9.01.2025

Seminarul 13

Geometrie analitică euclidiană

- pb de examen

$$d_1: \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases}$$

$$d_2: \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$$

a) d_1, d_2 necoplanare

b) ec $\perp$ comune a d_1, d_2

c) $dist(d_1, d_2)$

a) M1) $d_1: \begin{cases} x_3 = t \\ x_1 = -t \\ x_2 = t+1 \end{cases} \Leftrightarrow \frac{x_1}{-1} = \frac{x_2 - 1}{1} = \frac{x_3}{1} = t$

$A(0,1,0) \in d_1$ (pt $t=0$)

$U_1 = (-1,1,1)$ (de la numitor)

$d_2: \begin{cases} x_1 = s \\ x_3 = 0 = x_2 \end{cases} \Rightarrow \frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = s$ (dăm lui s o valoare, în acest caz $s=0$)

$B(0,0,0) \in d_2$

$U_2 = (1,0,0)$

Obs: $\mathcal{D}: \frac{x - x_1^0}{u_1} = \frac{x - x_2^0}{u_2} = \frac{x - x_3^0}{u_3}$

$A(x_1^0, x_2^0, x_3^0) \in \mathcal{D}$

$u = (u_1, u_2, u_3)$

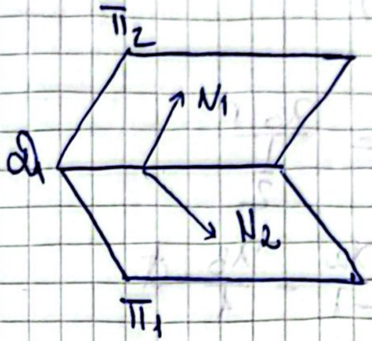
$$\Delta = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{vmatrix} = 1 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow d_1 \text{ și } d_2$$

necoplanare

$\vec{AB} = (0-0, 0-1, 0-0) = (0,-1,0)$

M2)

Fig:



$\pi: ax_1 + bx_2 + cx_3 + d = 0$

$N = (a,b,c)$

$N_1 = (1,0,1)$ (coef din sist)

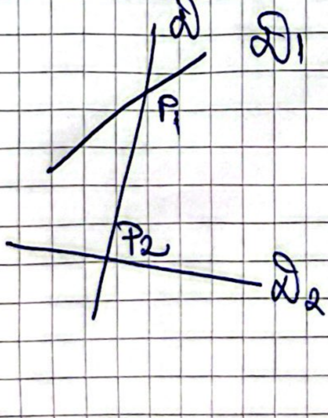
$N_2 = (0,1,-1)$

$u_{d_1} = N_1 \times N_2$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = e_1 \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} + e_2 \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} + e_3 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} =$$

$= (-1, 1, 1)$

b)



Fed comună

$d \cap d_k = \{P_k\}$, $k = \overline{1,2}$

$P_1(-t, ct+1, ct)$

$P_2(s, 0, 0)$

$\vec{P_1P_2} = (s+ct, -ct-1, -ct)$

$\vec{P_1P_2} \perp u_k$, $k = \overline{1,2}$

$\langle \overrightarrow{P_1P_2}, u_1 \rangle = 0$

$\langle \overrightarrow{P_1P_2}, u_2 \rangle = 0$

$-(s+t) - t - 1 - t = 0 \Rightarrow -2t = 1 \Rightarrow t = -\frac{1}{2} \Rightarrow s = \frac{1}{2}$

$s + t = 0 \Rightarrow s = -t$

$\overrightarrow{P_1P_2} = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

$d: \frac{x_1 - \frac{1}{2}}{0} = \frac{x_2 - 0}{-\frac{1}{2}} = \frac{x_3 - 0}{\frac{1}{2}}$

$\Rightarrow d: \frac{x_1 - \frac{1}{2}}{0} = \frac{x_2}{-1} = \frac{x_3}{1} = t$

$d: \begin{cases} x_1 - \frac{1}{2} = 0 \\ \cancel{x_2 = 0} \quad x_2 + x_3 = 0 \\ \cancel{x_3 = 0} \end{cases}$

Sau cu ajutorul ect parametrice

$\begin{cases} x_1 = \frac{1}{2} \\ x_2 = -t \\ x_3 = t \end{cases}$

c) Distanța dintre d_1 și d_2 = $dist(d_1, d_2) = \|\overrightarrow{P_1P_2}\| =$

$= \sqrt{0 + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2}$

$D_1: \frac{x_1-1}{1} = \frac{x_2-2}{-1} = \frac{x_3+2}{2}$

$D_2: \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0 \end{cases}$

a) D_1, D_2 coplanare

b) ec $D_1 D_2$

c) dist (D_1, D_2)

(y,z)

(x,z)

a) $N_1 = (2,0,-1)$ (coeficienti)

$N_2 = (0,2,1)$ $\Rightarrow N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} =$

$= e_1 \begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix} + e_2 \begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} + e_3 \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} =$

$= (2,-2,4) = 2(1,-1,2) = 2u_1$

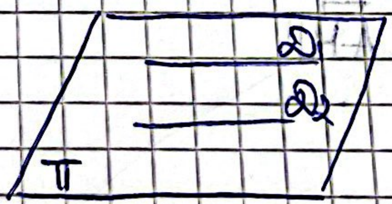
$V_{D_1} = \langle \{u_1\} \rangle$

$V_{D_2} = \langle \{u_2\} \rangle \Rightarrow D_1 \parallel D_2 \Rightarrow D_1, D_2$ coplanare

b) $\pi \begin{vmatrix} x_1-1 & 1 & 0 \\ x_2-2 & -2 & -4 \\ x_3+2 & 2 & 3 \end{vmatrix} = 0 \Rightarrow (x_1-1)5 - (x_2-2)\cdot 3 + (x_3+2)(-4) = 0$

$\pi: 5x_1 - 3x_2 - 4x_3 - 7 = 0$

$\vec{A} \quad \vec{u_1} \quad \vec{AB}$



Daca $\begin{cases} x_3 = 1 \\ x_1 = 1 \\ x_2 = -2 \end{cases} \Rightarrow B(1,-2,1)$
in D_2

$\vec{AB} = (0,-4,3)$

Obs.: $N_\pi = u_1 \times \vec{AB} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 2 \\ 0 & -4 & 3 \end{vmatrix} =$

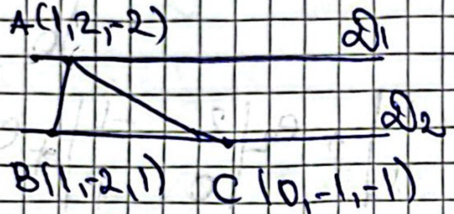
$= (5, -3, -4)$

$\pi : 5x_1 - 3x_2 - 4x_3 + d = 0$

$A(-1, 2, -2) \in \pi \Rightarrow 5 - 6 + 8 + d = 0$

$7 + d = 0 \Rightarrow d = -7$

c) $\text{dist}(\mathcal{D}_1, \mathcal{D}_2)$



$A_{\Delta ABC} = \frac{1}{2} \|\vec{AB} \times \vec{AC}\| = \frac{1}{2} \text{dist}(A, \mathcal{D}_2) \cdot \|\vec{BC}\|$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & -4 & 3 \\ -1 & -3 & 1 \end{vmatrix} = (5, -3, 4), \| \ \| = \sqrt{25 + 9 + 16} = \sqrt{50} =$

$= 5\sqrt{2}$

$\|\vec{BC}\| = \sqrt{1 + 1 + 4} = \sqrt{6}$

$\Rightarrow \text{dist}(A, \mathcal{D}_2) = \frac{5\sqrt{2}}{\sqrt{6}} = \frac{5\sqrt{3}}{3}$

(II) $\text{dist}(\mathcal{D}_1, \mathcal{D}_2) = \text{dist}(A, \mathcal{D}_2) = \|\vec{AH}\|$

$A \in \pi', \pi' \perp \mathcal{D}_2, \mathcal{D}_2 \cap \pi' = \{H\}$

$u_{D_2} = (2,-2,4) = N_{\pi'}$

$\pi' : 2 \cdot x_1 - 2x_2 + 4x_3 + d = 0$

$A(1,2,-2) \in \pi' \Rightarrow d = -2+4+8 = 10$

$$D_2 : \begin{cases} x_1 = \frac{1+t}{2} \\ x_2 = \frac{-t-3}{2} \\ x_3 = t \end{cases} \Rightarrow M : 2\left(\frac{1+t}{2}\right) - 2\,\frac{(t-3)}{2} + 4t + 10 = 0$$

$\Rightarrow t, M, \vec{AM}$

- $A(-1,0,0)$ $\quad \pi : x_1 + x_2 - x_3 + 2 = 0$

a) $dist(A, \pi)$

b) $pr_{\pi} A = A' = ?$

a) $dist(A,\pi) = \frac{|-1+0-0+2|}{\sqrt{1^2+1^2+(-1)^2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$A(x_1^0, x_2^0, x_3^0)$

$\pi : ax_1 + bx_2 + cx_3 + d = 0$

$dist(A,\pi) = \frac{|ax_1^0 + bx_2^0 + cx_3^0 + d|}{\sqrt{a^2+b^2+c^2}}$

$N = (1,1,-1) = u_D, \; D \perp \pi, \; A \in D$

$D : \frac{x_1+1}{1} = \frac{x_2-0}{1} = \frac{x_3-0}{-1} = t$

$$\begin{cases} x_1 = t-1 \\ x_2 = t \\ x_3 = -t \end{cases}, t \in \quad , D \cap \pi = \{N\}$$

$A': t-1+t+t+2=0$

$3t=-1$

$t=-\frac{1}{3}$

$A'\left(-\frac{4}{3}, -\frac{1}{3}, \frac{1}{3}\right)$

• $M(1,1,1)$

$d: \begin{cases} x_1-x_2+x_3+1=0 \\ x_1-2x_3-1=0 \end{cases}$

$\pi: x_1+2x_2+3x_3-1=0$

a) $\pi_1=?$ $M\in\pi_1$, ~~$\pi\pi$~~ $\pi_1 \parallel \pi_2$

b) d_1? $M\in d_1$, $d_1 \parallel d$

c) Studiați poz relativă a lui d față de π

a)

$\left.\begin{array}{l} N_\pi=(1,2,3) \\ \pi_1 \parallel \pi \end{array}\right| \Rightarrow N_\pi = N_{\pi_1}$

~~$\pi_1: x_1+2x_2+3x_3-6=0$~~

$\left.\begin{array}{l} \pi_1: x_1+2x_2+3x_3+d=0 \\ M\in\pi_1 \Rightarrow 1+2+3+d=0 \Rightarrow d=-6 \end{array}\right\} \Rightarrow \pi_1: x_1+2x_2+3x_3-6=0$

b) $N_1=(1,-1,1)$

$N_2=(1,0,-2)$

$$N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 1 \\ 1 & 0 & -3 \end{vmatrix}$$
$\parallel$
u_Δ

$d_1 : \frac{x_1-1}{2} = \frac{x_2-1}{3} = \frac{x_3-1}{1}$

$x_3 = 1$

$x_1 = x_2 = -1$

$\left.\begin{array}{l} A(-1,-1,1) \in d \\ u_d = (2,3,1) \end{array}\right| \Rightarrow d : \frac{x_1+1}{2} = \frac{x_2+1}{3} = \frac{x_3+1}{1} = t$

$$\begin{cases} x_1 = 2t-1 \\ x_2 = 3t-1 \\ x_3 = t-1 \end{cases}$$

c) $d \cap \pi : 2t-1+2(3t-1)+3(t-1)-1=0$

$11t-7=0 \Rightarrow t=\frac{7}{11}$

$d \cap \pi \neq \emptyset \Rightarrow$ Dreapta secanta

Obs. $at+b=0$

$a \neq 0 \quad t = -\frac{b}{a} \quad \exists!$ sol

$a=0, b\neq 0, \nexists$ sol

$a=0, b=0 \quad \infty$ infinitate de sol.