Seminar 10

1.

a) Stud cont lui f

b) Det $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$

c) Stud. dif.

unde:

i) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = \begin{cases} \frac{y^3}{x^4+y^2}; & (x,y) \neq 0 \\ 0; & (x,y) = (0,0) \end{cases}$

Sol

a) Vezi Seminar 9

b) Fie $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{0\cdot(x^4+y^2) - y^3 \cdot 4x^3}{(x^4+y^2)^2}$$

$$= \frac{-4x^3y^3}{(x^4+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{3x^4y^2+y^4}{(x^4+y^2)^2}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0} \frac{f((0,0)+t\cdot e_1) - f(0,0)}{t}$$

$$\lim_{t\to 0} \frac{f(t,0) - \overset{\to 0}{f(0,0)}}{t} = \lim_{t\to 0} \frac{\frac{0^3}{t^4+0^2}}{t} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0} \frac{f((0,0)+t(0,1)) - \overset{\to 0}{f(0,0)}}{t}$$

$$\lim_{t\to 0} \frac{f(0,t)}{t} = \lim_{t\to 0} \frac{\frac{t^3}{0^4+t^2}}{t} = 1$$

c) $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ cont pe $\mathbb{R}^2 \setminus \{(0,0)\}$ | $\xrightarrow{\text{Crit}}$ f dif pe

$\mathbb{R}^2 \setminus \{(0,0)\}$ mulț deschisă | de dif $\mathbb{R}^2 \setminus \{(0,0)\}$

Studiem diferențiala în (0,0)

Dacă f este dif în (0,0), atunci

$df(0,0)(\cancel{\;\;}) : \mathbb{R}^2 \to \mathbb{R} \qquad df(0,0)(u,v)$

$$= t\left[\left(\underbrace{\frac{\partial f}{\partial x}(0,0)}_{=0} \quad \underbrace{\frac{\partial f}{\partial y}(0,0)}_{=1}\right)\begin{pmatrix} u \\ v \end{pmatrix}\right]$$

$$= 0 \cdot u + 1 \cdot v = v$$

$$\lim_{(x,y)\to(0,0)} \frac{\overset{\to 0}{f(x,y)-f(0,0)} - df(0,0)((x,y)-(0,0))}{\|(x,y)-(0,0)\|}$$

$$\lim_{(x,y)\to(0,0)} \frac{\frac{y^3}{x^4+y^2} - 0 - y}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{\cancel{y^3} - x^4 y - \cancel{y^3}}{(x^4+y^2)\sqrt{x^2+y^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{-x^4 y}{(x^4+y^2)\sqrt{x^2+y^2}}$$

$$(x^2 - y)^2 \geq 0 \iff x^4 - 2x^2 y + y^2 \geq 0 \qquad \frac{|x|}{\sqrt{x^2+y^2}} \leq 1$$

$$x^4 + y^2 \geq 2x^2 y \qquad (\text{Expl } \sqrt{x^2+y^2} \geq \sqrt{x^2} = |x|)$$

$$\frac{1}{x^4+y^2} \leq \frac{1}{2x^2 y}$$

Fie $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

$$\left|\frac{-x^4 y}{(x^4+y^2)\sqrt{x^2+y^2}} - 0\right| = \frac{x^4 |y|}{(x^4+y^2)\sqrt{x^2+y^2}} = \underbrace{\frac{x^2|y|}{x^4+y^2}}_{\leq \frac{1}{2}} \cdot \underbrace{\frac{|x|}{\sqrt{x^2+y^2}}}_{\leq 1} \cdot |x|$$

$$\leq \frac{1}{2} \cdot \frac{x^2}{\sqrt{x^2+y^2}}$$

$$(\text{Expl } \frac{x^4+y^2}{2} \geq \sqrt{x^4 y^2} = |x^2 y| = x^2 |y|$$

$$\Rightarrow \frac{1}{2} \geq \frac{x^2|y|}{x^4+y^2}$$

Deci, $\lim\limits_{(x,y)\to(0,0)} \dfrac{-x^4 y}{(x^4+y^2)\sqrt{x^2+y^2}} = 0 \Rightarrow f$ dif în $(0,0)$

ii) $f:\mathbb{R}^2\to\mathbb{R}$ $\quad f(x,y)=\begin{cases} \dfrac{x^7y^8}{\sqrt{x^{28}+y^{28}}} & (x,y)\neq(0,0) \\ 0 & ;\ (x,y)=(0,0)\end{cases}$

a) f cont pe $\mathbb{R}^2\setminus\{(0,0)\}$ (op cu fct elem)
Stud. cont lui f în $(0,0)$

$$|f(x,y)-f(0,0)| = \left|\frac{x^7y^8}{\sqrt{x^{28}+y^{28}}}-0\right|$$

$$= \left|\frac{x^7y^8}{\sqrt{x^{28}+y^{28}}}\right| = \frac{|x|^7|y|^8}{\sqrt{x^{28}+y^{28}}} = |y|\cdot \frac{|x|^7|y|^7}{\sqrt{x^{28}+y^{28}}} \le \frac{1}{\sqrt2}$$

Expl.

$$\frac{x^{28}+y^{28}}{2} \ge \sqrt{x^{28}\cdot y^{28}} = x^{14}\cdot y^{14} \Big/ \sqrt{\ }$$

$$\frac{\sqrt{x^{28}+y^{28}}}{\sqrt2} \ge |x|^7|y|^7$$

$$\frac{1}{\sqrt2} \ge \frac{|x|^7|y|^7}{\sqrt{x^{28}+y^{28}}}$$

$$\le \frac{|y|}{\sqrt2} \xrightarrow[(x,y)\to(0,0)]{} 0 \Rightarrow f \text{ cont în } (0,0)$$

b) Fie $(x,y)\in\mathbb{R}^2\setminus\{(0,0)\}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{(x^7y^8)'_x\sqrt{x^{28}+y^{28}} - (x^7y^8)\left(\sqrt{x^{28}+y^{28}}\right)'_x}{x^{28}+y^{28}}$$

$$= \frac{7x^6y^8\sqrt{x^{28}+y^{28}} - x^7y^8\cdot\dfrac{1}{2\sqrt{x^{28}+y^{28}}}\cdot 28x^{27}}{x^{28}+y^{28}}$$

$$= \frac{7x^6y^8(x^{28}+y^{28}) - x^7y^8 \cdot 14x^{27}}{(x^{28}+y^{28})\sqrt{x^{28}+y^{28}}}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{(x^7y^8)'_y\sqrt{x^{28}+y^{28}} - (x^7y^8)(\sqrt{x^{28}+y^{28}})'_y}{x^{28}+y^{28}}$$

$$= \frac{8x^7y^7\sqrt{x^{28}+y^{28}} - (x^7y^8)\frac{1}{2\sqrt{x^{28}+y^{28}}}\cdot \overset{14}{\cancel{28}}y^{27}}{x^{28}+y^{28}}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0}\frac{f(t,0)-\overset{0}{f(0,0)}}{t}$$

$$= \lim_{t\to 0}\frac{t^7\cdot 0^8}{\sqrt{t^{28}+0^{28}}\cdot t} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0}\frac{f(0,t)-\overset{0}{f(0,0)}}{t}$$

$$\lim_{t\to 0}\frac{0^7\cdot t^8}{t\sqrt{0^{28}+t^{28}}} = 0$$

c) $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ cont pe $\mathbb{R}^2\setminus\{(0,0)\}$ $\Big|$ $\overset{\text{crit}}{\Rightarrow}$ de dif f dif pe $\mathbb{R}^2\setminus\{(0,0)\}$

Stud dif lui f în $(0;0)$

Dacă f ar fi dif în $(0,0)$, atunci

$\text{dif}(0,0) : \mathbb{R}^2 \to \mathbb{R}$

$$\text{dif}(0,0)(u,v) = \left[\left(\underset{\substack{\parallel\\0}}{\frac{\partial f}{\partial x}(0,0)}\ \ \underset{\substack{\parallel\\0}}{\frac{\partial f}{\partial y}(0,0)}\right)\begin{pmatrix}u\\v\end{pmatrix}\right]$$

$$= 0\cdot u + 0\cdot v = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - \overset{=0}{f(0,0)} - df(0,0)((x,y) - \overset{=0}{(0,0)})}{\|(x,y) - (0,0)\|}$$

$$\lim_{(x,y)\to(0,0)} \frac{\frac{x^7y^8}{\sqrt{x^{28}+y^{28}}} - 0 - 0}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{x^7y^8}{\sqrt{x^{28}+y^{28}}\cdot\sqrt{x^2+y^2}}$$

Alegem $(x_n, y_n) = \left(\frac{1}{n}, \frac{1}{n}\right)$ $\lim_{n\to\infty}(x_n, y_n) = (0,0)$

Avem $\lim_{\substack{\cancel{(x,y)\to(0,0)} \\ n\to\infty}} \frac{x_n^7 y_n^8}{\sqrt{x_n^{28}+y_n^{28}}\sqrt{x_n^2+y_n^2}} =$

$$\lim_{n\to\infty} \frac{\frac{1}{n^{15}}}{\sqrt{2\cdot\frac{1}{n^{28}}} \cdot \sqrt{2\cdot\frac{1}{n^2}}} = \lim_{n\to\infty} \frac{\frac{1}{n^{15}}}{\sqrt{2}\cdot\frac{1}{n^{14}}\cdot\sqrt{2}\cdot\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n^{15}}}{2\cdot\frac{1}{n^{15}}} = \frac{1}{2} \neq 0$$

Deci, f nu e dif în $(0,0)$

2. Fie $f: \mathbb{R}^2 \to \mathbb{R}$ $f(x,y) = \begin{cases} xy \sin\frac{1}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

a) Ar că f e cont

b) ~~Det~~ $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ și arăt că sunt cont în $\mathbb{R}^2 \setminus \{(0,0)\}$

c) Ar că f e dif

a) f e cont pe $\mathbb{R}^2 \setminus \{(0,0)\}$ (op cu fct elem)

Fie $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

$$|f(x,y)-f(0,0)| = \left| xy \cdot \sin\frac{1}{x^2+y^2} - 0\right| = |xy| \underbrace{\left|\sin\frac{1}{x^2+y^2}\right|}_{\le 1}$$

$$\le |xy| \xrightarrow[(x,y)\to(0,0)]{} 0$$

Deci, f cont în $(0,0)$

b)

$$\frac{\partial f}{\partial x}(x,y) = y \cdot \sin\frac{1}{x^2+y^2} + xy\left(\cos\frac{1}{x^2+y^2}\right)\cdot\frac{-2x}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = x \cdot \sin\frac{1}{x^2+y^2} + xy \cdot \cos\frac{1}{x^2+y^2}\cdot\frac{-2y}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0}\frac{f(t,0)-\overset{\to 0}{f(0,0)}}{t}$$

$$= \lim_{t\to 0}\frac{f(t,0)}{t} = \lim_{t\to 0}\frac{\cancel{t}\cdot 0\cdot \sin\frac{1}{t^2+0^2}}{\cancel{t}} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0}\frac{f(0,t)}{t} = \lim_{t\to 0}\frac{0\cdot t\cdot \sin\frac{1}{0^2+t^2}}{t}$$

$$=0$$

Avem

$$\frac{\partial f}{\partial x} = \begin{cases} y\cdot \sin\frac{1}{x^2+y^2} - 2y\overbrace{\left(\cos\frac{1}{x^2+y^2}\right)}^{\substack{\cos\\ \text{vreau } 2k\pi}}\cdot\frac{x^2y}{(x^2+y^2)^2} & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

$\frac{\partial f}{\partial x}$ cont pe $\mathbb{R}^2\setminus\{(0,0)\}$ – op. cu fct. elem.

Ar că $\frac{\partial f}{\partial x}$ nu este cont în $(0,0)$

Alegem $(x_n, y_n) = \left(\frac{1}{2\sqrt{n\pi}}, \frac{1}{2\sqrt{n\pi}}\right)$ $\forall n \in \mathbb{N}^*$

Avem $\lim\limits_{n\to\infty} (x_n, y_n) = (0,0)$

$$\lim_{n\to\infty} \frac{\partial f}{\partial x}(x_n, y_n) = \lim_{n\to\infty} x_n \cdot \sin\frac{1}{x_n^2+y_n^2} - \cos\frac{1}{x_n^2+y_n^2} \cdot \frac{2\, x_n^2\, y_n}{(x_n^2+y_n^2)^2}$$

$$= \lim_{n\to\infty} \frac{1}{2\sqrt{n\pi}} \cdot \sin\frac{1}{\left(\frac{1}{2\sqrt{n\pi}}\right)^2+\left(\frac{1}{2\sqrt{n\pi}}\right)^2} - \cos\frac{1}{\left(\frac{1}{2\sqrt{n\pi}}\right)^2+\left(\frac{1}{2\sqrt{n\pi}}\right)^2} \cdot \frac{2\left(\frac{1}{2\sqrt{n\pi}}\right)^2 \cdot \frac{1}{2\sqrt{n\pi}}}{\left(\left(\frac{1}{2\sqrt{n\pi}}\right)^2+\left(\frac{1}{2\sqrt{n\pi}}\right)^2\right)^2}$$

$$= 0 - \lim_{n\to\infty}\left(\cos 2n\pi \cdot \frac{\cancel{2}\left(\frac{1}{2\sqrt{n\pi}}\right)^3}{2^{\cancel{4}}\left(\frac{1}{2\sqrt{n\pi}}\right)^4}\right)$$

$$= -\lim_{n\to\infty} 1 \cdot \sqrt{n\pi} = -\infty \neq 0$$

$\Rightarrow \frac{\partial f}{\partial x}$ nu e cont în $(0,0)$

Analog se arată că $\frac{\partial f}{\partial y}$ este cont pe $\mathbb{R}^2 \setminus \{(0,0)\}$ (op cu fct elem) și că $\frac{\partial f}{\partial y}$ nu e cont în $(0,0)$ (folosind (x_n, y_n) ca mai sus)

c) $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ cont pe $\mathbb{R}^2 \setminus \{(0,0)\}$ $\Big|$ $\overset{\text{aif}}{\Rightarrow}$ f dif

$\mathbb{R}^2 \setminus \{(0,0)\}$ deschisă

f dif pe $\mathbb{R}^2 \setminus \{(0,0)\}$

Studiem dif în $(0,0)$

Dacă f ar fi dif în $(0,0)$, atunci

$df(0,0): \mathbb{R}^2 \to \mathbb{R}$

$$df(0,0)(u,v) = {}^t\left[\left(\underbrace{\frac{\partial f}{\partial x}(0,0)}_{0}\ \underbrace{\frac{\partial f}{\partial y}(0,0)}_{0}\right)\begin{pmatrix} u \\ v \end{pmatrix}\right]$$

$$= 0 \cdot u + 0 \cdot v = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - f(0,0) - df(0,0)((x,y)-(0,0))}{\|(x,y)-(0,0)\|}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy \sin\frac{1}{x^2+y^2} - 0 - 0}{\sqrt{x^2+y^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} \cdot \sin\frac{1}{x^2+y^2}$$

Fie $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

$$\left|\frac{xy}{\sqrt{x^2+y^2}} \cdot \sin\frac{1}{x^2+y^2} - 0\right| = \frac{|xy|}{\sqrt{x^2+y^2}} \cdot \underbrace{\left|\sin\frac{1}{x^2+y^2}\right|}_{\leq 1}$$

$$\leq \frac{|xy|}{\sqrt{x^2+y^2}} = |x| \cdot \frac{|y|}{\sqrt{x^2+y^2}}$$

≤ 1 (Expl: $\sqrt{x^2+y^2} \geq \sqrt{y^2} = |y|$

$\Rightarrow 1 \geq \frac{|y|}{\sqrt{x^2+y^2}}$)

Deci, $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} \sin \frac{1}{x^2+y^2} = 0$

Așadar, g e dif în $(0,0)$.

3. Fie $\cancel{f}$ $\varphi: \mathbb{R}^2 \to \mathbb{R}$ și $f: \mathbb{R}^3 \to \mathbb{R}$

Arătați că f e dif și $xz \frac{\partial f}{\partial x}(x,y,z) - yz \frac{\partial f}{\partial y}(x,y,z) + (x^2-y^2) \frac{\partial f}{\partial z}(x,y,z) = 0$

$\forall (x,y,z) \in \mathbb{R}^3$

Sol

Fie $g: \mathbb{R}^3 \to \mathbb{R}^2$ $\quad g(x,y,z) = (xy,\ x^2+y^2-z^2)$

$$f = \varphi \circ g$$

Fie $u, v: \mathbb{R}^3 \to \mathbb{R}$ $\quad u(x,y,z) = xy$

$\quad v(x,y,z) = x^2+y^2-z^2$

$\frac{\partial g}{\partial x}(x,y,z) = (y, 2x)$

$\frac{\partial g}{\partial y}(x,y,z) = (x, 2y)$

$\frac{\partial g}{\partial z}(x,y,z) = (0, -2z)$

$\left.\begin{array}{l} \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \text{ cont pe } \mathbb{R}^3 \\ \mathbb{R}^3 \text{ deschisă} \end{array}\right| \Rightarrow g \text{ dif pe } \mathbb{R}^3$

$$\begin{tikzcd} \underbrace{\mathbb{R}^3}_{(x,y,z)} \arrow[r, "g=(u,v)"] \arrow[rr, bend right, "f=\varphi\circ g"'] & \underbrace{\mathbb{R}^2}_{(u,v)} \arrow[r, "\varphi"] & \mathbb{R} \end{tikzcd}$$

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{\partial (f\circ g)}{\partial x}(x,y,z) = \frac{\partial f}{\partial u}(g(x,y,z)) \cdot \frac{\partial u}{\partial x}(x,y,z) + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot \frac{\partial v}{\partial x}(x,y,z)$$

$$= \frac{\partial f}{\partial u}(g(x,y,z))\, y + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot 2x$$

$$\frac{\partial f}{\partial y}(x,y,z) = \frac{\partial (f\circ g)}{\partial y}(x,y,z)$$

$$= \frac{\partial f}{\partial u}(g(x,y,z)) \cdot \frac{\partial u}{\partial y}(x,y,z) + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot \frac{\partial v}{\partial y}(x,y,z) = \frac{\partial f}{\partial u}(g(x,y,z)) \cdot x + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot 2y$$

$$\frac{\partial f}{\partial z}(x,y,z) = \frac{\partial f}{\partial u}(g(x,y,z)) \cdot \frac{\partial u}{\partial z}(x,y,z) + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot \frac{\partial v}{\partial z}(x,y,z)$$

$$= \cancel{\partial f}\ \frac{\partial f}{\partial u}(g(x,y,z)) \cdot 0 - 2\,\cancel{\partial z}\, z\, \frac{\partial f}{\partial v}(g(x,y,z))$$

$$xz \frac{\partial f}{\partial x}(x,y,z) - yz \frac{\partial f}{\partial y}(x,y,z) + (x^2-y^2)\frac{\partial f}{\partial z}(x,y,z)$$

$$= xz \cdot \left(\frac{\partial f}{\partial u}(g(x,y,z))\, y + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot 2x\right) - yz \left(\frac{\partial f}{\partial u}(g(x,y,z)) \cdot x + \frac{\partial f}{\partial v}(g(x,y,z)) \cdot 2y\right) +$$

$(x^2-y^2)\left(0 - \frac{\partial f}{\partial v}\left(g(x,y,z)\right)\right) \cdot 2z) = \dots = 0$

4\. Det. pct de extrem local ale fct de mai jos și precizați natura lor

a) $f: \mathbb{R}^2 \to \mathbb{R} \quad f(x,y) = 4xy - x^4 - y^4$

$\mathbb{R}^2$ - deschisă, f cont

$$\frac{\partial f}{\partial x} = 4y - 4x^3$$

$$\frac{\partial f}{\partial y} = 4x - 4y^3$$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ cont pe $\mathbb{R}^2$
$\mathbb{R}^2$ mult. deschisă
$\Rightarrow$ f de 2 ori dif

$$\left.\begin{array}{l} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{array}\right| \Rightarrow \begin{cases} 4y - 4x^3 = 0 \\ 4x - 4y^3 = 0 \end{cases}$$

$$4y - 4x^3 + 4x - 4y^3 = 0$$

$$4(x+y) - 4(x^3+y^3) = 0$$

$$4\left((x+y) - (x^3+y^3)\right) = 0$$

$$\begin{cases} y - x^3 = 0 \Rightarrow y - y^9 = 0 \Rightarrow y(1-y^8) = 0 \\ x - y^3 = 0 \Rightarrow x = y^3 \end{cases}$$

Deci $x=0, y=0$ sau $y=1, x=1$ sau $y=-1, x=-1$

Punctele critice ale lui f sunt (0,0), (1,1) (-1,-1)

$$\frac{\partial f^2}{\partial^2 x} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial f}{\partial x}(4y-4x^3) = -12x^2$$

$$\frac{\partial f^2}{\partial^2 y^2} = -12y^2$$

Observăm că f este de clasă C^2

$$\frac{\partial f^2}{\partial x \partial y} = \frac{\partial f^2}{\partial y \partial x} = 4$$

$$H_f(a) = \begin{pmatrix} \frac{\partial f^2}{\partial^2 x}(a) & \frac{\partial f^2}{\partial x \partial y}(a) \\ \frac{\partial f^2}{\partial y \partial x}(a) & \frac{\partial f^2}{\partial y \partial y}(a) \end{pmatrix}$$

$$H_f(x,y) = \begin{pmatrix} -12x^2 & 4 \\ \cancel{-12y^2}\ 4 & \cancel{4}\ -12x^2 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$$

$\Delta_1 = 0$

$\Delta_2 = -16$

$\Rightarrow$ (0,0) nu e pct de extrem

$$H_f(1,1) = \begin{pmatrix} -12 & 4 \\ 4 & -12 \end{pmatrix}$$

$\Delta_1 = -12 < 0$

$$\Delta_2 = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix} = 144 - 16 = 128 > 0$$

$\Rightarrow$ (1,1) e pct de max local al lui f

$$H_f(-1,-1) = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix}$$

$\Delta_1 = -12 < 0$

$$\Delta_2 = \begin{vmatrix} -12 & 4 \\ 4 & -12 \end{vmatrix} = 144 - 16 = 128 > 0$$

$\Rightarrow (-1,-1)$ pct de max local al lui f

b) $f: \mathbb{R}^2 \to \mathbb{R} \quad f(x,y) = x^3 + 8y^3 - 2xy$

Rez. voi!

c) $f: (0,\infty)^3 \to \mathbb{R} \quad f(x,y,z) = \frac{1}{x} + \frac{x}{y} + \frac{y}{z} +$

Sol

$(0,\infty)^3$ deschisă

f cont

$$\frac{\partial f}{\partial x}(x,y,z) = -\frac{1}{x^2} + \frac{1}{y}$$

$$\frac{\partial f}{\partial y} = x \cdot -\frac{1}{y^2} + \frac{1}{z}$$

$$\frac{\partial f}{\partial z} = y \cdot -\frac{1}{z^2} + 1$$

$\forall (x,y,z) \in (0;\infty)^3$

$\left.\begin{array}{l} \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \text{ cont pe } (0,\infty)^3 \\ (0;\infty)^3 \text{ desch} \end{array}\right| \Rightarrow$

f dif pe $(0;\infty)^3$

$$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \\ \frac{\partial f}{\partial z} = 0 \end{cases} \Leftrightarrow \begin{cases} -\frac{1}{x^2} + \frac{1}{y} = 0 \\ -\frac{x}{y^2} + \frac{1}{z} = 0 \\ -\frac{y}{z^2} + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} y = x^2 \\ -\frac{x}{y^2} + \frac{1}{z} = 0 \\ y = z^2 \end{cases}$$

$$\begin{cases} y = x^2 \\ y = z^2 \end{cases} \Rightarrow x = z$$

$$(x, y, z) \in (0, \infty)^3$$

$$-\frac{x}{x^4} + \frac{1}{x} = 0 \Leftrightarrow -\frac{1}{x^3} + \frac{1}{x} = 0$$

$$\frac{x^2 - 1}{x^3} = 0 \Rightarrow x = 1$$

$$x \in (0, \infty)$$

$$z = 1, \quad y = 1$$

$(1, 1, 1)$ pct critic

(și unic!)

d) $f: \mathbb{R}^3 \to \mathbb{R} \quad f(x, y, z) = x^2 + y^2 + z^2 - xy + x - 2z$