① $f: \mathbb{R}^2 \to \mathbb{R}^2$, $f(x_1, x_2) = (x_1 + x_2, -x_2)$

$f \in Aut(\mathbb{R}^2)$.

② $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, 2x_1 + 5x_2 + 3x_3, -3x_1 - 7x_2 - 4x_3)$

a) f liniară

b) $\ker f = ?$ Precizați un reper în $\ker f$.

c) $\operatorname{Im} f = ?$ — // — $\operatorname{Im} f$

d) $[f]_{R_0, R_0} = A = ?$, R_0 = reperul canonic în $\mathbb{R}^3$.

③ $f: \mathbb{R}^2 \to \mathbb{R}^3$, $f(x) = (3x_1 - 2x_2, 2x_1 - x_2, -x_1 + x_2)$

a) f liniară

b) f inj

c) $\operatorname{Im} f = ?$

d) $[f]_{R_0, R_0'} = A = ?$ R_0, R_0' repere canonice în $\mathbb{R}^2$, resp $\mathbb{R}^3$.

④ $f: \mathbb{R}_3[X] \to \mathbb{R}_2[X]$, $f(P) = P'$

a) $[f]_{R_0, R_0'} = A = ?$, R_0, R_0' repere canonice în $\mathbb{R}_3[X]$, resp. $\mathbb{R}_2[X]$

b) $\dim \ker f$, $\dim \operatorname{Im} f$.

⑤ $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, -x_1 - 2x_2 - x_3, x_1 + x_2 + x_3)$

a) $[f]_{R_0, R_0} = A = ?$

b) $\dim \ker f$, $\dim \operatorname{Im} f$

c) $V' = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 \,\middle|\, \begin{cases} x_1 - x_2 + x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases} \right\}$

$f(V') = ?$

(6) $(\mathbb{R}^2,+,\cdot)_{\mathbb{R}}$, $\mathcal{R}_0=\{e_1=(1,0), e_2=(0,1)\} \xrightarrow{C} \mathcal{R}'=\{e_1'=e_1-e_2, e_2'=e_1+2e_2\}$
repere

$((\mathbb{R}^2)^*,+,\cdot)_{\mathbb{R}}$ sp. vect. <u>dual</u>

$(\mathbb{R}^2)^*=\{f:\mathbb{R}^2\to\mathbb{R} \,/\, f$ liniară$\}$.

$\mathcal{R}^*=\{e_1^*, e_2^*\} \xrightarrow{D} \mathcal{R}'^*=\{e_1'^*, e_2'^*\}$ repere duale în sp. dual

$e_i^*(e_j)=\delta_{ij}$, $e_i'^*(e_j')=\delta_{ij}$, $\forall\, i,j=\overline{1,2}$, $\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j.\end{cases}$

Precizați legătura dintre matricele C și D.

(7) Fie $f\in End(V)$ a.î. $f^2=0$
Să se arate că $g=id_V+f\in Aut(V)$

(8) $f:\mathbb{R}_1[X]\longrightarrow\mathbb{R}^3$, $f(ax+b)=(a,b,a+b)$
Fie $\mathcal{R}=\{2X-1, -X+1\}$, $\mathcal{R}'=\{(1,1,1),(1,1,0),(1,0,0)\}$
repere în $\mathbb{R}_1[X]$, resp. $\mathbb{R}^3$

a) f liniară ; ~~det. expresia analitică pentru f.~~

b) $[f]_{\mathcal{R},\mathcal{R}'}=A=?$

c) $Ker f$, $Im f$

(9) $g:\mathbb{R}^3\longrightarrow\mathbb{R}^3$ liniară, $g(v_i)=u_i$, $i=\overline{1,3}$

$v_1=(-1,1,1)$, $v_2=(1,1,1)$, $v_3=(0,2,1)$

$u_1=2v_1+3v_2-v_3$, $u_2=v_1+3v_2+v_3$, $u_3=v_3$.

a) $g=?$

b) $[g]_{R_0,R_0}$

c) $Ker(g)$, $Im(g)$

(10) $f : \mathbb{R}_1[X] \longrightarrow \mathbb{R}_2[X]$, $f(aX+b) = aX^2 + (a+2b)X + a-b$

Fie $\mathcal{R} = \{1, X\}$ și $\mathcal{R}' = \{X^2, X^2+X+1, X^2+X\}$, repere în $\mathbb{R}_1[X]$, resp. $\mathbb{R}_2[X]$

a) f liniară

b) $[f]_{\mathcal{R},\mathcal{R}'}$

c) $\operatorname{Ker} f$, $\operatorname{Im} f$

(11) $f : \mathbb{R}_2[X] \longrightarrow \mathbb{R}_1[X]$ liniară

$f(X+2) = X+1$; $f(-X^2+3) = 2X+3$; $f(2X+5) = -X+1$

Determinați f.

(12) $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 + x_2 + 3x_3, x_1 + x_2 + a, x_1 + x_3 + a)$

$a \in \mathbb{R}$

$a = ?$ a.î. f este apl. liniară

(13) $f : \mathbb{R}_1[X] \longrightarrow \mathbb{R}_1[X]$ liniară.

Să se afle expresia analitică pentru f dacă

a) $[f]_{\mathcal{R}_0, \mathcal{R}_0} = A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

b) $[f]_{\mathcal{R}, \mathcal{R}} = A' = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, $\mathcal{R} = \{X-1, 2X+2\}$

(14) $f : \mathbb{R}^3 \to \mathbb{R}^3$

$f(x) = (x_1 - 2x_2 + 5x_3, mx_1 + 3x_2 - x_3, x_2 - 3x_3)$

a) $m = ?$ a.î. f inj

b) Pt $m = 1$ să se afle $\operatorname{Im} f$

c) Pt $m = -\frac{8}{3}$ să se afle $\dim \operatorname{Im} f$

(15) Fie apl. lin. $f_m : \mathbb{R}^3 \to \mathbb{R}^3$, $[f_m]_{\mathcal{R}_0, \mathcal{R}_0} = \begin{pmatrix} 1 & -1 & m \\ 2 & 3 & m \\ m & 1 & 0 \end{pmatrix}$

$m = ?$ a.î. $f_m \in \operatorname{Aut}(\mathbb{R}^3)$

6. $f : \mathbb{R}^3 \to \mathbb{R}^4$, $A = [f]_{R_0,R_0} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -3 \end{pmatrix}$

a) $f = ?$

b) Precizați câte o bază în $\ker f$, $\operatorname{Im} f$

Ex 7 $S : \mathbb{R}^3 \to \mathbb{R}^2$, $S(x) = (x_1 + x_2, \ 2x_1 - x_2 + x_3)$

$T : \mathbb{R}^2 \to \mathbb{R}^4$ $T(y) = (y_1, y_1 + y_2, y_2, y_1 - y_2)$

$[S]_{R_0,R_0'}$, $[T]_{R_0',R_0''}$, $[T \circ S]_{R_0,R_0''} = ?$

Ex 8 $A = [f]_{R_0,R_0} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

$A' = [f]_{R',R'} = ?$ $R' = \{ f_1 = (1,2,3), f_2 = (3,1,2), f_3 = (2,3,1) \}$

Ex 9 $f \in \operatorname{End}(\mathbb{R}^3)$ $f(u_i) = v_i$, $i = \overline{1,3}$,

$u_1 = (2,3,5)$, $u_2 = (0,1,2)$, $u_3 = (1,0,0)$

$v_1 = (1,1,1)$, $v_2 = (1,1,-1)$, $v_3 = (2,1,2)$

$f = ?$

Ex

$(\mathbb{R}^3, +, \cdot)/\mathbb{R}$ $\quad \mathcal{R} = \{e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\} \xrightarrow{C}$

$\mathcal{R}' = \{e_1' = e_1 + e_2 + e_3,\ e_2' = e_1 + e_2,\ e_3' = e_1\}$.

$((\mathbb{R}^3)^* = \{f : \mathbb{R}^3 \to \mathbb{R} \mid f \text{ lin}\}, +, \cdot)/\mathbb{R}$ sp. vect dual.

$\mathcal{R}^* = \{e_1^*, e_2^*, e_3^*\} \xrightarrow{D} (\mathcal{R}')^* = \{e_1'^*, e_2'^*, e_3'^*\}$ repere duale în sp. dual.

$e_i^*(e_j) = \delta_{ij}$, $e_i'^*(e_j') = \delta_{ij}$, $\forall i,j = \overline{1,3}$

$C, D = ?$

Ex. $f : \mathbb{R}^3 \to \mathbb{R}_2[X]$ liniară

$f(1,1,-1) = X^2 + 3X + 3$, $f(-1,1,0) = -X + 1$, $f(0,1,-1) = 2X^2$

a) $f = ?$

b) $[f]_{\mathcal{R}_0, \mathcal{R}_0'} = A = ?$ $\quad \mathcal{R}_0 = \{e_1, e_2, e_3\}$, $\mathcal{R}_0' = \{1, X, X^2\}$ repere canonice în $\mathbb{R}^3$, $\mathbb{R}_2[X]$

Ex. $f : \mathbb{R}_1[X] \to \mathcal{M}_{2,1}(\mathbb{R})$

$[f]_{\mathcal{R}_1, \mathcal{R}_2} = A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ $\quad \mathcal{R}_1 = \{1 + X, -X\}$, $\mathcal{R}_2 = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$

$f = ?$

Ex. $f : \mathbb{R}^n \to \mathbb{R}^n$ endomorfism a.î. $f^2 = f + id_{\mathbb{R}^n} \Rightarrow f \in Aut(\mathbb{R}^n)$

Ex. $f \in End(\mathbb{R}^4)$

$f(e_1 + e_2) = (1,1,-1,-1)$, $f(e_1 - e_2) = (-1,-1,1,1)$,

$f(e_3 + e_4) = (-1,1,-1,1)$, $f(e_3 - e_4) = (1,-1,1,-1)$, $\mathcal{R}_0 = \{e_1, e_2, e_3, e_4\}$.

a) $[f]_{\mathcal{R}_0, \mathcal{R}_0} = A = ?$

b) Precizați câte o bază în $Ker f$, $Im f$

Ex. $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1 + x_2 - 2x_3,\ x_2,\ x_1 - x_2)$

$f \in Aut(\mathbb{R}^3)$

Ex $f : \mathbb{R}_2[X] \to \mathbb{R}_2[X]$, $f(P) = P + P' + P''$, $P \to \tilde{P}$

a) Este apl. f bijectivă?

b) $[f]_{R_0, R_0} = ?$ $R_0 = \{1, X, X^2\}$

Ex $U \xrightarrow{f} V \xrightarrow{g} W$ f, g liniare a.î. $g \circ f = 0$

a) f surj $\Rightarrow g = 0$

b) g inj $\Rightarrow f = 0$