Tutoriat 10

Monday, 15 December 2025 09:54

Exercițiu examen 2023

- Ex are mai multe subpuncte, dar vom determina doar Forma canonică Jordan

$T:\mathbb{R}^4\to\mathbb{R}^4$

$$A=\begin{pmatrix} 1 & 1 & 0 & 0\\ -1 & 3 & 0 & 0\\ 1 & -3 & 1 & 1\\ -1 & -1 & -1 & 3\end{pmatrix}$$

a) Det pol caracteristic al lui T și val proprii ale sale

$$P_T(x)=\det(A-xI_4)=\begin{vmatrix} 1-x & 1 & 0 & 0\\ -1 & 3-x & 0 & 0\\ 1 & -3 & 1-x & 1\\ -1 & -1 & -1 & 3-x\end{vmatrix} \overset{\text{dezv după } L_1}{=} (1-x)(-1)^{1+1}\begin{vmatrix} 3-x & 0 & 0\\ -3 & 1-x & 1\\ -1 & -1 & 3-x\end{vmatrix} + 1\cdot(-1)^{1+2}\begin{vmatrix} -1 & 0 & 0\\ 1 & 1-x & 1\\ -1 & -1 & 3-x\end{vmatrix} = (1-x)(3-x)(-1)^{1+1}\begin{vmatrix} 1-x & 1\\ -1 & 3-x\end{vmatrix} + \underbrace{(-1)(-1)}_{1}\underbrace{(-1)^{1+1}}_{1}\begin{vmatrix} 1-x & 1\\ -1 & 3-x\end{vmatrix}$$

$$=(1-x)(3-x)\left[(1-x)(3-x)+1\right]+\left[(1-x)(3-x)+1\right]=\left[(1-x)(3-x)+1\right]\left[(1-x)(3-x)+1\right]=\left[(1-x)(3-x)+1\right]^2=\left(3-3x-x+x^2+1\right)^2=(x-4x+4)^2=(x-2)^3$$

$P_T(x)=(x-2)^4=0 \Rightarrow \underset{\substack{\downarrow\\ \text{valoare proprie}}}{\lambda=2},\ a(\lambda)=4.$

c) Să se det forma canonică Jordan.

$$(A-2I_4)=\begin{pmatrix} -1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 1 & -3 & -1 & 1\\ -1 & -1 & -1 & 1\end{pmatrix}$$

$\det(A-2I_4)=0$

$rg(A-2I_4)<4$

$rg(A-2I_4)=?$

Fie $\Delta_1=\begin{vmatrix} 1 & -3\\ -1 & -1\end{vmatrix}=-1-3=-4\neq 0 \Rightarrow rg(A-2I_4)\geq 2$

Fie $\Delta_2=\begin{vmatrix} -1 & 1 & 0\\ 1 & -3 & -1\\ -1 & -1 & -1\end{vmatrix}=-3+0+1-0+1+1=0$

$\Delta_3=\begin{vmatrix} -1 & 1 & 0\\ 1 & -3 & 1\\ -1 & -1 & 1\end{vmatrix}=3+0-1-0-1-1=0 \Rightarrow rg(A-2I_4)=2$

$rg(A-2I_4)=2=\dim Im(A-2I_4)$

$\dim\ker(A-2I_4)+\underbrace{\dim Im(A-2I_4)}_{2}=4 \Rightarrow \dim\ker(A-2I_4)=2=g(\lambda)$

$\left.\begin{array}{l} a(\lambda)=4\\ g(\lambda)=2\end{array}\right\} \Rightarrow a(\lambda)\neq g(\lambda) \Rightarrow T$ nu e diag.

Pasul 1: Caut $m\in\mathbb{N}^*$ a.î.

$\dim\ker(A-2I_4)^m=a(\lambda)=4 \Rightarrow \dim Im(A-2I_4)^m=4-\underbrace{4}_{\dim(\ker(A-2I_4)^m)}=0 \Rightarrow rg(A-2I_4)^m=0$

$$(A-2I_4)^2=\begin{pmatrix} -1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 1 & -3 & -1 & 1\\ -1 & -1 & -1 & 1\end{pmatrix}\begin{pmatrix} -1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 1 & -3 & -1 & 1\\ -1 & -1 & -1 & 1\end{pmatrix}=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}=O_4 \Rightarrow rg(A-2I_4)^2=0 \Rightarrow m=2$$

P_2. det γ_1,γ_2.

$2\underbrace{n}_{2}-\gamma_1=\underbrace{\dim\ker(A-2I_4)^2}_{4} \Rightarrow 4-\gamma_1=4 \Rightarrow \gamma_1=0$

$\gamma_1+\gamma_2=n \Rightarrow \gamma_2=2.$

Deci avem: $\left\{\begin{array}{l} \gamma_1=0 \text{ celule } J_1(2)\\ \gamma_2=2 \text{ celule } J_2(2)\end{array}\right.$

$$J=\begin{pmatrix} \boxed{\begin{matrix} 2 & 1\\ 0 & 2\end{matrix}} & \begin{matrix} 0 & 0\\ 0 & 0\end{matrix}\\ \begin{matrix} 0 & 0\\ 0 & 0\end{matrix} & \boxed{\begin{matrix} 2 & 1\\ 0 & 2\end{matrix}}\end{pmatrix}$$