$W_1 = \langle (0,1,3), (1,1,1) \rangle$

$W_2 = \langle (1,2,4), (3,5,0) \rangle$ dim + baza $W_1, W_2, W_1 + W_2, W_1 \cap W_2$

$\dim W_1 = 2$, $\dim W_2 = 2$

Fie $x \in W_1 \cap W_2 \Rightarrow x \in W_1$ şi $x \in W_2$

$$\left.\begin{aligned} x &= a(0,1,3) + b(1,1,1) \\ x &= c(1,2,4) + d(3,5,0) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &b - c - 3d = 0 \\ &a + b - 2c - 5d = 0 \\ &3a + b - 4c = 0 \end{aligned}\right\} \Rightarrow A^e = \left(\begin{array}{cccc|c} 0 & 1 & -1 & -3 & 0 \\ 1 & 1 & -2 & -5 & 0 \\ 3 & 1 & -4 & 0 & 0 \end{array}\right) \xrightarrow{L_1 \leftrightarrow L_2}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & -5 & 0 \\ 0 & 1 & -1 & -3 & 0 \\ 3 & 1 & -4 & 0 & 0 \end{array}\right) \xrightarrow{L_3 \to L_3 - 3L_1} \left(\begin{array}{cccc|c} 1 & 1 & -2 & -5 & 0 \\ 0 & 1 & -1 & -3 & 0 \\ 0 & -2 & 2 & 15 & 0 \end{array}\right) \xrightarrow[L_3 \to L_3 + 2L_2]{L_1 \to L_1 - L_2} \left(\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{array}\right) \xrightarrow{L_3 \cdot \frac{1}{9}}$$

$W_1 = \langle (0,1,3), (1,1,1) \rangle$

$W_2 = \langle (1,2,4), (3,5,0) \rangle$ dim + baza $W_1, W_2, W_1+W_2, W_1 \cap W_2$

$W_1 + W_2 = \langle W_1 \cup W_2 \rangle = \langle (0,1,3), (1,1,1), (1,2,4), (3,5,0) \rangle$

Fie $B = \begin{pmatrix} 0 & 1 & 1 & 3 \\ 1 & 1 & 2 & 5 \\ 3 & 1 & 4 & 0 \end{pmatrix}$. $\Delta = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \neq 0 \to rg\, B \geq 2$ $\Delta_1 = \begin{vmatrix} 0 & 1 & 3 \\ 1 & 1 & 5 \\ 3 & 1 & 0 \end{vmatrix} = 0+3+15-9 = 9 \neq 0 \Rightarrow$

$\Rightarrow \{(0,1,3), (1,1,1), (3,5,0)\}$ S.L.i $(1,2,4) = (0,1,3) + (1,1,1) + 0(3,5,0)$ S.G $\Rightarrow \{\underbrace{(0,1,3)}_{v_1}, \underbrace{(1,1,1)}_{v_2}, \underbrace{(3,5,0)}_{v_3}\}$

baza pt $W_1 + W_2$

$\Rightarrow \dim W_1 \cap W_2 = 1.$

$\dim(W_1 + W_2) = \underset{2}{\dim(W_1)} + \underset{2}{\dim(W_2)} - \underset{1}{\dim(W_1 \cap W_2)}$

$\dim(W_1 + W_2) = 3.$

$W_1 + W_2 = \langle v_1, v_2, v_3 \rangle.$

Def: Suma directă $\oplus$

Fie V un K sp vect $U_1, \dots U_n \leq V$. Sp că V e suma directă a subsp $U_1 \dots U_n$ dc $\forall x \in V$ se scrie în mod unic sub forma $x_1 + \dots + x_m$, $x_1 \in U_1 \dots x_m \in U_m$.

Not $V = U_1 \oplus U_2 \oplus \dots \oplus U_n$

Propoziție: Fie V un K sp vect, $U_1 \dots U_n \leq V$ subsp. S.E.

1. $V = U_1 \oplus U_2 \oplus \dots \oplus U_m$

pt $\forall i, j = \overline{1,m}$ $\quad U_i \cap \left(\sum_{\substack{j=1 \\ i \neq j}}^{m} U_j \right) = 0_V$

Fie $V_1 = \{(x,y,0) \mid x,y \in \mathbb{R}\}$.   a) $V_1, V_2 \leq \mathbb{R}^3$, dim V_1, V_2

$V_2 = \{(t,0,t) \mid t \in \mathbb{R}\}$ b) Dem $V_1 \oplus V_2 = \mathbb{R}^3$

$V_1 = \{(x,y,0) \mid x,y \in \mathbb{R}\} = \{(x,0,0) + (0,y,0) \mid x,y \in \mathbb{R}\} = \{x(1,0,0) + y(0,1,0) \mid x,y \in \mathbb{R}\}$.

$V_1 = \underset{\leq \mathbb{R}^3}{<(1,0,0),(0,1,0)>}$. $\begin{matrix}(1,0,0) = e_1 \\ (0,1,0) = e_2\end{matrix}$ vect. bazei can $\Rightarrow \{e_1, e_2\}$ bază pt V_1. $\Rightarrow \dim V_1 = 2$

$V_2 = \{t(1,0,1) \mid t \in \mathbb{R}\} = \underset{\leq \mathbb{R}^3}{<(1,0,1)>} \Rightarrow \{(1,0,1)\}$ bază $\Rightarrow \dim V_2 = 1$.

$V_1 \cap V_2 \ni v \Rightarrow v \in V_1$ și $v \in V_2$ $\Rightarrow \begin{matrix} v = a(1,0,0) + b(0,1,0), c, a, b \in \mathbb{R} \\ v = c(1,0,1) \end{matrix} \Rightarrow \left\{ \begin{matrix} a - c = 0 \\ b = 0 \\ c = 0 \end{matrix} \right\} \Rightarrow a = 0$

Cum $c = 0 \Rightarrow v = 0_{\mathbb{R}^3} \Rightarrow V_1 \cap V_2 = \{0_{\mathbb{R}^3}\}$

Fie $V_1 = \{(x,y,0) \mid x,y \in \mathbb{R}\}$. a) $V_1, V_2 \subseteq \mathbb{R}^3$, $\dim V_1, V_2$

$V_2 = \{(t,0,t) \mid t \in \mathbb{R}\}$ b) Dem $V_1 \oplus V_2 = \mathbb{R}^3$

$$\left.\begin{array}{l} V_1 \leq \mathbb{R}^3 \\ V_2 \leq \mathbb{R}^3 \end{array}\right\} \Rightarrow V_1, V_2 \subseteq \mathbb{R}^3$$

$$\dim(V_1+V_2) = \underset{\underset{2}{\parallel}}{\dim V_1} + \underset{\underset{1}{\parallel}}{\dim V_2} - \underset{\underset{0}{\parallel}}{\dim(V_1 \cap V_2)}$$

$$\dim(V_1+V_2) = 3.$$

$$\left.\begin{array}{l} \left.\begin{array}{l} \dim(V_1+V_2) = 3 \\ V_1, V_2 \subseteq \mathbb{R}^3 \end{array}\right\} \Rightarrow V_1 + V_2 = \mathbb{R}^3 \\ V_1 \cap V_2 = \{0_{\mathbb{R}^3}\} \end{array}\right\} \Rightarrow V_1 \oplus V_2 = \mathbb{R}^3.$$

Aplicații liniare

Definiție: Fie U, V două K - spații vectoriale $(\ast)$

o funcție $f: U \to V$ s.n. aplicație liniară dacă $f(ax+by) = a \cdot f(x) + b \cdot f(y)$; $\forall a, b \in K$
$x, y \in U$

Fie $A \in \mathcal{M}_{m,n}(K)$; $m, n \in \mathbb{N}^*$

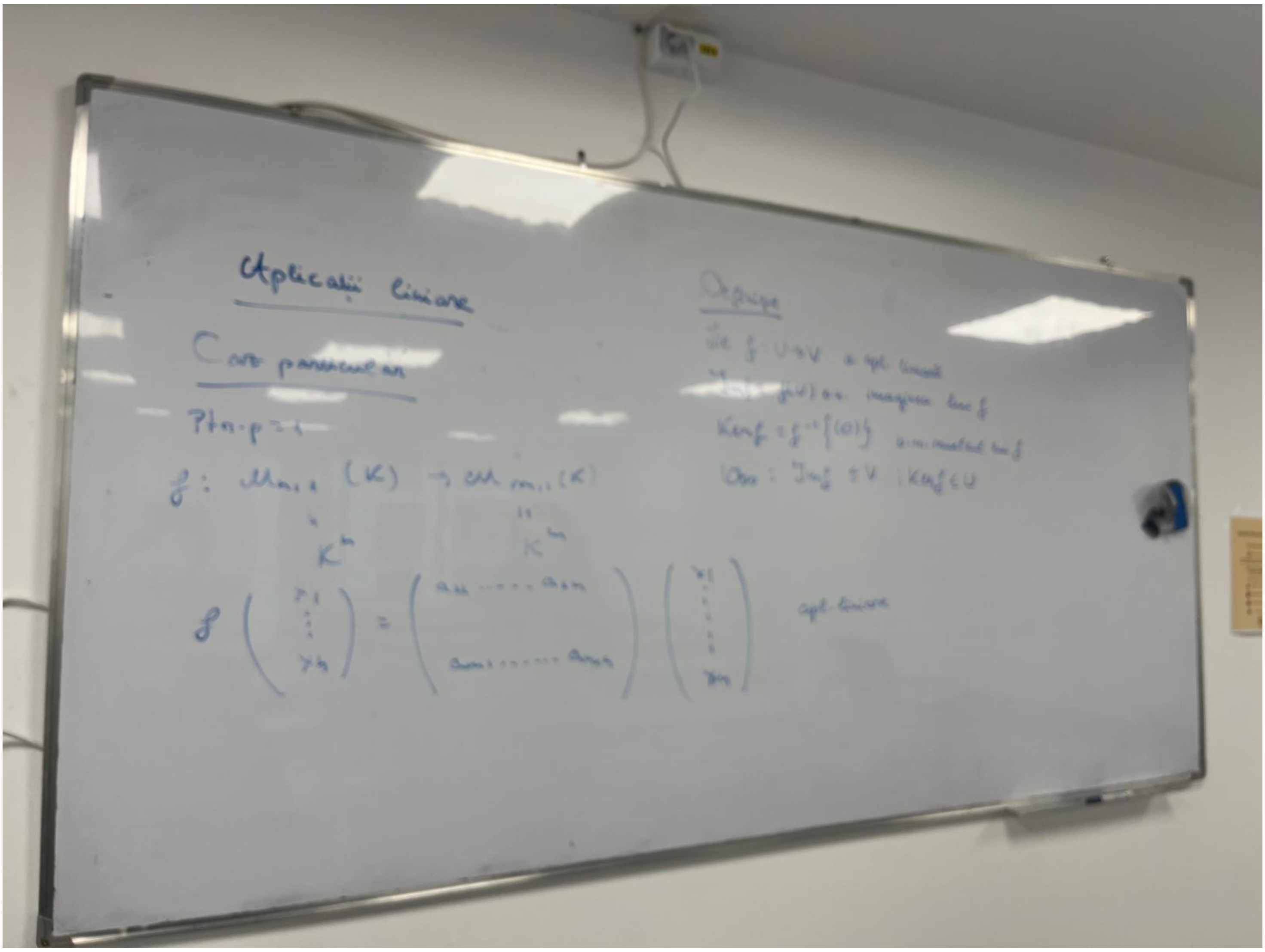
Fie $p \in \mathbb{N}^*$. Atunci

$f: \mathcal{M}_{n,p}(K) \to \mathcal{M}_{m,p}(K)$

$f(X) = A \cdot X$ apl. liniară ; $\forall X \in \mathcal{M}_{n,p}(K)$

$\downarrow$

$m,n \times n,p$


Aplicații liniare
Caz particular
apl. liniara

Aplicații liniare

Prop. Fie $f : U \to V$ apl. lin.

1) f surj. $\Leftrightarrow$ $\mathrm{Im} f = U$

2) f inj. $\Leftrightarrow$ $\mathrm{Ker} f = \{0\}$

Prop. Fie V un K sp. vect. de dim n. Atunci $V \simeq K^n$

Teoremă (propr. de universalitate)

Fie V un K-sp. vect. . Fie $B = \{x_1, \dots, x_n\}$ o bază a lui V

Fie W un K-sp. vect. . Atunci ptr. orice fam. de elem. $(y_i)_{i=\overline{1,n}}$ din W

$\exists$ o unică apl. liniară $f : V \to W$ ptr. care $f(x_i) = y_i$; $\forall i = \overline{1,n}$

Mai mult:

- f inj. $\Leftrightarrow$ $(y_i)_{i=\overline{1,n}}$ S.L.I. în W
- f surj. $\Leftrightarrow$ $(y_i)_{i=\overline{1,n}}$ S.G. în W
- f bij. $\Leftrightarrow$ $(y_i)_{i=\overline{1,n}}$ bază în W

Th. rang-defect Fie $f : V \to W$ apl. lin. Atunci:

$$\dim V = \underbrace{\dim \mathrm{Im} f}_{\text{rangul lui } f} + \underbrace{\dim \mathrm{Ker} f}_{\text{defectul lui } f}$$