Geom analit. euclid.

(1) ⊥ comună a 2 drepte necoplanare, Arii, Volume

a) D_1, D_2 necop, b) ec. ⊥ comune a D_1, D_2, dist(D_1, D_2)

Ex:

Fie $D_1: \begin{cases} x_1 + x_3 = 0 \\ x_2 - x_3 - 1 = 0 \end{cases}$

$D_2: \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

$D_1: \begin{cases} x_1 = -t \\ x_2 = t+1 \\ x_3 = t \end{cases}$ $\qquad D_2: \begin{cases} x_1 = s \\ x_2 = 0 \\ x_3 = 0 \end{cases}$

a) $\dfrac{x_1 - 0}{-1} = \dfrac{x_2 - 1}{1} = \dfrac{x_3 - 0}{1} = t$ $\qquad \dfrac{x_1 - 0}{1} = \dfrac{x_2 - 0}{0} = \dfrac{x_3 - 0}{0} = s$

$A(0, 1, 0) \in D_1$ $\qquad B(0, 0, 0)$

$u = (-1, 1, 1)$ $\qquad v = (1, 0, 0)$

$\Delta = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = (-1) \neq 0 \Rightarrow D_1, D_2$ necoplanare

$P_1(-t, t+1, t) \in D_1$

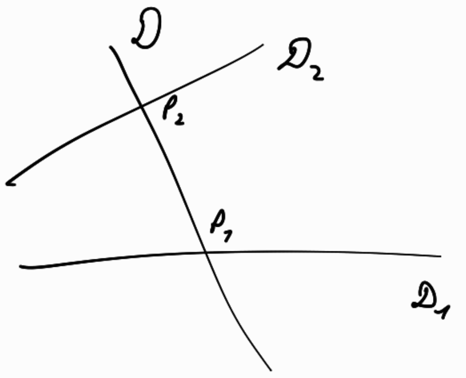
$P_2(s, 0, 0) \in D_2$

$\overrightarrow{P_1P_2} = (s+t, -t-1, -t)$

$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow \begin{cases} -s-t-t-1-t = 0 \\ s+t = 0 \end{cases}$

$P_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

$P_2\left(\frac{1}{2}, 0, 0\right)$



$\overrightarrow{P_1P_2}\ \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

$$D: \frac{x_1 - \frac{1}{2}}{0} = \frac{x_2 - 0}{-\frac{1}{2}} = \frac{x_3 - 0}{\frac{1}{2}}$$

$$D: \begin{cases} x_1 - \frac{1}{2} = 0 \\ x_2 + x_3 = 0 \end{cases}$$

c) $dist(D_1, D_2) = \|\overrightarrow{P_1 P_2}\|$

$$= \sqrt{0 + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2}$$

$$D_1: \frac{x_1 - 1}{2} = \frac{x_2 + 1}{3} = \frac{x_3 - 0}{1} = t$$

$$D_2: \frac{x_1 - 2}{4} = \frac{x_2 - 0}{6} = \frac{x_3 - 1}{\alpha} = s$$

a) $\alpha = ?$ a.î. $D_1 \parallel D_2$ ec. plan π det de D_1 și D_2

b) $dist(M, \pi)$; $M(0, 5, 1)$

$V_{D_1} = (2, 3, 1)$ $\quad D_1 \parallel D_2$

$V_{D_2} = (4, 6, \alpha) \Rightarrow \alpha = 2$

$A(2, 0, -1) \in D_2$

$B(1, -1, 0) \in D_1$

$\overrightarrow{AB} = (-1, -1, 1)$

$$N_\pi = \overrightarrow{AB} \times U = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = e_1 \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} - e_2 \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$+ e_3 \begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= -4e_1 + 3e_2 - e_3 = (-4, 3, -1)$$

$\pi: (x_1-2)\cdot(-4)+(x_2-0)\cdot 3+(x_3+1)\cdot(-1)=0$

$\pi: -4x_1+3x_2-x_3+7=0$

$$dist(M,\pi)=\frac{|-4\cdot 0+3\cdot 5-1+7|}{\sqrt{(-4)^2+3^2+(-1)^2}}=\frac{21}{26}=\frac{21\sqrt{26}}{26}$$

(7) $A(1,2,1)$, $B(2,1,3)$, $C(-2,1,3)$, $D(0,2,0)$

a) V_{ABCD}; b) $A_{\Delta BCD}$; c) $dist(A,(BCD))$;

Rez:

$$V_{ABCD}=\frac{1}{6}\cdot|\Delta|$$

$$\Delta=\begin{vmatrix}1&2&1&1\\2&1&3&1\\-2&1&3&1\\0&2&0&1\end{vmatrix}=\begin{vmatrix}1&0&1&1\\2&-1&3&1\\-2&-1&3&1\\0&0&0&1\end{vmatrix}=$$

$$=\begin{vmatrix}1&0&0\\2&-1&1\\-2&-1&5\end{vmatrix}=-5+1=(-4)$$

$$V_{ABCD}=\frac{1}{6}\cdot|-4|=\frac{4}{6}=\frac{2}{3}$$

b) $A_{\Delta BCD}=\frac{1}{2}\cdot\|\vec{BC}\times\vec{BD}\|$

$\vec{BC}=(-4,0,0)$, $\vec{BD}=(-2,1,-3)$

$$\vec{BC}\times\vec{BD}=\begin{vmatrix}e_1&e_2&e_3\\-4&0&0\\-2&1&-3\end{vmatrix}=(0,-12,-4)$$

$$A_{\Delta BCD}=\frac{1}{2}\cdot\sqrt{12^2+4^2}=\frac{\sqrt{144+16}}{2}=\frac{\sqrt{160}}{2}=\frac{4}{2}\cdot\sqrt{10}=2\sqrt{10}$$

c) $V_{ABCD} = \dfrac{A_{BCD} \cdot dist(A, (BCD))}{3}$

$\dfrac{\cancel{2}}{\cancel{3}} = \dfrac{\cancel{2}}{\cancel{3}} \cdot \sqrt{10} \cdot dist(A, (BCD)) \Rightarrow \dfrac{1}{\sqrt{10}} = dist(A, (BCD))$