Assignment 2

COS30023 - Languages in Software Development

Daniel Parker - 971328X

August 24, 2014

1. Problem 1

1.1. Hoare Triple

```
{a > 4, b = 7}
  x := b;
  y := a;
{a > 4, b = 7, x = 7, y > 4}
```

1.2. Rules

$$\text{C} =^{\text{def}} \text{C.Target} := \text{C.Source} : \frac{true}{\{Q[C.Target\ C.Source]\}C\{Q\}}$$

$$\text{C} =^{\text{def}} \text{C}_1; \text{C}_2 : \frac{\{P\}C_1\{R\} \quad \{R\}C_2\{Q\}}{\{P\}C_1; C_2\{Q\}}$$

1.3. Proof

$$\frac{\{a > 4, b = 7\}x := b\{R\} \quad \{R\}y := a\{a > 4, b = 7, x = 7, y > 4\}}{\{a > 4, b = 7\}x := b; y := a; \{a > 4, b = 7, x = 7, y > 4\}}$$

$\{R\}C_2 : y := a; \{a > 4, b = 7, x = 7, y > 4\}$

$$\begin{aligned}\{R\} &= \{a > 4, b = 7, x = 7, y > 4\}[y \backslash a] \\ &= \{a > 4, b = 7, x = 7, a > 4\} \\ &= \{a > 4, b = 7, x = 7, \cancel{a > 4}\}\end{aligned}$$

$\{R\} = \{a > 4, b = 7, x = 7\}$

$\{a > 4, b = 7\}C_1 : x := b; \{R : a > 4, b = 7, x = 7\}$

$$\begin{aligned}\{a > 4, b = 7\} &= \{a > 4, b = 7, x = 7\}[x \backslash b] \\ &= \{a > 4, b = 7, b = 7\} \\ &= \{a > 4, b = 7, \cancel{b = 7}\} \\ \{a > 4, b = 7\} &= \{a > 4, b = 7\}\end{aligned}$$

2. Problem 2

2.1. Hoare Triple

```
{true}
  if x < 0 then val := -x; else val := x;
{val = abs(x)}
```

2.2. Rules

C = $^{\text{def}}$ if C.Test then C.Then else C.Else : $\dfrac{\{P \wedge C.Test\}C.Then\{Q\} \quad \{P \wedge \neg C.Test\}C.Else\{Q\}}{\{P\}C\{Q\}}$

$$\frac{P \Rightarrow P' \quad \{P'\}C\{Q'\} \quad Q' \Rightarrow Q}{\{P\}C\{Q\}}$$

2.3. Proof

$$\frac{\{true \wedge x < 0\}val := -x; \{val = abs(x)\} \quad \{true \wedge x \geq 0\}val := x; \{val = abs(x)\}}{\{true\}\ if\ x < 0\ then\ val := -x;\ else\ val := x; \{val = abs(x)\}}$$

2.3.1. Premise 1

$$\frac{x < 0 \Rightarrow -x = abs(x) \quad \dfrac{true}{\{-x = abs(x)\}val := -x; \{val = abs(x)\}}}{\{x < 0\}val := -x; \{val = abs(x)\}}$$

2.3.2. Premise 2

$$\frac{x \geq 0 \Rightarrow x = abs(x) \quad \dfrac{true}{\{x = abs(x)\}val := x; \{val = abs(x)\}}}{\{x \geq 0\}val := x; \{val = abs(x)\}}$$

2.3.3. Full Proof

$$\dfrac{\dfrac{x < 0 \Rightarrow -x = abs(x) \qquad \dfrac{true}{\{-x = abs(x)\}val := -x;\{val = abs(x)\}}}{\{x < 0 \wedge true\}val := -x;\{val = abs(x)\}} \qquad \dfrac{x \geq 0 \Rightarrow x = abs(x) \qquad \dfrac{true}{\{x = abs(x)\}val := x;\{val = abs(x)\}}}{\{x \geq 0 \wedge true\}val := x;\{val = abs(x)\}}}{\{true\}\ if\ x < 0\ then\ val := -x;\ else\ val := x;\{val = abs(x)\}}$$