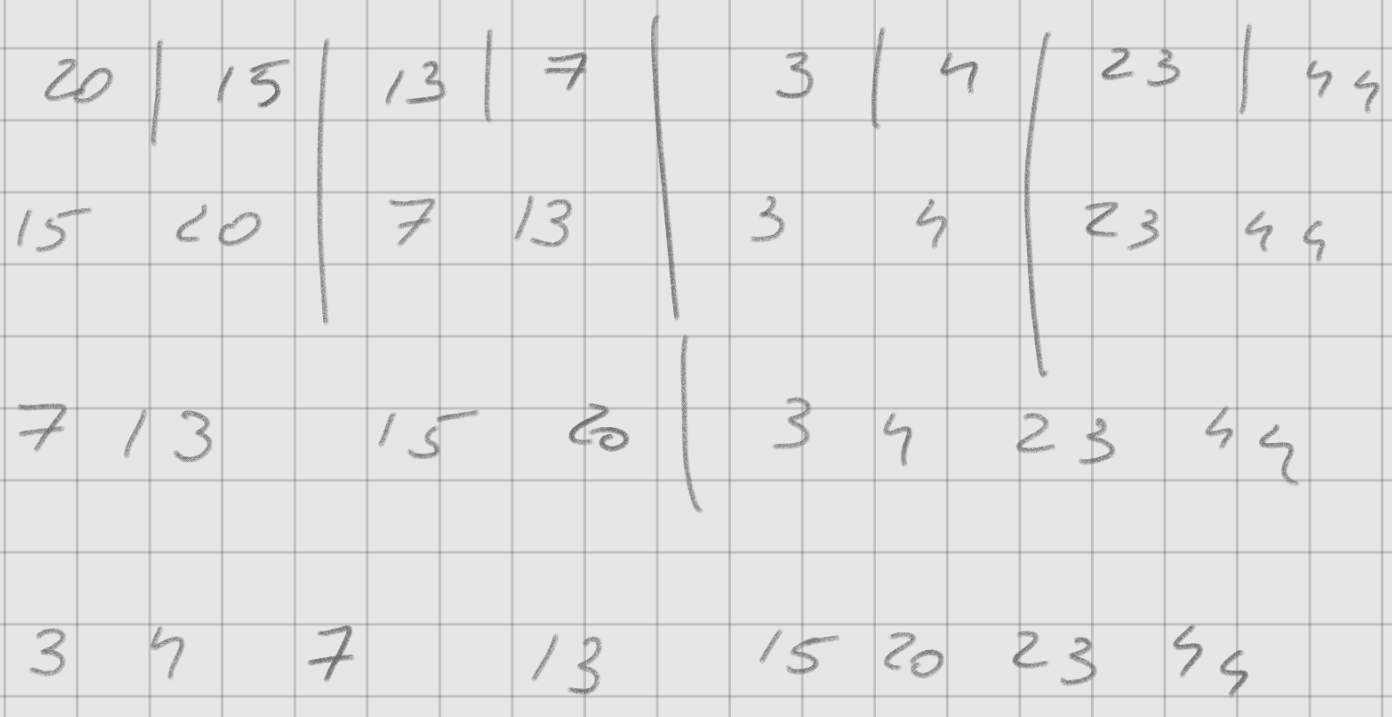
Sortarea merge sort



Pseudocod

```
mergeSort(A, l, r)
  if(l < r)
    mergeSort(A, ⌈(l+r)/2⌉+1, r);
    mergeSort(A, l, ⌈(l+r)/2⌉);
    merge(A, l, ⌈(l+r)/2⌉, r)
}
sortare...
```

$T(n) = 2T(n/2) + n = O(n \log n)$

Dem că $T(n) \leq c \cdot n \log n$

Inducție

Caz de bază ✓

Ipoteza de inducție: $T(n/2) \leq c \cdot \frac{n}{2} \log_2 \frac{n}{2}$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n \leq 2c\,\frac{n}{2}\log_2\frac{n}{2} + n =$$

$$= cn\log_2 n - cn\log_2 2 + n =$$

$$= cn\log_2 n - \underbrace{cn + n}_{\leq 0} \leq cn\log_2 n \quad \forall c \geq 1$$

Pentru ce valori ale lui c avem

$T(n) \leq cn\log_2 n$?

Teorema master

$$T(n) = aT(n/b) + f(n)$$

Caz 1 $f(n) \in O\left(n^{\log_b a - \varepsilon}\right) \Rightarrow$

$\Rightarrow T(n) \in \Theta\left(n^{\log_b a}\right)$

Caz 2 $f(n) \in \Theta\left(n^{\log_b a}\right) \Rightarrow$

$\rightarrow T(n) \in \Theta\left(n^{\log_b a}\log n\right)$

Caz 3 $f(n) \in \Omega\left(n^{\log_b a + \varepsilon}\right)$, $af(n/b) \leq cf(n)$ pt $c < 1$

$\Rightarrow T(n) \in \Theta(f(n))$

ex $T(n) = 2T(n/2) + n$

$f(n) = n \qquad n^{\log_a b} = n$

$a = 2$
$b = 2$

Caz 2 $f(n) \in \Theta\left(n^{\log_2 2}\right)$

ex $T(n) = 2T\left(\frac{n}{2}\right) + 1 = \Theta(n)$

$f(n) = 1$

$a = 2$

$b = 2$

$1 \in O\left(n^{\log_2 2 - \varepsilon}\right)$

ex $T(n) = 9T(n/3) + n$

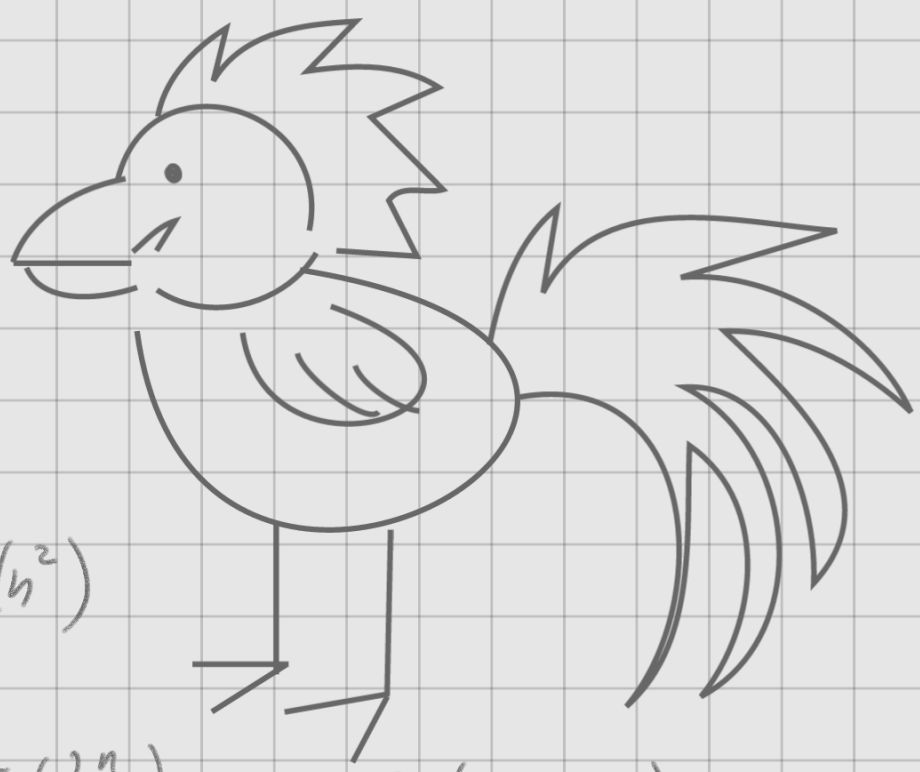
$f(n) = n$

$a = 9$

$b = 3$

$n^{\log_3 9} = n^3$

$n \in O\left(n^{2-\varepsilon}\right) \overset{\text{Caz 1}}{\Rightarrow} \Theta(n^2)$



$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n = \Theta(n \log n)$

$T(n) = T(n-1) + n = O(n^2)$