Seminar 2

Structuri algebrice în informatică

Dați exemple de funcție:

1) $f: \mathbb{R} \to \mathbb{R}$ injectivă și nu surjectivă

2) $g: \mathbb{R} \to \mathbb{R}$ neinjectivă și surjectivă

3) $h: \mathbb{Z} \to \mathbb{Z}$, inj, nusurj

4) $g: \mathbb{Z} \to \mathbb{Z}$, nuinj, surjectivă

1) $f: \mathbb{R} \to \mathbb{R}$ $f(x) = 2^x$, $(\forall) x \in \mathbb{R}$

$f \nearrow \Rightarrow f$ injectivă

$Im f = (0, \infty) \subset \mathbb{R} \Rightarrow f$ nu e surj

2) $g: \mathbb{R} \to \mathbb{R}$, $g(x) = x^3 - x$

$\lim_{x \to +\infty} g(x) = +\infty$

$\lim_{x \to -\infty} g(x) = -\infty$

$\Rightarrow Im g = \mathbb{R}$

g continuă

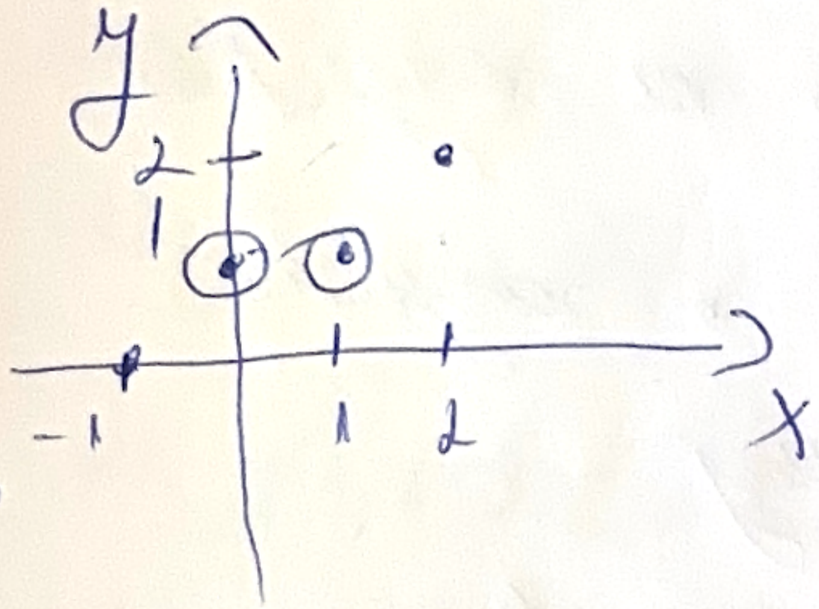
g surjectivă

$x^3 - x = 0 \Rightarrow g(0) = g(1) = g(-1) = 0 \Rightarrow$

$\Rightarrow$ g nu e injectivă

4). $g: \mathbb{Z} \to \mathbb{Z}$, $g(x) = \begin{cases} x+1, & x \leq 0 \\ x, & x > 0 \end{cases}$

$g(0) = g(1) = 1 \Rightarrow g$ - nu injectivă



Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} 2x+3, & x \geq 1 \\ -x^2+5x+2, & x < 1 \end{cases}$

Este f injectivă? Surjectivă? Bijectivă?

Dacă da, cine este f^{-1}.

I injectivitate

Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} 2x+1, & x \leq 1 \\ 3x-4, & x > 1 \end{cases}$

Este f inj? surjectivă? bijectivă?

Determinați:

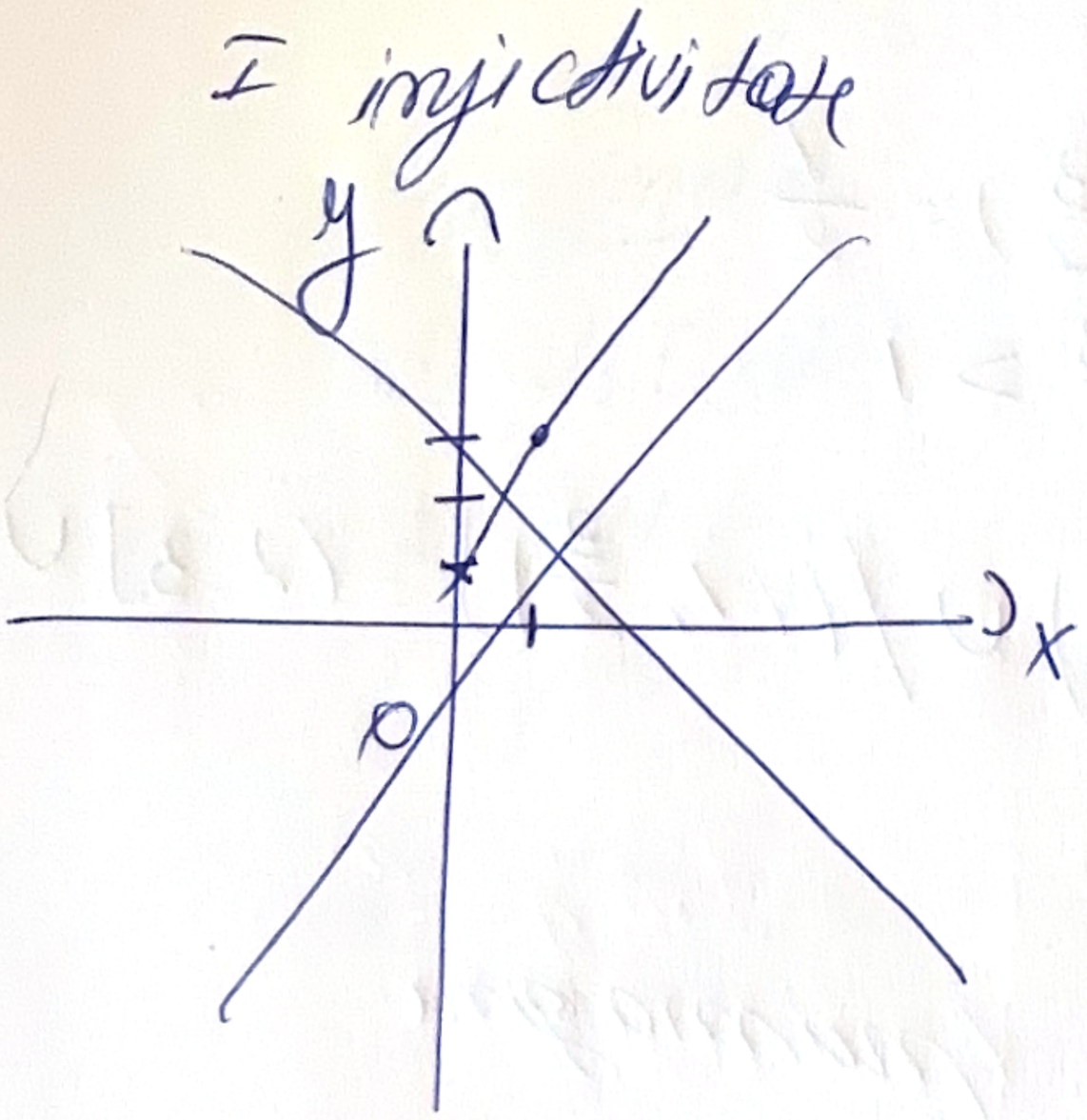
1) $f((-2,-1)) =$

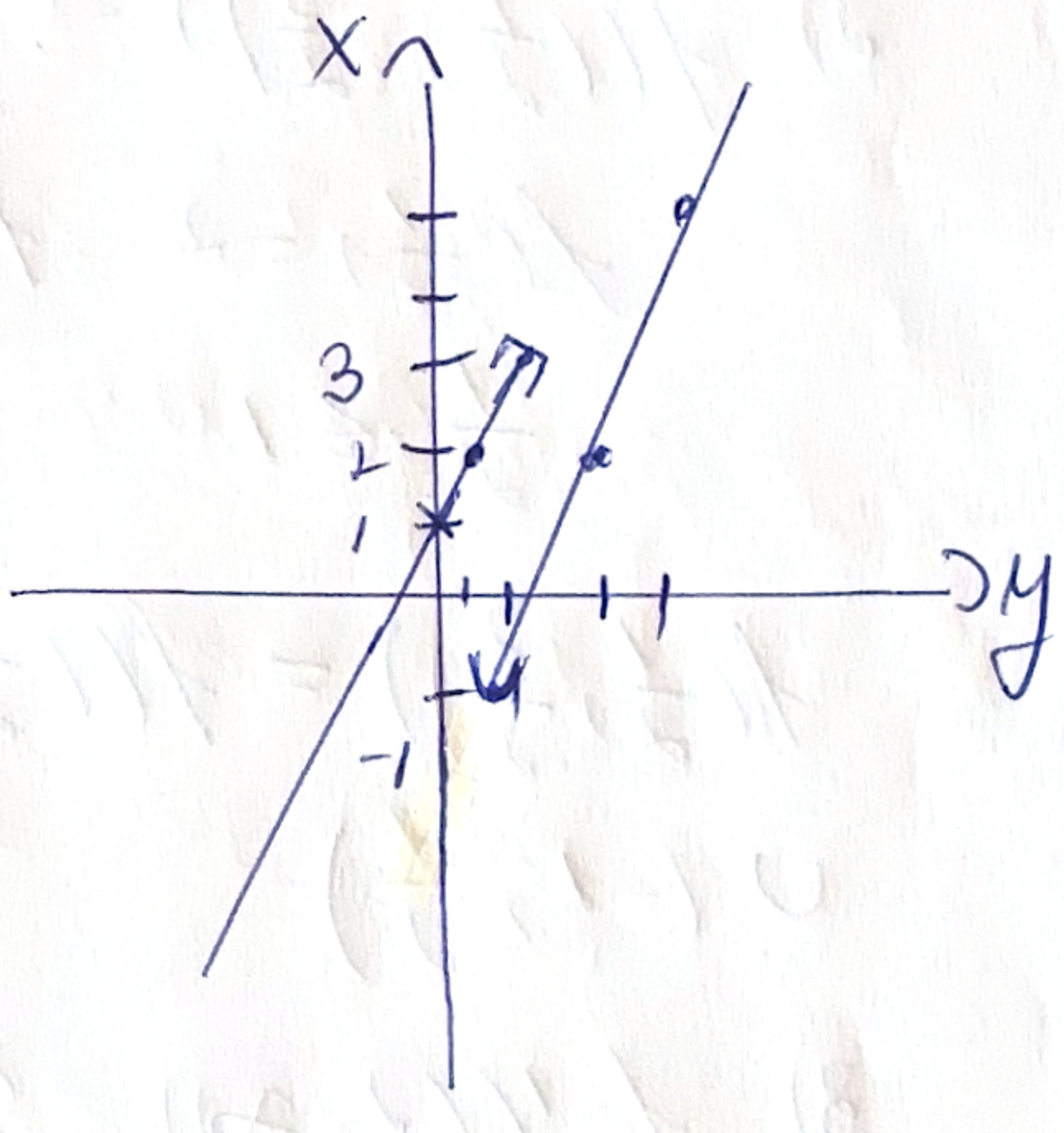
2) $f((\frac{1}{2}, \frac{3}{2})) =$

3) $f^{-1}((2,\infty)) =$

4) $f^{-1}([0,1]) =$

I injectivitate





$f(x)=0 \Leftrightarrow \begin{cases} 2x+1=0 \\ 3x-4=0 \end{cases} \Rightarrow \begin{cases} x=-\frac{1}{2} \\ x=\frac{4}{3} \end{cases} \Rightarrow$

$\Rightarrow f$ nu e injectivă

II surjectivitate

$Im f = f((-\infty, 1]) \cup f((1,+\infty)) =$

$= (-\infty, 3] \cup (-1, +\infty) = \mathbb{R} \Rightarrow f$ - surjectiv

1) $f((-2,-1)) = ?$

$f(-2) = -3$

$f(-1) = -1$

f - continuă

f - strict crescătoare pe $(-2,-1)$ $\Rightarrow$

$f((-2,-1)) = (-3,-1)$

2) $f((\frac{1}{2}, \frac{3}{2})) = ?$

$f(\frac{1}{2}) = 2$, $f(\frac{3}{2}) = \frac{1}{2}$

$\frac{1}{2} \leq 1$ $\quad$ $\frac{3}{2} > 1$

$f((\frac{1}{2}, \frac{3}{2})) = f((\frac{1}{2}, 1]) \cup f((1, \frac{3}{2})) = (2, 3] \cup$

$\cup (-1, \frac{1}{2})$

3) $f^{-1}((2, \infty)) = ?$ (mi imagine

$2x+1 = 2$, $x \leq 1$

$x = \frac{1}{2}$

$3x - 4 = 2$, $x > 1$

$x = 2$

$f^{-1}((2, \infty)) = (\frac{1}{2}, 1] \cup (2, \infty)$

4) $f^{-1}([0,1]) = [-\frac{1}{2}, 0] \cup [\frac{4}{3} \cup \frac{5}{3}]$

$2x+1=0, x \leq 1$ | $2x+1=1, x \leq 1$

$x = -\frac{1}{2}$ | $x = 0$

$3x-4=0, x \geq 1$ | $3x-4=1, x>1$

$x = \frac{4}{3}$ | $x = \frac{5}{3}$

Tema: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x+2, & x<2 \\ x^2-5x+6, & x \geq 2 \end{cases}$

f inj? surj? bij?

Determinați:

1) $f([1,2]) =$

2) $f((1,3)) =$

3) $f^{-1}((-\frac{1}{4}, 0)) =$

4) $f^{-1}((0,5]) =$