S12. Geometrie analitică euclidiană

I. ⊥ comună a 2 dr. necoplanare. Arii. Volume

Fie dreptele:

1. $D_1: \begin{cases} x_1+x_3=0 \\ x_2-x_3-1=0 \end{cases}$ $\qquad D_2: \begin{cases} x_2=0 \\ x_3=0 \end{cases}$

a) D_1, D_2, necoplanare

b) ⊥ comune

c) $dist(D_1, D_2)$

a) $D_1: \begin{cases} x_1=-t \\ x_2=t+1 \\ x_3=t \end{cases}$ $\qquad \frac{x_1-0}{-1} = \frac{x_2-1}{1} = \frac{x_3-0}{1} = t$

$\Rightarrow A(0,1,0) \in D_1$

$u = (-1,1,1)$

$D_2: \begin{cases} x_1=s \\ x_2=0 \\ x_3=0 \end{cases}$ $\qquad \frac{x_1-0}{1} = \frac{x_2-0}{0} = \frac{x_3-0}{0} = s$

$\Rightarrow B(0,0,0)$

$V = (1,0,0)$

$$\Delta_c = \begin{array}{ccc} u & v & \overrightarrow{AB} \end{array}$$
$$\Delta_c = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1 \neq 0 \Rightarrow D_1, D_2 \text{ -necoplanare}$$

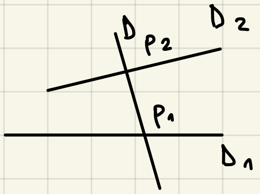
b) $P_1(-t, t+1, t) \in D_1$

$P_2(s, 0, 0) \in D_2$

$\overrightarrow{P_1P_2} = (s+t, -t-1, -t)$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow \begin{cases} -s-t-t-1-t=0 \\ s+t=0 \end{cases}$$

$t = -\frac{1}{2} \Rightarrow s = \frac{1}{2}$



$P_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

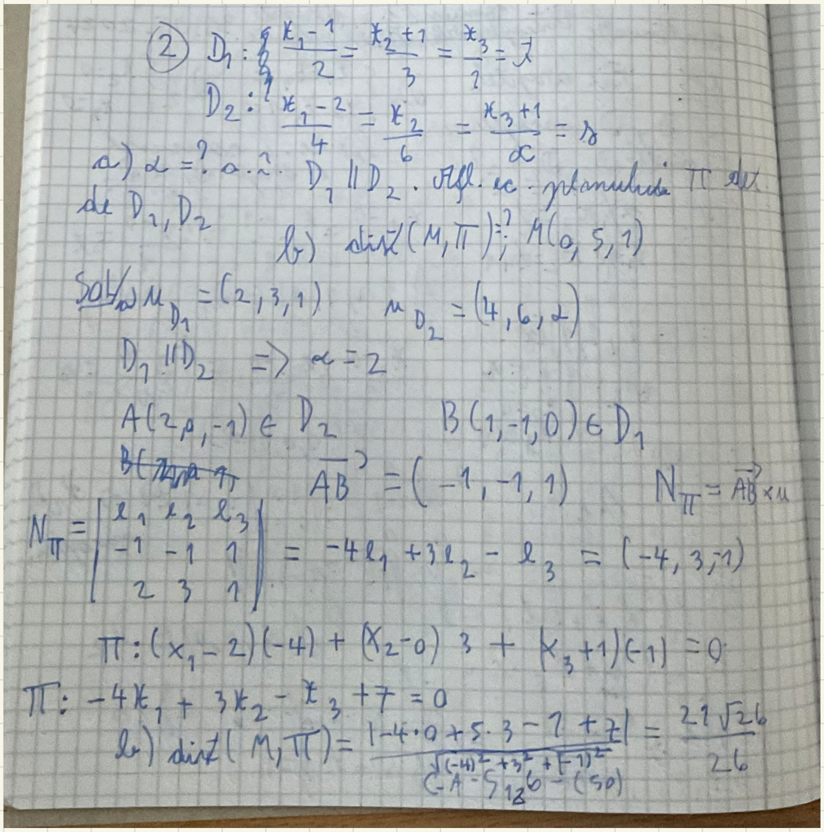
$P_2\left(\frac{1}{2}, 0, 0\right)$

$\overrightarrow{P_1P_2}\left(0, -\frac{1}{2}, \frac{1}{2}\right)$

$$D: \frac{x_1-\frac{1}{2}}{0} = \frac{x_2-0}{-\frac{1}{2}} = \frac{x_3-0}{\frac{1}{2}}$$

$$D: \begin{cases} x_1-\frac{1}{2}=0 \\ x_2+x_3=0 \end{cases}$$

c) $dist(D_1, D_2) = |\overrightarrow{P_1P_2}| = \sqrt{0+\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$



(2) $D_1: \frac{x_1-1}{2} = \frac{x_2+1}{3} = \frac{x_3}{1} = \lambda$

$D_2: \frac{x_1-2}{4} = \frac{x_2}{6} = \frac{x_3+1}{\alpha} = \delta$

a) $\alpha = ?$ a.î. $D_1 \parallel D_2$. Afl. ec. planului π det de D_1, D_2

b) $dist(M, \pi) = ?$; $M(0,5,1)$

Sol/ a) $u_{D_1} = (2,3,1)$ $\qquad u_{D_2} = (4,6,\alpha)$

$D_1 \parallel D_2 \Rightarrow \alpha = 2$

$A(2,0,-1) \in D_2$ $\qquad B(1,-1,0) \in D_1$

~~B(...)~~ $\qquad \overrightarrow{AB} = (-1,-1,1)$ $\qquad N_\pi = \overrightarrow{AB} \times u$

$$N_\pi = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = -4e_1 + 3e_2 - e_3 = (-4, 3, -1)$$

$\pi: (x_1-2)(-4) + (x_2-0)\,3 + (x_3+1)(-1) = 0$

$\pi: -4x_1 + 3x_2 - x_3 + 7 = 0$

b) $dist(M, \pi) = \frac{|-4\cdot 0 + 5\cdot 3 - 1 + 7|}{\sqrt{(-4)^2+3^2+(-1)^2}} = \frac{21\sqrt{26}}{26}$

C-A-S 1,2,6 - (50)

3. $A(1,2,1)$ $B(2,1,3)$ $C(-2,1,3)$ $D(0,2,0)$

a) V_{ABCD}

b) $A_{\Delta BCD}$

c) $dist(A,(BCD))$

$$V_{ABCD} = \frac{1}{6}|\Delta| = \frac{1}{6}|-4| = \frac{2}{3}$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 2 & -1 & 3 & 1 \\ -2 & -1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ -2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 4 & 0 & 0 \\ -2 & -1 & 3 \end{vmatrix} = -4\begin{vmatrix} 0 & 1 \\ -1 & 3 \end{vmatrix} = -4$$

$$A_{\Delta BCD} = \frac{1}{2}\|\overline{BC} \times \overline{BD}\| = \frac{1}{2}\sqrt{144+16} = \frac{4}{2}\sqrt{10} = 2\sqrt{10}$$

$$\overline{BC} \times \overline{BD} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -4 & 0 & 0 \\ -2 & 1 & -3 \end{vmatrix} = (0,-12,-4)$$

c) $V_{ABCD} = \dfrac{A_{BCD} \cdot dist(A,(BCD))}{3}$

$\frac{2}{3} = \frac{2\sqrt{10}}{3} \cdot dist \Rightarrow dist(A,(BCD)) = \frac{1}{\sqrt{10}}$

II. Aducerea la formă canonică a conicelor cu centru unic ($\delta \neq 0$)

Fie conica $\Gamma: f(x_1,x_2) = 5x_1^2 + 8x_1 + 5x_2^2 - 18x_1 - 18x_2 + 9 = 0$

a) form canonică efectuând izometrii

b) reprezentare grafică

$A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ $B = (-9,-9)$ $c = 9$

$\tilde{A} = \begin{pmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{pmatrix}$ $\Delta = \begin{vmatrix} -4 & -5 & 0 \\ -5 & -4 & 0 \\ -9 & -5 & 9 \end{vmatrix} = 9\begin{vmatrix} -4 & -5 \\ -5 & -4 \end{vmatrix} = -81$

$\delta = \begin{vmatrix} 5 & 4 \\ 4 & 5 \end{vmatrix} = 25-16 = 9$

$\delta \neq 0 \Rightarrow$ centru unic $\quad \delta > 0 \Rightarrow$ elipsă

$\Delta \neq 0 \Rightarrow$ nedegenerat

$R = \{0, e_1, e_2\} \xrightarrow{\theta} R' = \{P_0, e_1, e_2\}$

$\theta: X = X' + X_0$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} + \begin{pmatrix} x_{1,0} \\ x_{2,0} \end{pmatrix}$

$\frac{\partial f}{\partial x_1} = 0, \frac{\partial f}{\partial x_2} = 0$

$\Rightarrow \begin{cases} 10x_1 + 8x_2 - 18 = 0 \\ 8x_1 + 10x_2 - 18 = 0 \end{cases}$

$2x_1 - 2x_2 = 0 \Rightarrow x_1 = x_2 = 1$

$P_0(1,1)$

$\theta(\Gamma): 5x_1'^2 + 8x_1'x_2' + 5x_2'^2 + \frac{\Delta}{\delta} = 0$, $\frac{\Delta}{\delta} = -9$

$Q: \mathbb{R}^2 \to \mathbb{R}$

$Q(x) = 5x_1'^2 + 8x_1'x_2' + 5x_2'^2$

aducem la forma canonică util. met. val. proprii

$P(\lambda) = \lambda^2 - 10\lambda + 9 = 0$

$(\lambda-1)(\lambda-9) = 0$

$\lambda_1 = 1, \lambda_2 = 9$

$R' \xrightarrow{\tau} R'' = \{P_0, e_1', e_2'\}$

$V_{\lambda_1} = \{X \in \mathbb{R}^2 \mid AX = X\}$

$\Rightarrow (A - I_2)X = 0_{2,1}$

$\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_2 = -x_1$

$V_{\lambda_1} = \{(x_1,-x_1), x_1 \in \mathbb{R}\} = \langle\{(1,-1)\}\rangle$

$e_1' = \frac{1}{\sqrt{2}}(1,-1)$

. . . . .

$e_2' = \frac{1}{\sqrt{2}}(1,1)$

$\tau: X' = RX''$

$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \in SO(2)$

$Q(x) = 1 \cdot x_1''^2 + 9 \cdot x_2''^2$

$\tau(\theta(\Gamma)): 1 \cdot x_1''^2 + 9x_2''^2 - 9 = 0$

$\Rightarrow x_1''^2 + 9x_2''^2 = 9$

$\frac{x_1''^2}{9} + x_2''^2 = 1$

$\Rightarrow a = 3, b = 1$ (elipsă)