ex 1 $T(n) = T(n/2) + 1$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$

$\vdots$

$T(1) = 1$

$T(n) = \log n$

T. master

$a = 1 \qquad b = 2 \qquad f(n) = 1$

$$n^{\log_b a} = n^{\log_2 1} = n^0 = 1 = f(n) \xRightarrow{\text{caz } 2}$$

$$\Rightarrow T(n) \in \Theta(n^{\log_b a} \cdot \log n)$$

Dem. că $T(n) \le c \cdot \log n$

Plecăm cu $T\left(\frac{n}{2}\right) \le c \log \frac{n}{2}$ și dem că

$T(n) \le c \cdot \log(n)$

$$T(n) = T\left(\frac{n}{2}\right) + 1 \le c \cdot \log \frac{n}{2} + 1$$

$$\le c(\log_2 n - 1) + 1 \le c \cdot \log_2 n - (c - 1) \le$$

$$\le c \cdot \log n, \ \forall c \ge 1$$

1) $T(n) = 2T(n/4) + 1$

2) $T(n) = 2T(n/4) + \sqrt{n}$

3) $T(n) = 2T(n/4) + n$

4) $T(n) = 2T(n/4) + n^2$

1) $T(n) = 2T(n/4) + 1$

$a = 2 \quad b = 4 \quad f(n) = 1$

$n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$\Rightarrow f(n) \in O(n^{\frac{1}{2} - \varepsilon}) \Rightarrow T(n) = O(\sqrt{n})$

$\cancel{0} + \varepsilon = \frac{1}{2}$

2) $T(n) = 2T(n/4) + \sqrt{n}$

$a = 2 \quad b = 4 \quad f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\frac{1}{2}} = f(n) \Rightarrow T(n) = O(n^{\frac{1}{2}} \log n) =$

$= O(\sqrt{n} \log n)$

3) $T(n) = 2T(n/4) + n$

$n^{\log_b a} = \sqrt{n}$

$a \cdot f(\frac{n}{b}) \leq c \cdot f(n), \quad c < 1$

$2 \cdot f(\frac{n}{4}) \leq c \cdot n$

$2 \cdot \frac{n}{4} \leq c \cdot n \iff$

$$\left.\begin{array}{l} \iff \frac{n}{2} \leq c \cdot n, \ A, \forall c \geq 1 \\ \Rightarrow f(n) \in \Omega(n^{\frac{1}{2}+\varepsilon}) \end{array}\right\} \Rightarrow$$

$\Rightarrow T(n) \in \Theta(n)$

4) $T(n) = 2T(\frac{n}{4}) + n^2$

$2 \cdot f(\frac{n}{4}) \leq c \cdot f(n), \ c ? 1$

$2 \cdot \frac{n^2}{16} \leq c \cdot n^2 \iff$

$$\left.\begin{array}{l} \iff \frac{n^2}{8} \leq c \cdot n^2 \ (A) \\ f(n) \in \Omega(n^{\frac{1}{2}+\varepsilon}) \end{array}\right\} \Rightarrow$$

$\Rightarrow T(n) \in \Theta(n^2)$

1) $T(n) = T(n-1) + 1$

2) $T(n) = 2T(n-1) + 1$

3) $T(n) = T(n-1) + n$

1) $T(n) = T(n-1) + 1$

$T(n-1) = T(n-2) + 1$

$\vdots$

$\vdots$

$T(n) = T(1) + n$

$$\left.\begin{array}{c|c} n & 1 \\ \vdots & \vdots \\ n-1 & \vdots \\ \vdots & \vdots \\ 1 & 1 \end{array}\right\} \text{de } n \text{ ori} \Rightarrow O(n)$$

Arătăm că $T(n) \le cn$ pt un c constant

$Tn: T(n-1) \le c(n-1)$

$T(n) = T(n-1) + 1 \le c(n-1) + 1 =$

$= cn - c + 1$

$\Rightarrow T(n) \le cn$ pt $1 - c \le 0 \Leftrightarrow c \ge 1$

2) $T(n) = 2T(n-1) + n$

$$\begin{array}{c|c} n & 1 \\ 2(n-1) & 2 \\ 4(n-2) & 4 \\ \vdots & \vdots \\ 1 & 2^{n-1} \\ \hline & 2^n \end{array} \left.\vphantom{\begin{array}{c}1\\2\\4\\ \vdots \\ 2^{n-1}\end{array}}\right\} n$$

$T(n) = O(2^n)$

Dem că $T(n) \le c \cdot 2^n - b$

$T(n-1) \le c \cdot 2^{n-1} - b$

$T(n) = 2T(n-1) + 1 \le 2(c \cdot 2^{n-1} - b) + 1 =$

$= c \cdot 2^n - 2b + 1 = (c \cdot 2^n - b) + 1 - b$

$c \cdot 2^n - b + 1 - b \le c \cdot 2^n - b$, când $b \ge 1$

3) $T(n) = T(n-1) + n$

$$\begin{array}{c|c} n & n \\ n-1 & n \\ \vdots & \vdots \\ 1 & n \end{array} \left.\vphantom{\begin{array}{c} n \\ n \\ \vdots \\ n \end{array}}\right\} \text{de } n \text{ ori} \Rightarrow O(n^2)$$

Pp că $T(n-1) \le c(n-1)^2$

$T(n) = T(n-1) + n \le c(n-1)^2 + n =$

$= cn^2 - 2cn + c + n = cn^2 - n(2c-1) + c < cn^2$

Adv $\forall c \ge 1$

$T(n) = T(\sqrt{n}) + 1$

$m = \log n \Rightarrow n = 2^m \Rightarrow T(2^m) = T(2^{m/2}) + 1$

$$\begin{array}{c|c} \sqrt{n} & 1 \\ \sqrt[4]{n} & 1 \\ \sqrt[8]{n} & \vdots \\ \vdots & \vdots \\ 1 & 1 \end{array}$$

$\underbrace{\qquad}_{\log_2 n}$

$$T(2^m) = S(m)$$

$$S(m) = S(m/2) + 1 = O(\log m) \in \Theta(n$$

$$T(2^m) = T(n) = O(\log m) = O(\log \log n)$$

$*$ $T(n) = 2T(\sqrt{n}) + 1$

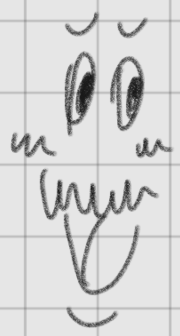
$$m = \log n$$

$$T(2^m) = 2\,T(2^{m/2}) + 1$$

$$T(2^m) = S(m)$$

$$S(m) = 2S(m/2) + 1 = O(\log^2 m)$$

$$T(n) = T(n/3) + T(2n/3) + n$$



$$T(n) = O(n \log n)$$

Dem cā $T(n) \le c \cdot n \log n - b$

Pp cā $T(n/3) + T(2n/3) + n \le \frac{c \cdot n \log n}{3} - b + \frac{2cn\log n}{3} - b + n$

$= cn\log n - b + n$

Pp. că $T(n/3) \leq c \cdot \frac{n}{3} \log_2 \frac{n}{3}$

$T(2n/3) \leq c \frac{2n}{3} \cdot \log_2 \frac{2n}{3}$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n \leq$

$\leq c \frac{n}{3} \log \frac{n}{3} + c \cdot \frac{2n}{3} \log \frac{2n}{3} + n \leq$

$\leq c \frac{n}{3} \log n -$

Andrei

Pp $T(x) \leq c x \log x \quad \forall x < n$

$T(n) \leq cn \log n$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n \leq$

$\leq \frac{cn}{3} \log \frac{n}{3} + c \frac{2n}{3} \log \frac{2n}{3} + n =$

$= \frac{cn}{3} \log n - \frac{cn}{3} \log 3 + \frac{2nc}{3} \log 2n -$

$- \frac{c2n}{3} \log 3 + n =$

$= c\frac{n}{3} \log n + c \frac{2n}{3} \log n + \frac{2cn}{3} - cn \log 3 + n =$

$= cn \log n + n\left(\frac{2c}{3} - c \log 3 + 1\right) \leq cn \log n$

i was close!

pt $\frac{2c}{3} - c \log 3 + 1 \leq 0 \Rightarrow c\left(\frac{2}{3} - \log 3\right) \leq -1 \Rightarrow$

$\Rightarrow c \geq \frac{1}{\log 3 - \frac{2}{3}}$