

MEGPP:

Rezolvati sistemul de ecuatii liniare:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

folosind MEGPP si metoda substitutiei descendente.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \underline{n=3}$$

$$\underline{k=1}:$$

$$\overline{A} \equiv \overline{A}^{(1)} = \left[A^{(1)} \quad \underline{b}^{(1)} \right] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array} \right]$$

$$\max_{j=1,3} |a_{j1}^{(1)}| = \max \{ |1|, |1|, |2| \}$$

$$= 2 = |a_{31}^{(1)}| =: |a_{\ell 1}^{(1)}| \Rightarrow \underline{\ell=3 > 1}$$

$$(E_3) \leftrightarrow (E_1):$$

Matricea permutare simplă:

$$P^{(1)} = P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (1a)$$

$$\begin{aligned} P^{(1)} \overline{A}^{(1)} &= P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & -1 & 1 \end{array} \right] \\ &=: [\tilde{A}^{(1)} \quad \tilde{b}^{(1)}] = \tilde{A}^{(1)} \end{aligned}$$

$$\tilde{a}_{11}^{(1)} = 2 \neq 0 \text{ (aplicăm MEGFP)}$$

$$\underline{j=2,3}: m_{ij}^{(1)} := \tilde{a}_{ij}^{(1)} / \tilde{a}_{11}^{(1)}$$

$$\bullet m_{21}^{(1)} := \tilde{a}_{21}^{(1)} / \tilde{a}_{11}^{(1)} = 1/2$$

$$(E_2 - m_{21}^{(1)} E_1) \rightarrow (E_2):$$

$$\underline{j=2,3}: \tilde{a}_{2j}^{(2)} := \tilde{a}_{2j}^{(1)} - m_{21}^{(1)} \tilde{a}_{1j}^{(1)}$$

$$\begin{aligned} \tilde{a}_{22}^{(2)} &:= \tilde{a}_{22}^{(1)} - m_{21}^{(1)} \tilde{a}_{12}^{(1)} \\ &= 1 - \frac{1}{2}(-1) = 3/2 \end{aligned}$$

$$a_{23}^{(2)} := \tilde{a}_{23}^{(1)} - w_2^{(1)} \tilde{a}_{13}^{(1)} \\ = 4 - \frac{1}{2} 2 = 3$$

$$a_{21}^{(2)} = 0 \text{ (Nu mai trebuie calculat!)} \\ w_2^{(1)} \text{ se alege a.i.t. } a_{21}^{(2)} = 0$$

$$b_2^{(2)} := \tilde{b}_2^{(1)} - w_2^{(1)} \tilde{b}_1^{(1)} \\ = 2 - \frac{1}{2} 3 = 1/2$$

$$\bullet w_3^{(1)} := \tilde{a}_{31}^{(1)} / \tilde{a}_{11}^{(1)} = 1/2$$

$$(E_3 - w_3^{(1)} E_1) \rightarrow (E_3):$$

$$\underline{j=2,3}: a_{3j}^{(2)} := \tilde{a}_{3j}^{(1)} - w_3^{(1)} \tilde{a}_{1j}^{(1)}$$

$$a_{32}^{(2)} := \tilde{a}_{32}^{(1)} - w_3^{(1)} \tilde{a}_{12}^{(1)} \\ = 1 - \frac{1}{2} (-1) = 3/2$$

$$a_{33}^{(2)} := \tilde{a}_{33}^{(1)} - w_3^{(1)} \tilde{a}_{13}^{(1)} \\ = -1 - \frac{1}{2} 2 = -2$$

$$a_{31}^{(2)} = 0 \text{ (această observație ca} \\ \text{cea 'pt } \tilde{a}_{21}^{(2)})$$

$$b_3^{(2)} := \tilde{b}_3^{(1)} - w_3^{(1)} \tilde{b}_1^{(1)} = 1 - \frac{1}{2} 3 = -1/2$$

Am obținut:

$k=2$:

$$\bar{A}^{(2)} = [A^{(2)} \quad \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 3/2 & -2 & -1/2 \end{array} \right]$$

Obs: Matricea core transformare

$$\begin{aligned} \bar{A}^{(1)} &= [\bar{A}^{(1)} \quad \underline{b}^{(1)}] = P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] \\ &= P^{(1)} \bar{A}^{(1)} \equiv P^{(1)} \bar{A} = P^{(1)} [A \quad \underline{b}] \end{aligned}$$

În matricea $\bar{A}^{(2)} = [A^{(2)} \quad \underline{b}^{(2)}]$ este

$$M^{(1)} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 0 & 1 \end{array} \right] \quad (1b)$$

Mai exact, are loc relația:

$$M^{(1)} P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] = [A^{(2)} \quad \underline{b}^{(2)}] \quad (1)$$

cu $P^{(1)}$ și $M^{(1)}$ date de (1a) și (1b).

$$\max_{i=2,3} |a_{i2}^{(2)}| = \max\{|3/2|, |3/2|\}$$

$$= 3/2 = a_{22}^{(2)} = a_{32}^{(2)} \Rightarrow 2 \in \{2,3\}$$

$$\Rightarrow 2 \in \{2,3\}$$

\Rightarrow Nu trebuie interschimbate
liniile matricei $\bar{A}^{(2)} \Rightarrow$

Matricea permutare simplă:

$$P^{(2)} = I_3 \quad (2a)$$

$$P^{(2)} \bar{A}^{(2)} = P^{(2)} [\bar{A}^{(2)} \quad \bar{b}^{(2)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 3/2 & -2 & -1/2 \end{array} \right]$$

$$= [\bar{A}^{(2)} \quad \bar{b}^{(2)}] = \bar{A}^{(2)}$$

$$\tilde{a}_{22}^{(2)} = 3/2 \neq 0 \text{ (aplicăm MEGFP)}$$

$$\underline{i=3,3}: m_{ij}^{(3)} := \frac{\tilde{a}_{ij}^{(2)}}{\tilde{a}_{22}^{(2)}}$$

$$\bullet m_{33}^{(3)} := \frac{\tilde{a}_{33}^{(2)}}{\tilde{a}_{22}^{(2)}} = \frac{3}{2} \cdot \frac{2}{3} = 1$$

$$(\bar{E}_3 - u_3^{(2)} \bar{E}_2) \rightarrow (\bar{E}_2):$$

$$\underline{j=3,3}: \tilde{a}_{3j}^{(3)} := \tilde{a}_{3j}^{(2)} - u_3^{(2)} \tilde{a}_{2j}^{(2)}$$

$$a_{33}^{(3)} := -2 - 1 \cdot 3 = -5$$

$$a_{23}^{(3)} = 0 \quad (\text{nu mai trebuie calculat!})$$

$$b_3^{(3)} := b_3^{(2)} - u_3^{(2)} b_2^{(2)} = -\frac{1}{2} - 1 \frac{1}{2} = -1$$

Am obtinut:

$$\bar{A}^{(3)} = [A^{(3)} \quad \underline{b}^{(3)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 0 & -5 & -1 \end{array} \right] = [U \quad \tilde{\underline{b}}]$$

Obs: Matricea core transformare

$$\bar{A}^{(2)} = [A^{(2)} \quad \underline{b}^{(2)}] = P^{(2)} [A^{(2)} \quad \underline{b}^{(2)}] = P^{(2)} \bar{A}^{(2)}$$

$$\text{in matricea } \bar{A}^{(3)} = [A^{(3)} \quad \underline{b}^{(3)}] = [U \quad \tilde{\underline{b}}]$$

este data de

$$M^{(2)} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{array} \right] \quad (2b)$$

Mai exact, are loc relatie:

$$M^{(2)} P^{(2)} [A^{(2)} \quad \underline{b}^{(2)}] = [A^{(3)} \quad \underline{b}^{(3)}] \quad (2)$$

cu $M^{(2)}$ și $P^{(2)}$ date de (2a) și (2b).

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)} P^{(2)} M^{(1)} P^{(1)} [A \ b] = [U \ \tilde{b}]}$$

Obs: Sistemul $Ax = b$ a devenit de forma $Ux = \tilde{b}$, ie

$$\begin{cases} 2x_1 - x_2 + 2x_3 = 3 \\ \frac{3}{2}x_2 + 3x_3 = \frac{1}{2} \\ -5x_3 = -1 \end{cases}$$

și acesta se rezolvă prin metode substituției descendente, ie de la ultima ecuație la prima:

$$-5x_3 = -1 \Rightarrow \boxed{x_3 = 1/5}$$

$$\frac{3}{2}x_2 = \frac{1}{2} - 3x_3 \Leftrightarrow \frac{3}{2}x_2 = \frac{1}{2} - \frac{3}{5} \Leftrightarrow$$

$$x_2 = \frac{2}{3} \frac{5-6}{10} \Rightarrow \boxed{x_2 = -1/15}$$

$$2x_1 = 3 + x_2 - 2x_3 \Rightarrow \dots \Rightarrow \boxed{x_1 = 19/15}$$