MEGPPS:

Rezolvați sistemul de ecuații liniare:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

folosind MEGPPS și metoda substituției descendente.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \underline{n=3}$$

$\underline{k=1}$:

$$\overline{A} \equiv \overline{A^{(1)}} = [A^{(1)}\ \underline{b}^{(1)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

$\underline{i=\overline{1,3}}$: $\quad s_i := \max\limits_{j=\overline{1,3}} |a_{ij}^{(1)}|$

$$s_1 = \max_{j=\overline{1,3}} |a_{1j}^{(1)}| = 1$$

$$s_2 := \max_{j=\overline{1,3}} |a^{(1)}_{2j}| = 4$$

$$s_3 := \max_{j=\overline{1,3}} |a^{(1)}_{3j}| = 2$$

$\underline{i=\overline{1,3}}$: $\tilde{a}^{(1)}_{i1} := a^{(1)}_{i1} / s_i$

$$\tilde{a}^{(1)}_{11} := a^{(1)}_{11} / s_1 = 1/1 = 1$$

$$\tilde{a}^{(1)}_{21} := a^{(1)}_{21} / s_2 = 1/4 = 1/4$$

$$\tilde{a}^{(1)}_{31} := a^{(1)}_{31} / s_3 = 2/2 = 1$$

$$\max_{i=\overline{1,3}} |\tilde{a}^{(1)}_{i1}| = \max\{|1|, |1/4|, |1|\}$$

$$= 1 \Longrightarrow \tilde{a}^{(1)}_{\ell 1} = \begin{cases} \tilde{a}^{(1)}_{11} \\ \tilde{a}^{(1)}_{31} \end{cases} \Longrightarrow \ell \in \{\underset{\underset{i}{\|}}{\underline{\underline{1}}}, 3\}$$

$1 \in \{1,3\} \Longrightarrow$ nu interschimbăm liniile lui $\overline{A}^{(1)}$ $\Longrightarrow$

$$P^{(1)} = I_3 \qquad (1a)$$

$$P^{(1)}\overline{A}^{(1)} = P^{(1)}[A^{(1)}\ \underline{b}^{(1)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

$$= [\tilde{A}^{(1)}\ \tilde{\underline{b}}^{(1)}] = \overline{\tilde{A}}^{(1)}$$

$\tilde{a}^{(1)}_{11} = 1 \neq 0$ (aplicăm MEGFP)

$\underline{i=\overline{2,3}}$: $m^{(1)}_i := \tilde{a}^{(1)}_{i1} / \tilde{a}^{(1)}_{11}$

• $m^{(1)}_2 := \tilde{a}^{(1)}_{21} / \tilde{a}^{(1)}_{11} = 1/1 = 1$

$(E_2 - m^{(1)}_2 E_1) \longrightarrow (E_2)$:

$\underline{j=\overline{2,3}}$: $\tilde{a}^{(2)}_{j2} := \tilde{a}^{(1)}_{j2} - m^{(1)}_2 \tilde{a}^{(1)}_{j2}$

$$\tilde{a}^{(2)}_{22} := \tilde{a}^{(1)}_{22} - m^{(1)}_2 \tilde{a}^{(1)}_{12}$$

$$= 1 - 1 \cdot 1 = 0$$

$$\tilde{a}^{(2)}_{23} := \tilde{a}^{(1)}_{23} - m^{(1)}_2 \tilde{a}^{(1)}_{13}$$

$$= 4 - 1 \cdot (-1) = 5$$

$\tilde{a}^{(2)}_{21} = 0$ (Nu mai trebuie calculat!)

$\tilde{b}^{(2)}_2 := \tilde{b}^{(1)}_2 - m^{(1)}_2 \tilde{b}^{(1)}_1 = 2 - 1 \cdot 1 = 1$

• $m^{(1)}_3 := \tilde{a}^{(1)}_{31} / \tilde{a}^{(1)}_{11} = 2/1 = 2$

$(E_3 - m^{(1)}_3 E_1) \longrightarrow (E_3)$:

$\underline{j=\overline{2,3}}$: $\tilde{a}^{(2)}_{j3} := \tilde{a}^{(1)}_{j3} - m^{(1)}_3 \tilde{a}^{(1)}_{j1}$

$$\tilde{a}_{32}^{(2)} := \tilde{a}_{32}^{(1)} - m_3^{(1)} \tilde{a}_{12}^{(1)}$$
$$= -1 - 2 \cdot 1 = -3$$
$$\tilde{a}_{33}^{(2)} := \tilde{a}_{33}^{(1)} - m_3^{(1)} \tilde{a}_{13}^{(1)}$$
$$= 2 - 2 \cdot (-1) = 4$$

$\tilde{a}_{31}^{(2)} = 0$ (nu mai trebuie calculat!)

$\tilde{b}_3^{(2)} := \tilde{b}_3^{(1)} - m_3^{(1)} \tilde{b}_1^{(1)} = 3 - 2 \cdot 1 = 1$

Am obținut:

$\underline{k=2}$:

$$\overline{A}^{(2)} = [A^{(2)} \ \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & 5 & 1 \\ 0 & -3 & 4 & 1 \end{array}\right]$$

Obs: Matricea care transformă

$$\overline{A}^{(1)} = [\tilde{A}^{(1)} \ \tilde{\underline{b}}^{(1)}] = P^{(1)} [A^{(1)} \ \underline{b}^{(1)}] = P^{(1)} \overline{A}^{(1)}$$
$$= P^{(1)} \overline{A} = P^{(1)} [A \ \underline{b}]$$

în matricea $\overline{A}^{(2)} = [A^{(2)} \ \underline{b}^{(2)}]$

este dată de:

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad (1b)$$

Mai exact, are loc relația:

$$M^{(1)} P^{(1)} \left[A^{(1)} \ \underline{b}^{(1)} \right] = \left[A^{(2)} \ \underline{b}^{(2)} \right] \quad (1)$$

cu $P^{(1)}$ și $M^{(1)}$ date de (1a) și (1b).

$\underline{i = \overline{2,3}}$: $s_i := \max\limits_{j=\overline{2,3}} |a_{ij}^{(2)}|$

$$s_2 := \max\{|0|, |5|\} = 5$$

$$s_3 := \max\{|-3|, |4|\} = 4$$

$\underline{i = \overline{2,3}}$: $\tilde{a}_{i2}^{(2)} := a_{i2}^{(2)} / s_i$

$$\tilde{a}_{22}^{(2)} := a_{22}^{(2)} / s_2 = 0/5 = 0$$

$$\tilde{a}_{32}^{(2)} := a_{32}^{(2)} / s_3 = -3/4$$

$$\max_{i=\overline{2,3}} |\tilde{a}_{i2}^{(2)}| = \max\{|0|, |-3/4|\} = 3/4$$

$\Rightarrow \tilde{a}_{\ell 2}^{(2)} = \tilde{a}_{32}^{(2)} \Rightarrow \ell = 3 > 2 = i$

$\Rightarrow$ Interschimbă $(E_2) \longleftrightarrow (E_3)$:

Matricea permutare simplă:

$$P^{(2)} = P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (2a)$$

$$P^{(2)}\overline{A}^{(2)} = P^{(2)}\left[A^{(2)}\ \underline{b}^{(2)}\right] = \underbrace{\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & 0 & 5 & 1 \end{array}\right]}$$

<u>**Obs**</u>:

$$= \overline{A}^{(3)} = \left[A^{(3)}\ \underline{b}^{(3)}\right] = \left[U\ \underline{\tilde{b}}\right]$$

Matricea $M^{(2)} = I_3$ (2b)

Din (2a) și (2b) rezultă:

$$M^{(2)}P^{(2)}\left[A^{(2)}\ \underline{b}^{(2)}\right] = \left[A^{(3)}\ \underline{b}^{(3)}\right] \quad (2)$$

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)}P^{(2)}\vec{M}^{(1)}P^{(1)}\left[A\ \underline{b}\right] = \left[U\ \underline{\tilde{b}}\right]}$$

i.e. $\begin{cases} x_1 + x_2 - x_3 = 1 \\ \quad -3x_2 + 4x_3 = 1 \\ \qquad 5x_3 = 1 \end{cases} \Rightarrow \boxed{\begin{aligned} x_3 &= 1/5 \\ x_2 &= -1/15 \\ x_1 &= 19/15 \end{aligned}}$