12.01.202

Seminar 13

1. $x \in \mathbb{Z}$

$$\begin{cases} x \equiv 1 \pmod 3 \\ 3x \equiv 4 \pmod 5 \\ 4x \equiv 1 \pmod 7 \end{cases}$$

- $(3,5)=1 \Rightarrow \hat{3}$ inversabil în $\mathbb{Z}_5$

  $[\hat{3}\cdot\hat{2}=\hat{1}]$

$\Rightarrow 3x \equiv 4 \quad | \cdot 2$

$x \equiv 3 \pmod 5$

- $(4,7)=1 \Rightarrow 4$ inversabil în $\mathbb{Z}_7$

  $[\hat{4}\cdot\hat{2}=\hat{1}]$

$\Rightarrow 4x \equiv 1 \quad | 2$

$x \equiv 2 \pmod 7$

$$\Rightarrow \begin{cases} x \equiv 1 \pmod 3 \\ x \equiv 3 \pmod 5 \\ x \equiv 2 \pmod 7 \end{cases}$$

$(3,5)=1$, $(3,7)=1$, $(5,7)=1$

$\Longrightarrow$ (LCR) $\exists x_0 \in \mathbb{Z}_{3\cdot5\cdot7} = \mathbb{Z}_{105}$ sol

$\cancel{9}$, $\cancel{16}$, $\cancel{23}$, $\cancel{30}$, $\cancel{37}$, $\cancel{44}$, $\cancel{51}$, 58 ✓

$\Rightarrow x_0 = 58$

$\Downarrow$

$x = 105\cdot n + 58$

Algoritm sistem de ec modulare:

$x \equiv a_1 \pmod{n_1}$

$x \equiv a_2 \pmod{n_2}$

$\vdots$

$x \equiv a_k \pmod{n_k}$

I) $n = n_1 \cdot n_2 \cdot \ldots \cdot n_k$

II) $n_i' = \frac{n}{n_i}$

III) t_i = inversul lui n_i' mod n_i

IV) $x_0 \equiv a_1 n_1' t_1 + a_2 n_2' t_2 + \ldots + a_k n_k' t_k \pmod{n}$

$\rightarrow x = n \cdot p + x_0, \ p \in \mathbb{Z}$

$\Rightarrow \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \end{cases}$

I) $n = 3 \cdot 5 \cdot 7 = 105$

II) $n_1' = \frac{105}{3} = 35$ III) $\equiv 2 \pmod{3} \Rightarrow t_1 = 2$ inversul lui 2

$n_2' = \frac{105}{5} = 21 \equiv 1 \pmod{5} \Rightarrow t_2 = 1$

$n_3' = \frac{105}{7} = 15 \equiv 1 \pmod{7} \Rightarrow t_3 = 1$

$\Rightarrow 1 \cdot \underbrace{35 \cdot 2}_{70} + \underbrace{3 \cdot 21 \cdot 1}_{63} + \underbrace{2 \cdot 15 \cdot 1}_{30} = 163$

$163 \equiv 58 \pmod{105}$

$\Rightarrow x_0 = 58 \Rightarrow x = 105 \cdot p + 58, \ p \in \mathbb{Z}$

(2) A = inel commutativ, $a \in A$ nilpotent

$\Longrightarrow a \cdot X^i$ nilpotent în $A[X]$

a nilpotent $\Rightarrow \exists\, t \in \mathbb{N}$ a.î. $a^t = 0$.

$(a \cdot X^i)^t = a^t \cdot X^{ti} = 0$

$\Rightarrow a \cdot X^i$ nilpotent

(3). A = inel

a = nilpotent, u = inversabil

$\Downarrow$

$u + a$ = inversabil

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a nilpotent $\Rightarrow \exists\, t \in \mathbb{N}$ a.î. $a^t = 0$

$\Rightarrow 1 - a^t = 1$

$\Leftrightarrow (1-a)(1 + a + a^2 + \ldots + a^{t-1}) = 1$

$\Rightarrow 1 - a$ inversabil $\Rightarrow u(1-a)$ inversabil

$\Rightarrow u - u \cdot a$ inversabil

$u + \underbrace{v}_{\text{nil}} \cdot a = \text{inv}$