

METODA GAUSS - JORDAN

Să se determine inversa matricii

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

folosind metoda Gauss - Jordan.

Folosim metoda Gauss - Jordan

împreună cu MEGFP:

k=1:

$$\bar{A} := [A \quad I_3] = \bar{A}^{(1)} := [A^{(1)} \quad I_3] =$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$a_{11}^{(1)} = 1 \neq 0 \Rightarrow \text{MEGFP}$$

$$\underline{i=2,3} : m_{i1}^{(1)} := a_{i1}^{(1)} / a_{11}^{(1)}$$

- $w_{21}^{(1)} := a_{21}^{(1)} / a_{11}^{(1)} = 2/1 = 2$

- $w_{31}^{(1)} := a_{31}^{(1)} / a_{11}^{(1)} = 3/1 = 3$

$$\left. \begin{aligned} (E_2 - w_{21}^{(1)} E_1) &\rightarrow (E_2) \\ (E_3 - w_{31}^{(1)} E_1) &\rightarrow (E_3) \end{aligned} \right\} \Rightarrow$$

$k=2$:

$$\bar{A}^{(2)} = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & -5 & 1 & 3 & 0 & 1 \end{array} \right]$$

$$a_{22}^{(2)} = -3 \neq 0 \Rightarrow \text{MEGFP}$$

$i=3, j=3$: $w_{i,j}^{(2)} := a_{ij}^{(2)} / a_{jj}^{(2)}$

- $w_{32}^{(2)} := a_{32}^{(2)} / a_{22}^{(2)} = -5/(-3) = 5/3$

$$(E_3 - w_{32}^{(2)} E_2) \rightarrow (E_3) \Rightarrow$$

$$\bar{A}^{(3)} = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & 0 & 8/3 & 1/3 & -5/3 & 1 \end{array} \right] \Rightarrow$$

Astfel, obținem 3 sisteme liniare
superior triunghiulare

$$\begin{cases} y_1 + 2y_2 = 1 & 0 & 0 \\ -3y_2 - y_3 = -2 & 1 & 0 \\ \frac{8}{3}y_3 = \frac{1}{3} & -\frac{5}{3} & 1 \end{cases}$$

• Sistemul 1 de coloana 1 a lui A^{-1} :

$$\begin{cases} x_{11} + 2x_{21} = 1 \\ -3x_{21} - x_{31} = -2 \\ \frac{8}{3}x_{31} = \frac{1}{3} \end{cases}$$

$$\boxed{x_{31} = \frac{1}{3} \cdot \frac{3}{8} = \frac{1}{8}}$$

$$\boxed{x_{21} = -\frac{1}{3}(-2 + x_{31}) = -\frac{1}{3}(-2 + \frac{1}{8})}$$

$$= -\frac{1}{3} \cdot \frac{-15}{8} = \frac{5}{8}$$

$$\boxed{x_{11} = 1 - 2x_{21} = 1 - 2 \cdot \frac{5}{8} = 1 - \frac{5}{4} = -\frac{1}{4}}$$

• Sistemul 2 dă coloana 2 a lui A^{-1} :

$$\begin{cases} x_{12} + 2x_{22} = 0 \\ -3x_{22} - x_{32} = 1 \\ \frac{8}{3}x_{32} = -\frac{5}{3} \end{cases}$$

$$x_{32} = \frac{3}{8} \cdot \frac{5}{8} = -\frac{5}{8}$$

$$x_{22} = -\frac{1}{3}(1 + x_{32}) = -\frac{1}{3}\left(1 - \frac{5}{8}\right) = -\frac{1}{8}$$

$$x_{12} = -2x_{22} = -2 \cdot \frac{-1}{8} = \frac{1}{4}$$

• Sistemul 3 dă coloana 3 a lui A^{-1} :

$$\begin{cases} x_{13} + 2x_{23} = 0 \\ -3x_{23} - x_{33} = 0 \\ \frac{8}{3}x_{33} = 1 \end{cases}$$

$$x_{33} = \frac{3}{8}$$

$$x_{23} = -\frac{1}{3}x_{33} = -\frac{1}{8}$$

$$x_{13} = -2x_{23} = \frac{1}{4}$$

Au obtient :

$$A^{-1} = \begin{bmatrix} -1/4 & 1/4 & 1/4 \\ 5/8 & -1/8 & -1/8 \\ 1/8 & -5/8 & 3/8 \end{bmatrix}$$