Seminar 1 - LFA

1) Fie $\Sigma = \{0,1\}$

Construiți DFA-uri pt următoarele limbaje

$L_1 = \{ w \mid w \text{ conține doar } 0 \}$

$L_2 = \{ w \mid |w|_0 \in 2\mathbb{Z} \}$

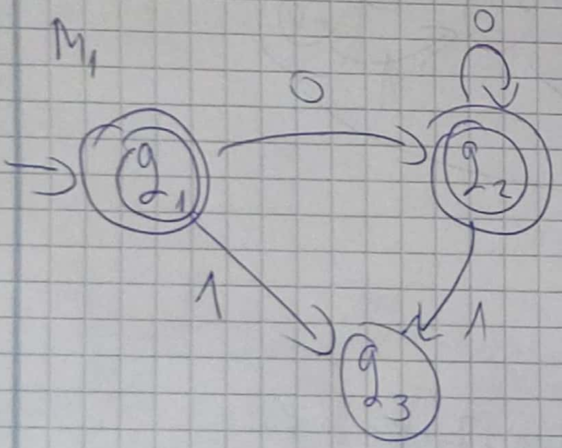
$L_3 = \{ w \mid |w|_1 \in 2\mathbb{Z}+1 \}$

$L_4 = \{ w \mid w \text{ nu conține } 101 \}$

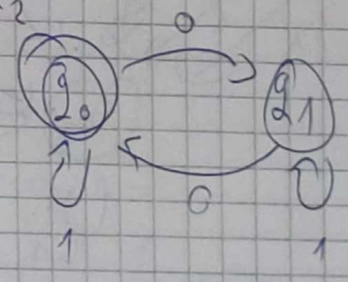
$L_5 = \{ 0^{2n} \mid n \geq 1 \}$, $L_6 = \{ 1^{2n+1} \mid n \geq 1 \}$

$L_7 = \{ 0^{2n} 1^{2m+1} \mid n, m \geq 1$

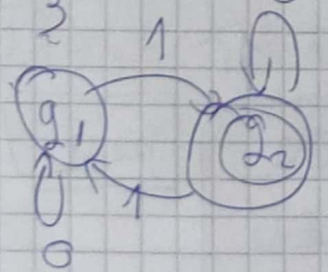
M_1



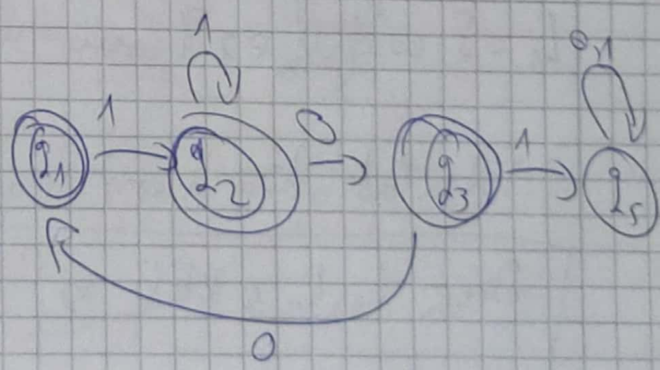
M_2

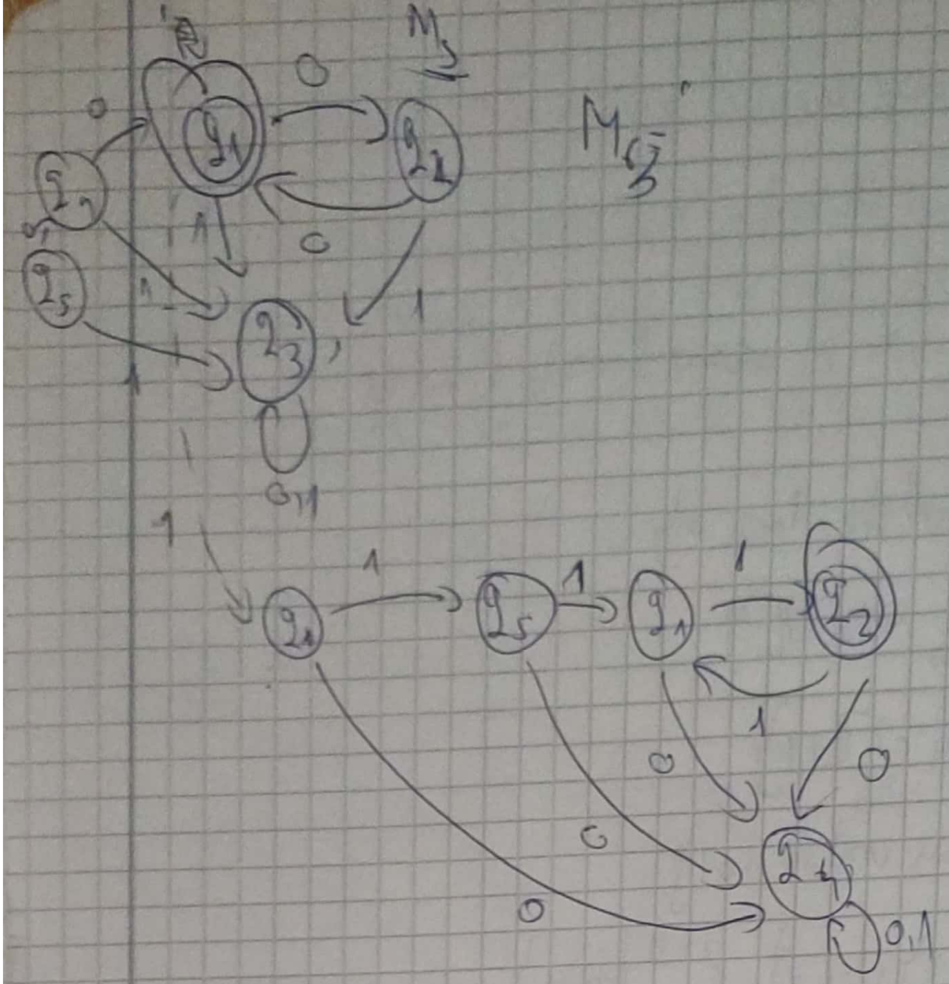


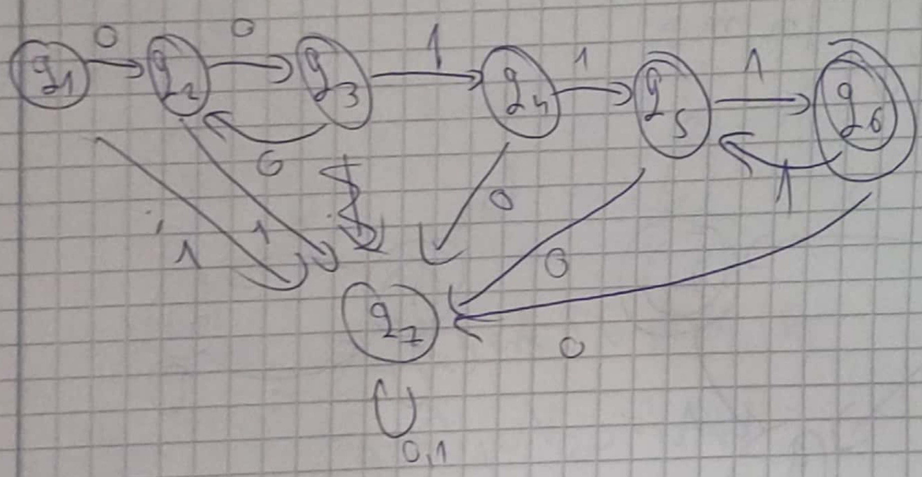
M_3



M_4







$L_8 = \{ w \mid |w|_0 \in 2\mathbb{Z} \text{ sau } |w|_1 \in 2\mathbb{Z}+1 \}$

2) Arătați, fără a construi automate, că $L_1, L_7, L_8 \in REG$

Dacă $L_1, L_2 \in REG \Rightarrow \begin{cases} L_1 \cup L_2 \in REG \\ L_1 L_2 \in REG \end{cases}$

$L \in REG \Rightarrow L^* \in REG$

Teorie!

11010

Pt L_1:

$\{0\} \in REG \Rightarrow \underbrace{\{0\}^*}_{=L_1} \in REG$

$\varepsilon \in L^*$

Pt L_7:

Din (1) $\Rightarrow L_5, L_6 \in REG \Rightarrow L_7 = L_5 L_6 \in REG$
(descen)

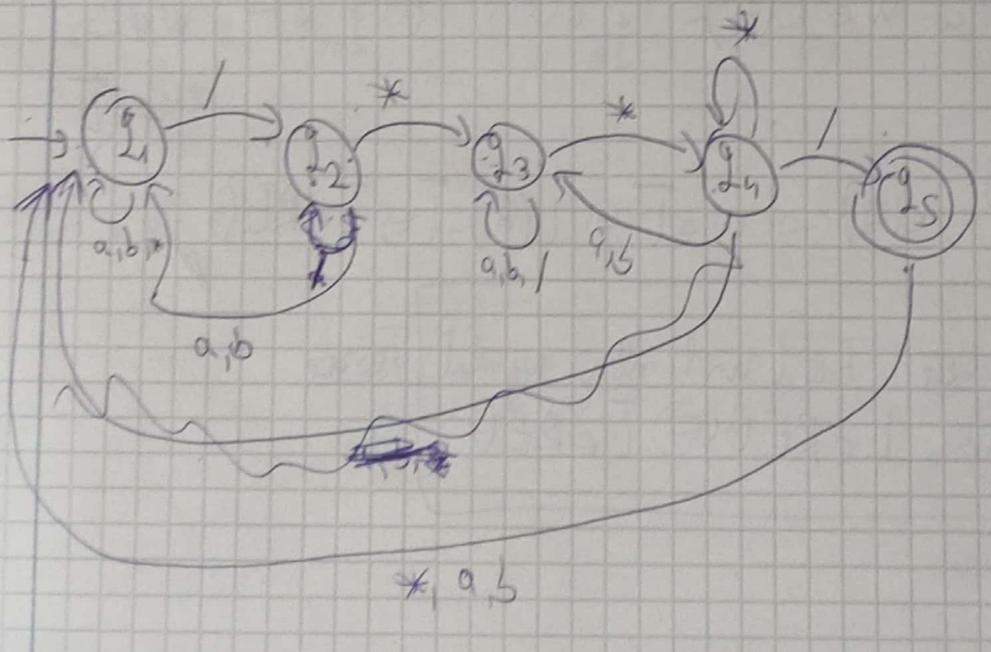
Pt L_8

Din (1) $\Rightarrow L_2, L_3 \in REG \Rightarrow L_8 = L_2 \cup L_3 \in REG$

3) Fie $\Sigma = \{$"/", "*", a, b$\}$

Scrieți un DFA care recunoaște comentariile bine formate peste Σ din C

/* ...... */



4) Fie Σ un alfabet oarecare și $L \subseteq \Sigma^*$ un limbaj

Să se arate că dacă $L \in REG \Rightarrow \overline{L} \in REG$

unde $\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$

$L \in REG \Rightarrow \overline{L} \in REG$

Fie $L \in REG$. Deci $\exists A = (\Sigma, Q, q_0, F, \delta)$ un DFA cu $L(A) = L$

Definim $A' = (\Sigma, Q, q_0, Q \setminus F, \delta)$

Vrem să arătăm că $L(A') = \overline{L}$

Fie ~~$w \in \Sigma^*$~~ $w = w_1 w_2 \dots w_n \in \Sigma^*$ cu $w_i \in \Sigma$

Vrem să arătăm că $w \in L(A') \Leftrightarrow w \in \overline{L}$

Definim „rularea lui A' pe w ca secvența $q_0, \dots, q_n$ unde:

q_0 - starea inițială stabilită

$q_{i+1} = \delta(q_i, w_{i+1})$ pt $i = 0, \dots, n-1$

Avem că $w \in L(A') \Leftrightarrow q_n \in Q \setminus F \Leftrightarrow q_n \notin F \Rightarrow w \notin L \Rightarrow$

$\Rightarrow w \in \overline{L}$

$w \in L(A') \Leftrightarrow w \in \overline{L} \Rightarrow L(A') = \overline{L} \Rightarrow \overline{L} \in REG$ q.e.d.

Concluzie: Fie L un limbaj regulat dacă $L \in REG \Leftrightarrow \overline{L} \in REG$ | Teorie

Seminar 2 - LFA

Scrieți automate pt

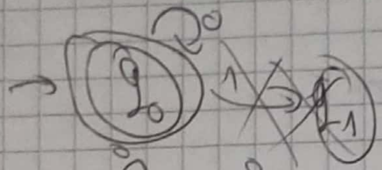
1) $\{0\}^*, \varepsilon, \{0\}$

2) $\{w \mid w$ conține cel puțin 3 de 1$\} = L_1$

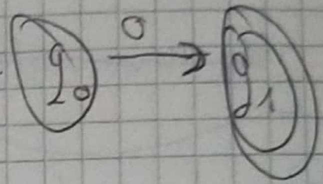
3) $\{w \mid w$ conține 0101$\}$

4) $\{w \mid w$ se termină cu 00$\}$

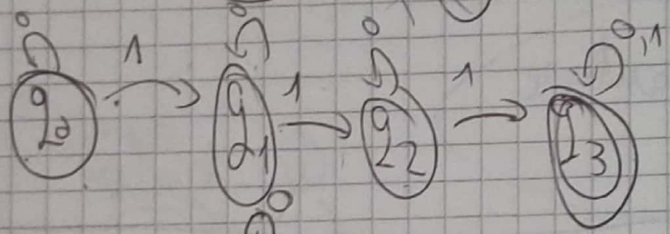
1) $\{0\}^*$



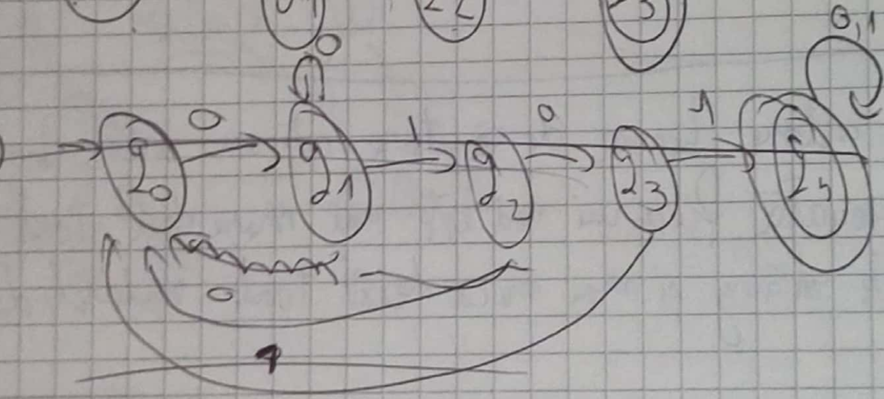
$\{0\}$

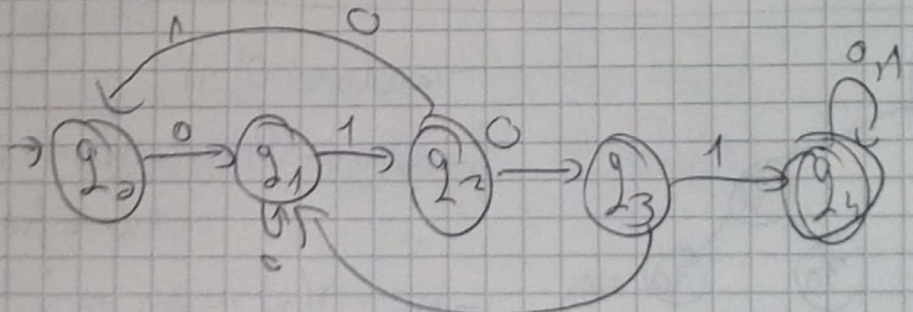


2)

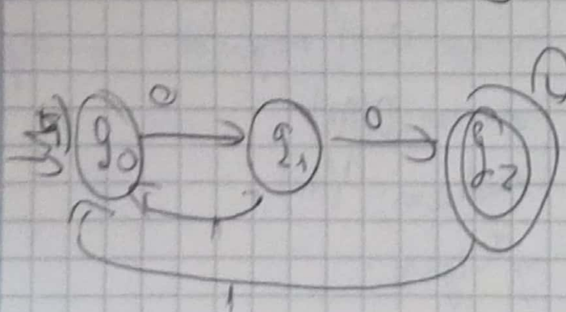


3)

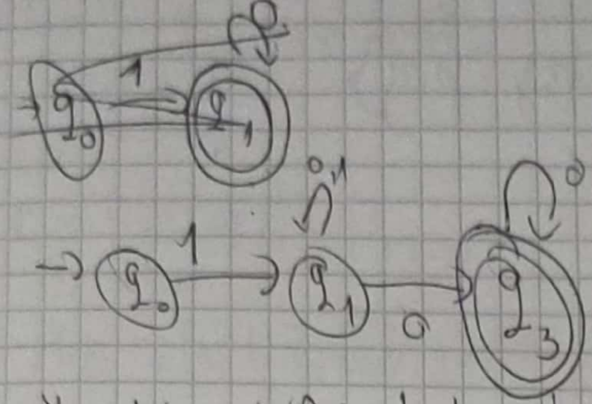




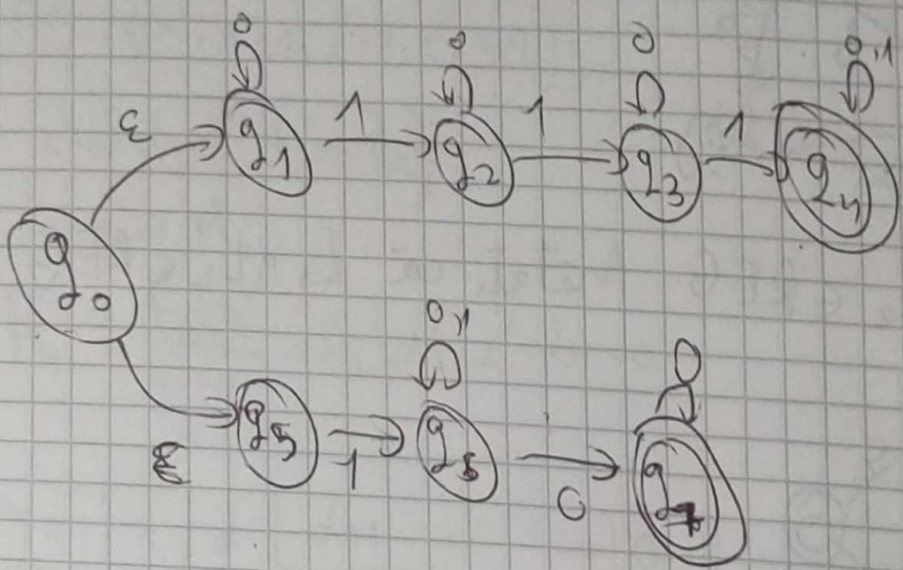
4)



5) $\{w \mid w$ începe cu 1 și se termină cu 0$\} = L_2$



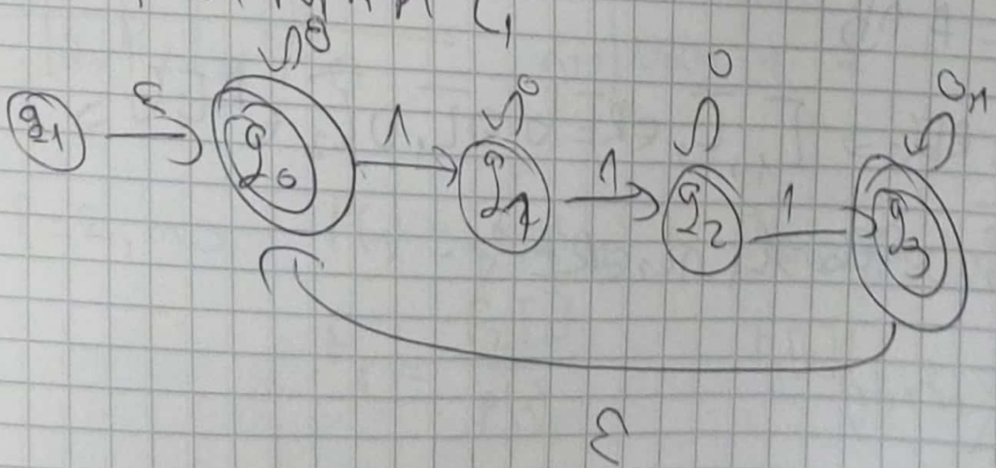
6) Scrieți un NFA pt $L_1 \cup L_2$



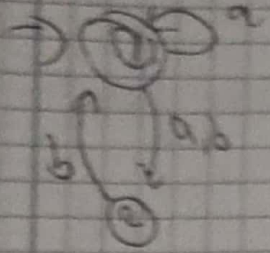
~~7) L* Scrieți un NFA pt L_1^*~~

Corectați:

Scrieți un NFA pt L_1^*



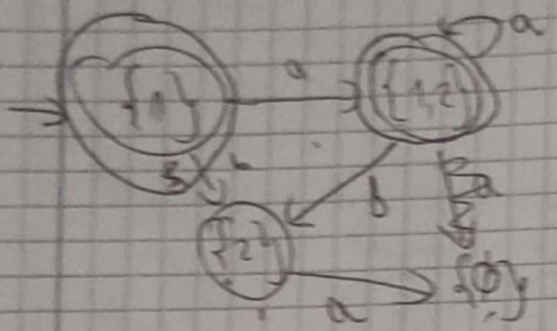
Convertiți, folosind algoritmul de la curs, NFA-ul într-un DFA



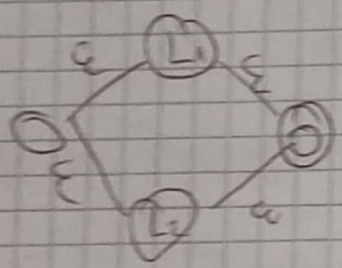
$M = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$E(\{1\}) = 1$

$F = \{1\}$



- Fie L_1 și $L_2 \in REG$. Arătați că $L_1 \cap L_2 \in REG$



$!(p \&\& q) = !p \,||\, !q$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$L_1, L_2 \in REG \Rightarrow \overline{L_1}, \overline{L_2} \in REG \Rightarrow \overline{L_1} \cup \overline{L_2} \in REG \Rightarrow$

$\Rightarrow \overline{\overline{L_1} \cup \overline{L_2}} \in REG \Rightarrow L_1 \cap L_2 \in REG$

Seminar 4 LFA

Lema de pompare

Fie $L \subseteq \Sigma^*$. Atunci $\exists p$ a.î. $\forall s \in L, |s| \geq p, \exists x, y, z \in \Sigma^*$ a.î.

i) $\forall i \geq 0, xy^i z \in L$

ii) $y \neq \varepsilon$

iii) $|xy| \leq p$

1) Arătați că urm. limbaje nu sunt regulate.

i) $L_1 = \{ w \in \{ "(", ")" \}^* \mid w$ sir de paranteze $\}$

Pres. $L_1 \in REG$. Atunci $\exists p$ lungime de pompare pt L_1

Alegem $s = (^p)^p \in L$ $|s| \geq p$. Fie x, y, z a.î. $s = xyz$

$|xy| \leq p \Rightarrow x = (^k, k \geq 0$

$y = (^m, m \geq 1$

$z = (^{p-(m+k)})^p$

Căutăm un $i \geq 0$ a.î. $xy^i z \notin L$

pt $i = 2$ $xy^2 z = (^k (^{2m} (^{p-(m+k)})^p = (^{p+m})^p$

dar cum $m \geq 1 \Rightarrow p+m > p$

deci avem mai multe (decât) $\Rightarrow xy^2 z \notin L_1 \Rightarrow$

$\Rightarrow L \notin REG$

ii) $L_2 = \{a^n b^n c^n \mid n \geq 0\}$

Pres $L_2 \in REG$. Atunci $\exists$ p lungime de pompare pt L_2

Alegem $s = a^p b^p c^p$ $|s| \geq p$ fie x, y, z a.î. $s = xyz$

$|xy| < p \Rightarrow x = a^k$ pt $k \geq 0$

$y = a^m$, pt $m \geq 1$ (cond ii)

$z = a^{p-(m+k)} b^p c^p$

Luăm un $i \geq 0$ a.î. $xy^i z \notin L_2$

pt $i = 2$

$xy^2z = a^k a^{2m} a^{p-(m+k)} b^p c^p = a^{p+m} b^p c^p$

Avem mai multe de a decât de b și c

$\Rightarrow p+m > p$ pt că $m \geq 1 \Rightarrow xy^2z \notin L_2 \Rightarrow L_2 \notin REG$

iii) $L_3 = \{www \mid w \in \{a,b\}^*\}$

Pres $L_3 \in REG \Rightarrow \exists p$ ~~a.î.~~ lungime de pompare pt $L_2 \Rightarrow$

$\Rightarrow$ alegem ~~$s = a^p a^p a^p$~~ $\in L_2$ a.î. $|s| \geq p$. Fie x, y, z a.î. $s = xyz$

$s = a^p b a^p b a^p b$

$|xy| < p \Rightarrow x = a^k$ $k \geq 0$

$y = a^m$; $m \geq 1$

$z = a^{p-(k+m)} b a^p b a^p b$

~~$i=2$: $xy^2z = a^k a^{2m} a^{p-(k+1)} a^p a^p =$~~

~~$= a^{p+m} a^p a^p = a^{p+m} a^p a^p \Rightarrow p+m > p$ pt că $m \geq 1 \Rightarrow$~~

~~$\Rightarrow s \notin L \Rightarrow L \notin REG$~~

pt $i=2$ $xy^2z = a^{p+m} b a^p b a^1 b$

$p+m > \frac{|xy^2z|}{3} \Rightarrow xy^2z = w_1 w_2 w_3$

$w_1 \in a^*$ dar $w_2, w_3 \notin a^*$

(w_1 conține doar a, dar w_2, w_3 conține și b-uri)

IV $L_4 = \{a^i b^j \mid i > j\}$

Pp $L_4 \in REG$, deci $\exists p$ ~~lungimea~~ lungimea de pompare pt L_4

~~Alegem $s = a^{p/2} b^{p/2 - 1}$, $s = a^{p+1} b^{p/2+1}$, $|s| \geq p$. Fie x, y, z a.î. $s = xyz$~~

~~$x = a$~~

$x = a$

Alegem $s = a^{p+1} b^{p}$. Fie x, y, z a.î. $s = xyz$

~~$x = a^k$~~

$|xy| \leq p \Rightarrow$

$x = a^k$ $k \geq 0$

$y = a^m$ ~~b, $m \geq 1$~~

~~$z = b^{p+3}$~~ $z = a^{p+1-(m+k)} b^p$

~~$xy^2z = a^k \cdot a^{2m} \cdot b$~~

$xy^iz = a^k \cdot a^{im} \cdot a^{p+1-(m+k)} \cdot b^p = a^{p+1+(i-1)m} b^p$

pt $i=0$

$xy^0z = a^{p+1-m} b^p$

$p+1-m \leq p \Rightarrow xy^0z \notin L_4 \Rightarrow L_4 \notin REG$

2) Adv / Fals?

1) Limbajele neregulate sunt închise la complement? (A)

Fie $L \notin REG$ Dacă $\overline{L} \in REG \Rightarrow \underbrace{\overline{\overline{L}}}_{L} \in REG$ (F)

2) ~~"~~ reuniuni

$\cancel{L_1 \notin REG \Rightarrow \overline{L_1} \notin REG}$
$\cancel{L_2 \notin REG \Rightarrow \overline{L_2} \notin REG}$ $\Rightarrow$

Fie $\Sigma^* \in REG$ Σ^* toate stringurile acceptate

Fie $L \notin REG$ oarecare

$\overline{L} \in REG$ (1)

$L \cup \overline{L} = \Sigma^* \in REG$

3) Fie $L = \{a^i b^j c^k \mid i, j, k \in \mathbb{N}$ și dacă $i=1$ atunci $j=k\}$

Arătați $L \notin REG$, dar satisface lema de pompare

Căutăm $L' \in REG$, a.î. $L \cap L' \notin REG$

$L = \{a b^j c^j \mid j \in \mathbb{N}\} \cup \{a^i b^* c^* \mid i \neq 1\}$

$L' = a b^* c^* \in REG$

$L \cap L' = \{a b^j c^j \mid j \in \mathbb{N}\} \notin REG \Rightarrow L \notin REG$

Seminar 5

Scrieți CFG-uri pt

i) $\{ a^n b^n \mid n \geq 0 \}$

ii) $\{ w \in \{ "[", "]" \}^* \mid w$ bine parantezat $\}$

iii) $\{ w \in \{ "[" "]", "(", ")" \}^* \mid w$ bine parantezat $\}$

IV $\{ w \in \{a, b\}^* \mid w = w^R \}$

V $\{ w w^R \mid w \in \{a, b\}^* \}$

I $S \Rightarrow aSb \mid \varepsilon$

~~$S_1 \Rightarrow bSa \mid \varepsilon$~~

II ~~$S \rightarrow [S] \mid \varepsilon$~~. $S \rightarrow SS \mid \varepsilon \mid [S]$

~~$S \Rightarrow [S \mid S] \mid \varepsilon$~~

~~$S \rightarrow [S_1 \mid \varepsilon$~~

~~$S_1 \rightarrow S_1] \mid \varepsilon$~~

ab ab ✓

aba aba

S S

III $S \rightarrow [S] \mid (S) \mid SS \mid \varepsilon$

IV $S \rightarrow a \mid b \mid aSa \mid bSb \mid \varepsilon$

V $S \rightarrow aSa \mid bSb \mid \varepsilon \mid$.

VI $\{w \in \{a,b\}^* \mid w$ începe și se termină cu același simbol $\}$

VII $\{w \in \{a,b\}^* \mid |w|$ pară $\}$

VIII $\{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$

VI $S = \varepsilon \mid aS_1a \mid bS_1b$

$S_1 = \cancel{ab} \; \varepsilon \mid S_1a \mid S_1b$

VII $w \in \{a,b\}^* \mid |w| \in 2\mathbb{Z}\}$

$S \to Sab \mid Saa \mid Sba \mid Sbb \mid \varepsilon$

VIII $w \in \{a,b\}^* \mid |w|_a = |w|_b\}$

$\cancel{S \to Saa \mid Sbb \mid \varepsilon \mid Sabab \mid Sabba \mid Sbaba}$

$\cancel{S \to Tab \mid Tba \mid \varepsilon}$

$\cancel{T \to T_1 \mid T_2}$ $T \to aTb$

$\cancel{T_1 \to aT_1 \mid \varepsilon}$

$\cancel{T_2 \to bT_2 \mid \varepsilon}$

$\cancel{S \to Sab \mid Sba \mid ab \; aSb \mid bSa \mid \varepsilon}$

$S \to aSb \mid bSa \mid SS \mid \varepsilon$

IX $\{w \in \{a,b\}^* \mid w$ conține cel puțin 3a$\}$

X $\{w \in \{a,b\}^* \mid |w| > |w|_b\}$

$\boxed{X}$ ~~$S \to aaaS_1 \mid aaS_1a \mid aS_1aa \mid S_1aaa \mid \varepsilon$~~

~~$S_1 \to a \mid b \mid S_1b \mid S_1a \mid \varepsilon$~~

$S \to TaTaTaT$

$T \to aT \mid bT \mid \varepsilon$

$\overline{X}$ ~~$S \to SaaSb \mid SabSa \mid SaSS \mid \varepsilon \mid Sa$~~

$T \to SaS$

$S \to aSb \mid bSa \mid SS \mid \varepsilon \mid a$

~~$T \to aT \mid Ta \mid \varepsilon$~~

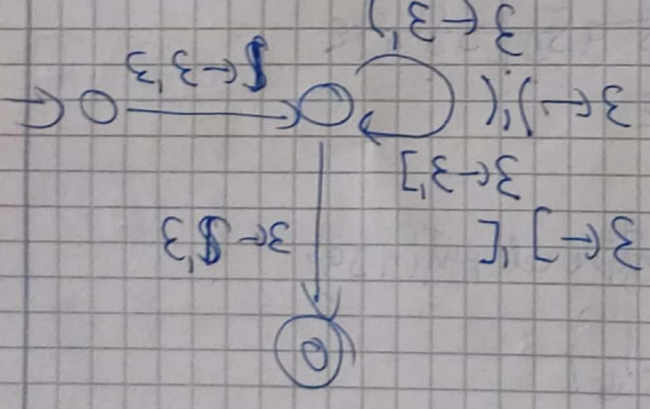
$\boxed{XI}$ $\overline{\{ a^n b^n \mid n \geq 0 \}}$

$S \to bT \mid Ta \mid aSb$

$T \to aT \mid bT \mid \varepsilon$

Scrieți PDA-uri pt un iii), x)

$w \in \{ "[", "]", "(", ")" \mid w$ bine parantezat $\}$



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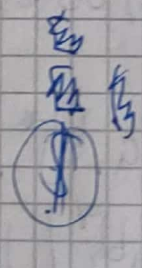


SEMINAR 7 - LFA

- Este clasa limbajelor independente de context închisă la intersecție? **NU**

Știm că $L=\{a^n b^n c^n \mid n \geq 0\} \notin CFL$

$L_1=\{a^n b^n c^m \mid n, m \geq 0\} \in CFL$

$L_2=\{a^m b^n c^n \mid n, m \geq 0\} \in CFL$

$\Rightarrow L_1 \cap L_2 = L \notin CFL$

$L_1: S \to AC$

$A \to aAb \mid \varepsilon$

$C \to cC \mid \varepsilon$

- Fie L un CFG și G o gramatică care generează L, având ca variabila de start S.

Fie G' gramatica după $S \to SS$

Generează o construcție bună pt L^*?

$L(G') \not\subseteq L^*$

$L^* \subseteq L(G')$ ✓

Fie $L=\{a^i b^j c^k \mid i=1 \Rightarrow j=k\}$. Scrieți un CFG pt L

$\cancel{S \to aS \mid Sb \mid bS \mid cS \mid SSc \mid Sbc \mid bSc \mid \varepsilon}$

$S \to aT \mid BC \mid aaABC$

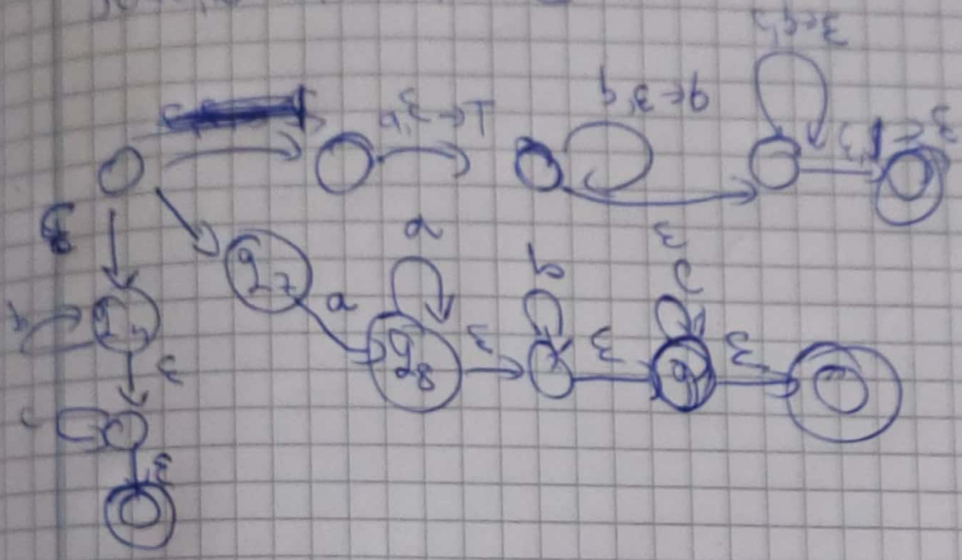
$T \to bTc \to \varepsilon$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

$C \to cC \mid \varepsilon$

Scrieți un PDA pt L



$L = \{w \in \{a,b\}^* \mid w$ conține cel puțin un b în a doua jumătate$\}$

Scrieți un PDA pt L