$$B = \{ (x,y) \in \mathbb{R}^2 \mid x = y \} \leq \mathbb{R}^2$$

Fie $v_1 = (x_1, y_1)$ și $v_2 = (x_2, y_2)$

$\alpha, \beta \in \mathbb{R}$

Vrem $\alpha v_1 + \beta v_2 \in B$

$x_1 = y_1 \Rightarrow x_1 - y_1 = 0$

$x_2 = y_2 \Rightarrow x_2 - y_2 = 0$

$$\alpha v_1 + \beta v_2 = \alpha (x_1, y_1) + \beta (x_2, y_2) = (\alpha x_1, \alpha y_1) + (\beta x_2, \beta y_2) = (\underbrace{\alpha x_1 + \beta x_2}_{x}, \underbrace{\alpha y_1 + \beta y_2}_{y})$$

$x - y = 0$

$$\alpha x_1 + \beta x_2 - \alpha y_1 - \beta y_2 = 0 \Leftrightarrow \alpha x_1 - \alpha y_1 + \beta x_2 - \beta y_2 = 0 \Leftrightarrow \alpha(\underbrace{x_1 - y_1}_{0}) + \beta(\underbrace{x_2 - y_2}_{0}) = 0 \Leftrightarrow \alpha \cdot 0 + \beta \cdot 0 = 0 \Leftrightarrow$$

$\Leftrightarrow 0 = 0$ adev.

$$\Delta = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathcal{M}_2(\mathbb{R}) \;\middle|\; \begin{matrix} a+b+c = 1 \\ (*) \end{matrix} \right\} \subseteq \mathcal{M}_2(\mathbb{R})$$

$$A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} ; \quad A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$A_1 + A_2 = \begin{pmatrix} \underbrace{a_1+a_2}_{a} & \underbrace{b_1+b_2}_{b} \\ \underbrace{c_1+c_2}_{c} & \underbrace{d_1+d_2}_{d} \end{pmatrix}$$

$$(a_1+a_2) + (b_1+b_2) + (c_1+c_2) = 1$$

Fie $\alpha \in \mathbb{R}$

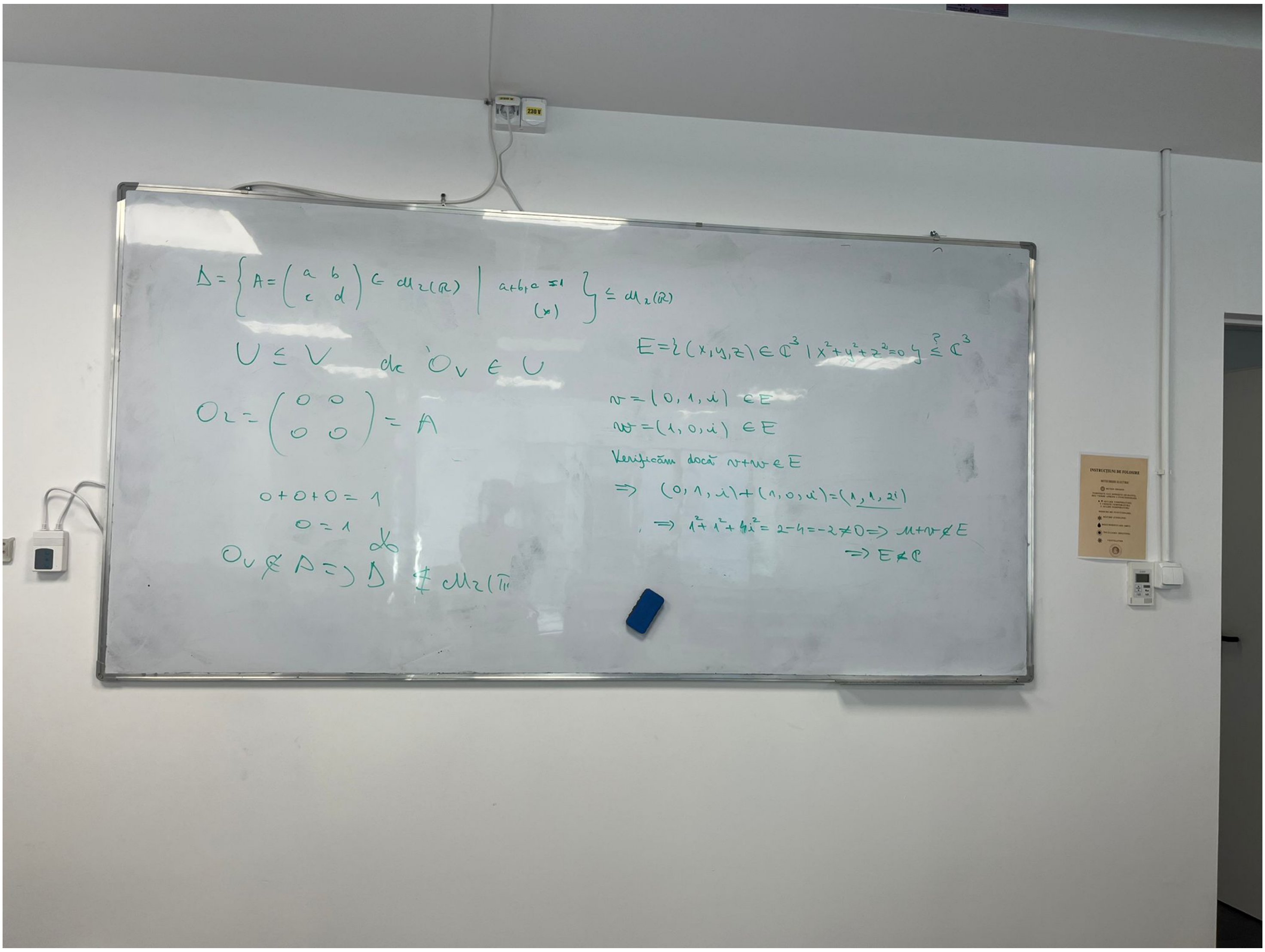
$$\alpha \cdot A = \alpha \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$\alpha \cdot A = \begin{pmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha d_1 \end{pmatrix}$$

$$\alpha a_1 + \alpha b_1 + \alpha c_1 = 1$$

$$\alpha(\underbrace{a_1+b_1+c_1}_{1}) = 1$$

$$\Rightarrow \alpha = 1$$



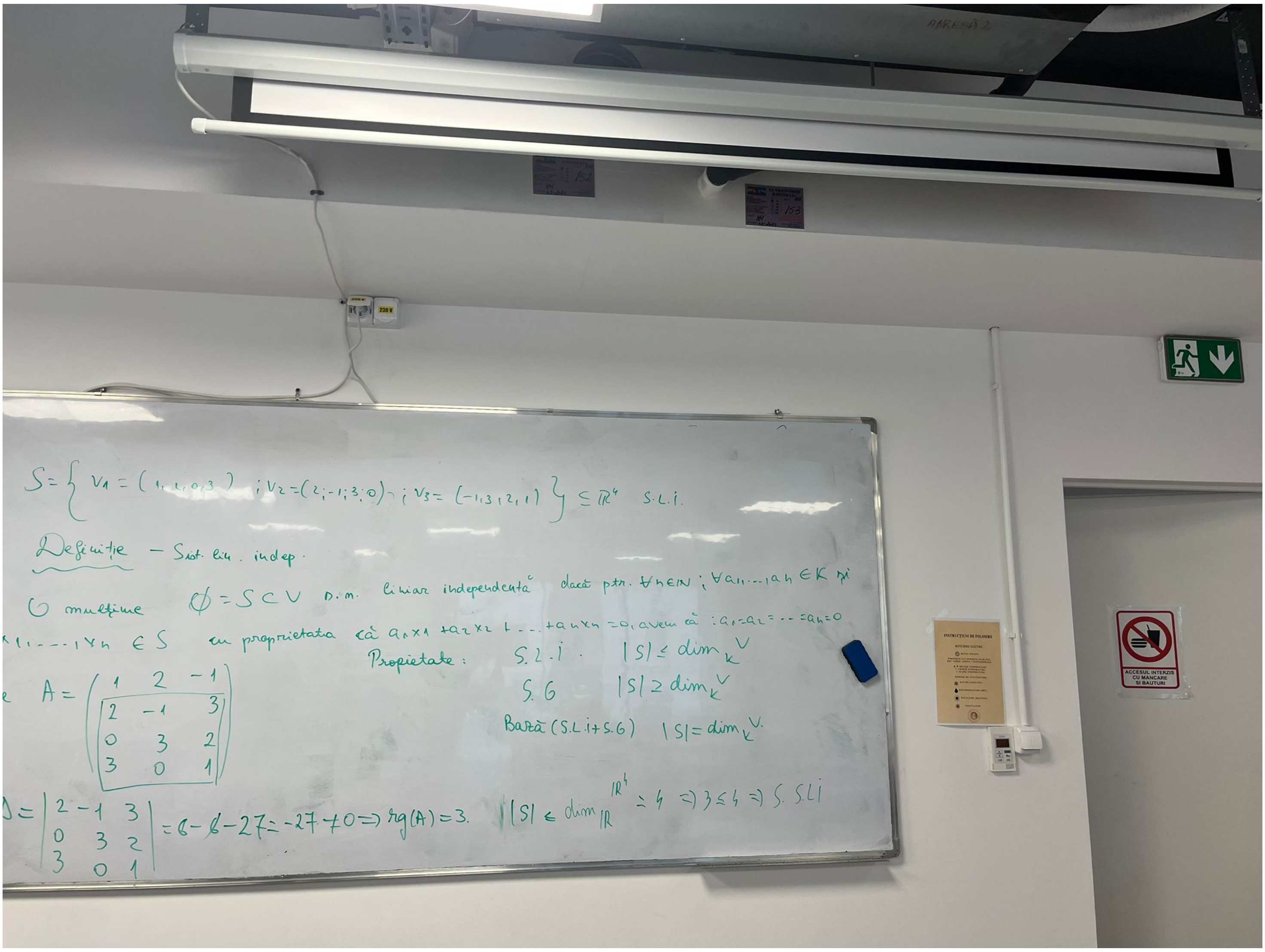
Δ = { A = (a b / c d) ∈ M2(R) | a+b+c = 1 (*) } ≤ M2(R)
U ≤ V dc 0V ∈ U
O2 = (0 0 / 0 0) = A
0+0+0 = 1
0 = 1
0V ∉ Δ => Δ ≰ M2(R)
E = {(x, y, z) ∈ C³ | x² + y² + z² = 0} ≤? C³
v = (0, 1, i) ∈ E
w = (1, 0, i) ∈ E
Verificăm dacă v+w ∈ E
=> (0, 1, i) + (1, 0, i) = (1, 1, 2i)
=> 1² + 1² + 4i² = 2−4 = −2 ≠ 0 => u+v ∉ E
=> E ≰ C

$U_1 = \{A \in \mathcal{M}_2(\mathbb{R}) \mid {}^tA = A\} \leq \mathcal{M}_2(\mathbb{R})$

Fie $A_1, A_2 \in \mathcal{M}_2(\mathbb{R})$, $\alpha, \beta \in \mathbb{R}$

Vr. $\alpha A_1 + \beta A_2 \in U_1$.

$$(\alpha A_1+\beta A_2)^T = (\alpha A_1)^T + (\beta A_2)^T = \alpha (A_1)^T + \beta (A_2)^T = \alpha A_1 + \beta A_2 \in U_1. \blacksquare$$


$S = \{ v_1 = (1,2,0,3) ; v_2 = (2;-1;3;0) ; v_3 = (-1,3,2,1) \} \subseteq \mathbb{R}^4$ S.L.i.
Definiție - Sist. lin. indep.
O mulțime $\emptyset \neq S \subset V$ s.n. liniar independentă dacă ptr. $\forall n \in \mathbb{N}$; $\forall a_1, \dots, a_n \in K$ și
$x_1, \dots, x_n \in S$ cu proprietatea că $a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$, avem că: $a_1 = a_2 = \dots = a_n = 0$
Propietate: S.L.i. $|S| \leq \dim_K V$
S.G $|S| \geq \dim_K V$
Bază (S.L.i+S.G) $|S| = \dim_K V$.
$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 0 & 3 & 2 \\ 3 & 0 & 1 \end{pmatrix}$
$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 0 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 6 - 6 - 27 = -27 \neq 0 \Rightarrow rg(A) = 3.$
$|S| \leq \dim_{\mathbb{R}} \mathbb{R}^4 = 4 \Rightarrow 3 \leq 4 \Rightarrow$ S. S.L.i
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ACCESUL INTERZIS CU MANCARE SI BAUTURI

$S = \{ v_1 = (1,2,0,3) ; v_2 = (2;-1;3;0) ; v_3 = (-1,3,2,1) \} \subseteq \mathbb{R}^4$ S.L.I.

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Fie $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ a.î. $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0_{\mathbb{R}^4} \to (0,0,0,0)$

$\alpha_1(1,2,0,3) + \alpha_2(2,-1,3,0) + \alpha_3(-1,3,2,1) = (0,0,0,0)$

$$\begin{cases} \alpha_1 + 2\alpha_2 - \alpha_3 = 0 \\ 2\alpha_1 - \alpha_2 + 3\alpha_3 = 0 \\ 3\alpha_2 + 2\alpha_3 = 0 \\ 3\alpha_1 + \alpha_3 = 0 \end{cases} \longrightarrow A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 0 & 3 & 2 \end{pmatrix}$$

S.L.I. $|S| \leq \dim_K V$

S.G $|S| \geq \dim_K V$

$|S| = \dim_K V$.

$\boxed{\text{OBS!}}$ 1) Dacă S.L.O. S are sol unică (sol nulă) atunci sistemul e sistem liniar independent

2) Dacă S.L.O S are și soluții nenule atunci avem s.l.d.

$$A^e = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & -1 & 3 & 0 \\ 0 & 3 & 2 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow[L_2-2L_1\to L_2]{L_4-3L_1\to L_4} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & -6 & 4 & 0 \end{array}\right) \xrightarrow{(-1)\cdot L_4} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 6 & -4 & 0 \end{array}\right) \xrightarrow{L_4+L_2\to L_2}$$

$$\to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 6 & -4 & 0 \end{array}\right) \xrightarrow[\substack{L_3\to L_3-3L_2 \\ L_4\to L_4-6L_2}]{L_1\to L_1-2L_2} \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -10 & 0 \end{array}\right) \xrightarrow{(-1)L_3} \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -10 & 0 \end{array}\right) \xrightarrow[\substack{L_2\to L_2-L_3 \\ L_4\to L_4+10L_3}]{L_1\to L_1+3L_3}$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} \alpha_1\cdot 1+\alpha_2\cdot 0+\alpha_3\cdot 0=0 \\ \alpha_1\cdot 0+\alpha_2\cdot 1+\alpha_3\cdot 0=0 \\ \alpha_1\cdot 0+\alpha_2\cdot 0+\alpha_3\cdot 1=0 \end{cases} \Rightarrow \begin{cases} \alpha_1=0 \\ \alpha_2=0 \\ \alpha_3=0 \end{cases} \Rightarrow S.L.I$$