25 Electric Charges and Forces

Electricity is one of the fundamental forces of nature. Lightning is a vivid manifestation of electric charges and forces.

▶ **Looking Ahead** The goal of Chapter 25 is to describe electric phenomena in terms of charges, forces, and fields.

Charge Model

Electric phenomena seem mysterious at first, but we'll find that we can understand them in terms of a **charge model:**

- There are two kinds of charge, called *positive* and *negative*.
- Two charges of the same kind repel; two opposite charges attract.
- Small neutral objects are attracted to a charge of either sign.

You'll learn how a comb rubbed through your hair picks up small pieces of paper.

Coulomb's Law

The law governing the electric force is called **Coulomb's law.** It tells us how the force between charged particles depends on their charge and on the distance between them.

You'll find that Coulomb's law, like Newton's law of gravity, is an *inverse-square law*.

◀ Looking Back
Sections 3.2–3.4 Vector addition
Sections 13.3–13.4 Newton's theory of gravity

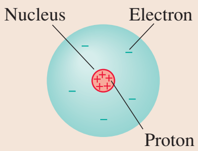
Field Model

How is a long-range force transmitted from one charge to another? We'll develop the idea that every charge alters the space around it by creating an **electric field.** It is the electric field that then exerts forces on other charges.

The liquid crystal displays (LCD) of your calculator, your digital watch, and your computer screen use electric fields to turn the pixels on and off.

Charges and Atoms

Electrons and protons—the constituents of atoms—are the basic charges of ordinary matter.



You'll learn that *charging* an object can be understood as the transfer of electrons from one material to another.

An object that is negative has an excess of electrons; a positively charged object is missing electrons.

Conductors and Insulators

There are two types of materials with very different electrical properties:

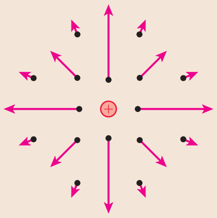
- **Conductors** are materials through or along which charge easily moves.
- **Insulators** are materials on or in which charge is immobile.

The metal wire—a conductor—carries a current of moving charges. It is separated from the support by a ceramic insulator.

Point Charges

A charged particle, with no physical size, is called a **point charge.** You'll learn that real objects can be modeled as point charges if they are very small compared to the distances between them.

The electric field of a point charge will be important throughout our study of electricity.



25.1 Developing a Charge Model

You can receive a mildly unpleasant shock and produce a little spark if you touch a metal doorknob after walking across a carpet. Vigorously brushing your freshly washed hair makes all the hairs fly apart. A plastic comb that you've run through your hair will pick up bits of paper and other small objects, but a metal comb won't.

The common factor in these observations is that two objects are *rubbed* together. Why should rubbing an object cause forces and sparks? What kind of forces are these? Why do metallic objects behave differently from nonmetallic? These are the questions with which we begin our study of electricity.

Our first goal is to develop a model for understanding electric phenomena in terms of *charges* and *forces.* We will later use our contemporary knowledge of atoms to understand electricity on a microscopic level, but the basic concepts of electricity make *no* reference to atoms or electrons. The theory of electricity was well established long before the electron was discovered.

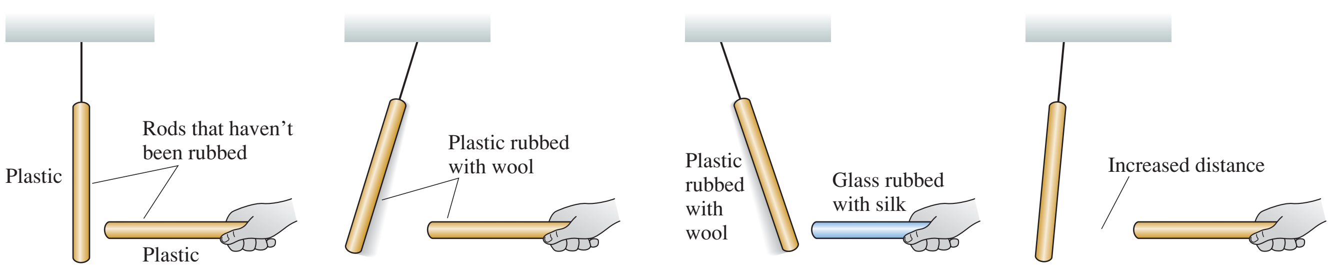
Experimenting with Charges

Let us enter a laboratory where we can make observations of electric phenomena. The major tools in the lab are:

- A variety of plastic and glass rods, each several centimeters long.
- A few metal rods with wood handles.
- Pieces of wool and silk.
- Small metal spheres, an inch or two in diameter, on wood stands.

Let's see what we can learn with these tools.

Discovering electricity I



Experiment 1

Take a plastic rod that has been undisturbed for a long period of time and hang it by a thread. Pick up another undisturbed plastic rod and bring it close to the hanging rod. Nothing happens to either rod.

Experiment 2

Rub both plastic rods with wool. Now the hanging rod tries to move away from the handheld rod when you bring the two close together. Two glass rods rubbed with silk also repel each other.

Experiment 3

Bring a glass rod that has been rubbed with silk close to a hanging plastic rod that has been rubbed with wool. These two rods *attract* each other.

Experiment 4

Further observations show that:

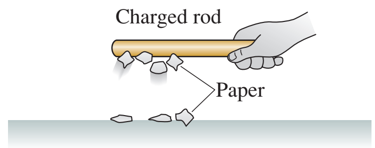
- These forces are greater for rods that have been rubbed more vigorously.
- The strength of the forces decreases as the separation between the rods increases.

No forces were observed in Experiment 1. We will say that the original objects are **neutral.** Rubbing the rods (Experiments 2 and 3) somehow causes forces to be exerted between them. We will call the rubbing process **charging** and say that a rubbed rod is *charged.* For now, these are simply descriptive terms. The terms don't tell us anything about the process itself.

Experiment 2 shows that there is a *long-range repulsive force*, requiring no contact, between two identical objects that have been charged in the *same* way. Furthermore, Experiment 4 shows that the force between two charged objects depends on the distance between them. This is the first long-range force we've encountered since gravity was introduced in Chapter 5. It is also the first time we've observed a repulsive force, so right away we see that new ideas will be needed to understand electricity.

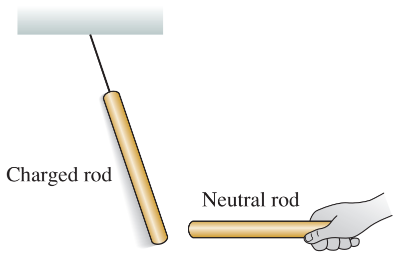
Experiment 3 is a puzzle. Two rods *seem* to have been charged in the same way, by rubbing, but these two rods *attract* each other rather than repel. Why does the outcome of Experiment 3 differ from that of Experiment 2? Back to the lab.

Discovering electricity II



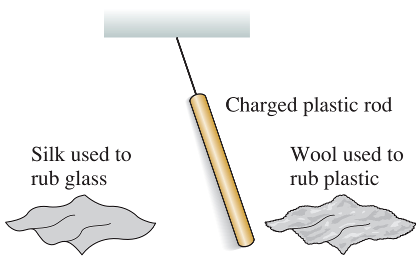
Experiment 5

Hold a charged (i.e., rubbed) plastic rod over small pieces of paper on the table. The pieces of paper leap up and stick to the rod. A charged glass rod does the same. However, a neutral rod has no effect on the pieces of paper.



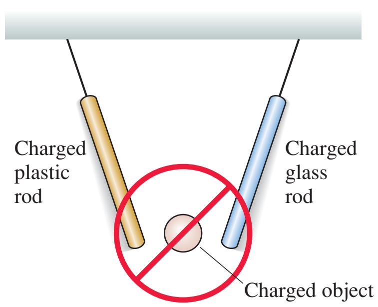
Experiment 6

Rub a plastic rod with wool and a glass rod with silk. Hang both by threads, some distance apart. Both rods are attracted to a *neutral* (i.e., unrubbed) plastic rod that is held close. Interestingly, both are also attracted to a *neutral* glass rod. In fact, the charged rods are attracted to *any* neutral object, such as a finger, a piece of paper, or a metal rod.



Experiment 7

Rub a hanging plastic rod with wool and then hold the *wool* close to the rod. The rod is weakly *attracted* to the wool. The plastic rod is *repelled* by a piece of silk that has been used to rub glass.



Experiment 8

Further experiments show that:

- Other objects, after being rubbed, attract one of the hanging charged rods (plastic or glass) and repel the other. These objects always pick up small pieces of paper.
- There appear to be *no* objects that, after being rubbed, pick up pieces of paper and attract *both* the charged plastic and glass rods.

A plastic comb that has been charged by running it through your hair attracts neutral objects—here drops of water.

Our first set of experiments found that charged objects exert forces on each other. The forces are sometimes attractive, sometimes repulsive. Experiments 5 and 6 show that there is an attractive force between a charged object and a *neutral* (uncharged) object. This discovery presents us with a problem: How can we tell if an object is charged or neutral? Because of the attractive force between a charged and a neutral object, simply observing an electric force does *not* imply that an object is charged.

However, an important characteristic of any *charged* object appears to be that **a charged object picks up small pieces of paper.** This behavior provides a straightforward test to answer the question, Is this object charged? An object that passes the test by picking up paper is charged; an object that fails the test is neutral.

These observations let us tentatively advance the first stages of a **charge model.**

Charge model, part I The basic postulates of our model are:

1. Frictional forces, such as rubbing, add something called **charge** to an object or remove it from the object. The process itself is called *charging*. More vigorous rubbing produces a larger quantity of charge.

2. There are two and only two kinds of charge. For now we will call these "plastic charge" and "glass charge." Other objects can sometimes be charged by rubbing, but the charge they receive is either "plastic charge" or "glass charge."
3. Two **like charges** (plastic/plastic or glass/glass) exert repulsive forces on each other. Two **opposite charges** (plastic/glass) attract each other.
4. The force between two charges is a long-range force. The size of the force increases as the quantity of charge increases and decreases as the distance between the charges increases.
5. *Neutral* objects have an *equal mixture* of both "plastic charge" and "glass charge." The rubbing process somehow manages to separate the two.

Postulate 2 is based on Experiment 8. If an object is charged (i.e., picks up paper), it always attracts one charged rod and repels the other. That is, it acts either "like plastic" or "like glass." If there were a third kind of charge, different from the first two, an object with that charge should pick up paper and attract *both* the charged plastic and glass rods. No such objects have ever been found.

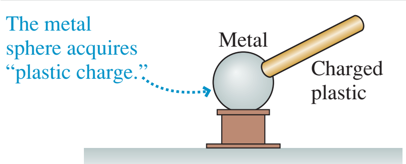
The basis for postulate 5 is the observation in Experiment 7 that a charged plastic rod is attracted to the wool used to rub it but repelled by silk that has rubbed glass. It appears that rubbing glass causes the silk to acquire "plastic charge." The easiest way to explain this is to hypothesize that the silk starts out with equal amounts of "glass charge" and "plastic charge" and that the rubbing somehow transfers "glass charge" from the silk to the rod. This leaves an excess of "glass charge" on the rod and an excess of "plastic charge" on the silk.

While the charge model is *consistent* with the observations, it is by no means proved. One could easily imagine other hypotheses that are just as consistent with the limited observations we have made so far. We still have some large unexplained puzzles, such as why charged objects exert attractive forces on neutral objects.

Electric Properties of Materials

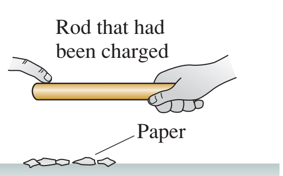
We still need to clarify how different types of materials respond to charges.

Discovering electricity III



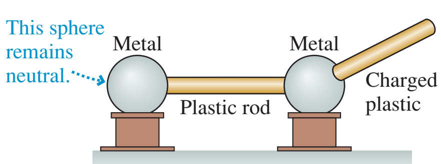
Experiment 9

Charge a plastic rod by rubbing it with wool. Touch a neutral metal sphere with the rubbed area of the rod. The metal sphere then picks up small pieces of paper and repels a charged, hanging plastic rod. The metal sphere appears to have acquired "plastic charge."



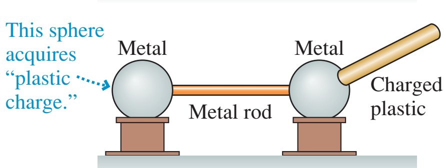
Experiment 10

Charge a plastic rod, then run your finger along it. After you've done so, the rod no longer picks up small pieces of paper or repels a charged, hanging plastic rod. Similarly, the metal sphere of Experiment 9 no longer repels the plastic rod after you touch it with your finger.



Experiment 11

Place two metal spheres close together with a plastic rod connecting them. Charge a second plastic rod, by rubbing, and touch it to one of the metal spheres. Afterward, the metal sphere that was touched picks up small pieces of paper and repels a charged, hanging plastic rod. The other metal sphere does neither.



Experiment 12

Repeat Experiment 11 with a metal rod connecting the two metal spheres. Touch one metal sphere with a charged plastic rod. Afterward, *both* metal spheres pick up small pieces of paper and repel a charged, hanging plastic rod.

Our final set of experiments has shown that

- Charge can be *transferred* from one object to another, but only when the objects *touch.* Contact is required. Removing charge from an object, which you can do by touching it, is called **discharging.**
- There are two types or classes of materials with very different electric properties. We call these *conductors* and *insulators.*

Experiment 12, in which a metal rod is used, is in sharp contrast to Experiment 11. Charge somehow *moves through* or along a metal rod, from one sphere to the other, but remains *fixed in place* on a plastic or glass rod. Let us define **conductors** as those materials through or along which charge easily moves and **insulators** as those materials on or in which charges remain immobile. Glass and plastic are insulators; metal is a conductor.

This information lets us add two more postulates to our charge model:

Charge model, part II

6. There are two types of materials. Conductors are materials through or along which charge easily moves. Insulators are materials on or in which charges remain fixed in place.
7. Charge can be transferred from one object to another by contact.

NOTE ▶ Both insulators and conductors can be charged. They differ in the *mobility* of the charge. ◀

We have by no means exhausted the number of experiments and observations we might try. Early scientific investigators were faced with all of these results, plus many others. Moreover, many of these experiments are hard to reproduce with much accuracy. How should we make sense of it all? The charge model seems promising, but certainly not proven. We have not yet explained how charged objects exert attractive forces on *neutral* objects, nor have we explained what charge is, how it is transferred, or *why* it moves through some objects but not others. Nonetheless, we will take advantage of our historical hindsight and continue to pursue this model. Homework problems will let you practice using the model to explain other observations.

EXAMPLE 25.1 Transferring charge

In Experiment 12, touching one metal sphere with a charged plastic rod caused a second metal sphere to become charged with the same type of charge as the rod. Use the postulates of the charge model to explain this.

SOLVE We need the following ideas from the charge model:

1. Charge is transferred upon contact.
2. Metal is a conductor.
3. Like charges repel.

The plastic rod was charged by rubbing with wool. The charge doesn't move around on the rod, because it is an insulator, but some of the "plastic charge" is transferred to the metal upon contact. Once in the metal, which is a conductor, the charges are free to move around. Furthermore, because like charges repel, these plastic charges quickly move as far apart as they possibly can. Some move through the connecting metal rod to the second sphere. Consequently, the second sphere acquires "plastic charge."

STOP TO THINK 25.1 To determine if an object has "glass charge," you need to

a. See if the object attracts a charged plastic rod.
b. See if the object repels a charged glass rod.
c. Do both a and b.
d. Do either a or b.

25.2 Charge

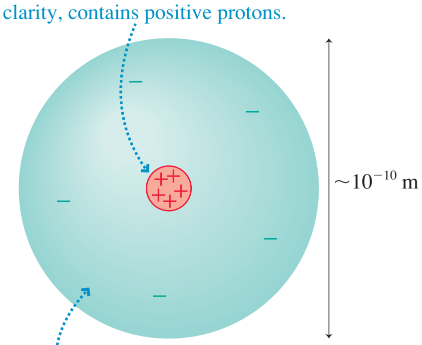
As you probably know, the modern names for the two types of charge are *positive charge* and *negative charge*. You may be surprised to learn that the names were coined by Benjamin Franklin. Franklin found that charge behaves like positive and negative numbers. If a plastic rod is charged twice, by rubbing, and twice transfers charge to a metal sphere, the electric forces exerted by the sphere are doubled. That is, $2 + 2 = 4$. But the sphere is found to be neutral after receiving equal amounts of "plastic charge" and "glass charge." This is like $2 + (-2) = 0$.

So what is positive and what is negative? It's entirely up to us! Franklin established the convention that **a glass rod that has been rubbed with silk is *positively* charged.** That's it. Any other object that repels a charged glass rod is also positively charged. Any charged object that attracts a charged glass rod is negatively charged. Thus **a plastic rod rubbed with wool is negative.** It was only long afterward, with the discovery of electrons and protons, that electrons were found to be attracted to a charged glass rod while protons were repelled. Thus *by convention* electrons have a negative charge and protons a positive charge.

Atoms and Electricity

Now let's fast forward to the 21st century. The theory of electricity was developed without knowledge of atoms, but there is no reason for us to continue to overlook this important part of our contemporary perspective. **FIGURE 25.1** shows that an atom consists of a very small and dense *nucleus* (diameter $\sim 10^{-14}$ m) surrounded by much less massive orbiting *electrons*. The electron orbital frequencies are so enormous ($\sim 10^{15}$ revolutions per second) that the electrons seem to form an **electron cloud** of diameter $\sim 10^{-10}$ m, a factor 10^4 larger than the nucleus. In fact, the wave–particle duality of quantum physics destroys any notion of a well-defined electron trajectory, and *all* we know about the electrons is the size and shape of the electron cloud.

FIGURE 25.1 An atom.



Experiments at the end of the 19th century revealed that electrons are particles with both mass and a negative charge. The nucleus is a composite structure consisting of *protons,* positively charged particles, and neutral *neutrons.* The atom is held together by the attractive electric force between the positive nucleus and the negative electrons.

One of the most important discoveries is that **charge, like mass, is an inherent property of electrons and protons.** It's no more possible to have an electron without charge than it is to have an electron without mass. As far as we know today, electrons and protons have charges of opposite sign but *exactly* equal magnitude. (Very careful experiments have never found any difference.) This atomic-level unit of charge, called the **fundamental unit of charge,** is represented by the symbol e. Table 25.1 shows the masses and charges of protons and electrons. We need to define a unit of charge, which we will do in Section 25.4, before we can specify how much charge e is.

TABLE 25.1 Protons and electrons

Particle	Mass (kg)	Charge
Proton	1.67×10^{-27}	$+e$
Electron	9.11×10^{-31}	$-e$

The Micro/Macro Connection

Electrons and protons are the basic charges of ordinary matter. Consequently, the various observations we made in Section 25.1 need to be explained in terms of electrons and protons.

NOTE ▶ Electrons and protons are particles of matter. Their motion is governed by Newton's laws. Electrons can move from one object to another when the objects are in contact, but neither electrons nor protons can leap through the air from one object to another. An object does not become charged simply from being close to a charged object. ◀

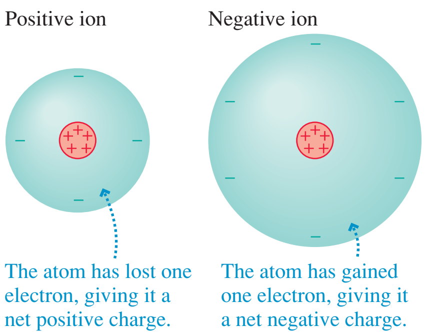
Charge is represented by the symbol q (or sometimes Q). A macroscopic object, such as a plastic rod, has charge

$$q = N_p e - N_e e = (N_p - N_e)e \tag{25.1}$$

where N_p and N_e are the number of protons and electrons contained in the object. Most macroscopic objects have an *equal number* of protons and electrons and therefore have $q = 0$. An object with no *net* charge (i.e., $q = 0$) is said to be *electrically neutral.*

NOTE ▶ *Neutral* does *not* mean "no charges" but, instead, means that there is no *net* charge. ◀

FIGURE 25.2 Positive and negative ions.

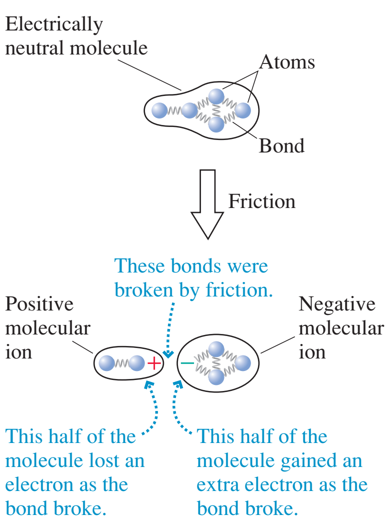


A charged object has an unequal number of protons and electrons. An object is positively charged if $N_p > N_e$. It is negatively charged if $N_p < N_e$. Notice that an object's charge is always an integer multiple of e. That is, the amount of charge on an object varies by small but discrete steps, not continuously. This is called **charge quantization.**

In practice, objects acquire a positive charge not by gaining protons, as you might expect, but by losing electrons. Protons are *extremely* tightly bound within the nucleus and cannot be added to or removed from atoms. Electrons, on the other hand, are bound rather loosely and can be removed without great difficulty. The process of removing an electron from the electron cloud of an atom is called **ionization.** An atom that is missing an electron is called a *positive ion.* Its *net* charge is $q = +e$.

Some atoms can accommodate an *extra* electron and thus become a *negative ion* with net charge $q = -e$. A saltwater solution is a good example. When table salt (the chemical sodium chloride, NaCl) dissolves, it separates into positive sodium ions Na^+ and negative chlorine ions Cl^-. **FIGURE 25.2** shows positive and negative ions.

FIGURE 25.3 Charging by friction usually creates molecular ions as bonds are broken.



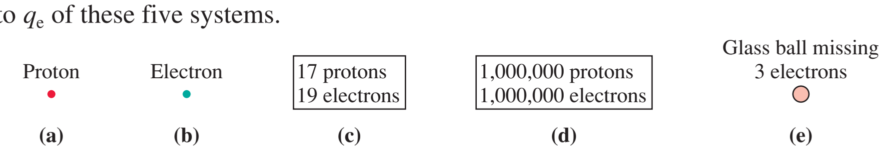
All the charging processes we observed in Section 25.1 involved rubbing and friction. The forces of friction cause molecular bonds at the surface to break as the two materials slide past each other. Molecules are electrically neutral, but **FIGURE 25.3** shows that *molecular ions* can be created when one of the bonds in a large molecule is broken. The positive molecular ions remain on one material and the negative ions on the other, so one of the objects being rubbed ends up with a net positive charge and the other with a net negative charge. This is the way in which a plastic rod is charged by rubbing with wool or a comb is charged by passing through your hair.

Charge Conservation and Charge Diagrams

One of the important discoveries about charge is the **law of conservation of charge:** Charge is neither created nor destroyed. Charge can be transferred from one object to another as electrons and ions move about, but the *total* amount of charge remains constant. For example, charging a plastic rod by rubbing it with wool transfers electrons from the wool to the plastic as the molecular bonds break. The wool is left with a positive charge equal in magnitude but opposite in sign to the negative charge of the rod: $q_{wool} = -q_{plastic}$. The *net* charge remains zero.

Diagrams are going to be an important tool for understanding and explaining charges and the forces on charged objects. As you begin to use diagrams, it will be important to make explicit use of charge conservation. The net number of plusses and minuses drawn on your diagrams should *not* change as you show them moving around.

STOP TO THINK 25.2 Rank in order, from most positive to most negative, the charges q_a to q_e of these five systems.

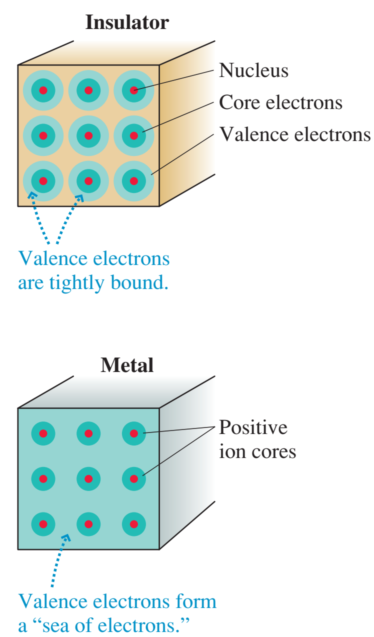


25.3 Insulators and Conductors

You have seen that there are two classes of materials as defined by their electrical properties: insulators and conductors. It's time for a closer look at these materials.

FIGURE 25.4 looks inside an insulator and a metallic conductor. The electrons in the insulator are all tightly bound to the positive nuclei and not free to move around. Charging an insulator by friction leaves patches of molecular ions on the surface, but these patches are immobile.

FIGURE 25.4 A microscopic look at insulators and conductors.



In metals, the outer atomic electrons (called the *valence electrons* in chemistry) are only weakly bound to the nuclei. As the atoms come together to form a solid, these outer electrons become detached from their parent nuclei and are free to wander about through the entire solid. The solid *as a whole* remains electrically neutral, because we have not added or removed any electrons, but the electrons are now rather like a negatively charged gas or liquid—what physicists like to call a **sea of electrons**—permeating an array of positively charged **ion cores.**

The primary consequence of this structure is that electrons in a metal are highly mobile. They can quickly and easily move through the metal in response to electric forces. The motion of charges through a material is what we will later call a **current,** and the charges that physically move are called the **charge carriers.** The charge carriers in metals are electrons.

Metals aren't the only conductors. Ionic solutions, such as salt water, are also good conductors. But the charge carriers in an ionic solution are the ions, not electrons. We'll focus on metallic conductors because of their importance in applications of electricity.

Charging

Insulators are often charged by rubbing. The charge diagrams of FIGURE 25.5 show that the charges on the rod are on the surface and that charge is conserved. The charge can be transferred to another object upon contact, but it doesn't move around on the rod.

FIGURE 25.5 An insulating rod is charged by rubbing.

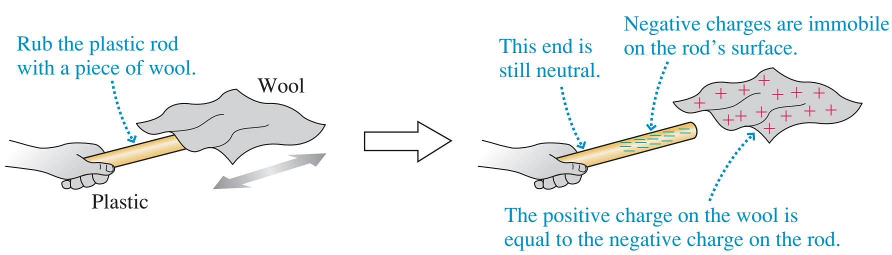
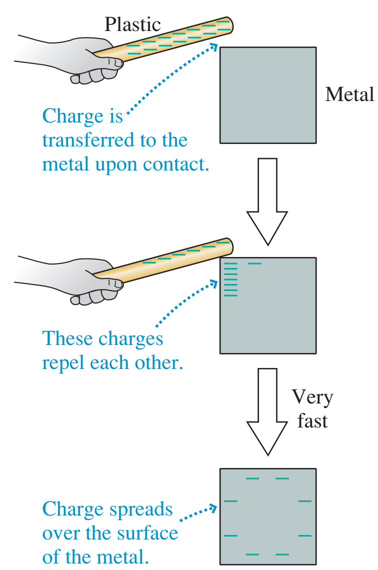


FIGURE 25.6 A conductor is charged by contact with a charged plastic rod.



Metals usually cannot be charged by rubbing, but Experiment 9 showed that a metal sphere can be charged by contact with a charged plastic rod. FIGURE 25.6 gives a pictorial explanation. An essential idea is that **the electrons in a conductor are free to move.** Once charge is transferred to the metal, repulsive forces between the negative charges cause the electrons to move apart from each other.

Note that the newly added electrons do not themselves need to move to the far corners of the metal. Because of the repulsive forces, the newcomers simply "shove" the entire electron sea a little to the side. The electron sea takes an extremely short time to adjust itself to the presence of the added charge, typically less than 10^{-9} s. For all practical purposes, a conductor responds *instantaneously* to the addition or removal of charge.

Other than this very brief interval during which the electron sea is adjusting, the charges in an *isolated* conductor are in static equilibrium. That is, the charges are at rest and there is no net force on any charge. This condition is called **electrostatic equilibrium.** If there *were* a net force on one of the charges, it would quickly move to an equilibrium point at which the force was zero.

Electrostatic equilibrium has an important consequence:

In an isolated conductor, any excess charge is located on the surface of the conductor.

To see this, suppose there *were* an excess electron in the interior of an isolated conductor. The extra electron would upset the electrical neutrality of the interior and exert forces on nearby electrons, causing them to move. But their motion would violate the assumption of static equilibrium, so we're forced to conclude that there cannot be any excess electrons in the interior. Any excess electrons push each other apart until they're all on the surface.

EXAMPLE 25.2 Charging an electroscope

Many electricity demonstrations are carried out with the help of an *electroscope* like the one shown in FIGURE 25.7. Touching the sphere at the top of an electroscope with a charged plastic rod causes the leaves to fly apart and remain hanging at an angle. Use charge diagrams to explain why.

MODEL We'll use the charge model and the model of a conductor as a material through which electrons move.

VISUALIZE FIGURE 25.8 uses a series of charge diagrams to show the charging of an electroscope.

FIGURE 25.7 A charged electroscope.

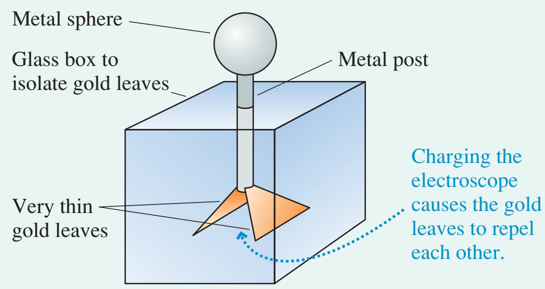


FIGURE 25.8 The process by which an electroscope is charged.

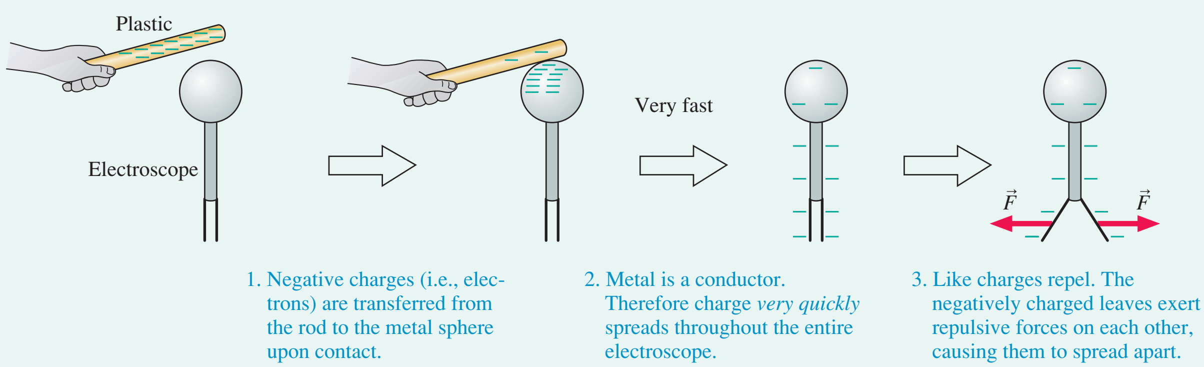
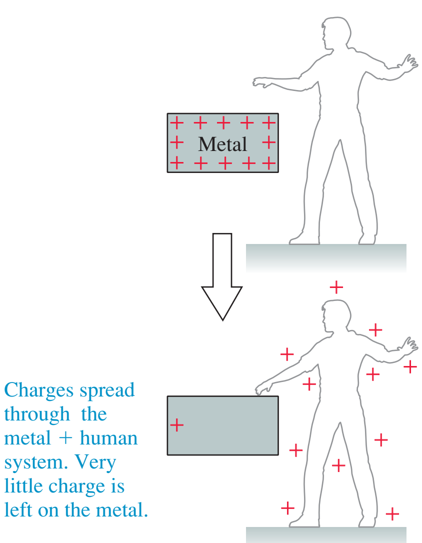


FIGURE 25.9 Touching a charged metal discharges it.



Discharging

Pure water is not a terribly good conductor, but nearly all water contains a variety of dissolved minerals that float around as ions. Dissolved table salt, as we noted previously, separates into Na^+ and Cl^- ions. These ions are the charge carriers, allowing salt water to be a fairly good conductor.

The human body consists largely of salt water. Consequently, and occasionally tragically, humans are reasonably good conductors. This fact allows us to understand how it is that *touching* a charged object discharges it, as we observed in Experiment 10. As FIGURE 25.9 shows, the net effect of touching a charged metal is that it and the conducting human together become a much larger conductor than the metal alone. Any excess charge that was initially confined to the metal can now spread over the larger metal + human conductor. This may not entirely discharge the metal, but in typical circumstances, where the human is much larger than the metal, the residual charge remaining on the metal is much reduced from the original charge. The metal, for most practical purposes, is discharged. In essence, two conductors in contact "share" the charge that was originally on just one of them.

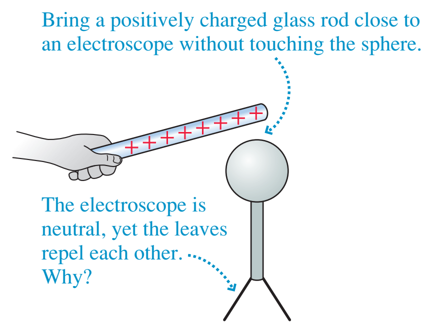
Moist air is a conductor, although a rather poor one. Charged objects in air slowly lose their charge as the object shares its charge with the air. The earth itself is a giant conductor because of its water, moist soil, and a variety of ions. Any object that is physically connected to the earth through a conductor is said to be **grounded.** The effect of being grounded is that the object shares any excess charge it has with the entire earth! But the earth is so enormous that any conductor attached to the earth will be completely discharged.

The purpose of *grounding* objects, such as circuits and appliances, is to prevent the buildup of any charge on the objects. The third prong on appliances and electronics that have a three-prong plug is the ground connection. The building wiring physically connects that third wire deep into the ground somewhere just outside the building, often by attaching it to a metal water pipe that goes underground.

Charge Polarization

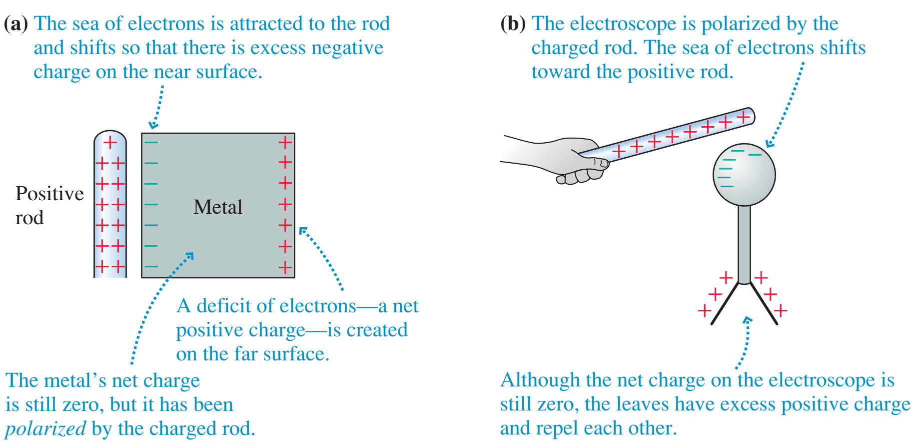
One observation from Section 25.1 still needs an explanation. How do charged objects of either sign exert an attractive force on a *neutral* object? To begin answering this question, **FIGURE 25.10** shows a positively charged rod held close to—but not touching—a *neutral* electroscope. The leaves move apart and stay apart as long as you hold the rod near, but they quickly collapse when it is removed. Can we understand this behavior?

FIGURE 25.10 A charged rod held close to an electroscope causes the leaves to repel each other.



We can, and **FIGURE 25.11a** shows how. Although the metal as a whole is still electrically neutral, we say that the object has been *polarized.* **Charge polarization** is a slight separation of the positive and negative charges in a neutral object. Charge polarization produces an excess positive charge on the leaves of the electroscope shown in **FIGURE 25.11b**, so they repel each other. But because the electroscope has no *net* charge, the electron sea quickly readjusts once the rod is removed.

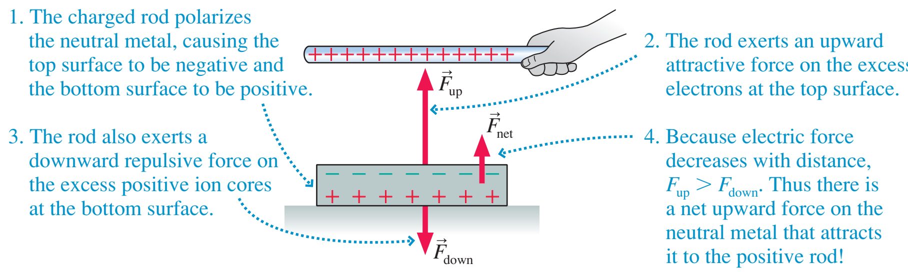
FIGURE 25.11 A charged rod polarizes a metal.



Why don't *all* the electrons in Figure 25.11a rush to the side near the positive charge? Once the electron sea shifts slightly, the stationary positive ions begin to exert a force, a restoring force, pulling the electrons back to the right. The equilibrium position for the sea of electrons is just far enough to the left that the forces due to the external charge and the positive ions are in balance. In practice, the displacement of the electron sea is usually *less than* 10^{-15} *m*!

Charge polarization explains not only why the electroscope leaves deflect but also how a charged object exerts an attractive force on a neutral object. **FIGURE 25.12** on the next page shows a positively charged rod near a neutral piece of metal. Because the electric force decreases with distance, the attractive force on the electrons at the top surface is *slightly greater* than the repulsive force on the ions at the bottom. The net force toward the charged rod is called a **polarization force.** The polarization force arises because the charges in the metal are separated, *not* because the rod and metal are oppositely charged.

FIGURE 25.12 The polarization force on a neutral piece of metal is due to the slight charge separation.

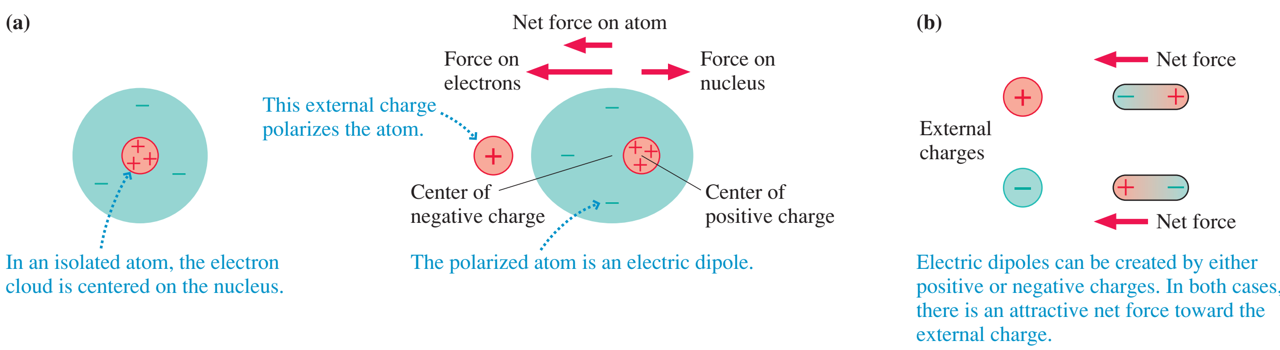


A negatively charged rod would push the electron sea slightly away, polarizing the metal to have a positive upper surface charge and a negative lower surface charge. Once again, these are the conditions for the charge to exert a *net attractive force* on the metal. Thus our charge model explains how a charged object of *either* sign attracts neutral pieces of metal.

The Electric Dipole

Now let's consider a slightly trickier situation. Why does a charged rod pick up paper, which is an insulator rather than a metal? First consider what happens when we bring a positive charge near an atom. As **FIGURE 25.13a** shows, the charge polarizes the atom. The electron cloud doesn't move far, because the force from the positive nucleus pulls it back, but the center of positive charge and the center of negative charge are now slightly separated.

FIGURE 25.13 A neutral atom is polarized by an external charge, forming an *electric dipole*.



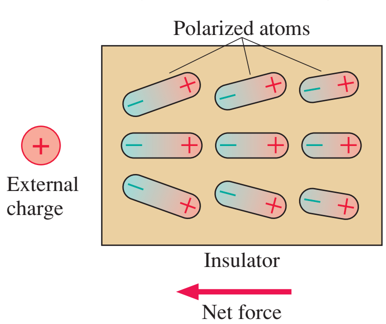
Two opposite charges with a slight separation between them form what is called an **electric dipole.** **FIGURE 25.13b** shows that an external charge of either sign polarizes the atom to produce an electric dipole with the near end opposite in sign to the charge. (The actual distortion from a perfect sphere is minuscule, nothing like the distortion shown in the figure.) The attractive force on the dipole's near end *slightly* exceeds the repulsive force on its far end because the near end is closer to the charge. The net force, an *attractive* force between the charge and the atom, is another example of a polarization force.

An insulator has no sea of electrons to shift if an external charge is brought close. Instead, as **FIGURE 25.14** shows, all the individual atoms inside the insulator become polarized. The polarization force acting *on each atom* produces a net polarization force toward the external charge. This solves the puzzle. A charged rod picks up pieces of paper by

- Polarizing the atoms in the paper,
- Then exerting an attractive polarization force on each atom.

This is important. Make sure you understand all the steps in the reasoning.

FIGURE 25.14 The atoms in an insulator are polarized by an external charge.



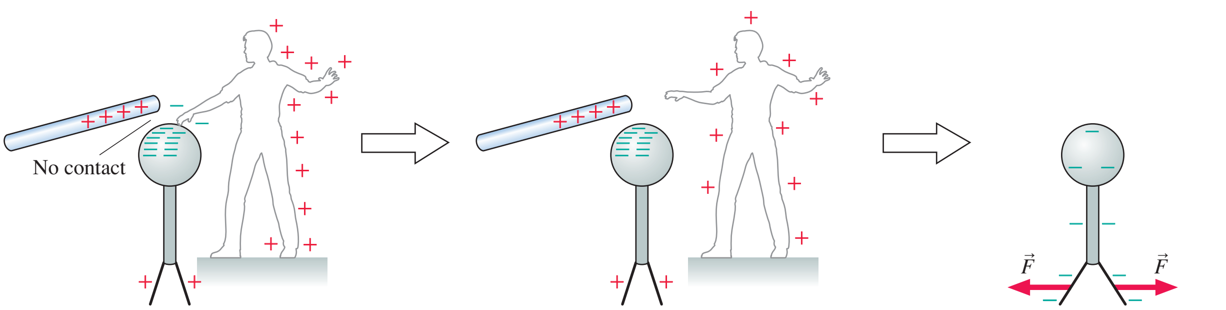
STOP TO THINK 25.3 An electroscope is positively charged by *touching* it with a positive glass rod. The electroscope leaves spread apart and the glass rod is removed. Then a negatively charged plastic rod is brought close to the top of the electroscope, but it doesn't touch. What happens to the leaves?

a. The leaves get closer together.
b. The leaves spread farther apart.
c. One leaf moves higher, the other lower.
d. The leaves don't move.

Charging by Induction

Charge polarization is responsible for an interesting and counterintuitive way of charging an electroscope. **FIGURE 25.15** shows a positively charged glass rod held near an electroscope but not touching it, while a person touches the electroscope with a finger. Unlike what happens in Figure 25.10, the electroscope leaves hardly move.

FIGURE 25.15 Charging by induction.



1. The charged rod polarizes the electroscope + person conductor. The leaves repel slightly due to polarization, but overall the electroscope has an excess of electrons and the person has a deficit of electrons.
2. The negative charge on the electroscope is isolated when contact is broken.
3. When the rod is removed, the leaves first collapse as the polarization vanishes, then repel as the excess negative charge spreads out. The electroscope has been *negatively* charged.

Charge polarization occurs, as it did in Figure 25.10, but this time in the much larger electroscope + person conductor. If the person removes his or her finger while the system is polarized, the electroscope is left with a *net* negative charge and the person has a net positive charge. The electroscope has been charged *opposite to the rod* in a process called **charging by induction.**

25.4 Coulomb's Law

The first three sections have established a *model* of charges and electric forces. This model has successfully provided a qualitative explanation of electric phenomena; now it's time to become quantitative. Experiment 4 in Section 25.1 found that the electric force decreases with distance. The force law that describes this behavior is known as *Coulomb's law*.

Charles Coulomb was one of many scientists investigating electricity in the late 18th century. Coulomb had the idea of studying electric forces using the torsion balance scheme by which Cavendish had measured the value of the gravitational constant G (see Section 13.4). This was a difficult experiment. Despite many obstacles, Coulomb announced in 1785 that the electric force obeys an *inverse-square law* analogous to Newton's law of gravity. Today we know it as **Coulomb's law.**

Coulomb's law:

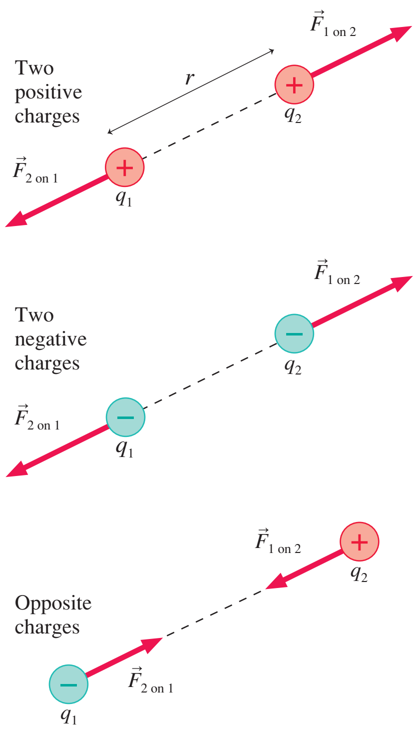
1. If two charged particles having charges q_1 and q_2 are a distance r apart, the particles exert forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2} \qquad (25.2)$$

where K is called the **electrostatic constant.** These forces are an action/reaction pair, equal in magnitude and opposite in direction.

2. The forces are directed along the line joining the two particles. The forces are *repulsive* for two like charges and *attractive* for two opposite charges.

FIGURE 25.16 Attractive and repulsive forces between charged particles.



We sometimes speak of the "force between charge q_1 and charge q_2," but keep in mind that we are really dealing with charged *objects* that also have a mass, a size, and other properties. Charge is not some disembodied entity that exists apart from matter. Coulomb's law describes the force between charged *particles,* which are also called **point charges.** A charged particle, which is an extension of the particle model we used in Part I, has a mass and a charge but has no size.

Coulomb's law looks much like Newton's law of gravity, but there is one important difference: The charge q can be either positive or negative. Consequently, the absolute value signs in Equation 25.2 are especially important. The first part of Coulomb's law gives only the *magnitude* of the force, which is always positive. The direction must be determined from the second part of the law. FIGURE 25.16 shows the forces between different combinations of positive and negative charges.

Units of Charge

Coulomb had no *unit* of charge, so he was unable to determine a value for K, whose numerical value depends on the units of both charge and distance. The SI unit of charge, the **coulomb** (C), is derived from the SI unit of *current,* so we'll have to await the study of current in Chapter 30 before giving a precise definition. For now we'll note that the fundamental unit of charge e has been measured to have the value

$$e = 1.60 \times 10^{-19}\text{ C}$$

This is a very small amount of charge. Stated another way, 1 C is the net charge of roughly 6.25×10^{18} protons.

NOTE ▶ The amount of charge produced by rubbing plastic or glass rods is typically in the range 1 nC (10^{-9} C) to 100 nC (10^{-7} C). This corresponds to an excess or deficit of 10^{10} to 10^{12} electrons. ◀

Once the unit of charge is established, torsion balance experiments such as Coulomb's can be used to measure the electrostatic constant K. In SI units

$$K = 8.99 \times 10^9\text{ N m}^2/\text{C}^2$$

It is customary to round this to $K = 9.0 \times 10^9\text{ N m}^2/\text{C}^2$ for all but extremely precise calculations, and we will do so.

Surprisingly, we will find that Coulomb's law is not explicitly used in much of the theory of electricity. While it *is* the basic force law, most of our future discussion and calculations will be of things called *fields* and *potentials.* It turns out that we can make many future equations easier to use if we rewrite Coulomb's law in a somewhat more complicated way. Let's define a new constant, called the **permittivity constant** ϵ_0 (pronounced "epsilon zero" or "epsilon naught"), as

$$\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$$

Rewriting Coulomb's law in terms of ϵ_0 gives us

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2} \tag{25.3}$$

It will be easiest when using Coulomb's law directly to use the electrostatic constant K. However, in later chapters we will switch to the second version with ϵ_0.

Using Coulomb's Law

Coulomb's law is a force law, and forces are vectors. It has been many chapters since we made much use of vectors and vector addition, but these mathematical techniques will be essential in our study of electricity and magnetism.

There are two important observations regarding Coulomb's law:

1. **Coulomb's law applies only to point charges.** A point charge is an idealized material object with charge and mass but with no size or extension. For practical purposes, two charged objects can be modeled as point charges if they are much smaller than the separation between them.
2. **Electric forces, like other forces, can be superimposed.** If multiple charges 1, 2, 3, . . . are present, the *net* electric force on charge j due to all other charges is

$$\vec{F}_{\text{net}} = \vec{F}_{1\text{ on }j} + \vec{F}_{2\text{ on }j} + \vec{F}_{3\text{ on }j} + \cdots \tag{25.4}$$

where each of the $\vec{F}_{i\text{ on }j}$ is given by Equation 25.2 or 25.3.

These conditions are the basis of a strategy for using Coulomb's law to solve electrostatic force problems.

PROBLEM-SOLVING STRATEGY 25.1 **Electrostatic forces and Coulomb's law** MP

MODEL Identify point charges or objects that can be modeled as point charges.

VISUALIZE Use a *pictorial representation* to establish a coordinate system, show the positions of the charges, show the force vectors on the charges, define distances and angles, and identify what the problem is trying to find. This is the process of translating words to symbols.

SOLVE The mathematical representation is based on Coulomb's law:

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$

- Show the directions of the forces—repulsive for like charges, attractive for opposite charges—on the pictorial representation.
- When possible, do graphical vector addition on the pictorial representation. While not exact, it tells you the type of answer you should expect.
- Write each force vector in terms of its x- and y-components, then add the components to find the net force. Use the pictorial representation to determine which components are positive and which are negative.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 26

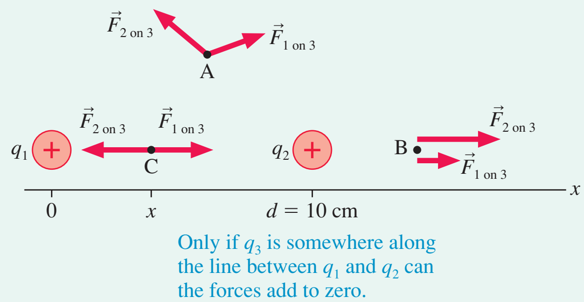
EXAMPLE 25.3 The point of zero force

Two positively charged particles q_1 and $q_2 = 3q_1$ are 10.0 cm apart on the x-axis. Where (other than at infinity) could a third charge q_3 be placed so as to experience no net force?

MODEL Model the charged particles as point charges.

VISUALIZE **FIGURE 25.17** establishes a coordinate system with q_1 at the origin. We first need to identify the region of space in which q_3 must be located. We have no information about the sign of q_3, so apparently the position for which we are looking will work for either sign. You can see from the figure that the forces at point A, above the axis, and at point B, outside the charges, cannot possibly add to zero. However, at point C on the x-axis *between* the charges, the two forces are oppositely directed.

FIGURE 25.17 A pictorial representation of the charges and forces.



SOLVE The mathematical problem is to find the position for which the forces $\vec{F}_{1\text{ on }3}$ and $\vec{F}_{2\text{ on }3}$ are equal in magnitude. If q_3 is distance x from q_1, it is distance $d - x$ from q_2. The *magnitudes* of the forces are

$$F_{1\text{ on }3} = \frac{Kq_1|q_3|}{r_{13}^2} = \frac{Kq_1|q_3|}{x^2}$$

$$F_{2\text{ on }3} = \frac{Kq_2|q_3|}{r_{23}^2} = \frac{K(3q_1)|q_3|}{(d-x)^2}$$

Charges q_1 and q_2 are positive and do not need absolute value signs. Equating the two forces gives

$$\frac{Kq_1|q_3|}{x^2} = \frac{3Kq_1|q_3|}{(d-x)^2}$$

The term $Kq_1|q_3|$ cancels. Multiplying by $x^2(d-x)^2$ gives

$$(d-x)^2 = 3x^2$$

which can be rearranged into the quadratic equation

$$2x^2 + 2dx - d^2 = 2x^2 + 20x - 100 = 0$$

where we used $d = 10$ cm and x is in cm. The solutions to this equation are

$$x = +3.66 \text{ cm and } -13.66 \text{ cm}$$

Both are points where the *magnitudes* of the two forces are equal, but $x = -13.66$ cm is a point where the magnitudes are equal while the directions are the same. The solution we want, which is between the charges, is $x = 3.66$ cm. Thus the point to place q_3 is 3.66 cm from q_1 along the line joining q_1 and q_2.

ASSESS q_1 is smaller than q_2, so we expect the point at which the forces balance to be closer to q_1 than to q_2. The solution seems reasonable. Note that the problem statement has no coordinates, so "$x = 3.66$ cm" is *not* an acceptable answer. You need to describe the position relative to q_1 and q_2.

EXAMPLE 25.4 Three charges

Three charged particles with $q_1 = -50$ nC, $q_2 = +50$ nC, and $q_3 = +30$ nC are placed on the corners of the 5.0 cm × 10.0 cm rectangle shown in **FIGURE 25.18**. What is the net force on charge q_3 due to the other two charges? Give your answer both in component form and as a magnitude and direction.

FIGURE 25.18 The three charges.

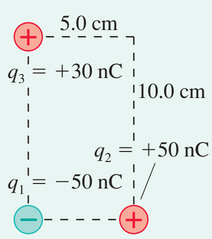
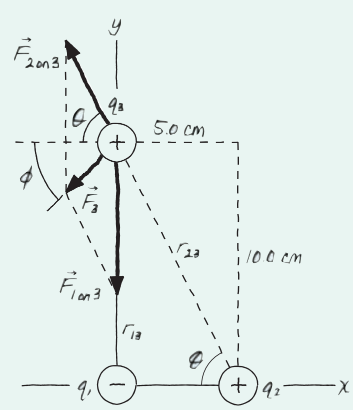


FIGURE 25.19 A pictorial representation of the charges and forces.



MODEL Model the charged particles as point charges.

VISUALIZE The pictorial representation of **FIGURE 25.19** establishes a coordinate system. q_1 and q_3 are opposite charges, so force vector $\vec{F}_{1\text{ on }3}$ is an attractive force toward q_1. q_2 and q_3 are like charges, so force vector $\vec{F}_{2\text{ on }3}$ is a repulsive force away from q_2. q_1 and q_2 have equal magnitudes, but $\vec{F}_{2\text{ on }3}$ has been drawn shorter than $\vec{F}_{1\text{ on }3}$ because q_2 is farther from q_3. Vector addition has been used to draw the net force vector and to define the angle ϕ.

SOLVE The question asks for a *force*, so our answer will be the *vector* sum $\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3}$. We need to write $\vec{F}_{1\text{ on }3}$ and $\vec{F}_{2\text{ on }3}$ in component form. The magnitude of force $\vec{F}_{1\text{ on }3}$ can be found using Coulomb's law:

$$F_{1\text{ on }3} = \frac{K|q_1||q_3|}{r_{13}^2}$$

$$= \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(50 \times 10^{-9}\text{ C})(30 \times 10^{-9}\text{ C})}{(0.100 \text{ m})^2}$$

$$= 1.35 \times 10^{-3} \text{ N}$$

where we used $r_{13} = 10.0$ cm.

The pictorial representation shows that $\vec{F}_{1\text{ on }3}$ points downward, in the negative y-direction, so

$$\vec{F}_{1\text{ on }3} = -1.35 \times 10^{-3}\,\hat{j}\text{ N}$$

To calculate $\vec{F}_{2\text{ on }3}$ we first need the distance r_{23} between the charges:

$$r_{23} = \sqrt{(5.0\text{ cm})^2 + (10.0\text{ cm})^2} = 11.2\text{ cm}$$

The magnitude of $\vec{F}_{2\text{ on }3}$ is thus

$$\begin{aligned} F_{2\text{ on }3} &= \frac{K|q_2||q_3|}{r_{23}^{\,2}} \\ &= \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(50\times10^{-9}\text{ C})(30\times10^{-9}\text{ C})}{(0.112\text{ m})^2} \\ &= 1.08\times10^{-3}\text{ N} \end{aligned}$$

This is only a magnitude. The *vector* $\vec{F}_{2\text{ on }3}$ is

$$\vec{F}_{2\text{ on }3} = -F_{2\text{ on }3}\cos\theta\,\hat{\imath} + F_{2\text{ on }3}\sin\theta\,\hat{\jmath}$$

where angle θ is defined in the figure and the signs (negative x-component, positive y-component) were determined from the pictorial representation. From the geometry of the rectangle,

$$\theta = \tan^{-1}\left(\frac{10.0\text{ cm}}{5.0\text{ cm}}\right) = \tan^{-1}(2.0) = 63.4°$$

Thus $\vec{F}_{2\text{ on }3} = (-4.83\hat{\imath} + 9.66\hat{\jmath})\times10^{-4}$ N. Now we can add $\vec{F}_{1\text{ on }3}$ and $\vec{F}_{2\text{ on }3}$ to find

$$\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3} = (-4.83\hat{\imath} - 3.84\hat{\jmath})\times10^{-4}\text{ N}$$

This would be an acceptable answer for many problems, but sometimes we need the net force as a magnitude and direction. With angle ϕ as defined in the figure, these are

$$F_3 = \sqrt{F_{3x}^{\,2} + F_{3y}^{\,2}} = 6.2\times10^{-4}\text{ N}$$

$$\phi = \tan^{-1}\left|\frac{F_{3y}}{F_{3x}}\right| = 38°$$

Thus $\vec{F}_3 = (6.2\times10^{-4}$ N, 38° below the negative x-axis).

ASSESS The forces are not large, but they are typical of electrostatic forces. Even so, you'll soon see that these forces can produce very large accelerations because the masses of the charged objects are usually very small.

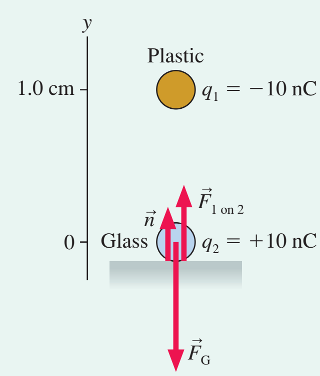
EXAMPLE 25.5 Lifting a glass bead

A small plastic sphere charged to −10 nC is held 1.0 cm above a small glass bead at rest on a table. The bead has a mass of 15 mg and a charge of +10 nC. Will the glass bead "leap up" to the plastic sphere?

MODEL Model the plastic sphere and glass bead as point charges.

VISUALIZE **FIGURE 25.20** establishes a y-axis, identifies the plastic sphere as q_1 and the glass bead as q_2, and shows a free-body diagram.

FIGURE 25.20 A pictorial representation of the charges and forces.



SOLVE If $F_{1\text{ on }2}$ is less than the gravitational force $F_G = m_{\text{bead}}g$, then the bead will remain at rest on the table with $\vec{F}_{1\text{ on }2} + \vec{F}_G + \vec{n} = \vec{0}$. But if $F_{1\text{ on }2}$ is greater than $m_{\text{bead}}g$, the glass bead will accelerate upward from the table. Using the values provided, we have

$$F_{1\text{ on }2} = \frac{K|q_1||q_2|}{r^2} = 9.0\times10^{-3}\text{ N}$$

$$F_G = m_{\text{bead}}g = 1.5\times10^{-4}\text{ N}$$

$F_{1\text{ on }2}$ exceeds $m_{\text{bead}}g$ by a factor of 60, so the glass bead will leap upward.

ASSESS The values used in this example are realistic for spheres ≈ 2 mm in diameter. In general, as in this example, electric forces are *significantly* larger than gravitational forces. Consequently, we can neglect gravity when working electric-force problems unless the particles are fairly massive.

EXAMPLE 25.6 A point charge and a charged wire

Coulomb's inverse-square law applies only to the electric force between two point charges. Your lab assignment for the week is to discover the law describing the force between a point charge and a long, straight, charged metal wire. It is postulated that the force on a point charge q is characterized by a *power law* $F \propto qr^n$, where r is the distance from the wire. To test this hypothesis and, if it is correct, to determine the exponent n, you first set up a long, straight metal wire and charge it by connecting it to a high-voltage

Continued

power supply. You then charge a small plastic ball and, using a sensitive force probe, measure the force on the ball at different distances from the wire. Your data are as follows:

Distance (cm)	Force (μN)
2.0	895
4.0	455
6.0	310
8.0	215
10.0	185

Is the force described by a power law? And if so, what is the exponent?

MODEL Model the small plastic ball as a point charge.

SOLVE A power law is represented by a linear log-log graph. To see why, we can write the postulated force law as

$$F = cqr^n$$

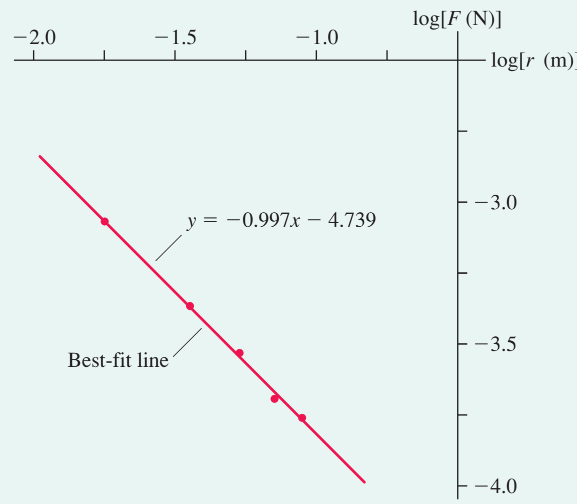
where c is an unknown proportionality constant. If we take the logarithm of both sides, applying the rules $\log(ab) = \log a + \log b$ and $\log a^n = n \log a$, we get

$$\log F = \log(cqr^n) = \log(cq) + \log r^n = \log(cq) + n \log r$$

If we plot $\log F$ on the y-axis against $\log r$ on the x-axis—a log-log graph—it should be a straight line with slope n. A nonlinear log-log graph would disprove the hypothesis that the force is characterized by a power law.

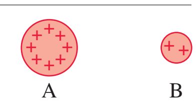
FIGURE 25.21 is a graph of $\log F$ versus $\log r$. It is clearly linear, which validates the postulated power-law force. And because distances were measured to only two significant figures, the experimental slope of -0.997 is consistent with the simpler $n = -1$. Thus our data show that the force between a point charge and a long, charged wire can be characterized as $F \propto q/r$.

FIGURE 25.21 A log-log graph of force versus distance.



ASSESS The force depends inversely on the distance. The inverse-square dependence of Coulomb's law describes only the force between two point charges.

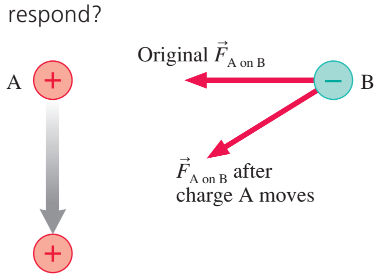
STOP TO THINK 25.4 Charged spheres A and B exert repulsive forces on each other. $q_A = 4q_B$. Which statement is true?



a. $F_{\text{A on B}} > F_{\text{B on A}}$ b. $F_{\text{A on B}} = F_{\text{B on A}}$ c. $F_{\text{A on B}} < F_{\text{B on A}}$

25.5 The Field Model

FIGURE 25.22 If charge A moves, how long does it take the force vector on B to respond?



Electric and magnetic forces, like gravity, are *long-range* forces; no contact is required for one charged particle to exert a force on another. But this raises some troubling issues. For example, consider the charged particles A and B in **FIGURE 25.22**. If A suddenly starts moving, as shown by the arrow, the force vector on B must pivot to follow A. Does this happen *instantly*? Or is there some *delay* between when A moves and when the force $\vec{F}_{\text{A on B}}$ responds?

Neither Coulomb's law nor Newton's law of gravity is dependent on time, so the answer from the perspective of Newtonian physics has to be "instantly." Yet most scientists found this troubling. What if A is 100,000 light years from B? Will B respond *instantly* to an event 100,000 light years away? The idea of instantaneous transmission of forces had become unbelievable to most scientists by the beginning of the 19th century. But if there is a delay, how long is it? How does the information to "change force" get sent from A to B? These were the issues when a young Michael Faraday appeared on the scene.

Michael Faraday is one of the most interesting figures in the history of science. Because of the late age at which he started his education—he was a teenager—he never became fluent in mathematics. In place of equations, Faraday's brilliant and insightful mind developed many ingenious *pictorial* methods for thinking about and describing physical phenomena. By far the most important of these was the field.

The Concept of a Field

Faraday was particularly impressed with the pattern that iron filings make when sprinkled around a magnet, as seen in **FIGURE 25.23**. The pattern's regularity and the curved lines suggested to Faraday that the *space itself* around the magnet is filled with some kind of magnetic influence. Perhaps the magnet in some way alters the space around it. In this view, a piece of iron near the magnet responds not directly to the magnet but, instead, to the alteration of space caused by the magnet. This space alteration, whatever it is, is the *mechanism* by which the long-range force is exerted.

FIGURE 25.23 Iron filings sprinkled around the ends of a magnet suggest that the influence of the magnet extends into the space around it.

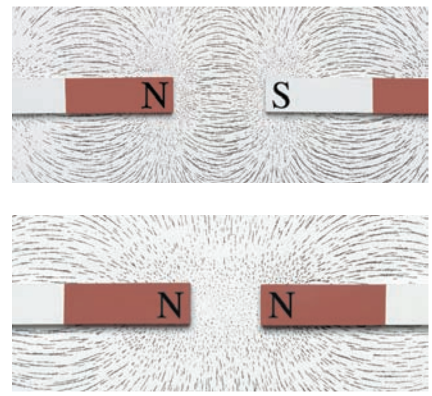
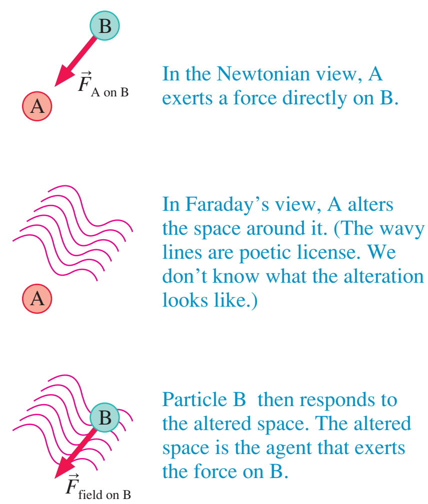


FIGURE 25.24 illustrates Faraday's idea. The Newtonian view was that A and B interact directly. In Faraday's view, A first alters or modifies the space around it, and particle B then comes along and interacts with this altered space. The alteration of space becomes the *agent* by which A and B interact. Furthermore, this alteration could easily be imagined to take a finite time to propagate outward from A, perhaps in a wave-like fashion. If A changes, B responds only when the new alteration of space reaches it. The interaction between B and this alteration of space is a *local* interaction, rather like a contact force.

FIGURE 25.24 Newton's and Faraday's ideas about long-range forces.



Faraday's idea came to be called a **field.** The term "field," which comes from mathematics, describes a function that assigns a vector to every point in space. When used in physics, a field conveys the idea that the physical entity exists at every point in space. That is, indeed, what Faraday was suggesting about how long-range forces operate. The charge makes an alteration *everywhere* in space. Other charges then respond to the alteration at their position. The alteration of the space around a mass is called the *gravitational field*. Similarly, the space around a charge is altered to create the **electric field.**

NOTE ▶ The concept of a field is in sharp contrast to the concept of a particle. A particle exists at *one* point in space. The purpose of Newton's laws of motion is to determine how the particle moves from point to point along a trajectory. A field exists simultaneously at *all* points in space. ◀

Faraday's idea was not taken seriously at first; it seemed too vague and nonmathematical to scientists steeped in the Newtonian tradition of particles and forces. But the significance of the concept of field grew as electromagnetic theory developed during the first half of the 19th century. What seemed at first a pictorial "gimmick" came to be seen as more and more essential for understanding electric and magnetic forces.

Faraday's field ideas were finally placed on a mathematical foundation in 1865 by James Clerk Maxwell. Maxwell was able to describe completely all the known behaviors of electric and magnetic fields in four equations, known today as Maxwell's equations. We will explore aspects of Maxwell's theory as we go along, then look at the full implications of Maxwell's equations in Chapter 34.

The Electric Field

We begin our investigation of electric fields by postulating a **field model** that describes how charges interact:

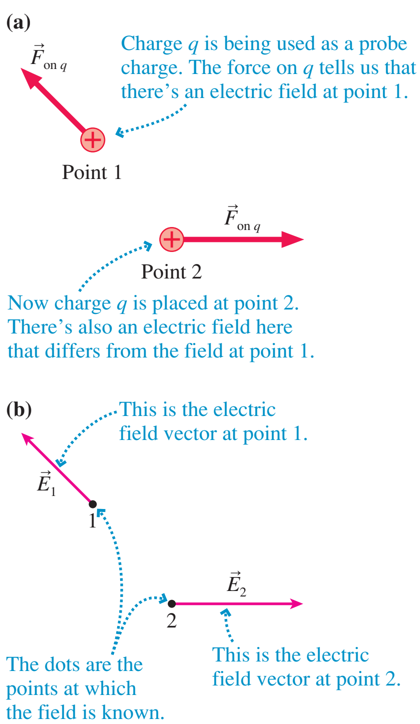
1. Some charges, which we will call the **source charges,** alter the space around them by creating an *electric field* $\vec{E}$.
2. A separate charge *in* the electric field experiences a force $\vec{F}$ exerted *by the field.*

Suppose some set of charges—the source charges—have created an electric field. We can learn about the field by using a different charge q as a *probe charge*. As we move the probe charge around from point to point in space, it experiences a changing electric force $\vec{F}_{\text{on } q}$ due to the other charges. This suggests that "something" is present at each point in space to cause the force that charge q experiences. We can use the force on the probe charge to define the electric field $\vec{E}$ at the point (x, y, z) as

$$\vec{E}(x, y, z) \equiv \frac{\vec{F}_{\text{on } q} \text{ at } (x, y, z)}{q} \tag{25.5}$$

We're *defining* the electric field as a force-to-charge ratio; hence the units of the electric field are newtons per coulomb, or N/C. The magnitude E of the electric field is called the **electric field strength.**

FIGURE 25.25 Charge q is a probe of the electric field.



If a probe charge q experiences an electric force at a point in space, as **FIGURE 25.25a** shows, we say that there is an electric field at that point causing the force. Further, we *define* the electric field at that point to be the vector given by Equation 25.5. **FIGURE 25.25b** shows the electric field only at two points, but you can imagine "mapping out" the electric field by moving charge q all through space.

NOTE ▶ Probe charge q also creates an electric field. But charges don't exert forces on themselves, so q is measuring only the electric field of *other* charges. ◀

The basic idea of the field model is that **the field is the agent that exerts an electric force on a charged particle.** Notice three important ideas about the field:

1. Equation 25.5 assigns a *vector* to *every point* in space. That is, the electric field is a *vector field.* Electric field diagrams will show a sample of the vectors, but there is an electric field vector at every point whether one is shown or not.
2. If q is positive, the electric field vector points in the same direction as the force on the charge.
3. Because q appears in Equation 25.5, it may seem that the electric field depends on the size of the charge used to probe the field. It doesn't. We know from Coulomb's law that the force $\vec{F}_{\text{on } q}$ is proportional to q. Thus the electric field defined in Equation 25.5 is *independent* of the charge q that probes the field. The electric field depends only on the source charges that create the field.

In practice we often want to turn Equation 25.5 around and find the force exerted by a known field. That is, a charged particle with charge q at a point in space where the electric field is $\vec{E}$ experiences an electric force

$$\vec{F}_{\text{on } q} = q\vec{E} \tag{25.6}$$

If q is positive, the force on the particle is in the direction of $\vec{E}$. The force on a negative charge is *opposite* the direction of $\vec{E}$.

EXAMPLE 25.7 Electric forces in a cell

Every cell in your body is electrically active in various ways. For example, nerve propagation occurs when large electric fields in the cell membranes of neurons cause ions to move through the cell walls. The field strength in a typical cell membrane is 1.0×10^7 N/C. What is the magnitude of the electric force on a singly charged calcium ion?

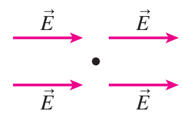
MODEL The ion is a point charge in an electric field. A singly charged ion is missing one electron and has net charge $q = +e$.

SOLVE A charged particle in an electric field experiences an electric force $\vec{F}_{\text{on } q} = q\vec{E}$. In this case, the magnitude of the force is

$$F = eE = (1.6 \times 10^{-19}\ \text{C})(1.0 \times 10^{7}\ \text{N/C}) = 1.6 \times 10^{-12}\ \text{N}$$

ASSESS This may seem like an incredibly tiny force, but it is applied to a particle with mass $m \sim 10^{-26}$ kg. The ion would have an unimaginable acceleration ($F/m \sim 10^{14}\ \text{m/s}^2$) were it not for resistive forces as it moves through the membrane. Even so, an ion can cross the cell wall in less than 1 μs.

STOP TO THINK 25.5 An electron is placed at the position marked by the dot. The force on the electron is



a. Zero. b. To the right. c. To the left.
d. There's not enough information to tell.

The Electric Field of a Point Charge

We will begin to put the definition of the electric field to full use in the next chapter. For now, to develop the ideas, we will determine the electric field of a single point charge q. **FIGURE 25.26a** shows charge q and a point in space at which we would like to know the electric field. We need a second charge, shown as q' in **FIGURE 25.26b**, to serve as a probe of the electric field.

For the moment, assume both charges are positive. The force on q', which is repulsive and directed straight away from q, is given by Coulomb's law:

$$\vec{F}_{\text{on } q'} = \left(\frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}, \text{ away from } q\right) \tag{25.7}$$

It's customary to use $1/4\pi\epsilon_0$ rather than K for field calculations. Equation 25.5 defined the electric field in terms of the force on a probe charge, thus the electric field at this point is

$$\vec{E} = \frac{\vec{F}_{\text{on } q'}}{q'} = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}, \text{ away from } q\right) \tag{25.8}$$

The electric field is shown in **FIGURE 25.26c**.

NOTE ▶ The expression for the electric field is similar to Coulomb's law. To distinguish the two, remember that Coulomb's law has a product of two charges in the numerator. It describes the force between *two* charges. The electric field has a single charge in the numerator. It is the field of *a* charge. ◀

If we calculate the field at a sufficient number of points in space, we can draw a **field diagram** such as the one shown in **FIGURE 25.27**. Notice that the field vectors all point straight away from charge q. Also notice how quickly the arrows decrease in length due to the inverse-square dependence on r.

Keep these three important points in mind when using field diagrams:

1. The diagram is just a representative sample of electric field vectors. The field exists at all the other points. A well-drawn diagram can tell you fairly well what the field would be like at a neighboring point.
2. The arrow indicates the direction and the strength of the electric field *at the point to which it is attached*—that is, at the point where the *tail* of the vector is placed. In this chapter, we indicate the point at which the electric field is measured with

FIGURE 25.26 Charge q' is used to probe the electric field of point charge q.

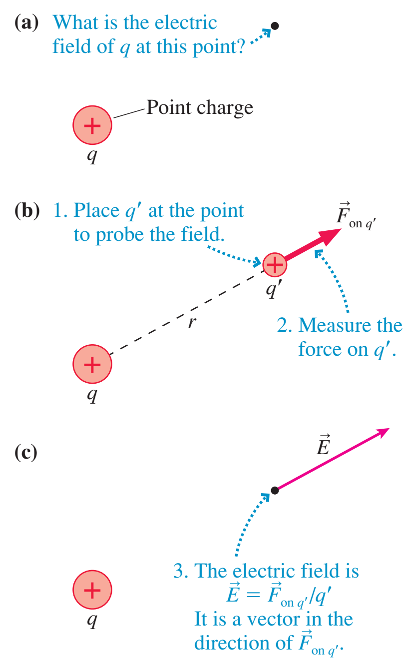
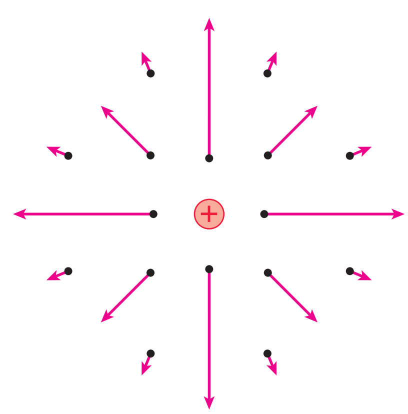


FIGURE 25.27 The electric field of a positive point charge.

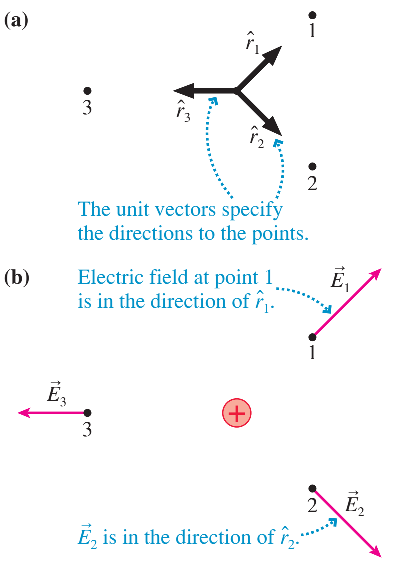


a dot. The length of any vector is significant only relative to the lengths of other vectors.

3. Although we have to draw a vector across the page, from one point to another, an electric field vector is *not* a spatial quantity. It does not "stretch" from one point to another. Each vector represents the electric field at *one point* in space.

Unit Vector Notation

FIGURE 25.28 Using the unit vector $\hat{r}$.



Equation 25.8 is precise, but it's not terribly convenient. Furthermore, what happens if the source charge q is negative? We need a more concise notation to write the electric field, a notation that will allow q to be either positive or negative.

The basic need is to express "away from q" in mathematical notation. "Away from q" is a *direction* in space. To guide us, recall that we already have a notation for expressing certain directions—namely, the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$. For example, unit vector $\hat{\imath}$ means "in the direction of the positive x-axis." With a minus sign, $-\hat{\imath}$ means "in the direction of the negative x-axis." Unit vectors, with a magnitude of 1 and no units, provide purely directional information.

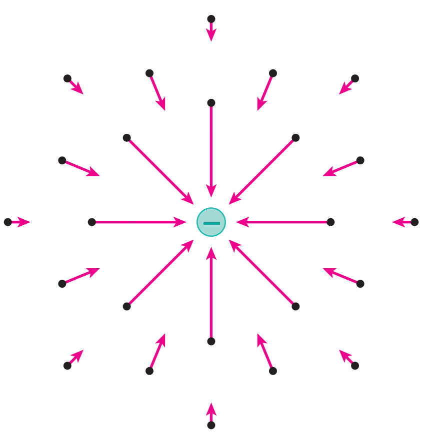
With this in mind, let's define the unit vector $\hat{r}$ to be a vector of length 1 that points from the origin to a point of interest. Unit vector $\hat{r}$ provides no information at all about the distance to the point; it merely specifies the direction.

FIGURE 25.28a shows unit vectors $\hat{r}_1$, $\hat{r}_2$, and $\hat{r}_3$ pointing toward points 1, 2, and 3. Unlike $\hat{\imath}$ and $\hat{\jmath}$, unit vector $\hat{r}$ does not have a fixed direction. Instead, unit vector $\hat{r}$ specifies the direction "straight outward from this point." But that's just what we need to describe the electric field vector. FIGURE 25.28b shows the electric fields at points 1, 2, and 3 due to a positive charge at the origin. No matter which point you choose, the electric field at that point is "straight outward" from the charge. In other words, the electric field $\vec{E}$ points in the direction of the unit vector $\hat{r}$.

With this notation, the electric field at distance r from a point charge q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)} \qquad (25.9)$$

FIGURE 25.29 The electric field of a negative point charge.



where $\hat{r}$ is the unit vector from the charge toward the point at which we want to know the field. Equation 25.9 is identical to Equation 25.8, but written in a notation in which the unit vector $\hat{r}$ expresses the idea "away from q."

Equation 25.9 works equally well if q is negative. A negative sign in front of a vector simply reverses its direction, so the unit vector $-\hat{r}$ points *toward* charge q. FIGURE 25.29 shows the electric field of a negative point charge. It looks like the electric field of a positive point charge except that the vectors point inward, toward the charge, instead of outward.

We'll end this chapter with three examples of the electric field of a point charge. Chapter 26 will expand these ideas to the electric fields of multiple charges and of extended objects.

EXAMPLE 25.8 Calculating the electric field

A -1.0 nC charged particle is located at the origin. Points 1, 2, and 3 have (x, y) coordinates (1 cm, 0 cm), (0 cm, 1 cm), and (1 cm, 1 cm), respectively. Determine the electric field $\vec{E}$ at these points, then show the vectors on an electric field diagram.

MODEL The electric field is that of a negative point charge.

VISUALIZE The electric field points straight *toward* the origin. It will be weaker at (1 cm, 1 cm), which is farther from the charge.

SOLVE The electric field is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

where $q = -1.0\text{ nC} = -1.0 \times 10^{-9}$ C. The distance r is 1.0 cm $=$ 0.010 m for points 1 and 2 and $\left(\sqrt{2} \times 1.0\text{ cm}\right) = 0.0141$ m for point 3. The *magnitude* of $\vec{E}$ at the three points is

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r_1^2}$$
$$= \frac{(9.0\times10^9\ \mathrm{N\,m^2/C^2})(1.0\times10^{-9}\ \mathrm{C})}{(0.010\ \mathrm{m})^2} = 90{,}000\ \mathrm{N/C}$$

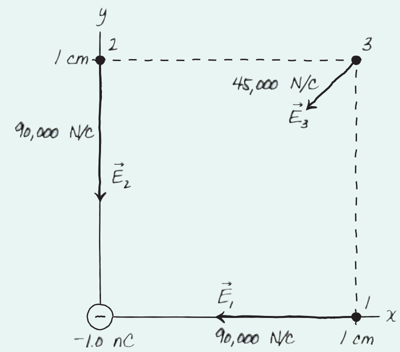
$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r_3^2}$$
$$= \frac{(9.0\times10^9\ \mathrm{N\,m^2/C^2})(1.0\times10^{-9}\ \mathrm{C})}{(0.0141\ \mathrm{m})^2} = 45{,}000\ \mathrm{N/C}$$

Because q is negative, the field at each of these positions points directly at charge q. The electric field vectors, in component form, are

$$\vec{E}_1 = -90{,}000\hat{\imath}\ \mathrm{N/C}$$
$$\vec{E}_2 = -90{,}000\hat{\jmath}\ \mathrm{N/C}$$
$$\vec{E}_3 = -E_3\cos45°\hat{\imath} - E_3\sin45°\hat{\jmath}$$
$$= (-31{,}800\hat{\imath} - 31{,}800\hat{\jmath})\ \mathrm{N/C}$$

These vectors are shown on the electric field diagram of **FIGURE 25.30**.

FIGURE 25.30 The electric field diagram of a -1.0 nC charged particle.



EXAMPLE 25.9 The electric field of a proton

The electron in a hydrogen atom orbits the proton at a radius of 0.053 nm.

a. What is the proton's electric field strength at the position of the electron?
b. What is the magnitude of the electric force on the electron?

SOLVE a. The proton's charge is $q = e$. Its electric field strength at the distance of the electron is

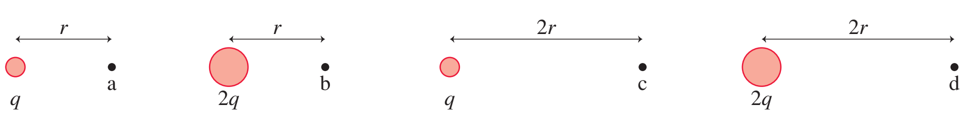
$$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{1.6\times10^{-19}\ \mathrm{C}}{(5.3\times10^{-11}\ \mathrm{m})^2} = 5.1\times10^{11}\ \mathrm{N/C}$$

Notice how large this field is in comparison to the field of Example 25.8.

b. We could use Coulomb's law to find the force on the electron, but the whole point of knowing the electric field is that we can use it directly to find the force on a charge in the field. The magnitude of the force on the electron is

$$F_{\text{on elec}} = |q_e|E_{\text{of proton}}$$
$$= (1.60\times10^{-19}\ \mathrm{C})(5.1\times10^{11}\ \mathrm{N/C})$$
$$= 8.2\times10^{-8}\ \mathrm{N}$$

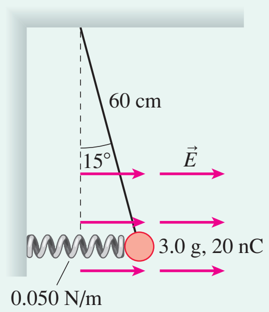
STOP TO THINK 25.6 Rank in order, from largest to smallest, the electric field strengths E_a to E_d at points a to d.



CHALLENGE EXAMPLE 25.10 A charge in static equilibrium

A horizontal electric field causes the charged ball in FIGURE 25.31 to hang at a 15° angle, as shown. The spring is plastic, so it doesn't discharge the ball, and in its equilibrium position the spring extends only to the vertical dashed line. What is the electric field strength?

FIGURE 25.31 A charged ball hanging in static equilibrium.



MODEL Model the ball as a point charge in static equilibrium. The electric force on the ball is $\vec{F}_E = q\vec{E}$. The charge is positive, so the force is in the same direction as the field.

VISUALIZE FIGURE 25.32 is a free-body diagram for the ball.

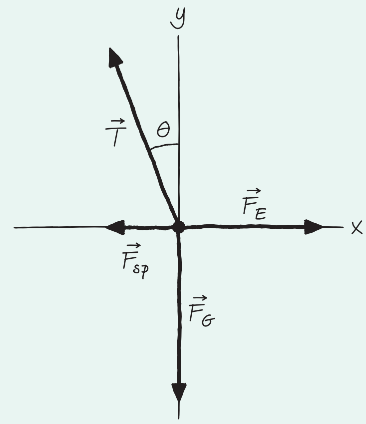
SOLVE The ball is in static equilibrium, so the net force on the ball must be zero. With the field applied, the spring is stretched by $\Delta x = L\sin\theta = (0.60\text{ m})(\sin 15°) = 0.16\text{ m}$, where L is the string length, and exerts a pulling force $F_{sp} = k\,\Delta x$ to the left.

Newton's first law, which we've not used in quite some time, is

$$\sum F_x = F_E - F_{sp} - T\sin\theta = 0$$

$$\sum F_y = T\cos\theta - F_G = T\cos\theta - mg = 0$$

FIGURE 25.32 The free-body diagram.



From the y-equation,

$$T = \frac{mg}{\cos\theta}$$

The x-equation is then

$$qE - k\,\Delta x - mg\tan\theta = 0$$

We can solve this for the electric field strength:

$$E = \frac{mg\tan\theta + k\,\Delta x}{q}$$

$$= \frac{(0.0030\text{ kg})(9.8\text{ m/s}^2)\tan 15° + (0.050\text{ N/m})(0.16\text{ m})}{20 \times 10^{-9}\text{ C}}$$

$$= 7.9 \times 10^5\text{ N/C}$$

ASSESS We don't yet have a way of judging whether this is a reasonable field strength, but we'll see in the next chapter that this is typical of the electric field strength near an object that has been charged by rubbing.

SUMMARY

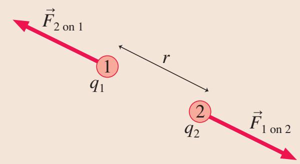
The goal of Chapter 25 has been to describe electric phenomena in terms of charges, forces, and fields.

General Principles

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$



These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the sum of the forces from all other charges.
- The unit of charge is the coulomb (C).
- The electrostatic constant is $K = 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$.

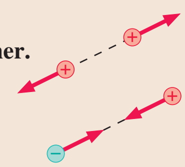
Important Concepts

The Charge Model

There are two kinds of charge, positive and negative.

- Fundamental charges are protons and electrons, with charge $\pm e$ where $e = 1.60 \times 10^{-19}$ C.
- Objects are charged by adding or removing electrons.
- The amount of charge is $q = (N_p - N_e)e$.
- An object with an equal number of protons and electrons is neutral, meaning no *net* charge.

Charged objects exert electric forces on each other.



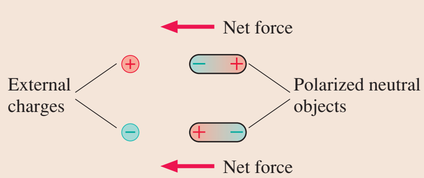
- Like charges repel, opposite charges attract.
- The force increases as the charge increases.
- The force decreases as the distance increases.

There are two types of material, insulators and conductors.

- Charge remains fixed in or on an insulator.
- Charge moves easily through or along conductors.
- Charge is transferred by contact between objects.

Charged objects attract neutral objects.

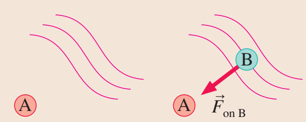
- Charge polarizes metal by shifting the electron sea.
- Charge polarizes atoms, creating electric dipoles.
- The polarization force is always an attractive force.



The Field Model

Charges interact with each other via the electric field $\vec{E}$.

- Charge A alters the space around it by creating an electric field.

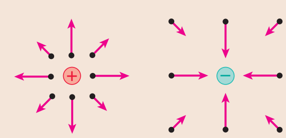


- The field is the agent that exerts a force. The force on charge q_B is $\vec{F}_{\text{on B}} = q_B\vec{E}$.

An electric field is identified and measured in terms of the force on a **probe charge** q:

$$\vec{E} = \vec{F}_{\text{on } q}/q$$

- The electric field exists at all points in space.
- An electric field vector shows the field only at one point, the point at the tail of the vector.



The electric field of a **point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Terms and Notation

neutral	electron cloud	electrostatic equilibrium	coulomb, C
charging	fundamental unit of charge, e	grounded	permittivity constant, ϵ_0
charge model	charge quantization	charge polarization	field
charge, q or Q	ionization	polarization force	electric field, $\vec{E}$
like charges	law of conservation of charge	electric dipole	field model
opposite charges	sea of electrons	charging by induction	source charge
discharging	ion core	Coulomb's law	electric field strength, E
conductor	current	electrostatic constant, K	field diagram
insulator	charge carriers	point charge	

CONCEPTUAL QUESTIONS

1. Can an insulator be charged? If so, how would you charge an insulator? If not, why not?
2. Can a conductor be charged? If so, how would you charge a conductor? If not, why not?
3. Four lightweight balls A, B, C, and D are suspended by threads. Ball A has been touched by a plastic rod that was rubbed with wool. When the balls are brought close together, without touching, the following observations are made:
   - Balls B, C, and D are attracted to ball A.
   - Balls B and D have no effect on each other.
   - Ball B is attracted to ball C.

   What are the charge states (glass, plastic, or neutral) of balls A, B, C, and D? Explain.
4. Charged plastic and glass rods hang by threads.
   a. An object repels the plastic rod. Can you predict what it will do to the glass rod? If so, what? If not, why not?
   b. A different object attracts the plastic rod. Can you predict what it will do to the glass rod? If so, what? If not, why not?
5. A lightweight metal ball hangs by a thread. When a charged rod is held near, the ball moves toward the rod, touches the rod, then quickly "flies away" from the rod. Explain this behavior.
6. Suppose there exists a third type of charge in addition to the two types we've called glass and plastic. Call this third type X charge. What experiment or series of experiments would you use to test whether an object has X charge? State clearly how each possible outcome of the experiments is to be interpreted.
7. A negatively charged electroscope has separated leaves.
   a. Suppose you bring a negatively charged rod close to the top of the electroscope, but not touching. How will the leaves respond? Use both charge diagrams and words to explain.
   b. How will the leaves respond if you bring a positively charged rod close to the top of the electroscope, but not touching? Use both charge diagrams and words to explain.
8. The two oppositely charged metal spheres in FIGURE Q25.8 have equal quantities of charge. They are brought into contact with a neutral metal rod. What is the final charge state of each sphere and of the rod? Use both charge diagrams and words to explain.

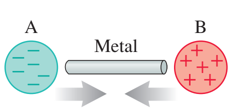


FIGURE Q25.8

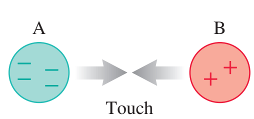


FIGURE Q25.9

9. Metal sphere A in FIGURE Q25.9 has 4 units of negative charge and metal sphere B has 2 units of positive charge. The two spheres are brought into contact. What is the final charge state of each sphere? Explain.
10. Metal spheres A and B in FIGURE Q25.10 are initially neutral and are touching. A positively charged rod is brought near A, but not touching. Is A now positive, negative, or neutral? Use both charge diagrams and words to explain.

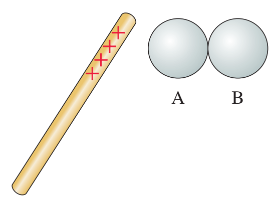


FIGURE Q25.10

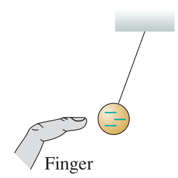


FIGURE Q25.11

11. If you bring your finger near a lightweight, negatively charged hanging ball, the ball swings over toward your finger as shown in FIGURE Q25.11. Use charge diagrams and words to explain this observation.
12. Reproduce FIGURE Q25.12 on your paper. Then draw a dot (or dots) on the figure to show the position (or positions) where an electron would experience no net force.

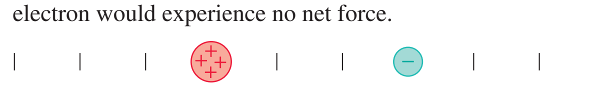


FIGURE Q25.12

13. Charges A and B in FIGURE Q25.13 are equal. Each charge exerts a force on the other of magnitude F. Suppose the charge of B is increased by a factor of 4, but everything else is unchanged. In terms of F, (a) what is the magnitude of the force on A, and (b) what is the magnitude of the force on B?

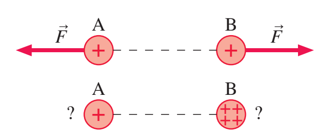


FIGURE Q25.13

14. The electric field strength at one point near a point charge is 1000 N/C. What is the field strength if (a) the distance from the point charge is doubled, and (b) the distance from the point charge is halved?
15. The electric force on a charged particle in an electric field is F. What will be the force if the particle's charge is tripled and the electric field strength is halved?

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 25.1 Developing a Charge Model

Section 25.2 Charge

1. A plastic rod is charged to -12 nC by rubbing.
   a. Have electrons been added to the rod or protons removed? Explain.
   b. How many electrons have been added or protons removed?
2. A glass rod is charged to $+8.0$ nC by rubbing.
   a. Have electrons been removed from the rod or protons added? Explain.
   b. How many electrons have been removed or protons added?
3. A glass rod that has been charged to $+12$ nC touches a metal sphere. Afterward, the rod's charge is $+8.0$ nC.
   a. What kind of charged particle was transferred between the rod and the sphere, and in which direction? That is, did it move from the rod to the sphere or from the sphere to the rod?
   b. How many charged particles were transferred?
4. A plastic rod that has been charged to -15 nC touches a metal sphere. Afterward, the rod's charge is -10 nC.
   a. What kind of charged particle was transferred between the rod and the sphere, and in which direction? That is, did it move from the rod to the sphere or from the sphere to the rod?
   b. How many charged particles were transferred?
5. What is the total charge of all the protons in 1.0 mol of He gas?
6. What is the total charge of all the electrons in 1.0 L of liquid water?

Section 25.3 Insulators and Conductors

7. Figure 25.8 showed how an electroscope becomes negatively charged. The leaves will also repel each other if you touch the electroscope with a positively charged glass rod. Use a series of charge diagrams to explain what happens and why the leaves repel each other.
8. A plastic balloon that has been rubbed with wool will stick to a wall.
   a. Can you conclude that the wall is charged? If not, why not? If so, where does the charge come from?
   b. Draw a series of charge diagrams showing how the balloon is held to the wall.
9. Two neutral metal spheres on wood stands are touching. A negatively charged rod is held directly above the top of the left sphere, not quite touching it. While the rod is there, the right sphere is moved so that the spheres no longer touch. Then the rod is withdrawn. Afterward, what is the charge state of each sphere? Use charge diagrams to explain your answer.
10. You have two neutral metal spheres on wood stands. Devise a procedure for charging the spheres so that they will have like charges of *exactly* equal magnitude. Use charge diagrams to explain your procedure.
11. You have two neutral metal spheres on wood stands. Devise a procedure for charging the spheres so that they will have opposite charges of *exactly* equal magnitude. Use charge diagrams to explain your procedure.

Section 25.4 Coulomb's Law

12. Two 1.0 kg masses are 1.0 m apart (center to center) on a frictionless table. Each has $+10\ \mu$C of charge.
    a. What is the magnitude of the electric force on one of the masses?
    b. What is the initial acceleration of this mass if it is released and allowed to move?
13. Two small plastic spheres each have a mass of 2.0 g and a charge of -50.0 nC. They are placed 2.0 cm apart (center to center).
    a. What is the magnitude of the electric force on each sphere?
    b. By what factor is the electric force on a sphere larger than its weight?
14. A small glass bead has been charged to $+20$ nC. A metal ball bearing 1.0 cm above the bead feels a 0.018 N downward electric force. What is the charge on the ball bearing?
15. Two protons are 2.0 fm apart.
    a. What is the magnitude of the electric force on one proton due to the other proton?
    b. What is the magnitude of the gravitational force on one proton due to the other proton?
    c. What is the ratio of the electric force to the gravitational force?
16. What is the net electric force on charge A in FIGURE EX25.16?

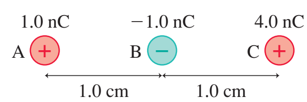


FIGURE EX25.16

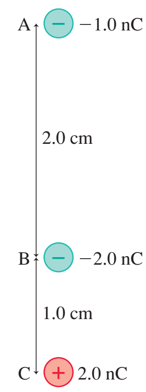


FIGURE EX25.17

17. What is the net electric force on charge B in FIGURE EX25.17?
18. Object A, which has been charged to $+4.0$ nC, is at the origin. Object B, which has been charged to -8.0 nC, is at $(x, y) = (0.0\text{ cm}, 2.0\text{ cm})$. Determine the electric force on each object. Write each force vector in component form.
19. A small plastic bead has been charged to -15 nC. What are the magnitude and direction of the acceleration of (a) a proton and (b) an electron that is 1.0 cm from the center of the bead?

Section 25.5 The Field Model

20. What are the strength and direction of the electric field 1.0 mm from (a) a proton and (b) an electron?
21. The electric field at a point in space is $\vec{E} = (400\hat{\imath} + 100\hat{\jmath})$ N/C.
    a. What is the electric force on a proton at this point? Give your answer in component form.
    b. What is the electric force on an electron at this point? Give your answer in component form.
    c. What is the magnitude of the proton's acceleration?
    d. What is the magnitude of the electron's acceleration?

22. ‖ What magnitude charge creates a 1.0 N/C electric field at a point 1.0 m away?

23. | What are the strength and direction of the electric field 4.0 cm from a small plastic bead that has been charged to −8.0 nC?

24. ‖ The electric field 2.0 cm from a small object points away from the object with a strength of 270,000 N/C. What is the object's charge?

25. ‖ What are the strength and direction of an electric field that will balance the weight of a 1.0 g plastic sphere that has been charged to −3.0 nC?

26. ‖ A +12 nC charge is located at the origin.

a. What are the electric fields at the positions $(x, y) = (5.0\text{ cm}, 0\text{ cm})$, $(-5.0\text{ cm}, 5.0\text{ cm})$, and $(-5.0\text{ cm}, -5.0\text{ cm})$? Write each electric field vector in component form.

b. Draw a field diagram showing the electric field vectors at these points.

27. ‖ A –12 nC charge is located at $(x, y) = (1.0\text{ cm}, 0\text{ cm})$. What are the electric fields at the positions $(x, y) = (5.0\text{ cm}, 0\text{ cm})$, $(-5.0\text{ cm}, 0\text{ cm})$, and $(0\text{ cm}, 5.0\text{ cm})$? Write each electric field vector in component form.

Problems

28. ‖ Pennies today are copper-covered zinc, but older pennies are 3.1 g of solid copper. What are the total positive charge and total negative charge in a solid copper penny that is electrically neutral?

29. | A 2.0 g plastic bead charged to −4.0 nC and a 4.0 g glass bead charged to +8.0 nC are 2.0 cm apart (center to center). What are the accelerations of (a) the plastic bead and (b) the glass bead?

30. ‖ The nucleus of a ^{125}Xe atom (an isotope of the element xenon with mass 125 u) is 6.0 fm in diameter. It has 54 protons and charge $q = +54e$.

a. What is the electric force on a proton 2.0 fm from the surface of the nucleus?

b. What is the proton's acceleration?

Hint: Treat the spherical nucleus as a point charge.

31. | Two 1.0 g spheres are charged equally and placed 2.0 cm apart. When released, they begin to accelerate at 150 m/s^2. What is the magnitude of the charge on each sphere?

32. ‖ Objects A and B are both positively charged. Both have a mass of 100 g, but A has twice the charge of B. When A and B are placed 10 cm apart, B experiences an electric force of 0.45 N.

a. What is the charge on A?

b. If the objects are released, what is the initial acceleration of A?

33. ‖ What is the force $\vec{F}$ on the 1.0 nC charge in FIGURE P25.33? Give your answer as a magnitude and a direction.

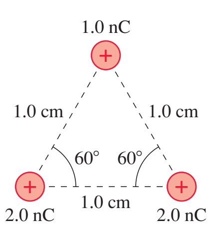


FIGURE P25.33

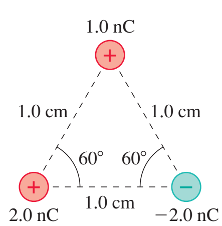


FIGURE P25.34

34. ‖ What is the force $\vec{F}$ on the 1.0 nC charge in FIGURE P25.34? Give your answer as a magnitude and a direction.

35. ‖ What is the force $\vec{F}$ on the −10 nC charge in FIGURE P25.35? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the +x-axis.

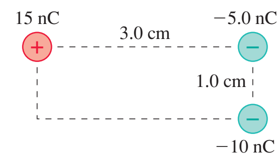


FIGURE P25.35

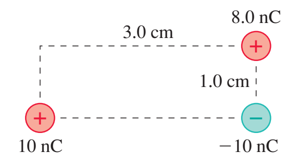


FIGURE P25.36

36. ‖ What is the force $\vec{F}$ on the −10 nC charge in FIGURE P25.36? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the +x-axis.

37. ‖ What is the force $\vec{F}$ on the 5.0 nC charge in FIGURE P25.37? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the +x-axis.

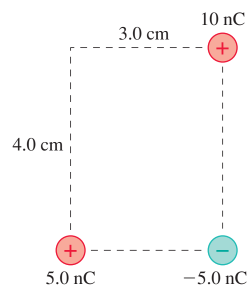


FIGURE P25.37

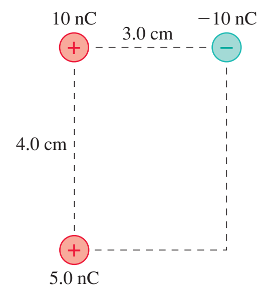


FIGURE P25.38

38. ‖ What is the force $\vec{F}$ on the 5.0 nC charge in FIGURE P25.38? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the +x-axis.

39. ‖‖ What is the force $\vec{F}$ on the 1.0 nC charge in the middle of FIGURE P25.39 due to the four other charges? Give your answer in component form.

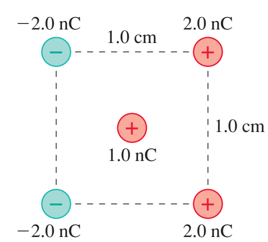


FIGURE P25.39

40. ‖ What is the force $\vec{F}$ on the 1.0 nC charge at the bottom in FIGURE P25.40? Give your answer in component form.

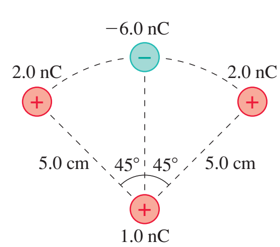


FIGURE P25.40

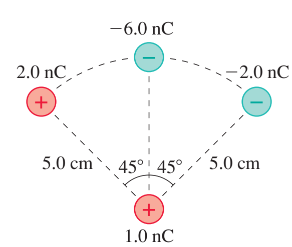


FIGURE P25.41

41. ‖ What is the force $\vec{F}$ on the 1.0 nC charge at the bottom in FIGURE P25.41? Give your answer in component form.

42. ||| A +2.0 nC charge is at the origin and a −4.0 nC charge is at $x = 1.0$ cm.
 a. At what x-coordinate could you place a proton so that it would experience no net force?
 b. Would the net force be zero for an electron placed at the same position? Explain.

43. || The net force on the 1.0 nC charge in FIGURE P25.43 is zero. What is q?

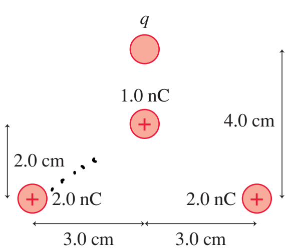


FIGURE P25.43

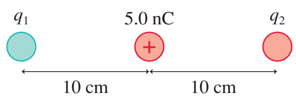


FIGURE P25.44

44. || Charge q_2 in FIGURE P25.44 is in static equilibrium. What is q_1?

45. || A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the y-axis. Find an expression for $(F_{net})_x$, the x-component of the net force on q.

46. || A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the x-axis. Find an expression for $(F_{net})_x$, the x-component of the net force on q, when (a) $|x| < a$ and (b) $|x| > a$.

47. ||| FIGURE P25.47 shows four charges at the corners of a square of side L. What is the magnitude of the net force on q?

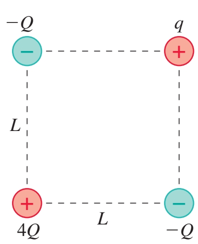


FIGURE P25.47

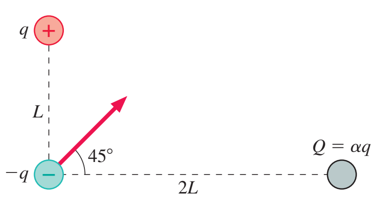


FIGURE P25.48

48. || FIGURE P25.48 shows three charges and the net force on charge $-q$. Charge Q is some multiple α of q. What is α?

49. || Two positive point charges q and $4q$ are at $x = 0$ and $x = L$, respectively, and free to move. A third charge is placed so that the entire three-charge system is in static equilibrium. What are the magnitude, sign, and x-coordinate of the third charge?

50. ||| Suppose the magnitude of the proton charge differs from the magnitude of the electron charge by a mere 1 part in 10^9.
 a. What would be the force between two 2.0-mm-diameter copper spheres 1.0 cm apart? Assume that each copper atom has an equal number of electrons and protons.
 b. Would this amount of force be detectable? What can you conclude from the fact that no such forces are observed?

51. || In a simple model of the hydrogen atom, the electron moves in a circular orbit of radius 0.053 nm around a stationary proton. How many revolutions per second does the electron make?

52. || You have two small, 2.0 g balls that have been given equal but opposite charges, but you don't know the magnitude of the charge. To find out, you place the balls distance d apart on a slippery horizontal surface, release them, and use a motion detector to measure the initial acceleration of one of the balls toward the other. After repeating this for several different separation distances, your data are as follows:

Distance (cm)	Acceleration (m/s^2)
2.0	0.74
3.0	0.30
4.0	0.19
5.0	0.10

Use an appropriate graph of the data to determine the magnitude of the charge.

53. || A 0.10 g honeybee acquires a charge of +23 pC while flying.
BIO
 a. The earth's electric field near the surface is typically (100 N/C, downward). What is the ratio of the electric force on the bee to the bee's weight?
 b. What electric field (strength and direction) would allow the bee to hang suspended in the air?

54. || As a science project, you've invented an "electron pump" that moves electrons from one object to another. To demonstrate your invention, you bolt a small metal plate to the ceiling, connect the pump between the metal plate and yourself, and start pumping electrons from the metal plate to you. How many electrons must be moved from the metal plate to you in order for you to hang suspended in the air 2.0 m below the ceiling? Your mass is 60 kg. **Hint:** Assume that both you and the plate can be modeled as point charges.

55. || You have a lightweight spring whose unstretched length is 4.0 cm. First, you attach one end of the spring to the ceiling and hang a 1.0 g mass from it. This stretches the spring to a length of 5.0 cm. You then attach two small plastic beads to the opposite ends of the spring, lay the spring on a frictionless table, and give each plastic bead the same charge. This stretches the spring to a length of 4.5 cm. What is the magnitude of the charge (in nC) on each bead?

56. || An electric dipole consists of two opposite charges $\pm q$ separated by a small distance s. The product $p = qs$ is called the *dipole moment*. FIGURE P25.56 shows an electric dipole perpendicular to an electric field $\vec{E}$. Find an expression in terms of p and E for the magnitude of the torque that the electric field exerts on the dipole.

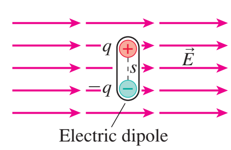


FIGURE P25.56

57. || You sometimes create a spark when you touch a doorknob after shuffling your feet on a carpet. Why? The air always has a few free electrons that have been kicked out of atoms by cosmic rays. If an electric field is present, a free electron is accelerated until it collides with an air molecule. It will transfer its kinetic energy to the molecule, then accelerate, then collide, then accelerate, collide, and so on. If the electron's kinetic energy just before a collision is 2.0×10^{-18} J or more, it has sufficient energy to kick an electron out of the molecule it hits. Where there was one free electron, now there are two! Each of these can then

accelerate, hit a molecule, and kick out another electron. Then there will be four free electrons. In other words, as FIGURE P25.57 shows, a sufficiently strong electric field causes a "chain reaction" of electron production. This is called a *breakdown* of the air. The current of moving electrons is what gives you the shock, and a spark is generated when the electrons recombine with the positive ions and give off excess energy as a burst of light.

a. The average distance an electron travels between collisions is 2.0 μm. What acceleration must an electron have to gain 2.0×10^{-18} J of kinetic energy in this distance?
b. What force must act on an electron to give it the acceleration found in part a?
c. What strength electric field will exert this much force on an electron? This is the *breakdown field strength.* **Note:** The measured breakdown field strength is a little less than your calculated value because our model of the process is a bit too simple. Even so, your calculated value is close.
d. Suppose a free electron in air is 1.0 cm away from a point charge. What minimum charge must this point charge have to cause a breakdown of the air and create a spark?

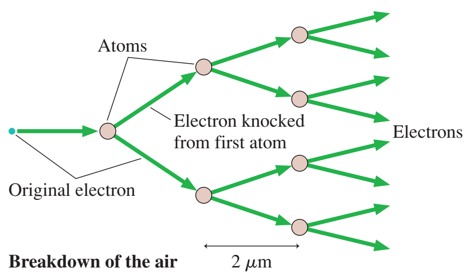


FIGURE P25.57

58. | Two 5.0 g point charges on 1.0-m-long threads repel each other after being charged to +100 nC, as shown in FIGURE P25.58. What is the angle θ? You can assume that θ is a small angle.

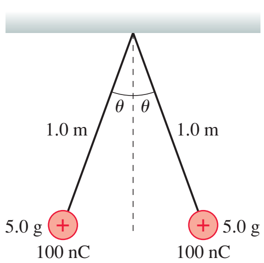


FIGURE P25.58

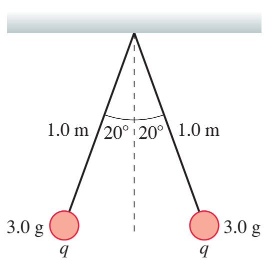


FIGURE P25.59

59. | Two 3.0 g point charges on 1.0-m-long threads repel each other after being equally charged, as shown in FIGURE P25.59. What is the charge q?

60. | What are the electric fields at points 1, 2, and 3 in FIGURE P25.60? Give your answer in component form.

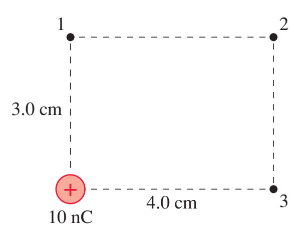


FIGURE P25.60

61. | What are the electric fields at points 1 and 2 in FIGURE P25.61? Give your answer as a magnitude and direction.

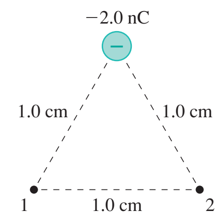


FIGURE P25.61

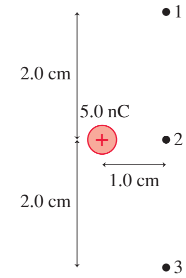


FIGURE P25.62

62. | What are the electric fields at points 1, 2, and 3 in FIGURE P25.62? Give your answer in component form.

63. | A −10.0 nC charge is located at position $(x, y) = (2.0$ cm, 1.0 cm$)$. At what (x, y) position(s) is the electric field
a. $-225{,}000\,\hat{\imath}$ N/C?
b. $(161{,}000\,\hat{\imath} - 80{,}500\,\hat{\jmath})$ N/C?
c. $(28{,}800\,\hat{\imath} + 21{,}600\,\hat{\jmath})$ N/C?

64. | A 10.0 nC charge is located at position $(x, y) = (1.0$ cm, 2.0 cm$)$. At what (x, y) position(s) is the electric field
a. $-225{,}000\,\hat{\imath}$ N/C?
b. $(161{,}000\,\hat{\imath} + 80{,}500\,\hat{\jmath})$ N/C?
c. $(21{,}600\,\hat{\imath} - 28{,}800\,\hat{\jmath})$ N/C?

65. | Three 1.0 nC charges are placed as shown in FIGURE P25.65. Each of these charges creates an electric field $\vec{E}$ at a point 3.0 cm in front of the middle charge.

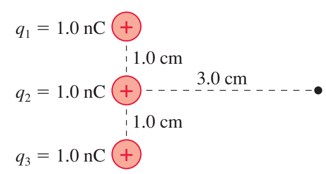


FIGURE P25.65

a. What are the three fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ created by the three charges? Write your answer for each as a vector in component form.
b. Do you think that electric fields obey a principle of superposition? That is, is there a "net field" at this point given by $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$? Use what you learned in this chapter and previously in our study of forces to argue why this is or is not true.
c. If it is true, what is $\vec{E}_{net}$?

66. | An electric field $\vec{E} = 100{,}000\,\hat{\imath}$ N/C causes the 5.0 g point charge in FIGURE P25.66 to hang at a 20° angle. What is the charge on the ball?

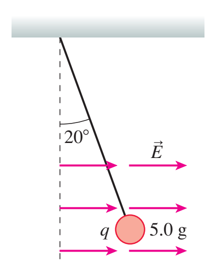


FIGURE P25.66

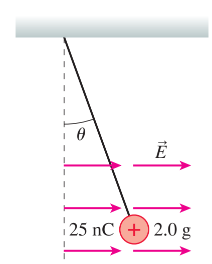


FIGURE P25.67

67. | An electric field $\vec{E} = 200{,}000\,\hat{\imath}$ N/C causes the point charge in FIGURE P25.67 to hang at an angle. What is θ?

In Problems 68 through 71 you are given the equation(s) used to solve a problem. For each of these,

a. Write a realistic problem for which this is the correct equation(s).
b. Finish the solution of the problem.

68. $$\frac{(9.0 \times 10^{9}\ \mathrm{N\,m^2/C^2}) \times N \times (1.60 \times 10^{-19}\ \mathrm{C})}{(1.0 \times 10^{-6}\ \mathrm{m})^2} = 1.5 \times 10^{6}\ \mathrm{N/C}$$

69. $$\frac{(9.0 \times 10^{9}\ \mathrm{N\,m^2/C^2})q^2}{(0.0150\ \mathrm{m})^2} = 0.020\ \mathrm{N}$$

70. $$\frac{(9.0 \times 10^{9}\ \mathrm{N\,m^2/C^2})(15 \times 10^{-9}\ \mathrm{C})}{r^2} = 54{,}000\ \mathrm{N/C}$$

71. $$\sum F_x = 2 \times \frac{(9.0 \times 10^{9}\ \mathrm{N\,m^2/C^2})(1.0 \times 10^{-9}\ \mathrm{C})q}{\big((0.020\ \mathrm{m})/\sin 30^\circ\big)^2} \times \cos 30^\circ = 5.0 \times 10^{-5}\ \mathrm{N}$$
$$\sum F_y = 0\ \mathrm{N}$$

Challenge Problems

72. A 2.0-mm-diameter copper ball is charged to $+50$ nC. What fraction of its electrons have been removed?
73. Three 3.0 g balls are tied to 80-cm-long threads and hung from a *single* fixed point. Each of the balls is given the same charge q. At equilibrium, the three balls form an equilateral triangle in a horizontal plane with 20 cm sides. What is q?
74. The identical small spheres shown in FIGURE CP25.74 are charged to $+100$ nC and -100 nC. They hang as shown in a 100,000 N/C electric field. What is the mass of each sphere?

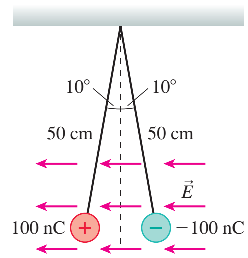


FIGURE CP25.74

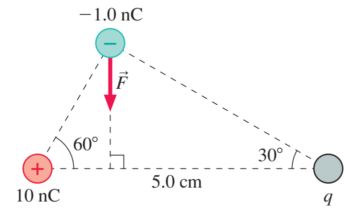


FIGURE CP25.75

75. The force on the -1.0 nC charge is as shown in FIGURE CP25.75. What is the magnitude of this force?
76. In Section 25.3 we claimed that a charged object exerts a net attractive force on an electric dipole. Let's investigate this. FIGURE CP25.76 shows a *permanent* electric dipole consisting of charges $+q$ and $-q$ separated by the fixed distance s. Charge $+Q$ is distance r from the center of the dipole. We'll assume, as is usually the case in practice, that $s \ll r$.
    a. Write an expression for the net force exerted on the dipole by charge $+Q$.
    b. Is this force toward $+Q$ or away from $+Q$? Explain.
    c. Use the *binomial approximation* $(1 + x)^{-n} \approx 1 - nx$ if $x \ll 1$ to show that your expression from part a can be written $F_{net} = 2KqQs/r^3$.
    d. How can an electric force have an inverse-cube dependence? Doesn't Coulomb's law say that the electric force depends on the inverse square of the distance? Explain.

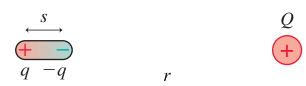


FIGURE CP25.76

STOP TO THINK ANSWERS

Stop to Think 25.1: b. Charged objects are attracted to neutral objects, so an attractive force is inconclusive. Repulsion is the only sure test.

Stop to Think 25.2: $q_e(+3e) > q_a(+1e) > q_d(0) > q_b(-1e) > q_c(-2e)$.

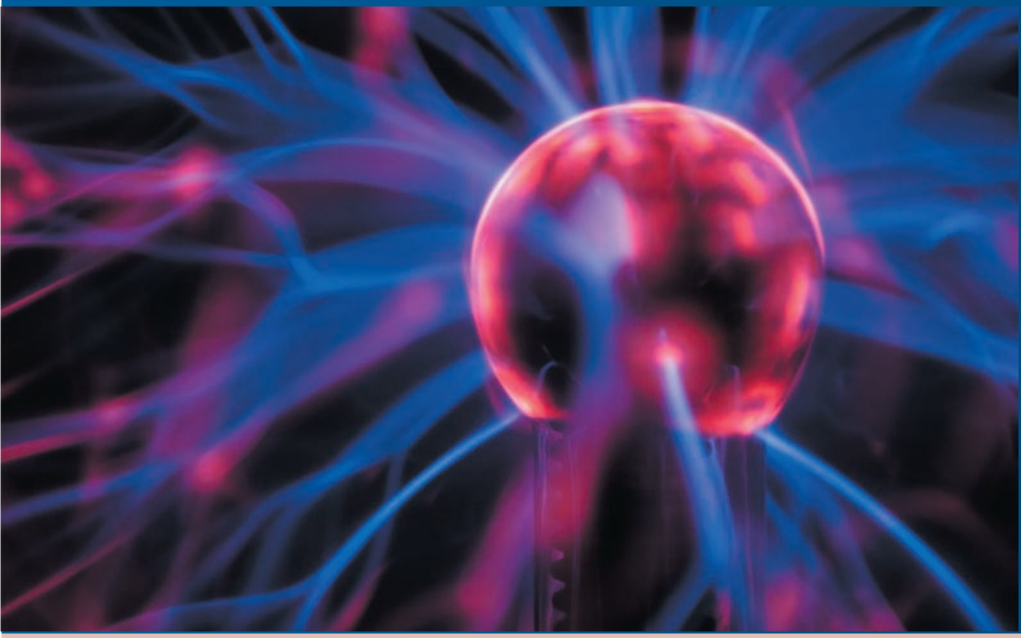
Stop to Think 25.3: a. The negative plastic rod will polarize the electroscope by pushing electrons down toward the leaves. This will partially neutralize the positive charge the leaves had acquired from the glass rod.

Stop to Think 25.4: b. The two forces are an action/reaction pair, opposite in direction but *equal* in magnitude.

Stop to Think 25.5: c. There's an electric field at *all* points, whether an $\vec{E}$ vector is shown or not. The electric field at the dot is to the right. But an electron is a negative charge, so the force of the electric field on the electron is to the left.

Stop to Think 25.6: $E_b > E_a > E_d > E_c$.

26 The Electric Field

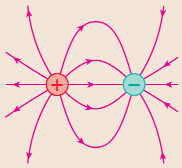


In a plasma ball, electrons follow the electric field lines outward from the center electrode. The streamers appear where gas atoms emit light after the high-speed electrons collide with them.

▶ **Looking Ahead** The goal of Chapter 26 is to learn how to calculate and use the electric field.

Fields of Multiple Charges

You'll learn that the electric field due to several point charges is the vector sum of the individual fields.



You'll also learn to use **electric field lines.** This figure shows the electric field lines of a *dipole*, two equal but opposite point charges.

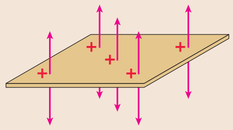
◀ **Looking Back**
Section 25.5 The electric field of a point charge

The Field of a Continuous Distribution of Charge

You'll learn a strategy for computing the electric field of a macroscopic charged object, such as a charged rod or a disk of charge.

- A charged object can be described by its **charge density,** the charge per unit length, area, or volume.
- The vector sum of electric fields will become an integral. We'll develop a step-by-step approach to setting up and evaluating these integrals.

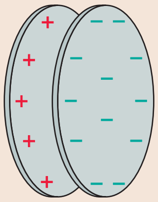
We'll calculate the electric field of charged wires, charged disks, planes of charge, and spheres of charge.

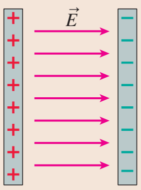


The electric field of a plane of charge is perpendicular to the plane. Many practical devices can be modeled as planes or lines of charge.

Uniform Electric Fields

Two parallel conducting plates with equal but opposite charges are called a **parallel-plate capacitor.**





You'll learn that parallel-plate capacitors are important for creating a **uniform electric field.**

Charges in Electric Fields

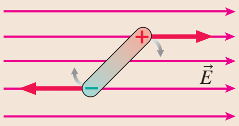
Electric fields exert forces on charged particles. You'll learn to calculate the trajectories of charged particles moving in electric fields.

Older televisions and computer monitors use a *cathode-ray tube*. The picture is formed as a changing electric field sweeps an electron beam back and forth across the screen.

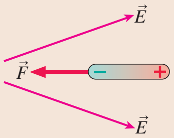
◀ **Looking Back**
Section 4.3 Projectile motion

Dipoles in Electric Fields

You learned in Chapter 25 that charged objects of either sign attract a neutral object. We'll understand better why this happens.



An electric field exerts a *torque* on a dipole, causing it to align with the field.



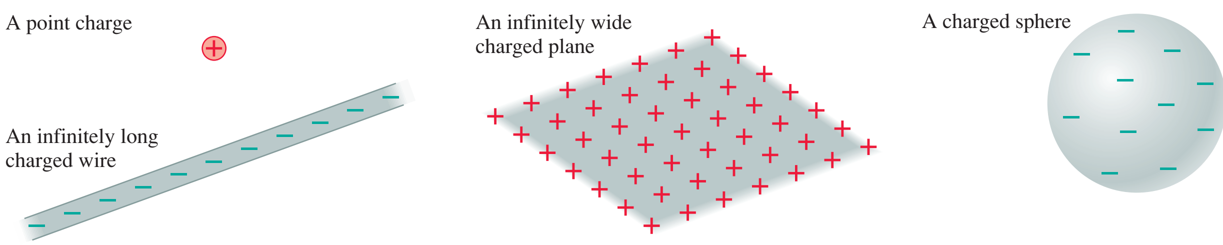
A nonuniform field exerts a force on a dipole, drawing it toward the stronger field.

26.1 Electric Field Models

Chapter 25 made a distinction between those charged particles that are the *sources* of an electric field and other charged particles that *experience* and move in the electric field. This is a very important distinction. Most of this chapter will be concerned with the *sources* of the electric field. Only at the end, once we know how to calculate the electric field, will we look at what happens to charges that find themselves *in* an electric field.

The electric fields used in science and engineering are often caused by fairly complicated distributions of charge. Sometimes these fields require exact calculations, but much of the time we can understand the essential physics on the basis of simplified *models* of the electric field.

FIGURE 26.1 Four basic electric field models.



Four widely used electric field models, illustrated in **FIGURE 26.1**, are:

- The electric field of a point charge.
- The electric field of an infinitely long charged wire.
- The electric field of an infinitely wide charged plane.
- The electric field of a charged sphere.

Small charged objects can often be modeled as point charges or charged spheres. Real wires aren't infinitely long, but in many practical situations this approximation is perfectly reasonable. As we derive and use these electric fields, we'll consider the conditions under which they are appropriate models.

Our starting point is the electric field of a point charge q:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)} \qquad (26.1)$$

where $\hat{r}$ is a unit vector pointing away from q and $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$ is the permittivity constant. **FIGURE 26.2** reminds you of the electric fields of point charges. Although we have to give each vector we draw a length, keep in mind that each arrow represents the electric field *at a point*. The electric field is not a spatial quantity that "stretches" from one end of the arrow to the other.

The electric field was defined as $\vec{E} = \vec{F}_{\text{on } q}/q$, where $\vec{F}_{\text{on } q}$ is the electric force on charge q. Forces add as vectors, so the net force on q due to a group of point charges is the vector sum

$$\vec{F}_{\text{on } q} = \vec{F}_{1 \text{ on } q} + \vec{F}_{2 \text{ on } q} + \cdots$$

Consequently, the net electric field due to a group of point charges is

$$\vec{E}_{\text{net}} = \frac{\vec{F}_{\text{on } q}}{q} = \frac{\vec{F}_{1 \text{ on } q}}{q} + \frac{\vec{F}_{2 \text{ on } q}}{q} + \cdots = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_i \vec{E}_i \qquad (26.2)$$

where $\vec{E}_i$ is the field from point charge i.

Equation 26.2, which is the primary tool for calculating electric fields, tells us that **the net electric field is the *vector sum* of the electric fields due to each charge.** In other words, electric fields obey the *principle of superposition.*

Knowing typical electric field strengths will also be helpful. The values in Table 26.1 on the next page will help you judge the reasonableness of your solutions to problems.

FIGURE 26.2 The electric field of a positive and a negative point charge.

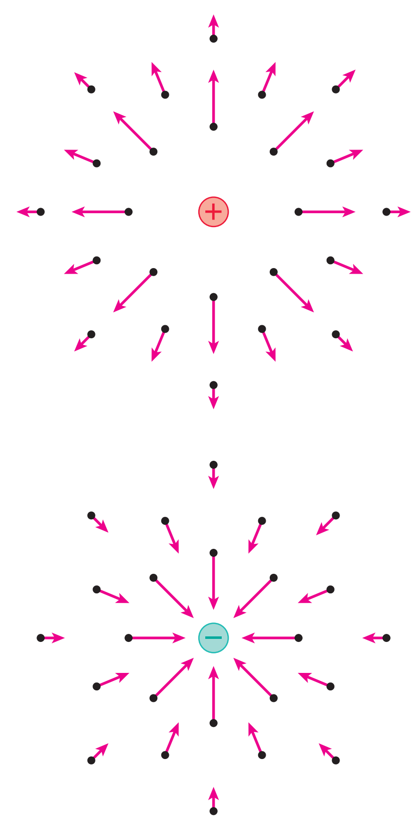


TABLE 26.1 Typical electric field strengths

Field location	Field strength (N/C)
Inside a current-carrying wire	10^{-3}–10^{-1}
Near the earth's surface	10^2–10^4
Near objects charged by rubbing	10^3–10^6
Electric breakdown in air, causing a spark	3×10^6
Inside an atom	10^{11}

26.2 The Electric Field of Multiple Point Charges

Suppose the source of an electric field is a group of point charges $q_1, q_2, \ldots$. According to Equation 26.2, the net electric field $\vec{E}_{net}$ at each point in space is a superposition of the electric fields due to each individual charge:

$$\begin{aligned}(E_{net})_x &= (E_1)_x + (E_2)_x + \cdots = \sum (E_i)_x \\ (E_{net})_y &= (E_1)_y + (E_2)_y + \cdots = \sum (E_i)_y \\ (E_{net})_z &= (E_1)_z + (E_2)_z + \cdots = \sum (E_i)_z\end{aligned} \tag{26.3}$$

Sometimes you'll want to write $\vec{E}_{net}$ in component form:

$$\vec{E}_{net} = (E_{net})_x \hat{\imath} + (E_{net})_y \hat{\jmath} + (E_{net})_z \hat{k}$$

At other times you will give $\vec{E}_{net}$ as a magnitude and a direction.

PROBLEM-SOLVING STRATEGY 26.1 **The electric field of multiple point charges**

MODEL Model charged objects as point charges.

VISUALIZE For the pictorial representation:

- Establish a coordinate system and show the locations of the charges.
- Identify the point P at which you want to calculate the electric field.
- Draw the electric field of each charge at P.
- Use symmetry to determine if any components of $\vec{E}_{net}$ are zero.

SOLVE The mathematical representation is $\vec{E}_{net} = \sum \vec{E}_i$.

- For each charge, determine its distance from P and the angle of $\vec{E}_i$ from the axes.
- Calculate the field strength of each charge's electric field.
- Write each vector $\vec{E}_i$ in component form.
- Sum the vector components to determine $\vec{E}_{net}$.
- If needed, determine the magnitude and direction of $\vec{E}_{net}$.

ASSESS Check that your result has the correct units, is reasonable, and agrees with any known limiting cases.

Exercise 16

EXAMPLE 26.1 **The electric field of three equal point charges**

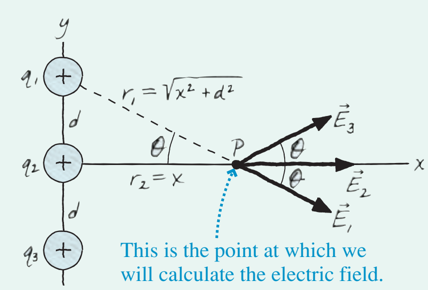
Three equal point charges q are located on the y-axis at $y = 0$ and at $y = \pm d$. What is the electric field at a point on the x-axis?

MODEL This problem is a step along the way to understanding the electric field of a charged wire. We'll assume that q is positive when drawing pictures, but the solution should allow for the possibility that q is negative. The question does not ask about any specific point, so we will be looking for a symbolic expression in terms of the unspecified position x.

VISUALIZE **FIGURE 26.3** shows the charges, the coordinate system, and the three electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$. Each of these fields points *away from* its source charge because of the assumption that q is positive. We need to find the vector sum $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$.

Before rushing into a calculation, we can make our task *much* easier by first thinking qualitatively about the situation. For example, the fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ all lie in the xy-plane, hence we can conclude without any calculations that $(E_{net})_z = 0$. Next,

FIGURE 26.3 Calculating the electric field of three equal point charges.



look at the y-components of the fields. The fields $\vec{E}_1$ and $\vec{E}_3$ have equal magnitudes and are tilted away from the x-axis by the same angle θ. Consequently, the y-components of $\vec{E}_1$ and $\vec{E}_3$ will *cancel* when added. $\vec{E}_2$ has no y-component, so we can conclude that $(E_{net})_y = 0$. The only component we need to calculate is $(E_{net})_x$.

SOLVE We're ready to calculate. The x-component of the field is

$$(E_{net})_x = (E_1)_x + (E_2)_x + (E_3)_x = 2(E_1)_x + (E_2)_x$$

where we used the fact that fields $\vec{E}_1$ and $\vec{E}_3$ have *equal* x-components. Vector $\vec{E}_2$ has *only* the x-component

$$(E_2)_x = E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}$$

where $r_2 = x$ is the distance from q_2 to the point at which we are calculating the field. Vector $\vec{E}_1$ is at angle θ from the x-axis, so its x-component is

$$(E_1)_x = E_1\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}\cos\theta$$

where r_1 is the distance from q_1. This expression for $(E_1)_x$ is correct, but it is not yet sufficient. Both the distance r_1 and the angle θ vary with the position x and need to be expressed as functions of x. From the Pythagorean theorem, $r_1 = (x^2 + d^2)^{1/2}$. Then from trigonometry,

$$\cos\theta = \frac{x}{r_1} = \frac{x}{(x^2+d^2)^{1/2}}$$

By combining these pieces, we see that $(E_1)_x$ is

$$(E_1)_x = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+d^2}\frac{x}{(x^2+d^2)^{1/2}} = \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+d^2)^{3/2}}$$

This expression is a bit complex, but notice that the dimensions of $x/(x^2 + d^2)^{3/2}$ are $1/\text{m}^2$, as they *must* be for the field of a point charge. Checking dimensions is a good way to verify that you haven't made algebra errors.

We can now combine $(E_1)_x$ and $(E_2)_x$ to write the x-component of $\vec{E}_{net}$ as

$$(E_{net})_x = 2(E_1)_x + (E_2)_x = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}\right]$$

The other two components of $\vec{E}_{net}$ are zero, hence the electric field of the three charges at a point on the x-axis is

$$\vec{E}_{net} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}\right]\hat{\imath}$$

ASSESS This is the electric field only at points *on the x-axis*. Furthermore, this expression is valid only for $x > 0$. The electric field to the left of the charges points in the opposite direction, but our expression doesn't change sign for negative x. (This is a consequence of how we wrote $(E_2)_x$.) We would need to modify this expression to use it for negative values of x. The good news, though, is that our expression *is* valid for both positive and negative q. A negative value of q makes $(E_{net})_x$ negative, which would be an electric field pointing to the left, toward the negative charges.

Let's explore this example a bit more. There are two limiting cases for which we know what the result should be. First, let x become really, really small. As the point in Figure 26.3 approaches the origin, the fields $\vec{E}_1$ and $\vec{E}_3$ become opposite to each other and cancel. Thus as $x \to 0$, the field *should* be that of the single point charge q at the origin, a field we already know. Is it? Notice that

$$\lim_{x\to 0}\frac{2x}{(x^2+d^2)^{3/2}} = 0 \tag{26.4}$$

Thus $E_{net} \to q/4\pi\epsilon_0 x^2$ as $x \to 0$, the expected field of a single point charge.

Now consider the opposite situation, where x becomes extremely large. From very far away, the three source charges will seem to merge into a single charge of size $3q$, just as three very distant lightbulbs appear to be a single light. Thus the field for $x \gg d$ *should* be that of a point charge $3q$. Is it?

The field is zero in the limit $x \to \infty$. That doesn't tell us much, so we don't want to go *that* far away. We simply want x to be very large in comparison to the spacing d between the source charges. If $x \gg d$, then the denominator of the second term of $\vec{E}_{net}$ is well approximated by $(x^2 + d^2)^{3/2} \approx (x^2)^{3/2} = x^3$. Thus

$$\lim_{x\gg d}\left[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}\right] = \frac{1}{x^2} + \frac{2x}{x^3} = \frac{3}{x^2} \tag{26.5}$$

Consequently, the net electric field far from the source charges is

$$\vec{E}_{net}(x \gg d) = \frac{1}{4\pi\epsilon_0}\frac{(3q)}{x^2}\hat{\imath} \tag{26.6}$$

As expected, this is the field of a point charge $3q$. These checks of limiting cases provide confidence in the result of the calculation.

FIGURE 26.4 is a graph of the field strength E_{net} for the three charges of Example 26.1. Although we don't have any numerical values, we can specify x as a multiple of the charge separation d. Notice how the graph matches the field of a single point charge when $x \ll d$ and matches the field of a point charge $3q$ when $x \gg d$.

FIGURE 26.4 The electric field strength along a line perpendicular to three equal point charges.

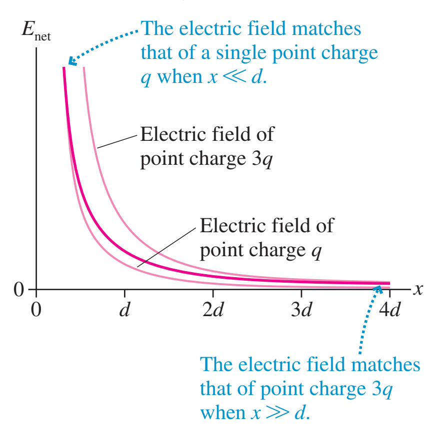
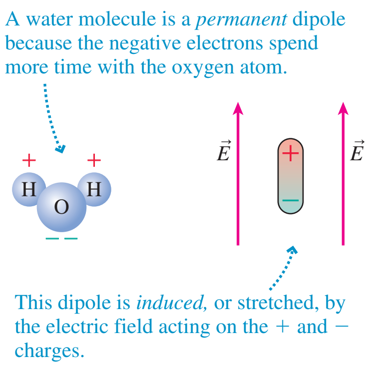


FIGURE 26.5 Permanent and induced electric dipoles.



The Electric Field of a Dipole

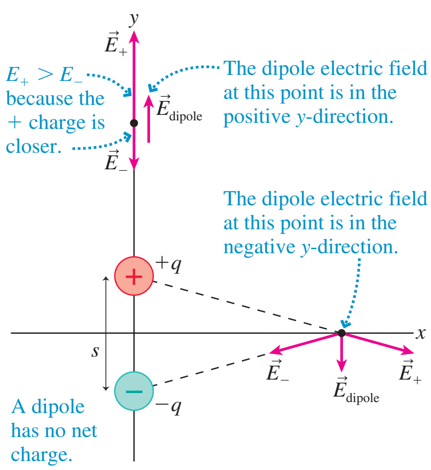
Two equal but opposite charges separated by a small distance form an *electric dipole*. **FIGURE 26.5** shows two examples. In a *permanent electric dipole,* such as the water molecule, the oppositely charged particles maintain a small permanent separation. We can also create an electric dipole, as you learned in Chapter 25, by polarizing a neutral atom with an external electric field. This is an *induced electric dipole*.

FIGURE 26.6 shows that we can represent an electric dipole, whether permanent or induced, by two opposite charges $\pm q$ separated by the small distance s. The dipole has zero net charge, but it *does* have an electric field. Consider a point on the positive y-axis. This point is slightly closer to $+q$ than to $-q$, so the fields of the two charges do not cancel. You can see in the figure that $\vec{E}_{\text{dipole}}$ points in the positive y-direction. Similarly, vector addition shows that $\vec{E}_{\text{dipole}}$ points in the negative y-direction at points along the x-axis.

Let's calculate the electric field of a dipole at a point on the axis of the dipole. This is the y-axis in Figure 26.6. The point is distance $r_+ = y - s/2$ from the positive charge and $r_- = y + s/2$ from the negative charge. The net electric field at this point has only a y-component, and the sum of the fields of the two point charges gives

$$\begin{aligned}(E_{\text{dipole}})_y = (E_+)_y + (E_-)_y &= \frac{1}{4\pi\epsilon_0}\frac{q}{(y-\frac{1}{2}s)^2} + \frac{1}{4\pi\epsilon_0}\frac{(-q)}{(y+\frac{1}{2}s)^2} \\ &= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y-\frac{1}{2}s)^2} - \frac{1}{(y+\frac{1}{2}s)^2}\right]\end{aligned} \tag{26.7}$$

FIGURE 26.6 The dipole electric field at two points.



Combining the two terms over a common denominator, we find

$$(E_{\text{dipole}})_y = \frac{q}{4\pi\epsilon_0}\left[\frac{2ys}{(y-\frac{1}{2}s)^2(y+\frac{1}{2}s)^2}\right] \tag{26.8}$$

We omitted some of the algebraic steps, but be sure you can do this yourself. Some of the homework problems will require similar algebra.

In practice, we almost always observe the electric field of a dipole only for distances $y \gg s$—that is, for distances much larger than the charge separation. In such cases, the denominator can be approximated $(y-\frac{1}{2}s)^2(y+\frac{1}{2}s)^2 \approx y^4$. With this approximation, Equation 26.8 becomes

$$(E_{\text{dipole}})_y \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{y^3} \tag{26.9}$$

It is useful to define the **dipole moment** $\vec{p}$, shown in **FIGURE 26.7**, as the vector

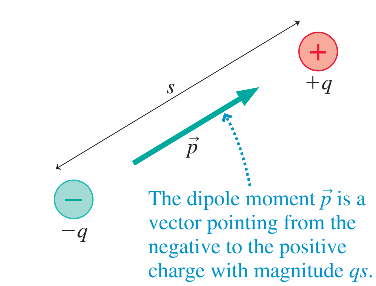
$$\vec{p} = (qs\text{, from the negative to the positive charge}) \tag{26.10}$$

The direction of $\vec{p}$ identifies the orientation of the dipole, and the dipole-moment magnitude $p = qs$ determines the electric field strength. The SI units of the dipole moment are C m.

We can use the dipole moment to write a succinct expression for the electric field at a point on the axis of a dipole:

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad \text{(on the axis of an electric dipole)} \tag{26.11}$$

FIGURE 26.7 The dipole moment.



where r is the distance measured from the *center* of the dipole. We've switched from y to r because we've now specified that Equation 26.11 is valid only along the axis of the dipole. Notice that the electric field along the axis points in the direction of the dipole moment $\vec{p}$.

A homework problem will let you calculate the electric field in the plane that bisects the dipole. This is the field shown on the x-axis in Figure 26.6, but it could equally well be the field on the z-axis as it comes out of the page. The field, for $r \gg s$, is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \quad \text{(bisecting plane)} \qquad (26.12)$$

This field is *opposite* to $\vec{p}$, and it is only half the strength of the on-axis field at the same distance.

NOTE ▶ Do these inverse-cube equations violate Coulomb's law? Not at all. Coulomb's law describes the force between two *point charges,* and from Coulomb's law we found that the electric field of a *point charge* varies with the inverse square of the distance. But a dipole is not a point charge. The field of a dipole decreases more rapidly than that of a point charge, which is to be expected because the dipole is, after all, electrically neutral. ◀

EXAMPLE 26.2 The electric field of a water molecule

The water molecule H_2O has a permanent dipole moment of magnitude 6.2×10^{-30} C m. What is the electric field strength 1.0 nm from a water molecule at a point on the dipole's axis?

MODEL The size of a molecule is $\approx$ 0.1 nm. Thus $r \gg s$, and we can use Equation 26.11 for the on-axis electric field of the molecule's dipole moment.

SOLVE The on-axis electric field strength at $r = 1.0$ nm is

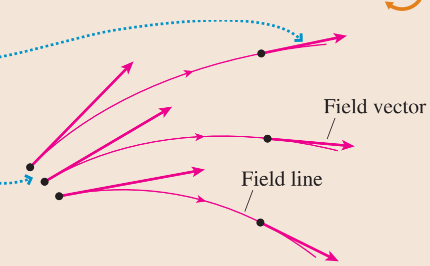
$$E \approx \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3} = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\frac{2(6.2 \times 10^{-30} \text{ C m})}{(1.0 \times 10^{-9} \text{ m})^3}$$
$$= 1.1 \times 10^8 \text{ N/C}$$

ASSESS By referring to Table 26.1 you can see that the field strength is "strong" compared to our everyday experience with charged objects but "weak" compared to the electric field inside the atoms themselves. This seems reasonable.

Picturing the Electric Field

We can't see the electric field. Consequently, we need pictorial tools to help us visualize it in a region of space. One method, introduced in Chapter 25, is to picture the electric field by drawing electric field vectors at various points in space. Another way to picture the field is to draw **electric field lines.**

TACTICS BOX 26.1 Drawing and using electric field lines



1. Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.
2. Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.
3. Electric field lines never cross.
4. Electric field lines start from positive charges and end on negative charges.

Exercises 2–4, 10–12

Step 3 is required to make sure that $\vec{E}$ has a unique direction at every point in space. Step 4 follows from the fact that electric fields are created by charges. However, we will have to modify step 4 in Chapter 33 when we find another way to create an electric field.

FIGURE 26.8a on the next page represents the electric field of a dipole as a field-vector diagram. **FIGURE 26.8b** shows the same field using electric field lines. Notice how the on-axis field points in the direction of $\vec{p}$, both above and below the dipole, while the field in the bisecting plane points opposite to $\vec{p}$. At most points, however, $\vec{E}$ has components both parallel to $\vec{p}$ and perpendicular to $\vec{p}$.

FIGURE 26.8 The electric field of a dipole: (a) field vectors, (b) field lines.

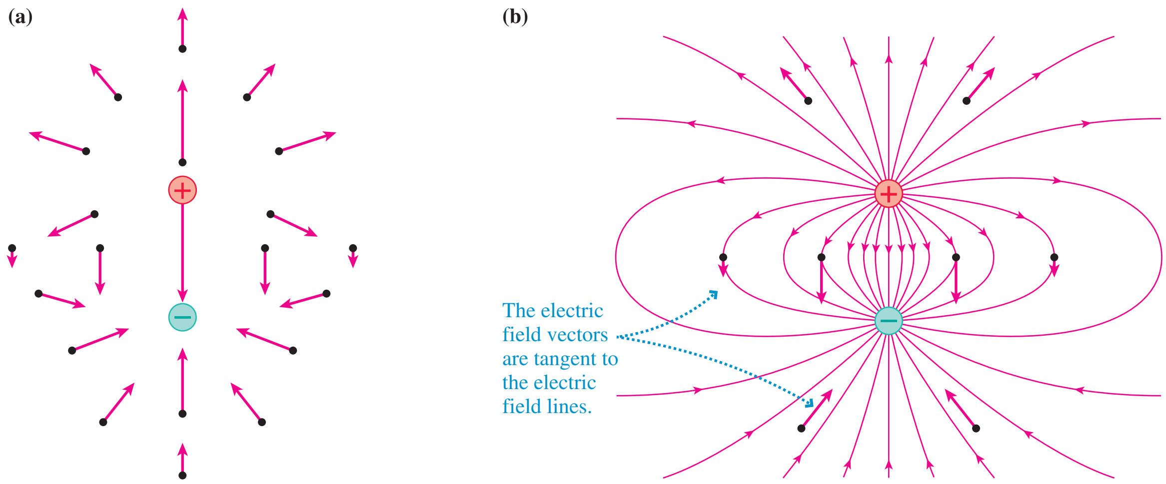


FIGURE 26.9 The electric field of two equal positive charges.

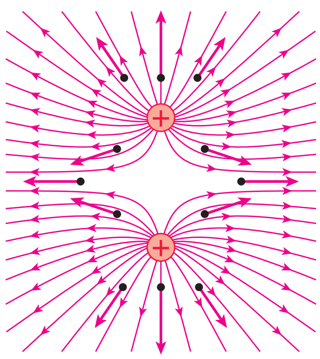
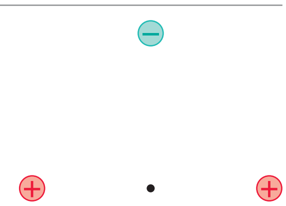


FIGURE 26.9 shows the electric field of two same-sign charges. A careful comparison of Figures 26.8b and 26.9 is worthwhile. Make sure you can explain the similarities and differences.

Neither field-vector diagrams nor field-line diagrams are perfect pictorial representations of an electric field. The field vectors are somewhat harder to draw, and they show the field at only a few points, but they do clearly indicate the direction and strength of the electric field at those points. Field-line diagrams perhaps look more elegant, and they're sometimes easier to sketch, but there's no formula for knowing where to draw the lines. We'll use both field-vector diagrams and field-line diagrams, depending on the circumstances.

STOP TO THINK 26.1 At the dot, the electric field points



a. Left. b. Right.
c. Up. d. Down.
e. The electric field is zero.

26.3 The Electric Field of a Continuous Charge Distribution

Ordinary objects—tables, chairs, beakers of water—seem to our senses to be continuous distributions of matter. There is no obvious evidence for an atomic structure, even though we have good reasons to believe that we would find atoms if we subdivided the matter sufficiently far. Thus it is easier, for many practical purposes, to consider matter to be continuous and to talk about the *density* of matter. Density—the number of kilograms of matter per cubic meter—allows us to describe the distribution of matter *as if* the matter were continuous rather than atomic.

Much the same situation occurs with charge. If a charged object contains a large number of excess electrons—for example, 10^{12} extra electrons on a metal rod—it is not practical to track every electron. It makes more sense to consider the charge to be *continuous* and to describe how it is distributed over the object.

FIGURE 26.10a shows an object of length L, such as a plastic rod or a metal wire, with charge Q spread uniformly along it. (We will use an uppercase Q for the total charge of an object, reserving lowercase q for individual point charges.) The **linear charge density** λ is defined to be

$$\lambda = \frac{Q}{L} \tag{26.13}$$

Linear charge density, which has units of C/m, is the amount of charge *per meter* of length. The linear charge density of a 20-cm-long wire with 40 nC of charge is 2.0 nC/cm or 2.0×10^{-7} C/m.

NOTE ▶ The linear charge density λ is analogous to the linear mass density μ that you used in Chapter 20 to find the speed of a wave on a string. ◀

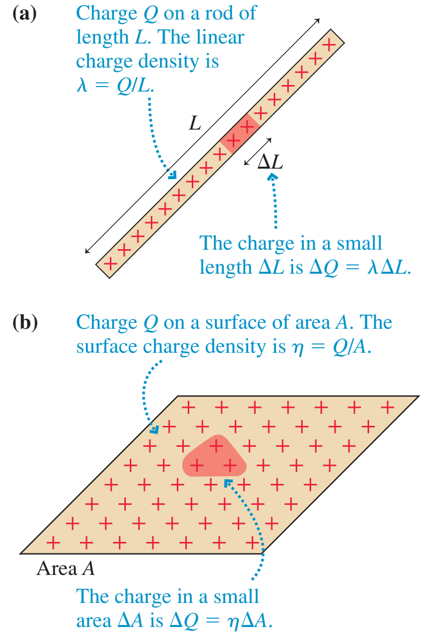
We'll also be interested in charged surfaces. **FIGURE 26.10b** shows a two-dimensional distribution of charge across a surface of area A. We define the **surface charge density** η (lowercase Greek eta) to be

$$\eta = \frac{Q}{A} \tag{26.14}$$

Surface charge density, with units of C/m^2, is the amount of charge *per square meter*. A 1.0 mm $\times$ 1.0 mm square on a surface with $\eta = 2.0 \times 10^{-4}$ C/m^2 contains 2.0×10^{-10} C or 0.20 nC of charge. (The volume charge density $\rho = Q/V$, measured in C/m^3, will be used in Chapter 27.)

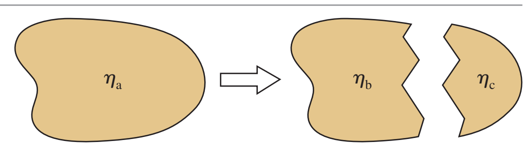
Figure 26.10 and the definitions of Equations 26.13 and 26.14 assume that the object is **uniformly charged,** meaning that the charges are evenly spread over the object. We will assume objects are uniformly charged unless noted otherwise.

FIGURE 26.10 One-dimensional and two-dimensional continuous charge distributions.



NOTE ▶ Some textbooks represent the surface charge density with the symbol σ. Because σ is also used to represent *conductivity,* an idea we'll introduce in Chapter 30, we've selected a different symbol for surface charge density. ◀

STOP TO THINK 26.2 A piece of plastic is uniformly charged with surface charge density η_a. The plastic is then broken into a large piece with surface charge density η_b and a small piece with surface charge density η_c. Rank in order, from largest to smallest, the surface charge densities η_a to η_c.



A Problem-Solving Strategy

Our goal is to find the electric field of a continuous distribution of charge, such as a charged rod or a charged disk. We have two basic tools to work with:

- The electric field of a point charge, and
- The principle of superposition.

We can apply these tools to a continuous distribution of charge if we follow a three-step strategy:

1. Divide the total charge Q into many small point-like charges ΔQ.
2. Use our knowledge of the electric field of a point charge to find the electric field of each ΔQ.
3. Calculate the net field $\vec{E}_{\text{net}}$ by summing the fields of all the ΔQ.

In practice, as you may have guessed, we'll let the sum become an integral.

The difficulty with electric field calculations is not the summation or integration itself, which is the last step, but setting up the calculation and knowing *what* to integrate. We will go step by step through several examples to illustrate the procedures. However, we first need to flesh out the steps of the problem-solving strategy. The aim of this problem-solving strategy is to break a difficult problem down into small steps that are individually manageable.

PROBLEM-SOLVING STRATEGY 26.2 The electric field of a continuous distribution of charge

MODEL Model the distribution as a simple shape, such as a line of charge or a disk of charge. Assume the charge is uniformly distributed.

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric field.
3. Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine $\vec{E}$. This is often, but not always, a division into point charges.
4. Draw the electric field vector at P for one or two small pieces of charge. This will help you identify distances and angles that need to be calculated.
5. Look for symmetries of the charge distribution that simplify the field. You may conclude that some components of $\vec{E}$ are zero.

SOLVE The mathematical representation is $\vec{E}_{\text{net}} = \sum \vec{E}_i$.

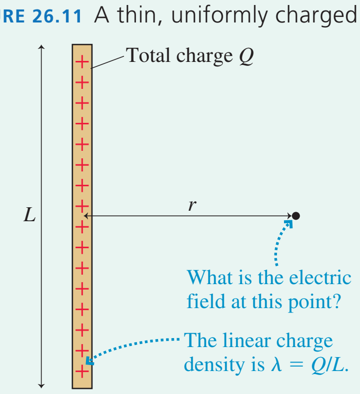
- Use superposition to form an algebraic expression for *each* of the three components of $\vec{E}$ (unless you are sure one or more is zero) at point P.
- Let the (x, y, z) coordinates of the point remain variables.
- Replace the small charge ΔQ with an equivalent expression involving a charge density and a coordinate, such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- Express all angles and distances in terms of the coordinates.
- Let the sum become an integral. The integration will be over the *one* coordinate variable that is related to ΔQ. The integration limits for this variable must "cover" the entire charged object.

ASSESS Check that your result is consistent with any limits for which you know what the field should be.

EXAMPLE 26.3 The electric field of a line of charge

FIGURE 26.11 shows a thin, uniformly charged rod of length L with total charge Q. Find the electric field strength at radial distance r in the plane that bisects the rod.

FIGURE 26.11 A thin, uniformly charged rod.



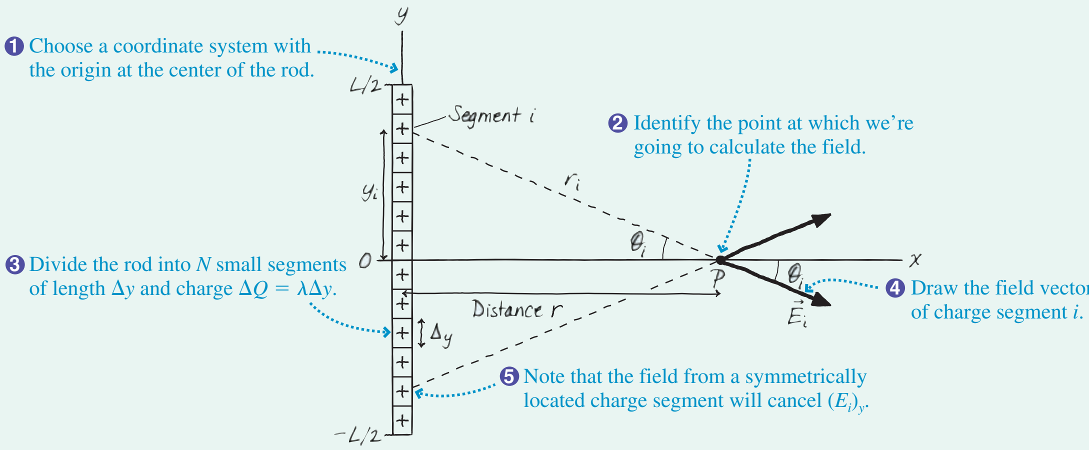
MODEL The rod is thin, so we'll assume the charge lies along a line and forms what we call a *line of charge*. This is an important charge distribution that models the electric field of a charged rod or a charged metal wire. The rod's linear charge density is $\lambda = Q/L$.

VISUALIZE **FIGURE 26.12** illustrates the five steps of the problem-solving strategy. We've chosen a coordinate system in which the rod lies along the y-axis and point P, in the bisecting plane, is on the x-axis. We've then divided the rod into N small segments of charge ΔQ, each of which can be modeled as a point charge. For every ΔQ in the bottom half of the wire with a field that points to the right and up, there's a matching ΔQ in the top half whose field points to the right and down. The y-components of these two fields cancel, hence the net electric field on the x-axis points straight away from the rod. The only component we need to calculate is E_x. (This is the same reasoning on the basis of symmetry that we used in Example 26.1.)

SOLVE Each of the little segments of charge can be modeled as a point charge. We know the electric field of a point charge, so we can write the x-component of $\vec{E}_i$, the electric field of segment i, as

$$(E_i)_x = E_i \cos\theta_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

FIGURE 26.12 Calculating the electric field of a line of charge.



where r_i is the distance from charge i to point P. You can see from the figure that $r_i = (y_i^2 + r^2)^{1/2}$ and $\cos\theta_i = r/r_i = r/(y_i^2 + r^2)^{1/2}$. With these, $(E_i)_x$ is

$$(E_i)_x = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{y_i^2 + r^2}\frac{r}{\sqrt{y_i^2 + r^2}} = \frac{1}{4\pi\epsilon_0}\frac{r\,\Delta Q}{(y_i^2 + r^2)^{3/2}}$$

Compare this result to the very similar calculation we did in Example 26.1. If we now sum this expression over all the charge segments, the net x-component of the electric field is

$$E_x = \sum_{i=1}^{N}(E_i)_x = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{r\,\Delta Q}{(y_i^2 + r^2)^{3/2}}$$

This is the same superposition we did for the $N = 3$ case in Example 26.1. The only difference is that we have now written the result as an explicit summation so that N can have any value. We want to let $N \to \infty$ and to replace the sum with an integral, but we can't integrate over Q; it's not a geometric quantity. This is where the linear charge density enters. The quantity of charge in each segment is related to its length Δy by $\Delta Q = \lambda\,\Delta y = (Q/L)\Delta y$. In terms of the linear charge density, the electric field is

$$E_x = \frac{Q/L}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{r\,\Delta y}{(y_i^2 + r^2)^{3/2}}$$

Now we're ready to let the sum become an integral. If we let $N \to \infty$, then each segment becomes an infinitesimal length $\Delta y \to dy$ while the discrete position variable y_i becomes the continuous integration variable y. The sum from $i = 1$ at the bottom end of the line of charge to $i = N$ at the top end will be replaced with an integral from $y = -L/2$ to $y = +L/2$. Thus in the limit $N \to \infty$,

$$E_x = \frac{Q/L}{4\pi\epsilon_0}\int_{-L/2}^{L/2}\frac{r\,dy}{(y^2 + r^2)^{3/2}}$$

This is a standard integral that you have learned to do in calculus and that can be found in Appendix A. Note that r is a *constant* as far as this integral is concerned. Integrating gives

$$E_x = \frac{Q/L}{4\pi\epsilon_0}\left.\frac{y}{r\sqrt{y^2 + r^2}}\right|_{-L/2}^{L/2} = \frac{Q/L}{4\pi\epsilon_0}\left[\frac{L/2}{r\sqrt{(L/2)^2 + r^2}} - \frac{-L/2}{r\sqrt{(-L/2)^2 + r^2}}\right] = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2 + (L/2)^2}}$$

Because E_x is the *only* component of the field, the electric field strength E_{rod} at distance r from the center of a charged rod is

$$E_{rod} = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

The field strength must be positive, so we added absolute value signs to Q to allow for the possibility that the charge could be negative. The only restriction is to remember that this is the electric field at a point in the plane that bisects the rod.

ASSESS Suppose we are at a point *very* far from the rod. If $r \gg L$, the length of the rod is not relevant and the rod appears to be a point charge Q in the distance. Thus in the *limiting case* $r \gg L$, we expect the rod's electric field to be that of a point charge. If $r \gg L$, the square root becomes $(r^2 + (L/2)^2)^{1/2} \approx (r^2)^{1/2} = r$ and the electric field strength at distance r becomes $E_{rod} \approx Q/4\pi\epsilon_0 r^2$, the field of a point charge. The fact that our expression of E_{rod} has the correct limiting behavior gives us confidence that we haven't made any mistakes in its derivation.

An Infinite Line of Charge

What if the rod or wire becomes very long, becoming a **line of charge,** while the linear charge density λ remains constant? To answer this question, we can rewrite the expression for E_{rod} by factoring $(L/2)^2$ out of the denominator:

$$E_{rod} = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r \cdot L/2}\frac{1}{\sqrt{1 + 4r^2/L^2}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1 + 4r^2/L^2}}$$

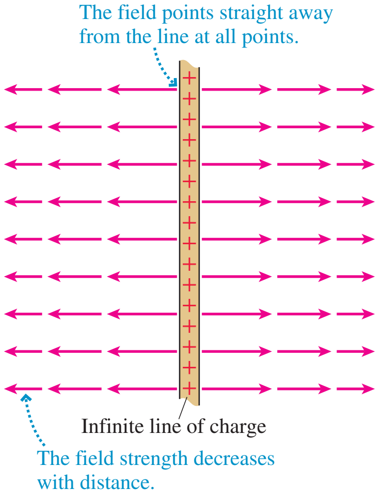


FIGURE 26.13 The electric field of an infinite line of charge.

where $|\lambda| = |Q|/L$ is the magnitude of the linear charge density. If we now let $L \to \infty$, the last term becomes simply 1 and we're left with

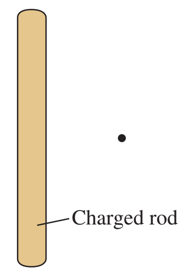
$$E_{\text{line}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r} \tag{26.15}$$

FIGURE 26.13 shows the electric field vectors of an infinite line of positive charge. The vectors would point inward for a negative line of charge.

NOTE ▶ Unlike a point charge, for which the field decreases as $1/r^2$, the field of an infinitely long charged wire decreases more slowly—as only $1/r$. ◀

Although no real wire is infinitely long, the fact that the field of a point charge decreases inversely with the square of the distance means that the electric field at any point is determined primarily by the nearest charges. Consequently, the field of a realistic finite-length wire is well approximated by Equation 26.15, the field of an infinitely long line of charge, except at points near the end of the wire.

STOP TO THINK 26.3 Which of the following actions will increase the electric field strength at the position of the dot?



a. Make the rod longer without changing the charge.
b. Make the rod shorter without changing the charge.
c. Make the rod wider without changing the charge.
d. Make the rod narrower without changing the charge.
e. Add charge to the rod.
f. Remove charge from the rod.
g. Move the dot farther from the rod.
h. Move the dot closer to the rod.

26.4 The Electric Fields of Rings, Disks, Planes, and Spheres

In this section we'll derive the electric fields for several important charge distributions.

EXAMPLE 26.4 The electric field of a ring of charge

A thin ring of radius R is uniformly charged with total charge Q. Find the electric field at a point on the axis of the ring (perpendicular to the ring).

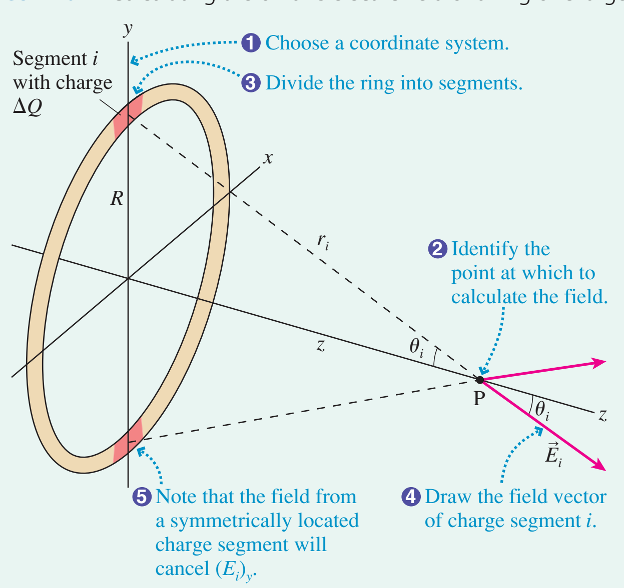
MODEL Because the ring is thin, we'll assume the charge lies along a circle of radius R. You can think of this as a line of charge of length $2\pi R$ wrapped into a circle. The linear charge density along the ring is $\lambda = Q/2\pi R$.

VISUALIZE FIGURE 26.14 shows the ring and illustrates the five steps of the problem-solving strategy. We've chosen a coordinate system in which the ring lies in the xy-plane and point P is on the z-axis. We've then divided the ring into N small segments of charge ΔQ, each of which can be modeled as a point charge. As you can see from the figure, the component of the field perpendicular to the axis cancels for two diametrically opposite segments. Thus we need to calculate only the z-component E_z.

SOLVE The z-component of the electric field due to segment i is

$$(E_i)_z = E_i\cos\theta_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

FIGURE 26.14 Calculating the on-axis electric field of a ring of charge.



You can see from the figure that *every* segment of the ring, independent of *i*, has

$$r_i = \sqrt{z^2 + R^2}$$
$$\cos\theta_i = \frac{z}{r_i} = \frac{z}{\sqrt{z^2 + R^2}}$$

Consequently, the field of segment *i* is

$$(E_i)_z = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{z^2 + R^2}\frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{4\pi\epsilon_0}\frac{z}{(z^2 + R^2)^{3/2}}\Delta Q$$

The net electric field is found by summing $(E_i)_z$ due to all N segments:

$$E_z = \sum_{i=1}^{N}(E_i)_z = \frac{1}{4\pi\epsilon_0}\frac{z}{(z^2 + R^2)^{3/2}}\sum_{i=1}^{N}\Delta Q$$

We were able to bring all terms involving z to the front because z is a constant as far as the summation is concerned. Surprisingly, we don't need to convert the sum to an integral to complete this calculation. The sum of all the ΔQ around the ring is simply the ring's total charge, $\sum \Delta Q = Q$, hence the field on the axis is

$$(E_{\text{ring}})_z = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2 + R^2)^{3/2}}$$

This expression is valid for both positive and negative z (i.e., on either side of the ring) and for both positive and negative charge.

ASSESS It will be left as a homework problem to show that this result gives the expected limit when $z \gg R$.

FIGURE 26.15 shows two representations of the on-axis electric field of a positively charged ring. **FIGURE 26.15a** shows that the electric field vectors point away from the ring, increasing in length until reaching a maximum when $|z| \approx R$, then decreasing. The graph of $(E_{\text{ring}})_z$ in **FIGURE 26.15b** confirms that the field strength has a maximum on either side of the ring. Notice that the electric field at the center of the ring is zero, even though this point is surrounded by charge. You might want to spend a minute thinking about why this has to be the case.

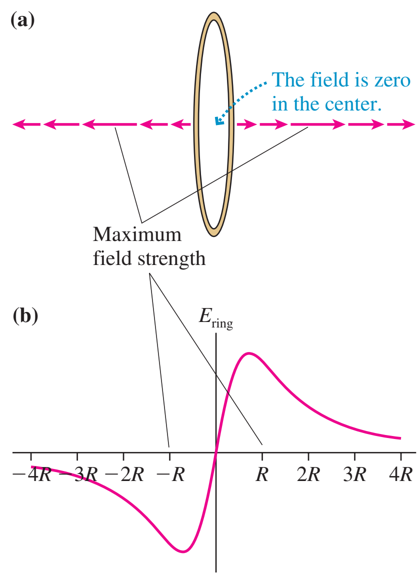


FIGURE 26.15 The on-axis electric field of a ring of charge.

A Disk of Charge

FIGURE 26.16 shows a disk of radius R that is uniformly charged with charge Q. This is a mathematical disk, with no thickness, and its surface charge density is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} \tag{26.16}$$

We would like to calculate the on-axis electric field of this disk. Our problem-solving strategy tells us to divide a continuous charge into segments for which we already know how to find $\vec{E}$. Because we now know the on-axis electric field of a ring of charge, let's divide the disk into N very narrow rings of radius r and width Δr. One such ring, with radius r_i and charge ΔQ_i, is shown.

We need to be careful with notation. The R in Example 26.4 was the radius of the ring. Now we have many rings, and the radius of ring i is r_i. Similarly, Q was the charge on the ring. Now the charge on ring i is ΔQ_i, a small fraction of the total charge on the disk. With these changes, the electric field of ring i, with radius r_i, is

$$(E_i)_z = \frac{1}{4\pi\epsilon_0}\frac{z\,\Delta Q_i}{(z^2 + r_i^2)^{3/2}} \tag{26.17}$$

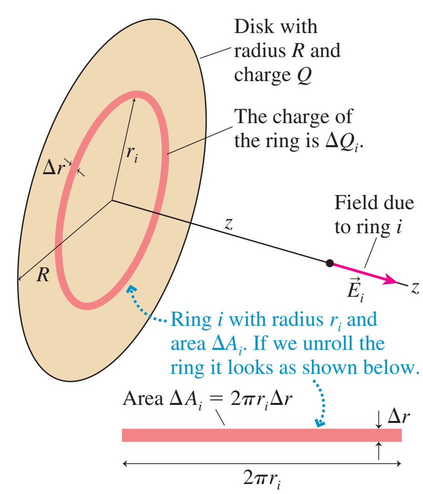
The on-axis electric field of the charged disk is the sum of the electric fields of all of the rings:

$$(E_{\text{disk}})_z = \sum_{i=1}^{N}(E_i)_z = \frac{z}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{\Delta Q_i}{(z^2 + r_i^2)^{3/2}} \tag{26.18}$$

The critical step, as always, is to relate ΔQ to a coordinate. Because we now have a surface, rather than a line, the charge in ring i is $\Delta Q = \eta\,\Delta A_i$, where ΔA_i is the area of ring i. We can find ΔA_i, as you've learned to do in calculus, by "unrolling" the ring to form a narrow rectangle of length $2\pi r_i$ and height Δr. Thus the area of ring i is $\Delta A_i = 2\pi r_i \Delta r$ and the charge is $\Delta Q_i = 2\pi\eta r_i \Delta r$. With this substitution, Equation 26.18 becomes

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0}\sum_{i=1}^{N}\frac{r_i\,\Delta r}{(z^2 + r_i^2)^{3/2}} \tag{26.19}$$

FIGURE 26.16 Calculating the on-axis field of a charged disk.



As $N \to \infty$, $\Delta r \to dr$ and the sum becomes an integral. Adding all the rings means integrating from $r = 0$ to $r = R$; thus

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0} \int_0^R \frac{r\,dr}{(z^2 + r^2)^{3/2}} \tag{26.20}$$

All that remains is to carry out the integration. This is straightforward if we make the variable change $u = z^2 + r^2$. Then $du = 2r\,dr$ or, equivalently, $r\,dr = \frac{1}{2}du$. At the lower integration limit $r = 0$, our new variable is $u = z^2$. At the upper limit $r = R$, the new variable is $u = z^2 + R^2$.

NOTE ▶ When changing variables in a definite integral, you *must* also change the limits of integration. ◀

With this variable change the integral becomes

$$(E_{\text{disk}})_z = \frac{\eta z}{2\epsilon_0}\frac{1}{2}\int_{z^2}^{z^2+R^2} \frac{du}{u^{3/2}} = \frac{\eta z}{4\epsilon_0}\frac{-2}{u^{1/2}}\bigg|_{z^2}^{z^2+R^2} = \frac{\eta z}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right] \tag{26.21}$$

If we multiply through by z, the on-axis electric field of a charged disk with surface charge density $\eta = Q/\pi R^2$ is

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] \tag{26.22}$$

NOTE ▶ This expression is valid only for $z > 0$. The field for $z < 0$ has the same magnitude but points in the opposite direction. ◀

It's a bit difficult see what Equation 26.22 is telling us, so let's compare it to what we already know. First, you can see that the quantity in square brackets is dimensionless. The surface charge density $\eta = Q/A$ has the same units as q/r^2, so $\eta/2\epsilon_0$ has the same units as $q/4\pi\epsilon_0 r^2$. This tells us that $\eta/2\epsilon_0$ really is an electric field.

Next, let's move very far away from the disk. At distance $z \gg R$, the disk appears to be a point charge Q in the distance and the field of the disk should approach that of a point charge. If we let $z \to \infty$ in Equation 26.22, so that $z^2 + R^2 \approx z^2$, we find $(E_{\text{disk}})_z \to 0$. This is true, but not quite what we wanted. We need to let z be very large in comparison to R, but not so large as to make E_{disk} vanish. That requires a little more care in taking the limit.

We can cast Equation 26.22 into a somewhat more useful form by factoring the z^2 out of the square root to give

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1+R^2/z^2}}\right] \tag{26.23}$$

Now $R^2/z^2 \ll 1$ if $z \gg R$, so the second term in the square brackets is of the form $(1 + x)^{-1/2}$ where $x \ll 1$. We can then use the *binomial approximation*

$$(1+x)^n \approx 1 + nx \quad \text{if} \quad x \ll 1 \qquad \text{(binomial approximation)}$$

to simplify the expression in square brackets:

$$1 - \frac{1}{\sqrt{1+R^2/z^2}} = 1 - (1 + R^2/z^2)^{-1/2} \approx 1 - \left(1 + \left(-\frac{1}{2}\right)\frac{R^2}{z^2}\right) = \frac{R^2}{2z^2} \tag{26.24}$$

This is a good approximation when $z \gg R$. Substituting this approximation into Equation 26.23, we find that the electric field of the disk for $z \gg R$ is

$$(E_{\text{disk}})_z \approx \frac{\eta}{2\epsilon_0}\frac{R^2}{2z^2} = \frac{Q/\pi R^2}{4\epsilon_0}\frac{R^2}{z^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{z^2} \quad \text{if} \quad z \gg R \tag{26.25}$$

This is, indeed, the field of a point charge Q, giving us confidence in Equation 26.22 for the on-axis electric field of a disk of charge.

NOTE ▶ The binomial approximation is an important tool for looking at the limiting cases of electric fields. ◀

EXAMPLE 26.5 The electric field of a charged disk

A 10-cm-diameter plastic disk is charged uniformly with an extra 10^{11} electrons. What is the electric field 1.0 mm above the surface at a point near the center?

MODEL Model the plastic disk as a uniformly charged disk. We are seeking the on-axis electric field. Because the charge is negative, the field will point *toward* the disk.

SOLVE The total charge on the plastic square is $Q = N(-e) = -1.60 \times 10^{-8}$ C. The surface charge density is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{-1.60 \times 10^{-8}\ \text{C}}{\pi (0.050\ \text{m})^2} = -2.04 \times 10^{-6}\ \text{C/m}^2$$

The electric field at $z = 0.0010$ m, given by Equation 26.23, is

$$E_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right] = -1.1 \times 10^5\ \text{N/C}$$

The minus sign indicates that the field points *toward,* rather than away from, the disk. As a vector,

$$\vec{E} = (1.1 \times 10^5\ \text{N/C, toward the disk})$$

ASSESS The total charge, -16 nC, is typical of the amount of charge produced on a small plastic object by rubbing or friction. Thus 10^5 N/C is a typical electric field strength near an object that has been charged by rubbing.

A Plane of Charge

Many electronic devices use charged, flat surfaces—disks, squares, rectangles, and so on—to steer electrons along the proper paths. These charged surfaces are called **electrodes.** Although any real electrode is finite in extent, we can often model an electrode as an infinite **plane of charge.** As long as the distance z to the electrode is small in comparison to the distance to the edges, we can reasonably treat the edges *as if* they are infinitely far away.

The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$. That is, a disk with infinite radius is an infinite plane. From Equation 26.22, we see that the electric field of a plane of charge with surface charge density η is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant} \qquad (26.26)$$

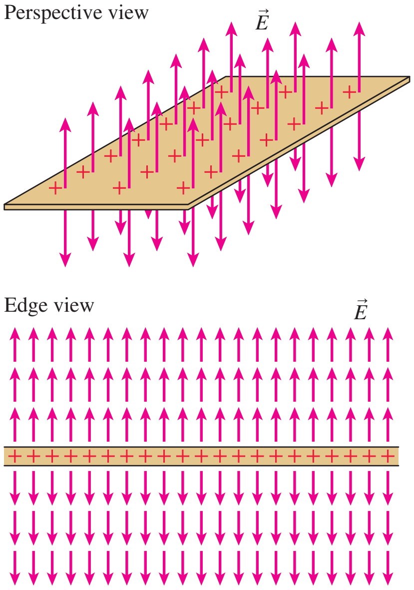
This is a simple result, but what does it tell us? First, the field strength is directly proportional to the charge density η: More charge, bigger field. Second, and more interesting, the field strength is the same at *all* points in space, independent of the distance z. The field strength 1000 m from the plane is the same as the field strength 1 mm from the plane.

How can this be? It seems that the field should get weaker as you move away from the plane of charge. But remember that we are dealing with an *infinite* plane of charge. What does it mean to be "close to" or "far from" an infinite object? For a disk of finite radius R, whether a point at distance z is "close to" or "far from" the disk is a comparison of z to R. If $z \ll R$, the point is close to the disk. If $z \gg R$, the point is far from the disk. But as $R \to \infty$, we have no *scale* for distinguishing near and far. In essence, *every* point in space is "close to" a disk of infinite radius.

No real plane is infinite in extent, but we can interpret Equation 26.26 as saying that the field of a surface of charge, regardless of its shape, is a constant $\eta/2\epsilon_0$ for those points whose distance z to the surface is much smaller than their distance to the edge. Eventually, when $z \gg R$, the charged surface will begin to look like a point charge Q and the field will have to decrease as $1/z^2$.

We do need to note that the derivation leading to Equation 26.26 considered only $z > 0$. For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane. This requires $E_z < 0$ ($\vec{E}$ pointing in the negative

FIGURE 26.17 Two views of the electric field of a plane of charge.



z-direction) on the side with $z < 0$. Thus a complete description of the electric field, valid for both sides of the plane and for either sign of η, is

$$(E_{\text{plane}})_z = \begin{cases} +\dfrac{\eta}{2\epsilon_0} & z > 0 \\ -\dfrac{\eta}{2\epsilon_0} & z < 0 \end{cases} \tag{26.27}$$

FIGURE 26.17 shows two views of the electric field of a positively charged plane. All the arrows would be reversed for a negatively charged plane. It would have been very difficult to anticipate this result from Coulomb's law or from the electric field of a single point charge, but step by step we have been able to use the concept of the electric field to look at increasingly complex distributions of charge.

A Sphere of Charge

The one last charge distribution for which we need to know the electric field is a **sphere of charge.** This problem is analogous to wanting to know the gravitational field of a spherical planet or star. The procedure for calculating the field of a sphere of charge is the same as we used for lines and planes, but the integrations are significantly more difficult. We will skip the details of the calculations and, for now, simply assert the result without proof. In Chapter 27 we'll use an alternative procedure to find the field of a sphere of charge.

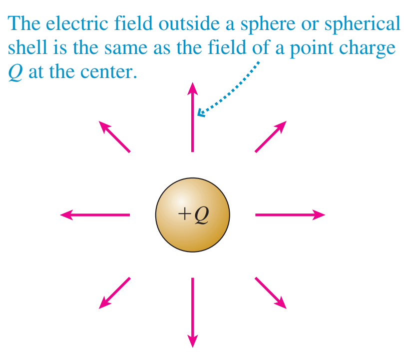
A sphere of charge Q and radius R, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere ($r \geq R$) that is exactly the same as that of a point charge Q located at the center of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R \tag{26.28}$$

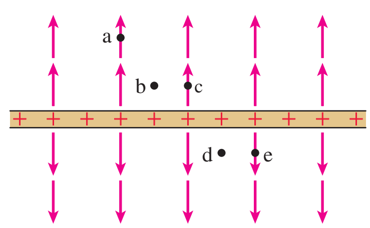
This assertion is analogous to our earlier assertion that the gravitational force between stars and planets can be computed as if all the mass is at the center.

FIGURE 26.18 shows the electric field of a sphere of positive charge. The field of a negative sphere would point inward.

FIGURE 26.18 The electric field of a sphere of positive charge.



STOP TO THINK 26.4 Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.



26.5 The Parallel-Plate Capacitor

FIGURE 26.19 A parallel-plate capacitor.

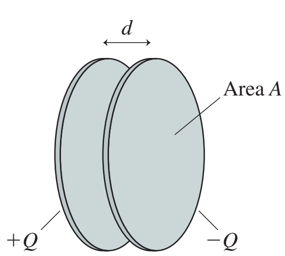


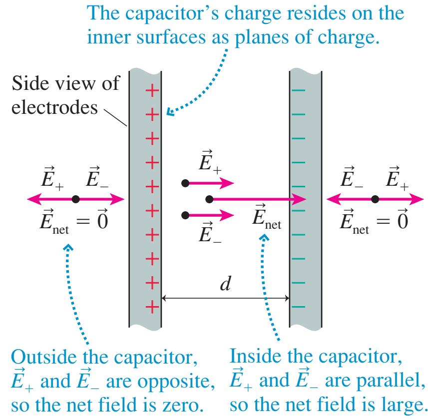
FIGURE 26.19 shows two electrodes, one with charge $+Q$ and the other with $-Q$, placed face-to-face a distance d apart. This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.** Capacitors play important roles in many electric circuits. Our goal is to find the electric field both inside the capacitor (i.e., between the plates) and outside the capacitor.

NOTE ▶ The *net* charge of a capacitor is zero. Capacitors are charged by transferring electrons from one plate to the other. The plate that gains N electrons has charge $-Q = N(-e)$. The plate that loses electrons has charge $+Q$. ◀

Let's begin with a qualitative investigation. **FIGURE 26.20** is an enlarged view of the capacitor plates, seen from the side. Because opposite charges attract, all of the charge is on the *inner* surfaces of the two plates. Thus the inner surfaces of the plates can be modeled as two planes of charge with equal but opposite surface charge densities. As you can see from the figure, at all points in space the electric field $\vec{E}_+$ of the positive plate points *away from* the plane of positive charges. Similarly, the field $\vec{E}_-$ of the negative plate everywhere points *toward* the plane of negative charges.

NOTE ▶ You might think the right capacitor plate would somehow "block" the electric field created by the positive plate and prevent the presence of an $\vec{E}_+$ field to the right of the capacitor. To see that it doesn't, consider an analogous situation with gravity. The strength of gravity above a table is the same as its strength below it. Just as the table doesn't block the earth's gravitational field, intervening matter or charges do not alter or block an object's electric field. ◀

FIGURE 26.20 The electric fields inside and outside a parallel-plate capacitor.



Inside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ are parallel and of equal strength. Their superposition creates a net electric field inside the capacitor that points from the positive plate to the negative plate. Outside the capacitor, $\vec{E}_+$ and $\vec{E}_-$ point in opposite directions and, because the field of a plane of charge is independent of the distance from the plane, have equal magnitudes. Consequently, the fields $\vec{E}_+$ and $\vec{E}_-$ add to zero outside the capacitor plates.

We can calculate the fields between the capacitor plates from the field of an infinite charged plane. Between the electrodes, $\vec{E}_+$ is of magnitude $\eta/2\epsilon_0$ and points from the positive toward the negative side. The field $\vec{E}_-$ is *also* of magnitude $\eta/2\epsilon_0$ and *also* points from positive to negative. Thus the electric field inside the capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$
$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right) \tag{26.29}$$

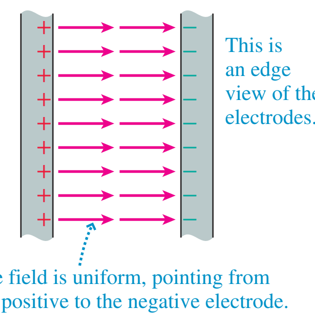
where A is the surface area of each electrode. Outside the capacitor plates, where $\vec{E}_+$ and $\vec{E}_-$ have equal magnitudes but *opposite* directions, $\vec{E} = \vec{0}$.

FIGURE 26.21a shows the electric field of an ideal parallel-plate capacitor constructed from two infinite charged planes. Now, it's true that no real capacitor is infinite in extent, but the ideal parallel-plate capacitor is a very good approximation for all but the most precise calculations as long as the electrode separation d is much smaller than the electrodes' size. **FIGURE 26.21b** shows that the interior field of a real capacitor is virtually identical to that of an ideal capacitor but that the exterior field isn't quite zero. This weak field outside the capacitor is called the **fringe field.** We will keep things simple by always assuming the plates are very close together and using Equation 26.29 for the field inside a parallel-plate capacitor.

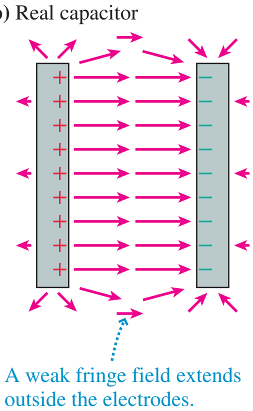
NOTE ▶ The shape of the electrodes—circular or square or any other shape—is not relevant as long as the electrodes are very close together. ◀

FIGURE 26.21 The electric field of a capacitor.

(a) Ideal capacitor



(b) Real capacitor



EXAMPLE 26.6 The electric field inside a capacitor

Two 1.0 cm × 2.0 cm rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength 2.0×10^6 N/C? How many electrons must be moved from one electrode to the other to accomplish this?

MODEL The electrodes can be modeled as a parallel-plate capacitor because the spacing between them is much smaller than their lateral dimensions.

SOLVE The electric field strength inside the capacitor is $E = Q/\epsilon_0 A$. Thus the charge to produce a field of strength E is

$$Q = (8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(2.0 \times 10^{-4}\ \text{m}^2)(2.0 \times 10^6\ \text{N/C})$$
$$= 3.5 \times 10^{-9}\ \text{C} = 3.5\ \text{nC}$$

The positive plate must be charged to +3.5 nC and the negative plate to −3.5 nC. In practice, the plates are charged by using a

Continued

battery to move electrons from one plate to the other. The number of electrons in 3.5 nC is

$$N = \frac{Q}{e} = \frac{3.5 \times 10^{-9}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = 2.2 \times 10^{10}\ \text{electrons}$$

Thus 2.2×10^{10} electrons are moved from one electrode to the other. Note that the capacitor *as a whole* has no net charge.

ASSESS The plate spacing does not enter the result. As long as the spacing is much smaller than the plate dimensions, as is true in this example, the field is independent of the spacing.

Uniform Electric Fields

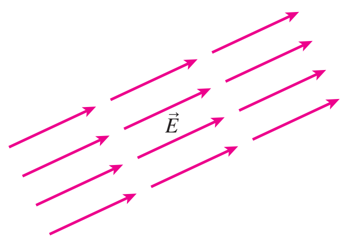


FIGURE 26.22 A uniform electric field.

FIGURE 26.22 shows an electric field that is the *same*—in strength and direction—at every point in a region of space. This is called a **uniform electric field.** A uniform electric field is analogous to the uniform gravitational field near the surface of the earth. Uniform fields are of great practical significance because, as you will see in the next section, computing the trajectory of a charged particle moving in a uniform electric field is a straightforward process.

The easiest way to produce a uniform electric field is with a parallel-plate capacitor, as you can see in Figure 26.21a. Indeed, our interest in capacitors is due in large measure to the fact that the electric field is uniform. Many electric field problems refer to a uniform electric field. Such problems carry an implicit assumption that the action is taking place *inside* a parallel-plate capacitor.

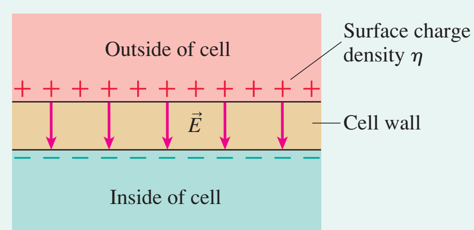
EXAMPLE 26.7 Charge density on a cell wall

Example 25.7 noted that the electric field strength in the cell wall of a neuron is typically 1.0×10^7 N/C. This electric field is established because the outer surface of the cell wall is positive and the inner surface negative. What is a typical surface charge density on the surface of a cell wall?

MODEL Although cells are roughly spherical, the wall thickness is much less than the radius of the cell. Locally, at a point inside the cell wall, the curvature is negligible, so we can model the cell wall as a parallel-plate capacitor.

VISUALIZE **FIGURE 26.23** shows a section of the cell wall. The charges are due to ions, not electrons, but that doesn't affect our analysis.

FIGURE 26.23 The electric field inside the cell wall is due to charges on the surfaces.

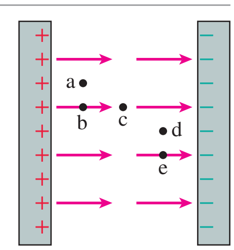


SOLVE The electric field strength inside a capacitor is $E = \eta/\epsilon_0$. The surface charge density needed to produce a known field is

$$\eta = \epsilon_0 E = (8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(1.0 \times 10^7\ \text{N/C})$$
$$= 8.9 \times 10^{-5}\ \text{C/m}^2$$

ASSESS The charge density may seem rather large, but cells are very small. A typical cell is $\approx 10\ \mu\text{m}$ in diameter, with a surface area of $\approx 3 \times 10^{-10}\ \text{m}^2$. At a surface charge density of $9 \times 10^{-5}\ \text{C/m}^2$, the total charge on the outer surface of the cell is $\approx 3 \times 10^{-14}$ C, or $\approx 200{,}000$ ions.

STOP TO THINK 26.5 Rank in order, from largest to smallest, the forces F_a to F_e a proton would experience if placed at points a to e in this parallel-plate capacitor.



26.6 Motion of a Charged Particle in an Electric Field

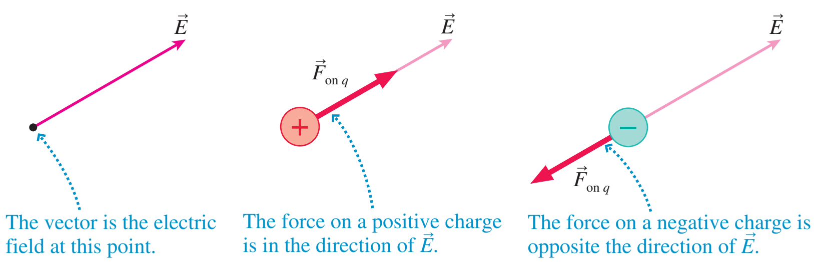
Our motivation for introducing the concept of the electric field was to understand the long-range electric interaction of charges. We said that some charges, the *source charges,* create an electric field. Other charges then respond to that electric field. The first five sections of this chapter have focused on the electric field of the source charges. Now we turn our attention to the second half of the interaction.

FIGURE 26.24 shows a particle of charge q and mass m at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges. The electric field exerts a force

$$\vec{F}_{\text{on } q} = q\vec{E}$$

on the charged particle. Notice that the force on a negatively charged particle is *opposite* in direction to the electric field vector. Signs are important!

FIGURE 26.24 The electric field exerts a force on a charged particle.



If $\vec{F}_{\text{on } q}$ is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E} \tag{26.30}$$

This acceleration is the *response* of the charged particle to the source charges that created the electric field. The ratio q/m is especially important for the dynamics of charged-particle motion. It is called the **charge-to-mass ratio.** Two *equal* charges, say a proton and a Na^+ ion, will experience *equal* forces $\vec{F} = q\vec{E}$ if placed at the same point in an electric field, but their accelerations will be *different* because they have different masses and thus different charge-to-mass ratios. Two particles with different charges and masses *but* with the same charge-to-mass ratio will undergo the same acceleration and follow the same trajectory.

"DNA fingerprints" are measured with the technique of *gel electrophoresis*. A solution of DNA fragments is placed in a well at one end of a plate covered with gel. The fragments are negatively charged when in solution, and they begin to migrate through the gel when a uniform electric field is established parallel to the surface of the plate. Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size. Thus gel electrophoresis sorts the DNA fragments by size, and fluorescent markers allow the results to be seen.

Motion in a Uniform Field

The motion of a charged particle in a *uniform* electric field is especially important for its basic simplicity and because of its many valuable applications. A uniform field is *constant* at all points—constant in both magnitude and direction—within the region of space where the charged particle is moving. It follows, from Equation 26.30, that **a charged particle in a uniform electric field will move with constant acceleration.** The magnitude of the acceleration is

$$a = \frac{qE}{m} = \text{constant} \tag{26.31}$$

where E is the electric field strength, and the direction of $\vec{a}$ is parallel or antiparallel to $\vec{E}$, depending on the sign of q.

Identifying the motion of a charged particle in a uniform field as being one of constant acceleration brings into play all the kinematic machinery that we developed in Chapters 2 and 4 for constant-acceleration motion. The basic trajectory of a charged particle in a uniform field is a *parabola,* analogous to the projectile motion of a mass in the near-earth uniform gravitational field. In the special case of a charged particle moving parallel to the electric field vectors, the motion is one-dimensional, analogous to the one-dimensional vertical motion of a mass tossed straight up or falling straight down.

NOTE ▶ The gravitational acceleration $\vec{a}_{\text{grav}}$ always points straight down. The electric field acceleration $\vec{a}_{\text{elec}}$ can point in *any* direction. You must determine the electric field $\vec{E}$ in order to learn the direction of $\vec{a}$. ◀

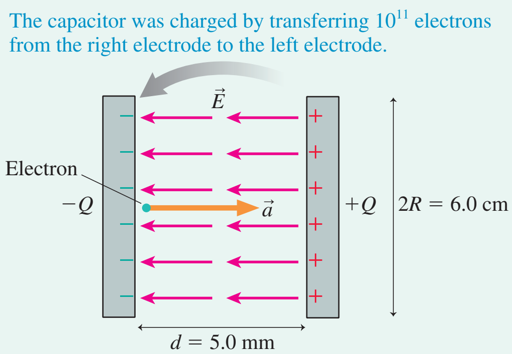
EXAMPLE 26.8 An electron moving across a capacitor

Two 6.0-cm-diameter electrodes are spaced 5.0 mm apart. They are charged by transferring 1.0×10^{11} electrons from one electrode to the other. An electron is released from rest at the surface of the negative electrode. How long does it take the electron to cross to the positive electrode? What is its speed as it collides with the positive electrode? Assume the space between the electrodes is a vacuum.

MODEL The electrodes form a parallel-plate capacitor. The electric field inside a parallel-plate capacitor is a uniform field, so the electron will have constant acceleration.

VISUALIZE **FIGURE 26.25** shows an edge view of the capacitor and the electron. The force on the negative electron is *opposite* the electric field, so the electron is repelled by the negative electrode as it accelerates across the gap of width d.

FIGURE 26.25 An electron accelerates across a capacitor (plate separation exaggerated).



SOLVE The electrodes are not point charges, so we cannot use Coulomb's law to find the force on the electron. Instead, we must analyze the electron's motion in terms of the electric field inside the capacitor. The field is the agent that exerts the force on the electron, causing it to accelerate. The electric field strength inside a parallel-plate capacitor with charge $Q = Ne$ is

$$E = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{Ne}{\epsilon_0 \pi R^2} = 639{,}000 \text{ N/C}$$

The electron's acceleration in this field is

$$a = \frac{eE}{m} = 1.1 \times 10^{17} \text{ m/s}^2$$

where we used the electron mass $m = 9.11 \times 10^{-31}$ kg. This is an enormous acceleration compared to accelerations we're familiar with for macroscopic objects. We can use one-dimensional kinematics, with $x_i = 0$ and $v_i = 0$, to find the time required for the electron to cross the capacitor:

$$x_f = d = \frac{1}{2}a(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2d}{a}} = 3.0 \times 10^{-10} \text{ s} = 0.30 \text{ ns}$$

The electron's speed as it reaches the positive electrode is

$$v = a\,\Delta t = 3.3 \times 10^7 \text{ m/s}$$

ASSESS We used e rather than $-e$ to find the acceleration because we already knew the direction; we needed only the magnitude. The electron's speed, after traveling a mere 5 mm, is approximately 10% the speed of light.

Parallel electrodes such as those in Example 26.8 are often used to accelerate charged particles. If the positive plate has a small hole in the center, a *beam* of electrons will pass through the hole, after accelerating across the capacitor gap, and emerge with a speed of 3.3×10^7 m/s. This is the basic idea of the *electron gun* used until quite recently in *cathode-ray tube* (CRT) devices such as televisions and computer display terminals. (A negatively charged electrode is called a *cathode,* so the physicists who first learned to produce electron beams in the late 19th century called them *cathode rays.*) The following example shows that parallel electrodes can also be used to deflect charged particles sideways.

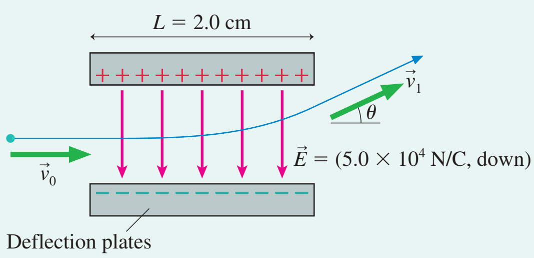
EXAMPLE 26.9 Deflecting an electron beam

An electron gun creates a beam of electrons moving horizontally with a speed of 3.3×10^7 m/s. The electrons enter a 2.0-cm-long gap between two parallel electrodes where the electric field is $\vec{E} = (5.0 \times 10^4\,\text{N/C, down})$. In which direction, and by what angle, is the electron beam deflected by these electrodes?

MODEL The electric field between the electrodes is uniform. Assume that the electric field outside the electrodes is zero.

VISUALIZE **FIGURE 26.26** shows an electron moving through the electric field. The electric field points down, so the force on the (negative) electrons is upward. The electrons will follow a parabolic trajectory, analogous to that of a ball thrown horizontally, except that the electrons "fall up" rather than down.

FIGURE 26.26 The deflection of an electron beam in a uniform electric field.



SOLVE This is a two-dimensional motion problem. The electron enters the capacitor with velocity *vector* $\vec{v}_0 = v_{0x}\hat{\imath} = 3.3 \times 10^7\,\hat{\imath}$ m/s and leaves with velocity $\vec{v}_1 = v_{1x}\hat{\imath} + v_{1y}\hat{\jmath}$. The electron's angle of travel upon leaving the electric field is

$$\theta = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right)$$

This is the *deflection angle.* To find θ we must compute the final velocity vector $\vec{v}_1$.

There is no horizontal force on the electron, so $v_{1x} = v_{0x} = 3.3 \times 10^7$ m/s. The electron's upward acceleration has magnitude

$$a = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\,\text{C})(5.0 \times 10^4\,\text{N/C})}{9.11 \times 10^{-31}\,\text{kg}}$$
$$= 8.78 \times 10^{15}\,\text{m/s}^2$$

We can use the fact that the horizontal velocity is constant to determine the time interval Δt needed to travel length 2.0 cm:

$$\Delta t = \frac{L}{v_{0x}} = \frac{0.020\,\text{m}}{3.3 \times 10^7\,\text{m/s}} = 6.06 \times 10^{-10}\,\text{s}$$

Vertical acceleration will occur during this time interval, resulting in a final vertical velocity

$$v_{1y} = v_{0y} + a\,\Delta t = 5.3 \times 10^6\,\text{m/s}$$

The electron's velocity as it leaves the capacitor is thus

$$\vec{v}_1 = (3.3 \times 10^7\,\hat{\imath} + 5.3 \times 10^6\,\hat{\jmath})\,\text{m/s}$$

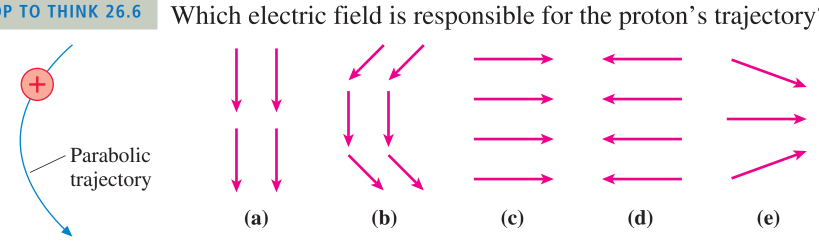
and the deflection angle θ is

$$\theta = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right) = 9.1°$$

ASSESS We know that the electron beam in a cathode-ray tube can be deflected enough to cover the screen, so a deflection angle of 9° seems reasonable. Our neglect of the gravitational force is seen to be justified because the acceleration of the electrons is enormous in comparison to the free-fall acceleration g.

Example 26.9 demonstrates how an electron beam is steered to a point on the screen of a cathode-ray tube. First, a high-speed electron beam is created by an electron gun like that of Example 26.8. The beam then passes first through a set of *vertical deflection plates,* as in Example 26.9, then through a second set of *horizontal deflection plates.* After leaving the deflection plates, it travels in a straight line (through vacuum, to eliminate collisions with air molecules) to the screen of the CRT, where it strikes a phosphor coating on the inside surface and makes a dot of light. Properly choosing the electric fields within the deflection plates steers the electron beam to any point on the screen.

STOP TO THINK 26.6 Which electric field is responsible for the proton's trajectory?



26.7 Motion of a Dipole in an Electric Field

Let us conclude this chapter by returning to one of the more striking puzzles we faced when making the observations at the beginning of Chapter 25. There you found that charged objects of *either* sign exert forces on neutral objects, such as when a comb used to brush your hair picks up pieces of paper. Our qualitative understanding of the *polarization force* was that it required two steps:

- The charge polarizes the neutral object, creating an induced electric dipole.
- The charge then exerts an attractive force on the near end of the dipole that is slightly stronger than the repulsive force on the far end.

We are now in a position to make that understanding more quantitative.

Dipoles in a Uniform Field

FIGURE 26.27 A dipole in a uniform electric field.

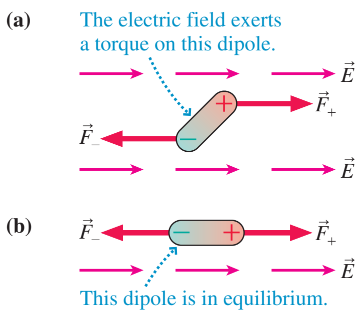


FIGURE 26.27a shows an electric dipole in a *uniform* external electric field $\vec{E}$ that has been created by source charges we do not see. That is, $\vec{E}$ is *not* the field of the dipole but, instead, is a field to which the dipole is responding. In this case, because the field is uniform, the dipole is presumably inside an unseen parallel-plate capacitor.

The net force on the dipole is the sum of the forces on the two charges forming the dipole. Because the charges $\pm q$ are equal in magnitude but opposite in sign, the two forces $\vec{F}_+ = +q\vec{E}$ and $\vec{F}_- = -q\vec{E}$, are also equal but opposite. Thus the net force on the dipole is

$$\vec{F}_{\text{net}} = \vec{F}_+ + \vec{F}_- = \vec{0} \tag{26.32}$$

There is no net force on a dipole in a uniform electric field.

There may be no net force, but the electric field *does* affect the dipole. Because the two forces in Figure 26.27a are in opposite directions but not aligned with each other, the electric field exerts a *torque* on the dipole and causes the dipole to *rotate*.

The torque causes the dipole to rotate until it is aligned with the electric field, as shown in FIGURE 26.27b. In this position, the dipole experiences not only no net force but also no torque. Thus Figure 26.27b represents the *equilibrium position* for a dipole in a uniform electric field. Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.

FIGURE 26.28 A sample of permanent dipoles is *polarized* in an electric field.

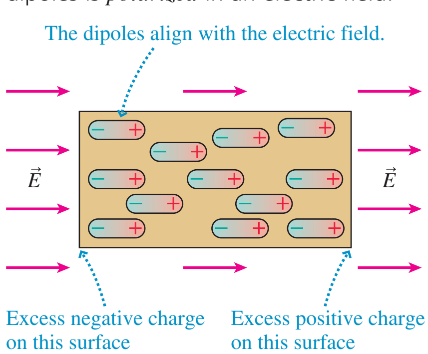
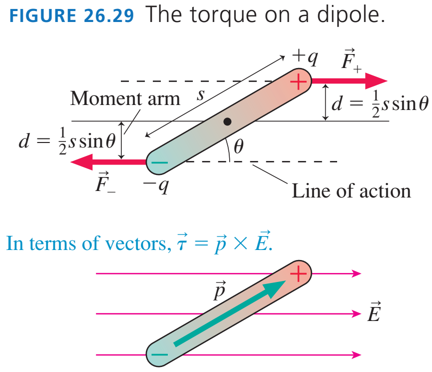


FIGURE 26.28 shows a sample of permanent dipoles, such as water molecules, in an external electric field. All the dipoles rotate until they are aligned with the electric field. This is the mechanism by which the sample becomes *polarized*. Once the dipoles are aligned, there is an excess of positive charge at one end of the sample and an excess of negative charge at the other end. The excess charges at the ends of the sample are the basis of the polarization forces we discussed in Section 25.3.

It's not hard to calculate the torque. Recall from Chapter 12 that the magnitude of a torque is the product of the force and the moment arm. FIGURE 26.29 shows that there are two forces of the same magnitude ($F_+ = F_- = qE$), each with the same moment arm ($d = \frac{1}{2}s\sin\theta$). Thus the torque on the dipole is

$$\tau = 2 \times dF_+ = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta \tag{26.33}$$

FIGURE 26.29 The torque on a dipole.



where $p = qs$ was our definition of the dipole moment. The torque is zero when the dipole is aligned with the field, making $\theta = 0$.

Recall from Chapter 12 that the torque can be written in a compact mathematical form as the cross product between two vectors. The terms p and E in Equation 26.33 are the magnitudes of vectors, and θ is the angle between them. Thus in vector notation, the torque exerted on a dipole moment $\vec{p}$ by an electric field $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{26.34}$$

The torque is greatest when $\vec{p}$ is perpendicular to $\vec{E}$, zero when $\vec{p}$ is aligned with or opposite to $\vec{E}$.

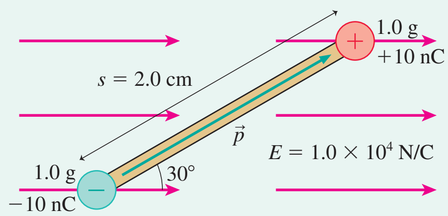
EXAMPLE 26.10 The angular acceleration of a dipole dumbbell

Two 1.0 g balls are connected by a 2.0-cm-long insulating rod of negligible mass. One ball has a charge of $+10$ nC, the other a charge of -10 nC. The rod is held in a 1.0×10^4 N/C uniform electric field at an angle of 30° with respect to the field, then released. What is its initial angular acceleration?

MODEL The two oppositely charged balls form an electric dipole. The electric field exerts a torque on the dipole, causing an angular acceleration.

VISUALIZE FIGURE 26.30 shows the dipole in the electric field.

FIGURE 26.30 The dipole of Example 26.10.



SOLVE The dipole moment is $p = qs = (1.0 \times 10^{-8}\text{ C}) \times (0.020\text{ m}) = 2.0 \times 10^{-10}\text{ C m}$. The torque exerted on the dipole moment by the electric field is

$$\tau = pE\sin\theta = (2.0 \times 10^{-10}\text{ C m})(1.0 \times 10^4\text{ N/C})\sin 30°$$
$$= 1.0 \times 10^{-6}\text{ N m}$$

You learned in Chapter 12 that a torque causes an angular acceleration $\alpha = \tau/I$, where I is the moment of inertia. The dipole rotates about its center of mass, which is at the center of the rod, so the moment of inertia is

$$I = m_1 r_1^{\,2} + m_2 r_2^{\,2} = 2m\left(\frac{1}{2}s\right)^2 = \frac{1}{2}ms^2 = 2.0 \times 10^{-7}\text{ kg m}^2$$

Thus the rod's angular acceleration is

$$\alpha = \frac{\tau}{I} = \frac{1.0 \times 10^{-6}\text{ N m}}{2.0 \times 10^{-7}\text{ kg m}^2} = 5.0\text{ rad/s}^2$$

ASSESS This value of α is the initial angular acceleration, when the rod is first released. The torque and the angular acceleration will decrease as the rod rotates toward alignment with $\vec{E}$.

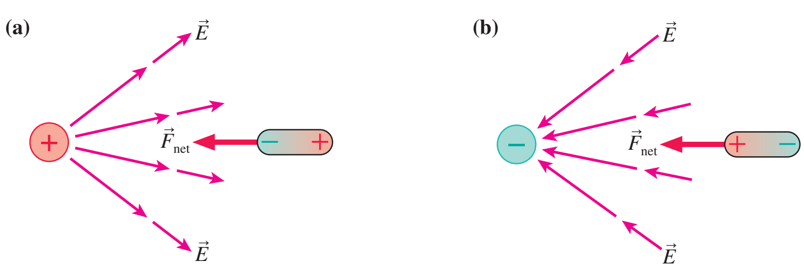
Dipoles in a Nonuniform Field

Suppose that a dipole is placed in a nonuniform electric field, one in which the field strength changes with position. For example, FIGURE 26.31 shows a dipole in the nonuniform field of a point charge. The first response of the dipole is to rotate until it is aligned with the field, with the dipole's positive end pointing in the same direction as the field. Now, however, there is a slight difference between the forces acting on the two ends of the dipole. This difference occurs because the electric field, which depends on the distance from the point charge, is stronger at the end of the dipole nearest the charge. This causes a net force to be exerted on the dipole.

Which way does the force point? FIGURE 26.31a shows a positive point charge. Once the dipole is aligned, the leftward attractive force on its negative end is slightly stronger than the rightward repulsive force on its positive end. This causes a net force to the *left,* toward the point charge. The dipole in FIGURE 26.31b aligns in the opposite orientation in the field of a negative point charge, but the net force is still to the left.

As you can see, **the net force on a dipole is toward the direction of the strongest field.** Because any finite-size charged object, such as a charged rod or a charged disk, has a field strength that increases as you get closer to the object, we can conclude that **a dipole will experience a net force toward any charged object.**

FIGURE 26.31 An aligned dipole is drawn toward a point charge.

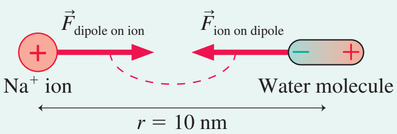


EXAMPLE 26.11 The force on a water molecule

The water molecule H_2O has a permanent dipole moment of magnitude 6.2×10^{-30} C m. A water molecule is located 10 nm from a Na^+ ion in a saltwater solution. What force does the ion exert on the water molecule?

VISUALIZE **FIGURE 26.32** shows the ion and the dipole. The forces are an action/reaction pair.

FIGURE 26.32 The interaction between an ion and a permanent dipole.



SOLVE A Na^+ ion has charge $q = +e$. The electric field of the ion aligns the water's dipole moment and exerts a net force on it. We could calculate the net force on the dipole as the small difference between the attractive force on its negative end and the repulsive force on its positive end. Alternatively, we know from Newton's third law that the force $\vec{F}_{\text{dipole on ion}}$ has the same magnitude as the force $\vec{F}_{\text{ion on dipole}}$ that we are seeking. We calculated the on-axis field of a dipole in Section 26.2. An ion of charge $q = e$ will experience a force of magnitude $F = qE_{\text{dipole}} = eE_{\text{dipole}}$ when placed in that field. The dipole's electric field, which we found in Equation 26.11, is

$$E_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$$

The force on the ion at distance $r = 1.0 \times 10^{-8}$ m is

$$F_{\text{dipole on ion}} = eE_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2ep}{r^3} = 1.8 \times 10^{-14}\,\text{N}$$

Thus the force on the water molecule is $F_{\text{ion on dipole}} = 1.8 \times 10^{-14}$ N.

ASSESS While 1.8×10^{-14} N may seem like a very small force, it is $\approx 10^{11}$ times larger than the size of the earth's gravitational force on these atomic particles. Forces such as these cause water molecules to cluster around any ions that are in solution. This clustering plays an important role in the microscopic physics of solutions studied in chemistry and biochemistry.

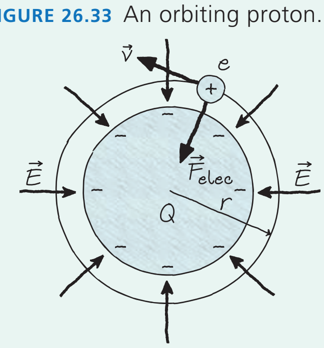
CHALLENGE EXAMPLE 26.12 An orbiting proton

In a vacuum chamber, a proton orbits a 1.0-cm-diameter metal ball 1.0 mm above the surface with a period of 1.0 μs. What is the charge on the ball?

MODEL Model the ball as a charged sphere. The electric field of a charged sphere is the same as that of a point charge at the center, so the radius of the ball is irrelevant. Assume that the gravitational force on the proton is extremely small compared to the electric force and can be neglected.

VISUALIZE **FIGURE 26.33** shows the orbit and the force on the proton.

FIGURE 26.33 An orbiting proton.



SOLVE The ball must be negative, with an inward electric field exerting an inward electric force on the positive proton. This is exactly the necessary condition for uniform circular motion. Recall from Chapter 8 that Newton's second law for uniform circular motion is $(F_{\text{net}})_r = mv^2/r$. Here the only radial force has magnitude $F_{\text{elec}} = eE$, so the proton will move in a circular orbit if

$$eE = \frac{mv^2}{r}$$

The electric field strength of a sphere of charge Q at distance r is $E = Q/4\pi\epsilon_0 r^2$. From Chapter 4, orbital speed and period are related by $v = \text{circumference/period} = 2\pi r/T$. With these substitutions, Newton's second law becomes

$$\frac{eQ}{4\pi\epsilon_0 r^2} = \frac{4\pi^2 m}{T^2} r$$

Solving for Q, we find

$$Q = \frac{16\pi^3\epsilon_0 m r^3}{eT^2} = 9.9 \times 10^{-12}\ \text{C}$$

where we used $r = 6.0$ mm as the radius of the proton's orbit. Q is the *magnitude* of the charge on the ball. Including the sign, we have

$$Q_{\text{ball}} = -9.9 \times 10^{-12}\,\text{C}$$

ASSESS This is not a lot of charge, but it shouldn't take much charge to affect the motion of something as light as a proton.

SUMMARY

The goal of Chapter 26 has been to learn how to calculate and use the electric field.

General Principles

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

Continuous distribution of charge

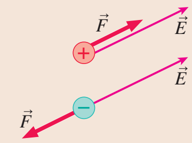
- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ.
- Find $\vec{E}$ by summing the fields of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$



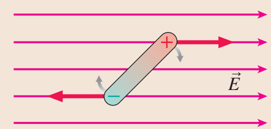
The force causes acceleration:

$$\vec{a} = (q/m)\vec{E}$$

Trajectories of charged particles are calculated with kinematics.

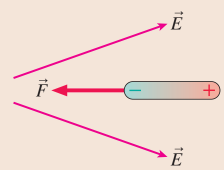
The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$



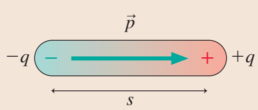
The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



Applications

Electric dipole



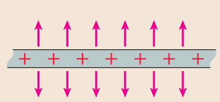
The electric dipole moment is

$\vec{p} = (qs$, from negative to positive)

Field on axis: $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

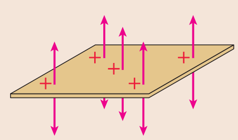
Field in bisecting plane: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{ perpendicular to line}\right)$$

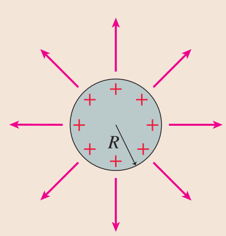
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{ perpendicular to plane}\right)$$

Sphere of charge

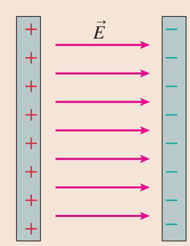
Same as a point charge Q for $r > R$



Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field:**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$



A real capacitor has a weak **fringe field** around it.

Terms and Notation

dipole moment, $\vec{p}$
electric field line
linear charge density, λ
surface charge density, η
uniformly charged
line of charge
electrode
plane of charge
sphere of charge
parallel-plate capacitor
fringe field
uniform electric field
charge-to-mass ratio, q/m

CONCEPTUAL QUESTIONS

1. You've been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.
2. Reproduce FIGURE Q26.2 on your paper. For each part, draw a dot or dots on the figure to show any position or positions (other than infinity) where $\vec{E} = \vec{0}$.

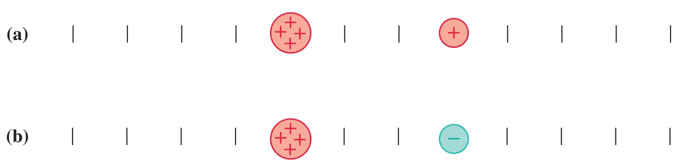


FIGURE Q26.2

3. Rank in order, from largest to smallest, the electric field strengths E_1 to E_4 at points 1 to 4 in FIGURE Q26.3. Explain.

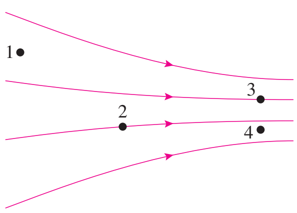


FIGURE Q26.3

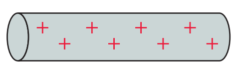


FIGURE Q26.4

4. A small segment of wire in FIGURE Q26.4 contains 10 nC of charge.
   a. The segment is shrunk to one-third of its original length. What is the ratio λ_f/λ_i, where λ_i and λ_f are the initial and final linear charge densities?
   b. A proton is very far from the wire. What is the ratio F_f/F_i of the electric force on the proton after the segment is shrunk to the force before the segment was shrunk?
   c. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be *added* to the wire to keep the linear charge density unchanged?
5. An electron experiences a force of magnitude F when it is 1 cm from a very long, charged wire with linear charge density λ. If the charge density is doubled, at what distance from the wire will a proton experience a force of the same magnitude F?
6. FIGURE Q26.6 shows a hollow soda straw that has been uniformly charged with positive charge. What is the electric field at the center (inside) of the straw? Explain.

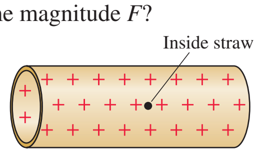


FIGURE Q26.6

7. The irregularly shaped area of charge in FIGURE Q26.7 has surface charge density η_i. Each dimension (x and y) of the area is reduced by a factor of 3.163.
   a. What is the ratio η_f/η_i, where η_f is the final surface charge density?
   b. An electron is very far from the area. What is the ratio F_f/F_i of the electric force on the electron after the area is reduced to the force before the area was reduced?

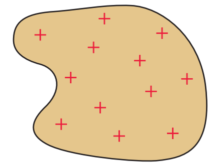


FIGURE Q26.7

8. A circular disk has surface charge density 8 nC/cm^2. What will the surface charge density be if the radius of the disk is doubled?
9. A sphere of radius R has charge Q. The electric field strength at distance $r > R$ is E_i. What is the ratio E_f/E_i of the final to initial electric field strengths if (a) Q is halved, (b) R is halved, and (c) r is halved (but is still $> R$)? Each part changes only one quantity; the other quantities have their initial values.
10. The ball in FIGURE Q26.10 is suspended from a large, uniformly charged positive plate. It swings with period T. If the ball is discharged, will the period increase, decrease, or stay the same? Explain.

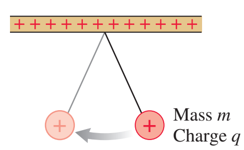


FIGURE Q26.10

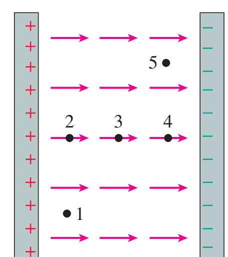


FIGURE Q26.11

$\frac{\sigma}{\varepsilon_0} = E$

11. Rank in order, from largest to smallest, the electric field strengths E_1 to E_5 at the five points in FIGURE Q26.11. Explain.
12. A parallel-plate capacitor consists of two square plates, size $L \times L$, separated by distance d. The plates are given charge $\pm Q$. What is the ratio E_f/E_i of the final to initial electric field strengths if (a) Q is doubled, (b) L is doubled, and (c) d is doubled? Each part changes only one quantity; the other quantities have their initial values.
13. A small object is released at point 3 in the center of the capacitor in FIGURE Q26.11. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?
    a. A positive object is released from rest.
    b. A neutral but polarizable object is released from rest.
    c. A negative object is released from rest.

14. A proton and an electron are released from rest in the center of a capacitor.
   a. Is the force ratio F_p/F_e greater than 1, less than 1, or equal to 1? Explain.
   b. Is the acceleration ratio a_p/a_e greater than 1, less than 1, or equal to 1? Explain.

15. Three charges are placed at the corners of the triangle in FIGURE Q26.15. The ++ charge has twice the quantity of charge of the two – charges; the net charge is zero. Is the triangle in equilibrium? If so, explain why. If not, draw the equilibrium orientation.

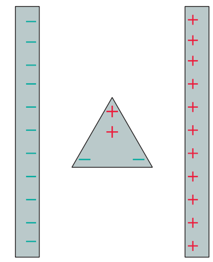


FIGURE Q26.15

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 26.2 The Electric Field of Multiple Point Charges

1. | What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.1? Specify the direction as an angle above or below horizontal.

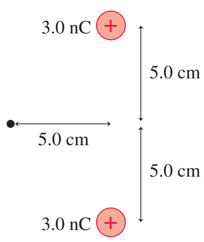


FIGURE EX26.1

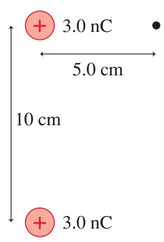


FIGURE EX26.2

2. | What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.2? Specify the direction as an angle above or below horizontal.
3. | What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.3? Specify the direction as an angle above or below horizontal.

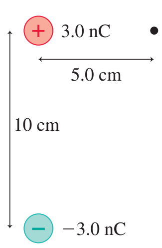


FIGURE EX26.3

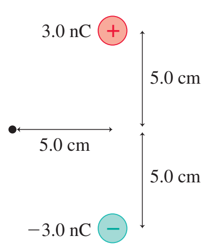


FIGURE EX26.4

4. | What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.4? Specify the direction as an angle above or below horizontal.
5. | An electric dipole is formed from ± 1.0 nC charges spaced 2.0 mm apart. The dipole is at the origin, oriented along the x-axis. What is the electric field strength at the points (a) $(x, y) =$ (10 cm, 0 cm) and (b) $(x, y) =$ (0 cm, 10 cm)?
6. | An electric dipole is formed from two charges, $\pm q$, spaced 1.0 cm apart. The dipole is at the origin, oriented along the y-axis. The electric field strength at the point $(x, y) =$ (0 cm, 10 cm) is 360 N/C.
   a. What is the charge q? Give your answer in nC.
   b. What is the electric field strength at the point $(x, y) =$ (10 cm, 0 cm)?

Section 26.3 The Electric Field of a Continuous Charge Distribution

7. | The electric field strength 10.0 cm from a very long charged wire is 2000 N/C. What is the electric field strength 5.0 cm from the wire?
8. | A 10-cm-long thin glass rod uniformly charged to +10 nC and a 10-cm-long thin plastic rod uniformly charged to −10 nC are placed side by side, 4.0 cm apart. What are the electric field strengths E_1 to E_3 at distances 1.0 cm, 2.0 cm, and 3.0 cm from the glass rod along the line connecting the midpoints of the two rods?
9. | Two 10-cm-long thin glass rods uniformly charged to +10 nC are placed side by side, 4.0 cm apart. What are the electric field strengths E_1 to E_3 at distances 1.0 cm, 2.0 cm, and 3.0 cm to the right of the rod on the left along the line connecting the midpoints of the two rods?
10. | A small glass bead charged to +6.0 nC is 4.0 cm from a thin, uniformly charged, 10-cm-long glass rod. The bead is repelled from the rod with a force of 840 μN. What is the total charge on the rod?

Section 26.4 The Electric Fields of Rings, Disks, Planes, and Spheres

11. | Two 10-cm-diameter charged rings face each other, 20 cm apart. The left ring is charged to −20 nC and the right ring is charged to +20 nC.
   a. What is the electric field $\vec{E}$, both magnitude and direction, at the midpoint between the two rings?
   b. What is the force $\vec{F}$ on a −1.0 nC charge placed at the midpoint?
12. | Two 10-cm-diameter charged rings face each other, 20 cm apart. Both rings are charged to +20 nC. What is the electric field strength at (a) the midpoint between the two rings and (b) the center of the left ring?
13. | Two 10-cm-diameter charged disks face each other, 20 cm apart. The left disk is charged to −50 nC and the right disk is charged to +50 nC.
   a. What is the electric field $\vec{E}$, both magnitude and direction, at the midpoint between the two disks?
   b. What is the force $\vec{F}$ on a −1.0 nC charge placed at the midpoint?

14. ‖ Two 10-cm-diameter charged disks face each other, 20 cm apart. Both disks are charged to $+50$ nC. What is the electric field strength at (a) the midpoint between the two disks and (b) a point on the axis 5.0 cm from one disk?

15. ‖ The electric field strength 2.0 cm from a 10-cm-diameter metal ball is 50,000 N/C. What is the charge (in nC) on the ball?

16. ‖ A 20 cm $\times$ 20 cm horizontal metal electrode is uniformly charged to $+80$ nC. What is the electric field strength 2.0 mm above the center of the electrode?

Section 26.5 The Parallel-Plate Capacitor

17. ‖ Two circular disks spaced 0.50 mm apart form a parallel-plate capacitor. Transferring 3.0×10^9 electrons from one disk to the other causes the electric field strength to be 2.0×10^5 N/C. What are the diameters of the disks?

18. ‖ A parallel-plate capacitor is formed from two 6.0-cm-diameter electrodes spaced 2.0 mm apart. The electric field strength inside the capacitor is 1.0×10^6 N/C. What is the charge (in nC) on each electrode?

19. ‖ Air "breaks down" when the electric field strength reaches 3.0×10^6 N/C, causing a spark. A parallel-plate capacitor is made from two 4.0 cm $\times$ 4.0 cm disks. How many electrons must be transferred from one disk to the other to create a spark between the disks?

Section 26.6 Motion of a Charged Particle in an Electric Field

20. ‖ A 0.10 g glass bead is charged by the removal of 1.0×10^{10} electrons. What electric field $\vec{E}$ (strength and direction) will cause the bead to hang suspended in the air?

21. | Two 2.0-cm-diameter disks face each other, 1.0 mm apart. They are charged to ± 10 nC.
   a. What is the electric field strength between the disks?
   b. A proton is shot from the negative disk toward the positive disk. What launch speed must the proton have to just barely reach the positive disk?

22. ‖ An electron in a uniform electric field increases its speed from 2.0×10^7 m/s to 4.0×10^7 m/s over a distance of 1.2 cm. What is the electric field strength?

23. ‖ The surface charge density on an infinite charged plane is -2.0×10^{-6} C/m^2. A proton is shot straight away from the plane at 2.0×10^6 m/s. How far does the proton travel before reaching its turning point?

24. ‖ A 1.0-μm-diameter oil droplet (density 900 kg/m^3) is negatively charged with the addition of 25 extra electrons. It is released from rest 2.0 mm from a very wide plane of positive charge, after which it accelerates toward the plane and collides with a speed of 3.5 m/s. What is the surface charge density of the plane?

Section 26.7 Motion of a Dipole in an Electric Field

25. | The permanent electric dipole moment of the water molecule (H_2O) is 6.2×10^{-30} C m. What is the maximum possible torque on a water molecule in a 5.0×10^8 N/C electric field?

26. ‖| A point charge Q is distance r from the center of a dipole consisting of charges $\pm q$ separated by distance s. The charge is located in the plane that bisects the dipole. At this instant, what are (a) the force (magnitude and direction) and (b) the magnitude of the torque on the dipole? You can assume $r \gg s$.

27. ‖ An ammonia molecule (NH_3) has a permanent electric dipole moment 5.0×10^{-30} C m. A proton is 2.0 nm from the molecule in the plane that bisects the dipole. What is the electric force of the molecule on the proton?

Problems

28. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.28? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.

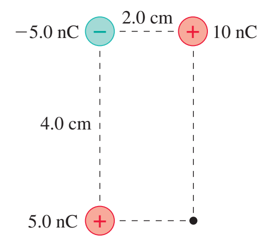


FIGURE P26.28

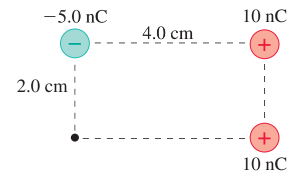


FIGURE P26.29

29. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.29? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.

30. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.30? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.

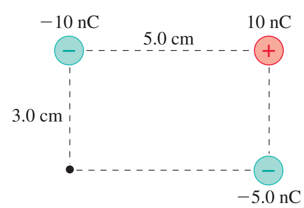


FIGURE P26.30

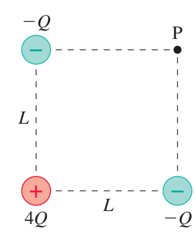


FIGURE P26.31

31. ‖ FIGURE P26.31 shows three charges at the corners of a square. Write the electric field at point P in component form.

32. ‖ Charges $-q$ and $+2q$ in FIGURE P26.32 are located at $x = \pm a$. Determine the electric field at points 1 to 4. Write each field in component form.

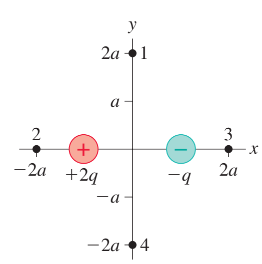


FIGURE P26.32

33. ‖ Two positive charges q are on the y-axis at $y = \pm\frac{1}{2}s$.
   a. Find an expression for the electric field strength at distance x on the axis that bisects the two charges.
   b. For $q = 1.0$ nC and $s = 6.0$ mm, evaluate E at $x = 0, 2, 4, 6,$ and 10 mm.

34. ‖ Derive Equation 26.12 for the field $\vec{E}_{\text{dipole}}$ in the plane that bisects an electric dipole.

35. ‖ Three charges are on the y-axis. Charges $-q$ are at $y = \pm d$ and charge $+2q$ is at $y = 0$.
   a. Find an expression for the electric field $\vec{E}$ at a point on the x-axis.
   b. Verify that your answer to part a has the expected behavior as x becomes very small and very large.

36. ‖ **FIGURE P26.36** is a cross section of two infinite lines of charge that extend out of the page. Both have linear charge density λ. Find an expression for the electric field strength E at height y above the midpoint between the lines.

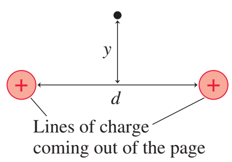


FIGURE P26.36

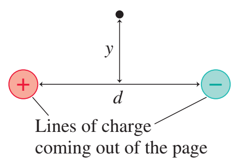


FIGURE P26.37

37. ||| **FIGURE P26.37** is a cross section of two infinite lines of charge that extend out of the page. The linear charge densities are $\pm\lambda$. Find an expression for the electric field strength E at height y above the midpoint between the lines.

38. ‖ Two infinite lines of charge, each with linear charge density λ, lie along the x- and y-axes, crossing at the origin. What is the electric field strength at position (x, y)?

39. ‖ The electric field 5.0 cm from a very long charged wire is (2000 N/C, toward the wire). What is the charge (in nC) on a 1.0-cm-long segment of the wire?

40. ‖ **FIGURE P26.40** shows a thin rod of length L with total charge Q.
   a. Find an expression for the electric field strength at point P on the axis of the rod at distance r from the center.
   b. Verify that your expression has the expected behavior if $r \gg L$.
   c. Evaluate E at $r = 3.0$ cm if $L = 5.0$ cm and $Q = 3.0$ nC.

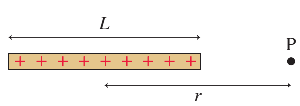


FIGURE P26.40

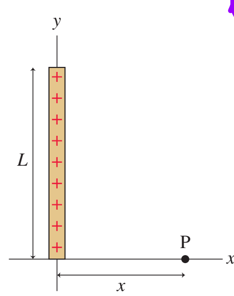


FIGURE P26.41

41. ||| **FIGURE P26.41** shows a thin rod of length L with total charge Q. Find an expression for the electric field $\vec{E}$ at point P. Give your answer in component form.

42. ‖ Show that the on-axis electric field of a ring of charge has the expected behavior when $z \ll R$ and when $z \gg R$.

43. ‖ A ring of radius R has total charge Q.
   a. At what distance along the z-axis is the electric field strength a maximum?
   b. What is the electric field strength at this point?

44. ‖ Charge Q is uniformly distributed along a thin, flexible rod of length L. The rod is then bent into the semicircle shown in **FIGURE P26.44**.
   a. Find an expression for the electric field $\vec{E}$ at the center of the semicircle.
   Hint: A small piece of arc length Δs spans a small angle $\Delta\theta = \Delta s/R$, where R is the radius.
   b. Evaluate the field strength if $L = 10$ cm and $Q = 30$ nC.

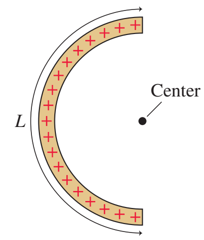


FIGURE P26.44

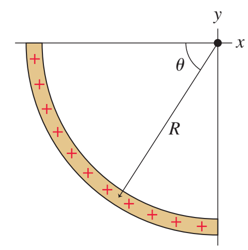


FIGURE P26.45

45. ‖ A plastic rod with linear charge density λ is bent into the quarter circle shown in **FIGURE P26.45**. We want to find the electric field at the origin.
   a. Write expressions for the x- and y-components of the electric field at the origin due to a small piece of charge at angle θ.
   b. Write, but do not evaluate, definite integrals for the x- and y-components of the net electric field at the origin.
   c. Evaluate the integrals and write $\vec{E}_{\text{net}}$ in component form.

46. ‖ You've hung two very large sheets of plastic facing each other with distance d between them, as shown in **FIGURE P26.46**. By rubbing them with wool and silk, you've managed to give one sheet a uniform surface charge density $\eta_1 = -\eta_0$ and the other a uniform surface charge density $\eta_2 = +3\eta_0$. What are the electric field vectors at points 1, 2, and 3?

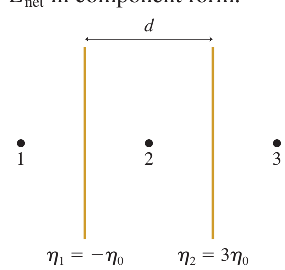


FIGURE P26.46

47. ‖ Two 2.0-cm-diameter insulating spheres have a 6.0 cm space between them. One sphere is charged to +10 nC, the other to −15 nC. What is the electric field strength at the midpoint between the two spheres?

48. ‖ Two parallel plates 1.0 cm apart are equally and oppositely charged. An electron is released from rest at the surface of the negative plate and simultaneously a proton is released from rest at the surface of the positive plate. How far from the negative plate is the point at which the electron and proton pass each other?

49. ‖ A parallel-plate capacitor has 2.0 cm × 2.0 cm electrodes with surface charge densities $\pm 1.0 \times 10^{-6}$ C/m^2. A proton traveling parallel to the electrodes at 1.0×10^6 m/s enters the center of the gap between them. By what distance has the proton been deflected sideways when it reaches the far edge of the capacitor? Assume the field is uniform inside the capacitor and zero outside the capacitor.

50. ‖ An electron is launched at a 45° angle and a speed of 5.0×10^6 m/s from the positive plate of the parallel-plate capacitor shown in **FIGURE P26.50**. The electron lands 4.0 cm away.
   a. What is the electric field strength inside the capacitor?
   b. What is the smallest possible spacing between the plates?

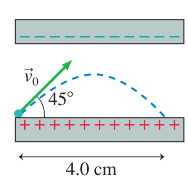


FIGURE P26.50

51. ||| The two parallel plates in FIGURE P26.51 are 2.0 cm apart and the electric field strength between them is 1.0×10^4 N/C. An electron is launched at a 45° angle from the positive plate. What is the maximum initial speed v_0 the electron can have without hitting the negative plate?

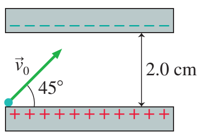


FIGURE P26.51

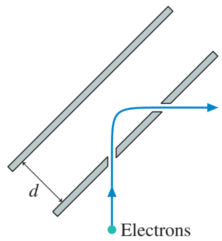


FIGURE P26.52

52. || A problem of practical interest is to make a beam of electrons turn a 90° corner. This can be done with the parallel-plate capacitor shown in FIGURE P26.52. An electron with kinetic energy 3.0×10^{-17} J enters through a small hole in the bottom plate of the capacitor.
 a. Should the bottom plate be charged positive or negative relative to the top plate if you want the electron to turn to the right? Explain.
 b. What strength electric field is needed if the electron is to emerge from an exit hole 1.0 cm away from the entrance hole, traveling at right angles to its original direction?
Hint: The difficulty of this problem depends on how you choose your coordinate system.
 c. What minimum separation d_{min} must the capacitor plates have?

53. | The combustion of fossil fuels produces micron-sized particles of soot, one of the major components of air pollution. The terminal speeds of these particles are extremely small, so they remain suspended in air for very long periods of time. Furthermore, very small particles almost always acquire small amounts of charge from cosmic rays and various atmospheric effects, so their motion is influenced not only by gravity but also by the earth's weak electric field. Consider a small spherical particle of radius r, density ρ, and charge q. A small sphere moving with speed v experiences a drag force $F_{drag} = 6\pi\eta r v$, where η is the viscosity of the air. (This differs from the drag force you learned in Chapter 6 because there we considered macroscopic rather than microscopic objects.)
 a. A particle falling at its terminal speed v_{term} is in dynamic equilibrium with no net force. Write Newton's first law for this particle falling in the presence of a *downward* electric field of strength E, then solve to find an expression for v_{term}.
 b. Soot is primarily carbon, and carbon in the form of graphite has a density of 2200 kg/m^3. In the absence of an electric field, what is the terminal speed in mm/s of a 1.0-μm-diameter graphite particle? The viscosity of air at 20°C is 1.8×10^{-5} kg/m s.
 c. The earth's electric field is typically (150 N/C, downward). In this field, what is the terminal speed in mm/s of a 1.0-μm-diameter graphite particle that has acquired 250 extra electrons?

54. || A 2.0-mm-diameter glass sphere has a charge of +1.0 nC. What speed does an electron need to orbit the sphere 1.0 mm above the surface?

55. || In a classical model of the hydrogen atom, the electron orbits the proton in a circular orbit of radius 0.053 nm. What is the orbital frequency? The proton is so much more massive than the electron that you can assume the proton is at rest.

56. ||| In a classical model of the hydrogen atom, the electron orbits a stationary proton in a circular orbit. What is the radius of the orbit for which the orbital frequency is 1.0×10^{12} s^{-1}?

57. || An electric field can *induce* an electric dipole in a neutral atom or molecule by pushing the positive and negative charges in opposite directions. The dipole moment of an induced dipole is directly proportional to the electric field. That is, $\vec{p} = \alpha\vec{E}$, where α is called the *polarizability* of the molecule. A bigger field stretches the molecule farther and causes a larger dipole moment.
 a. What are the units of α?
 b. An ion with charge q is distance r from a molecule with polarizability α. Find an expression for the force $\vec{F}_{\text{ion on dipole}}$.

58. || Show that an infinite line of charge with linear charge density λ exerts an attractive force on an electric dipole with magnitude $F = 2\lambda p/4\pi\epsilon_0 r^2$. Assume that r is much larger than the charge separation in the dipole.

In Problems 59 through 62 you are given the equation(s) used to solve a problem. For each of these
 a. Write a realistic problem for which this is the correct equation(s).
 b. Finish the solution of the problem.

59. $(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\dfrac{(2.0 \times 10^{-9}\text{ C})\,s}{(0.025\text{ m})^3} = 1150\text{ N/C}$

60. $(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\dfrac{2(2.0 \times 10^{-7}\text{ C/m})}{r} = 25{,}000\text{ N/C}$

61. $\dfrac{\eta}{2\epsilon_0}\left[1 - \dfrac{z}{\sqrt{z^2 + R^2}}\right] = \dfrac{1}{2}\dfrac{\eta}{2\epsilon_0}$

62. $2.0 \times 10^{12}\text{ m/s}^2 = \dfrac{(1.60 \times 10^{-19}\text{ C})E}{(1.67 \times 10^{-27}\text{ kg})}$

$$E = \frac{Q}{(8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2)(0.020\text{ m})^2}$$

Challenge Problems

63. Your physics assignment is to figure out a way to use electricity to launch a small 6.0-cm-long plastic drink stirrer. You decide that you'll charge the little plastic rod by rubbing it with fur, then hold it near a long, charged wire, as shown in FIGURE CP26.63. When you let go, the electric force of the wire on the plastic rod will shoot it away. Suppose you can uniformly charge the plastic stirrer to 10 nC and that the linear charge density of the long wire is 1.0×10^{-7} C/m. What is the net electric force on the plastic stirrer if the end closest to the wire is 2.0 cm away?

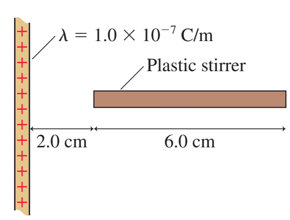


FIGURE CP26.63

64. Three 10-cm-long rods form an equilateral triangle in a plane. Two of the rods are charged to $+10$ nC, the third to -10 nC. What is the electric field strength at the center of the triangle?

65. A rod of length L lies along the y-axis with its center at the origin. The rod has a nonuniform linear charge density $\lambda = a|y|$, where a is a constant with the units C/m^2.
   a. Draw a graph of λ versus y over the length of the rod.
   b. Determine the constant a in terms of L and the rod's total charge Q.
   Hint: This requires an integration. Think about how to handle the absolute value sign.
   c. Find the electric field strength of the rod at distance x on the x-axis.

66. a. An infinitely long *sheet* of charge of width L lies in the xy-plane between $x = -L/2$ and $x = L/2$. The surface charge density is η. Derive an expression for the electric field $\vec{E}$ at height z above the centerline of the sheet.
   b. Verify that your expression has the expected behavior if $z \ll L$ and if $z \gg L$.
   c. Draw a graph of field strength E versus z.

67. a. An infinitely long *sheet* of charge of width L lies in the xy-plane between $x = -L/2$ and $x = L/2$. The surface charge density is η. Derive an expression for the electric field $\vec{E}$ along the x-axis for points outside the sheet ($x > L/2$).
   b. Verify that your expression has the expected behavior if $x \gg L$.
   Hint: $\ln(1 + u) \approx u$ if $u \ll 1$.
   c. Draw a graph of field strength E versus x for $x > L/2$.

68. One type of ink-jet printer, called an electrostatic ink-jet printer, forms the letters by using deflecting electrodes to steer charged ink drops up and down vertically as the ink jet sweeps horizontally across the page. The ink jet forms 30-μm-diameter drops of ink, charges them by spraying 800,000 electrons on the surface, and shoots them toward the page at a speed of 20 m/s. Along the way, the drops pass through two horizontal, parallel electrodes that are 6.0 mm long, 4.0 mm wide, and spaced 1.0 mm apart. The distance from the center of the electrodes to the paper is 2.0 cm. To form the tallest letters, which have a height of 6.0 mm, the drops need to be deflected upward (or downward) by 3.0 mm. What electric field strength is needed between the electrodes to achieve this deflection? Ink, which consists of dye particles suspended in alcohol, has a density of $800\ \text{kg/m}^3$.

69. A proton orbits a long charged wire, making 1.0×10^6 revolutions per second. The radius of the orbit is 1.0 cm. What is the wire's linear charge density?

70. A *positron* is an elementary particle identical to an electron except that its charge is $+e$. An electron and a positron can rotate about their center of mass as if they were a dumbbell connected by a massless rod. What is the orbital frequency for an electron and a positron 1.0 nm apart?

71. You have a summer intern position with a company that designs and builds nanomachines. An engineer with the company is designing a microscopic oscillator to help keep time, and you've been assigned to help him analyze the design. He wants to place a negative charge at the center of a very small, positively charged metal ring. His claim is that the negative charge will undergo simple harmonic motion at a frequency determined by the amount of charge on the ring.
   a. Consider a negative charge near the center of a positively charged ring centered on the z-axis. Show that there is a restoring force on the charge if it moves along the z-axis but stays close to the center of the ring. That is, show there's a force that tries to keep the charge at $z = 0$.
   b. Show that for *small* oscillations, with amplitude $\ll R$, a particle of mass m with charge $-q$ undergoes simple harmonic motion with frequency
   $$f = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\epsilon_0 m R^3}}$$
   R and Q are the radius and charge of the ring.
   c. Evaluate the oscillation frequency for an electron at the center of a 2.0-μm-diameter ring charged to 1.0×10^{-13} C.

STOP TO THINK ANSWERS

Stop to Think 26.1: c. From symmetry, the fields of the positive charges cancel. The net field is that of the negative charge, which is toward the charge.

Stop to Think 26.2: $\eta_c = \eta_b = \eta_a$. All pieces of a uniformly charged surface have the same surface charge density.

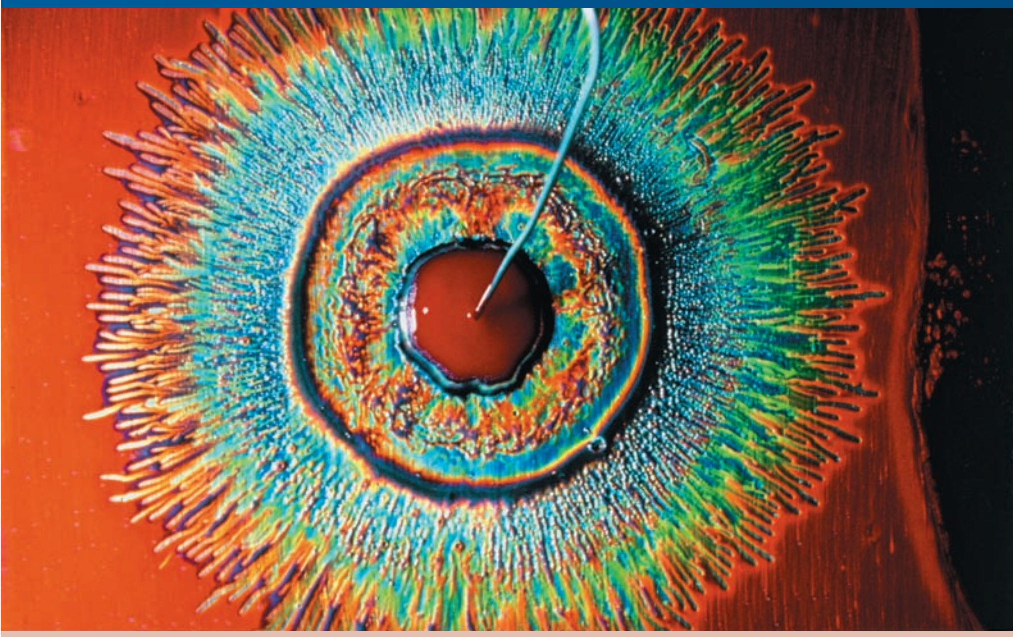
Stop to Think 26.3: b, e, and h. b and e both increase the linear charge density λ.

Stop to Think 26.4: $E_a = E_b = E_c = E_d = E_e$. The field strength of a charged plane is the same at all distances from the plane. An electric field diagram shows the electric field vectors at only a few points; the field exists at all points.

Stop to Think 26.5: $F_a = F_b = F_c = F_d = F_e$. The field strength inside a capacitor is the same at all points, hence the force on a charge is the same at all points. The electric field exists at all points whether or not a vector is shown at that point.

Stop to Think 26.6: c. Parabolic trajectories require *constant* acceleration and thus a *uniform* electric field. The proton has an initial velocity component to the left, but it's being pushed back to the right.

27 Gauss's Law



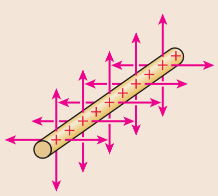
An electric field image of blood plasma from healthy blood. The wire in the center creates the electric field. Variations in the shape and color of the pattern can give early warning of cancer.

▶ **Looking Ahead** The goal of Chapter 27 is to understand and apply Gauss's law.

Symmetry

You'll learn how the shape of some important electric fields, those with a high degree of **symmetry,** can be deduced from the shape of the charge distribution. The idea of symmetry plays an important role in science and mathematics.

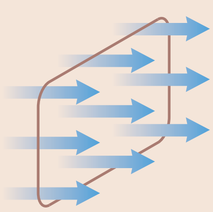
An infinitely long charged wire has *cylindrical symmetry*. The electric field of the wire must have the same symmetry.



Electric Flux

The amount of electric field passing through a surface is called the **electric flux.** You'll learn how to calculate the flux through open and closed surfaces.

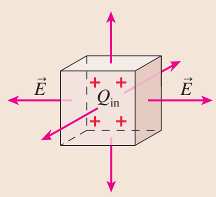
The electric flux is analogous to the amount of air or water flowing through a loop.



◀ **Looking Back**
Section 11.3 The vector dot product

Gauss's Law

In Chapter 26, you learned to calculate electric fields based on the superposition of the fields of point charges. In this chapter, you'll learn a different way to calculate electric fields based on the idea of electric flux.



Gauss's law says that the electric flux through a *closed surface* is proportional to the charge Q_{in} enclosed within the surface. This will be the basis of a powerful problem-solving strategy for finding the electric fields of highly symmetric charge distributions.

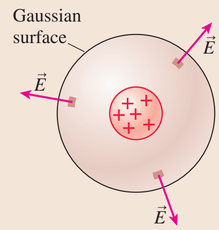
Gauss's law is a more general statement about the nature of electric fields than is Coulomb's law. It is the first of the four equations that we'll later call *Maxwell's equations*, the governing equations of electricity and magnetism.

◀ **Looking Back**
Section 25.5 The field of a point charge
Section 26.2 Electric field lines

Using Gauss's Law

You'll learn how Gauss's law can be used to find the electric field both inside and outside of charged spheres, cylinders, and planes. In these highly symmetric situations, Gauss's law is much easier to use than superposition.

To find the field of a sphere of charge, you'll draw a *Gaussian surface* around the sphere and then calculate the electric flux through the surface.



Conductors

Gauss's law can be used to establish several important properties of conductors in **electrostatic equilibrium.**

- Excess charge is on the surface.
- The interior electric field is zero.

The metal grid in the door of a microwave oven shields the room because the electric field inside the metal must be zero. It turns out that the holes don't matter because they are very small compared to the wavelength of the microwaves.

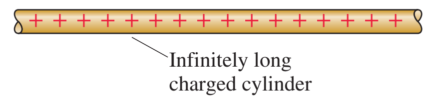
27.1 Symmetry

Suppose we knew only two things about electric fields:

1. The field points away from positive charges, toward negative charges, and
2. An electric field exerts a force on a charged particle.

From this information alone, what can we deduce about the electric field of the infinitely long charged cylinder shown in FIGURE 27.1?

FIGURE 27.1 A charge distribution with cylindrical symmetry.



We don't know if the cylinder's diameter is large or small. We don't know if the charge density is the same at the outer edge as along the axis. All we know is that the charge is positive and the charge distribution has *cylindrical symmetry*. We say that a charge distribution is **symmetric** if there is a group of *geometric transformations* that don't cause any *physical* change.

To make this idea concrete, suppose you close your eyes while a friend transforms a charge distribution in one of the following three ways. He or she can

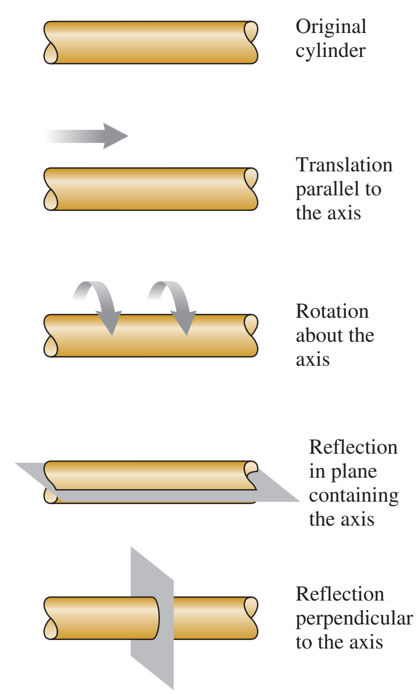
- *Translate* (that is, displace) the charge parallel to an axis,
- *Rotate* the charge about an axis, or
- *Reflect* the charge in a mirror.

When you open your eyes, will you be able to tell if the charge distribution has been changed? You might tell by observing a visual difference in the distribution. Or the results of an experiment with charged particles could reveal that the distribution has changed. If nothing you can see or do reveals any change, then we say that the charge distribution is symmetric under that particular transformation.

FIGURE 27.2 shows that the charge distribution of Figure 27.1 is symmetric with respect to

- Translation parallel to the cylinder axis. Shifting an infinitely long cylinder by 1 mm or 1000 m makes no noticeable or measurable change.
- Rotation by any angle about the cylinder axis. Turning a cylinder about its axis by 1° or 100° makes no detectable change.
- Reflections in any plane containing or perpendicular to the cylinder axis. Exchanging top and bottom, front and back, or left and right makes no detectable change.

FIGURE 27.2 Transformations that don't change an infinite cylinder of charge.



A charge distribution that is symmetric under these three groups of geometric transformations is said to be *cylindrically symmetric*. Other charge distributions have other types of symmetries. Some charge distributions have no symmetry at all.

Our interest in symmetry can be summed up in a single statement:

The symmetry of the electric field must match the symmetry of the charge distribution.

If this were not true, you could use the electric field to test whether the charge distribution had undergone a transformation.

Now we're ready to see what we can learn about the electric field in Figure 27.1. Could the field look like FIGURE 27.3a? (Imagine this picture rotated about the axis.) That is, is this a *possible* field? This field looks the same if it's translated parallel to the

FIGURE 27.3 Could the field of a cylindrical charge distribution look like this?

(a) Is this a possible electric field of an infinitely long charged cylinder? Suppose the charge and the field are reflected in a plane perpendicular to the axis.

(b) The charge distribution is not changed by the reflection, but the field is. This field doesn't match the symmetry of the cylinder, so the cylinder's field can't look like this.

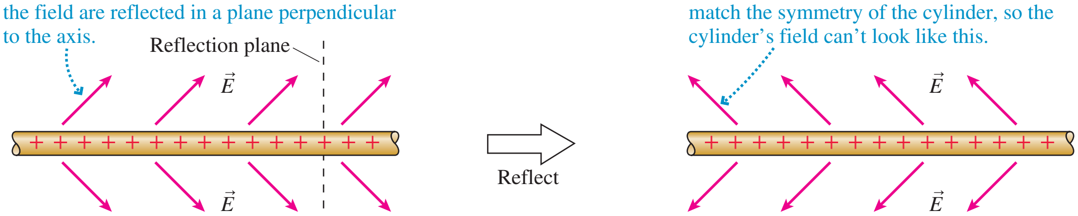
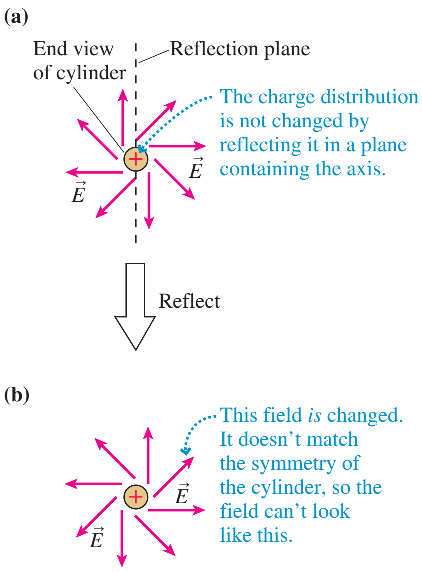


FIGURE 27.4 Or might the field of a cylindrical charge distribution look like this?



cylinder axis, if up and down are exchanged by reflecting the field in a plane coming out of the page, or if you rotate the cylinder about its axis.

But the proposed field fails one test: reflection in a plane perpendicular to the axis, a reflection that exchanges left and right. This reflection, which would *not* make any change in the charge distribution itself, produces the field shown in FIGURE 27.3b. This change in the field is detectable because a positively charged particle would now have a component of motion to the left instead of to the right.

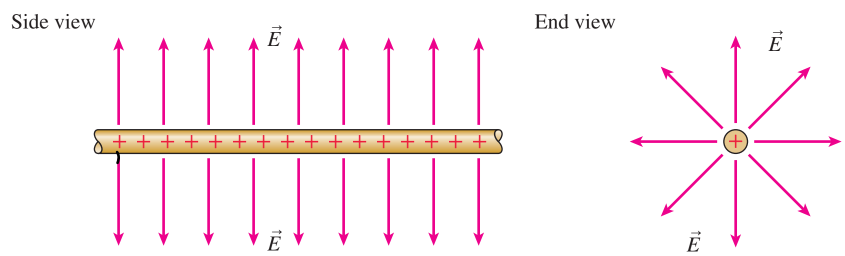
The field of Figure 27.3a, which makes a distinction between left and right, is not cylindrically symmetric and thus is *not* a possible field. In general, **the electric field of a cylindrically symmetric charge distribution cannot have a component parallel to the cylinder axis.**

Well then, what about the electric field shown in FIGURE 27.4a? Here we're looking down the axis of the cylinder. The electric field vectors are restricted to planes perpendicular to the cylinder and thus do not have any component parallel to the cylinder axis. This field is symmetric for rotations about the axis, but it's *not* symmetric for a reflection in a plane containing the axis.

The field of FIGURE 27.4b, after this reflection, is easily distinguishable from the field of Figure 27.4a. Thus **the electric field of a cylindrically symmetric charge distribution cannot have a component tangent to the circular cross section.**

FIGURE 27.5 shows the only remaining possible field shape. The electric field is radial, pointing straight out from the cylinder like the bristles on a bottle brush. This is the one electric field shape matching the symmetry of the charge distribution.

FIGURE 27.5 This is the only shape for the electric field that matches the symmetry of the charge distribution.



What Good Is Symmetry?

Given how little we assumed about Figure 27.1—that the charge distribution is cylindrically symmetric and that electric fields point away from positive charges—we've been able to deduce a great deal about the electric field. In particular, we've deduced the *shape* of the electric field.

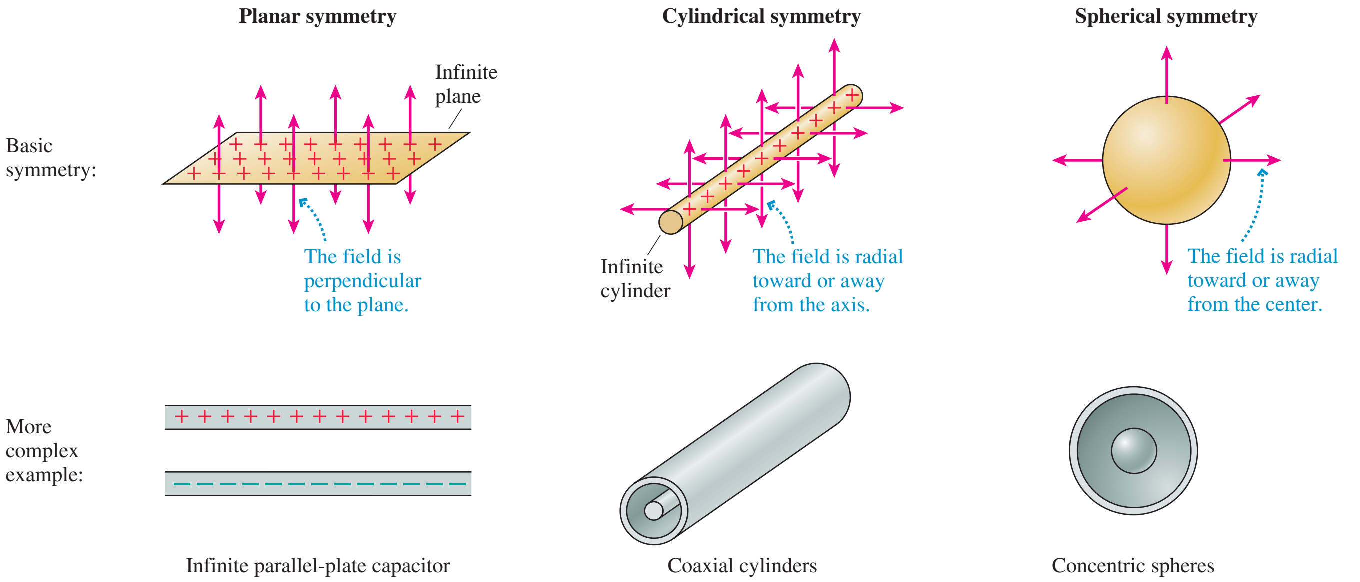
Now, shape is not everything. We've learned nothing about the strength of the field or how strength changes with distance. Is E constant? Does it decrease like $1/r$ or $1/r^2$? We don't yet have a complete description of the field, but knowing what shape the field *has* to have will make finding the field strength a much easier task.

That's the good of symmetry. Symmetry arguments allow us to *rule out* many conceivable field shapes as simply being incompatible with the symmetry of the charge distribution. Knowing what doesn't happen, or can't happen, is often as useful as knowing what does happen. By the process of elimination, we're led to the one and only shape the field can possibly have. Reasoning on the basis of symmetry is a sometimes subtle but always powerful means of reasoning.

Three Fundamental Symmetries

Three fundamental symmetries appear frequently in electrostatics. The first row of FIGURE 27.6 shows the simplest form of each symmetry. The second row shows a more complex, but more realistic, situation with the same symmetry.

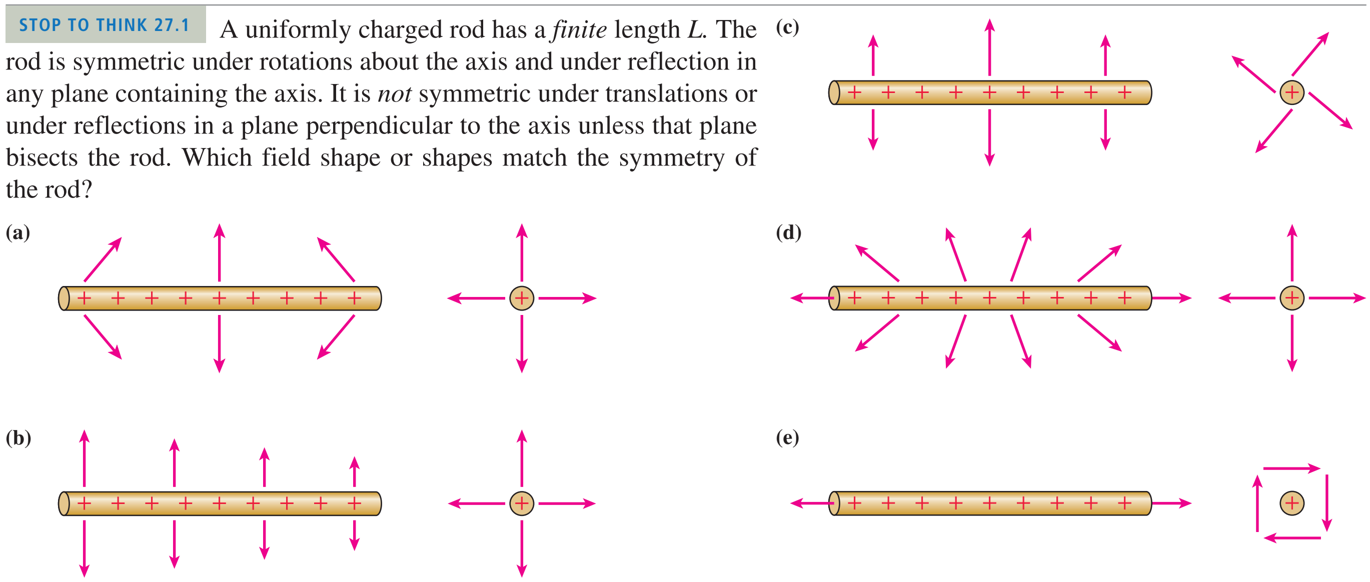
FIGURE 27.6 Three fundamental symmetries.



NOTE ▶ Figures must be finite in extent, but the planes and cylinders in Figure 27.6 are assumed to be infinite. ◀

Objects do exist that are extremely close to being perfect spheres, but no real cylinder or plane can be infinite in extent. Even so, the fields of infinite planes and cylinders are good models for the fields of finite planes and cylinders at points not too close to an edge or an end. The fields that we'll study in this chapter, even if idealized, have many important applications.

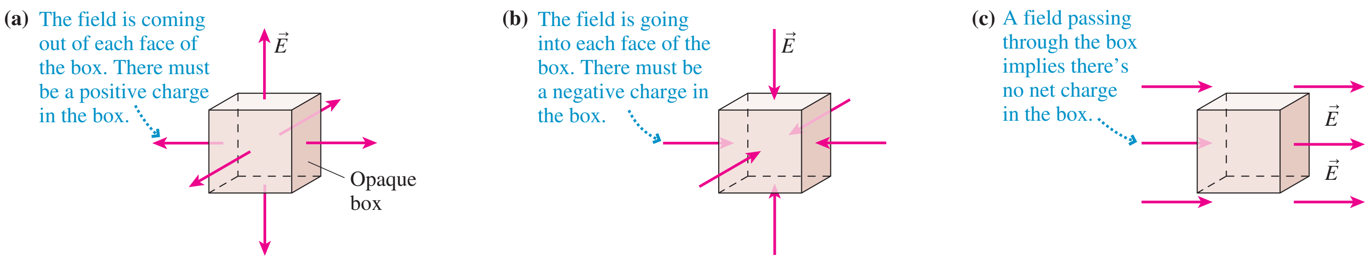
STOP TO THINK 27.1 A uniformly charged rod has a *finite* length *L*. The rod is symmetric under rotations about the axis and under reflection in any plane containing the axis. It is *not* symmetric under translations or under reflections in a plane perpendicular to the axis unless that plane bisects the rod. Which field shape or shapes match the symmetry of the rod?



27.2 The Concept of Flux

FIGURE 27.7a on the next page shows an opaque box surrounding a region of space. We can't see what's in the box, but there's an electric field vector coming out of each face of the box. Can you figure out what's in the box?

FIGURE 27.7 Although we can't see into the boxes, the electric fields passing through the faces tell us something about what's in them.

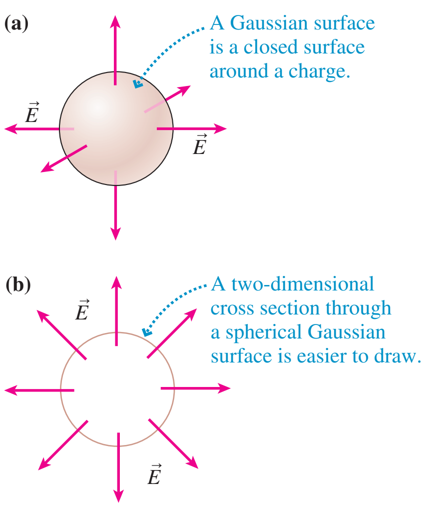


Of course you can. Because electric fields point away from positive charges, and the electric field is coming out of every face of the box, it seems clear that the box contains a positive charge or charges. Similarly, the box in FIGURE 27.7b certainly contains a negative charge.

What can we tell about the box in FIGURE 27.7c? The electric field points into the box on the left. An equal electric field points out on the right. This might be the electric field between a large positive electrode somewhere out of sight on the left and a large negative electrode off to the right. An electric field passes through the box, but we see no evidence there's any charge (or at least any net charge) inside the box.

These examples suggest that the electric field as it passes into, out of, or through the box is in some way connected to the charge within the box. However, these simple pictures don't tell us how much charge there is or where within the box the charge is located. Perhaps a better box would be more informative.

FIGURE 27.8 Gaussian surface surrounding a charge. A two-dimensional cross section is usually easier to draw.

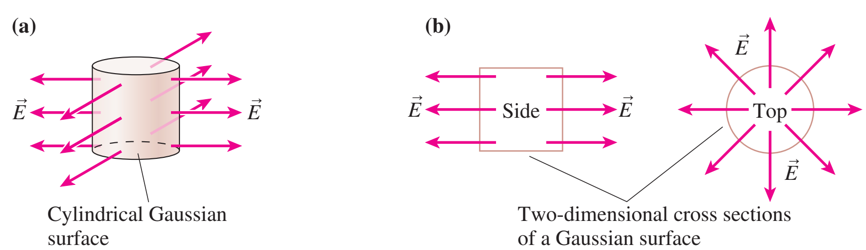


Suppose we surround a region of space with a *closed surface,* a surface that divides space into distinct inside and outside regions. Within the context of electrostatics, a closed surface through which an electric field passes is called a **Gaussian surface,** named after the 19th-century mathematician Karl Gauss who developed the mathematical foundations of geometry. This is an imaginary, mathematical surface, not a physical surface, although it might coincide with a physical surface. For example, FIGURE 27.8a shows a spherical Gaussian surface surrounding a charge.

A closed surface must, of necessity, be a surface in three dimensions. But three-dimensional pictures are hard to draw, so we'll often look at two-dimensional cross sections through a Gaussian surface, such as the one shown in FIGURE 27.8b. Now we can tell from the *spherical symmetry* of the electric field vectors poking through the surface that the positive charge inside must be spherically symmetric and centered at the *center* of the sphere. Notice two features that will soon be important: The electric field is everywhere *perpendicular* to the spherical surface and has the *same magnitude* at each point on the surface.

A Gaussian surface is most useful when it matches the shape and symmetry of the field. For example, FIGURE 27.9a shows a cylindrical Gaussian surface—a *closed* cylinder—surrounding some kind of cylindrical charge distribution, such as a charged wire. FIGURE 27.9b simplifies the drawing by showing two-dimensional end and side views. Because the Gaussian surface matches the symmetry of the charge distribution, the electric field is everywhere *perpendicular* to the side wall and *no* field passes through the top and bottom surfaces.

FIGURE 27.9 A Gaussian surface is most useful when it matches the shape of the field.



For contrast, consider the spherical surface in **FIGURE 27.10a**. This is also a Gaussian surface, and the protruding electric field tells us there's a positive charge inside. It might be a point charge located on the left side, but we can't really say. A Gaussian surface that doesn't match the symmetry of the charge distribution isn't terribly useful.

The nonclosed surface of **FIGURE 27.10b** doesn't provide much help either. What appears to be a uniform electric field to the right could be due to a large positive plate on the left, a large negative plate on the right, or both. A nonclosed surface doesn't provide enough information.

These examples lead us to two conclusions:

1. The electric field, in some sense, "flows" *out of* a closed surface surrounding a region of space containing a net positive charge and *into* a closed surface surrounding a net negative charge. The electric field may flow *through* a closed surface surrounding a region of space in which there is no net charge, but the *net flow* is zero.
2. The electric field pattern through the surface is particularly simple if the closed surface matches the symmetry of the charge distribution inside.

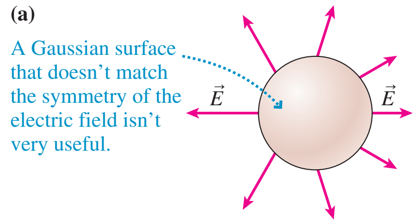
The electric field doesn't really flow like a fluid, but the metaphor is a useful one. The Latin word for flow is *flux,* and the amount of electric field passing through a surface is called the **electric flux.** Our first conclusions, stated in terms of electric flux, are

- There is an outward flux through a closed surface around a net positive charge.
- There is an inward flux through a closed surface around a net negative charge.
- There is no net flux through a closed surface around a region of space in which there is no net charge.

This chapter has been entirely qualitative thus far as we've established pictorially what we mean by symmetry, the idea of flux, and the fact that the electric flux through a closed surface has something to do with the charge inside. Understanding these qualitative ideas is essential, but to go further we need to make these ideas quantitative and precise. In the next section, you'll learn how to calculate the electric flux through a surface. Then, in the section following that, we'll establish a precise relationship between the net flux through a Gaussian surface and the enclosed charge. That relationship, Gauss's law, will allow us to determine the electric fields of some interesting and useful charge distributions.

FIGURE 27.10 Not every surface is useful for learning about charge.

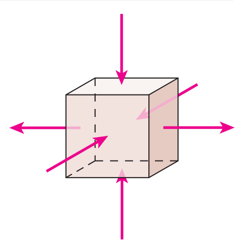
(a)



(b)

STOP TO THINK 27.2 This box contains

a. A positive charge.
b. A negative charge.
c. No charge.
d. A net positive charge.
e. A net negative charge.
f. No net charge.



27.3 Calculating Electric Flux

Let's start with a brief overview of where this section will take us. We'll begin with a definition of flux that is easy to understand, then we'll turn that simple definition into a formidable-looking integral. We need the integral because the simple definition applies only to uniform electric fields and flat surfaces. Those are good starting points, but we'll soon need to calculate the flux of nonuniform fields through curved surfaces.

Mathematically, the flux of a nonuniform field through a curved surface is described by a special kind of integral called a *surface integral.* It's quite possible that you have not yet encountered surface integrals in your calculus course, and the "novelty factor" contributes to making this integral look worse than it really is. We will emphasize over and over the idea that an integral is just a fancy way of doing a sum, in this case the sum of the small amounts of flux through many small pieces of a surface.

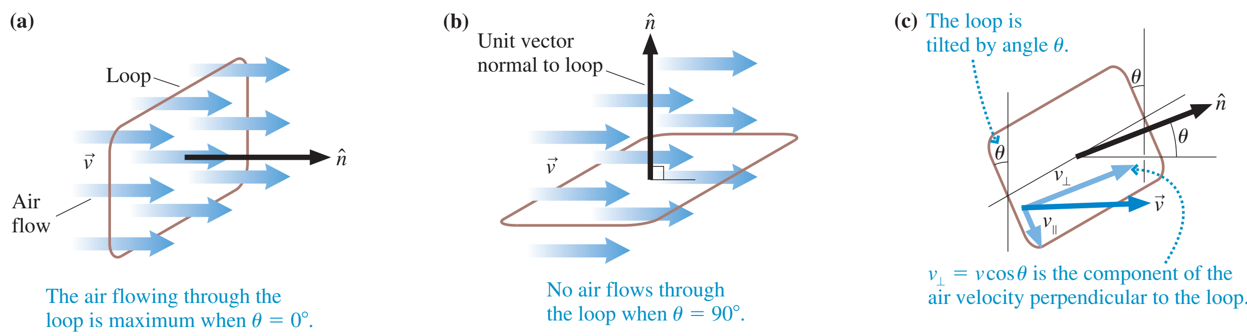
The good news is that *every* surface integral we need to evaluate in this chapter, or that you will need to evaluate for the homework problems, is either zero or is so easy that you will be able to do it in your head. This seems like an astounding claim, but you will soon see it is true. The key will be to make effective use of the *symmetry* of the electric field.

Now that you've been warned, you needn't panic at the sight of the mathematical notation that will be introduced. We'll go step by step, and you'll see that, at least as far as electrostatics is concerned, calculating the electric flux is not difficult.

The Basic Definition of Flux

Imagine holding a rectangular wire loop of area A in front of a fan. As **FIGURE 27.11** shows, the volume of air flowing through the loop each second depends on the angle between the loop and the direction of flow. The flow is maximum through a loop that is perpendicular to the airflow; no air goes through the same loop if it lies parallel to the flow.

FIGURE 27.11 The amount of air flowing through a loop depends on the angle between $\vec{v}$ and $\hat{n}$.



The flow direction is identified by the velocity vector $\vec{v}$. We can identify the loop's orientation by defining a unit vector $\hat{n}$ normal to the plane of the loop. Angle θ is then the angle between $\vec{v}$ and $\hat{n}$. The loop perpendicular to the flow in **FIGURE 27.11a** has $\theta = 0°$; the loop parallel to the flow in **FIGURE 27.11b** has $\theta = 90°$. You can think of θ as the angle by which a loop has been tilted away from perpendicular.

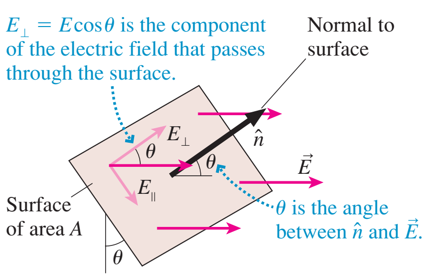
NOTE ▶ A surface has two sides, so $\hat{n}$ could point either way. We'll choose the side that makes $\theta \le 90°$. ◀

You can see from **FIGURE 27.11c** that the velocity vector $\vec{v}$ can be decomposed into components $v_\perp = v\cos\theta$ perpendicular to the loop and $v_\parallel = v\sin\theta$ parallel to the loop. Only the perpendicular component $v_\perp$ carries air *through* the loop. Consequently, the volume of air flowing through the loop each second is

$$\text{volume of air per second (m}^3\text{/s)} = v_\perp A = vA\cos\theta \qquad (27.1)$$

$\theta = 0°$ is the orientation for maximum flow through the loop, as expected, and no air flows through the loop if it is tilted 90°.

FIGURE 27.12 An electric field passing through a surface.



An electric field doesn't flow in a literal sense, but we can apply the same idea to an electric field passing through a surface. **FIGURE 27.12** shows a surface of area A in a uniform electric field $\vec{E}$. Unit vector $\hat{n}$ is normal to the surface, and θ is the angle between $\hat{n}$ and $\vec{E}$. Only the component $E_\perp = E\cos\theta$ passes *through* the surface.

With this in mind, and using Equation 27.1 as an analog, let's define the *electric flux* Φ_e (uppercase Greek phi) as

$$\Phi_e = E_\perp A = EA\cos\theta \qquad (27.2)$$

The electric flux measures the amount of electric field passing through a surface of area A if the normal to the surface is tilted at angle θ from the field.

Equation 27.2 looks very much like a vector dot product: $\vec{E} \cdot \vec{A} = EA\cos\theta$. For this idea to work, let's define an **area vector** $\vec{A} = A\hat{n}$ to be a vector in the direction of $\hat{n}$—that is, *perpendicular* to the surface—with a magnitude A equal to the area of the surface. Vector $\vec{A}$ has units of m^2. **FIGURE 27.13a** shows two area vectors.

FIGURE 27.13 The electric flux can be defined in terms of the area vector $\vec{A}$.

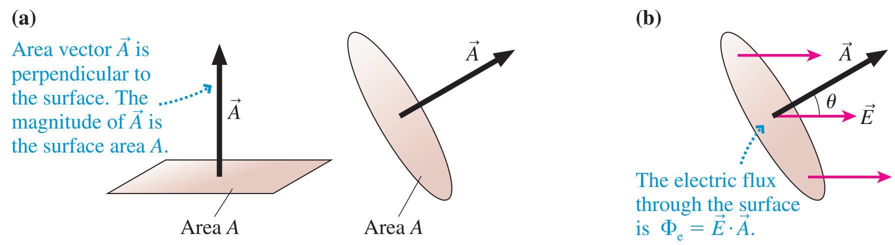


FIGURE 27.13b shows an electric field passing through a surface of area A. The angle between vectors $\vec{A}$ and $\vec{E}$ is the same angle used in Equation 27.2 to define the electric flux, so Equation 27.2 really is a dot product. We can define the electric flux more concisely as

$$\Phi_e = \vec{E} \cdot \vec{A} \quad \text{(electric flux of a constant electric field)} \qquad (27.3)$$

Writing the flux as a dot product helps make clear how angle θ is defined: θ is the angle between the electric field and a line *perpendicular* to the plane of the surface.

NOTE ▶ Figure 27.13b shows a circular area, but the shape of the surface is not relevant. However, Equation 27.3 is restricted to a *constant* electric field passing through a *planar* surface. ◀

EXAMPLE 27.1 The electric flux inside a parallel-plate capacitor

Two 100 cm^2 parallel electrodes are spaced 2.0 cm apart. One is charged to +5.0 nC, the other to −5.0 nC. A 1.0 cm × 1.0 cm surface between the electrodes is tilted to where its normal makes a 45° angle with the electric field. What is the electric flux through this surface?

MODEL Assume the surface is located near the center of the capacitor where the electric field is uniform. The electric flux doesn't depend on the shape of the surface.

VISUALIZE The surface is square, rather than circular, but otherwise the situation looks like Figure 27.13b.

SOLVE In Chapter 26, we found the electric field inside a parallel-plate capacitor to be

$$E = \frac{Q}{\epsilon_0 A_{\text{plates}}} = \frac{5.0 \times 10^{-9}\text{ C}}{(8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2)(1.0 \times 10^{-2}\text{ m}^2)}$$
$$= 5.65 \times 10^4 \text{ N/C}$$

A 1.0 cm × 1.0 cm surface has $A = 1.0 \times 10^{-4}\text{ m}^2$. The electric flux through this surface is

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$
$$= (5.65 \times 10^4 \text{ N/C})(1.0 \times 10^{-4}\text{ m}^2)\cos 45°$$
$$= 4.0 \text{ N m}^2/\text{C}$$

ASSESS The units of electric flux are the product of electric field and area units: N m^2/C.

The Electric Flux of a Nonuniform Electric Field

Our initial definition of the electric flux assumed that the electric field $\vec{E}$ was constant over the surface. How should we calculate the electric flux if $\vec{E}$ varies from point to point on the surface? We can answer this question by returning to the analogy of air flowing through a loop. Suppose the airflow varies from point to point. We can still find the total volume of air passing through the loop each second by dividing the loop into many small areas, finding the flow through each small area, then adding them. Similarly, **the electric flux through a surface can be calculated as the sum of the fluxes through smaller pieces of the surface.** Because flux is a scalar, adding fluxes is easier than adding electric fields.

FIGURE 27.14 A surface in a nonuniform electric field.

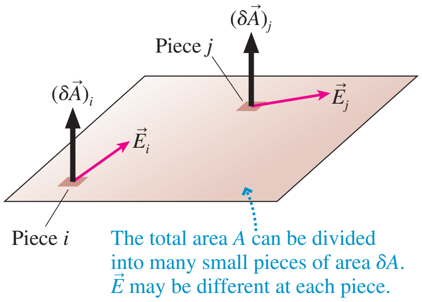


FIGURE 27.14 shows a surface in a nonuniform electric field. Imagine dividing the surface into many small pieces of area δA. Each little area has an area vector $\delta\vec{A}$ perpendicular to the surface. Two of the little pieces are shown in the figure. The electric fluxes through these two pieces differ because the electric fields are different.

Consider the small piece i where the electric field is $\vec{E}_i$. The small electric flux $\delta\Phi_i$ through area $(\delta\vec{A})_i$ is

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i \tag{27.4}$$

The flux through every other little piece of the surface is found the same way. The total electric flux through the entire surface is then the sum of the fluxes through each of the small areas:

$$\Phi_e = \sum_i \delta\Phi_i = \sum_i \vec{E}_i \cdot (\delta\vec{A})_i \tag{27.5}$$

Now let's go to the limit $\delta\vec{A} \rightarrow d\vec{A}$. That is, the little areas become infinitesimally small, and there are infinitely many of them. Then the sum becomes an integral, and the electric flux through the surface is

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \tag{27.6}$$

The integral in Equation 27.6 is called a **surface integral.**

Equation 27.6 may look rather frightening if you haven't seen surface integrals before. Despite its appearance, a surface integral is no more complicated than integrals you know from calculus. After all, what does $\int f(x)\,dx$ really mean? This expression is a shorthand way to say "Divide the x-axis into many little segments of length δx, evaluate the function $f(x)$ in each of them, then add up $f(x)\,\delta x$ for all the segments along the line." The integral in Equation 27.6 differs only in that we're dividing a surface into little pieces instead of a line into little segments. In particular, we're summing the fluxes through a vast number of very tiny pieces.

You may be thinking, "OK, I understand the idea, but I don't know what to *do*. In calculus, I learned formulas for evaluating integrals such as $\int x^2 dx$. How do I evaluate a surface integral?" This is a good question. We'll deal with evaluation shortly, and it will turn out that the surface integrals in electrostatics are quite easy to evaluate. But don't confuse *evaluating* the integral with understanding what the integral *means.* The surface integral in Equation 27.6 is simply a shorthand notation for the summation of the electric fluxes through a vast number of very tiny pieces of a surface.

The electric field might be different at every point on the surface, but suppose it isn't. That is, suppose the surface is in a uniform electric field $\vec{E}$. A field that is the same at every single point on a surface is a constant as far as the integration of Equation 27.6 is concerned, so we can take it outside the integral. In that case,

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E\cos\theta\, dA = E\cos\theta \int_{\text{surface}} dA \tag{27.7}$$

The integral that remains in Equation 27.7 tells us to add up all the little areas into which the full surface was subdivided. But the sum of all the little areas is simply the area of the surface:

$$\int_{\text{surface}} dA = A \tag{27.8}$$

This idea—that the surface integral of dA is the area of the surface—is one we'll use to evaluate most of the surface integrals of electrostatics. If we substitute Equation 27.8 into Equation 27.7, we find that the electric flux in a uniform electric field is $\Phi_e = EA\cos\theta$. We already knew this, from Equation 27.2, but it was important to see that the surface integral of Equation 27.6 gives the correct result for the case of a uniform electric field.

The Flux Through a Curved Surface

Most of the Gaussian surfaces we considered in the last section were curved surfaces. **FIGURE 27.15** shows an electric field passing through a curved surface. How do we find the electric flux through this surface? Just as we did for a flat surface!

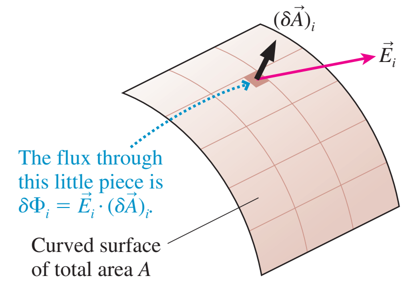


FIGURE 27.15 A curved surface in an electric field.

Divide the surface into many small pieces of area δA. For each, define the area vector $\delta\vec{A}$ perpendicular to the surface *at that point.* Compared to Figure 27.14, the only difference that the curvature of the surface makes is that the $\delta\vec{A}$ are no longer parallel to each other. Find the small electric flux $\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$ through each little area, then add them all up. The result, once again, is

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \tag{27.9}$$

We *assumed,* in deriving this expression the first time, that the surface was flat and that all the $\delta\vec{A}$ were parallel to each other. But that assumption wasn't necessary. The *meaning* of Equation 27.9—a summation of the fluxes through a vast number of very tiny pieces—is unchanged if the pieces lie on a curved surface.

We seem to be getting more and more complex, using surface integrals first for nonuniform fields and now for curved surfaces. But consider the two situations shown in **FIGURE 27.16**. The electric field $\vec{E}$ in **FIGURE 27.16a** is everywhere tangent, or parallel, to the curved surface. We don't need to know the magnitude of $\vec{E}$ to recognize that $\vec{E} \cdot d\vec{A}$ is *zero at every point* on the surface because $\vec{E}$ is perpendicular to $d\vec{A}$ at every point. Thus $\Phi_e = 0$. A tangent electric field never pokes through the surface, so it has no flux through the surface.

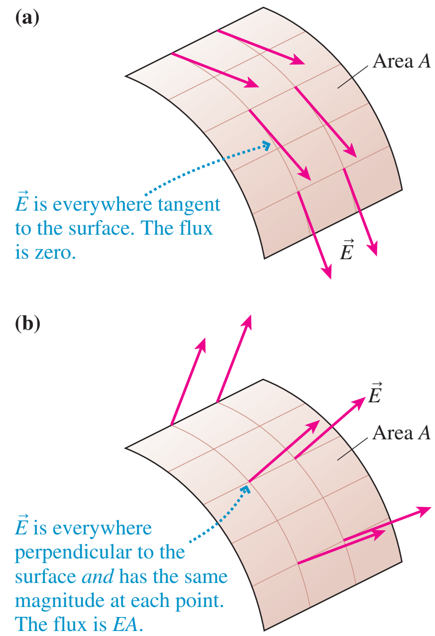


FIGURE 27.16 Electric fields that are everywhere tangent to or everywhere perpendicular to a curved surface.

The electric field in **FIGURE 27.16b** is everywhere perpendicular to the surface *and* has the same magnitude E at every point. $\vec{E}$ differs in direction at different points on a curved surface, but at any particular point $\vec{E}$ is parallel to $d\vec{A}$ and $\vec{E} \cdot d\vec{A}$ is simply $E\,dA$. In this case,

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E\,dA = E \int_{\text{surface}} dA = EA \tag{27.10}$$

As we evaluated the integral, the fact that E has the same magnitude at every point on the surface allowed us to bring the constant value outside the integral. We then used the fact that the integral of dA over the surface is the surface area A.

We can summarize these two situations with a Tactics Box.

TACTICS BOX 27.1 **Evaluating surface integrals** MP

1. If the electric field is everywhere tangent to a surface, the electric flux through the surface is $\Phi_e = 0$.
2. If the electric field is everywhere perpendicular to a surface *and* has the same magnitude E at every point, the electric flux through the surface is $\Phi_e = EA$.

These two results will be of immeasurable value for using Gauss's law because *every* flux we'll need to calculate will be one of these situations. This is the basis for our earlier claim that the evaluation of surface integrals is not going to be difficult.

The Electric Flux Through a Closed Surface

Our final step, to calculate the electric flux through a closed surface such as a box, a cylinder, or a sphere, requires nothing new. We've already learned how to calculate the electric flux through flat and curved surfaces, and a closed surface is nothing more than a surface that happens to be closed.

However, the mathematical notation for the surface integral over a closed surface differs slightly from what we've been using. It is customary to use a little circle on the integral sign to indicate that the surface integral is to be performed over a closed surface. With this notation, the electric flux through a closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} \tag{27.11}$$

Only the notation has changed. The electric flux is still the summation of the fluxes through a vast number of tiny pieces, pieces that now cover a closed surface.

NOTE ▶ A closed surface has a distinct inside and outside. The area vector $d\vec{A}$ is defined to always point *toward the outside*. This removes an ambiguity that was present for a single surface, where $d\vec{A}$ could point to either side. ◀

EXAMPLE 27.2 Calculating the electric flux through a closed cylinder

A charge distribution with cylindrical symmetry has created the electric field $\vec{E} = E_0(r^2/r_0^2)\hat{r}$, where E_0 and r_0 are constants and where unit vector $\hat{r}$ lies in the xy-plane. Calculate the electric flux through a closed cylinder of length L and radius R that is centered along the z-axis.

MODEL The electric field extends radially outward from the z-axis with cylindrical symmetry. The z-component is $E_z = 0$. The cylinder is a Gaussian surface.

VISUALIZE **FIGURE 27.17a** is a view of the electric field looking along the z-axis. The field strength increases with increasing radial distance, and it's symmetric about the z-axis. **FIGURE 27.17b** is the closed Gaussian surface for which we need to calculate the electric flux. We can place the cylinder anywhere along the z-axis because the electric field extends forever in that direction.

SOLVE To calculate the flux, we divide the closed cylinder into three surfaces: the top, the bottom, and the cylindrical wall. The electric field is tangent to the surface at every point on the top and bottom surfaces. Hence, according to step 1 in Tactics Box 27.1, the flux through those two surfaces is zero. For the cylindrical wall, the electric field is perpendicular to the surface at every point *and* has the constant magnitude $E = E_0(R^2/r_0^2)$ at every point on the surface. Thus, from step 2 in Tactics Box 27.1,

$$\Phi_{wall} = EA_{wall}$$

If we add the three pieces, the net flux through the closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \Phi_{top} + \Phi_{bottom} + \Phi_{wall} = 0 + 0 + EA_{wall}$$
$$= EA_{wall}$$

We've evaluated the surface integral, using the two steps in Tactics Box 27.1, and there was nothing to it! To finish, all we need to recall is that the surface area of a cylindrical wall is circumference × height, or $A_{wall} = 2\pi RL$. Thus

$$\Phi_e = \left(E_0 \frac{R^2}{r_0^2}\right)(2\pi RL) = \frac{2\pi LR^3}{r_0^2} E_0$$

ASSESS LR^3/r_0^2 has units of m^2, an area, so this expression for Φ_e has units of N m^2/C. These are the correct units for electric flux, giving us confidence in our answer. Notice the important role played by symmetry. The electric field was perpendicular to the wall and of constant value at every point on the wall *because* the Gaussian surface had the same symmetry as the charge distribution. We would not have been able to evaluate the surface integral in such an easy way for a surface of any other shape. Symmetry is the key.

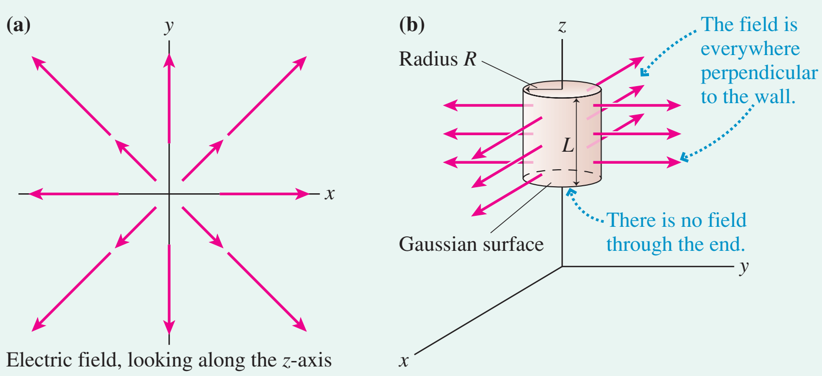


FIGURE 27.17 The electric field and the closed surface through which we will calculate the electric flux.

Example 27.2 illustrated a two-step approach to performing a flux integral over a closed surface. In summary:

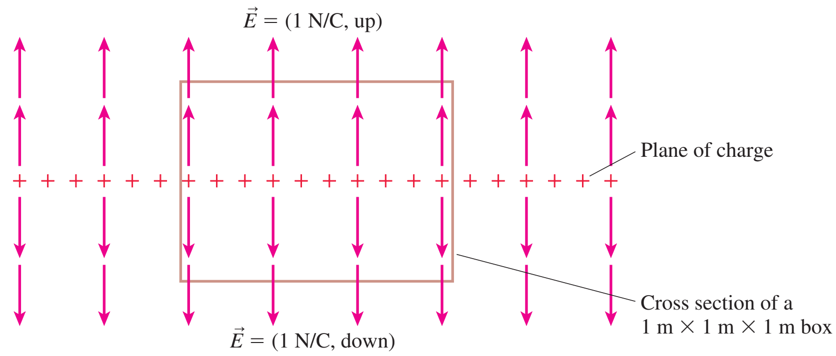
TACTICS BOX 27.2 Finding the flux through a closed surface

1. Divide the closed surface into pieces that are everywhere tangent to the electric field and everywhere perpendicular to the electric field.
2. Use Tactics Box 27.1 to evaluate the surface integrals over these surfaces, then add the results.

Exercise 11

STOP TO THINK 27.3 The total electric flux through this box is

a. $0\ \mathrm{N\,m^2/C}$
b. $1\ \mathrm{N\,m^2/C}$
c. $2\ \mathrm{N\,m^2/C}$
d. $4\ \mathrm{N\,m^2/C}$
e. $6\ \mathrm{N\,m^2/C}$
f. $8\ \mathrm{N\,m^2/C}$



27.4 Gauss's Law

The last section was long, but knowing how to calculate the electric flux through a closed surface is essential for the main topic of this chapter: Gauss's law. Gauss's law is equivalent to Coulomb's law for static charges, although Gauss's law will look very different.

The purpose of learning Gauss's law is twofold:

- Gauss's law allows the electric fields of some continuous distributions of charge to be found much more easily than does Coulomb's law.
- Gauss's law is valid for *moving* charges, but Coulomb's law is not (although it's a very good approximation for velocities that are much less than the speed of light). Thus Gauss's law is ultimately a more fundamental statement about electric fields than is Coulomb's law.

Let's start with Coulomb's law for the electric field of a point charge. **FIGURE 27.18** shows a spherical Gaussian surface of radius r centered on a positive charge q. Keep in mind that this is an imaginary, mathematical surface, not a physical surface. There is a net flux through this surface because the electric field points outward at every point on the surface. To evaluate the flux, given formally by the surface integral of Equation 27.11, notice that the electric field is perpendicular to the surface at every point on the surface *and,* from Coulomb's law, it has the same magnitude $E = q/4\pi\epsilon_0 r^2$ at every point on the surface. This simple situation arises because **the Gaussian surface has the same symmetry as the electric field.**

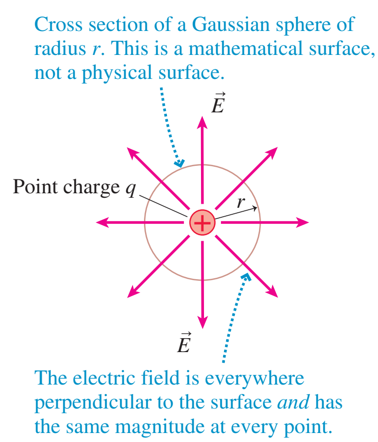
Thus we know, without having to do any hard work, that the flux integral is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = EA_{\text{sphere}} \tag{27.12}$$

The surface area of a sphere of radius r is $A_{\text{sphere}} = 4\pi r^2$. If we use A_{sphere} and the Coulomb-law expression for E in Equation 27.12, we find that the electric flux through the spherical surface is

$$\Phi_e = \frac{q}{4\pi\epsilon_0 r^2}\,4\pi r^2 = \frac{q}{\epsilon_0} \tag{27.13}$$

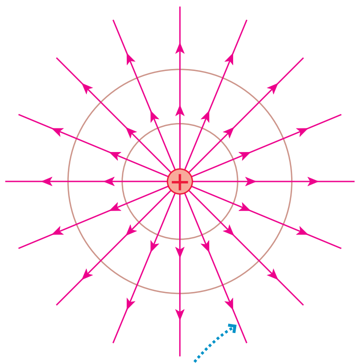
FIGURE 27.18 A spherical Gaussian surface surrounding a point charge.



You should examine the logic of this calculation closely. We really did evaluate the surface integral of Equation 27.11, although it may appear, at first, as if we didn't do much. The integral was easily evaluated, we reiterate for emphasis, because the closed surface on which we performed the integration matched the *symmetry* of the charge distribution.

NOTE ▶ We found Equation 27.13 for a positive charge, but it applies equally to negative charges. According to Equation 27.13, Φ_e is negative if q is negative. And that's what we would expect from the basic definition of flux, $\vec{E}\cdot\vec{A}$. The electric field of a negative charge points inward, while the area vector of a closed surface points outward, making the dot product negative. ◀

FIGURE 27.19 The electric flux is the same through *every* sphere centered on a point charge.

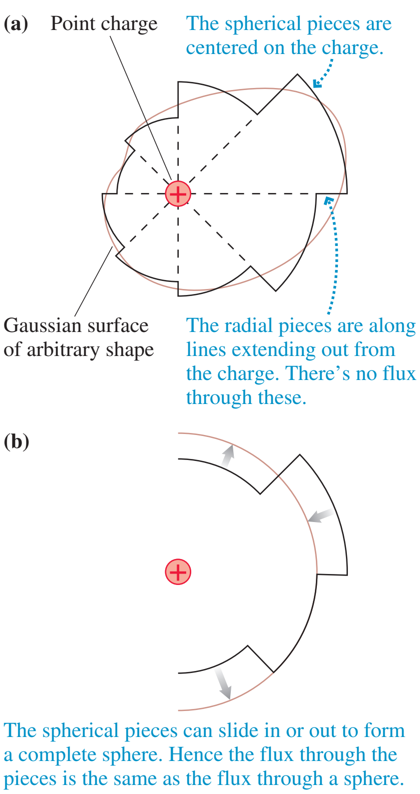


Electric Flux Is Independent of Surface Shape and Radius

Notice something interesting about Equation 27.13. The electric flux depends on the amount of charge but *not* on the radius of the sphere. Although this may seem a bit surprising, it's really a direct consequence of what we *mean* by flux. Think of the fluid analogy with which we introduced the term "flux." If fluid flows outward from a central point, all the fluid crossing a small-radius spherical surface will, at some later time, cross a large-radius spherical surface. No fluid is lost along the way, and no new fluid is created. Similarly, the point charge in **FIGURE 27.19** is the only source of electric field. Every electric field line passing through a small-radius spherical surface also passes through a large-radius spherical surface. Hence the electric flux is independent of r.

NOTE ▶ This argument hinges on the fact that Coulomb's law is an inverse-square force law. The electric field strength, which is proportional to $1/r^2$, decreases with distance. But the surface area, which increases in proportion to r^2, exactly compensates for this decrease. Consequently, the electric flux of a point charge through a spherical surface is independent of the radius of the sphere. ◀

FIGURE 27.20 An arbitrary Gaussian surface can be approximated with spherical and radial pieces.



This conclusion about the flux has an extremely important generalization. **FIGURE 27.20a** shows a point charge and a closed Gaussian surface of arbitrary shape and dimensions. All we know is that the charge is *inside* the surface. What is the electric flux through this arbitrary surface?

One way to answer the question is to approximate the surface as a patchwork of spherical and radial pieces. The spherical pieces are centered on the charge and the radial pieces lie along lines extending outward from the charge. (Figure 27.20 is a two-dimensional drawing so you need to imagine these arcs as actually being pieces of a spherical shell.) The figure, of necessity, shows fairly large pieces that don't match the actual surface all that well. However, we can make this approximation as good as we want by letting the pieces become sufficiently small.

The electric field is everywhere tangent to the radial pieces. Hence the electric flux through the radial pieces is zero. The spherical pieces, although at varying distances from the charge, form a *complete sphere.* That is, any line drawn radially outward from the charge will pass through exactly one spherical piece, and no radial lines can avoid passing through a spherical piece. You could even imagine, as **FIGURE 27.20b** shows, sliding the spherical pieces in and out *without changing the angle they subtend* until they come together to form a complete sphere.

Consequently, the electric flux through these spherical pieces that, when assembled, form a complete sphere must be exactly the same as the flux q/ϵ_0 through a spherical Gaussian surface. In other words, **the flux through *any* closed surface surrounding a point charge q is**

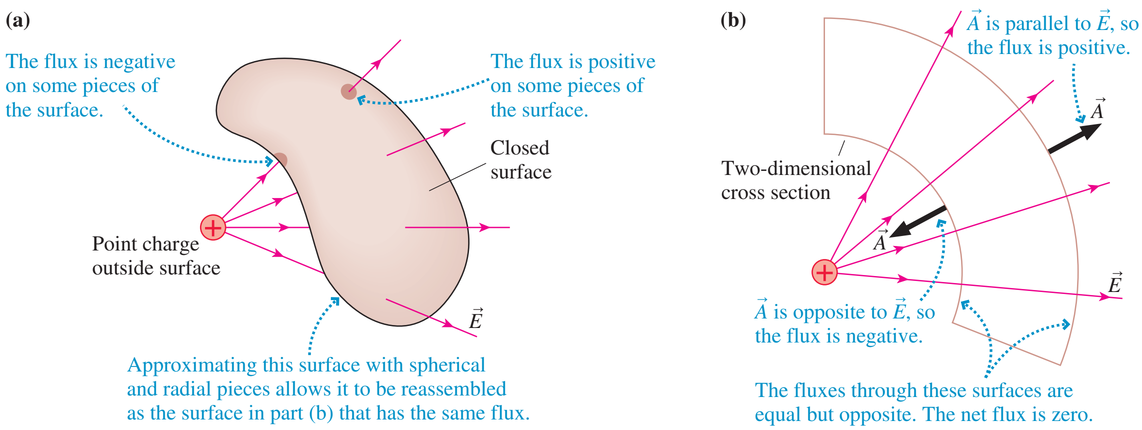
$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \tag{27.14}$$

This surprisingly simple result is a consequence of the fact that Coulomb's law is an inverse-square force law. Even so, the reasoning that got us to Equation 27.14 is rather subtle and well worth reviewing.

Charge Outside the Surface

The closed surface shown in **FIGURE 27.21a** has a point charge q outside the surface but no charges inside. Now what can we say about the flux? By approximating this surface with spherical and radial pieces *centered on the charge,* as we did in Figure 27.20, we can reassemble the surface into the equivalent surface of **FIGURE 27.21b**. This closed surface consists of sections of two spherical shells, and it is equivalent in the sense that the electric flux through this surface is the same as the electric flux through the original surface of Figure 27.21a.

FIGURE 27.21 A point charge outside a Gaussian surface.

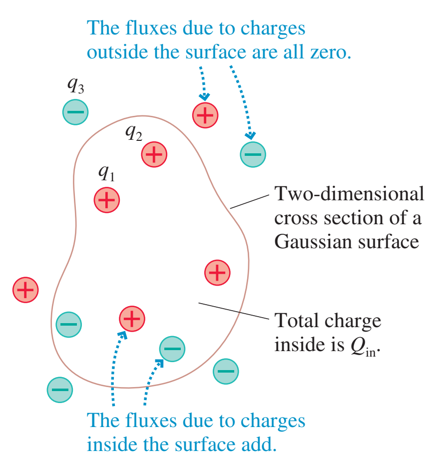


If the electric field were a fluid flowing outward from the charge, all the fluid *entering* the closed region through the first spherical surface would later *exit* the region through the second spherical surface. There is no *net* flow into or out of the closed region. Similarly, every electric field line entering this closed volume through one spherical surface exits through the other spherical surface.

Mathematically, the electric fluxes through the two spherical surfaces have the same magnitude because Φ_e is independent of r. But they have *opposite signs* because the outward-pointing area vector $\vec{A}$ is parallel to $\vec{E}$ on one surface but opposite to $\vec{E}$ on the other. The sum of the fluxes through the two surfaces is zero, and we are led to the conclusion that **the net electric flux is zero through a closed surface that does not contain any net charge.** Charges outside the surface do not produce a net flux through the surface.

This isn't to say that the flux through a small piece of the surface is zero. In fact, as Figure 27.21a shows, nearly every piece of the surface has an electric field either entering or leaving and thus has a nonzero flux. But some of these are positive and some are negative. When summed over the *entire* surface, the positive and negative contributions exactly cancel to give no *net* flux.

FIGURE 27.22 Charges both inside and outside a Gaussian surface.



Multiple Charges

Finally, consider an arbitrary Gaussian surface and a group of charges $q_1, q_2, q_3, \ldots$ such as those shown in **FIGURE 27.22**. What is the net electric flux through the surface?

By definition, the net flux is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

From the principle of superposition, the electric field is $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$, where $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$ are the fields of the individual charges. Thus the flux is

$$\begin{aligned}\Phi_e &= \oint \vec{E}_1 \cdot d\vec{A} + \oint \vec{E}_2 \cdot d\vec{A} + \oint \vec{E}_3 \cdot d\vec{A} + \cdots \qquad (27.15)\\ &= \Phi_1 + \Phi_2 + \Phi_3 + \cdots\end{aligned}$$

where $\Phi_1, \Phi_2, \Phi_3, \ldots$ are the fluxes through the Gaussian surface due to the individual charges. That is, the net flux is the sum of the fluxes due to individual charges. But we know what those are: q/ϵ_0 for the charges inside the surface and zero for the charges outside. Thus

$$\Phi_e = \left(\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \cdots + \frac{q_i}{\epsilon_0} \text{ for all charges inside the surface}\right) + (0 + 0 + \cdots + 0 \text{ for all charges outside the surface}) \tag{27.16}$$

We define

$$Q_{in} = q_1 + q_2 + \cdots + q_i \text{ for all charges inside the surface} \tag{27.17}$$

as the total charge enclosed *within* the surface. With this definition, we can write our result for the net electric flux in a very neat and compact fashion. For any *closed* surface enclosing total charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \tag{27.18}$$

This result for the electric flux is known as **Gauss's law.**

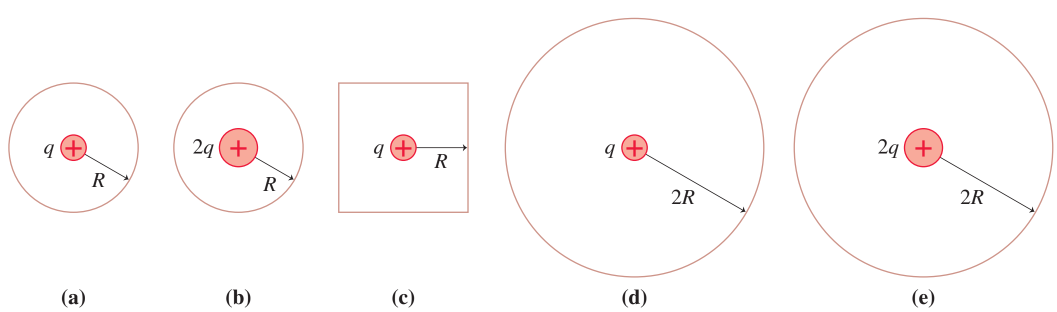
What Does Gauss's Law Tell Us?

In one sense, Gauss's law doesn't say anything new or anything that we didn't already know from Coulomb's law. After all, we derived Gauss's law from Coulomb's law. But in another sense, Gauss's law is more important than Coulomb's law. Gauss's law states a very general property of electric fields—namely, that charges create electric fields in just such a way that the net flux of the field is the same through *any* surface surrounding the charges, no matter what its size and shape may be. This fact may have been implied by Coulomb's law, but it was by no means obvious. And Gauss's law will turn out to be particularly useful later when we combine it with other electric and magnetic field equations.

Gauss's law is the mathematical statement of our observations in Section 27.2. There we noticed a net "flow" of electric field out of a closed surface containing charges. Gauss's law quantifies this idea by making a specific connection between the "flow," now measured as electric flux, and the amount of charge.

But is it useful? Although to some extent Gauss's law is a formal statement about electric fields, not a tool for solving practical problems, there are exceptions: Gauss's law will allow us to find the electric fields of some very important and very practical charge distributions much more easily than if we had to rely on Coulomb's law. We'll consider some examples in the next section.

STOP TO THINK 27.4 These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank in order, from largest to smallest, the electric fluxes Φ_a to Φ_e through surfaces a to e.



27.5 Using Gauss's Law

In this section, we'll use Gauss's law to determine the electric fields of several important charge distributions. Some of these you already know, from Chapter 26; others will be new. Three important observations can be made about using Gauss's law:

1. Gauss's law applies only to a *closed* surface, called a Gaussian surface.
2. A Gaussian surface is not a physical surface. It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.
3. We can't find the electric field from Gauss's law alone. We need to apply Gauss's law in situations where, from symmetry and superposition, we already can guess the *shape* of the field.

These observations and our previous discussion of symmetry and flux lead to the following strategy for solving electric field problems with Gauss's law.

PROBLEM-SOLVING STRATEGY 27.1 Gauss's law

MODEL Model the charge distribution as a distribution with symmetry.

VISUALIZE Draw a picture of the charge distribution.

- Determine the symmetry of its electric field.
- Choose and draw a Gaussian surface with the *same symmetry*.
- You need not enclose all the charge within the Gaussian surface.
- Be sure every part of the Gaussian surface is either tangent to or perpendicular to the electric field.

SOLVE The mathematical representation is based on Gauss's law

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

- Use Tactics Boxes 27.1 and 27.2 to evaluate the surface integral.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 19

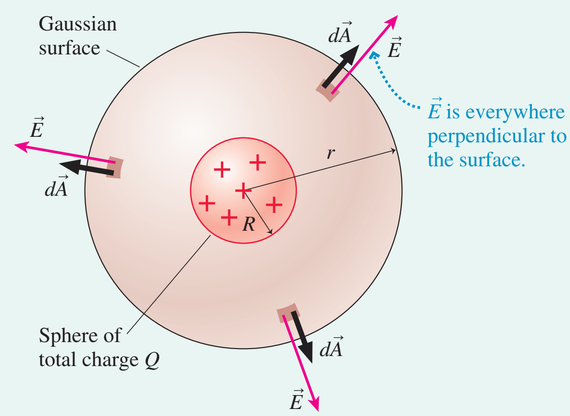
EXAMPLE 27.3 Outside a sphere of charge

In Chapter 26 we asserted, without proof, that the electric field outside a sphere of total charge Q is the same as the field of a point charge Q at the center. Use Gauss's law to prove this result.

MODEL The charge distribution within the sphere need not be uniform (i.e., the charge density might increase or decrease with r), but it must have spherical symmetry in order for us to use Gauss's law. We will assume that it does.

VISUALIZE **FIGURE 27.23** shows a sphere of charge Q and radius R. We want to find $\vec{E}$ outside this sphere, for distances $r > R$. The spherical symmetry of the charge distribution tells us that the electric field must point *radially outward* from the sphere. Although Gauss's law is true for any surface surrounding the charged sphere, it is useful only if we choose a Gaussian surface to match the spherical symmetry of the charge distribution and the field. Thus a spherical surface of radius $r > R$ *concentric with*

FIGURE 27.23 A spherical Gaussian surface surrounding a sphere of charge.



Continued

the charged sphere will be our Gaussian surface. Because this surface surrounds the entire sphere of charge, the enclosed charge is simply $Q_{in} = Q$.

SOLVE Gauss's law is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

To calculate the flux, notice that the electric field is everywhere perpendicular to the spherical surface. And although we don't know the electric field magnitude E, spherical symmetry dictates that E must have the same value at all points equally distant from the center of the sphere. Thus we have the simple result that the net flux through the Gaussian surface is

$$\Phi_e = EA_{sphere} = 4\pi r^2 E$$

where we used the fact that the surface area of a sphere is $A_{sphere} = 4\pi r^2$. With this result for the flux, Gauss's law is

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

Thus the electric field at distance r outside a sphere of charge is

$$E_{outside} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Or in vector form, making use of the fact that $\vec{E}$ is radially outward,

$$\vec{E}_{outside} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

where $\hat{r}$ is a radial unit vector.

ASSESS The field is exactly that of a point charge Q, which is what we wanted to show.

The derivation of the electric field of a sphere of charge depended crucially on a proper choice of the Gaussian surface. We would not have been able to evaluate the flux integral so simply for any other choice of surface. It's worth noting that the result of Example 27.3 can also be proven by the superposition of point-charge fields, but it requires a difficult three-dimensional integral and about a page of algebra. We obtained the answer using Gauss's law in just a few lines. Where Gauss's law works, it works *extremely* well! However, it works only in situations, such as this, with a very high degree of symmetry.

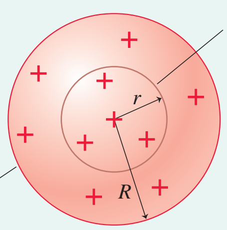
EXAMPLE 27.4 Inside a sphere of charge

What is the electric field *inside* a uniformly charged sphere?

MODEL We haven't considered a situation like this before. To begin, we don't know if the field strength is increasing or decreasing as we move outward from the center of the sphere. But the field inside must have spherical symmetry. That is, the field must point radially inward or outward, and the field strength can depend only on r. This is sufficient information to solve the problem because it allows us to choose a Gaussian surface.

VISUALIZE **FIGURE 27.24** shows a spherical Gaussian surface with radius $r \leq R$ *inside,* and *concentric with,* the sphere of charge. This surface matches the symmetry of the charge distribution, hence $\vec{E}$ is perpendicular to this surface and the field strength E has the same value at all points on the surface.

FIGURE 27.24 A spherical Gaussian surface inside a uniform sphere of charge.



SOLVE The flux integral is identical to that of Example 27.3:

$$\Phi_e = EA_{sphere} = 4\pi r^2 E$$

Consequently, Gauss's law is

$$\Phi_e = 4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

The difference between this example and Example 27.3 is that Q_{in} is no longer the total charge of the sphere. Instead, Q_{in} is the amount of charge *inside* the Gaussian sphere of radius r. Because the charge distribution is *uniform,* the volume charge density is

$$\rho = \frac{Q}{V_R} = \frac{Q}{\frac{4}{3}\pi R^3}$$

The charge enclosed in a sphere of radius r is thus

$$Q_{in} = \rho V_r = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\tfrac{4}{3}\pi r^3\right) = \frac{r^3}{R^3}Q$$

The amount of enclosed charge increases with the cube of the distance r from the center and, as expected, $Q_{in} = Q$ if $r = R$. With this expression for Q_{in}, Gauss's law is

$$4\pi r^2 E = \frac{(r^3/R^3)Q}{\epsilon_0}$$

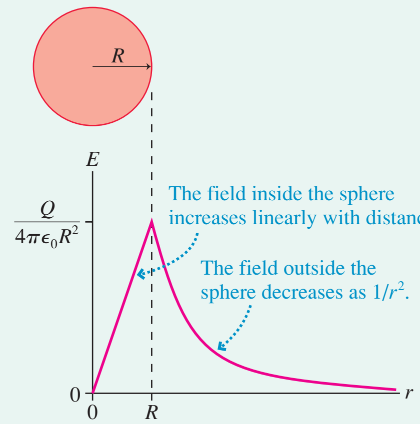
Thus the electric field at radius r inside a uniformly charged sphere is

$$E_{\text{inside}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r$$

The electric field strength inside the sphere increases *linearly* with the distance r from the center.

ASSESS The field inside and the field outside a sphere of charge match at the boundary of the sphere, $r = R$, where both give $E = Q/4\pi\epsilon_0 R^2$. In other words, the field strength is *continuous* as we cross the boundary of the sphere. These results are shown graphically in **FIGURE 27.25**.

FIGURE 27.25 The electric field strength of a uniform sphere of charge of radius R.



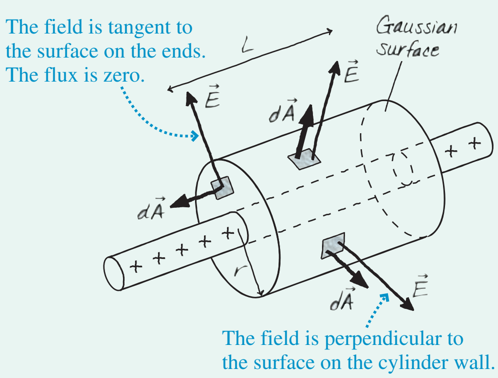
EXAMPLE 27.5 The electric field of a long, charged wire

In Chapter 26, we used superposition to find the electric field of an infinitely long line of charge with linear charge density (C/m) λ. It was not an easy derivation. Find the electric field using Gauss's law.

MODEL A long, charged wire can be modeled as an infinitely long line of charge.

VISUALIZE **FIGURE 27.26** shows an infinitely long line of charge. We can use the symmetry of the situation to see that the only possible shape of the electric field is to point straight into or out from the wire, rather like the bristles on a bottle brush. The shape of the field suggests that we choose our Gaussian surface to be a cylinder of radius r and length L, centered on the wire. Because Gauss's law refers to *closed* surfaces, we must include the ends of the cylinder as part of the surface.

FIGURE 27.26 A Gaussian surface around a charged wire.



SOLVE Gauss's law is

$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

where Q_{in} is the charge *inside* the closed cylinder. We have two tasks: to evaluate the flux integral, and to determine how much charge is inside the closed surface. The wire has linear charge density λ, so the amount of charge inside a cylinder of length L is simply

$$Q_{\text{in}} = \lambda L$$

Finding the net flux is just as straightforward. We can divide the flux through the entire closed surface into the flux through each end plus the flux through the cylindrical wall. The electric field $\vec{E}$, pointing straight out from the wire, is tangent to the end surfaces at every point. Thus the flux through these two surfaces is zero. On the wall, $\vec{E}$ is perpendicular to the surface and has the same strength E at every point. Thus

$$\Phi_e = \Phi_{\text{top}} + \Phi_{\text{bottom}} + \Phi_{\text{wall}} = 0 + 0 + EA_{\text{cyl}} = 2\pi rLE$$

where we used $A_{\text{cyl}} = 2\pi rL$ as the surface area of a cylindrical wall of radius r and length L. Once again, the proper choice of the Gaussian surface reduces the flux integral merely to finding a surface area. With these expressions for Q_{in} and Φ_e, Gauss's law is

$$\Phi_e = 2\pi rLE = \frac{Q_{\text{in}}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

Thus the electric field at distance r from a long, charged wire is

$$E_{\text{wire}} = \frac{\lambda}{2\pi\epsilon_0 r}$$

ASSESS This agrees exactly with the result of the more complex derivation in Chapter 26. Notice that the result does not depend on our choice of L. A Gaussian surface is an imaginary device, not a physical object. We needed a finite-length cylinder to do the flux calculation, but the electric field of an *infinitely* long wire can't depend on the length of an imaginary cylinder.

Example 27.5, for the electric field of a long, charged wire, contains a subtle but important idea, one that often occurs when using Gauss's law. The Gaussian cylinder of length L encloses only some of the wire's charge. The pieces of the charged wire outside the cylinder are not enclosed by the Gaussian surface and consequently do not contribute anything to the net flux. Even so, *they are essential* to the use of Gauss's law because it takes the *entire* charged wire to produce an electric field with cylindrical symmetry. In other words, the wire outside the cylinder may not contribute to the flux, but it affects the *shape* of the electric field. Our ability to write $\Phi_e = EA_{cyl}$ depended on knowing that E is the same at every point on the wall of the cylinder. That would not be true for a charged wire of finite length, so we cannot use Gauss's law to find the electric field of a finite-length charged wire.

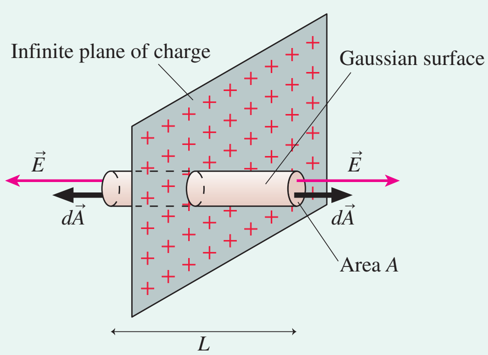
EXAMPLE 27.6 The electric field of a plane of charge

Use Gauss's law to find the electric field of an infinite plane of charge with surface charge density (C/m^2) η.

MODEL A uniformly charged flat electrode can be modeled as an infinite plane of charge.

VISUALIZE **FIGURE 27.27** shows a uniformly charged plane with surface charge density η. We will assume that the plane extends infinitely far in all directions, although we obviously have to show "edges" in our drawing. The planar symmetry allows the electric field to point only straight toward or away from the plane. With this in mind, choose as a Gaussian surface a cylinder with length L and faces of area A centered on the plane of charge. Although we've drawn them as circular, the shape of the faces is not relevant.

FIGURE 27.27 The Gaussian surface extends to both sides of a plane of charge.



SOLVE The electric field is perpendicular to both faces of the cylinder, so the total flux through both faces is $\Phi_{faces} = 2EA$. (The fluxes add rather than cancel because the area vector $\vec{A}$ points *outward* on each face.) There's *no* flux through the wall of the cylinder because the field vectors are tangent to the wall. Thus the net flux is simply

$$\Phi_e = 2EA$$

The charge inside the cylinder is the charge contained in area A of the plane. This is

$$Q_{in} = \eta A$$

With these expressions for Q_{in} and Φ_e, Gauss's law is

$$\Phi_e = 2EA = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$$

Thus the electric field of an infinite charged plane is

$$E_{plane} = \frac{\eta}{2\epsilon_0}$$

This agrees with the result in Chapter 26.

ASSESS This is another example of a Gaussian surface enclosing only some of the charge. Most of the plane's charge is outside the Gaussian surface and does not contribute to the flux, but it does affect the shape of the field. We wouldn't have planar symmetry, with the electric field exactly perpendicular to the plane, without all the rest of the charge on the plane.

The plane of charge is an especially good example of how powerful Gauss's law can be. Finding the electric field of a plane of charge via superposition was a difficult and tedious derivation. With Gauss's law, once you see how to apply it, the problem is simple enough to solve in your head!

You might wonder, then, why we bothered with superposition at all. The reason is that Gauss's law, powerful though it may be, is effective only in a limited number of situations where the field is highly symmetric. Superposition always works, even if the derivation is messy, because superposition goes directly back to the fields of individual point charges. It's good to use Gauss's law when you can, but superposition is often the only way to attack real-world charge distributions.

STOP TO THINK 27.5 Which Gaussian surface would allow you to use Gauss's law to determine the electric field outside a uniformly charged cube?

a. A sphere whose center coincides with the center of the charged cube
b. A cube whose center coincides with the center of the charged cube and that has parallel faces
c. Either a or b
d. Neither a nor b

27.6 Conductors in Electrostatic Equilibrium

Consider a charged conductor, such as a charged metal electrode, in electrostatic equilibrium. That is, there is no current through the conductor and the charges are all stationary. One very important conclusion is that **the electric field is zero at all points within a conductor in electrostatic equilibrium.** That is, $\vec{E}_{in} = \vec{0}$. If this weren't true, the electric field would cause the charge carriers to move and thus violate the assumption that all the charges are at rest. Let's use Gauss's law to see what else we can learn.

At the Surface of a Conductor

FIGURE 27.28 shows a Gaussian surface just barely inside the physical surface of a conductor that's in electrostatic equilibrium. The electric field is zero at all points within the conductor, hence the electric flux Φ_e through this Gaussian surface must be zero. But if $\Phi_e = 0$, Gauss's law tells us that $Q_{in} = 0$. That is, there's no net charge within this surface. There are charges—electrons and positive ions—but no *net* charge.

If there's no net charge in the interior of a conductor in electrostatic equilibrium, then **all the excess charge on a charged conductor resides on the exterior surface of the conductor.** Any charges added to a conductor quickly spread across the surface until reaching positions of electrostatic equilibrium, but there is no net charge *within* the conductor.

There may be no electric field within a charged conductor, but the presence of net charge requires an exterior electric field in the space outside the conductor. FIGURE 27.29 shows that **the electric field right at the surface of the conductor has to be perpendicular to the surface.** To see that this is so, suppose $\vec{E}_{surface}$ had a component tangent to the surface. This component of $\vec{E}_{surface}$ would exert a force on the surface charges and cause a surface current, thus violating the assumption that all charges are at rest. The only exterior electric field consistent with electrostatic equilibrium is one that is perpendicular to the surface.

We can use Gauss's law to relate the field strength at the surface to the charge density on the surface. FIGURE 27.30 shows a small Gaussian cylinder with faces very slightly above and below the surface of a charged conductor. The charge inside this Gaussian cylinder is ηA, where η is the surface charge density at this point on the conductor. There's a flux $\Phi = AE_{surface}$ through the outside face of this cylinder but, unlike Example 27.6 for the plane of charge, *no* flux through the inside face because $\vec{E}_{in} = \vec{0}$ within the conductor. Furthermore, there's no flux through the wall of the cylinder because $\vec{E}_{surface}$ is perpendicular to the surface. Thus the net flux is $\Phi_e = AE_{surface}$. Gauss's law is

$$\Phi_e = AE_{surface} = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0} \tag{27.19}$$

from which we can conclude that the electric field at the surface of a charged conductor is

$$\vec{E}_{surface} = \left(\frac{\eta}{\epsilon_0}, \text{perpendicular to surface}\right) \tag{27.20}$$

FIGURE 27.28 A Gaussian surface just inside a conductor that's in electrostatic equilibrium.

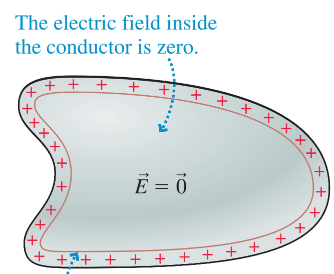


FIGURE 27.29 The electric field at the surface of a charged conductor.

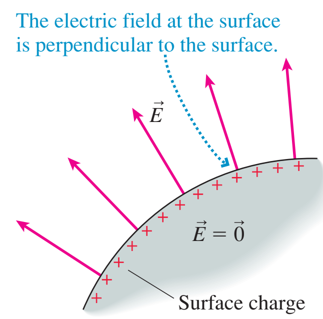
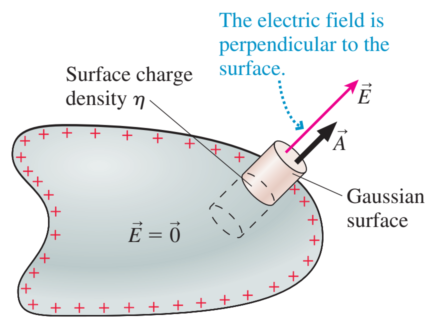


FIGURE 27.30 A Gaussian surface extending through the surface of the conductor has a flux only through the outer face.



In general, the surface charge density η is *not* constant on the surface of a conductor but depends on the shape of the conductor. If we can determine η, by either calculating it or measuring it, then Equation 27.20 tells us the electric field at that point on the surface. Alternatively, we can use Equation 27.20 to deduce the charge density on the conductor's surface if we know the electric field just outside the conductor.

Charges and Fields Within a Conductor

FIGURE 27.31 A Gaussian surface surrounding a hole inside a conductor in electrostatic equilibrium.

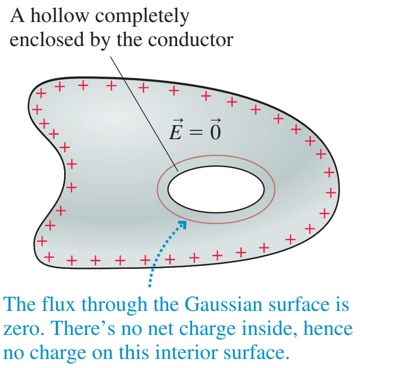
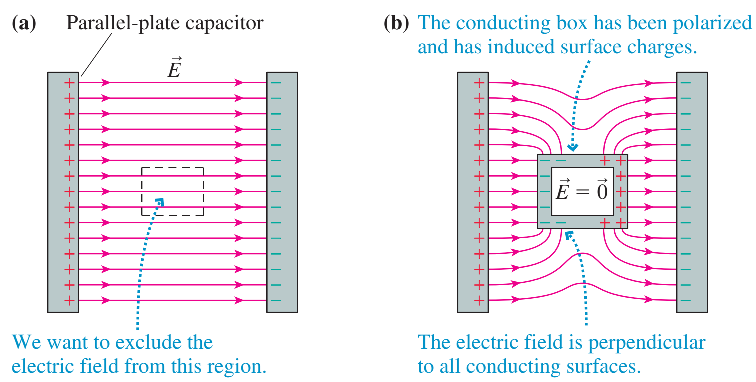


FIGURE 27.31 shows a charged conductor with a hole inside. Can there be charge on this interior surface? To find out, we place a Gaussian surface around the hole, infinitesimally close but entirely within the conductor. The electric flux Φ_e through this Gaussian surface is zero because the electric field is zero everywhere inside the conductor. Thus we must conclude that $Q_{in} = 0$. There's no net charge inside this Gaussian surface and thus no charge on the surface of the hole. Any excess charge resides on the *exterior* surface of the conductor, not on any interior surfaces.

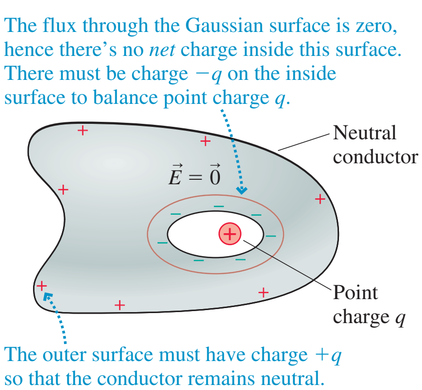
Furthermore, because there's no electric field inside the conductor and no charge inside the hole, the electric field inside the hole must also be zero. This conclusion has an important practical application. For example, suppose we need to exclude the electric field from the region in **FIGURE 27.32a** enclosed within dashed lines. We can do so by surrounding this region with the neutral conducting box of **FIGURE 27.32b**.

FIGURE 27.32 The electric field can be excluded from a region of space by surrounding it with a conducting box.



This region of space is now a hole inside a conductor, thus the interior electric field is zero. The use of a conducting box to exclude electric fields from a region of space is called **screening.** Solid metal walls are ideal, but in practice wire screen or wire mesh—sometimes called a *Faraday cage*—provides sufficient screening for all but the most sensitive applications. The price we pay is that the exterior field is now very complicated.

FIGURE 27.33 A charge in the hole causes a net charge on the interior and exterior surfaces.



Finally, **FIGURE 27.33** shows a charge q inside a hole within a neutral conductor. The electric field *within* the conductor is still zero, hence the electric flux through the Gaussian surface is zero. But $\Phi_e = 0$ requires $Q_{in} = 0$. Consequently, the charge inside the hole attracts an equal charge of opposite sign, and charge $-q$ now lines the inner surface of the hole.

The conductor as a whole is neutral, so moving $-q$ to the surface of the hole must leave $+q$ behind somewhere else. Where is it? It can't be in the interior of the conductor, as we've seen, and that leaves only the exterior surface. In essence, an internal charge polarizes the conductor just as an external charge would. Net charge $-q$ moves to the inner surface and net charge $+q$ is left behind on the exterior surface.

In summary, conductors in electrostatic equilibrium have the properties described in Tactics Box 27.3.

TACTICS BOX 27.3 Finding the electric field of a conductor in electrostatic equilibrium

1. The electric field is zero at all points within the volume of the conductor.
2. Any excess charge resides entirely on the *exterior* surface.
3. The external electric field at the surface of a charged conductor is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density at that point.
4. The electric field is zero inside any hole within a conductor unless there is a charge in the hole.

Exercises 20–24

EXAMPLE 27.7 The electric field at the surface of a charged metal sphere

A 2.0-cm-diameter brass sphere has been given a charge of 2.0 nC. What is the electric field strength at the surface?

MODEL Brass is a conductor. The excess charge resides on the surface.

VISUALIZE The charge distribution has spherical symmetry. The electric field points radially outward from the surface.

SOLVE We can solve this problem in two ways. One uses the fact that a sphere is the one shape for which any excess charge will spread out to a *uniform* surface charge density. Thus

$$\eta = \frac{q}{A_{\text{sphere}}} = \frac{q}{4\pi R^2} = \frac{2.0 \times 10^{-9}\ \text{C}}{4\pi(0.010\ \text{m})^2} = 1.59 \times 10^{-6}\ \text{C/m}^2$$

From Equation 27.20, we know the electric field at the surface has strength

$$E_{\text{surface}} = \frac{\eta}{\epsilon_0} = \frac{1.59 \times 10^{-6}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2} = 1.8 \times 10^5\ \text{N/C}$$

Alternatively, we could have used the result, obtained earlier in the chapter, that the electric field strength outside a sphere of charge Q is $E_{\text{outside}} = Q_{\text{in}}/(4\pi\epsilon_0 r^2)$. But $Q_{\text{in}} = q$ and, at the surface, $r = R$. Thus

$$E_{\text{surface}} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} = (9.0 \times 10^9\ \text{N m}^2/\text{C}^2)\frac{2.0 \times 10^{-9}\ \text{C}}{(0.010\ \text{m})^2}$$

$$= 1.8 \times 10^5\ \text{N/C}$$

As we can see, both methods lead to the same result.

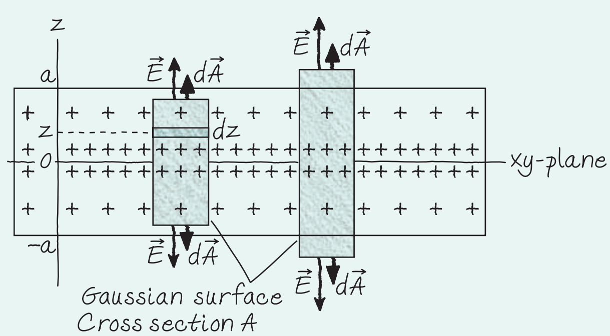
CHALLENGE EXAMPLE 27.8 The electric field of a slab of charge

An infinite slab of charge of thickness $2a$ is centered in the xy-plane. The charge density is $\rho = \rho_0(1 - |z|/a)$. Find the electric field strengths inside and outside this slab of charge.

MODEL The charge density is not uniform. Starting at ρ_0 in the xy-plane, it decreases linearly with distance above and below the xy-plane until reaching zero at $z = \pm a$, the edges of the slab.

VISUALIZE **FIGURE 27.34** shows an edge view of the slab of charge and, as Gaussian surfaces, side views of two cylinders with cross-section area A. By symmetry, the electric field must point away from the xy-plane; the field cannot have an x- or y-component.

FIGURE 27.34 Two cylindrical Gaussian surfaces for an infinite slab of charge.



SOLVE Gauss's law is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

With symmetry, finding the net flux is straightforward. The electric field is perpendicular to the faces of the cylinders and pointing outward, so the total flux through the faces is $\Phi_{\text{faces}} = 2EA$, where E may depend on distance z. The field is parallel to the walls of the cylinders, so $\Phi_{\text{wall}} = 0$. Thus the net flux is simply

$$\Phi_e = 2EA$$

Because the charge density is not uniform, we need to integrate to find Q_{in}, the charge *inside* the cylinder. We can slice the cylinder into small slabs of infinitesimal thickness dz and volume $dV = A\,dz$. Figure 27.34 shows one such little slab at distance z from the xy-plane. The charge in this little slab is

$$dq = \rho\, dV = \rho_0\left(1 - \frac{z}{a}\right)A\, dz$$

where we assumed that z is positive. Because the charge is symmetric about $z = 0$, we can avoid difficulties with the absolute value sign in the charge density by integrating from 0 and

Continued

multiplying by 2. For the Gaussian cylinder that ends inside the slab of charge, at distance z, the total charge inside is

$$Q_{\text{in}} = \int dq = 2\int_0^z \rho_0\left(1 - \frac{z}{a}\right)A\,dz$$

$$= 2\rho_0 A\left[z\Big|_0^z - \frac{1}{2a}z^2\Big|_0^z\right]$$

$$= 2\rho_0 Az\left(1 - \frac{z}{2a}\right)$$

Gauss's law inside the slab is then

$$\Phi_e = 2E_{\text{inside}}A = \frac{Q_{\text{in}}}{\epsilon_0} = \frac{2\rho_0 Az}{\epsilon_0}\left(1 - \frac{z}{2a}\right)$$

The area A cancels, as it must because it was an arbitrary choice, leaving

$$E_{\text{inside}} = \frac{\rho_0 z}{\epsilon_0}\left(1 - \frac{z}{2a}\right)$$

The field strength is zero at $z = 0$, then increases as z increases. This expression is valid only above the xy-plane, for $z > 0$, but the field strength is symmetric on the other side.

For the Gaussian cylinder that extends outside the slab of charge, the integral for Q has to end at $z = a$. Thus

$$Q_{\text{in}} = 2\rho_0 Aa\left(1 - \frac{a}{2a}\right) = \rho_0 Aa$$

independent of distance z. With this, Gauss's law gives

$$E_{\text{outside}} = \frac{Q_{\text{in}}}{2\epsilon_0 A} = \frac{\rho_0 a}{2\epsilon_0}$$

This matches E_{inside} at the surface, $z = a$, so the field is continuous as it crosses the boundary.

ASSESS Outside a sphere of charge, the field is the same as that of a point charge at the center. Similarly, the field outside an infinite slab of charge should be the same as that of an infinite charged plane. We found, by integration, that the total charge in an area A of the slab is $Q = \rho_0 Aa$. If we squished this charge into a plane, the surface charge density would be $\eta = Q/A = \rho_0 a$. Thus our expression for E_{outside} could be written $\eta/2\epsilon_0$, which matches the field we found in Example 27.6 for a plane of charge.

SUMMARY

The goal of Chapter 27 has been to understand and apply Gauss's law.

General Principles

Gauss's Law

For any *closed* surface enclosing net charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

The electric flux Φ_e is the same for *any* closed surface enclosing charge Q_{in}.

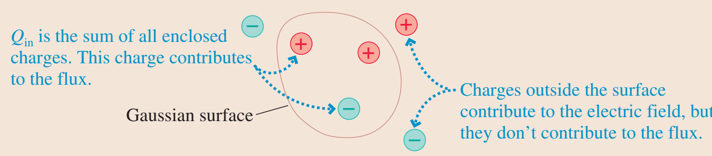
Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.

In practice, Φ_e is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

Important Concepts

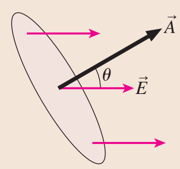
Charge creates the electric field that is responsible for the electric flux.



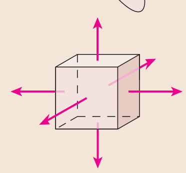
Flux is the amount of electric field passing through a surface of area A:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

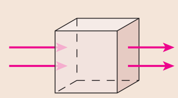
where $\vec{A}$ is the **area vector.**



For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge.



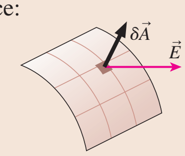
Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



Surface integrals calculate the flux by summing the fluxes through many small pieces of the surface:

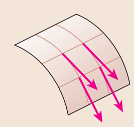
$$\Phi_e = \sum \vec{E} \cdot \delta\vec{A}$$

$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



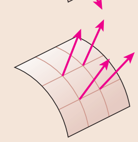
Two important situations:
If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$



If the electric field is everywhere perpendicular to the surface *and* has the same strength E at all points, then

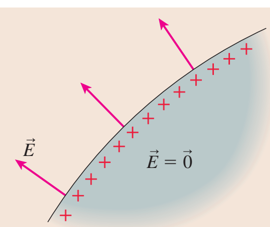
$$\Phi_e = EA$$



Applications

Conductors in electrostatic equilibrium

- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



Terms and Notation

symmetric
Gaussian surface
electric flux, Φ_e
area vector, $\vec{A}$
surface integral
Gauss's law
screening

CONCEPTUAL QUESTIONS

1. Suppose you have the uniformly charged cube in FIGURE Q27.1. Can you use symmetry alone to deduce the *shape* of the cube's electric field? If so, sketch and describe the field shape. If not, why not?

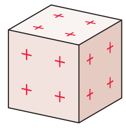


FIGURE Q27.1

2. FIGURE Q27.2 shows cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. For each, does the surface enclose a net positive charge, a net negative charge, or no net charge? Explain.

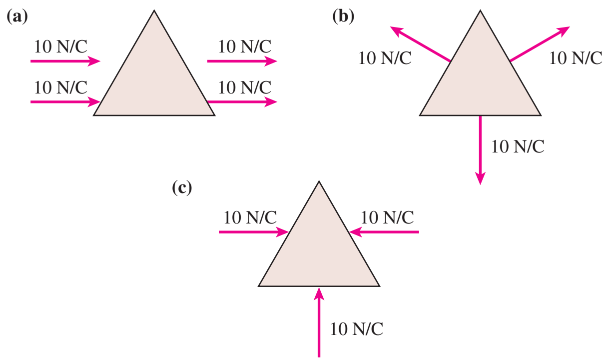


FIGURE Q27.2

3. The square and circle in FIGURE Q27.3 are in the same uniform field. The diameter of the circle equals the edge length of the square. Is Φ_{square} larger than, smaller than, or equal to Φ_{circle}? Explain.

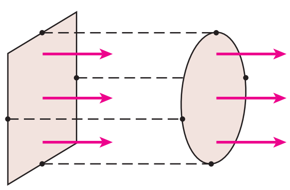


FIGURE Q27.3

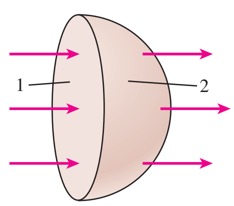


FIGURE Q27.4

4. In FIGURE Q27.4, where the field is uniform, is Φ_1 larger than, smaller than, or equal to Φ_2? Explain.
5. What is the electric flux through each of the surfaces in FIGURE Q27.5? Give each answer as a multiple of q/ϵ_0.

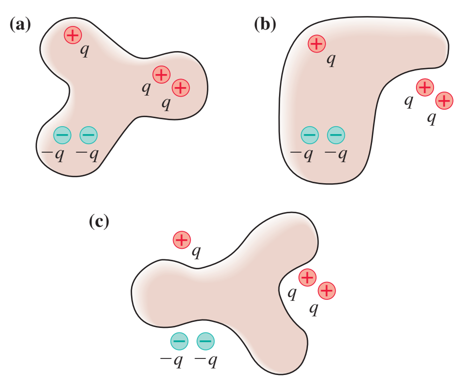


FIGURE Q27.5

6. What is the electric flux through each of the surfaces A to E in FIGURE Q27.6? Give each answer as a multiple of q/ϵ_0.

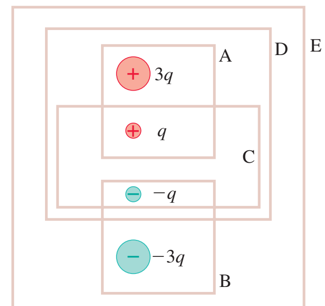


FIGURE Q27.6

7. The charged balloon in FIGURE Q27.7 expands as it is blown up, increasing in size from the initial to final diameters shown. Do the electric field strengths at points 1, 2, and 3 increase, decrease, or stay the same? Explain your reasoning for each.

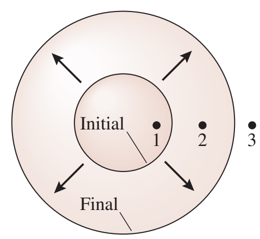


FIGURE Q27.7

8. The two spheres in FIGURE Q27.8 surround equal charges. Three students are discussing the situation.
Student 1: The fluxes through spheres A and B are equal because they enclose equal charges.
Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. The flux depends on the electric field strength, so the flux through A is larger than the flux through B.
Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.
Which of these students, if any, do you agree with? Explain.

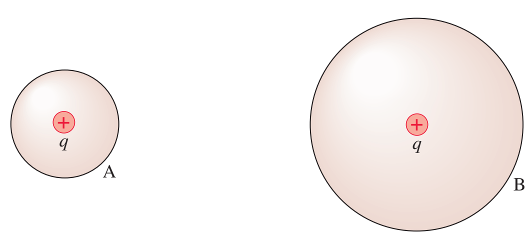


FIGURE Q27.8

9. The sphere and ellipsoid in FIGURE Q27.9 surround equal charges. Four students are discussing the situation.
Student 1: The fluxes through A and B are equal because the average radius is the same.
Student 2: I agree that the fluxes are equal, but that's because they enclose equal charges.
Student 3: The electric field is not perpendicular to the surface for B, and that makes the flux through B less than the flux through A.
Student 4: I don't think that Gauss's law even applies to a situation like B, so we can't compare the fluxes through A and B.
Which of these students, if any, do you agree with? Explain.

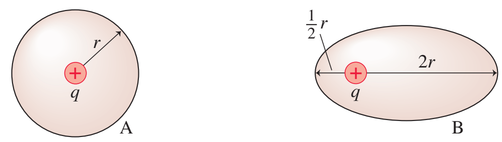


FIGURE Q27.9

10. A small, metal sphere hangs by an insulating thread within the larger, hollow conducting sphere of FIGURE Q27.10. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the protruding wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged? Explain.
a. The small sphere.
b. The inner surface of the hollow sphere.
c. The outer surface of the hollow sphere.

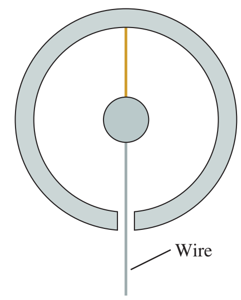


FIGURE Q27.10

EXERCISES AND PROBLEMS

Exercises

Section 27.1 Symmetry

1. | FIGURE EX27.1 shows two cross sections of two infinitely long coaxial cylinders. The inner cylinder has a positive charge, the outer cylinder has an equal negative charge. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.

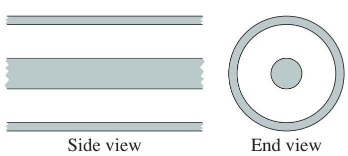


FIGURE EX27.1

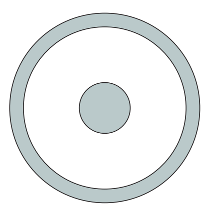


FIGURE EX27.2

2. | FIGURE EX27.2 shows a cross section of two concentric spheres. The inner sphere has a negative charge. The outer sphere has a positive charge larger in magnitude than the charge on the inner sphere. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.
3. | FIGURE EX27.3 shows a cross section of two infinite parallel planes of charge. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.

FIGURE EX27.3

Section 27.2 The Concept of Flux

4. | The electric field is constant over each face of the cube shown in FIGURE EX27.4. Does the box contain positive charge, negative charge, or no charge? Explain.

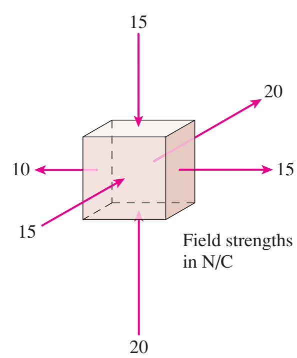


FIGURE EX27.4

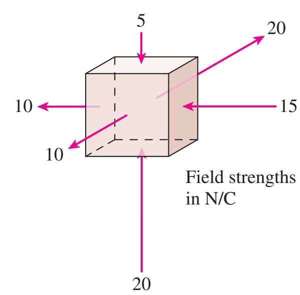


FIGURE EX27.5

5. | The electric field is constant over each face of the cube shown in FIGURE EX27.5. Does the box contain positive charge, negative charge, or no charge? Explain.
6. | The cube in FIGURE EX27.6 contains negative charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What strength must this field exceed?

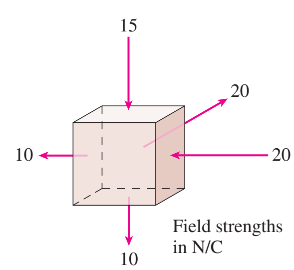


FIGURE EX27.6

7. | The cube in FIGURE EX27.7 contains negative charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What strength must this field exceed?

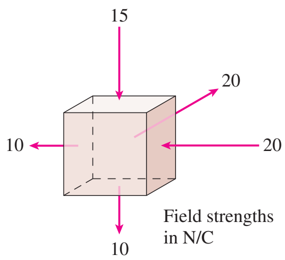


FIGURE EX27.7

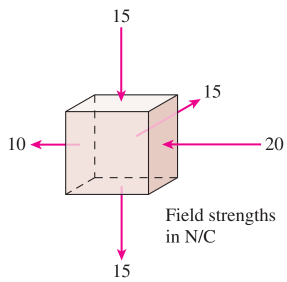


FIGURE EX27.8

8. | The cube in FIGURE EX27.8 contains no net charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What is the field strength?

Section 27.3 Calculating Electric Flux

9. || What is the electric flux through the surface shown in FIGURE EX27.9?

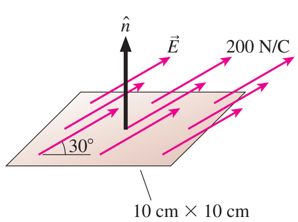


FIGURE EX27.9

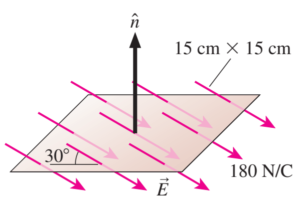


FIGURE EX27.10

10. || What is the electric flux through the surface shown in FIGURE EX27.10?

11. || The electric flux through the surface shown in FIGURE EX27.11 is 25 N m^2/C. What is the electric field strength?

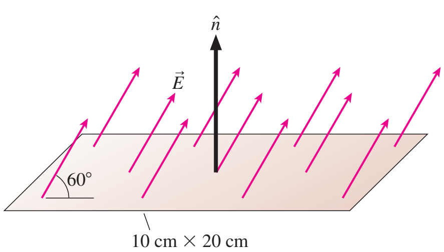


FIGURE EX27.11

12. || A 2.0 cm × 3.0 cm rectangle lies in the xy-plane. What is the electric flux through the rectangle if

a. $\vec{E} = (100\hat{\imath} + 50\hat{k})$ N/C?

b. $\vec{E} = (100\hat{\imath} + 50\hat{\jmath})$ N/C?

13. || A 2.0 cm × 3.0 cm rectangle lies in the xz-plane. What is the electric flux through the rectangle if

a. $\vec{E} = (100\hat{\imath} + 50\hat{k})$ N/C?

b. $\vec{E} = (100\hat{\imath} + 50\hat{\jmath})$ N/C?

14. || A 3.0-cm-diameter circle lies in the xz-plane in a region where the electric field is $\vec{E} = (1500\hat{\imath} + 1500\hat{\jmath} - 1500\hat{k})$ N/C. What is the electric flux through the circle?

15. || A 1.0 cm × 1.0 cm × 1.0 cm box with its edges aligned with the xyz-axes is in the electric field $\vec{E} = (350x + 150)\hat{\imath}$ N/C, where x is in meters. What is the net electric flux through the box?

16. | What is the net electric flux through the two cylinders shown in FIGURE EX27.16? Give your answer in terms of R and E.

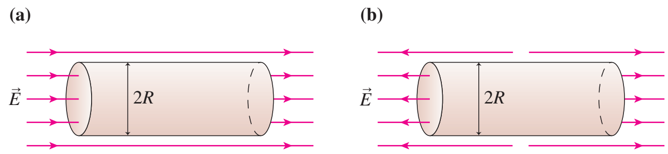


FIGURE EX27.16

Section 27.4 Gauss's Law

Section 27.5 Using Gauss's Law

17. | FIGURE EX27.17 shows three charges. Draw these charges on your paper four times. Then draw two-dimensional cross sections of three-dimensional closed surfaces through which the electric flux is (a) $2q/\epsilon_0$, (b) q/ϵ_0, (c) 0, and (d) $5q/\epsilon_0$.

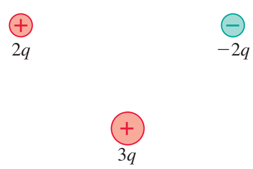


FIGURE EX27.17

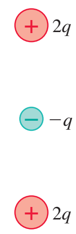


FIGURE EX27.18

18. | FIGURE EX27.18 shows three charges. Draw these charges on your paper four times. Then draw two-dimensional cross sections of three-dimensional closed surfaces through which the electric flux is (a) $-q/\epsilon_0$, (b) q/ϵ_0, (c) $3q/\epsilon_0$, and (d) $4q/\epsilon_0$.

19. | FIGURE EX27.19 shows three Gaussian surfaces and the electric flux through each. What are the three charges q_1, q_2, and q_3?

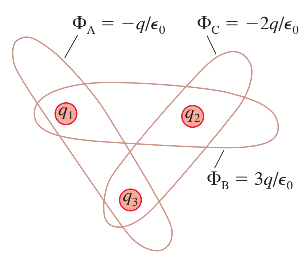


FIGURE EX27.19

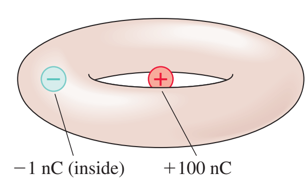


FIGURE EX27.20

20. || What is the net electric flux through the torus (i.e., doughnut shape) of FIGURE EX27.20?

21. What is the net electric flux through the cylinder of FIGURE EX27.21?

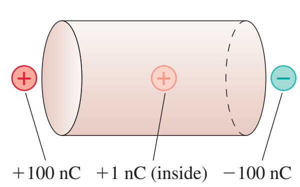


FIGURE EX27.21

22. || The net electric flux through an octahedron is −1000 N m^2/C. How much charge is enclosed within the octahedron?

23. || 55.3 million excess electrons are inside a closed surface. What is the net electric flux through the surface?

Section 27.6 Conductors in Electrostatic Equilibrium

24. | The electric field strength just above one face of a copper penny is 2000 N/C. What is the surface charge density on this face of the penny?

25. | A spark occurs at the tip of a metal needle if the electric field strength exceeds 3.0×10^6 N/C, the field strength at which air breaks down. What is the minimum surface charge density for producing a spark?

26. | The conducting box in FIGURE EX27.26 has been given an excess negative charge. The surface density of excess electrons at the center of the top surface is 5.0×10^{10} electrons/m^2. What are the electric field strengths E_1 to E_3 at points 1 to 3?

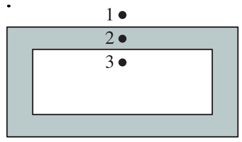


FIGURE EX27.26

27. | A thin, horizontal, 10-cm-diameter copper plate is charged to 3.5 nC. If the electrons are uniformly distributed on the surface, what are the strength and direction of the electric field
a. 0.1 mm above the center of the top surface of the plate?
b. at the plate's center of mass?
c. 0.1 mm below the center of the bottom surface of the plate?

28. | FIGURE EX27.28 shows a hollow cavity within a neutral conductor. A point charge Q is inside the cavity. What is the net electric flux through the closed surface that surrounds the conductor?

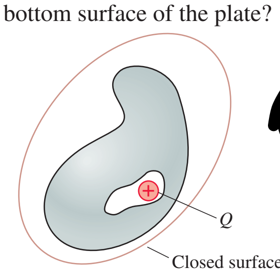


FIGURE EX27.28

Problems

29. | FIGURE P27.29 shows four sides of a 3.0 cm × 3.0 cm × 3.0 cm cube.
a. What are the electric fluxes Φ_1 to Φ_4 through sides 1 to 4?
b. What is the net flux through these four sides?

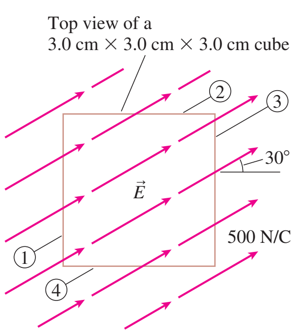


FIGURE P27.29

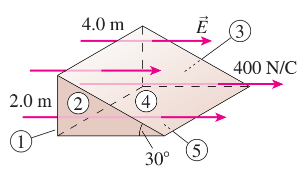


FIGURE P27.30

30. ||| Find the electric fluxes Φ_1 to Φ_5 through surfaces 1 to 5 in FIGURE P27.30.

31. || A tetrahedron has an equilateral triangle base with 20-cm-long edges and three equilateral triangle sides. The base is parallel to the ground, and a vertical uniform electric field of strength 200 N/C passes upward through the tetrahedron.
a. What is the electric flux through the base?
b. What is the electric flux through each of the three sides?

32. Charges $q_1 = -4Q$ and $q_2 = +2Q$ are located at $x = -a$ and $x = +a$, respectively. What is the net electric flux through a sphere of radius $2a$ centered (a) at the origin and (b) at $x = 2a$?

33. | A 10 nC point charge is at the center of a 2.0 m × 2.0 m × 2.0 m cube. What is the electric flux through the top surface of the cube?

34. || The electric flux is 300 N m^2/C through two opposing faces of a 2.0 cm × 2.0 cm × 2.0 cm box. The flux through each of the other faces is 100 N m^2/C. How much charge is inside the box?

35. || A spherically symmetric charge distribution produces the electric field $\vec{E} = (200/r)\hat{r}$ N/C, where r is in m.
a. What is the electric field strength at $r = 10$ cm?
b. What is the electric flux through a 20-cm-diameter spherical surface that is concentric with the charge distribution?
c. How much charge is inside this 20-cm-diameter spherical surface?

36. || A spherically symmetric charge distribution produces the electric field $\vec{E} = (5000r^2)\hat{r}$ N/C, where r is in m.
a. What is the electric field strength at $r = 20$ cm?
b. What is the electric flux through a 40-cm-diameter spherical surface that is concentric with the charge distribution?
c. How much charge is inside this 40-cm-diameter spherical surface?

37. || A neutral conductor contains a hollow cavity in which there is a +100 nC point charge. A charged rod then transfers −50 nC to the conductor. Afterward, what is the charge (a) on the inner wall of the cavity wall, and (b) on the exterior surface of the conductor?

38. || A hollow metal sphere has inner radius a and outer radius b. The hollow sphere has charge $+2Q$. A point charge $+Q$ sits at the center of the hollow sphere.
a. Determine the electric fields in the three regions $r \le a$, $a < r < b$, and $r \ge b$.
b. How much charge is on the inside surface of the hollow sphere? On the exterior surface?

39. || A 20-cm-radius ball is uniformly charged to 80 nC.
a. What is the ball's volume charge density (C/m^3)?
b. How much charge is enclosed by spheres of radii 5, 10, and 20 cm?
c. What is the electric field strength at points 5, 10, and 20 cm from the center?

40. || FIGURE P27.40 shows a solid metal sphere at the center of a hollow metal sphere. What is the total charge on (a) the exterior of the inner sphere, (b) the inside surface of the hollow sphere, and (c) the exterior surface of the hollow sphere?

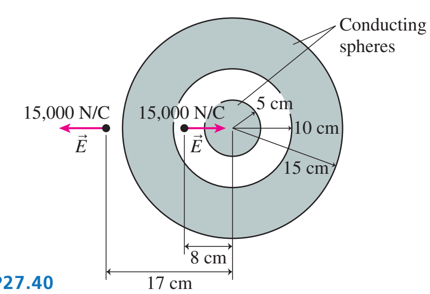


FIGURE P27.40

41. | The earth has a vertical electric field at the surface, pointing down, that averages 100 N/C. This field is maintained by various atmospheric processes, including lightning. What is the excess charge on the surface of the earth?

42. ‖ Figure 27.32b showed a conducting box inside a parallel-plate capacitor. The electric field inside the box is $\vec{E} = \vec{0}$. Suppose the surface charge on the exterior of the box could be frozen. Draw a picture of the electric field inside the box after the box, with its frozen charge, is removed from the capacitor.
Hint: Superposition.

43. ‖ A hollow metal sphere has 6 cm and 10 cm inner and outer radii, respectively. The surface charge density on the inside surface is $-100\ \text{nC/m}^2$. The surface charge density on the exterior surface is $+100\ \text{nC/m}^2$. What are the strength and direction of the electric field at points 4, 8, and 12 cm from the center?

44. ‖ A positive point charge q sits at the center of a hollow spherical shell. The shell, with radius R and negligible thickness, has net charge $-2q$. Find an expression for the electric field strength (a) inside the sphere, $r < R$, and (b) outside the sphere, $r > R$. In what direction does the electric field point in each case?

45. ‖ Find the electric field inside and outside a hollow plastic ball of radius R that has charge Q uniformly distributed on its outer surface.

46. ‖ A uniformly charged ball of radius a and charge $-Q$ is at the center of a hollow metal shell with inner radius b and outer radius c. The hollow sphere has net charge $+2Q$. Determine the electric field strength in the four regions $r \le a$, $a < r < b$, $b \le r \le c$, and $r > c$.

47. | The three parallel planes of charge shown in FIGURE P27.47 have surface charge densities $-\frac{1}{2}\eta$, η, and $-\frac{1}{2}\eta$. Find the electric fields $\vec{E}_1$ to $\vec{E}_4$ in regions 1 to 4.

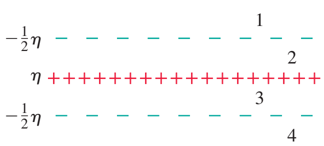


FIGURE P27.47

48. ‖ An infinite slab of charge of thickness $2z_0$ lies in the xy-plane between $z = -z_0$ and $z = +z_0$. The volume charge density ρ (C/m^3) is a constant.
   a. Use Gauss's law to find an expression for the electric field strength inside the slab ($-z_0 \le z \le z_0$).
   b. Find an expression for the electric field strength above the slab ($z \ge z_0$).
   c. Draw a graph of E from $z = 0$ to $z = 3z_0$.

49. ‖ FIGURE P27.49 shows an infinitely wide conductor parallel to and distance d from an infinitely wide plane of charge with surface charge density η. What are the electric fields $\vec{E}_1$ to $\vec{E}_4$ in regions 1 to 4?

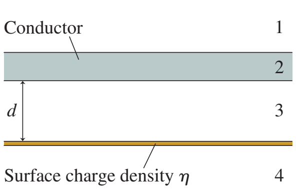


FIGURE P27.49

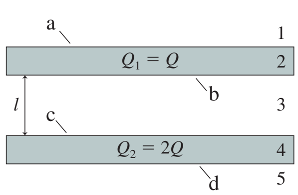


FIGURE P27.50

50. ‖ FIGURE P27.50 shows two very large slabs of metal that are parallel and distance l apart. Each slab has a total surface area (top + bottom) A. The thickness of each slab is so small in comparison to its lateral dimensions that the surface area around the sides is negligible. Metal 1 has total charge $Q_1 = Q$ and metal 2 has total charge $Q_2 = 2Q$. Assume Q is positive. In terms of Q and A, determine
   a. The electric field strengths E_1 to E_5 in regions 1 to 5.
   b. The surface charge densities η_a to η_d on the four surfaces a to d.

51. ‖ A long, thin straight wire with linear charge density λ runs down the center of a thin, hollow metal cylinder of radius R. The cylinder has a net linear charge density 2λ. Assume λ is positive. Find expressions for the electric field strength (a) inside the cylinder, $r < R$, and (b) outside the cylinder, $r > R$. In what direction does the electric field point in each of the cases?

52. ‖ A very long, uniformly charged cylinder has radius R and linear charge density λ. Find the cylinder's electric field (a) outside the cylinder, $r \ge R$, and (b) inside the cylinder, $r \le R$. (c) Show that your answers to parts a and b match at the boundary, $r = R$.

53. ‖ A spherical shell has inner radius R_{in} and outer radius R_{out}. The shell contains total charge Q, uniformly distributed. The interior of the shell is empty of charge and matter.
   a. Find the electric field outside the shell, $r \ge R_{out}$.
   b. Find the electric field in the interior of the shell, $r \le R_{in}$.
   c. Find the electric field within the shell, $R_{in} \le r \le R_{out}$.
   d. Show that your solutions match at both the inner and outer boundaries.

54. ‖ An early model of the atom, proposed by Rutherford after his discovery of the atomic nucleus, had a positive point charge $+Ze$ (the nucleus) at the center of a sphere of radius R with uniformly distributed negative charge $-Ze$. Z is the atomic number, the number of protons in the nucleus and the number of electrons in the negative sphere.
   a. Show that the electric field inside this atom is

$$E_{in} = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

   b. What is E at the surface of the atom? Is this the expected value? Explain.
   c. A uranium atom has $Z = 92$ and $R = 0.10$ nm. What is the electric field strength at $r = \frac{1}{2}R$?

Challenge Problems

55. All examples of Gauss's law have used highly symmetric surfaces where the flux integral is either zero or EA. Yet we've claimed that the net $\Phi_e = Q_{in}/\epsilon_0$ is independent of the surface. This is worth checking. FIGURE CP27.55 shows a cube of edge length L centered on a long thin wire with linear charge density λ. The flux through one face of the cube is *not* simply EA because, in this case, the electric field varies in both strength and direction. But you can calculate the flux by actually doing the flux integral.

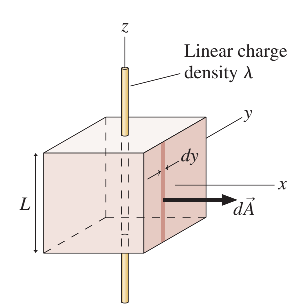


FIGURE CP27.55

a. Consider the face parallel to the yz-plane. Define area $d\vec{A}$ as a strip of width dy and height L with the vector pointing in the x-direction. One such strip is located at position y. Use the known electric field of a wire to calculate the electric flux $d\Phi$ through this little area. Your expression should be written in terms of y, which is a variable, and various constants. It should not explicitly contain any angles.

b. Now integrate $d\Phi$ to find the total flux through this face.

c. Finally, show that the net flux through the cube is $\Phi_e = Q_{in}/\epsilon_0$.

56. An infinite cylinder of radius R has a linear charge density λ. The volume charge density (C/m^3) within the cylinder ($r \le R$) is $\rho(r) = r\rho_0/R$, where ρ_0 is a constant to be determined.

a. Draw a graph of ρ versus x for an x-axis that crosses the cylinder perpendicular to the cylinder axis. Let x range from $-2R$ to $2R$.

b. The charge within a small volume dV is $dq = \rho\, dV$. The integral of $\rho\, dV$ over a cylinder of length L is the total charge $Q = \lambda L$ within the cylinder. Use this fact to show that $\rho_0 = 3\lambda/2\pi R^2$.

Hint: Let dV be a cylindrical shell of length L, radius r, and thickness dr. What is the volume of such a shell?

c. Use Gauss's law to find an expression for the electric field E inside the cylinder, $r \le R$.

d. Does your expression have the expected value at the surface, $r = R$? Explain.

57. A sphere of radius R has total charge Q. The volume charge density (C/m^3) within the sphere is $\rho(r) = C/r^2$, where C is a constant to be determined.

a. The charge within a small volume dV is $dq = \rho\, dV$. The integral of $\rho\, dV$ over the entire volume of the sphere is the total charge Q. Use this fact to determine the constant C in terms of Q and R.

Hint: Let dV be a spherical shell of radius r and thickness dr. What is the volume of such a shell?

b. Use Gauss's law to find an expression for the electric field E inside the sphere, $r \le R$.

c. Does your expression have the expected value at the surface, $r = R$? Explain.

58. A sphere of radius R has total charge Q. The volume charge density (C/m^3) within the sphere is

$$\rho = \rho_0\left(1 - \frac{r}{R}\right)$$

This charge density decreases linearly from ρ_0 at the center to zero at the edge of the sphere.

a. Show that $\rho_0 = 3Q/\pi R^3$.

b. Show that the electric field inside the sphere points radially outward with magnitude

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - 3\frac{r}{R}\right)$$

c. Show that your result of part b has the expected value at $r = R$.

59. A spherical ball of charge has radius R and total charge Q. The electric field strength inside the ball ($r \le R$) is $E(r) = E_{max}(r^4/R^4)$.

a. What is E_{max} in terms of Q and R?

b. Find an expression for the volume charge density $\rho(r)$ inside the ball as a function of r.

c. Verify that your charge density gives the total charge Q when integrated over the volume of the ball.

STOP TO THINK ANSWERS

Stop to Think 27.1: a and d. Symmetry requires the electric field to be unchanged if front and back are reversed, if left and right are reversed, or if the field is rotated about the wire's axis. Fields a and d both have the proper symmetry. Other factors would now need to be considered to determine the correct field.

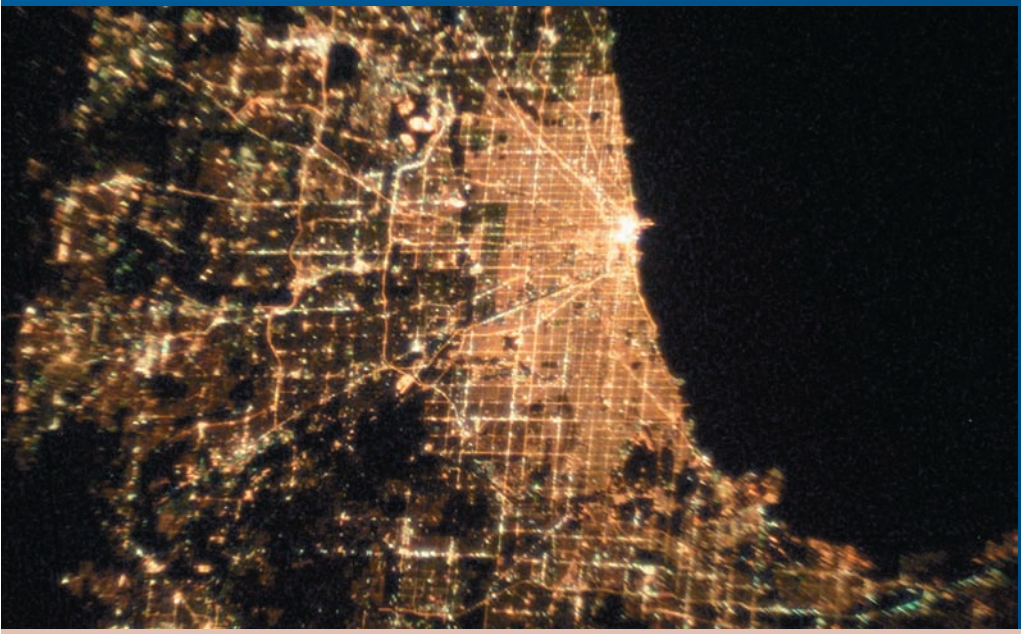
Stop to Think 27.2: e. The net flux is into the box.

Stop to Think 27.3: c. There's no flux through the four sides. The flux is positive 1 N m^2/C through both the top and bottom because $\vec{E}$ and $\vec{A}$ both point outward.

Stop to Think 27.4: $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$. The flux through a closed surface depends only on the amount of enclosed charge, not the size or shape of the surface.

Stop to Think 27.5: d. A cube doesn't have enough symmetry to use Gauss's law. The electric field of a charged cube is *not* constant over the face of a cubic Gaussian surface, so we can't evaluate the surface integral for the flux.

28 The Electric Potential



City lights seen from space show where millions of lightbulbs are transforming electric energy into light and thermal energy.

▶ **Looking Ahead** The goals of Chapter 28 are to calculate and use the electric potential and electric potential energy.

Electric Energy

Energy allows things to happen. You want your lights to light, your computer to compute, and your stereo to keep your neighbors awake. All these require energy—*electric* energy.

This is the first of two chapters that explore electric energy and its connection to electric forces and fields.

Lightning is a dramatic example of the transformation of electric energy into light, sound, and thermal energy.

You'll learn to calculate the electric potential energy of charged particles and to solve problems using conservation of mechanical energy.

There's a close connection between electric potential energy and gravitational potential energy because both forces obey inverse-square laws.

◀ **Looking Back**
Sections 10.2–10.5 Kinetic energy, potential energy, and conservation

◀ **Looking Back**
Sections 11.2–11.5 Work and potential energy

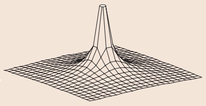
The Electric Potential

Just as source charges create an electric field, they also create an **electric potential.** A charge moving in an electric potential has an electric potential energy.

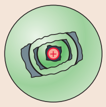
The unit of electric potential is the **volt,** perhaps the most well known of all electrical units. A voltmeter reads the *potential difference* between two points.

Calculating Electric Potential

You'll learn how to calculate the electric potential for several important charge distributions.



Elevation graph



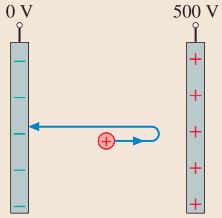
Equipotential surfaces

You'll also learn to use several different representations of the electric potential.

◀ **Looking Back**
Section 26.3 Calculating electric fields

Using Electric Potential

Charged particles *accelerate* as they move through a potential difference.



You'll learn to use the electric potential and a conservation of energy problem-solving strategy to solve problems about the motion of charged particles.

◀ **Looking Back**
Section 10.6 Energy diagrams

Sources of Electric Potential

In practice, electric potential is created by separating positive and negative charges—an idea we'll explore more thoroughly in Chapter 29.

A battery is the most common source of electric potential. As you'll learn, its *voltage* is the potential difference between separated charges—the plus and minus terminals.

28.1 Electric Potential Energy

In electricity, just as in mechanics, it takes energy to make things happen. It's been many chapters since we dealt much with work and energy, but these ideas will now be *essential* to our story. Consequently, the Looking Back recommendations in the chapter preview are especially important. You will recall that a system's mechanical energy $E_{\text{mech}} = K + U$ is conserved for particles that interact with each other via *conservative forces,* where K and U are the kinetic and potential energy. That is,

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0 \tag{28.1}$$

We need to be careful with notation because we are now using E to represent the electric field strength. To avoid confusion, we will represent mechanical energy either as the explicit sum $K + U$ or as E_{mech}, with an explicit subscript.

NOTE ▶ Recall that for any X, the *change* in X is $\Delta X = X_{\text{final}} - X_{\text{initial}}$. ◀

The kinetic energy $K = \sum K_i$, where $K_i = \frac{1}{2}m_i v_i^2$, is the sum of the kinetic energies of all the particles in the system. The potential energy U is the *interaction energy* of the system. In particular, we defined the *change* in potential energy in terms of the work W done by the forces of interaction as the system moves from an initial position or configuration i to a final position or configuration f:

$$\Delta U = U_{\text{f}} - U_{\text{i}} = -W_{\text{interaction forces}} \qquad (\text{position i} \rightarrow \text{position f}) \tag{28.2}$$

This formal definition of ΔU is rather abstract and will make more sense when we see specific applications.

A *constant* force does work

$$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r\cos\theta \tag{28.3}$$

on a particle that undergoes a linear displacement $\Delta\vec{r}$, where θ is the angle between the force $\vec{F}$ and $\Delta\vec{r}$. **FIGURE 28.1** reminds you of the three special cases $\theta = 0°$, $90°$, and $180°$. It also shows that, in general, the work is done by the force component F_r in the direction of motion.

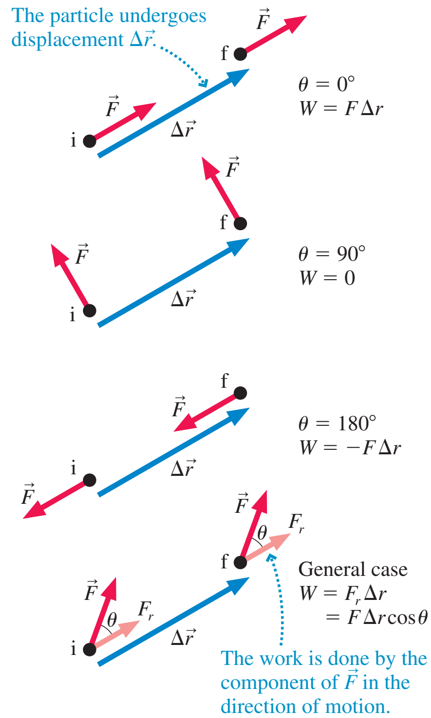


FIGURE 28.1 The work done by a constant force.

NOTE ▶ Work is *not* the oft-remembered "force times distance." Work is force times distance only in the one very special case in which the force is both constant *and* parallel to the displacement. ◀

If the force is *not* constant or the displacement is *not* along a linear path, we can calculate the work by dividing the path into many small segments. **FIGURE 28.2** shows how this is done. The work done as the particle moves distance ds is $F_s\,ds$, where F_s is the force component parallel to ds (i.e., the component in the direction of motion). The total work done on the particle is

$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_{\text{i}}^{\text{f}} \vec{F} \cdot d\vec{s} \tag{28.4}$$

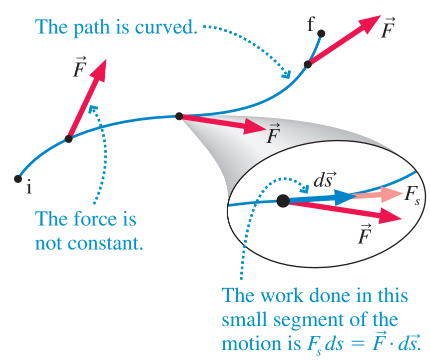


FIGURE 28.2 The work done along a curved path or by a variable force.

The second integral recognizes that $F_s\,ds = F\cos\theta\,ds$ is equivalent to the dot product $\vec{F} \cdot d\vec{s}$, allowing us to write the work in vector notation. As with Gauss's law, this integral looks more formidable than it really is. We'll look at examples shortly.

Finally, recall that a *conservative force* is one for which the work done as a particle moves from position i to position f is *independent of the path followed.* In other words, the integral in Equation 28.4 gives the same value for *any* path between points i and f. We'll assert for now, and prove later, that **the electric force is a conservative force.**

Uniform Fields

Gravity, like electricity, is a long-range force. Much as we defined the electric field $\vec{E} = \vec{F}_{\text{on } q}/q$, we can also define a gravitational field—the agent that exerts gravitational forces on masses—as $\vec{F}_{\text{on } m}/m$. But $\vec{F}_{\text{on } m} = m\vec{g}$ near the earth's surface; thus the familiar $\vec{g} = (9.80\text{ N/kg, down})$ is really the gravitational field! Notice how we've written the units of $\vec{g}$ as N/kg, as is appropriate for a field, but you can easily show that $\text{N/kg} = \text{m/s}^2$. The gravitational field near the earth's surface is a *uniform* field in the downward direction.

FIGURE 28.3 Potential energy is transformed into kinetic energy as a particle moves in a gravitational field.

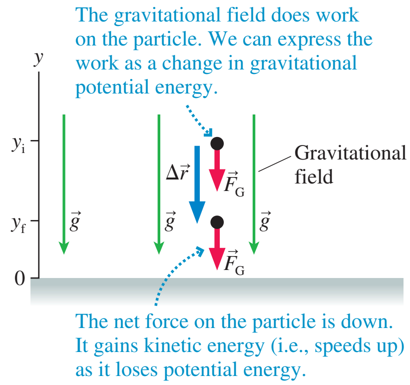


FIGURE 28.3 shows a particle of mass m falling in the gravitational field. The gravitational force is in the same direction as the particle's displacement, so the gravitational field does a *positive* amount of work on the particle. The gravitational force is constant, hence the work done by gravity is

$$W_{\text{grav}} = F_G\,\Delta r\cos 0° = mg|y_f - y_i| = mgy_i - mgy_f \tag{28.5}$$

We have to be careful with signs because Δr, the magnitude of the displacement vector, must be a positive number.

Now we can see how the definition of ΔU in Equation 28.2 makes sense. The *change* in gravitational potential energy is

$$\Delta U_{\text{grav}} = U_f - U_i = -W_{\text{grav}}(i \rightarrow f) = mgy_f - mgy_i \tag{28.6}$$

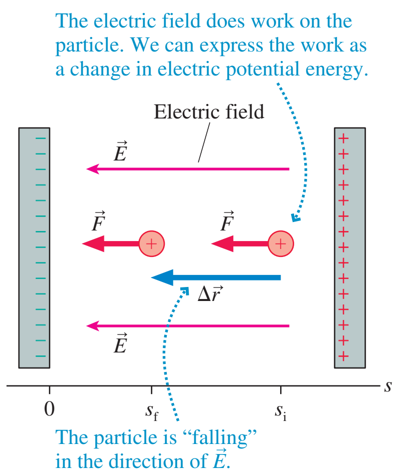
Comparing the initial and final terms on the two sides of the equation, we see that the gravitational potential energy near the earth is the familiar quantity

$$U_{\text{grav}} = U_0 + mgy \tag{28.7}$$

where U_0 is the value of U_{grav} at $y = 0$. We usually choose $U_0 = 0$, in which case $U_{\text{grav}} = mgy$, but such a choice is not necessary. The zero point of potential energy is an arbitrary choice because we have defined ΔU rather than U.

The uniform electric field between the plates of the parallel-plate capacitor of FIGURE 28.4 looks very much like the uniform gravitational field near the earth's surface. The one difference is that $\vec{g}$ always points down whereas the positive-to-negative electric field can point in any direction. To deal with this, let's define a coordinate axis s that points *from* the negative plate, which we define to be $s = 0$, *toward* the positive plate. The electric field $\vec{E}$ then points in the negative s-direction, just as the gravitational field $\vec{g}$ points in the negative y-direction. This s-axis, which is valid no matter how the capacitor is oriented, is analogous to the y-axis used for gravitational potential energy.

FIGURE 28.4 The electric field does work on the charged particle.



A positive charge q inside the capacitor speeds up and gains kinetic energy as it "falls" toward the negative plate. Is the charge losing potential energy as it gains kinetic energy? Indeed it is, and the calculation of the potential energy is just like the calculation of gravitational potential energy. The electric field exerts a *constant* force $F = qE$ on the charge in the direction of motion; thus the work done on the charge by the electric field is

$$W_{\text{elec}} = F\,\Delta r\cos 0° = qE|s_f - s_i| = qEs_i - qEs_f \tag{28.8}$$

where we again have to be careful with the signs because $s_f < s_i$.

The work done by the electric field causes the charge to experience a change in *electric* potential energy given by

$$\Delta U_{\text{elec}} = U_f - U_i = -W_{\text{elec}}(i \rightarrow f) = qEs_f - qEs_i \tag{28.9}$$

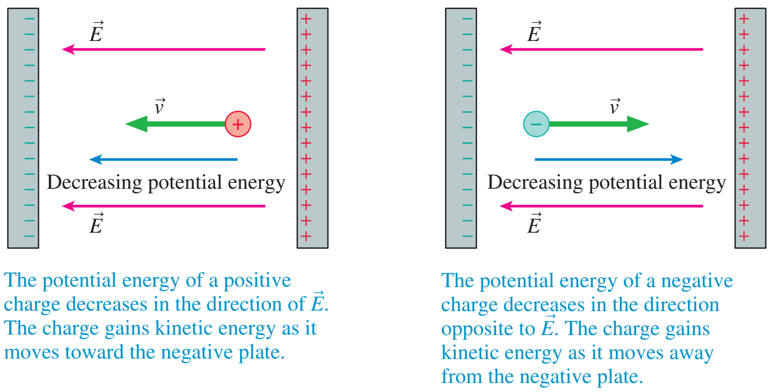
Comparing the initial and final terms on the two sides of the equation, we see that the **electric potential energy** of charge q in a uniform electric field is

$$U_{\text{elec}} = U_0 + qEs \tag{28.10}$$

where s is measured from the negative plate and U_0 is the potential energy at the negative plate ($s = 0$). It will often be convenient to choose $U_0 = 0$, but the choice has no physical consequences because it doesn't affect ΔU_{elec}, the *change* in the electric potential energy. Only the *change* is significant.

Equation 28.10 was derived with the assumption that q is positive, but it is valid for either sign of q. A negative value for q in Equation 28.10 causes the potential energy U_{elec} to become *more negative* as s increases. As **FIGURE 28.5** shows, a negative charge gains kinetic energy as it moves *away from* the negative plate of the capacitor.

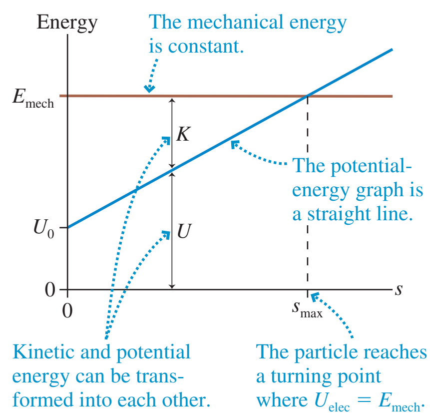
FIGURE 28.5 A charged particle of either sign gains kinetic energy as it moves in the direction of decreasing potential energy.



NOTE ▶ Although Equation 28.10 is often called "the potential energy of charge q," it is really the potential energy of the charge + capacitor system. To the extent that the charges on the capacitor plate stay fixed, we're justified in thinking of this as the potential energy of just the charge q. ◀

FIGURE 28.6 is the *energy diagram* for a positively charged particle in a uniform electric field. Recall that an energy diagram is a graphical representation of how the kinetic and potential energy are transformed as a particle moves. The potential energy, given by Equation 28.10, increases linearly with distance, but the particle's total mechanical energy E_{mech} is fixed. If a positively charged particle is projected against a uniform field, it gradually slows (transforming kinetic to potential energy) until reaching the *turning point* where $U_{elec} = E_{mech}$.

FIGURE 28.6 The energy diagram for a positively charged particle in a uniform electric field.



EXAMPLE 28.1 Conservation of energy

A 2.0 cm × 2.0 cm parallel-plate capacitor with a 2.0 mm spacing is charged to ±1.0 nC. First a proton, then an electron are released from rest at the midpoint of the capacitor.

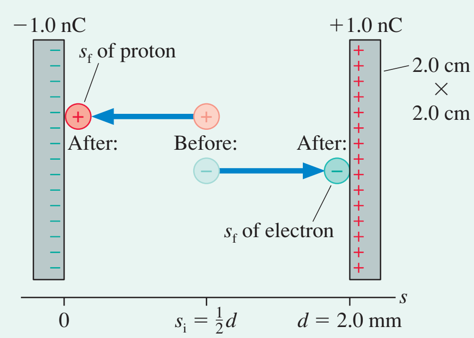
a. What is each particle's change in electric potential energy from its release until it collides with one of the plates?
b. What is each particle's speed as it reaches the plate?

MODEL The mechanical energy of each particle is conserved. A parallel-plate capacitor has a uniform electric field.

VISUALIZE **FIGURE 28.7** is a before-and-after pictorial representation, as you learned to draw in Part II. On the energy diagram of Figure 28.6, each particle is released at the turning point ($K = 0$) and moves toward lower potential energy. Thus the proton moves toward the negative plate, the electron toward the positive plate.

SOLVE a. The s-axis was defined to point from the negative toward the positive plate of the capacitor. Both charged particles have $s_i = \frac{1}{2}d$, where $d = 2.0$ mm is the plate separation. The positive proton loses potential energy and gains kinetic energy

FIGURE 28.7 A proton and an electron in a capacitor.



as it moves toward the negative plate. For the proton, with $q = +e$ and $s_f = 0$, the change in potential energy is

$$\Delta U_p = U_f - U_i = (U_0 + 0) - \left(U_0 + eE\frac{d}{2}\right) = -\frac{1}{2}eEd$$

Continued

where we used the electric potential energy for a charge in a uniform electric field. ΔU_p is negative, as expected. Notice that U_0 cancels when ΔU is calculated.

The electron moves toward the positive plate, which is the direction of decreasing potential energy for a negative charge. The electron has $q = -e$ and ends at $s_f = d$. Thus

$$\Delta U_e = U_f - U_i = (U_0 + (-e)Ed) - \left(U_0 + (-e)E\frac{d}{2}\right)$$
$$= -\frac{1}{2}eEd$$

Both particles have the *same* change in potential energy. The capacitor's electric field is

$$E = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = 2.82 \times 10^5 \text{ N/C}$$

Using $d = 0.0020$ m, we find

$$\Delta U_p = \Delta U_e = -4.5 \times 10^{-17} \text{ J}$$

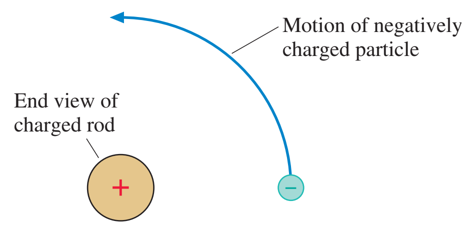
b. The law of conservation of energy is $\Delta K + \Delta U = 0$. Both particles are released from rest; hence $\Delta K = K_f - 0 = \frac{1}{2}mv_f^2$. Thus $\frac{1}{2}mv_f^2 = -\Delta U$, or

$$v_f = \sqrt{\frac{-2\,\Delta U}{m}} = \begin{cases} 2.3 \times 10^5 \text{ m/s for the proton} \\ 1.0 \times 10^7 \text{ m/s for the electron} \end{cases}$$

where we used the masses of the proton and the electron.

ASSESS Even though both particles have the same ΔU, the electron reaches a much faster final speed due to its much smaller mass.

STOP TO THINK 28.1 A glass rod is positively charged. The figure shows an end view of the rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod's electric field positive, negative, or zero?



28.2 The Potential Energy of Point Charges

Now that we've introduced the idea of electric potential energy, let's look at *the* fundamental interaction of electricity—the force between two point charges. This force, given by Coulomb's law, varies with the distance between the two charges; hence we need to use the integral expression of Equation 28.4 to calculate the work done.

FIGURE 28.8 The interaction between two point charges.

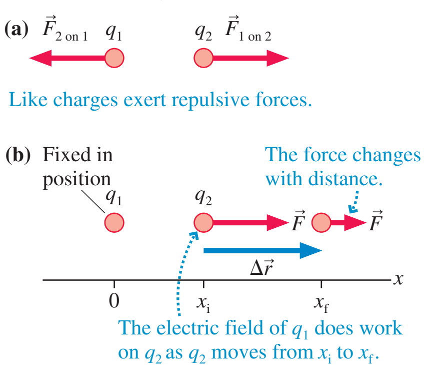


FIGURE 28.8a shows two charges q_1 and q_2, which we will assume to be like charges. The potential energy of their interaction can be found by calculating the work done by the electric field of q_1 on q_2 as q_2 moves from position x_i to position x_f. We'll assume that q_1 has been glued down and is unable to move, as shown in **FIGURE 28.8b**.

The force is entirely in the direction of motion, so $F_s\,ds = F_{1 \text{ on } 2}\,dx$. Thus

$$W_{elec} = \int_{x_i}^{x_f} F_{1 \text{ on } 2}\,dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\,dx = Kq_1q_2 \frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i} \tag{28.11}$$

The potential energy of the two charges is related to the work done by

$$\Delta U_{elec} = U_f - U_i = -W_{elec}(i \to f) = \frac{Kq_1q_2}{x_f} - \frac{Kq_1q_2}{x_i} \tag{28.12}$$

By comparing the left and right sides of the equation we see that the potential energy of the two-point-charge system is

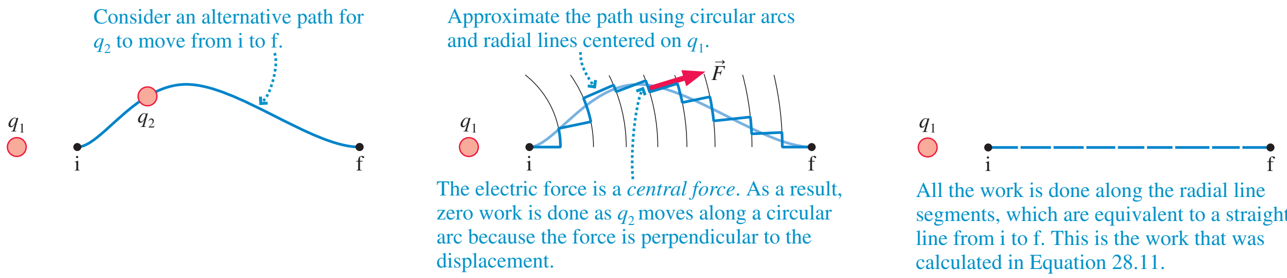
$$U_{elec} = \frac{Kq_1q_2}{x} \tag{28.13}$$

We could include a constant U_0, as we did in Equation 28.10, for the potential energy of a charge in a uniform electric field, but it is customary to set $U_0 = 0$.

The Electric Force Is a Conservative Force

Potential energy can be defined only if the force is *conservative,* meaning that the work done on the particle as it moves from position i to position f is independent of the path followed between i and f. FIGURE 28.10 demonstrates that electric force is indeed conservative.

FIGURE 28.10 The work done on q_2 is independent of the path from i to f.



EXAMPLE 28.2 Approaching a charged sphere

A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

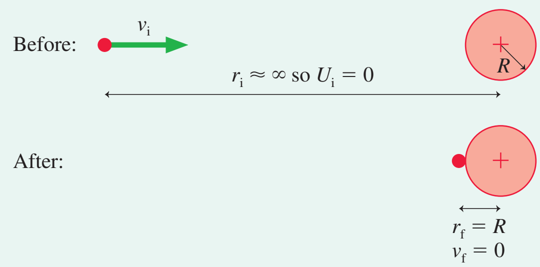
MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

VISUALIZE FIGURE 28.11 shows the before-and-after pictorial representation. To "just reach" the glass sphere means that the proton comes to rest, $v_f = 0$, as it reaches $r_f = 0.50$ mm, the *radius* of the sphere.

SOLVE Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + \frac{Kq_p q_{sphere}}{r_f} = \frac{1}{2}mv_i^2 + 0$$

FIGURE 28.11 A proton approaching a glass sphere.



The proton charge is $q_p = e$. With this, we can solve for the proton's initial speed:

$$v_i = \sqrt{\frac{2Keq_{sphere}}{mr_f}} = 1.86 \times 10^7 \text{ m/s}$$

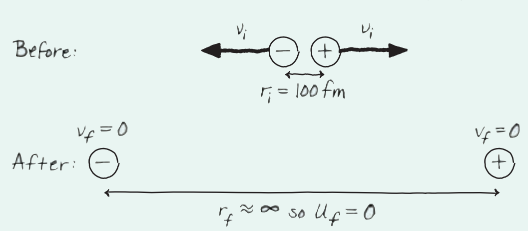
EXAMPLE 28.3 Escape velocity

An interaction between two elementary particles causes an electron and a positron (a positive electron) to be shot out back to back with equal speeds. What minimum speed must each have when they are 100 fm apart in order to escape each other?

MODEL Energy is conserved. The particles end "far apart," which we interpret as sufficiently far to make $U_f \approx 0$.

VISUALIZE FIGURE 28.12 shows the before-and-after pictorial representation. The minimum speed to escape is the speed that allows the particles to reach $r_f = \infty$ with $v_f = 0$.

FIGURE 28.12 An electron and a positron flying apart.



SOLVE Here it is essential to interpret U_{elec} as the potential energy of the electron + positron system. Similarly, K is the *total* kinetic energy of the system. The electron and the positron, with equal masses and equal speeds, have equal kinetic energies. Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + 0 + 0 = \frac{1}{2}mv_i^2 + \frac{1}{2}mv_i^2 + \frac{Kq_e q_p}{r_i} = mv_i^2 - \frac{Ke^2}{r_i}$$

Using $r_i = 100 \text{ fm} = 1.0 \times 10^{-13}$ m, we can calculate the minimum initial speed to be

$$v_i = \sqrt{\frac{Ke^2}{mr_i}} = 5.0 \times 10^7 \text{ m/s}$$

ASSESS v_i is a little more than 10% the speed of light, just about the limit of what a "classical" calculation can predict. We would need to use the theory of relativity if v_i were any larger.

Multiple Point Charges

If more than two charges are present, the potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{elec} = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}} \tag{28.15}$$

where r_{ij} is the distance between q_i and q_j. The summation contains the $i < j$ restriction to ensure that each pair of charges is counted only once.

NOTE ▶ For energy conservation problems, it's necessary to calculate only the potential energy for those pairs of charges for which the distance r_{ij} changes. The potential energy of charges that don't move is an additive constant with no physical consequences. ◀

EXAMPLE 28.4 Launching an electron

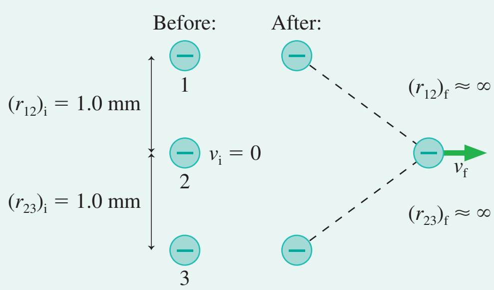
Three electrons are spaced 1.0 mm apart along a vertical line. The outer two electrons are fixed in position.

a. Is the center electron at a point of stable or unstable equilibrium?
b. If the center electron is displaced horizontally by a small distance, what will its speed be when it is very far away?

MODEL Energy is conserved. The outer two electrons don't move, so we don't need to include the potential energy of their interaction.

VISUALIZE **FIGURE 28.13** shows the situation.

FIGURE 28.13 Three electrons.



SOLVE a. The center electron is in equilibrium *exactly* in the center because the two electric forces on it balance. But if it moves a little to the right or left, no matter how little, then the horizontal components of the forces from both outer electrons will push the center electron farther away. This is an unstable equilibrium for horizontal displacements, like being on the top of a hill.

b. A small displacement will cause the electron to move away. If the displacement is only infinitesimal, the initial conditions are $(r_{12})_i = (r_{23})_i = 1.0$ mm and $v_i = 0$. "Far away" is interpreted as $r_f \rightarrow \infty$, where $U_f \approx 0$. There are now *two* terms in the potential energy, so conservation of energy $K_f + U_f = K_i + U_i$ gives

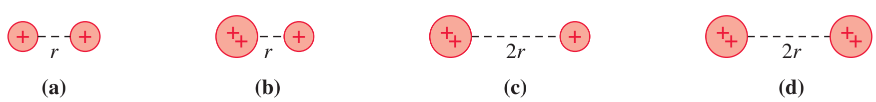
$$\frac{1}{2}mv_f^2 + 0 + 0 = 0 + \left[\frac{Kq_1q_2}{(r_{12})_i} + \frac{Kq_2q_3}{(r_{23})_i}\right]$$

$$= \left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]$$

This is easily solved to give

$$v_f = \sqrt{\frac{2}{m}\left[\frac{Ke^2}{(r_{12})_i} + \frac{Ke^2}{(r_{23})_i}\right]} = 1000 \text{ m/s}$$

STOP TO THINK 28.2 Rank in order, from largest to smallest, the potential energies U_a to U_d of these four pairs of charges. Each + symbol represents the same amount of charge.

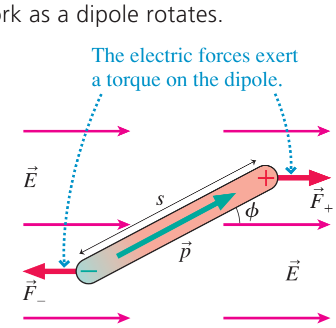


28.3 The Potential Energy of a Dipole

The electric dipole has been our model for understanding how charged objects interact with neutral objects. In Chapter 26 we found that an electric field exerts a *torque* on a dipole. We can complete the picture by calculating the potential energy of an electric dipole in a uniform electric field.

FIGURE 28.14 shows a dipole in an electric field $\vec{E}$. Recall that the dipole moment $\vec{p}$ is a vector that points from $-q$ to q with magnitude $p = qs$. The forces $\vec{F}_+$ and $\vec{F}_-$ exert a torque on the dipole, but now we're interested in calculating the *work* done by these forces as the dipole rotates from angle ϕ_i to angle ϕ_f.

FIGURE 28.14 The electric field does work as a dipole rotates.



When a force component F_s acts through a small displacement ds, the force does work $dW = F_s\,ds$. If we exploit the rotational-linear motion analogy from Chapter 12, where torque τ is the analog of force and angular displacement $\Delta\phi$ is the analog of linear displacement, then a torque acting through a small angular displacement $d\phi$ does work $dW = \tau\,d\phi$. From Chapter 26, the torque on the dipole in Figure 28.14 is $\tau = -pE\sin\phi$, where the minus sign is due to the torque trying to cause a clockwise rotation. Thus the work done by the electric field on the dipole as it rotates through the small angle $d\phi$ is

$$dW_{\text{elec}} = -pE\sin\phi\,d\phi \tag{28.16}$$

The total work done by the electric field as the dipole turns from ϕ_i to ϕ_f is

$$W_{\text{elec}} = -pE\int_{\phi_i}^{\phi_f}\sin\phi\,d\phi = pE\cos\phi_f - pE\cos\phi_i \tag{28.17}$$

The potential energy associated with the work done on the dipole is

$$\Delta U_{\text{dipole}} = U_f - U_i = -W_{\text{elec}}(i \to f) = -pE\cos\phi_f + pE\cos\phi_i \tag{28.18}$$

By comparing the left and right sides of Equation 28.18, we see that the potential energy of an electric dipole $\vec{p}$ in a uniform electric field $\vec{E}$ is

$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p}\cdot\vec{E} \tag{28.19}$$

FIGURE 28.15 The energy of a dipole in an electric field.

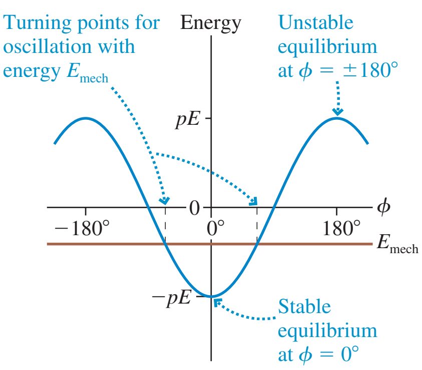


FIGURE 28.15 shows the energy diagram of a dipole. The potential energy is minimum at $\phi = 0°$ where the dipole is aligned with the electric field. This is a point of stable equilibrium. A dipole exactly opposite $\vec{E}$, at $\phi = \pm180°$, is at a point of unstable equilibrium. Any disturbance will cause it to flip around. A frictionless dipole with mechanical energy E_{mech} will oscillate back and forth between turning points on either side of $\phi = 0°$.

EXAMPLE 28.5 Rotating a molecule

The water molecule is a permanent electric dipole with dipole moment 6.2×10^{-30} C m. A water molecule is aligned in an electric field with field strength 1.0×10^7 N/C. How much energy is needed to rotate the molecule 90°?

MODEL The molecule is at the point of minimum energy. It won't spontaneously rotate 90°. However, an external force that supplies energy, such as a collision with another molecule, can cause the water molecule to rotate.

SOLVE The molecule starts at $\phi_i = 0°$ and ends at $\phi_f = 90°$. The increase in potential energy is

$$\Delta U_{\text{dipole}} = U_f - U_i = -pE\cos 90° - (-pE\cos 0°)$$
$$= pE = 6.2 \times 10^{-23}\ \text{J}$$

This is the energy needed to rotate the molecule 90°.

ASSESS ΔU_{dipole} is significantly less than $k_B T$ at room temperature. Thus collisions with other molecules can easily supply the energy to rotate the water molecules and keep them from staying aligned with the electric field.

This battery is labeled 1.5 Volts. As we'll soon see, a battery is a source of electric potential.

28.4 The Electric Potential

We introduced the concept of the *electric field* in Chapter 25 because action at a distance raised concerns and difficulties. The field provides an intermediary through which two charges exert forces on each other. Charge q_1 somehow alters the space around it by creating an electric field $\vec{E}_1$. Charge q_2 then responds to the field, experiencing force $\vec{F} = q_2\vec{E}_1$.

We face the same kinds of difficulties when we try to understand electric potential energy. For a mass on a spring, we can *see* how the energy is stored in the stretched or compressed spring. But when we say two charged particles have a potential energy, an energy that can be converted to a tangible kinetic energy of motion, *where is the energy?* It's indisputable that two positive charges fly apart when you release them, gaining kinetic energy, but there's no obvious place that the energy had been stored.

In defining the electric field, we chose to separate the charges that are the *source* of the field from the charge *in* the field. The force on charge q is related to the electric field of the source charges by

force on q by sources = [charge q] $\times$ [alteration of space by the source charges]

Let's try a similar procedure for the potential energy. The electric potential energy is due to the interaction of charge q with other charges, so let's write

potential energy of q + sources

= [charge q] $\times$ [*potential* for interaction of the source charges]

FIGURE 28.16 Source charges alter the space around them by creating an electric potential.

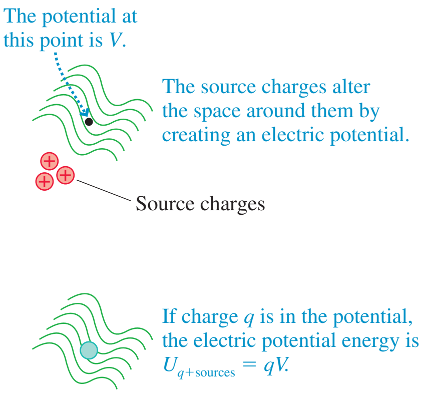


FIGURE 28.16 shows this idea schematically.

In analogy with the electric field, we will define the **electric potential** V (or, for brevity, just *the potential*) as

$$V \equiv \frac{U_{q+\text{sources}}}{q} \tag{28.20}$$

Charge q is used as a probe to determine the electric potential, but the value of V is *independent of q*. **The electric potential, like the electric field, is a property of the source charges.**

In practice, we're usually more interested in knowing the potential energy if a charge q happens to be at a point in space where the electric potential of the source charges is V. Turning Equation 28.20 around, we see that the electric potential energy is

$$U_{q+\text{sources}} = qV \tag{28.21}$$

Once the potential has been determined, it's very easy to find the potential energy.

The unit of electric potential is the joule per coulomb, which is called the **volt** V:

$$1 \text{ volt} = 1 \text{ V} \equiv 1 \text{ J/C}$$

This unit is named for Alessandro Volta, who invented the electric battery in the year 1800. Microvolts (μV), millivolts (mV), and kilovolts (kV) are commonly used units.

NOTE ▶ Once again, commonly used symbols are in conflict. The symbol V is widely used to represent *volume,* and now we're introducing the same symbol to represent *potential.* To make matters more confusing, V is the abbreviation for *volts.* In printed text, V for potential is italicized and V for volts is not, but you can't make such a distinction in handwritten work. This is not a pleasant state of affairs, but these are the commonly accepted symbols. It's incumbent upon you to be especially alert to the *context* in which a symbol is used. ◀

Using the Electric Potential

The electric potential is an abstract idea, and it will take some practice to see just what it means and how it is useful. We'll use multiple representations—words, pictures, graphs, and analogies—to explain and describe the electric potential.

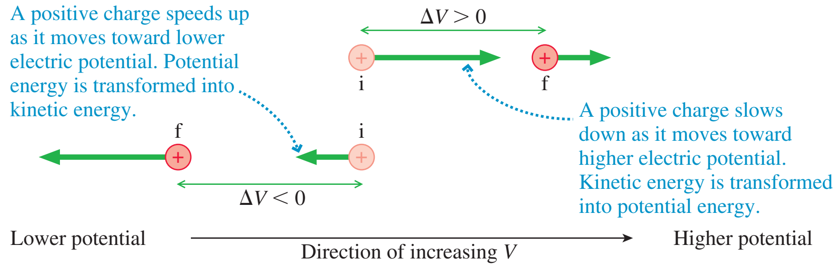
NOTE ▶ It is unfortunate that the terms *potential* and *potential energy* are so much alike. Despite the similar names, they are very different concepts and are not interchangeable. Table 28.1 will help you to distinguish between the two. ◀

TABLE 28.1 Distinguishing electric potential and potential energy

The *electric potential* is a property of the source charges and, as you'll soon see, is related to the electric field. The electric potential is present whether or not a charged particle is there to experience it. Potential is measured in J/C, or V.
The *electric potential energy* is the interaction energy of a charged particle with the source charges. Potential energy is measured in J.

Basically, knowing the electric potential in a region of space allows us to determine whether a charged particle speeds up or slows down as it moves through that region. FIGURE 28.17 on the next page illustrates this idea. Here a group of source charges, which remains hidden offstage, has created an electric potential V that increases from left to right. A charged particle q, which for now we'll assume to be positive, has electric

FIGURE 28.17 A charged particle speeds up or slows down as it moves through a potential difference.



potential energy $U = qV$. If the particle moves to the right, its potential energy increases and so, by energy conservation, its kinetic energy must decrease. **A positive charge slows down as it moves into a region of higher electric potential.**

It is customary to say that the particle moves through a **potential difference** $\Delta V = V_f - V_i$. The potential difference between two points is often called the **voltage.** The particle moving to the right moves through a positive potential difference ($\Delta V > 0$ because $V_f > V_i$), so we can say that a positively charged particle slows down as it moves through a positive potential difference.

The particle moving to the left in Figure 28.17 travels in the direction of decreasing electric potential—through a negative potential difference—and is losing potential energy. It speeds up as it transforms potential energy into kinetic energy. A negatively charged particle would slow down because its potential energy qV would increase as V decreases. Table 28.2 summarizes these ideas.

TABLE 28.2 Charged particles moving in an electric potential

	Electric potential	
	Increasing ($\Delta V > 0$)	Decreasing ($\Delta V < 0$)
+ charge	Slows down	Speeds up
− charge	Speeds up	Slows down

If a particle moves through a potential difference ΔV, its electric potential energy changes by $\Delta U = q\,\Delta V$. We can write the conservation of energy equation in terms of the electric potential as $\Delta K + \Delta U = \Delta K + q\,\Delta V = 0$ or, as is often more practical,

$$K_f + qV_f = K_i + qV_i \qquad (28.22)$$

Conservation of energy is the basis of a powerful problem-solving strategy.

PROBLEM-SOLVING STRATEGY 28.1 **Conservation of energy in charge interactions**

MP

MODEL Check whether there are any dissipative forces that would keep the mechanical energy from being conserved.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Is the electric potential given in the problem statement? If not, you'll need to use a known potential, such as that of a point charge, or calculate the potential using the procedure given later, in Problem-Solving Strategy 28.2.
- K_i and K_f are the sums of the kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 22

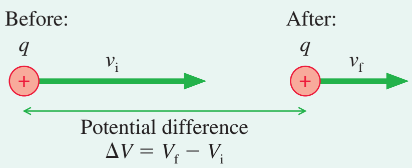
EXAMPLE 28.6 Moving through a potential difference

A proton with a speed of 2.0×10^5 m/s enters a region of space in which source charges have created an electric potential. What is the proton's speed after it moves through a potential difference of 100 V? What will be the final speed if the proton is replaced by an electron?

MODEL Energy is conserved. The electric potential determines the potential energy.

VISUALIZE FIGURE 28.18 is a before-and-after pictorial representation of a charged particle moving through a potential difference. A positive charge *slows down* as it moves into a region of higher potential ($K \rightarrow U$). A negative charge *speeds up* ($U \rightarrow K$).

FIGURE 28.18 A charged particle moving through a potential difference.



SOLVE The potential energy of charge q is $U = qV$. Conservation of energy, now expressed in terms of the electric potential V, is $K_f + qV_f = K_i + qV_i$, or

$$K_f = K_i - q\,\Delta V$$

where $\Delta V = V_f - V_i$ is the potential difference through which the particle moves. In terms of the speeds, energy conservation is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - q\,\Delta V$$

We can solve this for the final speed:

$$v_f = \sqrt{v_i^2 - \frac{2q}{m}\Delta V}$$

For a proton, with $q = e$, the final speed is

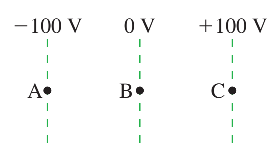
$$(v_f)_p = \sqrt{(2.0 \times 10^5\text{ m/s})^2 - \frac{2(1.60 \times 10^{-19}\text{ C})(100\text{ V})}{1.67 \times 10^{-27}\text{ kg}}}$$
$$= 1.4 \times 10^5\text{ m/s}$$

An electron, though, with $q = -e$ and a different mass, speeds up to $(v_f)_e = 5.9 \times 10^6$ m/s.

ASSESS The electric potential *already existed* in space due to other charges that are not explicitly seen in the problem. The electron and proton have nothing to do with creating the potential. Instead, they *respond* to the potential by having potential energy $U = qV$.

STOP TO THINK 28.3 A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton

a. Remains at rest at B.
b. Moves toward A with a steady speed.
c. Moves toward A with an increasing speed.
d. Moves toward C with a steady speed.
e. Moves toward C with an increasing speed.



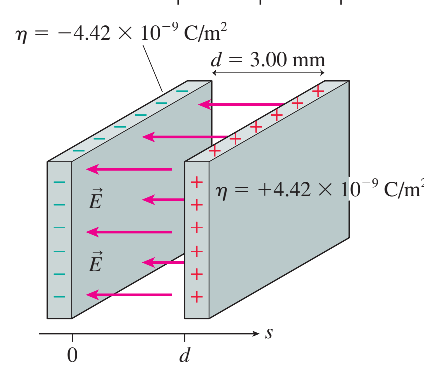
28.5 The Electric Potential Inside a Parallel-Plate Capacitor

We began this chapter with the potential energy of a charge inside a parallel-plate capacitor. Now let's investigate the electric potential. FIGURE 28.19 shows two parallel electrodes, separated by distance d, with surface charge density $\pm\eta$. As a specific example, we'll let $d = 3.00$ mm and $\eta = 4.42 \times 10^{-9}$ C/m^2. The electric field inside the capacitor, as you learned in Chapter 26, is

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive toward negative}\right)$$
$$= (500\text{ N/C, from right to left}) \qquad (28.23)$$

This electric field is due to the *source charges* on the capacitor plates.

FIGURE 28.19 A parallel-plate capacitor.



In Section 28.1, we found that the electric potential energy of a charge q in the uniform electric field of a parallel-plate capacitor is

$$U_{\text{elec}} = U_{q+\text{sources}} = qEs \tag{28.24}$$

We've set the constant term U_0 to zero. U_{elec} is the energy of q interacting with the source charges on the capacitor plates.

Our new view of the interaction is to separate the role of charge q from the role of the source charges by defining the electric potential $V = U_{q+\text{sources}}/q$. Thus the electric potential inside a parallel-plate capacitor is

$$V = Es \qquad \text{(electric potential inside a parallel-plate capacitor)} \tag{28.25}$$

where **s is the distance from the *negative* electrode.** The electric potential, like the electric field, exists at *all points* inside the capacitor. The electric potential is created by the source charges on the capacitor plates and exists whether or not charge q is inside the capacitor.

FIGURE 28.20 The electric potential of a parallel-plate capacitor increases linearly from the negative to the positive plate.

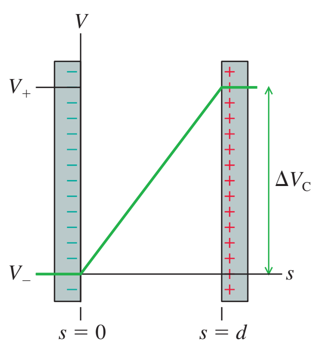


FIGURE 28.20 illustrates the important point that the electric potential increases linearly from the negative plate, where $V_- = 0$, to the positive plate, where $V_+ = Ed$. Let's define the *potential difference* ΔV_C between the two capacitor plates to be

$$\Delta V_C = V_+ - V_- = Ed \tag{28.26}$$

In our specific example, $\Delta V_C = (500\ \text{N/C})(0.0030\ \text{m}) = 1.5\ \text{V}$. The units work out because $1.5\ (\text{N m})/\text{C} = 1.5\ \text{J/C} = 1.5\ \text{V}$.

NOTE ▶ People who work with circuits would call ΔV_C "the voltage across the capacitor" or simply "the capacitor voltage." ◀

Equation 28.26 has an interesting implication. Thus far, we've determined the electric field inside a capacitor by specifying the surface charge density η on the plates. Alternatively, we could specify the capacitor voltage ΔV_C (i.e., the potential difference between the capacitor plates) and then determine the electric field strength as

$$E = \frac{\Delta V_C}{d} \tag{28.27}$$

In fact, this is how E is determined in practical applications because it's easy to measure ΔV_C with a voltmeter but difficult, in practice, to know the value of η.

Equation 28.27 implies that the units of electric field are volts per meter, or V/m. We have been using electric field units of newtons per coulomb. In fact, as you can show as a homework problem, these units are equivalent to each other. That is,

$$1\ \text{N/C} = 1\ \text{V/m}$$

NOTE ▶ Volts per meter are the electric field units used by scientists and engineers in practice. We will now adopt them as our standard electric field units. ◀

Returning to the electric potential, we can substitute Equation 28.27 for E into Equation 28.25 for V. Thus the electric potential inside the capacitor is

$$V = Es = \frac{s}{d}\Delta V_C \tag{28.28}$$

The potential increases linearly from $V_- = 0\ \text{V}$ at the negative plate ($s = 0$) to $V_+ = \Delta V_C$ at the positive plate ($s = d$).

Let's explore the electric potential inside the capacitor by looking at several different, but related, ways that the potential can be represented graphically.

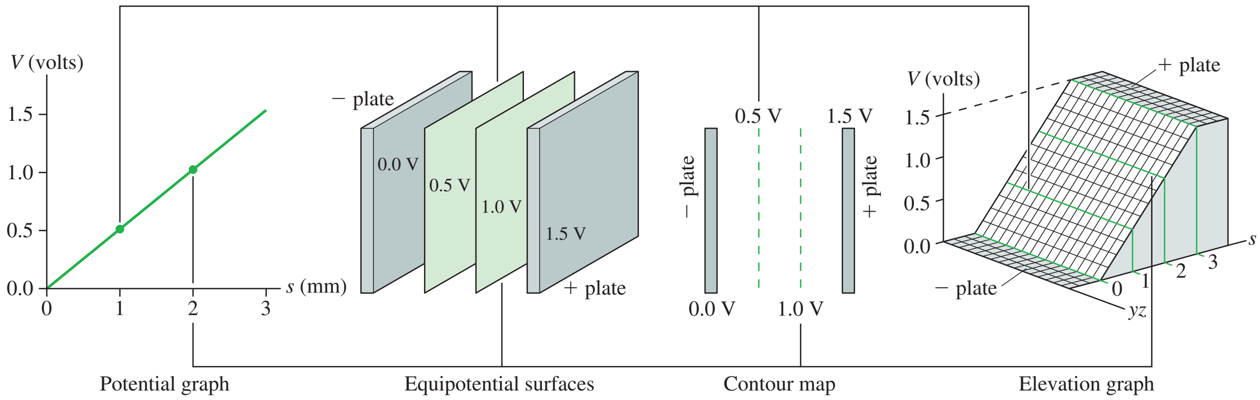
Graphical representations of the electric potential inside a capacitor

A graph of potential versus *s*. You can see the potential increasing from 0.0 V at the negative plate to 1.5 V at the positive plate.

A three-dimensional view showing **equipotential surfaces.** These are mathematical surfaces, not physical surfaces, with the same value of *V* at every point. The equipotential surfaces of a capacitor are planes parallel to the capacitor plates. The capacitor plates are also equipotential surfaces.

A two-dimensional **contour map.** The capacitor plates and the equipotential surfaces are seen edge-on, so you need to imagine them extending above and below the plane of the page.

A three-dimensional **elevation graph.** The potential is graphed vertically versus the *s*-coordinate on one axis and a generalized "*yz*-coordinate" on the other axis. Viewing the right face of the elevation graph gives you the potential graph.



These four graphical representations show the same information from different perspectives, and the connecting lines help you see how they are related. If you think of the elevation graph as a "mountain," then the contour lines on the contour map are like the lines of a topographic map.

The potential graph and the contour map are the two representations most widely used in practice because they are easy to draw. Their limitation is that they are trying to convey three-dimensional information in a two-dimensional presentation. When you see graphs or contour maps, you need to imagine the three-dimensional equipotential surfaces or the three-dimensional elevation graph.

There's nothing special about showing equipotential surfaces or contour lines every 0.5 V. We chose these intervals because they were convenient. As an alternative, **FIGURE 28.21** shows how the contour map looks if the contour lines are spaced every 0.3 V. Contour lines and equipotential surfaces are *imaginary* lines and surfaces drawn to help us visualize how the potential changes in space. Drawing the map more than one way reinforces the idea that there is an electric potential at *every* point inside the capacitor, not just at the points where we happened to draw a contour line or an equipotential surface.

Figure 28.21 also shows the electric field vectors. Notice that

- The electric field vectors are perpendicular to the equipotential surfaces.
- The electric field points in the direction of decreasing potential. In other words, the electric field points "downhill" on a graph or map of the electric potential.

Chapter 29 will present a more in-depth exploration of the connection between the electric field and the electric potential. There you will find that these observations are always true. They are not unique to the parallel-plate capacitor.

Finally, you might wonder how we can arrange a capacitor to have a surface charge density of precisely 4.42×10^{-9} C/m^2. Simple! As **FIGURE 28.22** shows, we use wires to attach the capacitor plates to a 1.5 V battery. This is another topic that we'll explore in Chapter 29, but it's worth noting now that **a battery is a source of potential.** That's why batteries are labeled in volts, and it's a major reason we need to thoroughly understand the concept of potential.

FIGURE 28.21 The contour lines of the electric potential and the electric field vectors inside a parallel-plate capacitor.

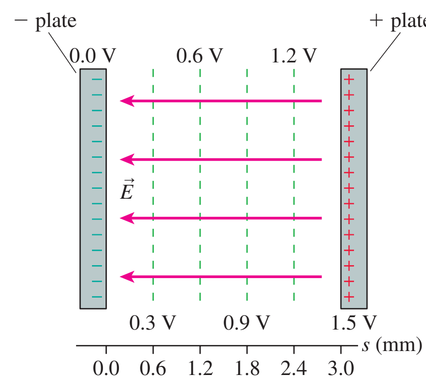
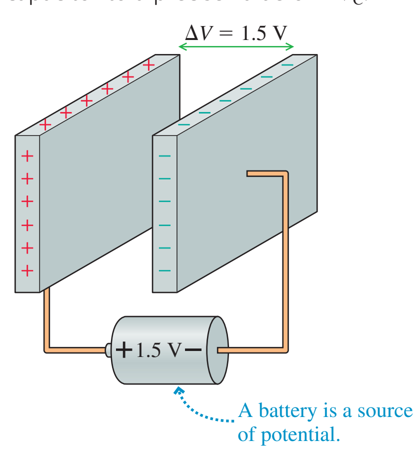


FIGURE 28.22 Using a battery to charge a capacitor to a precise value of ΔV_C.



EXAMPLE 28.7 Measuring the speed of a proton

The lab in which you work has a small proton accelerator. You've been assigned the task of measuring the speed of the protons as they emerge from the accelerator. To do so, you decide to measure how much voltage is needed across a parallel-plate capacitor to stop the protons. The capacitor you choose has a 2.0 mm plate separation and a small hole in one plate that you shoot the protons through. By filling the space between the plates with a low-density gas, you can see (with a microscope) a slight glow from the region where the protons collide with and excite the gas molecules. The width of the glow tells you how far the protons travel before being stopped and reversing direction. Varying the voltage across the capacitor gives the following data:

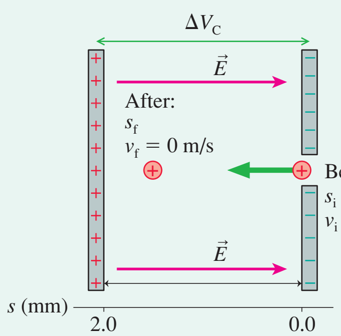
Capacitor voltage (V)	Glow width (mm)
1000	1.7
1250	1.3
1500	1.1
1750	1.0
2000	0.8

What value will you report for the speed of the protons?

MODEL Energy is conserved. The proton's potential energy can be found from the capacitor's electric potential.

VISUALIZE FIGURE 28.23 shows a before-and-after pictorial representation of the proton entering the capacitor with speed v_i, which we want to find, and later reaching a turning point with $v_f = 0$ m/s after traveling distance s_f = glow width. For the protons to slow and stop, the hole through which they pass has to be in the negative plate. We've established an s-axis with $s = 0$ at this point.

FIGURE 28.23 A proton being stopped in a capacitor.



SOLVE The conservation of energy equation, with the proton having charge $q = e$, is $K_f + eV_f = K_i + eV_i$. The initial potential energy is zero, because the capacitor's electric potential is zero at $s_i = 0$, and the final kinetic energy is zero. Using Equation 28.28 for the potential inside the capacitor, we have

$$eV_f = e\left(\frac{s_f}{d}\Delta V_C\right) = K_i = \frac{1}{2}mv_i^2$$

Solving for the distance traveled, we find

$$s_f = \frac{dmv_i^2}{2e}\frac{1}{\Delta V_C}$$

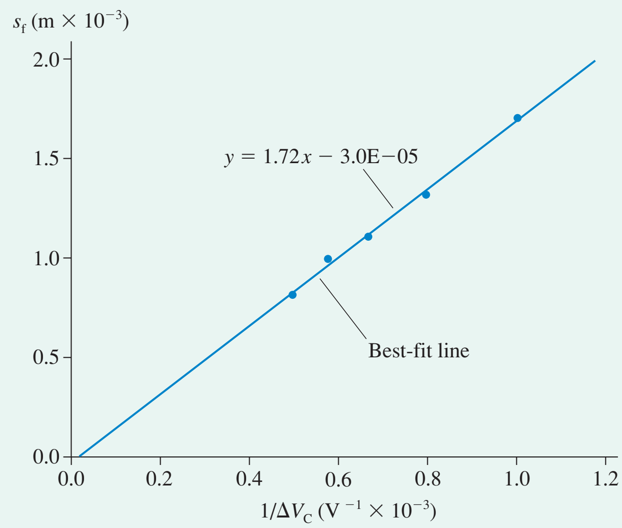
Thus a graph of the distance traveled versus the *inverse* of the capacitor voltage should be a straight line with zero y-intercept and slope $dmv_i^2/2e$. We can use the experimentally determined slope to find the proton speed.

FIGURE 28.24 is a graph of s_f versus $1/\Delta V_C$. It has the expected shape, and the slope of the best-fit line is seen to be 1.72 V m. The units are those of the rise-over-run. Using the slope, we calculate the proton speed:

$$v_i = \sqrt{\frac{2e}{dm}\times \text{slope}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(1.72\text{ V m})}{(0.0020\text{ m})(1.67\times10^{-27}\text{ kg})}}$$

$$= 4.1\times10^5\text{ m/s}$$

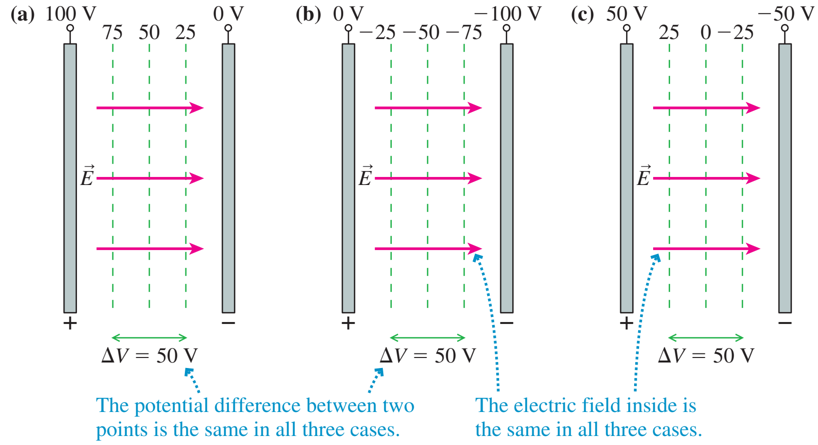
FIGURE 28.24 A graph of the data.



ASSESS This would be a very high speed for a macroscopic object but is quite typical of the speeds of charged particles.

In writing the electric potential inside a parallel-plate capacitor, we made the choice that $V_- = 0$ V at the negative plate. But that is not the only possible choice. FIGURE 28.25 shows three parallel-plate capacitors, each having the same capacitor voltage $\Delta V_C = V_+ - V_- = 100$ V, but each with a different choice for the location of the zero point of the electric potential. Notice the *terminal symbols* (lines with small circles at the end) showing how the potential, from a battery or a power supply, is applied to each plate; these symbols are common in electronics.

FIGURE 28.25 These three choices for $V = 0$ represent the same physical situation. These are contour maps, showing the edges of the equipotential surfaces.



The important thing to notice is that the three contour maps in Figure 28.25 represent the *same physical situation.* The potential difference between any two points is the same in all three maps. The electric field is the same in all three. We may *prefer* one of these figures over the others, but there is no measurable physical difference between them.

EXAMPLE 28.8 The force on an ion

Example 26.7 noted that a cell wall can be modeled as a parallel-plate capacitor, with the outer surface of the cell wall being positive while the inner surface is negative. The potential difference between the inside of the cell and the outside is called the *membrane potential*. Suppose a molecular ion with charge $5e$ is embedded within the 5.0-nm-thick wall of a cell with a membrane potential of -70 mV, typical for a nerve cell in its resting state. What is the force on the molecular ion?

MODEL Model the cell wall as a parallel-plate capacitor with the inner surface being the negative plate. Although the walls are actually curved, and not large flat planes, the parallel-plate approximation is valid if the wall thickness is much less than the radius of the cell. The capacitor voltage is $\Delta V_C = 70$ mV $= 0.070$ V. The membrane potential is negative because the potential inside the cell is less than the potential outside, but ΔV_C, the capacitor voltage, is the *magnitude* of the potential difference and thus always positive.

SOLVE The force on a charged particle is $\vec{F} = q\vec{E}$. The electric field strength inside the parallel-plate capacitor of the cell wall is

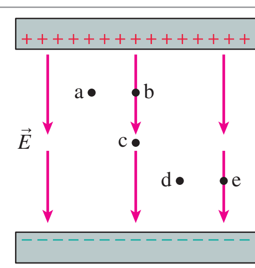
$$E = \frac{\Delta V_C}{d} = \frac{0.070 \text{ V}}{5.0 \times 10^{-9} \text{ m}} = 1.4 \times 10^7 \text{ V/m}$$

Notice that we're now using V/m rather than N/C as the units of electric field. Because the field points from positive to negative, the field vector is $\vec{E} = (1.4 \times 10^7$ V/m, toward inside). Thus the force on an ion with $q = 5e = 8.0 \times 10^{-19}$ C is

$$\vec{F} = q\vec{E} = (1.1 \times 10^{-11} \text{ N, toward inside})$$

ASSESS For cells to function, a steady flow of molecules must pass back and forth through the cell wall. Although the details of how this happens are very complex, a key idea is that a potential difference between the inside and outside of the cell creates an electric field that pushes positive ions toward the inside, negative ions toward the outside.

STOP TO THINK 28.4 Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e.



28.6 The Electric Potential of a Point Charge

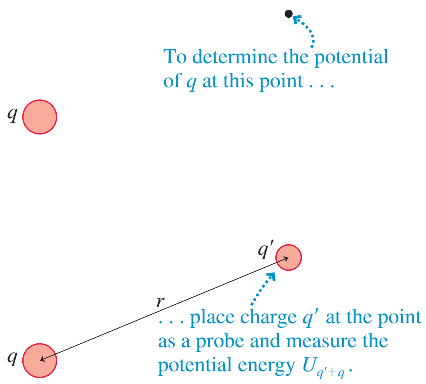


FIGURE 28.26 Measuring the electric potential of charge q.

Another important electric potential is that of a point charge. Let q in **FIGURE 28.26** be the source charge, and let a second charge q' probe the electric potential of q. The potential energy of the two point charges is

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r} \tag{28.29}$$

Thus, by definition, the electric potential of charge q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(electric potential of a point charge)} \tag{28.30}$$

The potential of Equation 28.30 extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$. This expression for V assumes that we have chosen $V = 0$ V to be at $r = \infty$. This is the most logical choice for a point charge because the influence of charge q ends at infinity.

The expression for the electric potential of charge q is similar to that for the electric field of charge q. The difference most quickly seen is that V depends on $1/r$ whereas $\vec{E}$ depends on $1/r^2$. But it is also important to notice that **the potential is a scalar** whereas the field is a vector. Thus the mathematics of using the potential are much easier than the vector mathematics using the electric field requires.

EXAMPLE 28.9 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a +1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

SOLVE The potential at $r = 1.0$ cm is

$$V_{1\,\text{cm}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = (9.0\times10^{9}\ \text{N m}^2/\text{C}^2)\frac{1.0\times10^{-9}\ \text{C}}{0.010\ \text{m}}$$
$$= 900\ \text{V}$$

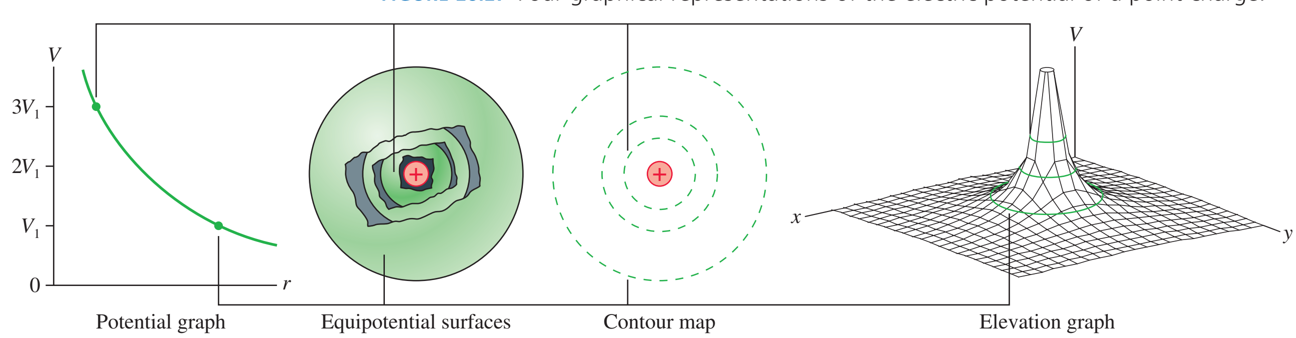
We can similarly calculate $V_{3\,\text{cm}} = 300$ V. Thus the potential difference between these two points is $\Delta V = V_{1\,\text{cm}} - V_{3\,\text{cm}} = 600$ V.

ASSESS 1 nC is typical of the electrostatic charge produced by rubbing, and you can see that such a charge creates a fairly large potential nearby. Why are we not shocked and injured when working with the "high voltages" of such charges? The sensation of being shocked is a result of current, not potential. Some high-potential sources simply do not have the ability to generate much current. We will look at this issue in Chapter 31.

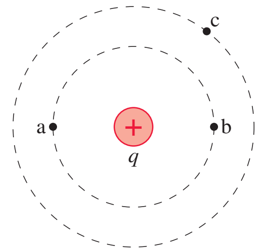
Visualizing the Potential of a Point Charge

FIGURE 28.27 shows four graphical representations of the electric potential of a point charge. These match the four representations of the electric potential inside a capacitor, and a comparison of the two is worthwhile. This figure assumes that q is positive; you may want to think about how the representations would change if q were negative.

FIGURE 28.27 Four graphical representations of the electric potential of a point charge.



STOP TO THINK 28.5 Rank in order, from largest to smallest, the potential differences ΔV_{ab}, ΔV_{ac}, and ΔV_{bc} between points a and b, points a and c, and points b and c.



The Electric Potential of a Charged Sphere

In practice, you are more likely to work with a charged sphere, of radius R and total charge Q, than with a point charge. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge Q at the center. That is,

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad \text{(sphere of charge, } r \geq R\text{)} \tag{28.31}$$

We can cast this result in a more useful form. It is customary to speak of charging an electrode, such as a sphere, "to" a certain potential, as in "Bob charged the sphere to a potential of 3000 volts." This potential, which we will call V_0, is the potential right on the surface of the sphere. We can see from Equation 28.31 that

$$V_0 = V(\text{at } r = R) = \frac{Q}{4\pi\epsilon_0 R} \tag{28.32}$$

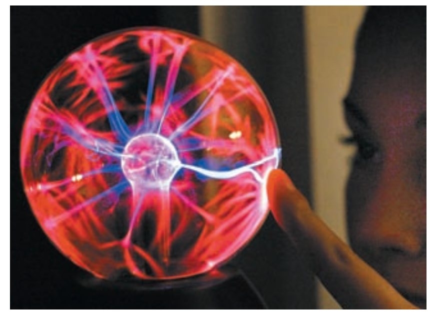
Consequently, a sphere of radius R that is charged to potential V_0 has total charge

$$Q = 4\pi\epsilon_0 R V_0 \tag{28.33}$$

If we substitute this expression for Q into Equation 28.31, we can write the potential outside a sphere that is charged to potential V_0 as

$$V = \frac{R}{r}V_0 \quad \text{(sphere charged to potential } V_0\text{)} \tag{28.34}$$

Equation 28.34 tells us that the potential of a sphere is V_0 on the surface and decreases inversely with the distance. The potential at $r = 3R$ is $\frac{1}{3}V_0$.



A *plasma ball* consists of a small metal ball charged to a potential of about 2000 V inside a hollow glass sphere. The glass sphere is filled with gas—typically neon or argon because of the colors they produce—at a pressure of about 0.01 atm. The electric field of the high-voltage ball is sufficient to cause a gas breakdown at this pressure, creating "lightning bolts" between the ball and the glass sphere.

EXAMPLE 28.10 A proton and a charged sphere

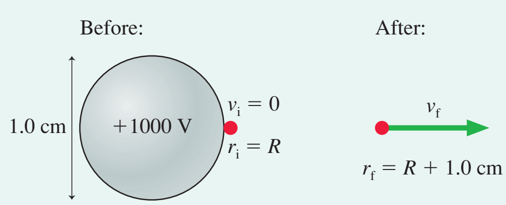
A proton is released from rest at the surface of a 1.0-cm-diameter sphere that has been charged to +1000 V.

a. What is the charge of the sphere?
b. What is the proton's speed at 1.0 cm from the sphere?

MODEL Energy is conserved. The potential outside the charged sphere is the same as the potential of a point charge at the center.

VISUALIZE **FIGURE 28.28** shows the situation.

FIGURE 28.28 A sphere and a proton.



SOLVE a. The charge of the sphere is

$$Q = 4\pi\epsilon_0 R V_0 = 0.56 \times 10^{-9}\text{ C} = 0.56\text{ nC}$$

b. A sphere charged to $V_0 = +1000$ V is positively charged. The proton will be repelled by this charge and move away from the sphere. The conservation of energy equation $K_f + eV_f = K_i + eV_i$, with Equation 28.34 for the potential of a sphere, is

$$\frac{1}{2}mv_f^2 + \frac{eR}{r_f}V_0 = \frac{1}{2}mv_i^2 + \frac{eR}{r_i}V_0$$

The proton starts from the surface of the sphere, $r_i = R$, with $v_i = 0$. When the proton is 1.0 cm from the *surface* of the sphere, it has $r_f = 1.0\text{ cm} + R = 1.5$ cm. Using these, we can solve for v_f:

$$v_f = \sqrt{\frac{2eV_0}{m}\left(1 - \frac{R}{r_f}\right)} = 3.6 \times 10^5\text{ m/s}$$

ASSESS This example illustrates how the ideas of electric potential and potential energy work together, yet they are *not* the same thing.

28.7 The Electric Potential of Many Charges

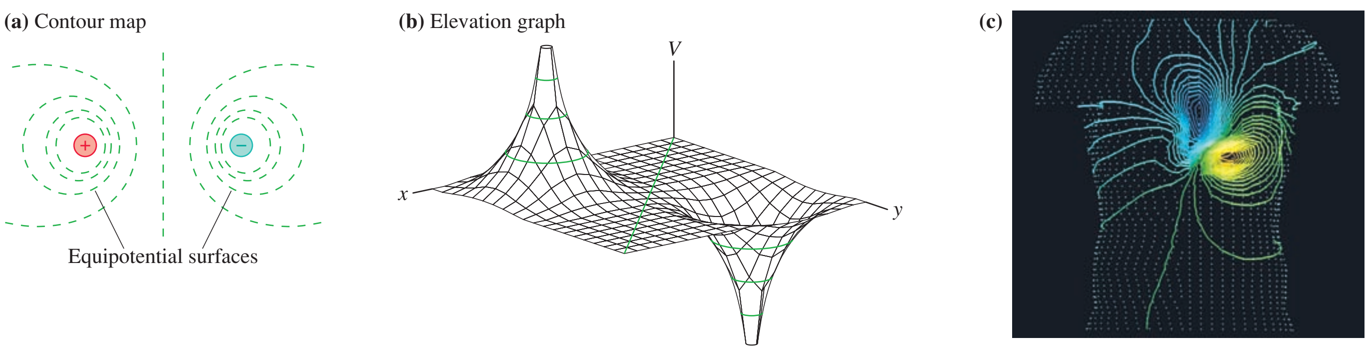
Suppose there are many source charges $q_1, q_2, \ldots$. The electric potential V at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} \tag{28.35}$$

where r_i is the distance from charge q_i to the point in space where the potential is being calculated. In other words, **the electric potential, like the electric field, obeys the principle of superposition.**

As an example, the contour map and elevation graph in FIGURE 28.29 show that the potential of an electric dipole is the sum of the potentials of the positive and negative charges. Potentials such as these have many practical applications. For example, electrical activity within the body can be monitored by measuring equipotential lines on the skin. Figure 28.29c shows that the equipotentials near the heart are a slightly distorted but recognizable electric dipole.

FIGURE 28.29 The electric potential of an electric dipole.

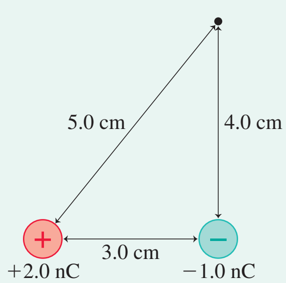


Equipotentials on the chest of a human are a slightly distorted electric dipole.

EXAMPLE 28.11 The potential of two charges

What is the electric potential at the point indicated in FIGURE 28.30?

FIGURE 28.30 Finding the potential of two charges.



MODEL The potential is the sum of the potentials due to each charge.

SOLVE The potential at the indicated point is

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} \\ &= (9.0 \times 10^9\ \mathrm{N\,m^2/C^2})\left(\frac{2.0 \times 10^{-9}\ \mathrm{C}}{0.050\ \mathrm{m}} + \frac{-1.0 \times 10^{-9}\ \mathrm{C}}{0.040\ \mathrm{m}}\right) \\ &= 135\ \mathrm{V} \end{aligned}$$

ASSESS The potential is a *scalar,* so we found the net potential by adding two numbers. We don't need any angles or components to calculate the potential.

A Continuous Distribution of Charge

Equation 28.35 is the basis for determining the potential of a continuous distribution of charge, such as a charged rod or a charged disk. The procedure is much like the one you learned in Chapter 26 for calculating the electric field of a continuous distribution of charge, but *easier* because the potential is a scalar. We will continue to assume that the object is *uniformly charged,* meaning that the charges are evenly spaced over the object.

PROBLEM-SOLVING STRATEGY 28.2

The electric potential of a continuous distribution of charge

MODEL Model the charges as a simple shape, such as a line or a disk. Assume the charge is uniformly distributed.

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric potential.
3. Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine V. This division is often, but not always, into point charges.
4. Identify distances that need to be calculated.

SOLVE The mathematical representation is $V = \sum V_i$.

- Use superposition to form an algebraic expression for the potential at P.
- Let the (x, y, z) coordinates of the point remain as variables.
- Replace the small charge ΔQ with an equivalent expression involving a *charge density* and a *coordinate,* such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- All distances must be expressed in terms of the coordinates.
- Let the sum become an integral. The integration will be over the coordinate variable that is related to ΔQ. The integration limits for this variable will depend on the coordinate system you have chosen. Carry out the integration and simplify the result.

ASSESS Check that your result is consistent with any limits for which you know what the potential should be.

Exercise 29

EXAMPLE 28.12 The potential of a ring of charge

A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.

MODEL Because the ring is thin, we'll assume the charge lies along a circle of radius R.

VISUALIZE FIGURE 28.31 illustrates the four steps of the problem-solving strategy. We've chosen a coordinate system in which the ring lies in the xy-plane and point P is on the z-axis. We've then divided the ring into N small segments of charge ΔQ, each of which can be modeled as a point charge. The distance r_i between segment i and point P is

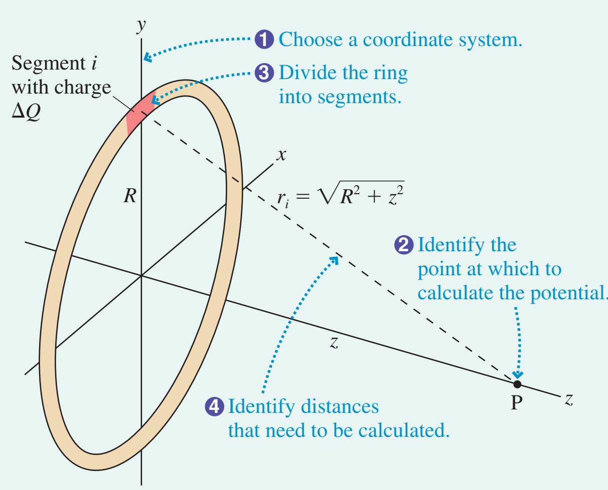
$$r_i = \sqrt{R^2 + z^2}$$

Note that r_i is a constant distance, the same for every charge segment.

SOLVE The potential V at P is the sum of the potentials due to each segment of charge:

$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{R^2 + z^2}}\sum_{i=1}^{N} \Delta Q$$

FIGURE 28.31 Finding the potential of a ring of charge.



Continued

We were able to bring all terms involving z to the front because z is a constant as far as the summation is concerned. Surprisingly, we don't need to convert the sum to an integral to complete this calculation. The sum of all the ΔQ charge segments around the ring is simply the ring's total charge, $\sum(\Delta Q) = Q$; hence the electric potential on the axis of a charged ring is

$$V_{\text{ring on axis}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2 + z^2}}$$

ASSESS From far away, the ring appears as a point charge Q in the distance. Thus we expect the potential of the ring to be that of a point charge when $z \gg R$. You can see that $V_{\text{ring}} \approx Q/4\pi\epsilon_0 z$ when $z \gg R$, which is, indeed, the potential of a point charge Q.

CHALLENGE EXAMPLE 28.13 The potential of a charged dime

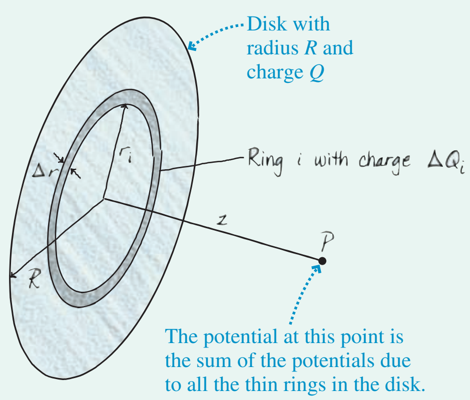
A 17.5-mm-diameter dime is charged to +5.00 nC.

a. What is the potential of the dime?
b. What is the potential energy of an electron 1.00 cm above the dime?

MODEL Model the dime as a thin, uniformly charged disk of radius R and charge Q. The disk has uniform surface charge density $\eta = Q/A = Q/\pi R^2$. We can take advantage of now knowing the on-axis potential of a ring of charge.

VISUALIZE Orient the disk in the xy-plane, as shown in FIGURE 28.32, with point P at distance z. Then divide the disk into *rings* of equal width Δr. Ring i has radius r_i and charge ΔQ_i.

FIGURE 28.32 Finding the potential of a disk of charge.



SOLVE We can use the result of Example 28.12 to write the potential at distance z of ring i as

$$V_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

The potential at P due to all the rings is the sum

$$V = \sum_i V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

The critical step is to relate ΔQ_i to a coordinate. Because we now have a surface, rather than a line, the charge in ring i is $\Delta Q_i = \eta\,\Delta A_i$, where ΔA_i is the area of ring i. We can find ΔA_i, as you've learned to do in calculus, by "unrolling" the ring to form a narrow rectangle of length $2\pi r_i$ and height Δr. Thus the area of ring i is $\Delta A_i = 2\pi r_i\,\Delta r$ and the charge is

$$\Delta Q_i = \eta\,\Delta A_i = \frac{Q}{\pi R^2}2\pi r_i\,\Delta r = \frac{2Q}{R^2}r_i\,\Delta r$$

With this substitution, the potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{2Q}{R^2}\frac{r_i\,\Delta r_i}{\sqrt{r_i^2 + z^2}} \to \frac{Q}{2\pi\epsilon_0 R^2}\int_0^R \frac{r\,dr}{\sqrt{r^2 + z^2}}$$

where, in the last step, we let $N \to \infty$ and the sum become an integral. This integral can be found in Appendix A, but it's not hard to evaluate with a change of variables. Let $u = r^2 + z^2$, in which case $r\,dr = \frac{1}{2}du$. Changing variables requires that we also change the integration limits. You can see that $u = z^2$ when $r = 0$, and $u = R^2 + z^2$ when $r = R$. With these changes, the on-axis potential of a charged disk is

$$V_{\text{disk on axis}} = \frac{Q}{2\pi\epsilon_0 R^2}\int_{z^2}^{R^2+z^2}\frac{\frac{1}{2}du}{u^{1/2}} = \frac{Q}{2\pi\epsilon_0 R^2}u^{1/2}\Big|_{z^2}^{R^2+z^2}$$

$$= \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{R^2 + z^2} - z\right)$$

We can find the potential V_0 of the disk itself by setting $z = 0$, giving $V_0 = Q/2\pi\epsilon_0 R$. In other words, placing charge Q on a disk of radius R charges it to potential V_0. The on-axis potential of the disk can be written in terms of V_0 as

$$V_{\text{disk on axis}} = V_0\left[\sqrt{1 + (z/R)^2} - (z/R)\right]$$

Now we can evaluate the case of the charged dime.

a. The potential of the dime is the potential of a disk at $z = 0$:

$$V_0 = \frac{Q}{2\pi\epsilon_0 R} = 10{,}300\ \text{V}$$

b. To calculate the potential energy $U = qV$ of charge q, we first need to determine the potential of the disk at $z = 1.0$ cm. This is

$$V = V_0\left[\sqrt{1 + (z/R)^2} - (z/R)\right] = 3870\ \text{V}$$

The electron's charge is $q = -e = -1.60 \times 10^{-19}$ C, so its potential energy at $z = 1.00$ cm is $U = qV = -6.19 \times 10^{-16}$ J.

ASSESS Although we had to go through a number of steps, this procedure is easier than evaluating the electric field because we do not have to worry about vector components.

SUMMARY

The goals of Chapter 28 have been to calculate and use the electric potential and electric potential energy.

General Principles

Sources of V

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Consequences of V

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + qV_f = K_i + qV_i$$

The potential energy of **two point charges** separated by distance r is

$$U_{q_1 + q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

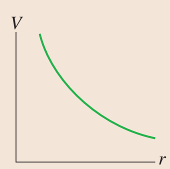
The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

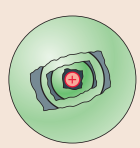
$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Applications

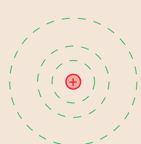
Graphical representations of the potential:



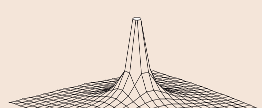
Potential graph



Equipotential surfaces



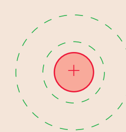
Contour map



Elevation graph

Sphere of charge Q

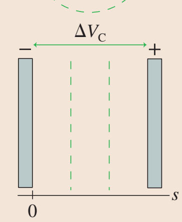
Same as a point charge if $r \geq R$



Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_C}{d}$$



Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C

Terms and Notation

electric potential energy, U
electric potential, V
volt, V
potential difference, ΔV
voltage, ΔV
equipotential surface
contour map
elevation graph

CONCEPTUAL QUESTIONS

1. a. Charge q_1 is distance r from a positive point charge Q. Charge $q_2 = q_1/3$ is distance $2r$ from Q. What is the ratio U_1/U_2 of their potential energies due to their interactions with Q?
   b. Charge q_1 is distance s from the negative plate of a parallel-plate capacitor. Charge $q_2 = q_1/3$ is distance $2s$ from the negative plate. What is the ratio U_1/U_2 of their potential energies?
2. **FIGURE Q28.2** shows the potential energy of a proton ($q = +e$) and a lead nucleus ($q = +82e$). The horizontal scale is in units of *femtometers*, where 1 fm $= 10^{-15}$ m.
   a. A proton is fired toward a lead nucleus from very far away. How much initial kinetic energy does the proton need to reach a turning point 10 fm from the nucleus? Explain.
   b. How much kinetic energy does the proton of part a have when it is 20 fm from the nucleus and moving toward it, before the collision?

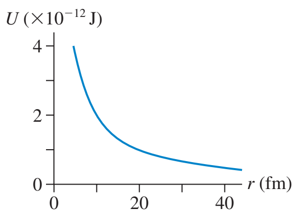


FIGURE Q28.2

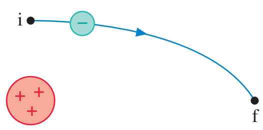


FIGURE Q28.3

3. An electron moves along the trajectory of **FIGURE Q28.3** from i to f.
   a. Does the electric potential energy increase, decrease, or stay the same? Explain.
   b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.
4. Two protons are launched with the same speed from point 1 inside the parallel-plate capacitor of **FIGURE Q28.4**. Points 2 and 3 are the same distance from the negative plate.
   a. Is $\Delta U_{1\rightarrow 2}$, the change in potential energy along the path $1 \rightarrow 2$, larger than, smaller than, or equal to $\Delta U_{1\rightarrow 3}$?
   b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to v_3? Explain.

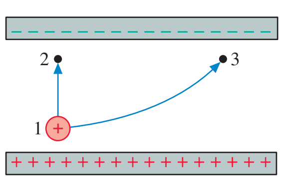


FIGURE Q28.4

5. Rank in order, from most positive to most negative, the potential energies U_a to U_f of the six electric dipoles in the uniform electric field of **FIGURE Q28.5**. Explain.

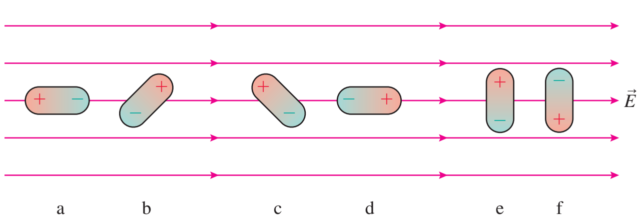


FIGURE Q28.5

6. **FIGURE Q28.6** shows the electric potential along the x-axis.
   a. Draw a graph of the potential energy of a 0.1 C charged particle. Provide a numerical scale for both axes.
   b. If the charged particle is shot toward the right from $x = 1$ m with 1.0 J of kinetic energy, where is its turning point? Use your graph to explain.

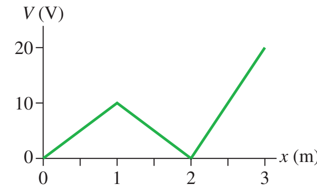


FIGURE Q28.6

7. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.
   a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?
   b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?
   c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?
8. Rank in order, from largest to smallest, the electric potentials V_a to V_e at points a to e in **FIGURE Q28.8**. Explain.

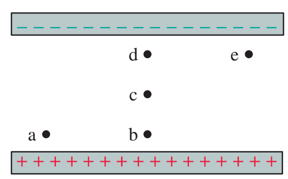


FIGURE Q28.8

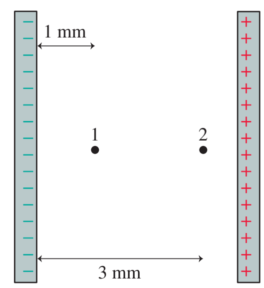


FIGURE Q28.9

9. **FIGURE Q28.9** shows two points inside a capacitor. Let $V = 0$ V at the negative plate.
   a. What is the ratio V_2/V_1 of the electric potentials? Explain.
   b. What is the ratio E_2/E_1 of the electric field strengths?
10. **FIGURE Q28.10** shows two points near a positive point charge.
    a. What is the ratio V_2/V_1 of the electric potentials? Explain.
    b. What is the ratio E_2/E_1 of the electric field strengths?

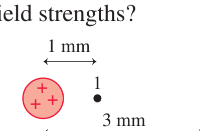


FIGURE Q28.10

11. **FIGURE Q28.11** shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from most positive to most negative, the potentials V_a to V_c.

FIGURE Q28.11

12. Reproduce **FIGURE Q28.12** on your paper. Then draw a dot (or dots) on the figure to show the position (or positions) at which the electric potential is zero.

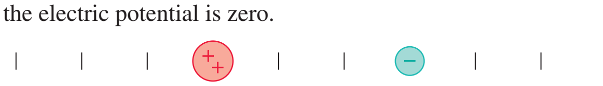


FIGURE Q28.12

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 28.1 Electric Potential Energy

1. | The electric field strength is 50,000 N/C inside a parallel-plate capacitor with a 2.0 mm spacing. A proton is released from rest at the positive plate. What is the proton's speed when it reaches the negative plate?
2. || The electric field strength is 20,000 N/C inside a parallel-plate capacitor with a 1.0 mm spacing. An electron is released from rest at the negative plate. What is the electron's speed when it reaches the positive plate?
3. | A proton is released from rest at the positive plate of a parallel-plate capacitor. It crosses the capacitor and reaches the negative plate with a speed of 50,000 m/s. What will be the final speed of an electron released from rest at the negative plate?
4. | A proton is released from rest at the positive plate of a parallel-plate capacitor. It crosses the capacitor and reaches the negative plate with a speed of 50,000 m/s. The experiment is repeated with a He^+ ion (charge e, mass 4 u). What is the ion's speed at the negative plate?

Section 28.2 The Potential Energy of Point Charges

5. | What is the electric potential energy of the proton in FIGURE EX28.5? The electrons are fixed and cannot move.

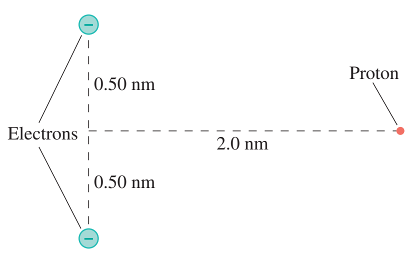


FIGURE EX28.5

6. || What is the electric potential energy of the group of charges in FIGURE EX28.6?

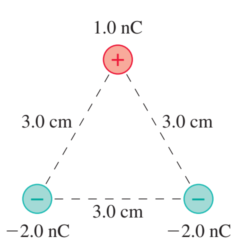


FIGURE EX28.6

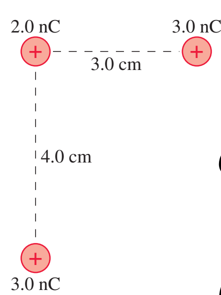


FIGURE EX28.7

7. || What is the electric potential energy of the group of charges in FIGURE EX28.7?

Section 28.3 The Potential Energy of a Dipole

8. | A water molecule perpendicular to an electric field has 1.0×10^{-21} J more potential energy than a water molecule aligned with the field. The dipole moment of a water molecule is 6.2×10^{-30} C m. What is the strength of the electric field?
9. | FIGURE EX28.9 shows the potential energy of an electric dipole. Consider a dipole that oscillates between $\pm 60°$.
   a. What is the dipole's mechanical energy?
   b. What is the dipole's kinetic energy when it is aligned with the electric field?

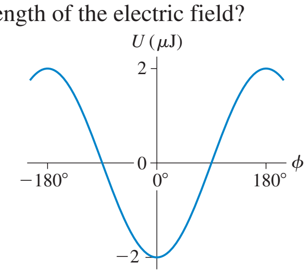


FIGURE EX28.9

Section 28.4 The Electric Potential

10. | What is the speed of a proton that has been accelerated from rest through a potential difference of −1000 V?
11. | What is the speed of an electron that has been accelerated from rest through a potential difference of 1000 V?
12. | What potential difference is needed to accelerate an electron from rest to a speed of 2.0×10^6 m/s?
13. || What potential difference is needed to accelerate a He^+ ion (charge $+e$, mass 4 u) from rest to a speed of 2.0×10^6 m/s?
14. | A proton with an initial speed of 800,000 m/s is brought to rest by an electric field.
    a. Did the proton move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the proton?
15. || An electron with an initial speed of 500,000 m/s is brought to rest by an electric field.
    a. Did the electron move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the electron?

Section 28.5 The Electric Potential Inside a Parallel-Plate Capacitor

16. | Show that 1 V/m = 1 N/C.
17. | a. What is the potential of an ordinary AA or AAA battery? (If you're not sure, find one and look at the label.)
    b. An AA battery is connected to a parallel-plate capacitor having 4.0 cm × 4.0 cm plates spaced 1.0 mm apart. How much charge does the battery supply to each plate?
18. | Two 2.00 cm × 2.00 cm plates that form a parallel-plate capacitor are charged to ±0.708 nC. What are the electric field strength inside and the potential difference across the capacitor if the spacing between the plates is (a) 1.00 mm and (b) 2.00 mm?
19. | A 3.0-cm-diameter parallel-plate capacitor has a 2.0 mm spacing. The electric field strength inside the capacitor is 1.0×10^5 V/m.
    a. What is the potential difference across the capacitor?
    b. How much charge is on each plate?

20. ‖ Two 2.0-cm-diameter disks spaced 2.0 mm apart form a parallel-plate capacitor. The electric field between the disks is 5.0×10^5 V/m.
   a. What is the voltage across the capacitor?
   b. An electron is launched from the negative plate. It strikes the positive plate at a speed of 2.0×10^7 m/s. What was the electron's speed as it left the negative plate?

Section 28.6 The Electric Potential of a Point Charge

21. | a. What is the electric potential at points A, B, and C in FIGURE EX28.21?
   b. What are the potential differences ΔV_{AB} and ΔV_{BC}?

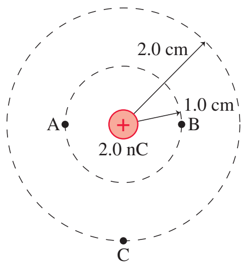


FIGURE EX28.21

22. ‖ A 1.0-mm-diameter ball bearing has 2.0×10^9 excess electrons. What is the ball bearing's potential?
23. | In a semiclassical model of the hydrogen atom, the electron orbits the proton at a distance of 0.053 nm.
   a. What is the electric potential of the proton at the position of the electron?
   b. What is the electron's potential energy?

Section 28.7 The Electric Potential of Many Charges

24. | What is the electric potential at the point indicated with the dot in FIGURE EX28.24?

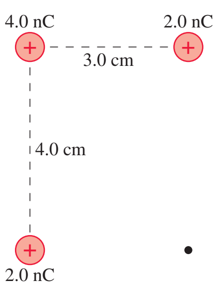


FIGURE EX28.24

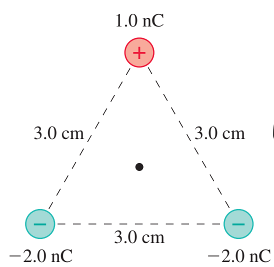


FIGURE EX28.25

25. | What is the electric potential at the point indicated with the dot in FIGURE EX28.25?
26. ‖ The electric potential at the dot in FIGURE EX28.26 is 3140 V. What is charge q?

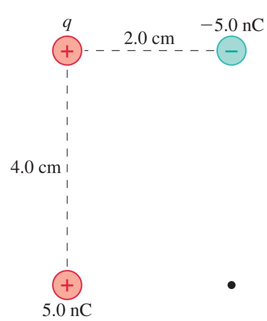


FIGURE EX28.26

27. ‖ A −2.0 nC charge and a +2.0 nC charge are located on the x-axis at $x = -1.0$ cm and $x = +1.0$ cm, respectively.
   a. Other than at infinity, is there a position or positions on the x-axis where the electric field is zero? If so, where?
   b. Other than at infinity, at what position or positions on the x-axis is the electric potential zero?
   c. Sketch graphs of the electric field strength and the electric potential along the x-axis.
28. ‖ Two point charges q_a and q_b are located on the x-axis at $x = a$ and $x = b$. FIGURE EX28.28 is a graph of E_x, the x-component of the electric field.
   a. What are the signs of q_a and q_b?
   b. What is the ratio $|q_a/q_b|$?
   c. Draw a graph of V, the electric potential, as a function of x.

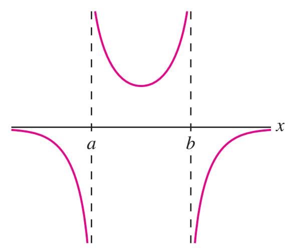


FIGURE EX28.28

29. ‖ Two point charges q_a and q_b are located on the x-axis at $x = a$ and $x = b$. FIGURE EX28.29 is a graph of V, the electric potential.
   a. What are the signs of q_a and q_b?
   b. What is the ratio $|q_a/q_b|$?
   c. Draw a graph of E_x, the x-component of the electric field, as a function of x.

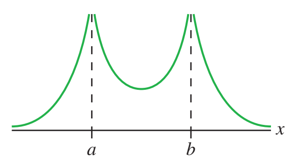


FIGURE EX28.29

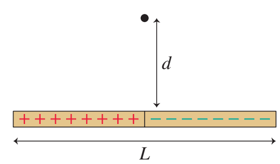


FIGURE EX28.30

30. | The two halves of the rod in FIGURE EX28.30 are uniformly charged to $\pm Q$. What is the electric potential at the point indicated by the dot?

Problems

31. | Two positive point charges are 5.0 cm apart. If the electric potential energy is 72 μJ, what is the magnitude of the force between the two charges?
32. ‖| Two point charges 2.0 cm apart have an electric potential energy −180 μJ. The total charge is 30 nC. What are the two charges?
33. ‖ A −10.0 nC point charge and a +20.0 nC point charge are 15.0 cm apart on the x-axis.
   a. What is the electric potential at the point on the x-axis where the electric field is zero?
   b. What is the magnitude of the electric field at the point on the x-axis, between the charges, where the electric potential is zero?
34. ‖| A +3.0 nC charge is at $x = 0$ cm and a −1.0 nC charge is at $x = 4$ cm. At what point or points on the x-axis is the electric potential zero?
35. ‖| A −3.0 nC charge is on the x-axis at $x = -9$ cm and a +4.0 nC charge is on the x-axis at $x = 16$ cm. At what point or points on the y-axis is the electric potential zero?

36. | Two small metal cubes with masses 2.0 g and 4.0 g are tied together by a 5.0-cm-long massless string and are at rest on a frictionless surface. Each is charged to $+2.0\ \mu C$.
   a. What is the energy of this system?
   b. What is the tension in the string?
   c. The string is cut. What is the speed of each cube when they are far apart?

Hint: There are *two* conserved quantities. Make use of both.

37. | The four 1.0 g spheres shown in **FIGURE P28.37** are released simultaneously and allowed to move away from each other. What is the speed of each sphere when they are very far apart?

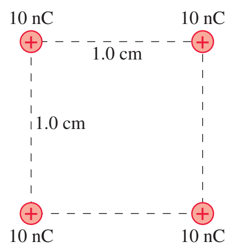


FIGURE P28.37

38. | A proton's speed as it passes point A is 50,000 m/s. It follows the trajectory shown in **FIGURE P28.38**. What is the proton's speed at point B?

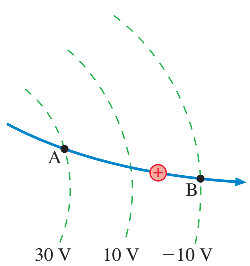


FIGURE P28.38

39. | Living cells "pump" singly ionized sodium ions, Na^+, from
BIO the inside of the cell to the outside to maintain a membrane potential $\Delta V_{membrane} = V_{in} - V_{out} = -70$ mV. It is called *pumping* because work must be done to move a positive ion from the negative inside of the cell to the positive outside, and it must go on continuously because sodium ions "leak" back through the cell wall by diffusion.
   a. How much work must be done to move one sodium ion from the inside of the cell to the outside?
   b. At rest, the human body uses energy at the rate of approximately 100 W to maintain basic metabolic functions. It has been estimated that 20% of this energy is used to operate the sodium pumps of the body. Estimate—to one significant figure—the number of sodium ions pumped per second.

40. | An arrangement of source charges produces the electric potential $V = 5000x^2$ along the x-axis, where V is in volts and x is in meters. What is the maximum speed of a 1.0 g, 10 nC charged particle that moves in this potential with turning points at ± 8.0 cm?

41. | A proton moves along the x-axis, where an arrangement of source charges has created the electric potential $V = 6000x^2$, where V is in volts and x is in meters. By exploiting the analogy with the potential energy of a mass on a spring, determine the proton's oscillation frequency.

42. | In **FIGURE P28.42**, a proton is fired with a speed of 200,000 m/s from the midpoint of the capacitor toward the positive plate.
   a. Show that this is insufficient speed to reach the positive plate.
   b. What is the proton's speed as it collides with the negative plate?

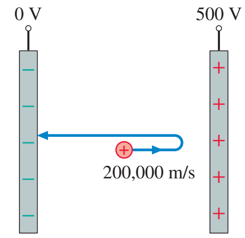


FIGURE P28.42

43. | The electron gun in an old TV picture tube accelerates electrons between two parallel plates 1.2 cm apart with a 25 kV potential difference between them. The electrons enter through a small hole in the negative plate, accelerate, then exit through a small hole in the positive plate. Assume that the holes are small enough not to affect the electric field or potential.
   a. What is the electric field strength between the plates?
   b. With what speed does an electron exit the electron gun if its entry speed is close to zero?

NOTE ▶ The exit speed is so fast that we really need to use the theory of relativity to compute an accurate value. Your answer to part b is in the right range but a little too big. ◀

44. | An uncharged parallel-plate capacitor with spacing d is horizontal. A small bead with mass m and positive charge q is shot straight up from the bottom plate with speed v_0. It reaches maximum height y_{max} before falling back. Then the capacitor is charged with the bottom plate negative. Find an expression for the capacitor voltage ΔV_C for which the bead's maximum height is reduced to $\frac{1}{2}y_{max}$. Ignore air resistance.

45. | A room with 3.0-m-high ceilings has a metal plate on the floor with $V = 0$ V and a separate metal plate on the ceiling. A 1.0 g glass ball charged to $+4.9$ nC is shot straight up at 5.0 m/s. How high does the ball go if the ceiling voltage is (a) $+3.0 \times 10^6$ V and (b) -3.0×10^6 V?

46. | In *proton-beam therapy*, a high-energy beam of protons is
BIO fired at a tumor. As the protons stop in the tumor, their kinetic energy breaks apart the tumor's DNA, thus killing the tumor cells. For one patient, it is desired to deposit 0.10 J of proton energy in the tumor. To create the proton beam, protons are accelerated from rest through a 10,000 kV potential difference. What is the total charge of the protons that must be fired at the tumor?

47. | What is the escape speed of an electron launched from the surface of a 1.0-cm-diameter glass sphere that has been charged to 10 nC?

48. | An electric dipole consists of 1.0 g spheres charged to ± 2.0 nC at the ends of a 10-cm-long massless rod. The dipole rotates on a frictionless pivot at its center. The dipole is held perpendicular to a uniform electric field with field strength 1000 V/m, then released. What is the dipole's angular velocity at the instant it is aligned with the electric field?

49. ||| Three electrons form an equilateral triangle 1.0 nm on each side. A proton is at the center of the triangle. What is the potential energy of this group of charges?

50. ||| A 2.0-mm-diameter glass bead is positively charged. The potential difference between a point 2.0 mm from the bead and a point 4.0 mm from the bead is 500 V. What is the charge on the bead?

51. ||| Your lab assignment for the week is to measure the amount of charge on the 6.0-cm-diameter metal sphere of a Van de Graaff generator. To do so, you're going to use a spring with spring constant 0.65 N/m to launch a small, 1.5 g bead horizontally toward the sphere. You can reliably charge the bead to 2.5 nC, and your plan is to use a video camera to measure the bead's closest approach to the sphere as you change the compression of the spring. Your data are as follows:

Compression (cm)	Closest approach (cm)
1.6	5.5
1.9	2.6
2.2	1.6
2.5	0.4

Use an appropriate graph of the data to determine the sphere's charge in nC. You can assume that the bead's motion is entirely horizontal and that the spring is so far away that the bead has no interaction with the sphere as it's launched.

52. || A proton is fired from far away toward the nucleus of an iron atom. Iron is element number 26, and the diameter of the nucleus is 9.0 fm. What initial speed does the proton need to just reach the surface of the nucleus? Assume the nucleus remains at rest.

53. || A proton is fired from far away toward the nucleus of a mercury atom. Mercury is element number 80, and the diameter of the nucleus is 14.0 fm. If the proton is fired at a speed of 4.0×10^7 m/s, what is its closest approach to the surface of the nucleus? Assume the nucleus remains at rest.

54. || In the form of radioactive decay known as *alpha decay,* an unstable nucleus emits a helium-atom nucleus, which is called an *alpha particle.* An alpha particle contains two protons and two neutrons, thus having mass $m = 4$ u and charge $q = 2e$. Suppose a uranium nucleus with 92 protons decays into thorium, with 90 protons, and an alpha particle. The alpha particle is initially at rest at the surface of the thorium nucleus, which is 15 fm in diameter. What is the speed of the alpha particle when it is detected in the laboratory? Assume the thorium nucleus remains at rest.

55. || One form of nuclear radiation, *beta decay,* occurs when a neutron changes into a proton, an electron, and a neutral particle called a *neutrino:* $\mathrm{n} \rightarrow \mathrm{p}^+ + \mathrm{e}^- + \nu$ where ν is the symbol for a neutrino. When this change happens to a neutron within the nucleus of an atom, the proton remains behind in the nucleus while the electron and neutrino are ejected from the nucleus. The ejected electron is called a *beta particle.* One nucleus that exhibits beta decay is the isotope of hydrogen ^{3}H, called *tritium,* whose nucleus consists of one proton (making it hydrogen) and two neutrons (giving tritium an atomic mass $m = 3$ u). Tritium is radioactive, and it decays to helium: $^3\mathrm{H} \rightarrow {}^3\mathrm{He} + \mathrm{e}^- + \nu$.
   a. Is charge conserved in the beta decay process? Explain.
   b. Why is the final product a helium atom? Explain.
   c. The nuclei of both ^{3}H and ^{3}He have radii of 1.5×10^{-15} m. With what minimum speed must the electron be ejected if it is to escape from the nucleus and not fall back?

56. || The sun is powered by *fusion*, with four protons fusing together to form a helium nucleus (two of the protons turn into neutrons) and, in the process, releasing a large amount of thermal energy. The process happens in several steps, not all at once. In one step, two protons fuse together, with one proton then becoming a neutron, to form the "heavy hydrogen" isotope *deuterium* (^{2}H). A proton is essentially a 2.4-fm-diameter sphere of charge, and fusion occurs only if two protons come into contact with each other. This requires extraordinarily high temperatures due to the strong repulsion between the protons. Recall that the average kinetic energy of a gas particle is $\frac{3}{2}k_B T$.
   a. Suppose two protons, each with exactly the average kinetic energy, have a head-on collision. What is the minimum temperature for fusion to occur?
   b. Your answer to part a is much hotter than the 15 million K in the core of the sun. If the temperature were as high as you calculated, every proton in the sun would fuse almost instantly and the sun would explode. For the sun to last for billions of years, fusion can occur only in collisions between two protons with kinetic energies much higher than average. Only a very tiny fraction of the protons have enough kinetic energy to fuse when they collide, but that fraction is enough to keep the sun going. Suppose two protons with the same kinetic energy collide head-on and just barely manage to fuse. By what factor does each proton's energy exceed the average kinetic energy at 15 million K?

57. || Two 10-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. After a long time, the capacitor is disconnected from the battery but is not discharged. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
   a. Right after the battery is disconnected?
   b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart?
   c. After the original electrodes (not the modified electrodes of part b) are expanded until they are 20 cm in diameter?

58. || Two 10-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
   a. While the capacitor is attached to the battery?
   b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart? The electrodes remain connected to the battery during this process.
   c. After the original electrodes (not the modified electrodes of part b) are expanded until they are 20 cm in diameter while remaining connected to the battery?

59. || a. Find an algebraic expression for the electric field strength E_0 at the surface of a charged sphere in terms of the sphere's potential V_0 and radius R.
   b. What is the electric field strength at the surface of a 1.0-cm-diameter marble charged to 500 V?

60. || Two spherical drops of mercury each have a charge of 0.10 nC and a potential of 300 V at the surface. The two drops merge to form a single drop. What is the potential at the surface of the new drop?

61. || A Van de Graaff generator is a device for generating a large electric potential by building up charge on a hollow metal sphere. A typical classroom-demonstration model has a diameter of 30 cm.
   a. How much charge is needed on the sphere for its potential to be 500,000 V?
   b. What is the electric field strength just outside the surface of the sphere when it is charged to 500,000 V?

62. || A thin spherical shell of radius R has total charge Q. What is the electric potential at the center of the shell?

63. | **FIGURE P28.63** shows two uniformly charged spheres. What is the potential difference between points a and b? Which point is at the higher potential?

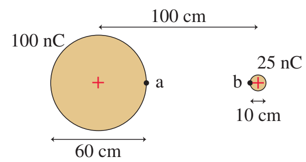


FIGURE P28.63

Hint: The potential at any point is the superposition of the potentials due to *all* charges.

64. || An electric dipole with dipole moment p is oriented along the y-axis.
    a. Find an expression for the electric potential on the y-axis at a point where y is much larger than the charge spacing s. Write your expression in terms of the dipole moment p.
    b. The dipole moment of a water molecule is 6.2×10^{-30} C m. What is the electric potential 1.0 nm from a water molecule along the axis of the dipole?

65. || Two positive point charges q are located on the y-axis at $y = \pm\frac{1}{2}s$.
    a. Find an expression for the potential along the x-axis.
    b. Draw a graph of V versus x for $-\infty < x < \infty$. For comparison, use a dotted line to show the potential of a point charge $2q$ located at the origin.

66. || The arrangement of charges shown in **FIGURE P28.66** is called a *linear electric quadrupole*. The positive charges are located at $y = \pm s$. Notice that the net charge is zero. Find an expression for the electric potential on the y-axis at distances $y \gg s$.

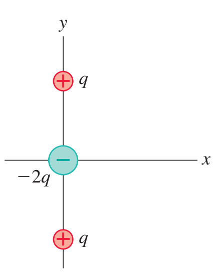


FIGURE P28.66

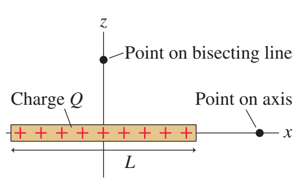


FIGURE P28.67

67. || **FIGURE P28.67** shows a thin rod of length L and charge Q. Find an expression for the electric potential a distance x away from the center of the rod on the axis of the rod.

68. ||| **FIGURE P28.67** showed a thin rod of length L and charge Q. Find an expression for the electric potential a distance z away from the center of rod on the line that bisects the rod.

69. | **FIGURE P28.69** shows a thin rod with charge Q that has been bent into a semicircle of radius R. Find an expression for the electric potential at the center.

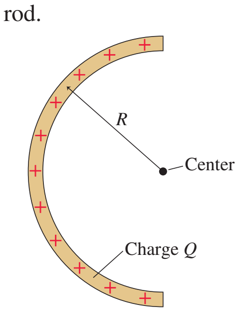


FIGURE P28.69

70. || A disk with a hole has inner radius R_{in} and outer radius R_{out}. The disk is uniformly charged with total charge Q. Find an expression for the on-axis electric potential at distance z from the center of the disk. Verify that your expression has the correct behavior when $R_{in} \to 0$.

In Problems 71 through 73 you are given the equation(s) used to solve a problem. For each of these,
  a. Write a realistic problem for which this is the correct equation(s).
  b. Finish the solution of the problem.

71. $$\frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})q_1 q_2}{0.030\ \mathrm{m}} = 90 \times 10^{-6}\ \mathrm{J}$$
$$q_1 + q_2 = 40\ \mathrm{nC}$$

72. $$\tfrac{1}{2}(1.67 \times 10^{-27}\ \mathrm{kg})(2.5 \times 10^6\ \mathrm{m/s})^2 + 0 = \tfrac{1}{2}(1.67 \times 10^{-27}\ \mathrm{kg})v_i^2 + \frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(2.0 \times 10^{-9}\ \mathrm{C})(1.60 \times 10^{-19}\ \mathrm{C})}{0.0010\ \mathrm{m}}$$

73. $$\frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(3.0 \times 10^{-9}\ \mathrm{C})}{0.030\ \mathrm{m}} + \frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(3.0 \times 10^{-9}\ \mathrm{C})}{(0.030\ \mathrm{m}) + d} = 1200\ \mathrm{V}$$

Challenge Problems

74. A proton and an alpha particle ($q = +2e, m = 4$ u) are fired directly toward each other from far away, each with an initial speed of $0.010c$. What is their distance of closest approach, as measured between their centers?

75. Bead A has a mass of 15 g and a charge of −5.0 nC. Bead B has a mass of 25 g and a charge of −10.0 nC. The beads are held 12 cm apart (measured between their centers) and released. What maximum speed is achieved by each bead?

76. Two 2.0-mm-diameter beads, C and D, are 10 mm apart, measured between their centers. Bead C has mass 1.0 g and charge 2.0 nC. Bead D has mass 2.0 g and charge −1.0 nC. If the beads are released from rest, what are the speeds v_C and v_D at the instant the beads collide?

77. An electric dipole has dipole moment p. If $r \gg s$, where s is the separation between the charges, show that the electric potential of the dipole can be written
$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$
where r is the distance from the center of the dipole and θ is the angle from the dipole axis.

78. Electrodes of area A are spaced distance d apart to form a parallel-plate capacitor. The electrodes are charged to $\pm q$.
    a. What is the infinitesimal increase in electric potential energy dU if an infinitesimal amount of charge dq is moved from the negative electrode to the positive electrode?
    b. An uncharged capacitor can be charged to $\pm Q$ by transferring charge dq over and over and over. Use your answer to part a to show that the potential energy of a capacitor charged to $\pm Q$ is $U_{cap} = \frac{1}{2}Q\,\Delta V_C$.

79. A sphere of radius R has charge q.
    a. What is the infinitesimal increase in electric potential energy dU if an infinitesimal amount of charge dq is brought from infinity to the surface of the sphere?
    b. An uncharged sphere can acquire total charge Q by the transfer of charge dq over and over and over. Use your answer to part a to find an expression for the potential energy of a sphere of radius R with total charge Q.
    c. Your answer to part b is the amount of energy needed to assemble a charged sphere. It is often called the *self-energy* of the sphere. What is the self-energy of a proton, assuming it to be a charged sphere with a diameter of 1.0×10^{-15} m?
80. The wire in **FIGURE CP28.80** has linear charge density λ. What is the electric potential at the center of the semicircle?

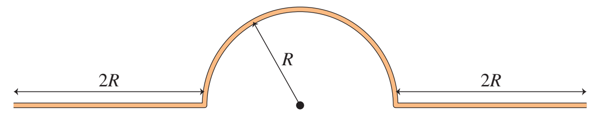


FIGURE CP28.80

81. A circular disk of radius R and total charge Q has the charge distributed with surface charge density $\eta = cr$, where c is a constant. Find an expression for the electric potential at distance z on the axis of the disk. Your expression should include R and Q, but not c.
82. A hollow cylindrical shell of length L and radius R has charge Q uniformly distributed along its length. What is the electric potential at the center of the cylinder?

STOP TO THINK ANSWERS

Stop to Think 28.1: Zero. The motion is always perpendicular to the electric force.

Stop to Think 28.2: $U_b = U_d > U_a = U_c$. The potential energy depends inversely on r. The effects of doubling the charge and doubling the distance cancel each other.

Stop to Think 28.3: c. The proton gains speed by losing potential energy. It loses potential energy by moving in the direction of decreasing electric potential.

Stop to Think 28.4: $V_a = V_b > V_c > V_d = V_e$. The potential decreases steadily from the positive to the negative plate. It depends only on the distance from the positive plate.

Stop to Think 28.5: $\Delta V_{ac} = \Delta V_{bc} > \Delta V_{ab}$. The potential depends only on the *distance* from the charge, not the direction. $\Delta V_{ab} = 0$ because these points are at the same distance.