$$\int_0^{\frac{\pi}{2}} \sqrt[3]{\sin^{11}x\cos^7 x}\,dx = \int_0^{\frac{\pi}{2}} \sin^{\frac{11}{3}}x\cos^{\frac{7}{3}}x\,dx = B\left(\frac{7}{3},\frac{5}{3}\right) = \frac{\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{5}{3}\right)}{\Gamma\left(\frac{7}{3}+\frac{5}{3}\right)} = \frac{\frac{4}{3}\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\frac{2}{3}\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{7+5}{3}\right)} =$$

$$2p-1=\frac{11}{3} \Leftrightarrow 2p=\frac{14}{3} \Leftrightarrow p=\frac{7}{3}$$

$$2q-1=\frac{7}{3} \Leftrightarrow 2q=\frac{10}{3} \Leftrightarrow q=\frac{5}{3}$$

$$= \frac{8\Gamma\left(\frac{1}{3}\right)\Gamma\left(1-\frac{1}{3}\right)}{27\Gamma(4)} = \frac{8\frac{\pi}{\sin\left(\frac{\pi}{3}\right)}}{27*3!} = \frac{4\frac{\sqrt{3}\pi}{2}}{81} = \frac{2\sqrt{3}\pi}{81}$$

$$\int_0^1 (1-x)^3\sqrt{x^2(1-x)}\,dx = \int_0^1 (1-x)^3 x(1-x)^{\frac{1}{2}}\,dx = \int_0^1 (1-x)^{3+\frac{1}{2}}x\,dx = B\left(4+\frac{1}{2},2\right) =$$

$$p-1=3+\frac{1}{2} \Leftrightarrow p=4+\frac{1}{2}$$

$$q-1=1 \Leftrightarrow q=2$$

$$= \frac{\Gamma\left(4+\frac{1}{2}\right)\Gamma(2)}{\Gamma\left(4+\frac{1}{2}+2\right)} = \frac{\left(3+\frac{1}{2}\right)\left(2+\frac{1}{2}\right)\Gamma\left(2+\frac{1}{2}\right)\Gamma(2)}{\Gamma\left(6+\frac{1}{2}\right)} = \frac{\left(3+\frac{1}{2}\right)\left(2+\frac{1}{2}\right)\frac{\sqrt{\pi}}{2^{2*2-1}}\Gamma(2*2)}{\left(5+\frac{1}{2}\right)\left(4+\frac{1}{2}\right)\left(3+\frac{1}{2}\right)\left(2+\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\frac{1}{2}\sqrt{\pi}} =$$

$$= \frac{2*3!}{(10+1)(8+1)(2+1)} = \frac{4}{99}$$

$$\int_0^\infty \frac{\sqrt{x^5}}{(1+x^2)^3}\,dx =$$

$$x^2=t \Leftrightarrow x=\sqrt{t} \Rightarrow dx=(\sqrt{t})'dt \Rightarrow dx=\frac{1}{2\sqrt{t}}dt$$

$$x=0 \Rightarrow t=0$$

$$x=1 \Rightarrow t=1$$

$$= \int_0^1 \frac{\sqrt[4]{t^5}}{(1+t)^3}\frac{1}{2\sqrt{t}}\,dt = \int_0^1 \frac{t^{\frac{5}{4}}}{(1+t)^3}t^{-\frac{1}{2}}\frac{1}{2}\,dt = \frac{1}{2}\int_0^1 \frac{t^{\frac{3}{4}}}{(1+t)^3}\,dt =$$

$$p-1=\frac{3}{4} \Leftrightarrow p=\frac{7}{4}$$

$$p+q=3 \Leftrightarrow q=3-\frac{7}{4} \Leftrightarrow q=\frac{5}{4}$$

$$= B\left(\frac{7}{4},\frac{5}{4}\right) = \frac{\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{5}{4}\right)}{\Gamma(3)} = \frac{\frac{3}{4}\Gamma\left(\frac{3}{4}\right)\frac{1}{4}\Gamma\left(\frac{1}{4}\right)}{2} = \frac{5\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)}{32} = \frac{5\frac{\pi}{\sin\left(\frac{\pi}{4}\right)}}{32} = \frac{5\frac{\pi}{\frac{1}{2}}}{32} = \frac{5\pi}{16}$$