$O, o, \omega, \Theta, \Omega$

$\boxed{1}$ $O \rightarrow$ esențial, compararea a 2 funcții

$f \in O(g) \rightarrow f(n) \leq c \cdot g(n)$

$n \in O(n^2)$

$n_0 = 1,\ c = 1 \Rightarrow \forall n \geq n_0,\ n \leq n^2$

$2^{n+1} \in O(2^n)$

$\hookrightarrow \underbrace{2 \cdot 2^n}_{f} \leq 2 \cdot 2^n$

$c \qquad g(n_0)$

O - mare

$\frac{n}{2} \in O(n)$

$\boxed{2}$ $f \in \Omega(g) \longrightarrow f(n) \geq c \cdot g(n)$

$n^2 \in \Omega(n)$

Ω - small

submissive...

$n \log n \in \Omega(n)$

$n \log \log n \in \Omega(n)$

$\boxed{3}$ $f \in \Theta(g) \rightarrow c_2 \cdot g(n) \leq f(n) \leq c_1 g(n)$

ex: $2n \in \Theta(n) \longrightarrow 1 \cdot n \leq 2n \leq 3n$

$n^2 \in \Theta\left(\frac{n^2}{100}\right) \quad 1 \cdot \frac{n^2}{100} \leq n^2 \leq 200 \cdot \frac{n^2}{100}$

$\Theta \rightarrow$ fracțional

$\boxed{4}$ $f \in o(g) \rightarrow f(n) < c \cdot g(n)$

O dar strict

<u>ex</u> $n \in o(n^2)$, $n \notin (7n)$

$n \in o(n \log n)$

$n \in o(n \log \log \dots \log n)$

$\rightarrow$ <u>nu</u> înmulțiri, împărțiri....

$\boxed{5}$ $f \in \omega(g) \rightarrow f(n) > c \cdot g(n)$

Ω dar strict (all in the name!)

<u>ex</u> $n^2 \in \omega(n)$

$n \log n \in \omega(n)$

$7n \notin \omega(n)$

$\rightarrow$ O opus Ω

$\downarrow$ $\downarrow$

o opus ω

Θ - separat

ex model examen

1.1. în Θ

a) $\lg n^{10} \rightarrow 10 \lg n \in \Theta(\lg n)$

b) $(n-2^{2023})^7 \in \Theta(n^7)$

c) $(\lg n^{100})^3 + n^{1.3} \in \Theta(n^{1.3})$

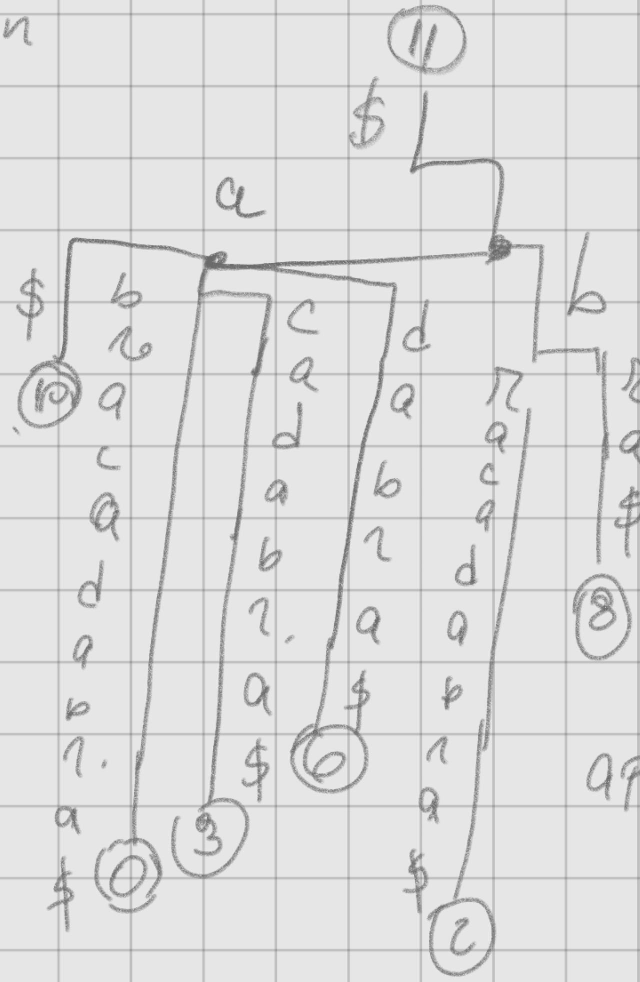
$n^{1.3} \leq 100^3 \log^3 n + n^{1.3} \leq c \cdot n^{1.3}$

d) $\lg(n!) \in \Theta(\lg n)$

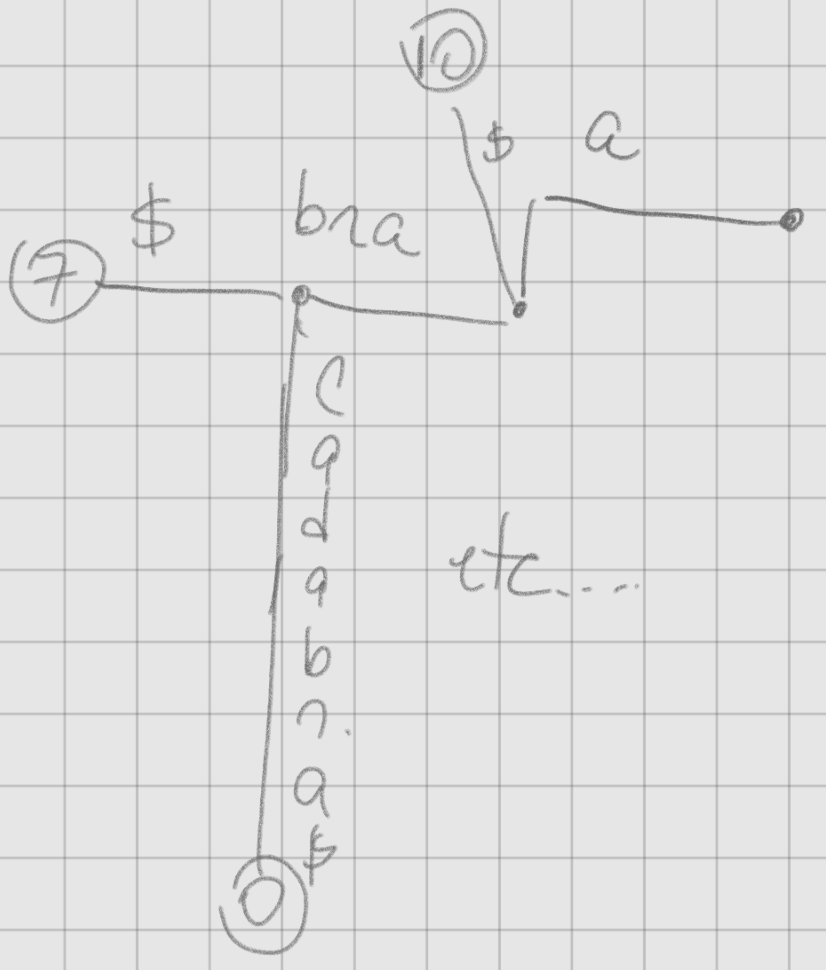
$\lg 1 \cdot 2 \cdot 3 \cdots n$

1.2. suffixtree

0 1 2 3 4 5 6 7 8 9 10
abracadabra



aproape



etc...

suffix array:
→ numerotezi sufixele
→ pui lexicografic
→ scrii renumerotarea

0 1 2 3 4 5 6 7 8 9 10

0. abracadabra

1. bracadabra

. . . . .

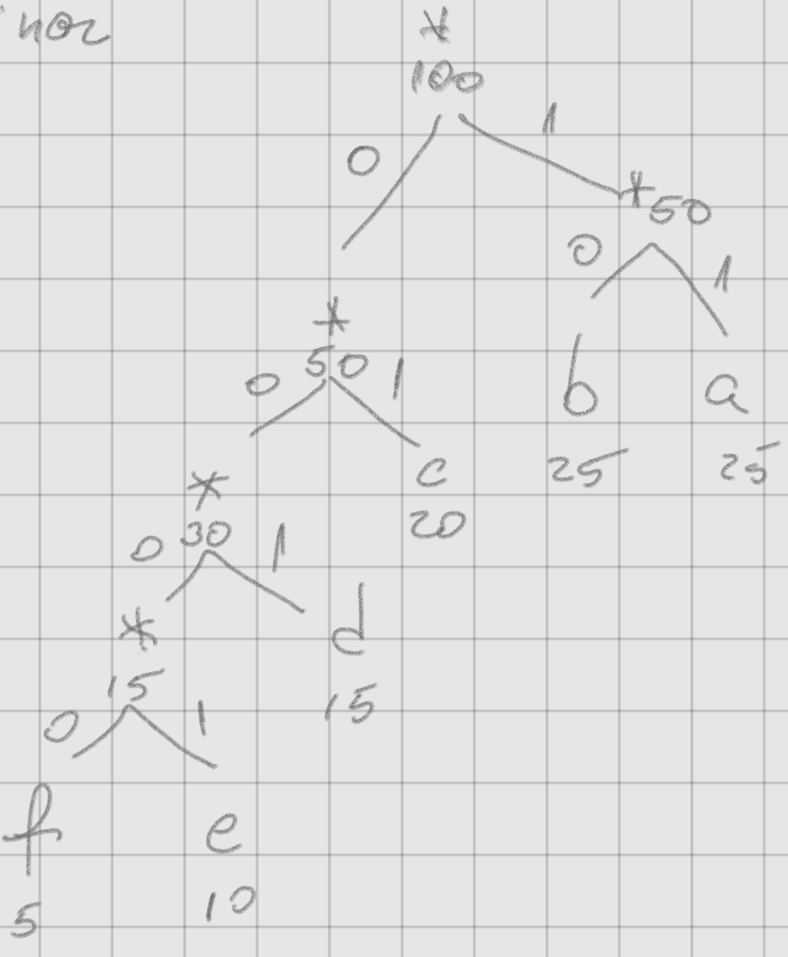
:

10. a

10	a
7	abra
0	abracadabra
3	acadabra
5	adabra
8	bra
	⋮

1.3 Huffman pt frecv. a=25, b=25, c=20, d=15, e=10, f=5

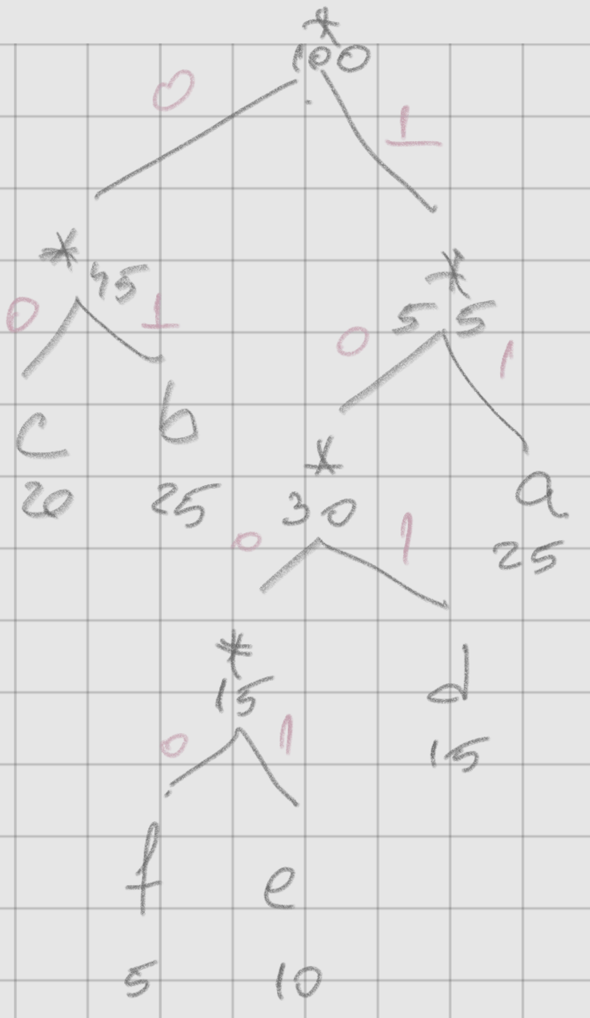
f cod binar



a	11
b	10
c	01
d	001
e	0001
f	0000

30 *
20 c → 45 *
25 b   25 a
25 a   30 *

55 *
45 *



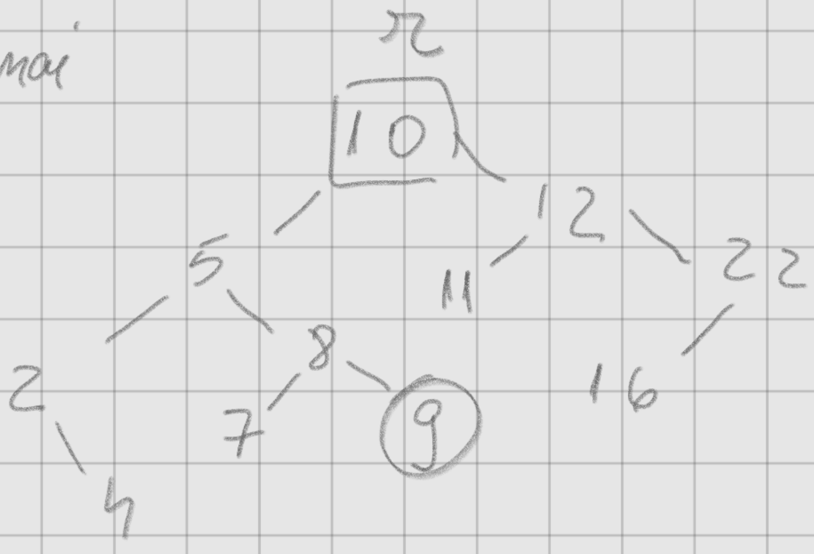
a 11
b 01
c 00
d 101
e 1001
f 1000

1.4 Binary search tree

chei: 10 5 2 4 8 9 12 22 11 16 7

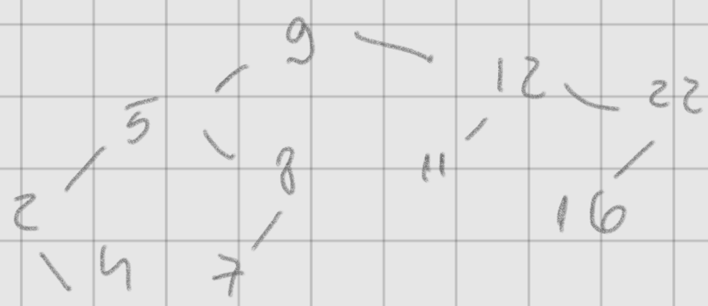
la stânga mai mic

la dreapta mai mare



rădăcina extrasă → cel mai mic element din subarborele drept

sau cel mai mare din cel stâng



2.1 Dem că $f(n) + o(f(n)) = \Theta(f(n))$

$g \in o(f(n))$ dacă $\forall c > 0, \exists n_0 > 0$ a.î. $\forall n \geq n_0$ avem

$$g(n) < c \cdot f(n)$$

$g \in \Theta(f(n))$ dacă $\forall c_1, c_2, n_0$ a.î. $\forall n \geq n_0$ avem

$$c_1 \cdot f(n) \leq g(n) \leq c_2 \cdot f(n)$$

Fie $g(n) \in ?(f(n))$

$\rightarrow c_1 \cdot f(n) \leq f(n) + g(n) \leq c_2 \cdot f(n)$

$f(n) + g(n) \geq f(n) \Rightarrow$ alegem $c_1 = 1$

$f(n) + g(n) \leq 2 f(n) \rightarrow$ ~~deoarece~~ din def o $\exists n_0'$ a.î. $g(n) < c \cdot f(n)$

$\Rightarrow f(n) + g(n) \leq \underbrace{(c+1)} f(n)$

îl putem lua ca $c_2 = c + 1$

$\rightarrow$ Dem gen $c_1 \cdot f \leq f + g \leq (c+1) \cdot f$

$f+g \geq f-g$

$g \leq c \cdot f \Rightarrow -g \geq -c \cdot f$

$\rightarrow f+g \geq f-g \geq f-c \cdot f = (1-c)f$

$\hookrightarrow c_1 = 1-c,$

da mrg.

2.2. $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + 1$

Dem că rez. e corect

Arătăm că $T(n) = c \cdot n - b$

Pp. că $T(n/4) \leq c \cdot \frac{n}{4} - b$

$T\left(\frac{3n}{4}\right) \leq 3c \cdot \frac{n}{4} - b$

$T(n) = T\left(\frac{3n}{4}\right) + T\left(\frac{n}{4}\right) + 1 \leq 3c \cdot \frac{n}{4} - c\frac{n}{4} - 2b + 1$

$\leq c \cdot n - 2b + 1 = (c \cdot n - b) + (1-b)$

$\leq (c \cdot n - b) \quad \forall b$ a.î. $1-b \leq 0$

$b \geq 1$

$T(n) = T$