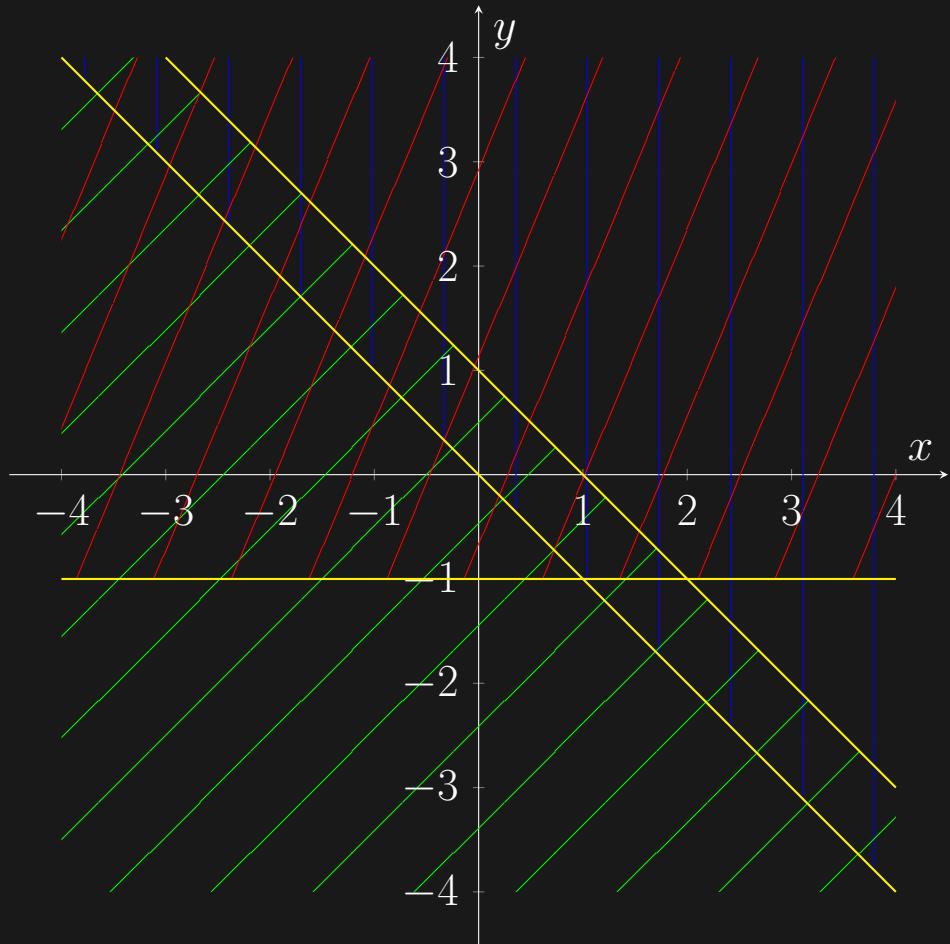


$$\int \int_D \sqrt{x+y} dx dy \quad D = \{(x,y) \in \mathbb{R}^2 \mid 0 \leq x+y \leq 1, y \geq -1\}$$

$$\begin{cases} 0 \leq x+y \\ x+y \leq 1 \\ y \geq -1 \end{cases} \Leftrightarrow \begin{cases} -y \leq x \\ x \leq 1-y \\ y \geq -1 \end{cases} \Leftrightarrow \begin{cases} -y \leq x \leq 1-y \\ y \geq -1 \end{cases}$$



$$\begin{aligned} & \int_{-1}^{\infty} \int_{-y}^{1-y} \sqrt{x+y} dx dy = \int_{-1}^{\infty} \int_{-y}^{1-y} \sqrt{x+y}(x+y)' dx dy = \\ &= \int_{-1}^{\infty} \frac{(x+y)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_{-y}^{1-y} dy = \int_{-1}^{\infty} \frac{2}{3} - \frac{0}{3} dy = \frac{2}{3}y \Big|_{-1}^{\infty} = \infty \end{aligned}$$

$$\int \int_D x \, dx \, dy \quad D = \{(x, y) \in \mathbb{R}^2 \mid x \geq y^2 - 3, x \leq 9 + y, x \leq 9 - y\}$$

$$\begin{cases} x \geq y^2 - 3 \\ x \leq 9 + y \\ x \leq 9 - y \end{cases} \Leftrightarrow \begin{cases} y^2 - 3 \leq x \leq 9 - y \\ y^2 - 3 \leq x \leq 9 + y \end{cases} \Leftrightarrow \begin{cases} y^2 - 3 \leq 9 - y \\ y^2 - 3 \leq 9 + y \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y^2 - 3 + y - 9 \leq 0 \\ y^2 - 3 - y - 9 \leq 0 \end{cases} \Rightarrow \begin{cases} y^2 + y - 12 = 0 \\ y^2 - y - 12 = 0 \end{cases}$$

$$y^2 - y - 12 = 0$$

$$\delta = (-1)^2 - 4 * 1 * (-12) = 1 + 48 = 49 = 7^2$$

$$\frac{-(-1) \pm \sqrt{\delta}}{2*1} = \frac{1 \pm 7}{2} = \begin{cases} \frac{1-7}{2} = \frac{-6}{2} = -3 \\ \frac{1+7}{2} = \frac{8}{2} = 4 \end{cases}$$

$$\begin{array}{c|cccccc} y & & -3 & & 4 & & \\ \hline y^3 - y - 12 & + + + + & 0 & - - - - & 0 & + + + + & \\ \Rightarrow y \in [-3, 4] \end{array}$$

$$y^3 + \underbrace{y}_{4y-3y} - 12 = 0$$

$$\begin{array}{c|cccccc} y & & -4 & & 3 & & \\ \hline y^3 + y - 12 & + + + + & 0 & - - - - & 0 & + + + + & \\ \Rightarrow y \in [-4, 3] \\ \Rightarrow y \in [-3, 3] \end{array}$$

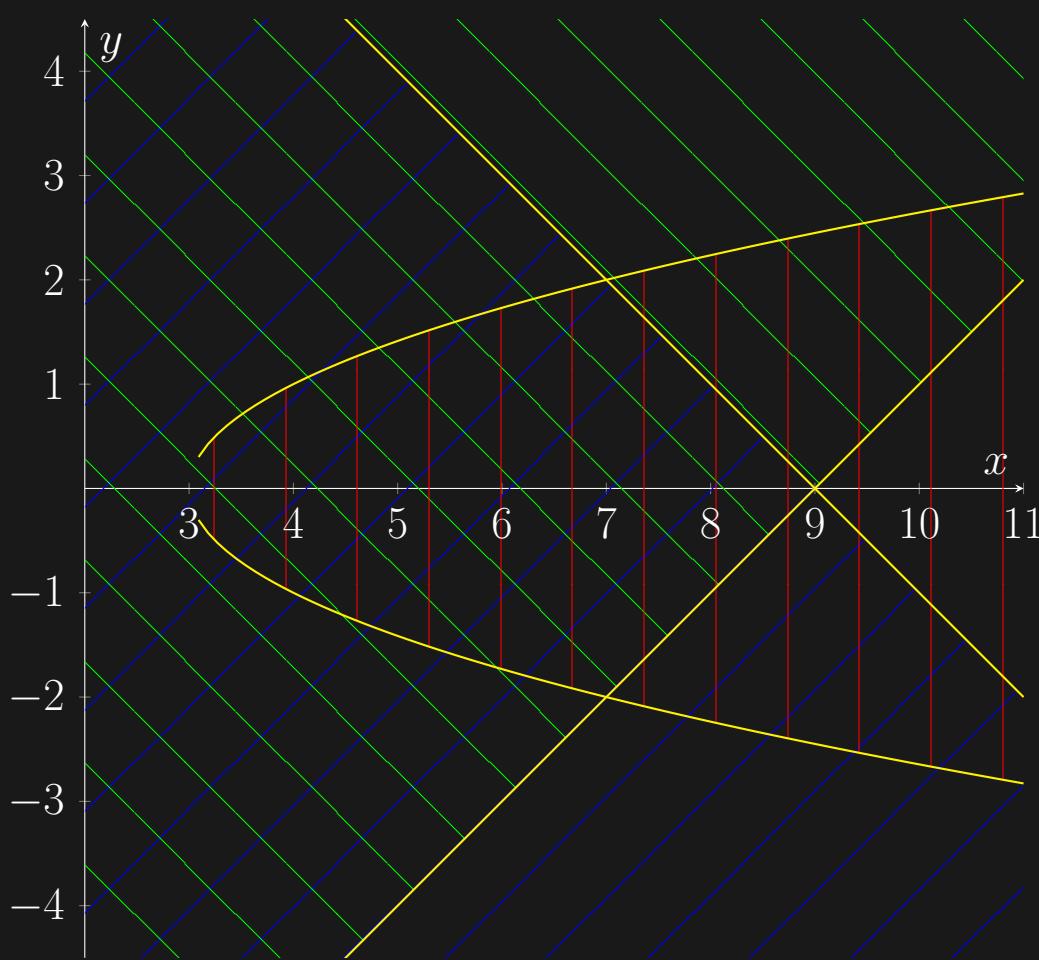
$$y^2 - 3 \leq x \leq \min(9 - y, 9 + y)$$

$$g : [-3, 3] \rightarrow \mathbb{R}, g(y) = \min(9 - y, 9 + y)$$

$$9 - y \leq 9 + y \Leftrightarrow 0 \leq y \Rightarrow y \in [0, 3]$$

$$9 - y \geq 9 + y \Leftrightarrow 0 \geq y \Rightarrow y \in [-3, 0]$$

$$D = \begin{cases} y^2 - 3 \leq x \leq 9 + y, -3 \leq y \leq 0 \\ y^2 - 3 \leq x \leq 9 - y, 0 \leq y \leq 3 \end{cases}$$



$$\begin{aligned} & \int_{-3}^0 \int_{y^2-3}^{9+y} x \, dx \, dy + \int_0^3 \int_{y^2-3}^{9-y} x \, dx \, dy = \int_{-3}^0 \frac{x^2}{2} \Big|_{y^2-3}^{9+y} \, dy + \\ & + \int_0^3 \frac{x^2}{2} \Big|_{y^2-3}^{9-y} \, dy = \frac{1}{2} \int_{-3}^0 (y^2 - 3)^2 - (9 + y)^2 \, dy + \\ & + \frac{1}{2} \int_0^3 (y^2 - 3)^2 - (9 - y)^2 \, dy = \frac{1}{2} \int_{-3}^0 y^4 - 6y^2 + 9 - 81 - 18y - y^2 \, dy + \\ & + \frac{1}{2} \int_0^3 y^4 - 6y^2 + 9 - 81 + 18y - y^2 \, dy = \frac{1}{2} \int_{-3}^0 y^4 - 7y^2 - 18y - 72 \, dy + \\ & + \frac{1}{2} \int_0^3 y^4 - 5y^2 - 18y - 72 \, dy = \frac{1}{2} \left(\frac{y^5}{5} - 7 \frac{y^3}{3} - 18 \frac{y^2}{2} - 72y \right) \Big|_{-3}^0 + \\ & + \frac{1}{2} \left(\frac{y^5}{5} - 5 \frac{y^3}{3} - 18 \frac{y^2}{2} - 72y \right) \Big|_0^3 = -\frac{1}{2} \left(\frac{(-3)^5}{5} - 7 \frac{(-3)^3}{3} - 9(-3)^2 - 72(-3) \right) + \\ & + \frac{1}{2} \left(\frac{3^5}{5} - 5 \frac{3^3}{3} - 9 * 3^2 - 72 * 3 \right) = \frac{\frac{3^5}{5} - 7 * 3^2 + 3^4 - 216 + \frac{3^5}{5} - 5 \frac{3^3}{3} - 3^4 - 216}{2} = \\ & = \frac{2 \frac{3^5}{5} - 12 * 3^2 - 432}{2} = \frac{3^5}{5} - 2 * 3^3 - 8 * 3^3 = \frac{3^5}{5} - 10 * 3^3 \end{aligned}$$

WIP