Spherical solution for gmtselect -L

A and **B** are two points on the sphere; they define a great circle (**A**, **B**) whose normal (pole) is

$$\mathbf{P} = \mathbf{A} \times \mathbf{B} \big/ \|\mathbf{A} \times \mathbf{B}\|$$

C is a third point. We want to find the intersection **X** between the great circle (**A**, **B**) and the great circle that goes through **C** and is normal to the great circle (**A**, **B**). We know that the great circle (**C**, **X**) has a normal (pole) that we will call **E**. The following three identities define **E**:

$$\mathbf{E} \cdot \mathbf{P} = 0 \quad (1)$$

$$\mathbf{E} \cdot \mathbf{C} = 0 \quad (2)$$

$$\|\mathbf{E}\| = 1 \quad (3)$$

Writing **V** = (v_1, v_2, v_3) we can write

$$e_1 \cdot p_1 + e_2 \cdot p_2 + e_3 \cdot p_3 = 0 \quad (1)$$

$$e_1 \cdot c_1 + e_2 \cdot c_2 + e_3 \cdot c_3 = 0 \quad (2)$$

$$e_1 \cdot e_1 + e_2 \cdot e_2 + e_3 \cdot e_3 = 1 \quad (3)$$

First we reduce (1) to

$$e_3 = -\frac{e_1 \cdot p_1 + e_2 \cdot p_2}{p_3} \quad (4)$$

Then substitute (4) into (2) gives

$$e_1 \cdot c_1 + e_2 \cdot c_2 - \left(\frac{e_1 \cdot p_1 + e_2 \cdot p_2}{p_3} \right) \cdot c_3 = 0$$

which reduces to

$$e_2 = \frac{\dfrac{p_1 \cdot c_3}{p_2} - c_1}{c_2 - \dfrac{p_2 c_3}{p_3}} e_1 = \frac{p_1 p_3 c_3 - p_2 p_3 c_1}{p_2 p_3 c_2 - p_2^2 c_3} = a e_1 \quad (5)$$

and further simplifies (4) to

$$e_3 = -\frac{e_1 \cdot p_1 + a \cdot e_1 \cdot p_2}{p_3} = -\left(\frac{p_1 + a \cdot p_2}{p_3} \right) e_1 = b e_1 \quad (6)$$

Now solve (3) to give

$$e_1 = \left(1 + a^2 + b^2\right)^{-1/2} \quad (7)$$

with e_2, e_3 given by (5-6). Then

$$\mathbf{X} = \mathbf{E} \times \mathbf{P} \big/ \|\mathbf{E} \times \mathbf{P}\|$$

This works except when $p_3 = 0$ or $p_2 = 0$. We need special case tests for those situations.
Case $p_3 = 0$. This occurs when the great circle (**A**, **B**) is a meridian.