Teoremă: $f : V \to V$ endomorfism diag $\Leftrightarrow$

1) $m_a(\lambda_1) + \dots + m_a(\lambda_p) = \dim_K V$, λ_i - val proprii $i = \overline{1,p}$

$\Downarrow$

$a(\lambda_1)$ - multiplicitatea algebrica

2) $m_a(\lambda_i) = m_g(\lambda_i)$

$\parallel$

$g(\lambda_i)$ - multiplicitatea geometrica

- $\det(A - \lambda I_n) = 0. \Rightarrow \lambda$
- Sp. vect. proprii $V_\lambda = \{ v \in \mathbb{R}^n \mid (A - \lambda I_n) \cdot v = 0 \}$

Fie $T:\mathbb{R}^4\to\mathbb{R}^4$, $T(x,y,z,t)=(x+y+z+t,\ x+y-z-t,\ x-y+z-t,\ x-y-z+t)$

a) Găsiți matr lui T în BC.

$$A=\begin{pmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&1&-1\\1&-1&-1&1\end{pmatrix}\in\mathcal{M}_4(\mathbb{R})$$

b) Calculați val. proprii + vectorii proprii + decideți de A diagon.

$$P_T(\lambda)=\det(A-\lambda I_4)=\begin{vmatrix}1-\lambda&1&1&1\\1&1-\lambda&-1&-1\\1&-1&1-\lambda&-1\\1&-1&-1&1-\lambda\end{vmatrix}\overset{C_1+C_2\to C_1}{=}\begin{vmatrix}2-\lambda&1&1&1\\2-\lambda&1-\lambda&-1&-1\\0&-1&1-\lambda&-1\\0&-1&-1&1-\lambda\end{vmatrix}\overset{L_1-L_2\to L_2}{=}\begin{vmatrix}2-\lambda&1&1&1\\0&\lambda&2&2\\0&-1&1-\lambda&-1\\0&-1&-1&1-\lambda\end{vmatrix}$$

$$=(2-\lambda)(-1)^{1+1}\begin{vmatrix}\lambda&2&2\\-1&1-\lambda&-1\\-1&-1&1-\lambda\end{vmatrix}=(2-\lambda)\left[\lambda(1-\lambda)^2+2+2+2(1-\lambda)-\lambda+2(1-\lambda)\right]$$

$$=(2-\lambda)\left[\lambda(\lambda^2-2\lambda+1)+4+2-2\lambda-\lambda+2-2\lambda\right]=$$

$$=(2-\lambda)\left[\lambda(\lambda^2-2\lambda+1)+8-5\lambda\right]$$

: $\mathbb{R}^4 \to \mathbb{R}^4$, $T(x,y,z,t) = (x+y+z+t,\ x+y-z-t,\ x-y+z-t,\ x-y-z+t)$

matr lui T în BC.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \in \mathcal{M}_4(\mathbb{R})$$

ulați val. proprii + vectorii proprii + decideți de A diagon.

$$\lambda| = \det(A - \lambda I_4) = \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & -1 & -1 \\ 1 & -1 & 1-\lambda & -1 \\ 1 & -1 & -1 & 1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(-1)^{1+1} \begin{vmatrix} \lambda & 2 & 2 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} = (2-\lambda)\left[\lambda(1-\lambda)^2 + 2 + 2 + 2(1-\lambda) - \lambda + 2(1-\lambda)\right]$$

$$= (2-\lambda)\left[\lambda(\lambda^2 - 2\lambda + 1) + 4 + 2 - 2\lambda - \lambda + 2 - 2\lambda\right] =$$

$$= (2-\lambda)\left[\lambda(\lambda^2 - 2\lambda + 1) + 8 - 5\lambda\right] = (2-\lambda)(\lambda^3 - 2\lambda^2 - 4\lambda + 8)$$

$$= (2-\lambda)\left[\lambda^2(\lambda-2) - 4(\lambda-2)\right] = (2-\lambda)(\lambda-2)^2(\lambda+2) =$$

$$= -(\lambda-2)^3 \cdot (\lambda+2)$$

$$P_T(\lambda) = (\lambda-2)^3(\lambda+2) = 0 \Rightarrow \begin{array}{l} \lambda_1 = 2,\ m_a(\lambda_1) = 3 \\ \lambda_2 = -2,\ m_a(\lambda_2) = 1 \end{array}$$

Fie $T : \mathbb{R}^4 \to \mathbb{R}^4$, $T(x,y,z,t) = (x+y+z+t,\ x+y-z-t,\ x-y+z-t,\ x-y-z+t)$

a) Valori matr lui T în BC.

Pt. $\lambda_1 = 2$

$V_{\lambda_1} = \{ v \in \mathbb{R}^4 \mid (A - 2I_4)v = 0_{\mathbb{R}^4} \}$

Fie $v = (a,b,c,d)^T \in \mathbb{R}^4$

$$(A-2I_4)\cdot v = \begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & -1 & -1 \\ 1 & -1 & 1-\lambda & -1 \\ 1 & -1 & -1 & 1-\lambda \end{vmatrix}$$

$$\begin{cases} -a+b+c+d=0 \\ a-b-c-d=0 \\ a-b-c-d=0 \\ a-b-c-d=0 \Rightarrow a=b+c+d \end{cases} \quad a,b,c,d \in \mathbb{R}.$$

$V_{\lambda_1} = \{ (b+c+d, b, c, d) \mid b,c,d \in \mathbb{R} \} = \{ (b,b,0,0) + (c,0,c,0) + (d,0,0,d) \mid b,c,d \in \mathbb{R} \}$

$= \{ b(1,1,0,0) + c(1,0,1,0) + d(1,0,0,1) \mid b,c,d \in \mathbb{R} \}$

$= \langle (1,1,0,0), (1,0,1,0), (1,0,0,1) \rangle.$

Fie $T:\mathbb{R}^4\to\mathbb{R}^4$, $T(x,y,z,t)=(x+y+z+t,\ x+y-z-t,\ x-y+z-t,\ x-y-z+t)$

a) Găsiți matr. lui T în BC.

Pt. $\lambda_2=-2$

$$V_{\lambda_2}=\{v\in\mathbb{R}^4 \mid (A+2I_4)v=0_{\mathbb{R}^4}\}$$

Fie $v=(a,b,c,d)^T\in\mathbb{R}^4$

$$(A-2I_4)\cdot v=\begin{pmatrix}3&1&1&1\\1&3&-1&-1\\1&-1&3&-1\\1&-1&-1&3\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$\begin{cases}3a+b+c+d=0\ (1)\\ a+3b-c-d=0\ (2)\\ a-b+3c-d=0\ (3)\\ a-b-c+3d=0\ (4)\end{cases}$$

$\overset{(1)+(2)}{\Longrightarrow} 4a+4b=0\Rightarrow a=-b$

$\overset{(1)+(3)}{\Longrightarrow} 4a+4c=0\Rightarrow a=-c$

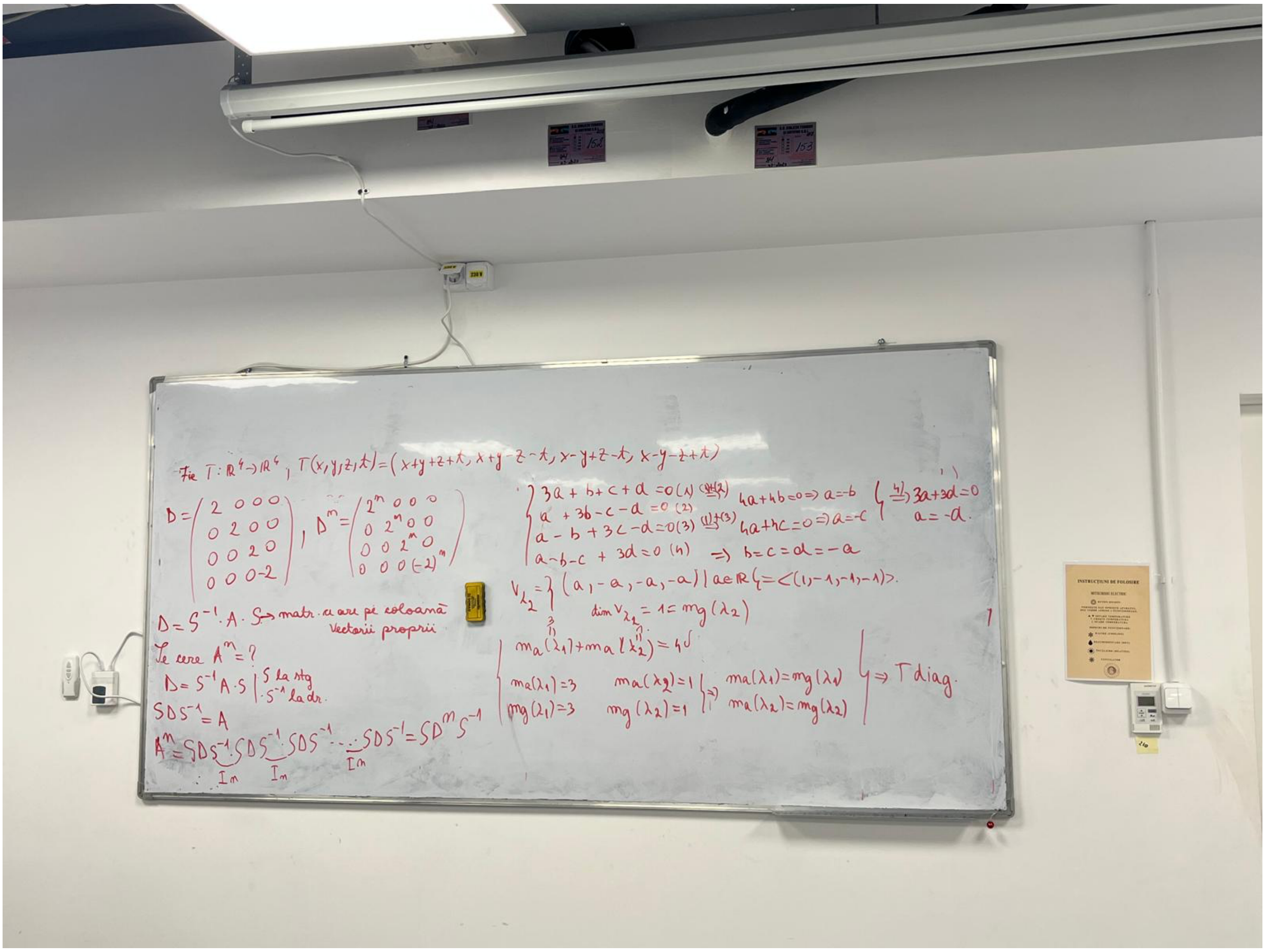
$\overset{(4)}{\Longrightarrow} 3a+3d=0,\ a=-d$

$\Rightarrow b=c=d=-a$

$$V_{\lambda_2}=\{(a,-a,-a,-a) \mid a\in\mathbb{R}\}=\langle(1,-1,-1,-1)\rangle.$$

$\dim V_{\lambda_2}=1=m_g(\lambda_2)$

$$\left.\begin{array}{l} m_a(\lambda_1)+m_a(\lambda_2)=4 \checkmark \\ \left.\begin{array}{ll} m_a(\lambda_1)=3 & m_a(\lambda_2)=1 \\ m_g(\lambda_1)=3 & m_g(\lambda_2)=1\end{array}\right\}\Rightarrow \begin{array}{l} m_a(\lambda_1)=m_g(\lambda_1)\\ m_a(\lambda_2)=m_g(\lambda_2)\end{array}\end{array}\right\}\Rightarrow T \text{ diag.}$$



Fie $T:\mathbb{R}^4 \to \mathbb{R}^4$, $T(x,y,z,t) = (x+y+z+t, x+y-z-t, x-y+z-t, x-y-z+t)$
$D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$, $D^n = \begin{pmatrix} 2^n & 0 & 0 & 0 \\ 0 & 2^n & 0 & 0 \\ 0 & 0 & 2^n & 0 \\ 0 & 0 & 0 & (-2)^n \end{pmatrix}$
$D = S^{-1} \cdot A \cdot S \Rightarrow$ matr. ce are pe coloană vectorii proprii
Se cere $A^n = ?$
$D = S^{-1} A \cdot S \mid \cdot S$ la stg, $\cdot S^{-1}$ la dr.
$SDS^{-1} = A$
$A^n = SD\underbrace{S^{-1} S}_{I_n} D \underbrace{S^{-1} S}_{I_n} DS^{-1} \cdots \underbrace{\cdots}_{I_n} SDS^{-1} = SD^n S^{-1}$
$\begin{cases} 3a + b + c + d = 0 \ (1) \\ a + 3b - c - d = 0 \ (2) \\ a - b + 3c - d = 0 \ (3) \\ a - b - c + 3d = 0 \ (4) \end{cases}$
$\xRightarrow{(1)+(2)} 4a + 4b = 0 \Rightarrow a = -b$
$\xRightarrow{(1)+(3)} 4a + 4c = 0 \Rightarrow a = -c$
$\xRightarrow{(4)} 3a + 3d = 0$, $a = -d$.
$\Rightarrow b = c = d = -a$
$V_{\lambda_2} = \{(a, -a, -a, -a) \mid a \in \mathbb{R}\} = \langle (1,-1,-1,-1) \rangle$.
$\dim V_{\lambda_2} = 1 = mg(\lambda_2)$
$ma(\lambda_1) + ma(\lambda_2) = 4$
$ma(\lambda_1) = 3$, $ma(\lambda_2) = 1$
$mg(\lambda_1) = 3$, $mg(\lambda_2) = 1$
$\Rightarrow ma(\lambda_1) = mg(\lambda_1)$, $ma(\lambda_2) = mg(\lambda_2)$
$\Rightarrow T$ diag.

ALGORITMUL DE DETERMINARE A FORMEI YORDAN

$\gamma_1 \begin{cases} \gamma_1(\lambda) \longrightarrow \boxed{\lambda} \\ \gamma_1(\lambda) \longrightarrow \boxed{\lambda} \end{cases}$ $\quad \gamma_2 \begin{cases} \dots \boxed{\begin{matrix} \lambda & 1 \\ 0 & \lambda \end{matrix}} \\ \text{——}, \boxed{\begin{matrix} \lambda & 1 \\ 0 & \lambda \end{matrix}} \end{cases}$

* Se det polinomul caracteristic

* Se det valorile proprii

Pt fiecare valoare proprie λ:

1) Determinăm m minim a.î. $V^\lambda = Ker(A-\lambda I)^m$ i.e. determinăm m minim a.î. $\dim(Ker(A-\lambda I)^m) = ma(\lambda)$

2) $r = \dim(\underbrace{Ker(A-\lambda I)}_{V_\lambda})$

$\tilde{N} = A - \lambda I$

$N = \tilde{N}|_{V^\lambda}$

$r = \dim(Ker N)$

3) Determinăm $\gamma_1, \gamma_2 \dots \gamma_m$

$2r - \gamma_1 = \dim(Ker N^2)$

$3r - 2\gamma_1 - \gamma_2 = \dim(Ker N^3)$

$\vdots$

$mr - (m-1)\gamma_1 - \dots - \gamma_{m-1} = \dim(Ker N^m)$

și $\gamma_m = r - \gamma_1 - \dots - \gamma_{m-1}$

4) Asociate lui λ vor fi

γ_1 celule Jordan $J_1(\lambda)$

γ_2 —— " ——

γ_m —— " ——

$\gamma_1 + \dots + \gamma_m = \underline{mg(\lambda) = r}$

$\gamma_1 + 2\gamma_2 + \dots + m\gamma_m = \dim(Ker N^m) = \dim V^\lambda = ma(\lambda)$

$\gamma_m \neq 0$

1 $A = \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$P(A) = \det(A - \lambda I_4) = 0$

$\begin{vmatrix} 1-\lambda & 1 & 1 & -2 \\ 0 & 1-\lambda & 0 & -1 \\ 0 & 0 & 1-\lambda & 1 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^4$

$\lambda = 1 \quad Spec\, A = \{1\}$

$\Rightarrow m_a(\lambda) = 4$

$m_g = \dim V_\lambda = \dim Ker(A - \lambda I_4)$

$= \dim Ker(\underbrace{A - I_4}_{N})$

$N = \begin{pmatrix} 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad N^2 = N \cdot N = O_4 \Rightarrow m = 2$

Avem de calculat γ_1, γ_2

$\dim Ker(A - I_4) + \underbrace{\dim Im(A - I_4)}_{rang(A-I) = 2} = \dim V = 4$

$\Rightarrow \dim Ker(A - I_4) = 2 = r$

Determinăm m minim a.î. $\dim Ker N^m = m_a(\lambda) = 4$

$\underbrace{\dim Ker N^m}_{4} + \dim Im N^m = 4 \Rightarrow rg N^m = 0 \Leftrightarrow N^m = 0$

$\dim Im N^m = rg N^m$

(1) $A = \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

forma Jordan a lui A:

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$N = \begin{pmatrix} 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $N^2 = N \cdot N = O_4 \Rightarrow m = 2$

Avem de calculat γ_1, γ_2

$2r - \gamma_1 = \dim \operatorname{Ker} N^2$

$2 \cdot 2 - \gamma_1 = 4 \Rightarrow 4 - \gamma_1 = 4 \Leftrightarrow \gamma_1 = 0$

$\left\{ \gamma_1 + \gamma_2 = r = 2 \Rightarrow \gamma_2 = 2 \right.$

0 celule Jordan $J_1(1)$

2 celule Jordan $\underline{J_2(1)}$ $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$\dim \operatorname{Ker}(A - I_4) + \underbrace{\dim \operatorname{Im}(A - I_4)}_{\substack{\operatorname{rang}(A - I) \\ \| \\ 2}} = \dim V$, $\dim V = 4$

$\Rightarrow \dim \operatorname{Ker}(A - I_4) = 2 = r$

Determinăm m minim a.î. $\dim \operatorname{Ker} N^m = m_a(\lambda) = 4$

$\underbrace{\dim \operatorname{Ker} N^m}_{4} + \dim \operatorname{Im} N^m = 4 \Rightarrow rg\, N^m = 0 \Leftrightarrow N^m = 0$

$\dim \operatorname{Im} N^m = rg\, N^m$

Examen 2025

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 5 & -2 & 2 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix},$$

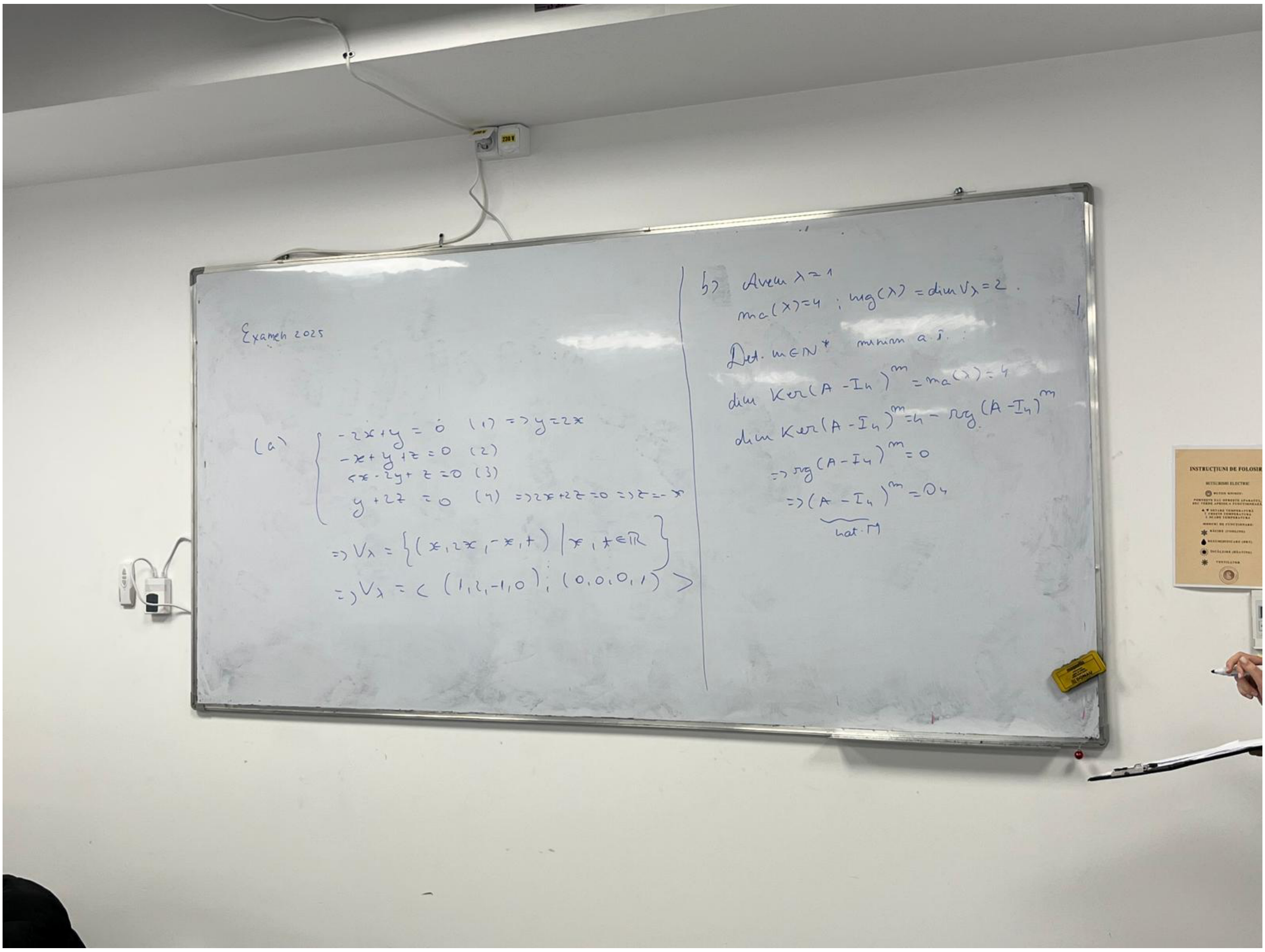
(a) $$p_A(\lambda) = \det(A - \lambda I_4) = \begin{vmatrix} -1-\lambda & 1 & 0 & 0 \\ -1 & 2-\lambda & 1 & 0 \\ 5 & -2 & 2-\lambda & 0 \\ 0 & 1 & 2 & 1-\lambda \end{vmatrix} \overset{\text{dezv. } C_4}{=} (1-\lambda) \begin{vmatrix} -1-\lambda & 1 & 0 \\ -1 & 2-\lambda & 1 \\ 5 & -2 & 2-\lambda \end{vmatrix}$$

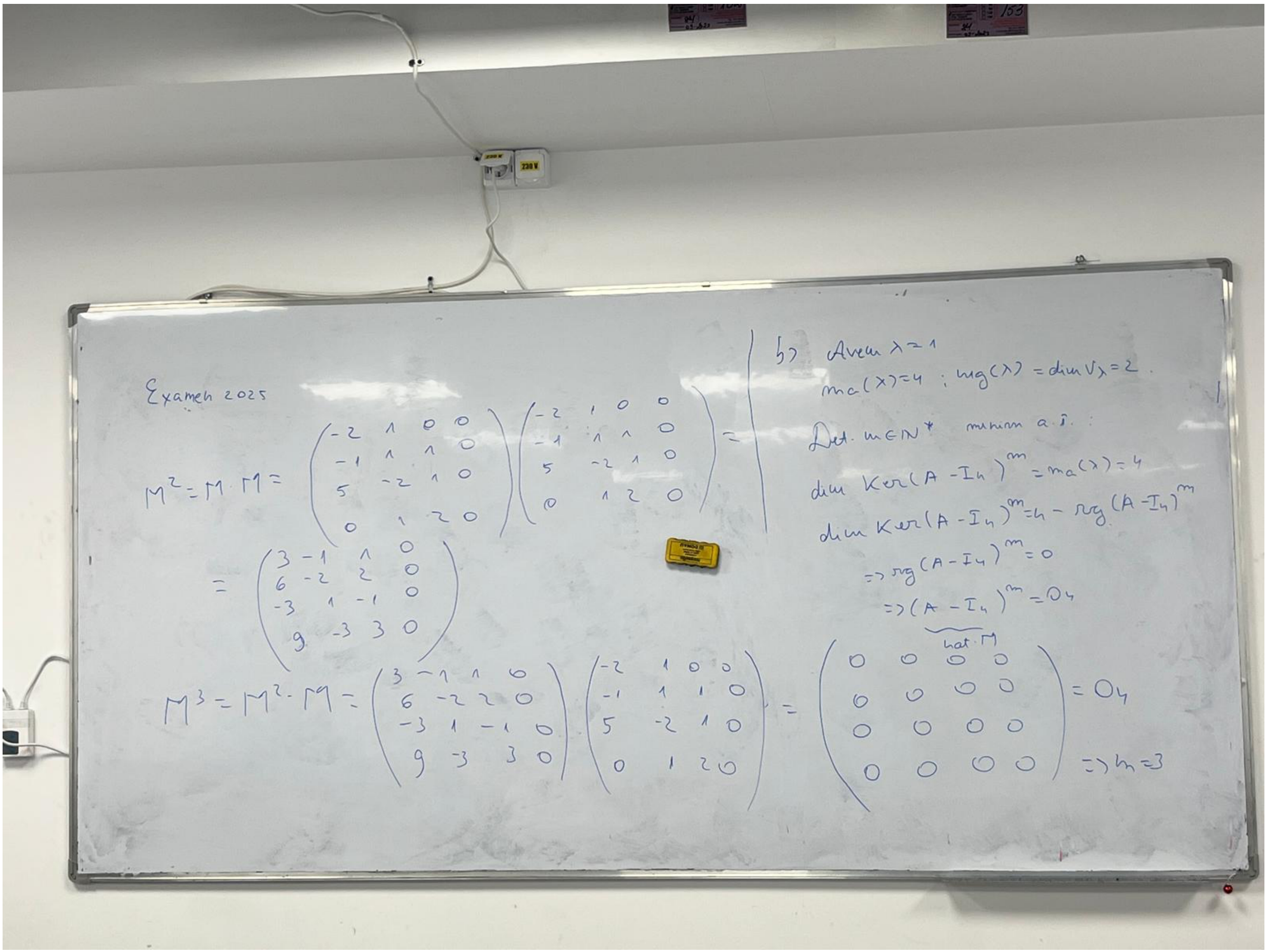
$$= -(1-\lambda)(\lambda-1)^3 = (\lambda-1)^4$$

$(\lambda-1)^4 = 0 \Rightarrow \lambda = 1$ cu $m_a(\lambda) = 4$

Căutăm $v \in \mathbb{R}^4 \setminus \{0\}$ a. î.:

$$(A - I_4) \cdot v = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad v = (x, y, z, t)^T$$


Examen 2025
(a) -2x + y = 0 (1) => y = 2x
-x + y + z = 0 (2)
5x - 2y + z = 0 (3)
y + 2z = 0 (4) => 2x + 2z = 0 => z = -x
=> Vλ = {(x, 2x, -x, t) | x, t ∈ R}
=> Vλ = < (1, 2, -1, 0); (0, 0, 0, 1) >
b) Avem λ = 1
ma(λ) = 4 ; mg(λ) = dim Vλ = 2.
Det. m ∈ N* minim a.î.:
dim Ker(A - I4)^m = ma(λ) = 4
dim Ker(A - I4)^m = 4 - rg(A - I4)^m
=> rg(A - I4)^m = 0
=> (A - I4)^m = O4
nat. M



Examen 2025
$M^2 = M \cdot M = \begin{pmatrix} -2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 5 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 5 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 1 & 0 \\ 6 & -2 & 2 & 0 \\ -3 & 1 & -1 & 0 \\ 9 & -3 & 3 & 0 \end{pmatrix}$
$M^3 = M^2 \cdot M = \begin{pmatrix} 3 & -1 & 1 & 0 \\ 6 & -2 & 2 & 0 \\ -3 & 1 & -1 & 0 \\ 9 & -3 & 3 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 5 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = O_4$
$\Rightarrow m = 3$
b) Avem $\lambda = 1$
$m_a(\lambda) = 4$; $m_g(\lambda) = \dim V_\lambda = 2$.
Det. $m \in \mathbb{N}^*$ minim a.î. :
$\dim Ker(A - I_4)^m = m_a(\lambda) = 4$
$\dim Ker(A - I_4)^m = 4 - rg(A - I_4)^m$
$\Rightarrow rg(A - I_4)^m = 0$
$\Rightarrow \underbrace{(A - I_4)}_{\text{not. } M}{}^m = O_4$

Examen 2025

Avem $\begin{cases} \gamma_1 + \gamma_2 + \gamma_3 = m_g(\lambda) = 2 = r \\ \gamma_1 + 2\gamma_2 + 3\gamma_3 = m_a(\lambda) = 4 \end{cases}$

$$2r - \gamma_1 = 4 - \underbrace{rg(A - I_4)^2}_{= \, rg\, M^2} \Rightarrow 4 - \gamma_1 = 3 \Rightarrow \boxed{\gamma_1 = 1}$$

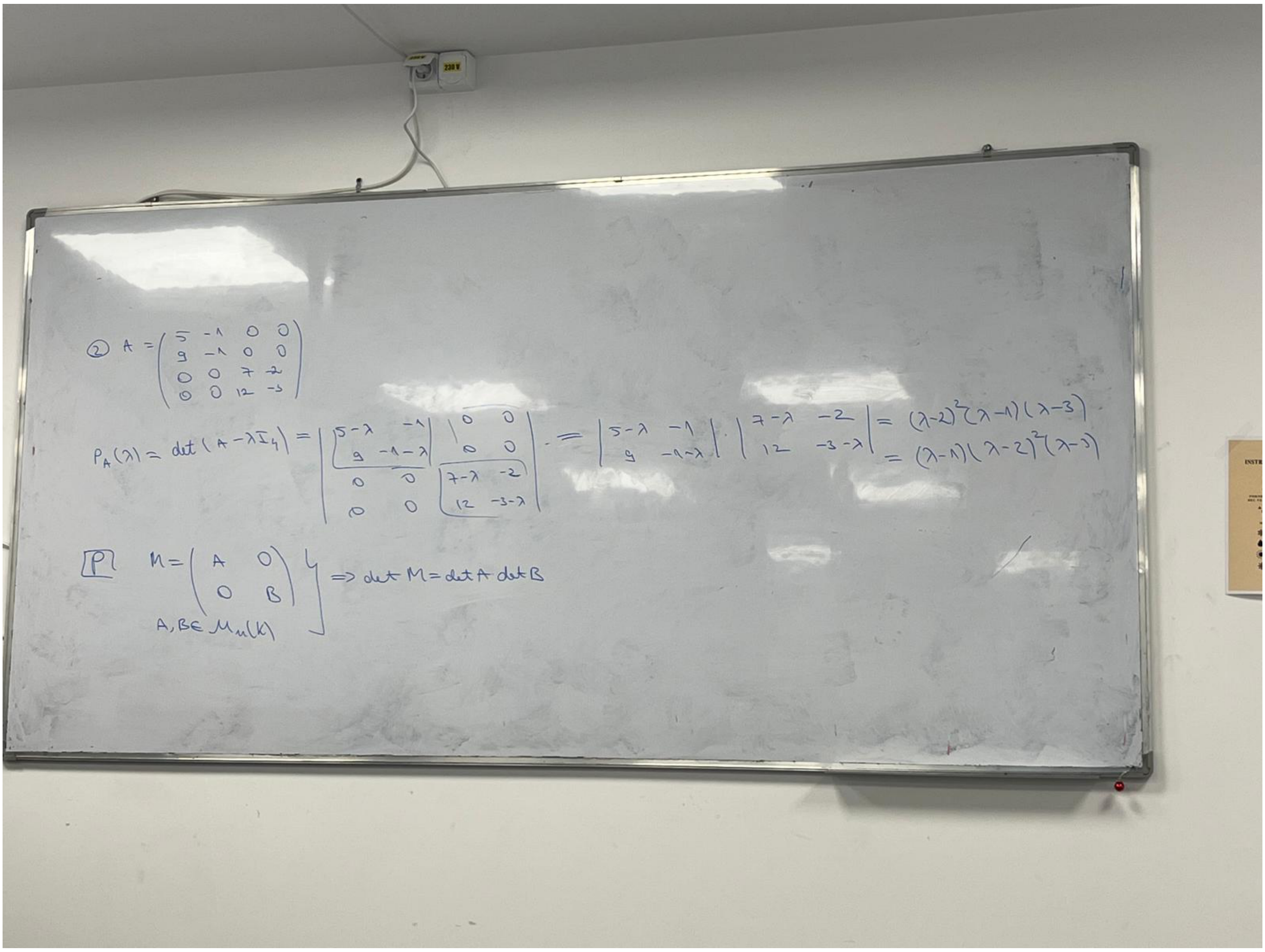
$$3r - 2\gamma_1 - \gamma_2 = 4 - \underbrace{rg(A - I_4)^3}_{= \, 0}$$

$$6 - 2 - \gamma_2 = 4$$

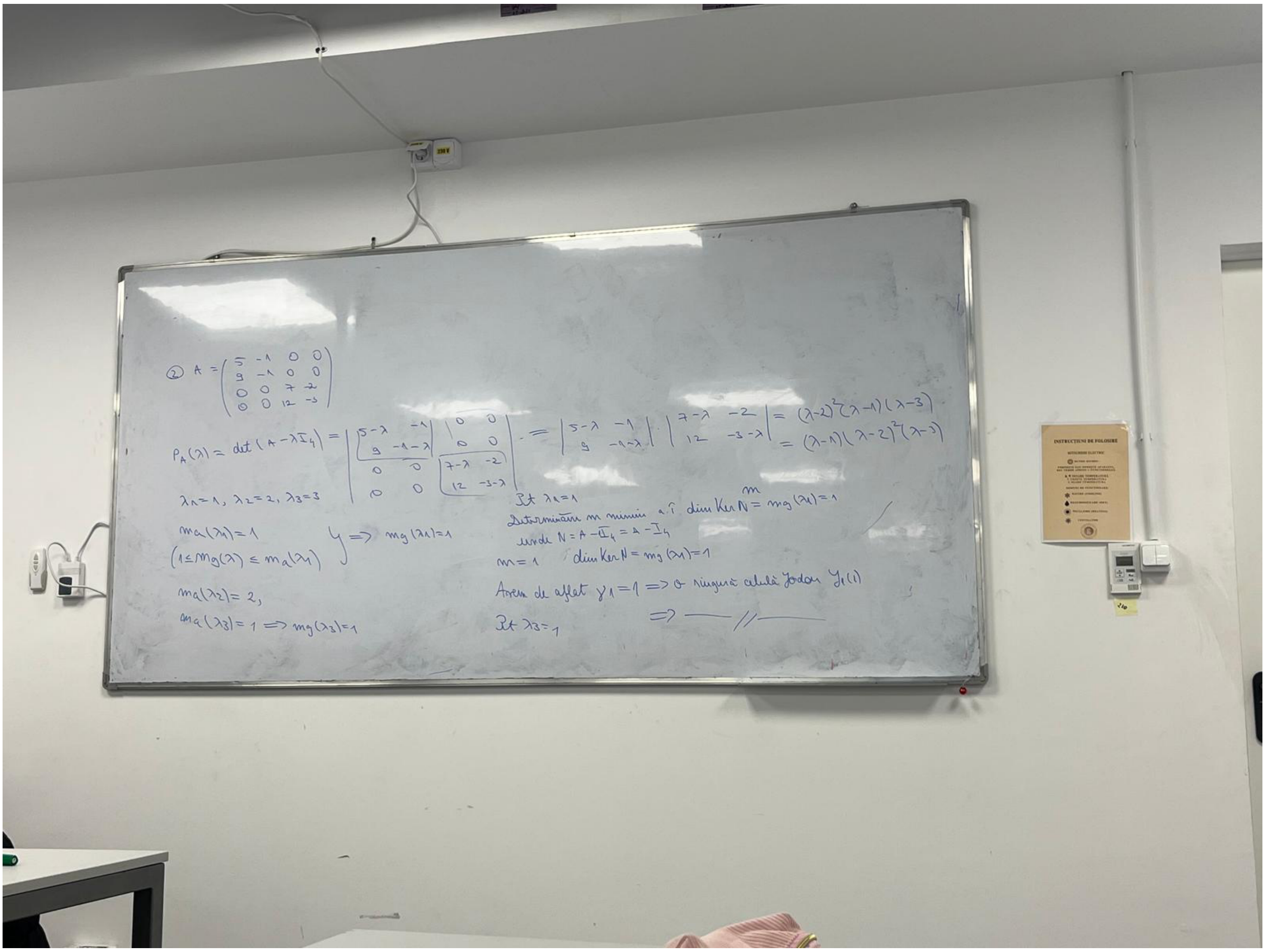
$$\Rightarrow \gamma_2 = 0 \Rightarrow \boxed{\gamma_3 = 1}$$

$\Rightarrow$ Forma can. Jordan a lui T este:

$$\begin{pmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



② $A = \begin{pmatrix} 5 & -1 & 0 & 0 \\ 9 & -1 & 0 & 0 \\ 0 & 0 & 7 & -2 \\ 0 & 0 & 12 & -3 \end{pmatrix}$
$P_A(\lambda) = \det(A - \lambda I_4) = \begin{vmatrix} 5-\lambda & -1 & 0 & 0 \\ 9 & -1-\lambda & 0 & 0 \\ 0 & 0 & 7-\lambda & -2 \\ 0 & 0 & 12 & -3-\lambda \end{vmatrix} = \begin{vmatrix} 5-\lambda & -1 \\ 9 & -1-\lambda \end{vmatrix} \cdot \begin{vmatrix} 7-\lambda & -2 \\ 12 & -3-\lambda \end{vmatrix} = (\lambda-2)^2(\lambda-1)(\lambda-3)$
$= (\lambda-1)(\lambda-2)^2(\lambda-3)$
$\boxed{P}$ $\left.\begin{matrix} M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \\ A, B \in \mathcal{M}_n(K) \end{matrix}\right\} \Rightarrow \det M = \det A \det B$



② $A = \begin{pmatrix} 5 & -1 & 0 & 0 \\ 9 & -1 & 0 & 0 \\ 0 & 0 & 7 & -2 \\ 0 & 0 & 12 & -3 \end{pmatrix}$
$P_A(\lambda) = \det(A - \lambda I_4) = \begin{vmatrix} 5-\lambda & -1 & 0 & 0 \\ 9 & -1-\lambda & 0 & 0 \\ 0 & 0 & 7-\lambda & -2 \\ 0 & 0 & 12 & -3-\lambda \end{vmatrix} = \begin{vmatrix} 5-\lambda & -1 \\ 9 & -1-\lambda \end{vmatrix} \cdot \begin{vmatrix} 7-\lambda & -2 \\ 12 & -3-\lambda \end{vmatrix} = (\lambda-2)^2(\lambda-1)(\lambda-3)$
$= (\lambda-1)(\lambda-2)^2(\lambda-3)$
$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$
$m_a(\lambda_1) = 1$
$(1 \le m_g(\lambda) \le m_a(\lambda_1))$ $\Rightarrow m_g(\lambda_1) = 1$
$m_a(\lambda_2) = 2,$
$m_a(\lambda_3) = 1 \Rightarrow m_g(\lambda_3) = 1$
Pt $\lambda_1 = 1$
Determinăm m minim a.î. $\dim \operatorname{Ker} N^m = m_g(\lambda_1) = 1$
unde $N = A - \lambda I_4 = A - I_4$
$m = 1$ $\dim \operatorname{Ker} N = m_g(\lambda_1) = 1$
Avem de aflat $\gamma_1 = 1 \Rightarrow$ o singură celulă Jordan $J_1(1)$
Pt $\lambda_3 = 1$
$\Rightarrow$ — // —

② $A = \begin{pmatrix} 5 & -1 & 0 & 0 \\ 9 & -1 & 0 & 0 \\ 0 & 0 & 7 & -2 \\ 0 & 0 & 12 & -3 \end{pmatrix}$

Fie $\lambda_2 = 2$

Det m minim a.î. $\dim \operatorname{Ker} N^m = m_a(\lambda_2) = 2$

$N = A - 2I_4$

$\dim \operatorname{Ker} N = 4 - rg\, N = 1 = r$

$\dim \operatorname{Ker} N^2 = 4 - \underbrace{rg\, N^2}_{=2} = 2 = m_a(\lambda_2)$

$N^2 = O_4$

$\begin{cases} \gamma_1 + \gamma_2 = r = 1 \\ \gamma_1 + 2\gamma_2 = 2 \end{cases} \Rightarrow \gamma_1 = 0,\ \gamma_2 = 1 \Rightarrow J_2(2) \longrightarrow \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

$\begin{pmatrix} \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} & & \\ & \boxed{1} & \\ & & \boxed{3} \end{pmatrix}$

$\begin{pmatrix} \boxed{1} & & \\ & \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} & \\ & & \boxed{3} \end{pmatrix}$