Ex $(\mathbb{R}^2, +, \cdot)/\mathbb{R}$

$\mathcal{R}_0 = \{e_1, e_2\} \xrightarrow{C} \mathcal{R}' = \{e_1' = e_1 - e_2,\ e_2' = e_1 + 2e_2\}$ $\quad C = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$

$\mathcal{R}^* = \{e_1^*, e_2^*\} \xrightarrow{D} \mathcal{R}'^* = \{e_1'^*, e_2'^*\}$.

$e_i^* : \mathbb{R}^2 \longrightarrow \mathbb{R}$ liniare, $i = \overline{1,2}$; $e_i'^* : \mathbb{R}^2 \longrightarrow \mathbb{R}$ liniare

$e_i^*(e_j) = \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$ $\qquad e_i'^*(e_j') = \delta_{ij}$

• $e_1'^* = a e_1^* + b e_2^*$

1) $\underbrace{e_1'^*(e_1')}_{=1} = a e_1^*(e_1 - e_2) + b e_2^*(e_1 - e_2) = a \underbrace{e_1^*(e_1)}_{1} - a \underbrace{e_1^*(e_2)}_{0} + b \underbrace{e_2^*(e_1)}_{0} - b \underbrace{e_2^*(e_2)}_{1}$

$\Rightarrow \boxed{1 = a - b}$

2) $\underbrace{e_1'^*(e_2')}_{=0} = a e_1^*(e_1 + 2e_2) + b e_2^*(e_1 + 2e_2) = a \underbrace{e_1^*(e_1)}_{1} + 2a \underbrace{e_1^*(e_2)}_{0} + b \underbrace{e_2^*(e_1)}_{0} + 2b \underbrace{e_2^*(e_2)}_{1}$

$\Rightarrow \boxed{0 = a + 2b}$

$\begin{cases} a - b = 1 \\ a + 2b = 0 \end{cases}$

$/ \quad 3b = -1$

$b = -\frac{1}{3}$

$a = \frac{2}{3}$

$D = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$

•• $e_2'^* = c e_1^* + d e_2^*$

1) $e_2'^*(e_1') = c e_1^*(e_1 - e_2) + d e_2^*(e_1 - e_2) \Rightarrow 0 = c - d$

2) $e_2'^*(e_2') = c e_1^*(e_1 + 2e_2) + d e_2^*(e_1 + 2e_2) \Rightarrow 1 = c + 2d$

$\begin{cases} c - d = 0 \\ c + 2d = 1 \end{cases}$

$/ \quad 3d = 1$

$d = \frac{1}{3}$

$c = \frac{1}{3}$

$C = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$, $\det C = 3$

$C^T = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$ $\quad C^* = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

$C^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $\quad (C^{-1})^T = D$

OBS $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f(x_1, x_2) = (x_1 - x_2, x_1 + 3x_2)$

expresie analitică a lui f

Scanat cu CamScanner