

Assignment 2

COS30023 - Languages in Software Development

Daniel Parker - 971328X

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1. Problem 1

1.1. Hoare Triple

```
{a > 4, b = 7}  
  x := b;  
  y := a;  
{a > 4, b = 7, x = 7, y > 4}
```

1.2. Rules

$$C = \stackrel{\text{def}}{=} C.\text{Target} := C.\text{Source} : \frac{\text{true}}{\{Q[C.\text{Target} \ C.\text{Source}]\}C\{Q\}}$$

$$C = \stackrel{\text{def}}{=} C_1; C_2 : \frac{\{P\}C_1\{R\} \quad \{R\}C_2\{Q\}}{\{P\}C_1; C_2\{Q\}}$$

1.3. Proof

$$\frac{\{a > 4, b = 7\}x := b\{R\} \quad \{R\}y := a\{a > 4, b = 7, x = 7, y > 4\}}{\{a > 4, b = 7\}x := b; y := a; \{a > 4, b = 7, x = 7, y > 4\}}$$

$$\{R\}C_2 : y := a; \{a > 4, b = 7, x = 7, y > 4\}$$

$$\{R\} = \{a > 4, b = 7, x = 7, y > 4\}[y \backslash a]$$

$$= \{a > 4, b = 7, x = 7, a > 4\}$$

$$= \{a > 4, b = 7, x = 7, a > 4\}$$

$$\{R\} = \{a > 4, b = 7, x = 7\}$$

$$\{a > 4, b = 7\}C_1 : x := b; \{R : a > 4, b = 7, x = 7\}$$

$$\begin{aligned}\{a > 4, b = 7\} &= \{a > 4, b = 7, x = 7\}[x \backslash b] \\ &= \{a > 4, b = 7, b = 7\} \\ &= \{a > 4, b = 7, \cancel{b = 7}\}\end{aligned}$$

$$\{a > 4, b = 7\} = \{a > 4, b = 7\}$$

2. Problem 2

2.1. Hoare Triple

```
{true}
  if x < 0 then val := -x; else val := x;
{val = abs(x)}
```

2.2. Rules

$$C = \stackrel{\text{def}}{=} \text{if } C.\text{Test} \text{ then } C.\text{Then} \text{ else } C.\text{Else} : \frac{\{P \wedge C.\text{Test}\}C.\text{Then}\{Q\} \quad \{P \wedge \neg C.\text{Test}\}C.\text{Else}\{Q\}}{\{P\}C\{Q\}}$$

$$\frac{P \Rightarrow P' \quad \{P'\}C\{Q'\} \quad Q' \Rightarrow Q}{\{P\}C\{Q\}}$$

2.3. Proof

$$\frac{\{true \wedge x < 0\}val := -x; \{val = abs(x)\} \quad \{true \wedge x \geq 0\}val := x; \{val = abs(x)\}}{\{true\} \text{ if } x < 0 \text{ then } val := -x; \text{ else } val := x; \{val = abs(x)\}}$$

2.3.1. Premise 1

$$\frac{x < 0 \Rightarrow -x = abs(x) \quad \frac{true}{\{-x = abs(x)\}val := -x; \{val = abs(x)\}}}{\{x < 0\}val := -x; \{val = abs(x)\}}$$

2.3.2. Premise 2

$$\frac{x \geq 0 \Rightarrow x = abs(x) \quad \frac{true}{\{x = abs(x)\}val := x; \{val = abs(x)\}}}{\{x \geq 0\}val := x; \{val = abs(x)\}}$$

2.3.3. Full Proof

$$\begin{array}{c}
 \frac{x < 0 \Rightarrow -x = \text{abs}(x) \quad \frac{\text{true}}{\{-x = \text{abs}(x)\} \text{val} := -x; \{\text{val} = \text{abs}(x)\}}}{\{x < 0 \wedge \text{true}\} \text{val} := -x; \{\text{val} = \text{abs}(x)\}} \quad \frac{x \geq 0 \Rightarrow x = \text{abs}(x) \quad \frac{\text{true}}{\{x = \text{abs}(x)\} \text{val} := x; \{\text{val} = \text{abs}(x)\}}}{\{x \geq 0 \wedge \text{true}\} \text{val} := x; \{\text{val} = \text{abs}(x)\}} \\
 \hline
 \{true\} \text{ if } x < 0 \text{ then } \text{val} := -x; \text{ else } \text{val} := x; \{\text{val} = \text{abs}(x)\}
 \end{array}$$