$$f_n : (2,5] \to \mathbb{R}, f_n(x) = \frac{n * x^3}{n + x^2}, \forall x \in (2,5], \forall n \in \mathbb{N}$$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{n * x^3}{n + x^2} = x^3 > 0$$

$$f : (2,5] \to \mathbb{R}, f(x) = x^3, \forall x \in (2,5]$$

$$f_n \xrightarrow[(2,5]]{s} f$$

$$\lim_{n\to\infty} \sup_{x\in(2,5]} |f_n(x) - f(x)| = \lim_{n\to\infty} \sup_{x\in(2,5]} \left| \frac{n * x^3}{n + x^2} - x^3 \right| = \lim_{n\to\infty} \sup_{x\in(2,5]} \left| \frac{n * x^3 - n * x^3 - x^5}{n + x^2} \right| = \lim_{n\to\infty} \sup_{x\in(2,5]} \left| - \frac{x^5}{n + x^2} \right| =$$

$$= \lim_{n\to\infty} \sup_{x\in(2,5]} \underbrace{\frac{x^5}{n + x^2}}_{g(x)}$$

$$g'(x) = \left( \frac{x^5}{n + x^2} \right)' = \frac{\left(x^5\right)' \left(n + x^2\right) - \left(n + x^2\right)' \left(x^5\right)}{\left(n + x^2\right)^2} = \frac{5nx^4 + 3x^6}{\left(n + x^2\right)^2} = 0 \Leftrightarrow 5nx^4 + 3x^6 = 0 \Rightarrow$$

$$\Rightarrow \quad \begin{cases} x^4 = 0 \Leftrightarrow x = 0 \\ 3x^2 = -5n \Leftrightarrow x^2 = -\frac{5n}{3} \Rightarrow x \in \mathbb{C} \end{cases}$$

| $x$ | $(2$ | | $5]$ |
|---|---|---|---|
| $g'(x)$ | $+$ | $+$ | $+$ |
| $g(x)$ | $\frac{32}{n+4}$ | $\nearrow$ | $\frac{5^5}{n+25}$ |

$$\lim_{n\to\infty} \sup_{x\in(2,5]} |f_n(x) - f(x)| = \lim_{n\to\infty} \sup_{x\in(2,5]} g(x) = \lim_{n\to\infty} \frac{5^5}{n + 25} = 0 \Rightarrow f_n \xrightarrow[(2,5]]{u} f$$

$$f_n : \mathbb{R} \to \mathbb{R}, f_n(x) = \frac{n^3 x}{x^4 + n^4}, \forall x \in \mathbb{R}, \forall n \in \mathbb{N}$$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{n^3 x}{x^4 + n^4} = 0$$

$$f : \mathbb{R} \to \mathbb{R}, f(x) = 0, \forall x \in \mathbb{R}$$

$$f_n \xrightarrow[\mathbb{R}]{s} f$$

$$\lim_{n\to\infty} \sup_{x\in\mathbb{R}} |f_n(x) - f(x)| = \lim_{n\to\infty} \sup_{x\in\mathbb{R}} \left| \frac{n^3 x}{n^4 + x^4} - 0 \right| = \lim_{n\to\infty} \sup_{x\in\mathbb{R}} \underbrace{\left| \frac{n^3 x}{n^4 + x^4} \right|}_{g(x)} =$$

$$g(1) = \frac{n^3}{n^4 + 1}$$

$$g(n) = \frac{n^3 n}{n^4 + n^4} = \frac{n^4}{2n^4} = \frac{1}{2} \Rightarrow \sup_{x\in\mathbb{R}} |g(x)| \geq \frac{1}{2} \Rightarrow \sup_{x\in\mathbb{R}} |f_n(x) - f(x)| \geq \frac{1}{2} \Rightarrow \lim_{n\to\infty} \sup_{x\in\setminus} |f_n(x) - f(x)| \geq \frac{1}{2} \neq 0 \Rightarrow$$

$$\Rightarrow f_n \cancel{\xrightarrow[(2,5]]{u} f}$$