Sob Sob

① Să dem că $2^{n+1} \in O(2^n)$

Def. $f \in O(g)$ dacă $\exists\, c, n_0 > 0$ a.î. $\forall n \geq n_0$

avem $f(n) \leq c \cdot g(n)$

$f = 2^{n+1}$ $\quad$ $g = 2^n$

Alegem $c = 2$, $n_0 = 1 \Rightarrow 2^{n+1} \leq 2 \cdot 2^n$, $\forall n \geq 1$

$2^{2n} \in O(2n)$

$\exists\, c$ a.î. $2^{2n} \leq c \cdot 2^n$ $\forall n > n_0 \Leftrightarrow 2^n \leq c$ $\forall n \geq n_0$

Fals deoarece $2n \to \infty$ și c e o constantă

Teoremă $f \in \Theta(g) \Leftrightarrow f \in O(g)$ și $f \in \Omega(g)$

Dem: "$\Leftarrow$"

$f \in O(g) \Rightarrow \exists\, c_1$ a.î. $f(n) \leq c_1 \cdot g(n)$ $\forall n \geq n_{0_1}$

$f \in \Omega(g) \Rightarrow \exists\, c_2$ a.î. $f(n) \geq c_2 \cdot g(n)$ $\forall n \geq n_{0_2}$

Alegem $n_0 = \max(n_{01}, n_{02})$

$\Rightarrow c_1 \cdot g(n) \geq f(n) \geq c_2 \cdot g(n)$, $\forall n \geq n_0$

"$\Rightarrow$"

$f \in \Theta(g) \Rightarrow \exists\, c_1, c_2, n_0 > 0$ a.î. $c_1 \cdot g(n) \geq f(n) \geq c_2 \cdot g(n)$, $\forall n \geq n_0$

$\Rightarrow$ 1) $c_1 \cdot g(n) \geq f(n) \quad \forall\, n \geq n_0 \Rightarrow f \in O(g)$

2) $f(n) \geq c_2 \cdot g(n) \quad \forall n \geq n_0 \Rightarrow f \in \Omega(g)$

ex 2 $\quad f(n) + g(n) \in \Theta(\min\{f(n), g(n)\})$

nu e corect. contraex:

$$\left.\begin{array}{l} f(n) = 1 \\ g(n) = n^2 \end{array}\right\} \Rightarrow n^2 + 1 \in \Theta(1)$$

$c_1 \cdot 1 \geq n^2 \geq c_2 \cdot 1 \rightarrow$ fals $\quad 1 + n^2 \notin \Theta(1)$

$f(n) + g(n) \in \Theta(\max\{f(n), g(n)\})$

$f(n) + g(n) \in \Theta(\max\{f(n), g(n)\}) ? \Leftrightarrow$

$\Leftrightarrow \exists\, c_1, c_2, n_0$ a.î. $\forall n \geq n_0 \Rightarrow$

$\Rightarrow c_1 \cdot \max(..) \geq f(n) + g(n) \geq c_2 \cdot \max(...)$

Alegem $c_1 = 2$, $c_2 = 1$ și inegalitatea este adevărată

(pt $f(n) = 1$, $g(n) = n^2$)

ex $\quad n! \qquad o(n^n)$

$\Theta(n^n)$

$n \in o(n^2)$

$\Leftrightarrow \exists c > 0 \quad c \cdot n^2 > n \quad \forall n \geq n_0 \Rightarrow \frac{1}{n} < c \Rightarrow n > \frac{1}{c}$

$\exists n_0 > 0$ a.î.

$\forall n \geq n_0$

$n_0 = \frac{1}{c} + 1$

$n! \in o(n^n)$

$\Leftrightarrow \exists c > 0$ a.î. $\exists n_0 > 0$ a.î. $\forall n \geq n_0 \Rightarrow c \cdot n^n > n!$

(A) $\forall c > 1$ dar cum dem ⁝

$n! < c \cdot n^n \quad | : n^{n-1}$

$\frac{n!}{n^{n-1}} < c \cdot n \quad | : c$

$\frac{\frac{n!}{n^{n-1}}}{c} < n \Rightarrow n > \frac{\frac{n!}{n^{n-1}}}{c}$

$n_0 = \frac{1}{c}$ satisface

n_0 trb să fie în fct. de c

ex $\log_2 n! \in \theta(n \log_2 n)$

o sau θ?

$\Leftrightarrow \exists c_1, c_2, n_0 > 0$ a.î. $\forall n \geq n_0$ și.

$c_1 \cdot g(n) \geq f(n) \geq c_2 \cdot g(n)$

$\Rightarrow c_1 \cdot \log_2 n^n \geq \log_2 n! \geq c_2 \cdot \log_2 n^n \quad \forall n \geq n_0$

$\log_2 n! = \log_2 (1 \cdot 2 \cdot 3 \cdot \ldots \cdot n) = \log_2 1 + \log_2 2 + \ldots + \log_2 n =$

$= \log_2 1 + \log_2 2 + \ldots + \log_2 \left(\frac{n}{2} - 1\right) + \log_2 \frac{n}{2} + \ldots + \log_2 n \geq$

$\geq \log_2 \frac{n}{2} + \ldots + \log_2 n \geq \frac{n}{2} \cdot \log_2 \frac{n}{2} \rightarrow$

$\rightarrow = \frac{n}{2} \log_2 n - \frac{n}{2} \log_2 2 \geq \frac{n \log_2 n}{4}$

ex $f(n) + o(f(n)) \in \Theta(f(n))$

$c_1 \cdot f(n) \geq f(n) + o(f(n)) \geq c_2 \cdot f(n)$

$g \in o(f(n))$ dacă $\forall c > 0 \ \exists n_0 > 0$ a.î. $\forall n \geq n_0$,

$g(n) < c \cdot f(n)$

Alegem $c = 1$ - deci $f(n) + g < 2 f(n) \quad \forall n \geq n_0$

$\exists n_0$ a.î. $g(n) < f(n) \ \forall n \geq n_0$

$\forall n \geq n_0 \ f(n) + o(f(n)) \geq f(n)$

$f(n) + o(f(n)) \leq 2 f(n)$