additional libraries and do not change code outside of the specified sections. If it does not compile on our machines, you will receive no points.
- Instructions for **text assignments**:
  - Prepare your solution as a single pdf file per group. Submissions on paper will not be accepted.
  - If you write your solution by hand, write neatly! Anything we cannot decipher will receive zero points. No exceptions!
  - Add the names and student ID numbers of all team members to every pdf.
  - Unless explicitly asked otherwise, always justify your answer.
  - Be concise!
  - Submit your solution via Moodle, together with your coding submission.

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Exercise 1 Vector Bases **[8 Points]**

In the lecture you learned that points and vectors are always expressed with respect to a basis. In this task a basis $\mathcal{B}$ of $\mathbb{R}^3$ is given by:

$$\mathbf{b}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \mathbf{b}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

(a) **[1 Points]**

Is the basis $\mathcal{B}$ an orthogonal basis? Justify your answer.

(b) **[1.5 Points]**

Write down the matrix $\mathbf{B} \in R^{3\times3}$ that transforms vectors from basis $\mathcal{B}$ to the standard basis $\left(\begin{smallmatrix} 1 \\ 0 \\ 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 \\ 1 \\ 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 \\ 0 \\ 1 \end{smallmatrix}\right)$.

(c) **[1.5 Points]**

Use your result from part a) to transform the vectors $\left(\begin{smallmatrix} 1 \\ 0 \\ 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 \\ 1 \\ 0 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 0 \\ 2 \\ 1 \end{smallmatrix}\right)$ into the standard basis.

(d) **[4 Points]**

We now want to decompose an unknown vector $\mathbf{v} \in R^3$ (given in the standard basis) into a parallel component $\mathbf{v}^{||}$ and an orthogonal component $\mathbf{v}^{\perp}$ with respect to the known vector $\mathbf{b}_1$. Derive the matrices $\mathbf{B}_1^{||}$ and $\mathbf{B}_1^{\perp} \in \mathbb{R}^{3\times3}$ that perform these operations via matrix-vector multiplication with $\mathbf{v}$. Use that $||\mathbf{b}_1|| = 1$.

Exercise 2 Implicit & Parametric Representations **[6 Points]**

In the lecture you were introduced to parametric and implicit representations of objects such as lines and planes.
Given are two points in $\mathbb{R}^3$:

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

(a) **[1 Point]**

Give a parametric expression $\mathbf{S}(\alpha)$ for the straight line **segment** between $\mathbf{p}_1$ and $\mathbf{p}_2$.

(b) **[2 Point]**

Give an implicit expression $\mathbf{P}(\mathbf{p}) \in \mathbb{R}$ for the plane passing through $\mathbf{p}_1$, $\mathbf{p}_2$ and the origin.

(c) **[1 Point]**

Is your expression from part (b) unique? Justify your answer.

(d) **[2 Point]**

For which use cases would you prefer using a parametric representation? When would you rather use an implicit representation?

Exercise 3 Implicit & Parametric Representations **[8 Points]**

In this task, we are interested in parametric and implicit representations for the surface of a sphere with center $\mathbf{c} \in \mathbb{R}^3$ and radius $r \in \mathbb{R}$.

(a) **[4 Points]**

Derive a parametric representation for the surface of the sphere described above.

(b) **[4 Points]**

Derive an implicit representation for the surface of the sphere described above.

Hint: Remember that deriving a representations implies explaining why your formula does indeed describe a sphere. As in all the assignments, you will not get any points for simply writing down a formula without any explanation (except we explicitly ask you to only specify the formula)!