C6: Active and reactive power

$$m\ddot{x} = -r\dot{x} - kx + F_0\cos\Omega t \qquad \Big|\ \frac{r}{m} = 2b$$

$$\ddot{x} + 2b\dot{x} + \omega^2 x = \frac{F_0}{m}\cos\Omega t$$

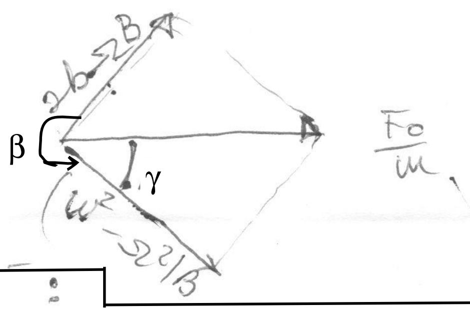
$$x = B\cos(\Omega t + \beta)$$

$$v = -B\Omega\sin(\Omega t+\beta) = B\Omega\cos\left(\Omega t+\beta+\frac{\pi}{2}\right)$$

$$(\omega^2-\Omega^2)B\cos(\Omega t+\beta) - 2b\Omega B\underbrace{\sin(\Omega t+\beta)}_{-\cos\left(\Omega t+\beta+\frac{\pi}{2}\right)} = \frac{F_0}{m}\cos\Omega t$$

$$(\omega^2-\Omega^2)B\cos(\Omega t+\beta) + 2b\Omega B\cos\left(\Omega t+\beta+\frac{\pi}{2}\right) = \frac{F_0}{m}\cos\Omega t$$

$$\sin\beta = \ominus\frac{2b\Omega B\cdot m}{F_0} = \ominus\frac{\Omega B r}{F_0}$$



see the diagram of forced oscillations

Active power

$$P_F = \frac{dL_F}{dt} = F\cdot\frac{dx}{dt} = Fv \Rightarrow$$

$$\overline{P_F} = \frac{1}{T}\int_0^T Fv\,dt = \frac{1}{T}\int_0^T F_0\cos\Omega t \times \Omega B\cos\left(\Omega t+\beta+\frac{\pi}{2}\right)dt$$

$$= \frac{1}{T}F_0\,\Omega B\left[\cancel{\frac{1}{2}\int_0^T \cos\left(2\Omega t+\beta+\frac{\pi}{2}\right)dt}^{\,0} + 2^{-1}\int_0^T\cos\left(\beta+\frac{\pi}{2}\right)dt\right]$$

$$= \frac{F_0\,\Omega B}{2T}T\cos\left(\beta+\frac{\pi}{2}\right) = -\frac{F_0\Omega B}{2}\sin\beta =$$

$$= \frac{\Omega B F_0}{2}\cdot\frac{\Omega B r}{F_0} = \frac{\Omega^2 B^2 r}{2}$$

$$P_F > 0$$

Reactive power

(2)

$$P_R = \frac{dL_R}{dt} = F_r \cdot \frac{dx}{dt} = F_r \cdot v = -r v^2$$

$$\overline{P_R} = -\frac{1}{T} r B^2 \Omega^2 \int_0^T \cos^2\left(\Omega t + \beta + \frac{\pi}{2}\right) = -\frac{r B^2 \Omega^2}{2} \Rightarrow \overline{P_R} < 0$$

$T/2$

$$= \boxed{\overline{P_F} = -\overline{P_R} = -\frac{r B^2 \Omega^2}{2}}$$

$$\int_0^T \cos^2(\Omega t + \alpha)\,dt = \int_0^T \frac{1 + \cos[2(\Omega t + \alpha)]}{2}\,dt =$$

$$= \frac{T}{2} + \frac{1}{2} \underbrace{\int_0^T \cos[\overbrace{2(\Omega t + \alpha)}^{u}]\,dt}_{\int_{2\alpha}^{4\pi+2\alpha} \cos u \frac{du}{2\Omega} = 0.} = \frac{T}{2}$$

$$\int_0^T \cos(2\Omega t + \beta + \pi/2)dt = \int_{\beta+\pi/2}^{2\Omega T+\beta+\pi/2} \cos u \frac{du}{2\Omega} = \int_{\beta+\pi/2}^{2\frac{2\pi}{T}T+\beta+\pi/2} \cos u \frac{du}{2\Omega}$$

$$= \int_{\beta+\pi/2}^{4\pi+\beta+\pi/2} \cos u \frac{du}{2\Omega} = \frac{\sin u}{2\Omega}\Bigg|_{\beta+\pi/2}^{4\pi+\beta+\pi/2}$$

$$= \frac{\sin(4\pi+\beta+\pi/2) - \sin(\beta+\pi/2)}{2\Omega}$$

$$= \frac{\sin(\beta+\pi/2) - \sin(\beta+\pi/2)}{2\Omega} = 0$$