Tutoriat 9 CDI

Funcțiile Beta și Gamma

$\Gamma : (0,\infty) \to \mathbb{R}$, $\Gamma(p) = \int_{0+0}^{+\infty} x^{p-1} e^{-x}\, dx, \forall\, p \in (0,\infty)$

$B : (0,\infty) \times (0,\infty)$, $B(p,q) = \int_{0+0}^{1-0} x^{p-1} (1-x)^{q-1}\, dx, \quad \forall\, p \in (0,\infty)$

$$B(p,q) = 2\int_{0+0}^{\frac{\pi}{2}-0} (\sin x)^{2p-1} (\cos x)^{2q-1}\, dx$$

$$B(p,q) = \int_{0+0}^{+\infty} \frac{x^{p-1}}{(1+x)^{p+q}}\, dx$$

$\Gamma(1) = 1$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma(p+1) = p\,\Gamma(p), \forall\, p > 0$

$\Gamma(n+1) = n!, \forall\, n \in \mathbb{N}$

$\Gamma(p) \cdot \Gamma(1-p) = \dfrac{\pi}{\sin(p\pi)}, \forall\, p \in (0,1)$

$\Gamma(p) \cdot \Gamma\left(p + \frac{1}{2}\right) = \dfrac{\sqrt{\pi}}{2^{2p-1}} \cdot \Gamma(2p), \forall\, p > 0$

$B(p,q) = \dfrac{\Gamma(p) \cdot \Gamma(q)}{\Gamma(p+q)}, \forall\, p, q \in (0,\infty)$

$B(p,q) = B(q,p), \forall\, p,q \in (0,\infty)$

$\Gamma \to$ generalizare a noțiunii de factorial

(+) $\Gamma(2p) = \pi^{-\frac{1}{2}}\, 2^{2p-1}\, \Gamma(p)\, \Gamma\left(p+\frac{1}{2}\right)$

$\lim\limits_{\substack{x \to 0 \\ x > 0}} x\, \Gamma(x) = 1$ (! Γ nu e mărginită)

$\lim\limits_{x \to \infty} \dfrac{\Gamma(x+a)}{x^a\, \Gamma(x)} = 1, \forall\, a \in (0,\infty)$

$\Gamma <$ C^∞ / convexă