$f(x,y)=2x^3+3yx^2+6y^2x-4, \forall(x,y)\in\mathbb{R}^2$

f cont. pe $\mathbb{R}^2$

$Df=$ mult punct de discont.

$Df=\emptyset$

$$\frac{\partial f}{\partial x}(x,y)=(2x^3+3yx^2+6y^2x-4)'_x=6x^2+6yx+6y^2$$

$$\frac{\partial f}{\partial y}(x,y)=(2x^3+3yx^2+6y^2x-4)'_y=3x^2+12yx$$

$\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}$ functii cont pe $\mathbb{R}^2$

$\mathbb{R}^2$ mult desc

$\Rightarrow f$ dif pe $\mathbb{R}^2$

$D_1=$ mult pct in care functia este dif

$D_1=\emptyset$

$$\begin{cases}\frac{\partial f}{\partial x}(x,y)=0\\ \frac{\partial f}{\partial x}(x,y)=0\end{cases}\Leftrightarrow\begin{cases}6x^2+6yx+6y^2=0\\ 3x^2+12yx=0\end{cases}\Leftrightarrow\begin{cases}x^2+xy+y^2=0\Rightarrow\begin{cases}y^2=0\Rightarrow y=0\\ 16y^2-4y^2+y^2=0\Rightarrow 13y^2=0\end{cases}\\ x^2+4yx=0\Rightarrow\begin{cases}x=0\\ x+4y=0\Leftrightarrow x=-4y\end{cases}\end{cases}$$

$(0,0)\in\mathbb{R}^2$

$C=\{(0,0)\}$

$$\frac{\partial^2 f}{\partial^2 x}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(6x^2+6xy+6y^2\right)'_x=12x+6y$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(3x^2+12yx\right)'_x=6x+12y$$

$$\frac{\partial^2 f}{\partial^2 y}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(3x^2+12yx\right)'_x=12x$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(6x^2+6xy+6y^2\right)'_y=6x+12y$$

$\mathbb{R}^2$ mult desc

$\frac{\partial^2 f}{\partial^2 x},\frac{\partial^2 f}{\partial x\partial y},\frac{\partial^2 f}{\partial^2 y},\frac{\partial^2 f}{\partial y\partial x}$ functii cont pe $\mathbb{R}^2$

Δ_1

$$H_f(0,0)=\begin{pmatrix}\frac{\partial^2 f}{\partial^2 x}(0,0) & \frac{\partial^2 f}{\partial x\partial y}(0,0)\\ \frac{\partial^2 f}{\partial y\partial x}(0,0) & \frac{\partial^2 f}{\partial^2 y}(0,0)\end{pmatrix}=\begin{pmatrix}\boxed{0} & 0\\ 0 & 0\end{pmatrix}$$

Δ_2

$\Delta_1=0$

$\Delta_2=0$

$f(x,y)-f(0,0)=2x^3+3yx^2+6y^2x-4+4=2x^3+3yx^2+6y^2x$

$f(x,0)-f(0,0)=2x^3>0$

$f(-x,0)-f(0,0)=-2x^3<0$

$\Rightarrow(0,0)$ nu este punct de extrem local

$f(x,y)=x\ln(x^2+y^2), \forall(x,y)\in(0,+\infty)\times\mathbb{R}$

f cont. pe $\mathbb{R}^2$

$Df=$ mult punct de discont.

$Df=\emptyset$

$$\frac{\partial f}{\partial x}(x,y)=(x\ln(x^2+y^2))'_x=\ln(x^2+y^2)+x\frac{1}{x^2+y^2}2x=\ln(x^2+y^2)+\frac{2x^2}{x^2+y^2}$$

$$\frac{\partial f}{\partial y}(x,y)=(x\ln(x^2+y^2))'_y=x\frac{1}{x^2+y^2}2y=\frac{2xy}{x^2+y^2}$$

$\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}$ functii cont pe $\mathbb{R}^2$

$\mathbb{R}^2$ mult desc

$\Rightarrow f$ dif pe $\mathbb{R}^2$

$D_1=$ mult pct in care functia este dif

$D_1=\emptyset$

$$\begin{cases}\frac{\partial f}{\partial x}(x,y)=0\\ \frac{\partial f}{\partial x}(x,y)=0\end{cases}\Leftrightarrow\begin{cases}\ln(x^2+y^2)+\frac{2x^2}{x^2+y^2}=0\Rightarrow\begin{cases}\ln y^2+0=0\Rightarrow\ln y^2=0\Rightarrow y^2=1\Rightarrow y=\pm1\\ \ln x^2+2=0\Rightarrow\ln x^2=-2\Rightarrow x^2=e^{-2}\Rightarrow x=\pm e^{-1}\end{cases}\\ \frac{2xy}{x^2+y^2}=0\Rightarrow 2xy=0\Rightarrow\begin{cases}x=0\\ y=0\end{cases}\end{cases}$$

$(-e^{-1},0),(0,-1),(0,1)\notin(0,+\infty)\times\mathbb{R}$

$(e^{-1},0)\in(0,+\infty)\times\mathbb{R}$

$C=\{(e^{-1},0)\}$

$$\frac{\partial^2 f}{\partial^2 x}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(\ln(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right)'_x=\frac{2x}{x^2+y^2}+\frac{4x(x^2+y^2)-4x^3}{(x^2+y^2)^2}=$$
$$=\frac{2x(x^2+y^2)+4x^3+4xy^2-4x^3}{(x^2+y^2)^2}=\frac{2x^3+6xy^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(\frac{2xy}{x^2+y^2}\right)'_x=\frac{2y(x^2+y^2)-4x^2y}{(x^2+y^2)^2}=\frac{2x^2y+2y^3-4x^2y}{(x^2+y^2)^2}=\frac{2y^3-2x^2y}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial^2 y}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(\frac{2xy}{x^2+y^2}\right)'_x=\frac{2x(x^2+y^2)-4xy^2}{(x^2+y^2)^2}=\frac{2x^3+2xy^2-4xy^2}{(x^2+y^2)^2}=\frac{2x^3-2xy^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(\ln(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right)'_y=\frac{2y}{x^2+y^2}-\frac{2x^2}{(x^2+y^2)^2}2y=$$
$$=\frac{2y(x^2+y^2)-4x^2y}{(x^2+y^2)^2}=\frac{2y^3-2x^2y}{(x^2+y^2)^2}$$

$\mathbb{R}^2$ mult desc

$\frac{\partial^2 f}{\partial^2 x},\frac{\partial^2 f}{\partial x\partial y},\frac{\partial^2 f}{\partial^2 y},\frac{\partial^2 f}{\partial y\partial x}$ functii cont pe $\mathbb{R}^2$

$$H_f(e^{-1},0)=\begin{pmatrix}\frac{\partial^2 f}{\partial^2 x}(e^{-1},0) & \frac{\partial^2 f}{\partial x\partial y}(e^{-1},0)\\ \frac{\partial^2 f}{\partial y\partial x}(e^{-1},0) & \frac{\partial^2 f}{\partial^2 y}(e^{-1},0)\end{pmatrix}=\begin{pmatrix}\boxed{2e} & 0\\ 0 & 2e\end{pmatrix}$$

Δ_1

Δ_2

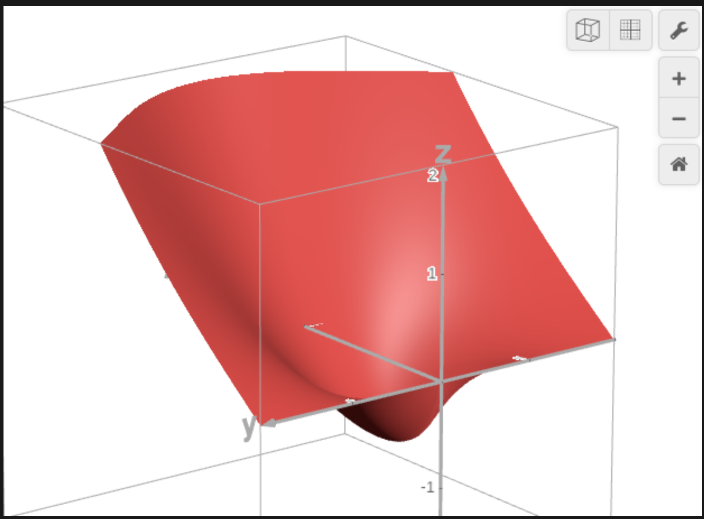
$\Delta_1=2e>0$

$\Delta_2=4e^2>0$

$\Rightarrow(e^{-1},0)$ punct de min local

Exemplu de cum o reprezentare 3d poate fi folositoare in exercitiu de acasa:

In cazul functiei anterioare putem vedea dreapta oX si clarul punct de minim local. Pe langa asta deducel ca $(e^{-1}, 0)$este minim local, si daca calculele ne dau altfel inseamna ca am gresit undeva.



Nu putem face asta in cazul examenului deoarece lucram in 3d, mult noroc cu reprezentari 3d pe foaia 2d. Dar acasa putem folosi diferite aplicatii, precum desmos in browser, din cate stiu aplicatia nu iti permite, 3d functions ploter sau 3d calculator, pentru a ne verifica munca stiind ca am ajuns la raspunsul corect.

$f(x,y)=\dfrac{y^3}{3}+\dfrac{y^2x}{2}+x^2, \forall(x,y)\in\mathbb{R}^2$

f cont. pe $\mathbb{R}^2$

Df = mult punct de discont.

$Df=\emptyset$

$$\frac{\partial f}{\partial x}(x,y)=\left(\frac{y^3}{3}+\frac{y^2x}{2}+x^2\right)'_x=\frac{y^2}{2}+2x$$

$$\frac{\partial f}{\partial y}(x,y)=\left(\frac{y^3}{3}+\frac{y^2x}{2}+x^2\right)'_y=y^2+yx$$

$\dfrac{\partial f}{\partial x},\dfrac{\partial f}{\partial y}$ functii cont pe $\mathbb{R}^2$

$\mathbb{R}^2$ mult desc

$\Rightarrow f$ dif pe $\mathbb{R}^2$

D_1 = mult pct in care functia este dif

$D_1=\emptyset$

$$\begin{cases}\frac{\partial f}{\partial x}(x,y)=0\\ \frac{\partial f}{\partial x}(x,y)=0\end{cases}\Leftrightarrow\begin{cases}\frac{y^2}{2}+2x=0\Rightarrow\begin{cases}2x=0\Rightarrow x=0\\ \frac{y^2}{2}-2y=0\Rightarrow\begin{cases}y=0\\ \frac{y}{2}-2=0\Leftrightarrow\frac{y}{2}=2\Leftrightarrow y=4\end{cases}\end{cases}\\ y^2+yx=0\Rightarrow\begin{cases}y=0\\ x+y=0\Leftrightarrow x=-y\end{cases}\end{cases}$$

$(0,0),(-4,4)\in\mathbb{R}^2$

$C=\{(0,0),(-4,4)\}$

$$\frac{\partial^2 f}{\partial^2 x}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(\frac{y^2}{2}+2x\right)'_x=2$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y)=\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(y^2+yx\right)'_x=y$$

$$\frac{\partial^2 f}{\partial^2 y}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial y}\right)(x,y)=\left(y^2+yx\right)'_y=2y+x$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y)=\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)(x,y)=\left(\frac{y^2}{2}+2x\right)'_y=y$$

$\mathbb{R}^2$ mult desc

$\dfrac{\partial^2 f}{\partial^2 x},\dfrac{\partial^2 f}{\partial x\partial y},\dfrac{\partial^2 f}{\partial^2 y},\dfrac{\partial^2 f}{\partial y\partial x}$ functii cont pe $\mathbb{R}^2$

Δ_1

$$H_f(0,0)=\begin{pmatrix}\frac{\partial^2 f}{\partial^2 x}(0,0) & \frac{\partial^2 f}{\partial x\partial y}(0,0)\\ \frac{\partial^2 f}{\partial y\partial x}(0,0) & \frac{\partial^2 f}{\partial^2 y}(0,0)\end{pmatrix}=\begin{pmatrix}\boxed{2} & 0\\ 0 & 0\end{pmatrix}$$

Δ_2

$\Delta_1 = 0$

$\Delta_2 = 0$

$f(x,y) - f(0,0) = \dfrac{y^3}{3} + \dfrac{y^2x}{2} + x^2 - 0 = \dfrac{y^3}{3} + \dfrac{y^2x}{2} + x^2$

$f(0,y) - f(0,0) = \dfrac{y^3}{3} > 0$

$f(0,-y) - f(0,0) = -\dfrac{y^3}{3} < 0$

$\Rightarrow (0,0)$ nu este punct de extrem local

$$H_f(-4,4) = \begin{pmatrix} \frac{\partial^2 f}{\partial^2 x}(-4,4) & \frac{\partial^2 f}{\partial x \partial y}(-4,4) \\ \frac{\partial^2 f}{\partial y \partial x}(-4,4) & \frac{\partial^2 f}{\partial^2 y}(-4,4) \end{pmatrix} = \begin{array}{c} \Delta_1 \\ \begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix} \\ \Delta_2 \end{array}$$

$\Delta_1 = 2 > 0$

$\Delta_2 = 2 * 4 - 4^2 = 8 - 16 = -8 < 0$

$\Rightarrow (-4,4)$ nu este punct de extrem local