S13

$\mathbb{R}^3$

1

a) $\mathcal{D}: \frac{x-3}{-3} = \frac{y+1}{5} = \frac{z-3}{-6} \iff \begin{cases} x_1 = -3t+3 \\ x_2 = 5t-1 \\ x_3 = -6t+3 \end{cases} \quad t \in \mathbb{R}$

b) $AB: \frac{x-3}{1} = \frac{y+1}{1} = \frac{z-3}{-2}$, $\overrightarrow{AB} = (2, 2, -4) = 2(1,1,-2)$

c) $\mathcal{D} \cap xOy: z=0 \Rightarrow \frac{x-3}{-3} = \frac{y+1}{5} = \frac{1}{2} \Rightarrow x = -\frac{3}{2}+3 = \frac{3}{2}$,

$A = (\frac{3}{2}, \frac{3}{2}, 0)$ ✓

$y = \frac{5}{2} - 1 = \frac{3}{2}$

$\mathcal{D} \cap xOz: y=0 \Rightarrow \frac{x-3}{-3} = \frac{z-3}{-6} = \frac{1}{5} \Rightarrow x = -\frac{3}{5}+3 = \frac{12}{5}$

$B = (\frac{12}{5}, 0, \frac{9}{5})$ ✓

$z = -\frac{6}{5}+3 = \frac{9}{5}$

$\mathcal{D} \cap yOz = C = (0, 4, -3)$

$yOz: x=0 \Rightarrow 1 = \frac{z-3}{-6} = \frac{y+1}{5} \Rightarrow y=4, \quad z=-3$

2 M2 $\mathcal{D} = ?$, $A = (2, -5, 3) \in \mathcal{D}$

$\mathcal{D} \parallel \mathcal{D}'$, $\mathcal{D}': \begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 \\ 5x_1 + 4x_2 - x_3 + 1 = 0 \end{cases}$

$x_3 = t$

$2x_1 - x_2 = -1 - 3t \quad | \cdot 4$ — " — $| \cdot 5$

$5x_1 + 4x_2 = -1 + t$ — " — $| \cdot 2$

$+$ $\quad$ $-$

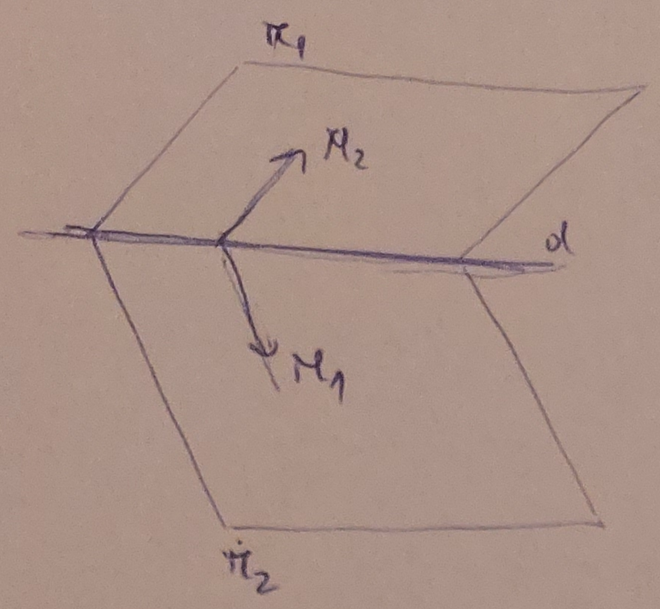
$13x_1 = -5 - 11t$ $\quad$ $13x_2 = -2 + 2t - (-5 - 15t) = 3 - 13t$

? calcule

$u = (-\frac{11}{13}, -1, 1)$

$\mathcal{D}: \frac{x_1 - 2}{-\frac{11}{13}} = \frac{x_2 + 5}{\frac{17}{13}} = \frac{x_3 - 3}{1}$

2 ✓



$\pi: ax_1 + bx_2 + cx_3 + d = 0$

$N = (a, b, c)$

pt a afla $d \to$ prod. vectorial

$(2,-5,3) = A \xrightarrow{u} M(x_1, x_2, x_3)$

$\exists t \in \mathbb{R}$ a.î. $\overline{AM} = tu$

$(x_1-2, x_2+5, x_3-3) = tu = t(-11, 17, 13)$

$\overset{\shortparallel}{(-11, 17, 13)}$

2. $\mathcal{D}': \begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 \\ 5x_1 + 4x_2 - x_3 + 1 = 0 \end{cases}$

$N_1 = (2, -1, 3)$

$N_2 = (5, 4, -1)$

$u_{\mathcal{D}'} = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} = e_1 \begin{vmatrix} -1 & 3 \\ 4 & -1 \end{vmatrix} - e_2 \begin{vmatrix} 2 & 3 \\ 5 & -1 \end{vmatrix} + e_3 \begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix}$

$= -11e_1 + 17e_2 + 13e_3 = (-11, 17, 13)$

$\mathcal{D} \parallel \mathcal{D}' \iff V_{\mathcal{D}} = V_{\mathcal{D}'} = \langle \{u_{\mathcal{D}'}\} \rangle$

$\mathcal{D}: \dfrac{x_1 - 2}{-11} = \dfrac{x_2 + 5}{17} = \dfrac{x_3 - 3}{13}$

3. $\pi: x_1 + x_2 + x_3 = 1,$

$M = (1, 2, -1), \quad \mathcal{D}: \dfrac{x_1 - 1}{2} = \dfrac{x_2 - 1}{-1} = \dfrac{x_3}{3}$

a) $\mathcal{D}' \ni M, \ \mathcal{D}' \perp \pi$

$u_{\mathcal{D}'} = (1, 1, 1) \Rightarrow \mathcal{D}': x_1 - 1 = x_2 - 2 = x_3 + 1$

b) $\pi', \ M \in \pi', \ \pi' \perp \mathcal{D}$

$u_{\mathcal{D}} = (2, -1, 3)$

$\pi': 2x_1 - x_2 + 3x_3 + c = 0$

$M \in \pi' \Rightarrow \cancel{2} - \cancel{2} - 3 + c = 0 \Rightarrow c = 3$

$\pi': 2x_1 - x_2 + 3x_3 + 3 = 0$

c) $\pi'' = ?$

$M \in \pi''$, $\mathcal{D} \subset \pi''$

$\Downarrow$

$u_{\mathcal{D}} \subset u_{\pi''}$

etc... (vezi S6-9)

4. $\mathcal{D}_1 : \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases} \Rightarrow \mathcal{D}_1 : \begin{cases} x_1 = -t \\ x_2 = t+1 \\ x_3 = t \end{cases} \Rightarrow u_1 = (-1, 1, 1)$

$\mathcal{D}_2 : \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases} \left(\Rightarrow \mathcal{D}_2 = Ox_1 \right) \Rightarrow \mathcal{D}_2 : \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow u_2 = (1, 0, 0)$

a) scriem direct $\hookrightarrow \Delta_c = \begin{vmatrix} | & | & | \\ \text{dir}(\mathcal{D}_1) & \text{dir}(\mathcal{D}_2) & u \\ | & | & | \end{vmatrix}$, $u = \overrightarrow{A_1 A_2}$, $A_1 \in \mathcal{D}_1$, $A_2 \in \mathcal{D}_2$

$\Delta_c = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -(-1) = 1 \neq 0 \Rightarrow$ sist. incompatibil

$\Downarrow$

$\mathcal{D}_1, \mathcal{D}_2$ sunt necoplanare

$(0, 1, 0), (0, 0, 0)$

$(0, 1, 0)$

b) Fie $P_1 = (-t, t+1, t) \in \mathcal{D}_1$

$P_2 = (s, 0, 0) \in \mathcal{D}_2$

$\overrightarrow{P_1P_2} = (s+t, -t-1, -t)$

$\langle \overrightarrow{P_1P_2}, u_1 \rangle = 0 \Leftrightarrow -s-t-t-1-t = 0 \Leftrightarrow 2s = 1 \Leftrightarrow s = \frac{1}{2}, \quad t = -\frac{1}{2}$

$\langle \overrightarrow{P_1P_2}, u_2 \rangle = 0 \Leftrightarrow (s+t) \cdot 1 + 0 + 0 = 0 \Leftrightarrow s = -t$

$P_1 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$, $P_2 = \left(\frac{1}{2}, 0, 0\right)$

$\begin{cases} x = 0 \\ y = -z \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = t \\ x_3 = -t \end{cases}$, $\quad \text{dist}(\mathcal{D}_1, \mathcal{D}_2) = \|\overrightarrow{P_1P_2}\| = \ldots = \sqrt{\frac{1}{2}}$

$y + z = 0$

9. $\mathcal{D}_1 : \begin{cases} x_1 = 2t+1 \\ x_2 = 3t-1 \\ x_3 = t \end{cases}$, $\mathcal{D}_2 = \begin{cases} x_1 = 4t+2 \\ x_2 = 6t \\ x_3 = \alpha t - 1 \end{cases}$

$u_1 = (2,3,1)$ $\qquad u_2 = (a_1, b_1, \alpha) = 2(2, 3, \frac{\alpha}{2})$

a) $\mathcal{D}_1 \parallel \mathcal{D}_2 \iff u_2 = \lambda u_1 \iff 2(2, 3, \frac{\alpha}{2}) = \lambda(2,3,1) \iff \begin{cases} \lambda = 2 \\ \frac{\alpha}{2} = 1 \end{cases} \iff \alpha = 2$

$\underline{\alpha = 2}$

$A = (1,-1,0) \in \mathcal{D}_1$, $\quad B = (2,0,-1) \in \mathcal{D}_2$

$\overrightarrow{AB} = (1, 1, -1)$

$\pi = \langle u_1, \overrightarrow{AB} \rangle \to$ sp. gen. de ...

$\pi : \begin{cases} x_1 = 2t + s + c_1 \leftarrow x_{1A} \\ x_2 = 3t + s + c_2 \leftarrow x_{2A} \\ x_3 = t - s + c_3 \leftarrow x_{3A} \end{cases}$ $\quad A \in \pi \Rightarrow \ldots$

e) de tem.

5. $\mathcal{D}_1 : \begin{cases} x_1 = t+1 \\ x_2 = -t+2 \\ x_3 = 2t-2 \end{cases}$ $\quad \mathcal{D}_2 : \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0 \end{cases} \Rightarrow \mathcal{D}_2 : \begin{cases} x_1 = t \\ x_2 = -t-1 \\ x_3 = 2t-1 \end{cases}$

$\to u_1 = (1,-1,2)$ $\qquad \to u_2 = (1,-1,2)$

a) $M_1 = (2, 0, -1)$, $M_2 = (0, 2, 4)$

$u_2 = M_1 \times M_2 = \ldots = 2e_1 - 2e_2 + 4e_2 = 2(1,-1,2) = 2u_1$

$A_1 = (1, 2, -2)$, $\cancel{A_2 = (0, -1, -1) \Rightarrow \overrightarrow{A_2A_1} = (1, 3, -1)}$

$A_2 = (\frac{1}{2}, -\frac{3}{2}, 0)$

$\overrightarrow{A_1A_2} = \frac{1}{2}(\underbrace{-1, -7, 4}_{w})$

$\pi = \pi(A, u_1, w) \Rightarrow \forall M \in \pi$
$\overrightarrow{A_1M} = t u_1 + s w \Rightarrow \begin{cases} (x_1-1, x_2-2, x_3+2) = t\cdot(1,-1,2) + \\ \qquad + s(-1,-7,4) \end{cases}$

$\pi : \begin{cases} x_1 = t - s + 1 \\ x_2 = -t - 7s + 2 \\ x_3 = 2t + 4s - 2 \end{cases}$, $t, s \in \mathbb{R}$

Dem

$$\pi: \begin{vmatrix} x_1-1 & 1 & -1 \\ x_2-2 & -1 & 7 \\ x_3+2 & 2 & 4 \end{vmatrix} = 0$$

$$(x_1-1)\cdot\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix} - (x_2-2)\cdot 6 + (x_3+2)\cdot 8 = 0$$

$$-18x_1 + 18 - 6x_2 + 12 + 8x_3 + 16 = 0 \qquad \cdot\frac{1}{6}$$

$$-3x_1 - x_2 + \frac{4}{3}x_3 + 5 + \frac{8}{3} = 0$$

$$\pi: -3x_1 - 3x_2 + 4x_3 + 23 = 0$$

+ calculé !!

$$\pi: 5x_1 - x_2 - 3x_3 - 9 = 0 \qquad \pi: 5x_1 - 3x_2 - 4x_3 - 7 = 0$$

Dem

$$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ x_1^A & x_2^A & x_3^A & 1 \\ x_1^B & x_2^B & x_3^C & 1 \\ x_1^C & x_2^C & x_3^C & 1 \end{vmatrix} = 0 \qquad \text{cu 3 pcte}$$

4 pcte capt. $\begin{cases} x_1 \to x_1^D \\ x_2 \to x_2^D \\ x_3 \to x_3^D \end{cases}$

ex: $A=(1,3,0)$, $B=(3,-2,1)$, $C=(\alpha,1,-3)$, $D=(7,-2,3)$

$\alpha=?$ a.î. A, B, C, D copl.

$$\Leftrightarrow \begin{vmatrix} 1 & 3 & 0 & 1 \\ 3 & -2 & 1 & 1 \\ \alpha & 1 & -3 & 1 \\ 7 & -2 & 3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -2 & 1 & 1 \\ 1 & -3 & 1 \\ -2 & 3 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 1 & 1 \\ \alpha & -3 & 1 \\ 7 & 3 & 1 \end{vmatrix} - \begin{vmatrix} 3 & -2 & 1 \\ \alpha & 1 & -3 \\ 7 & -2 & 3 \end{vmatrix} = 0$$

$$(4-1) - 3(3\alpha - 2 + 12 - \alpha) - (9 + 42 - 2\alpha - 7 + 6\alpha - 18) = 0$$

$$6 - 6\alpha - 30 - (4\alpha + 27) = 0$$

$$-10\alpha = 57 - 6 = 50$$

$$\alpha = -5$$

ex. $M=(1,1,1)$, $d: \begin{cases} x_1 - x_2 + x_3 + 1 = 0 \\ x_1 - 2x_3 - 1 = 0 \end{cases}$, $\pi: x_1 + 2x_2 + 3x_3 - 11 = 0$

a) $\pi_1 \parallel \pi$, $M \in \pi_1$

$\Downarrow$

$N_1 = N = (1,2,3) \Rightarrow \pi_1: x_1 + 2x_2 + 3x_3 + d = 0$

$M \in \pi_1 \Rightarrow 1 + 2 + 3 + d = 0 \Rightarrow d = -6$

$\pi_1: x_1 + 2x_2 + 3x_3 - 6 = 0$

a) M_1 $\begin{cases} \pi_1 \ldots \\ d: \begin{cases} - \\ - \end{cases} \end{cases}$ → nu. det

M_2 $d: \begin{cases} x_1 = 2t + 1 \\ x_2 = 3t + 2 \\ x_3 = t \end{cases}$

$d \cap \pi: 2t + 2 + 2(3t + 2) + 3t - 11 = 0$

$\Downarrow$

$11t = -4, \ t = -\frac{4}{11}$

$u_d = (2,3,1)$

c) $d' = ?$

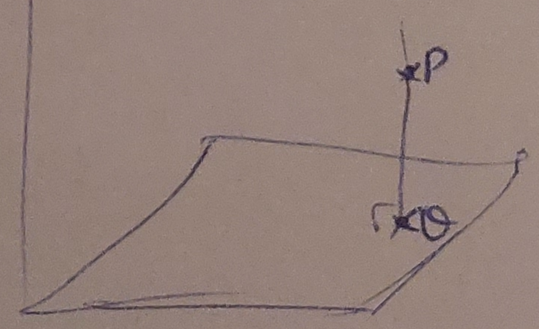
$M \in d'$, $d' \perp \pi \Rightarrow u_{d'} = N$

$d': \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$

$\Downarrow$

$d': \begin{cases} x = t + 1 \\ y = 2t + 1 \\ z = 3t + 1 \end{cases}$

$dist(P, \pi) = dist(P, Q) = \|\overrightarrow{PQ}\|$



(S14)

G. analitică eucl.

Conice. Forma canonică

1. $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

$A = (1,2,1)$, $B = (2,1,3)$, $C = (-2,1,3)$, $D = (0,2,0)$

a)

$V_{ABCD} = \frac{1}{6}|\Delta|$; $V_{ABCD} = S_{\Delta BCD} \cdot h \cdot \frac{1}{3}$; $h = dist(A,(BCD)) = \frac{3V}{S}$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 \\ -2 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 4 \cdot (-1) = -4$$

$V_{ABCD} = \frac{2}{3}$

b) $S_{\Delta BCD} = ?$

$S_{\Delta BCD} = \frac{1}{2}\|\vec{BC} \times \vec{BD}\|$

$\vec{BC} = (-4, 0, 0)$, $\vec{BD} = (-2, 1, -3)$

$$\vec{BC} \times \vec{BD} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -4 & 0 & 0 \\ -2 & 1 & -3 \end{vmatrix} = 4 \cdot \begin{vmatrix} e_2 & e_3 \\ 1 & -3 \end{vmatrix} = 4(-3e_2 - e_3) = 0 - 12e_2 - 4e_3 = (0, -12, -4)$$

$\|\vec{BC} \times \vec{BD}\| = \sqrt{12^2 + 4^2} = \sqrt{144+16} = \sqrt{160} = 4\sqrt{10}$

$S_{\Delta BCD} = 2\sqrt{10}$

c) $h = 3 \cdot \frac{\frac{2}{3}}{2\sqrt{10}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

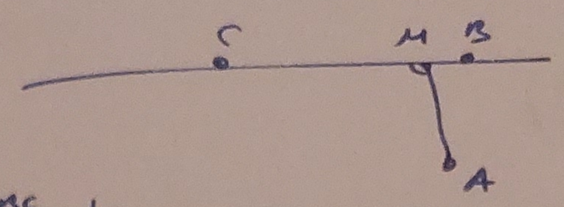
! exam + 1 combinată a 2 dr. necopl.

2. $A=(1,1,1)$

$d: \begin{cases} x_1+x_2-x_3+1=0 \\ 2x_1+x_2-3x_3+2=0 \end{cases}$

$\pi: x_1+x_2+x_3=0$

sau a)



S_{ABC}, BC $\Rightarrow AM$

a) $d: \begin{cases} x_1+x_2=t-1 \\ 2x_1+x_2=3t-2 \end{cases}$

$x_1 = 2t-1$

$x_2=-t$

(II) pt a afla dom dir (d)

2 plane care dau d: facem prod. vectorial

$N_A=(1,1,-1)$, $N_2=(2,1,-3)$, $u_d=N_1\times N_2=\dots=(-3,1,-1)$

$(-3,1,-1)=(-1)\cdot(3,-1,1)$

(I)

$d: \begin{cases} x_1=2t-1 \\ x_2=-t \\ x_3=t \end{cases}$ $\Rightarrow \pi^\perp: 2x_1-x_2+x_3+c=0$

$\pi' \perp d$, $A\in\pi'$ $\quad A\in\pi^\perp \Rightarrow \pi^\perp: 2x_1-x_2+x_3-2=0$

$d\cap\pi^\perp: 2(2t-1)-(-t)+t-2=0$

$4t-2+2t-2=0$

$3t=2 \Rightarrow t=\frac{2}{3}$

$B=\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$

~~dist $(A,d)=\sqrt{\left(\frac{2}{3}\right)^2+2\cdot\left(\frac{1}{3}\right)^2}=\sqrt{\frac{4+2}{9}}=\sqrt{\frac{2}{3}}$~~ dist $(A,d)=\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{5}{3}\right)^2+\left(\frac{1}{3}\right)^2}=\sqrt{\frac{30}{9}}=\frac{\sqrt{10}}{}$

b) $d(A,\pi)=\frac{|1+1+1|}{\sqrt{3}}=\sqrt{3}$

sau $d'\perp\pi$, $A\in d'$

$u_{d'}=N_\pi=(1,1,1) \Rightarrow d': x-1=y-1=z-1 \Rightarrow x=y=z=t+1$

$\pi\cap d': 3(t+1)=0 \Rightarrow t=-1 \Rightarrow Q=(0,0,0)$

dist $(A,\pi)=\|AQ\|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

! formă canonică

(?+ să se precize ce s-a făcut / ce conică e)

↳ izom ↳ cu Δ și δ

- $\delta \neq 0 \Rightarrow$ centru unic

$P_0(x_1, x_2)$: $\begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases}$

Ex 5 $(\mathbb{R}^2, (\mathbb{R}^2, g_0), \varphi)$

Fie conica:

$\Gamma: f(x_1, x_2) = 5x_1^2 + 8x_1x_2 + 5x_2^2 - 18x_1 - 18x_2 + 9 = 0$

Să se aducă la o formă canonică prin izometrii:

$^TA = A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -9 \\ -9 \end{pmatrix}$, $c = 9$

$\tilde{A} = \begin{pmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{pmatrix}$

$\Delta \neq 0 \rightarrow$ conică nedegenerată:

- elipsă $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- hiperbolă: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- parabolă: $x_2^2 = 2px_1$

$F(\frac{p}{2}, 0)$

~~$\delta = 25 - 64 \neq 0$, $\delta < 0$~~ $\delta = 9$, $\Delta = -81$

$\Delta = \begin{vmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{vmatrix} = \begin{vmatrix} -4 & -1 & 0 \\ -1 & -4 & 0 \\ -9 & -9 & 9 \end{vmatrix} = 9 \begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} = \ldots = -81 < 0$

$\frac{\partial f}{\partial x_1}(x_1, x_2) = 10x_1 + 8x_2 - 18 = 0 \Rightarrow 5x_1 + 4x_2 = 9$

$\frac{\partial f}{\partial x_2}(x_1, x_2) = 10x_2 + 8x_1 - 18 = 0 \Rightarrow 4x_1 + 5x_2 = 9$

$9(x_1 + x_2) = 9 \cdot 2 \Rightarrow x_1 + x_2 = 2 \Rightarrow x_1 = 9 - 8 = 1$

$x_2 = 1$

$P_0 = (1, 1)$

$$R:\{O, e_1, e_2\} \xrightarrow[\text{transl.}]{\theta} R' = \{P_0, e_1, e_2\} \xrightarrow[\text{rot.}]{\tau} R'' = \{P_0, e_1', e_2'\}$$

$\theta: X = X' + X_0$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\theta(\Gamma): {}^T X' A X' + \frac{D}{\delta} = 0$

$\theta(\Gamma): 5x_1'^2 + 8x_1'x_2' + 5x_2'^2 - 9 = 0$

$\tau: X' = RX'', \quad R \in SO(2)$

$Q: \mathbb{R}^2 \to \mathbb{R} \quad Q(x_1, x_2) = 5x_1^2 + 8x_1x_2 + 5x_2$

Aduc Q la formă canonică cu met. val. pr.

$P(A) = \det(A - \lambda I_2) = 0$

$P(A): (1-\lambda)(9-\lambda) = 0$

$\lambda_1 = 1, \ \lambda_2 = 9$

simetrică $\Rightarrow v_{\lambda_1} \perp v_{\lambda_2}$

$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = X\}$

$(A - I_2)X = O_{2,1} \Rightarrow \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_2 = -x_1 \Rightarrow V_{\lambda_1} = \{(x_1, -x_1) \mid x_1 \in \mathbb{R}\}$

$V_{\lambda_1} = \langle (1,-1) \rangle \Rightarrow e_1' = \frac{1}{\sqrt{2}}(1,-1)$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid Ax = 9x\} \Rightarrow \begin{pmatrix} -4 & 4 \\ 4 & -4 \end{pmatrix} X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow V_{\lambda_2} = \langle (1,1) \rangle \Rightarrow e_2 = \frac{1}{\sqrt{2}}(1,1)$

$R = \begin{pmatrix} | & | \\ e_1' & e_2' \\ | & | \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \Rightarrow \det R = 1 \Rightarrow R \in SO(2)$

$\left(\lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0\right)$

$\tau \circ \theta: \; x = R x'' + x_0 \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\tau \circ \theta(\Gamma): \; 1 \cdot x_1''^2 + 9 \cdot x_2''^2 - 9 = 0$

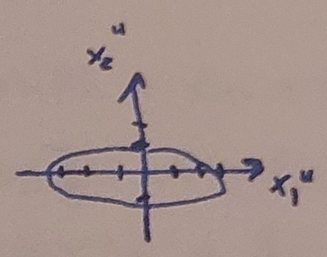
$\frac{x_1''^2}{9} + \frac{x_2''^2}{1} = 1$ elipsa

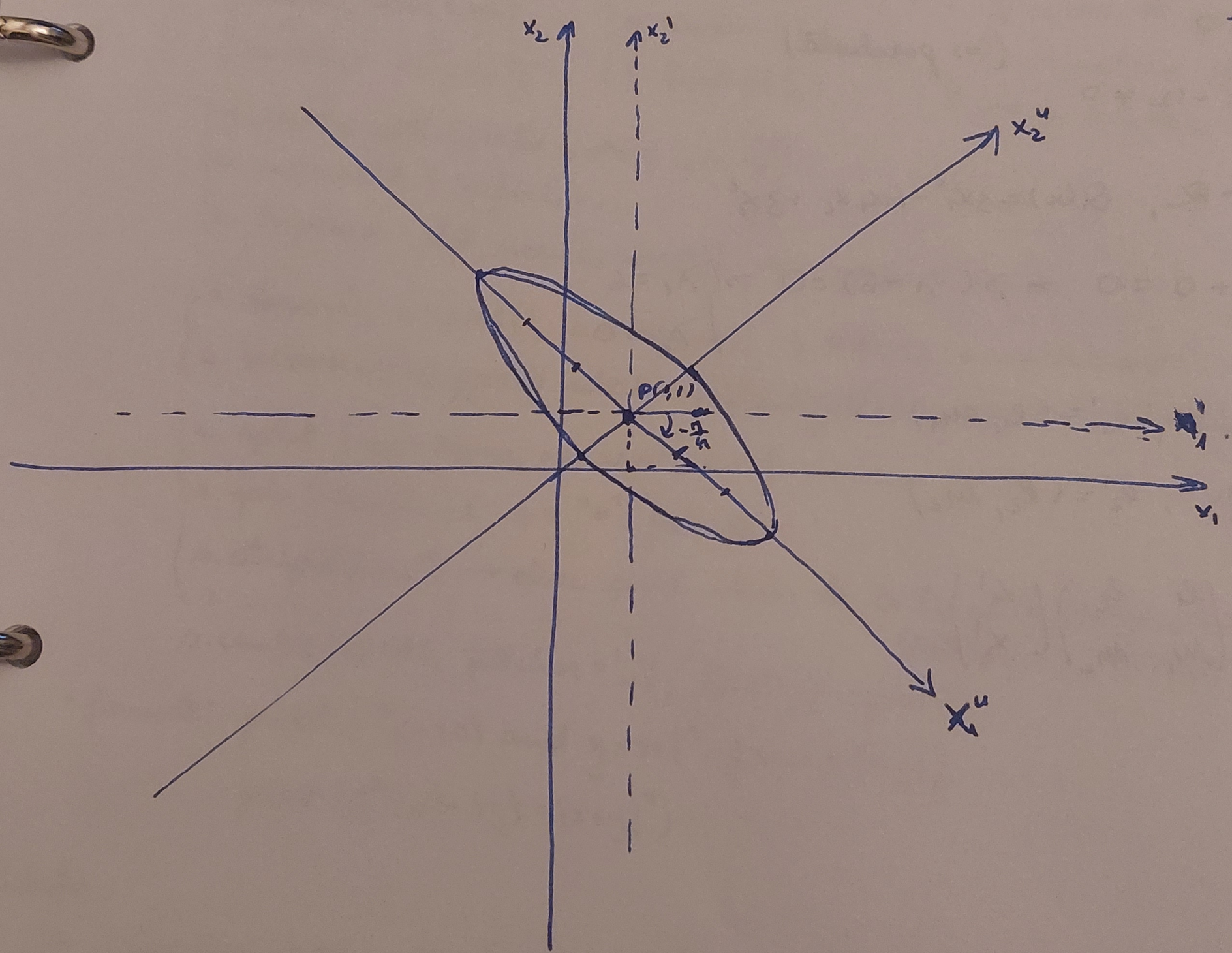
$\varphi = 2\pi - \frac{\pi}{4} = -\frac{\pi}{4}$

<u>Repr. grafica</u>:

$e_1' = \frac{1}{\sqrt{2}}(1,-1), \quad e_2' = \frac{1}{\sqrt{2}}(1,1) \qquad R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

$\lambda_1 = 1, \; \lambda_2 = 9, \quad \delta = 9, \; \Delta = -81, \; P_0 = (1,1)$





$\delta = 0$

$R = \{O; e_1, e_2\} \xrightarrow{\tau} R' = \{O, e_1', e_2'\} \xrightarrow{\theta} R'' = \{P_0, e_1', e_2'\}$ (nie sensu)

$\tau : X = RX'$, $\quad \theta : X' = X'' + X_0 \Rightarrow \theta \circ \tau : X = R(X'' + X_0) = RX'' + RX_0$

$RX_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \overset{\Delta}{=} P_0(\alpha, \beta)$

$\Gamma : \varphi(x_1, x_2) = 3x_1^2 - 6x_1x_2 + 3x_2^2 + 2x_1 + 2x_2 - 2 = 0$

$A = \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 3 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

$\delta = 0$

$\Delta = -12 \neq 0$ $\quad$ ($\Rightarrow$ parabola)

$Q : \mathbb{R}^2 \to \mathbb{R}, \quad Q(x) = 3x_1^2 - 6x_1x_2 + 3x_2^2$

$\lambda^2 - 6\lambda + 0 = 0 \Rightarrow \lambda(\lambda - 6) = 0 \Rightarrow \begin{cases} \lambda_1 = 6 \\ \lambda_2 = 0 \end{cases}$

$V_{\lambda_1} = \langle e_1' \rangle, \quad e_1' = (l_1, m_1)$

$V_{\lambda_2} = \langle e_2' \rangle, \quad e_2' = (l_2, m_2)$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$