MEGPT:

Rezolvați sistemul de ecuații liniare:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

folosind MEGPT și metoda substituției descendente.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \underline{n=3}$$

$\underline{k=1}$:

$$\bar{A} = \bar{A}^{(1)} = [A^{(1)} \; \underline{b}^{(1)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

$$\max_{i,j=\overline{1,3}} |a_{ij}^{(1)}| = 4 = |a_{23}^{(1)}| =: |a_{lm}^{(1)}| \Rightarrow$$

$$\Rightarrow \begin{cases} l = 2 > 1 = k \Rightarrow (E_l) \leftrightarrow (E_k) \\ m = 3 > 1 = k \Rightarrow (C_m) \leftrightarrow (C_k) \end{cases}$$

$(E_1) \longleftrightarrow (E_2)$:

$$P^{(1)} = P_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1a)$$

$(C_1) \longleftrightarrow (C_3)$:

$$Q^{(1)} = P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad (1b)$$

$$P^{(1)} \left[A^{(1)} \ \ \underline{b}^{(1)} \right] = \left[\begin{array}{ccc|c} 1 & 1 & 4 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

$$\left[P^{(1)} A^{(1)} Q^{(1)} \ \ P^{(1)} \underline{b}^{(1)} \right] = \left[\begin{array}{ccc|c} 4 & 1 & 1 & 2 \\ -1 & 1 & 1 & 1 \\ 2 & -1 & 2 & 3 \end{array}\right] =:$$

$$=: \left[\tilde{A}^{(1)} \ \ \underline{\tilde{b}}^{(1)} \right] \equiv \overline{\tilde{A}}^{(1)}$$

$\tilde{a}^{(1)}_{11} = 4 \neq 0$ (aplicăm MEGFP)

$\underline{i = \overline{2,3}}$: $m^{(1)}_i := \tilde{a}^{(1)}_{i1} / \tilde{a}^{(1)}_{11}$

- $m^{(1)}_2 := \tilde{a}^{(1)}_{21} / \tilde{a}^{(1)}_{11} = -1/4$

$(E_2 - m^{(1)}_2 E_1) \longrightarrow (E_2)$:

$\underline{j = \overline{2,3}}$: $\tilde{a}^{(2)}_{2j} := \tilde{a}^{(1)}_{2j} - m^{(1)}_2 \tilde{a}^{(1)}_{1j}$

$$\tilde{a}^{(2)}_{22} := \tilde{a}^{(1)}_{22} - m^{(1)}_2 \tilde{a}^{(1)}_{12}$$
$$= 1 - \left(-\frac{1}{4}\right) 1 = 5/4$$
$$\tilde{a}^{(2)}_{23} := \tilde{a}^{(1)}_{23} - m^{(1)}_2 \tilde{a}^{(1)}_{13}$$
$$= 1 - \left(-\frac{1}{4}\right) 1 = 5/4$$

$\tilde{a}^{(2)}_{21} = 0$ (nu trebuie calculat)

$$\tilde{b}^{(2)}_2 := \tilde{b}^{(1)}_2 - m^{(1)}_2 \tilde{b}^{(1)}_1 = 1 - \left(\frac{-1}{4}\right) 2 = 3/2$$

- $m^{(1)}_3 := \tilde{a}^{(1)}_{31} / \tilde{a}^{(1)}_{11} = 2/4 = 1/2$

$(E_3 - m^{(1)}_3 E_1) \longrightarrow (E_3)$:

$\underline{j = \overline{2,3}}$: $\tilde{a}^{(2)}_{3j} := \tilde{a}^{(1)}_{3j} - m^{(1)}_3 \tilde{a}^{(1)}_{1j}$

$$\tilde{a}^{(2)}_{32} := \tilde{a}^{(1)}_{32} - m^{(1)}_3 \tilde{a}^{(1)}_{13}$$
$$= -1 - \frac{1}{2} \cdot 1 = -3/2$$

$$\tilde{a}_{33}^{(2)} := \tilde{a}_{33}^{(1)} - m_3^{(1)} \tilde{a}_{13}^{(1)}$$
$$= 2 - \frac{1}{2} 1 = 3/2$$

$\tilde{a}_{31}^{(2)} = 0$ (nu trebuie calculat)

$$\tilde{b}_3^{(2)} := \tilde{b}_3^{(1)} - m_3^{(1)} \tilde{b}_1^{(1)} = 3 - \frac{1}{2} 2 = 2$$

Am obținut:

<u>$k=2$</u>:

$$\overline{A}^{(2)} = [A^{(2)} \ \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 4 & 1 & 1 & 2 \\ 0 & 5/4 & 5/4 & 3/2 \\ 0 & -3/2 & 3/2 & 2 \end{array}\right]$$

<u>Obs</u>: Matricea care transformă

$$\overline{A}^{(1)} = [P^{(1)} A^{(1)} Q^{(1)} \quad P^{(1)} \underline{b}^{(1)}]$$
$$= [P^{(1)} A Q^{(1)} \quad P^{(1)} \underline{b}]$$

în matricea $\overline{A}^{(2)} = [A^{(2)} \ \underline{b}^{(2)}]$ este

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} \qquad (\mathrm{C})$$

Mai exact are loc relația:

$$M^{(1)} P^{(1)} \left[A^{(1)} Q^{(1)} \quad \underline{b}^{(1)} \right] = \left[A^{(2)} \quad \underline{b}^{(2)} \right] \quad (1)$$

cu $P^{(1)}$, $Q^{(1)}$ și $M^{(1)}$ date de (1a)-(1c).

$$\max_{i,j=\overline{2,3}} |a_{ij}^{(2)}| = 3/2 = \begin{cases} |a_{32}^{(2)}| \\ |a_{33}^{(2)}| \end{cases} =: |a_{\ell m}^{(2)}|$$

<u>Cazul favorabil</u>:

$$\begin{cases} \ell = 3 > 2 = k \\ m = 2 = 2 = k \end{cases} \Rightarrow \begin{cases} (E_\ell) \longleftrightarrow (E_k) \\ \text{fără intersch.} \\ \text{de coloane} \end{cases}$$

$(E_2) \longleftrightarrow (E_3)$:

$$P^{(2)} = P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (2a)$$

Fără interschimb de coloane:

$$Q^{(2)} = I_3 \quad (2b)$$

$$P^{(2)} \left[A^{(2)} \quad \underline{b}^{(2)} \right] = \left[\begin{array}{ccc|c} 4 & 1 & 1 & 2 \\ 0 & -3/2 & 3/2 & 2 \\ 0 & 5/4 & 5/4 & 3/2 \end{array} \right] \Rightarrow$$

$$\left[P^{(2)} A^{(2)} Q^{(2)} \quad P^{(2)} \underline{b}^{(2)}\right] = \left[\begin{array}{ccc|c} 4 & 1 & 1 & 2 \\ 0 & -3/2 & 3/2 & 2 \\ 0 & 5/4 & 5/4 & 3/2 \end{array}\right] =:$$

$$=: \left[\tilde{A}^{(2)} \quad \underline{b}^{(2)}\right] = \overline{\tilde{A}}^{(2)}$$

$\tilde{a}^{(2)}_{22} = 3/2 \neq 0$ (aplicăm MEGPP)

$\underline{i = \overline{3,3}}$: $m_i^{(2)} := \tilde{a}^{(2)}_{2i} / \tilde{a}^{(2)}_{22}$

$\cdot\ m_3^{(2)} := \tilde{a}^{(2)}_{23} / \tilde{a}^{(2)}_{22} = \frac{5}{4}\left(-\frac{2}{3}\right) = -5/6$

$(E_3 - m_3^{(2)} E_2) \longrightarrow (E_3)$:

$\underline{j = \overline{3,3}}$: $\tilde{a}^{(3)}_{3j} := \tilde{a}^{(2)}_{3j} - m_3^{(2)} \tilde{a}^{(2)}_{2j}$

$\tilde{a}^{(3)}_{33} := \tilde{a}^{(2)}_{33} - m_3^{(2)} \tilde{a}^{(3)}_{23}$

$= \frac{5}{4} + \frac{5}{6}\,\frac{3}{2} = \frac{5}{4} + \frac{5}{4} = 5/2$

$\tilde{a}^{(3)}_{32} = 0$ (nu trebuie calculat!)

$\tilde{b}^{(3)}_3 := \tilde{b}^{(2)}_3 - m_3^{(2)} \tilde{b}^{(2)}_2 = \frac{3}{2} + \frac{5}{6}\,2 = \frac{3}{2} + \frac{5}{3}$

$= 19/6$

Am obținut:

$$\overline{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}] = \left[\begin{array}{ccc|c} 4 & 1 & 1 & 2 \\ 0 & -3/2 & 3/2 & 2 \\ 0 & 0 & 5/2 & 19/6 \end{array}\right] = [U\ \underline{\tilde{b}}]$$

<u>Obs</u>: Matricea care transformă

$$\overline{A}^{(2)} = [P^{(2)} A^{(2)} Q^{(2)}\ \ P^{(2)} \underline{b}^{(2)}]$$

în matricea $\overline{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}]$ este

$$M^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5/6 & 1 \end{bmatrix} \quad (2c)$$

Mai exact, are loc relația:

$$M^{(2)} P^{(2)} [A^{(2)} Q^{(2)}\ \ \underline{b}^{(2)}] = [A^{(3)}\ \underline{b}^{(3)}] \quad (2)$$

unde $P^{(2)}, Q^{(2)}$ și $M^{(2)}$ sunt date de (2a) – (2c).

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)} P^{(2)} M^{(1)} P^{(1)} \left[A\, Q^{(1)} Q^{(2)} \quad \underline{b} \right] = \left[U \ \underline{\tilde{b}} \right]}$$

<u>Obs</u>: Sistemul $A\underline{x} = \underline{b}$ a devenit de forma $U\underline{\tilde{x}} = \underline{b}$, $\underline{\tilde{x}} = (x_3\ x_2\ x_1)^T$:

$$\begin{cases} 4x_3 + x_2 + x_1 = 2 \\ -\frac{3}{2}x_2 + \frac{3}{2}x_1 = 2 \\ \frac{5}{2}x_1 = \frac{19}{6} \end{cases}$$

$$x_1 = \frac{19}{6}\,\frac{2}{5} \Rightarrow \boxed{x_1 = 19/15}$$

$$x_2 = -\frac{2}{3}\left(2 - \frac{3}{2}x_1\right) = -\frac{4}{3} + \frac{19}{15} = -1/15$$

$$\Rightarrow \boxed{x_2 = -1/15}$$

$$x_3 = \frac{1}{4}(2 - x_2 - x_1) = \frac{1}{4}\left(2 + \frac{1}{15} - \frac{19}{15}\right)$$

$$= \frac{1}{4}\left(2 - \frac{18}{15}\right) = \frac{1}{4}\left(2 - \frac{6}{5}\right) \Rightarrow \boxed{x_3 = 1/5}$$