3. $\bar{A} = \left(\begin{array}{cccc|c} 1 & 1 & m & -1 & 0 \\ 2 & 1 & -1 & 1 & 0 \\ 3 & -1 & -1 & -1 & 0 \\ m & -2 & 0 & -2 & 0 \end{array}\right)$

Sisteme Spații vectoriale SLI SLD

SG Baze

$m = ?$ a.i. SLO are și soluții nenule

SCN $\Leftrightarrow \Delta = \det A = 0 \Leftrightarrow 4(m-1) = 0 \Leftrightarrow m = 1$

$$\det A = \begin{vmatrix} 1 & 1 & m & -1 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -1 \\ m & -2 & 0 & -2 \end{vmatrix} \overset{C_4 - C_2}{=} \begin{vmatrix} 1 & 1 & m & -2 \\ 2 & 1 & -1 & 0 \\ 3 & -1 & -1 & 0 \\ m & -2 & 0 & 0 \end{vmatrix} =$$

$$= (-2)\cdot(-1)^{1+4} \begin{vmatrix} 2 & 1 & -1 \\ 3 & -1 & -1 \\ m & -2 & 0 \end{vmatrix} \overset{L_2 - L_1}{=} 2 \begin{vmatrix} 2 & 1 & -1 \\ 1 & -2 & 0 \\ m & -2 & 0 \end{vmatrix} = 2\cdot(-1)(-1)^{2+1} \begin{vmatrix} 1 & -2 \\ m & -2 \end{vmatrix} = -4(1-m)$$

4. $\begin{cases} 3x+2y+5z+4t=-1 \\ 2x+y+3z+3t=0 \\ x+2y+3z=-3 \end{cases}$

Sisteme Spații vector

Să se rezolve utilizând met. Gauss-Jordan

$$\bar{A} = (A\,|\,B) = \left(\begin{array}{cccc|c} 3 & 2 & 5 & 4 & -1 \\ 2 & 1 & 3 & 3 & 0 \\ 1 & 2 & 3 & 0 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 0 & -3 \\ 2 & 1 & 3 & 3 & 0 \\ 3 & 2 & 5 & 4 & -1 \end{array}\right)$$

$L_2' = L_2 - 2\cdot L_1$

$L_3' = L_3 - 3L_1$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & -3 & -3 & 3 & 6 \\ 0 & -4 & -4 & 4 & 8 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & -1 & -2 \end{array}\right) \overset{L_3' = L_3 - L_2}{\sim} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & 1 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Sisteme Spații vectoriale

4. $S: \begin{cases} 3x + 2y + 5z + 4t = -1 \\ 2x + y + 3z + 3t = 0 \\ x + 2y + 3z = -3 \end{cases}$

Să se rezolve utilizând met. Gauss-Jordan

$$\bar{A} = (A|B) = \left(\begin{array}{cccc|c} 3 & 2 & 5 & 4 & -1 \\ 2 & 1 & 3 & 3 & 0 \\ 1 & 2 & 3 & 0 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 0 & -3 \\ 2 & 1 & 3 & 3 & 0 \\ 3 & 2 & 5 & 4 & -1 \end{array}\right)$$

$L_2' = L_2 - 2 \cdot L_1$

$L_3' = L_3 - 3L_1$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & -3 & -3 & 3 & 6 \\ 0 & -4 & -4 & 4 & 8 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & -1 & -2 \end{array}\right) \underset{L_3 - L_2}{\sim} \left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 0 & -3 \\ 0 & \boxed{1} & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\begin{cases} x + 2y + 3z = -3 \\ y + z - t = -2 \Rightarrow y = -2 - \alpha + \beta \end{cases}$

$x = -2(-2 - \alpha + \beta) - 3\alpha - 3$, $x = -\alpha - 2\beta + 1$

$(x, y, z, t) \in \{(-\alpha - 2\beta + 1, -2 - \alpha + \beta, \alpha, \beta) \mid \alpha, \beta \in \mathbb{R}\} = S(A)$

$rg A = 2 = rg \bar{A}$ SC dublu N

$z = \alpha$

$t = \beta$

$(\mathbb{R}^3, +, \cdot)\big|_{\mathbb{R}}$, $S = \{u = (1,2,3), v = (2,3,1), w = (\alpha+3, \alpha+1, \alpha+2)\}$, $\alpha \in \mathbb{R}$

$\alpha = ?$ a.î. a) S este SLI

b) S este SLD

a) Fie $a, b, c \in \mathbb{R}$

$au + bv + cw = 0_{\mathbb{R}^3} \iff (a, 2a, 3a) + (2b, 3b, b) + (c(\alpha+3), c(\alpha+1), c(\alpha+2)) = (0, 0, 0)$

$$\iff (*) \begin{cases} a + 2b + c(\alpha+3) = 0 \\ 2a + 3b + c(\alpha+1) = 0 \\ 3a + b + c(\alpha+2) = 0 \end{cases}, \quad A = \left(\begin{array}{ccc|c} 1 & 2 & \alpha+3 & 0 \\ 2 & 3 & \alpha+1 & 0 \\ 3 & 1 & \alpha+2 & 0 \end{array}\right)$$

S este SLI $\iff$ (*) are sol unică nulă $(a, b, c) = (0, 0, 0) \iff$ SCD, $\det(A) \neq 0$

$C_3' = C_1 + C_2 + C_3$

$$\begin{vmatrix} 1 & 2 & \alpha+6 \\ 2 & 3 & \alpha+6 \\ 3 & 1 & \alpha+6 \end{vmatrix} = (\alpha+6)\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 1 \end{vmatrix} = (\alpha+6)\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & -1 & 0 \end{vmatrix} = -3(\alpha+6)$$

S este SLI $\Leftrightarrow \alpha \in \mathbb{R} \setminus \{-6\}$

S este SLD $\Leftrightarrow \alpha = -6$

a) $B_0 = \{1, x, x^2\}$ repr. baza canonică a lui $\mathbb{R}_2[X]$.

~~Dim. lui~~

$\dim_{\mathbb{R}}(\mathbb{R}_2[X]) = 3 = |B_0| = |B|$

B bază $\Leftrightarrow \begin{cases} 1.\ B \text{ e SLI} \\ 2.\ B \text{ e SG} \end{cases}$

Arătăm B SLI:

Fie $a_1, a_2, a_3 \in \mathbb{R}$ a.î.

$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$

SLO cu necunoscutele a_1, a_2, a_3 şi $A = \begin{pmatrix} 0 & 1 & 4 \\ -3 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$.

$\det A = \begin{vmatrix} 0 & 1 & 4 \\ -3 & 0 & -4 \\ 2 & 0 & -1 \end{vmatrix} = -11 \neq 0 \Rightarrow$ SCD $\Rightarrow a_1 = a_2 = a_3 = 0$

Prop: $(V, +, \cdot)|_K$ sp. vect. $\dim_K V = n$

Fie $S = \{v_1, \dots, v_n\} \subset V$. UAE:

1) S bază
2) S SLI
3) S e SG.

Obs $\underset{\mathbb{R}_2[X]}{\overset{\cap}{P}} = a_0 + a_1 X + a_2 X^2 \equiv (a_0, a_1, a_2) \in \mathbb{R}^3$

$\boxed{B \text{ e SLI} \overset{\text{Prop}}{\Longrightarrow} B \text{ e bază.}}$

5) $(\mathbb{R}_2[X], +, \cdot)$

a) $B = \{ v_1 = \underset{(0,-3,2)}{2x^2 - 3x}, v_2 = \underset{(1,1,0)}{X+1}, v_3 = \underset{(4,0,-1)}{-x^2+4} \}$

Dem. bază

b) $S = \{ v_1' = X+3, v_2' = X^2 - 2X, v_3' = \alpha X^2 - 6 \}$

$\alpha = ?$ a.î. S este SLI / SLD

a) B

Dim

$\dim_{\mathbb{R}} (\mathbb{R}$

B b

Prop:

Fie

bază.

$+4\}$

$,0,-1)$

$-6\}$

$$\Delta = \begin{vmatrix} 3 & 0 & -6 \\ 1 & -2 & \alpha \\ 0 & 1 & \alpha \end{vmatrix} = -6\alpha - 6 = -6(\alpha+1)$$

S e $SCI \iff \Delta \neq 0 \iff \alpha \in \mathbb{R} \setminus \{-1\}$

S e $SCD \iff \alpha = -1$

$\begin{matrix} 1 & -1 \\ 0 & 1 \end{matrix}\Big),$

$\left(\begin{matrix} \alpha & 1 \\ 0 & -2 \end{matrix}\right)\Big\}$

SLI.

ază

$\begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}\Big)\Big\}$

<u>Obs</u> Matricea $\underset{M_2(\mathbb{R})}{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \equiv (a_{11}, a_{12}, a_{21}, a_{22}) \in \mathbb{R}^4$

$S' = \{(1,0,\alpha,0), (1,-1,0,1), (0,1,0,-3), (\alpha,1,0,-2)\}$

S' e SLI $\Leftrightarrow rg\begin{pmatrix} 1 & 1 & 0 & \alpha \\ 0 & -1 & 1 & 1 \\ \alpha & 0 & 0 & 0 \\ 0 & 1 & -3 & -2 \end{pmatrix} = \max(4)$

$\det M = \alpha(-1)^{3+1}\begin{vmatrix} 1 & 0 & \alpha \\ -1 & 1 & 1 \\ 1 & -3 & -2 \end{vmatrix} = \alpha\begin{vmatrix} 0 & 3 & \alpha+2 \\ 0 & -2 & -1 \\ 1 & -3 & -2 \end{vmatrix} = \alpha\cdot(-1)^{1+3}((-3)+2\alpha+4) =$

$= \alpha(2\alpha+1) \neq 0 \Leftrightarrow \alpha \in \mathbb{R}\setminus\{0, -\frac{1}{2}\}$

b) $\dim_{\mathbb{R}} M_2(\mathbb{R}) = 4 \underset{prop}{=} |S|$

S e bază $\Leftrightarrow$ S e SLI

$$(\mathcal{M}_2(\mathbb{R}), +, \cdot)\big|_{\mathbb{R}}$$

$$S = \left\{ E_1 = \begin{pmatrix} 1 & 2 \\ \alpha & 0 \end{pmatrix}, E_2 \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}, E_4 = \begin{pmatrix} \alpha & 1 \\ 0 & -2 \end{pmatrix} \right\}$$

$\alpha = ?$ a.î. a) S e SLI.

b) S e bază

$$\mathcal{B}_0 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

baza canonică în $\mathcal{M}_2(\mathbb{R})$

Obs M

$S' = \{($

S' e

det

b) $\dim_{\mathbb{R}} \mathcal{M}_2(\mathbb{R}$

S e ba

$(\mathbb{R}^3, +, \cdot)_{\mathbb{R}}$

Ex $W = \{ x \in \mathbb{R}^3 \mid \begin{cases} x_1 + x_2 - x_3 = 0 \\ 3x_1 - 2x_2 + 2x_3 = 0 \\ 6x_1 + x_2 - x_3 = 0 \end{cases} \}$ $\quad A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & -2 & 2 & 0 \\ 6 & 1 & -1 & 0 \end{array} \right)$

$\| $

$S(A)$

$\det W = \begin{vmatrix} 0 & 0 & -1 \\ 5 & 0 & 2 \\ 7 & 0 & -1 \end{vmatrix} = 0 \Rightarrow rg\, A = 2$

$(x_1, x_2, x_3) \in \{ (0, \underbrace{\alpha, \alpha) \mid \alpha \in \mathbb{R} \}}_{\alpha(0,1,1)} \Rightarrow$

$\dim W = 3 - rg\, A = 3 - 2 = 1$ $\quad \Rightarrow B \text{ e } SG$

$\begin{cases} x_1 + x_2 = \alpha \\ 3x_1 - 2x_2 = -2\alpha \end{cases} \Big| \, 2$ $\quad x_1 = 0, \; x_2 = \alpha, \; x_3 = \alpha$

$5x_1 \quad / = 0$

dar $B = \{(0,1,1)\}$ SLI $\} \Rightarrow B$ bază

$$(\mathcal{M}_2(\mathbb{R}), +, \cdot)|_{\mathbb{R}}$$

$$S = \left\{ E_1 = \begin{pmatrix} 1 & 0 \\ \alpha & 0 \end{pmatrix}, E_2 \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}, E_4 = \begin{pmatrix} \alpha & 1 \\ 0 & -2 \end{pmatrix} \right\}$$

$\alpha = ?$ a.î. a) S e SLI.

b) S e bază

$$B_0 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

bază canonică în $\mathcal{M}_2(\mathbb{R})$

$(\mathbb{R}^3, +, \cdot)|_{\mathbb{R}}$

Ex $W = \{ x \in \mathbb{R}^3$

$\parallel$

$S(A)$

$\det W = |$

$\dim W = 3 -$

$$\begin{cases} x_1 + x_2 = 0 \\ 3x_1 - 2x_2 = \end{cases}$$

$5x_1 \;/ =$