**Amplitude calculations in grdseamount (8/1/2020)**

The **grdseamount** module can compute the amplitude $h(\mathbf{x},\mathbf{t})$ of a growing seamount for four different types of shape

1. A cone
2. A paraboloid
3. A Gaussian
4. A disc

Except for the disc, all shapes may have a truncated top determined by a flattening parameter $f$ that goes from 0 (no flattening) to 1 (no seamount at all). All shapes may have an elliptical or circular basal area. Given a normalized volume fraction function $v(t)$ that goes from 0 to 1 over the lifespan of the seamount, we wish to determine the parameters of the shape that yields the actual volume at time $t$, which is simply

$$V(t) = V_0 v(t), \tag{1}$$

where $V_0$ is the final volume of the fully grown seamount of the given shape. To yield a unique result, we relate the radius $r(t)$ and height $h(t)$ at a given time to a constant slope, so height is a function of the radius. With this conformality we can solve for the radius (and major, minor axes in the elliptical case). Below, we define $r_0$ as either the final circular radius or the final major axis ($a$), with $e = b/a$ as the eccentricity (which is 1 for circular). In all cases, the minor axis is simply $b(t) = ea(t)$, and $h^*$ is the projected height in the absence of truncation.

**1. Growing conical seamount**

We wish to determine $r(t)$ from $V(t)$. First, the final volume is given by

$$V_0 = \frac{e\pi\alpha(1-f^3)}{3} r_0^3, \tag{2}$$

where the constant slope of the cone is given by

$$\alpha = \frac{h^*}{r_0} = \frac{h_0}{(1-f)r_0} = \frac{h(t)}{(1-f)r(t)}. \tag{3}$$

We then wish to solve

$$V(t) = \frac{e\pi\alpha(1-f^3)}{3} r(t)^3, \tag{4}$$

yielding

$$r(t) = \left( \frac{3V(t)}{e\pi\alpha(1-f^3)} \right)^{1/3}. \tag{5}$$

This then yields

$$h(t) = \alpha(1-f)r(t). \tag{6}$$

### 1. Growing paraboloid

We wish to determine $r(t)$ from $V(t)$. First, the final volume is given by

$$V_0 = \frac{e\pi\alpha(1-f^4)}{2} r_0^3, \tag{7}$$

where the constant slope of the paraboloid is given by

$$\alpha = \frac{h^*}{r_0} = \frac{h_0}{(1-f^2)r_0} = \frac{h(t)}{(1-f^2)r(t)}. \tag{8}$$

We then wish to solve

$$V(t) = \frac{e\pi\alpha(1-f^4)}{2} r(t)^3, \tag{9}$$

yielding

$$r(t) = \left( \frac{2V(t)}{e\pi\alpha(1-f^4)} \right)^{1/3}. \tag{10}$$

This then yields

$$h(t) = \alpha(1-f^2)r(t). \tag{11}$$

### 1. Growing Gaussian

We wish to determine $r(t)$ from $V(t)$. Because of truncation, we need to compute the height scale

$$\gamma = e^{\frac{9}{2}} f^2. \tag{12}$$

which of course is 1 if no truncation, and where the constant slope of the Gaussian is given by

$$\alpha = \frac{h^*}{r_0} = \frac{h_0}{\gamma r_0} = \frac{h(t)\gamma}{r(t)}. \tag{13}$$

First, the final volume is given by

$$V_0 = \frac{2e\pi\alpha(1+\frac{9}{2}f^2)}{9\gamma} r_0^3. \tag{14}$$

We then wish to solve

$$V(t) = \frac{e\pi\alpha(1+\frac{9}{2}f^2)}{9\gamma} r(t)^3, \tag{15}$$

yielding

$$r(t) = \left(\frac{9\gamma V(t)}{2e\pi\alpha(1+\frac{9}{2}f^2))}\right)^{1/3}. \tag{16}$$

This then yields

$$h(t) = \frac{\alpha}{\gamma} r(t). \tag{17}$$

In all cases, inserting (1) into the expressions for $r(t)$ gives

$$r(t) = v(t)^{1/3} r_0. \tag{18}$$