~ Tutoriat I ~

a) $$A=\begin{pmatrix} \boxed{1} & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_2 \to L_2 - 2L_1} \begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & -3 & 0 & -3 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_2 \cdot \left(-\frac{1}{3}\right)} \begin{pmatrix} \boxed{1} & 2 & 1 & 2 & 1 \\ 0 & \boxed{1} & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow[L_3 \to L_3 - L_2]{L_1 \to L_1 - 2L_2}$$

$$= \begin{pmatrix} \boxed{1} & 0 & 1 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ nr de pivoți este $= rg\, A$

b) $$A=\begin{pmatrix} 1 & 2 & 0 & 5 \\ 2 & 3 & 1 & 4 \\ -1 & -1 & -1 & 1 \end{pmatrix} \xrightarrow[L_3 \to L_3 + L_1]{L_2 \to L_2 - 2L_1} \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & -1 & 1 & -6 \\ 0 & 1 & -1 & 6 \end{pmatrix} \xrightarrow{L_2 \cdot (-1)} \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & -1 & 6 \\ 0 & 1 & -1 & 6 \end{pmatrix} \xrightarrow[L_3 \to L_3 - L_2]{L_1 \to L_1 - 2L_2} \begin{pmatrix} 1 & 0 & 2 & -7 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

~ Tutoriat I ~

OPERAȚII ELEMENTARE PT MATRICI

1) Înmulțirea unei linii cu un scalar
2) Adunarea unei multiple a unei linii la altă linie
3) Permutarea a două linii

c) $\begin{cases} x_1 + 2x_2 + 2x_3 + 2x_4 = -3 \\ x_2 + x_3 = 1 \\ -2x_1 + 2x_2 + 2x_3 + 3x_4 = 5 \end{cases}$

$$A^e = \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & -3 \\ 0 & 1 & 1 & 0 & 1 \\ -2 & 2 & 2 & 3 & 5 \end{array}\right) \xrightarrow{L_3+2L_1} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & -3 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 6 & 6 & 7 & -1 \end{array}\right)$$

$$\xrightarrow[L_1-2L_2]{L_3-6L_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -5 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 7 & -7 \end{array}\right) \xrightarrow{\frac{1}{7}\cdot L_3 \to L_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -5 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow{L_1-2L_3 \to L_1}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right) \to \begin{cases} x_1 = -3 \\ x_2 + x_3 = 1 \\ x_4 = -1 \end{cases}$$

Nec pr.: x_1, x_2, x_4

Nec secundare: $x_3 = \alpha$, $\alpha \in \mathbb{R}$

$$\iff \begin{cases} x_1 = -3 \\ x_2 = 1 - \alpha \\ x_4 = -1 \end{cases}$$

$$S = \{(-3, 1-\alpha, \alpha, -1) \mid \alpha \in \mathbb{R}\}$$

~ Tutoriat I ~

Determinarea inversei unei matrici pătratice dacă există, utilizând Gauss-Jordan:

Fie $A \in \mathcal{M}_n(\mathbb{C})$

Considerăm $(A \mid I_n) \in \mathcal{M}_{n,2n}(\mathbb{C})$ – FORMA EȘALON REDUSĂ

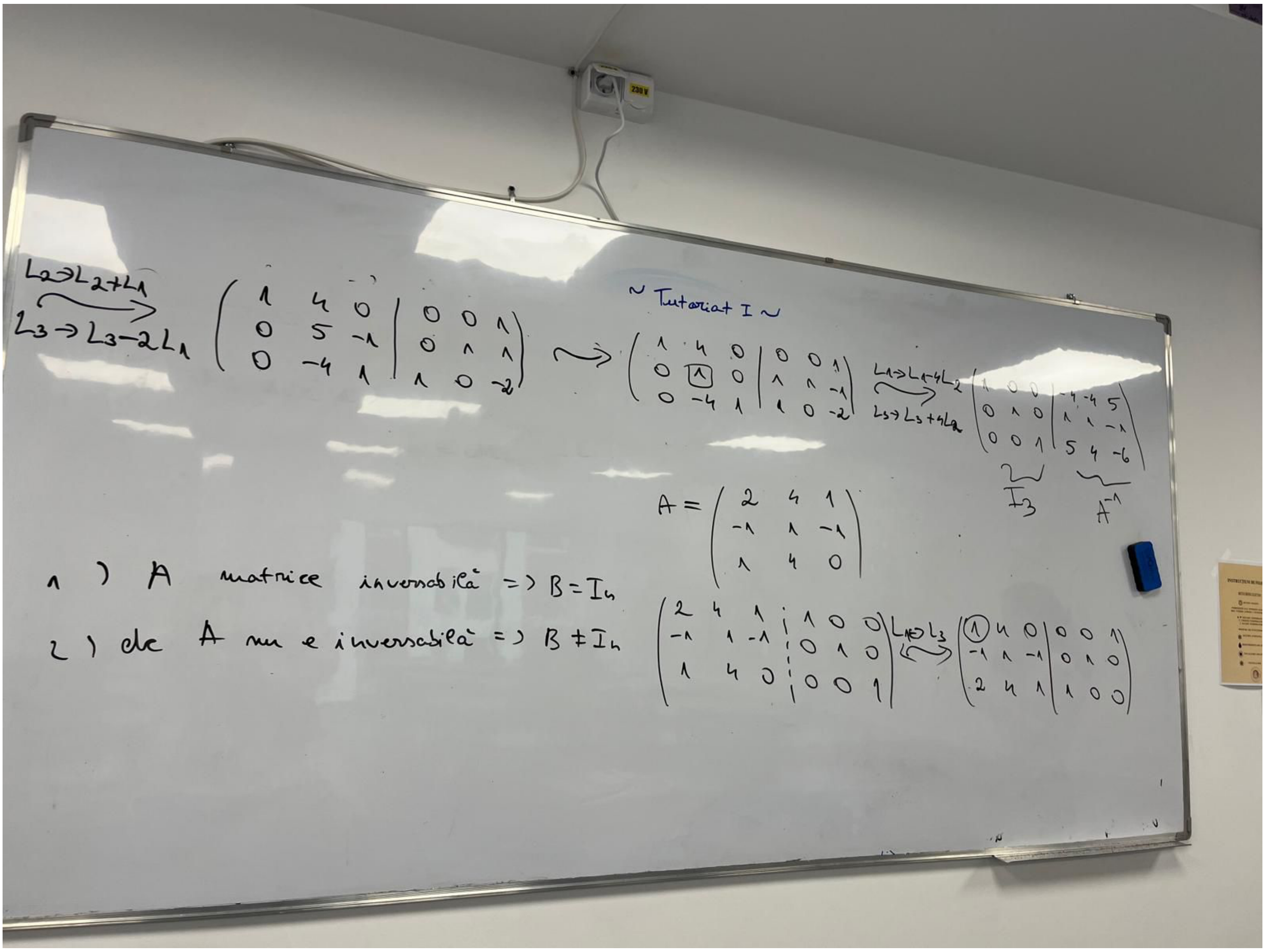
$E(B/C)$

1) A matrice inversabilă $\Rightarrow B = I_n$

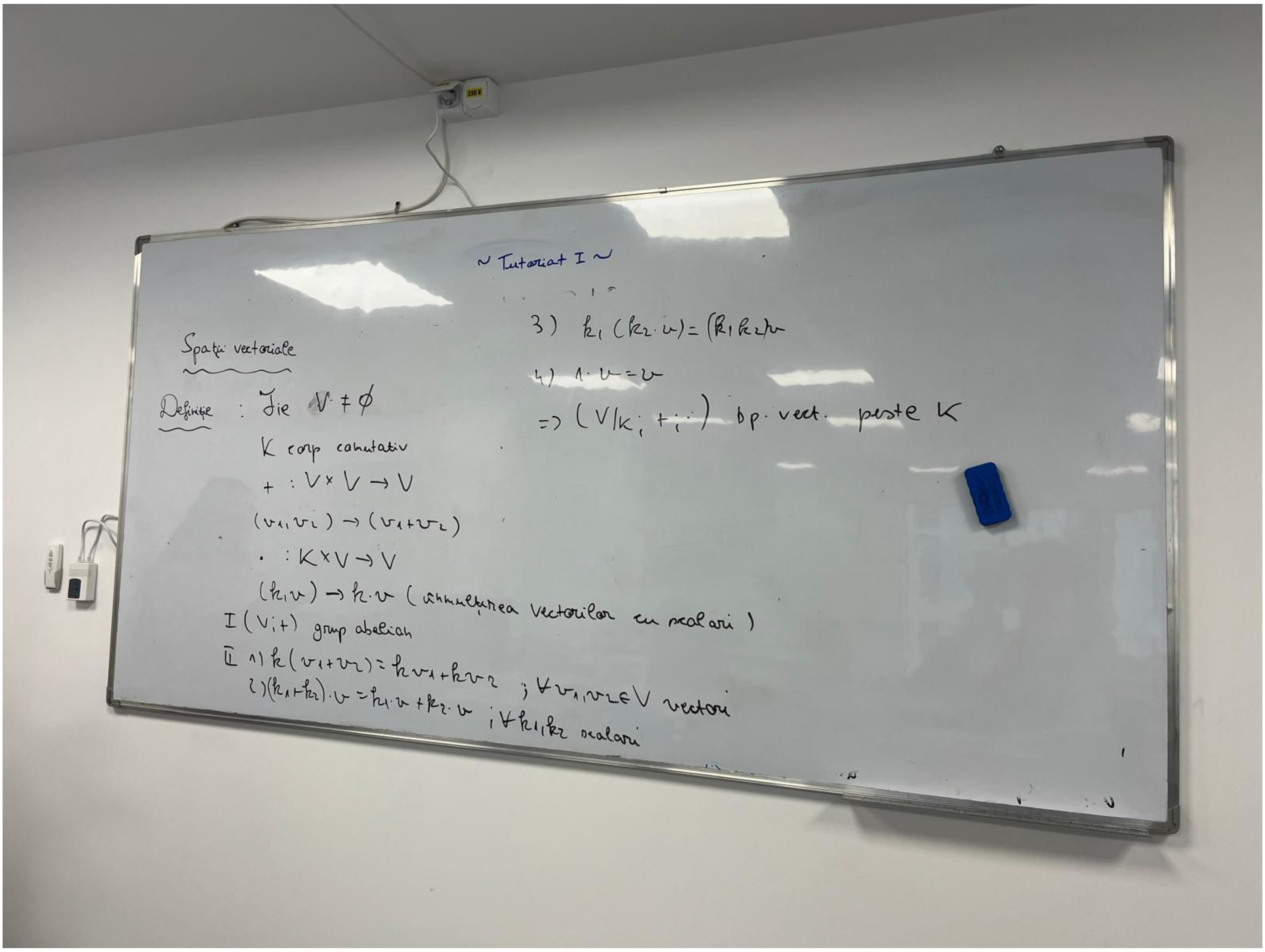
2) de A nu e inversabilă $\Rightarrow B \neq I_n$

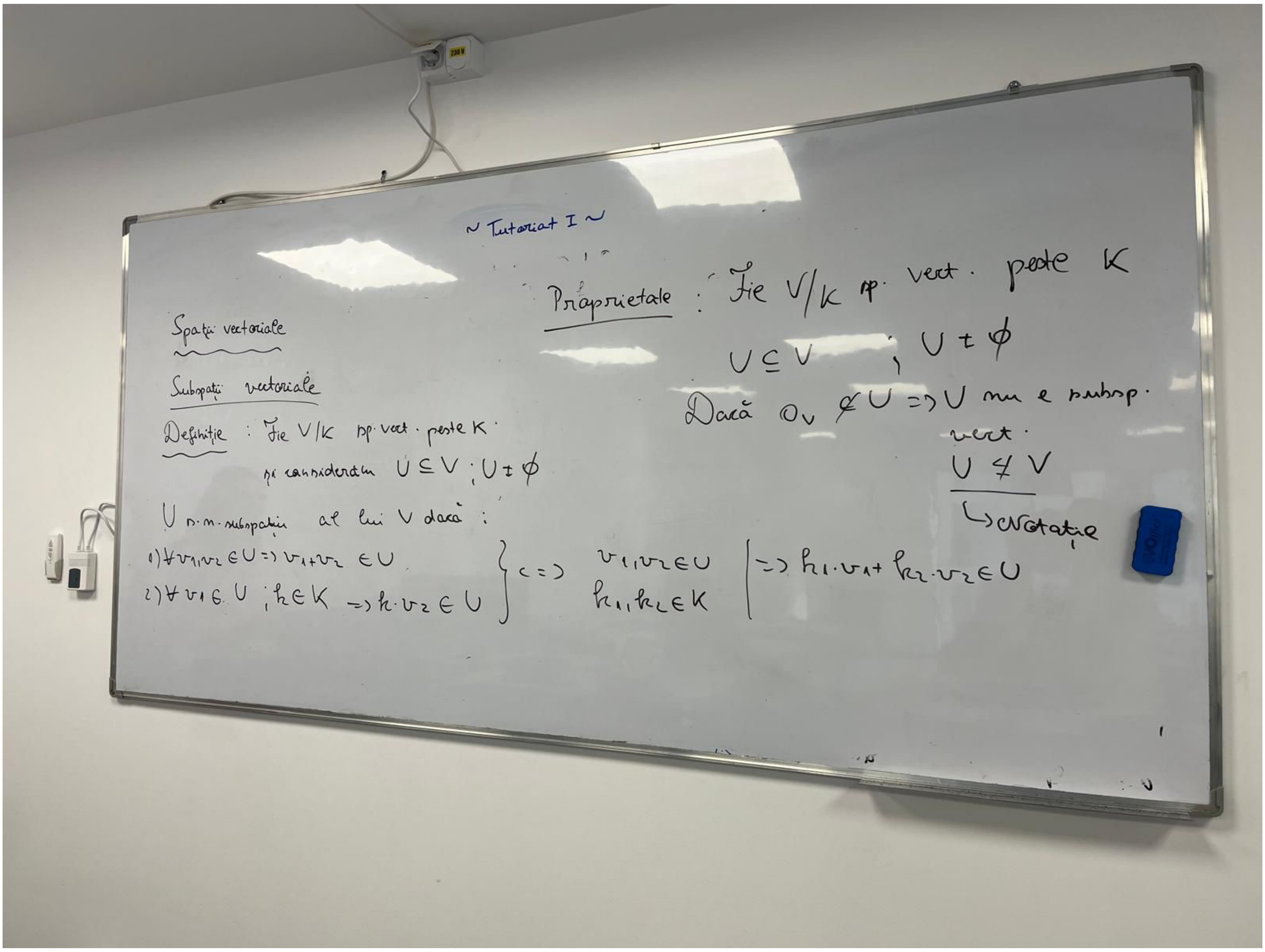
$$A = \begin{pmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{L_1 \leftrightarrow L_3} \left(\begin{array}{ccc|ccc} 1 & 4 & 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 2 & 4 & 1 & 1 & 0 & 0 \end{array}\right)$$



~ Tutoriat I ~
L2 → L2 + L1
L3 → L3 − 2L1
1 4 0 | 0 0 1
0 5 −1 | 0 1 1
0 −4 1 | 1 0 −2
1 4 0 | 0 0 1
0 1 0 | 1 1 −1
0 −4 1 | 1 0 −2
L1 → L1 − 4L2
L3 → L3 + 4L2
1 0 0 | −4 −4 5
0 1 0 | 1 1 −1
0 0 1 | 5 4 −6
I3
A−1
A =
2 4 1
−1 1 −1
1 4 0
1) A matrice inversabilă => B = In
2) dc A nu e inversabilă => B ≠ In
2 4 1 | 1 0 0
−1 1 −1 | 0 1 0
1 4 0 | 0 0 1
L1 ↔ L3
1 4 0 | 0 0 1
−1 1 −1 | 0 1 0
2 4 1 | 1 0 0
230 V


~ Tutoriat I ~
Spații vectoriale
Definiție : Fie $V \neq \emptyset$
K corp comutativ
$+ : V \times V \to V$
$(v_1, v_2) \to (v_1 + v_2)$
$\cdot : K \times V \to V$
$(k, v) \to k \cdot v$ (înmulțirea vectorilor cu scalari)
I $(V, +)$ grup abelian
II 1) $k(v_1 + v_2) = kv_1 + kv_2$; $\forall v_1, v_2 \in V$ vectori
2) $(k_1 + k_2) \cdot v = k_1 \cdot v + k_2 \cdot v$; $\forall k_1, k_2$ scalari
3) $k_1 (k_2 \cdot v) = (k_1 k_2) v$
4) $1 \cdot v = v$
$\Rightarrow (V/K, +, \cdot)$ sp. vect. peste K


~ Tutoriat I ~
Spații vectoriale
Subspații vectoriale
Definiție : Fie V/K sp. vect. peste K.
și considerăm $U \subseteq V$; $U \neq \emptyset$
U s.n. subspațiu al lui V dacă :
1) $\forall v_1, v_2 \in U \Rightarrow v_1 + v_2 \in U$
2) $\forall v_1 \in U$; $k \in K \Rightarrow k \cdot v_2 \in U$
$\Leftrightarrow$ $v_1, v_2 \in U$, $k_1, k_2 \in K$ $\Rightarrow k_1 \cdot v_1 + k_2 \cdot v_2 \in U$
Proprietăți : Fie V/K sp. vect. peste K
$U \subseteq V$; $U \neq \emptyset$
Dacă $0_V \notin U \Rightarrow U$ nu e subsp. vect.
$U \not\leq V$
notație

$$A^e = \left(\begin{array}{ccccc|c} \boxed{1} & -2 & 1 & -1 & 1 & 1 \\ 2 & 1 & -1 & 2 & -2 & 4 \\ 2 & -5 & 1 & -2 & 2 & 0 \\ 3 & -2 & -1 & 1 & -1 & m \end{array}\right) \xrightarrow[\substack{L_3 \to L_3 - 2L_1 \\ L_4 \to L_4 - 3L_1}]{L_2 \to L_2 - 2L_1}$$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 4 & -4 & 2 \\ 0 & -1 & -1 & 0 & 0 & -2 \\ 0 & 4 & -4 & 4 & -4 & m-3 \end{array}\right) \xleftrightarrow{L_3 \leftrightarrow L_2} \left(\begin{array}{ccccc|c} \boxed{1} & -2 & 1 & -1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 & -2 \\ 0 & 5 & -3 & 4 & -4 & 2 \\ 0 & 4 & -4 & 4 & -4 & m-3 \end{array}\right)$$

$$\xleftrightarrow{L_2 \cdot (-1)} \left(\begin{array}{ccccc|c} \boxed{1} & -2 & 1 & -1 & 1 & 1 \\ 0 & \boxed{1} & 1 & 0 & 0 & 2 \\ 0 & 5 & -3 & 4 & -4 & 2 \\ 0 & 4 & -4 & 4 & -4 & m-3 \end{array}\right) \xrightarrow[\substack{L_3 \to L_3 - 5L_2 \\ L_4 \to L_4 - 4L_2}]{L_1 \to L_1 + 2L_2}$$

$$\left(\begin{array}{ccccc|c} \boxed{1} & 0 & 3 & -1 & 1 & 5 \\ 0 & \boxed{1} & 1 & 0 & 0 & 2 \\ 0 & 0 & -8 & 4 & -4 & -8 \\ 0 & 0 & -8 & 4 & -4 & m-11 \end{array}\right) \xleftrightarrow{L_4 \to L_3 - L_4} \left(\begin{array}{ccccc|c} 1 & 0 & 3 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & -8 & 4 & -4 & -8 \\ 0 & 0 & 0 & 0 & 0 & m-3 \end{array}\right)$$

$$\xrightarrow{L_3\left(-\frac{1}{8}\right)} \left(\begin{array}{ccccc|c} \boxed{1} & 0 & 3 & -1 & 1 & 5 \\ 0 & \boxed{1} & 1 & 0 & 0 & 2 \\ 0 & 0 & \boxed{1} & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & m-3 \end{array}\right) \xrightarrow[L_2 \to L_2 - L_3]{L_1 \to L_1 - 3L_3}$$

$$\left(\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 1/2 & -1/2 & 2 \\ 0 & \boxed{1} & 0 & 1/2 & -1/2 & 1 \\ 0 & 0 & \boxed{1} & -1/2 & 1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & m-3 \end{array}\right)$$

sist comp $\Leftrightarrow m-3=0 \Leftrightarrow m=3$

Rezolvați sist în $\mathbb{R}$ și det soluțiile pt care $x_1, x_2, x_3, x_4, x_5 \in \mathbb{N}$

x_1, x_2, x_3 nec pp

x_4, x_5 nec sec $\qquad x_4 = \alpha,\ x_5 = \beta \qquad \alpha, \beta \in \mathbb{N}^*$

$$\begin{cases} x_1 + \frac{1}{2}(\alpha-\beta) = 2 \Rightarrow x_1 = 2 - \frac{1}{2}(\alpha-\beta) \in \mathbb{N}^* \\ x_2 + \frac{1}{2}(\alpha-\beta) = 1 \Rightarrow x_2 = 1 - \frac{1}{2}(\alpha-\beta) \in \mathbb{N}^* \\ x_3 - \frac{1}{2}(\alpha-\beta) = 1 \Rightarrow x_3 = 1 + \frac{1}{2}(\alpha-\beta) \in \mathbb{N}^* \end{cases}$$

notăm $\alpha - \beta = 2k \in \mathbb{N}$

$x_1 = 2 - k \in \mathbb{N}^*$

$x_2 = 1 - k \in \mathbb{N}^*$ (1)

$x_3 = 1 + k \in \mathbb{N}^*$ (2)

Din (1) $\Rightarrow 1 - k > 0 \Rightarrow 1 > k$

$1 + k > 0 \Rightarrow k > -1 \Big\} \Rightarrow -1 < k < 1$, $k \in \mathbb{N} \Big\} \Rightarrow k = 0$

$x_1 = 2 - \frac{1}{2} \cdot 2 \cdot 0 = 2$

$x_2 = 1 - \frac{1}{2} \cdot 2 \cdot 0 = 1$

$x_3 = 1 + \frac{1}{2} \cdot 2 \cdot 0 = 1$

$\alpha - \beta = 0 \Rightarrow \alpha = \beta$

$S = \{ (2, 1, 1, \alpha, \alpha) \mid \alpha \in \mathbb{N}^* \}$