6.11.2024

Seminar 5
Structuri Algebrice în Informatică

1) Determinați $d = \gcd(128, 36)$
Aflați toți $u, v \in \mathbb{Z}$, $128u + 36v = d$

$128 = 2^7$
$36 = 2^2 \cdot 3^2$

128	2
64	2
32	2
16	2
8	2
4	2
2	
1	

36	2
18	2
9	3
3	3
1	

$\gcd(128, 36) = 2^2 = 4$

$128 = 3 \cdot 36 + 20$
$36 = 1 \cdot 20 + 16$
$20 = 1 \cdot 16 + 4 \Rightarrow \gcd(128, 36)$
$16 = 4 \cdot 4$

$4 = 20 - 16 = 36 - 16$
~~$3 \cdot 3$~~ $128 - 3 \cdot 36 - 36 + 20$
$= 128 - 4 \cdot 36 + 128 - 3 \cdot 36$
$= 2 \cdot 128 - 7 \cdot 36$

$128 \cdot 2 + 36(-7) = 4 \Rightarrow u_0 = 2$
$v_0 = -7$

$\frac{128 \cdot u + 36 \cdot v = 4}{128(u-2) + 36(v+7) = 0} \Leftrightarrow 36(v+7) = 128(2-u)$

$\Leftrightarrow 9(\underbrace{v+7}_{\in \mathbb{Z}}) = 32(\underbrace{2-u}_{\in \mathbb{Z}})$

$\gcd(9,32)=1$
$9 \mid 32(2-u)$ $\Big| \Rightarrow 9 \mid 2-u \Rightarrow 2-u=9\cdot k, k\in\mathbb{Z}$

$\Rightarrow u = 2-9\cdot k$

$9(v+7) = 32(9k)$

$v+7 = 32k$

$v = 32k-7$

$S = \{(2-9k, 32k-7) \mid k\in\mathbb{Z}\}$

2) $\gcd(2^{128}-1, 2^{36}-1)$

$(2^{128}-1) : (2^{36}-1) =$

$2^{128}-1 = 2^{4\cdot 32}-1 = (2^4)^{32}-1^{32}$

$2^{36}-1 = 2^{4\cdot 9}-1 = (2^4)^9 - 1^9$

$x^n-y^n = (x-y)(x^{n-1}+x^{n-2}y+\dots+xy^{n-2}+y^{n-1})$

$(2^4)^{32}-1 = (2^4-1)((2^4)^{31}+(2^4)^{30}+\dots+1)$

$(2^4)^9-1 = (2^4-1)((2^4)^8+\dots+1)$

$(24-1) \mid \gcd(2^{128}-1, 2^{36}-1)$

$(2^{128}-1)-(2^{36}-1) = 2^{128}-2^{36} = 2^{36}(2^{92}-1)$

Fie d un div. comun al $2^{128}-1, 2^{36}-1$

$\Rightarrow d \mid (2^{128}-1)-(2^{36}-1) \Rightarrow d \mid 2^{36}(2^{92}-1)$

$2^{12\ell}-1,\ 2^{36}-1$ sunt impare $\Rightarrow d \mid (2^{92}-1)$

d - impar

$d \mid 2^{92}-1$ și $d \mid 2^{36}-1 \Rightarrow d \mid [(2^{92}-1)-(2^{36}-1)] \Rightarrow$

$\Rightarrow d \mid 2^{92}-2^{36} \Rightarrow d \mid 2^{36}(2^{56}-1)$

$\Rightarrow d \mid 2^{92} = 2^{36}(2^{56}-1) \Rightarrow d \mid 2^{56}-1$

$d \mid 2^{12\ell-2\cdot 36}-1$

$\Rightarrow d \mid 2^{56}-1$

$d \mid 2^{12\ell-2\cdot 36}-1$ (!)

$d \mid 2^{20}-1$

$d \mid 2^{36}-1$ și $d \mid 2^{20}-1 \Rightarrow d \mid 2^{16}-1$

(!!)

$d \mid 2^4-1 \Rightarrow \gcd(2^{12\ell}-1, 2^{36}-1) = 2^4-1$

$\Rightarrow \gcd(2^{12\ell}-1, 2^{36}-1) = 2^4-1 = 15$

~~3) Fie~~ ! Fie $a, b \in \mathbb{N}^*$

$\gcd(2^a-1, 2^b-1) = 2^{\gcd(a,b)}-1$

$x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + \ldots + y^{n-1})$

$x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 \mp \ldots \pm y^{n-1})$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$(\mathbb{N}, /)$ este multime ordonata

$\Rightarrow 3 \times 5 \neq 5 \times 3 \Rightarrow$ multimea nu e total ordonata cu "/"

$n \in \mathbb{N}^*, n \geq 2$

$\Delta(n)$ = divizorii lui $n \neq 0$

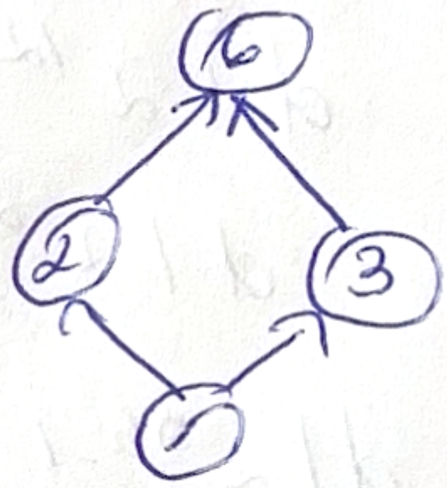
$\Delta P(n)$ = divizorii proprii ai lui $n = D(n) \setminus \{1, n\}$

3) Desenati diagrama Hasse pentru $(P(n), /)$

i) $(DP(n), /)$ $n = 6, 8, 10, 12, 36, \ldots$

$D(6) = \{1, 2, 3, 6\}$

$(D(6), /)$



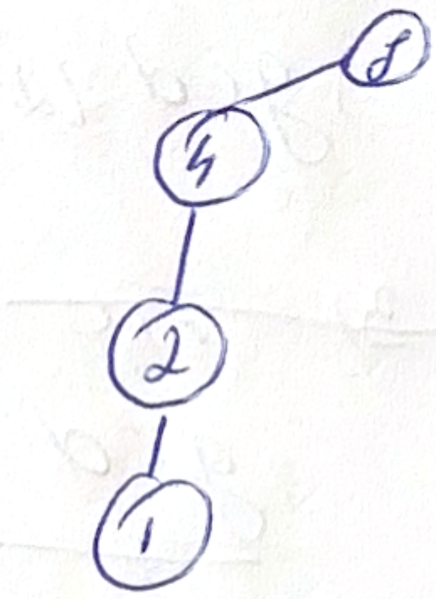
$DP(6) = \{2, 3\}$

(2) (3)

$(DP(6), /)$

$n = 8$

$D(8) = \{1, 2, 4, 8\}$

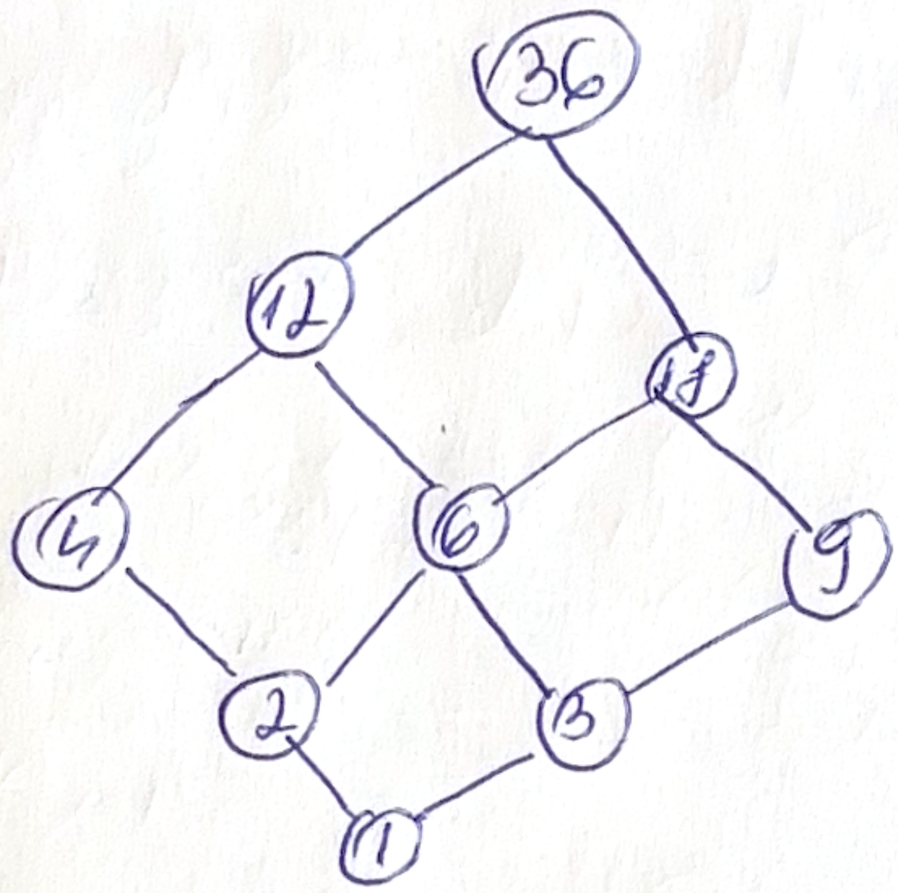


$DP(8) = \{2, 4\}$

$(PP(8), /)$

$D_{36} = 2^2 \cdot 3^2$

$D(36) = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$



$(D(36), /)$

$P(n)$ $a \to b$

$a \mid b$

$b = a$

$b = a \cdot p$ cu p prim

$b = a \cdot k_1 \cdot k_2$

$\neq_1 \quad \neq_1$

$a \mid a \cdot k_1 \mid a \cdot k_1 k_2 = b$

$\neq \quad \neq$