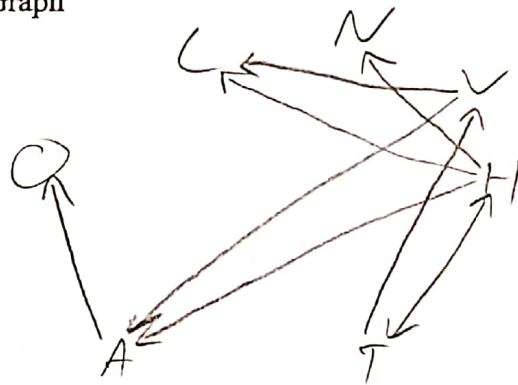


$V(\text{StateGraph}) = \{\text{Oregon, Alaska, Texas, Hawaii, Vermont, New York, California}\}$
 $E(\text{StateGraph}) = \{(\text{Alaska, Oregon}), (\text{Hawaii, Alaska}), (\text{Hawaii, Texas}), (\text{Texas, Hawaii}), (\text{Hawaii, California}), (\text{Hawaii, New York}), (\text{Texas, Vermont}), (\text{Vermont, California}), (\text{Vermont, Alaska})\}$

1. Draw the StateGraph



A, C, H, N, O, T, V

✓ Describe the graph pictured above, using the formal graph notation.

$V(\text{StateGraph}) =$

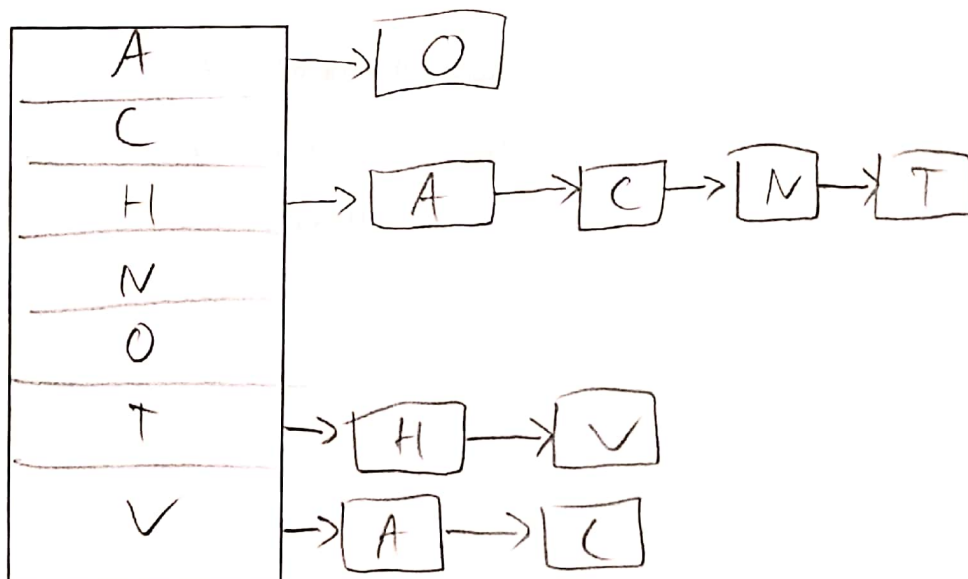
$E(\text{StateGraph}) =$

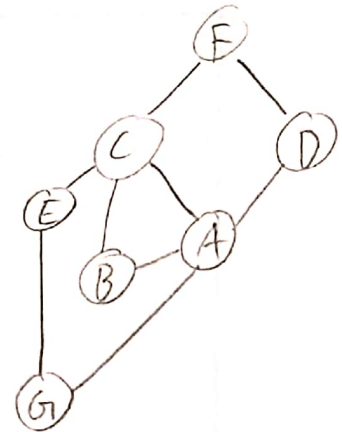
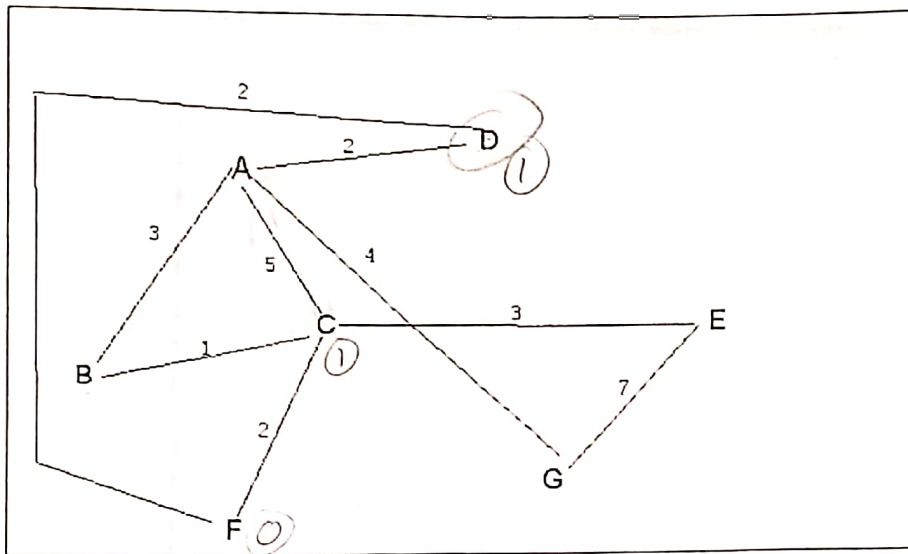
2. a. Is there a path from Oregon to any other state in the graph? \checkmark
- b. Is there a path from Hawaii to every other state in the graph? \checkmark
(can be transitive)
- c. From which state(s) in the graph is there a path to Hawaii? T

3. a. Show the adjacency matrix that would describe the edges in the graph.
Store the vertices in alphabetical order

States	A	C	H	N	O	T	V
A	0	0	0	0	1	0	0
C	0	0	0	0	0	0	0
H	1	1	0	1	0	1	0
N	0	0	0	0	0	0	0
O	0	0	0	0	0	0	0
T	0	0	1	0	0	0	1
V	1	1	0	0	0	0	0

3. b. Show the adjacency lists
that would describe the edges in the graph



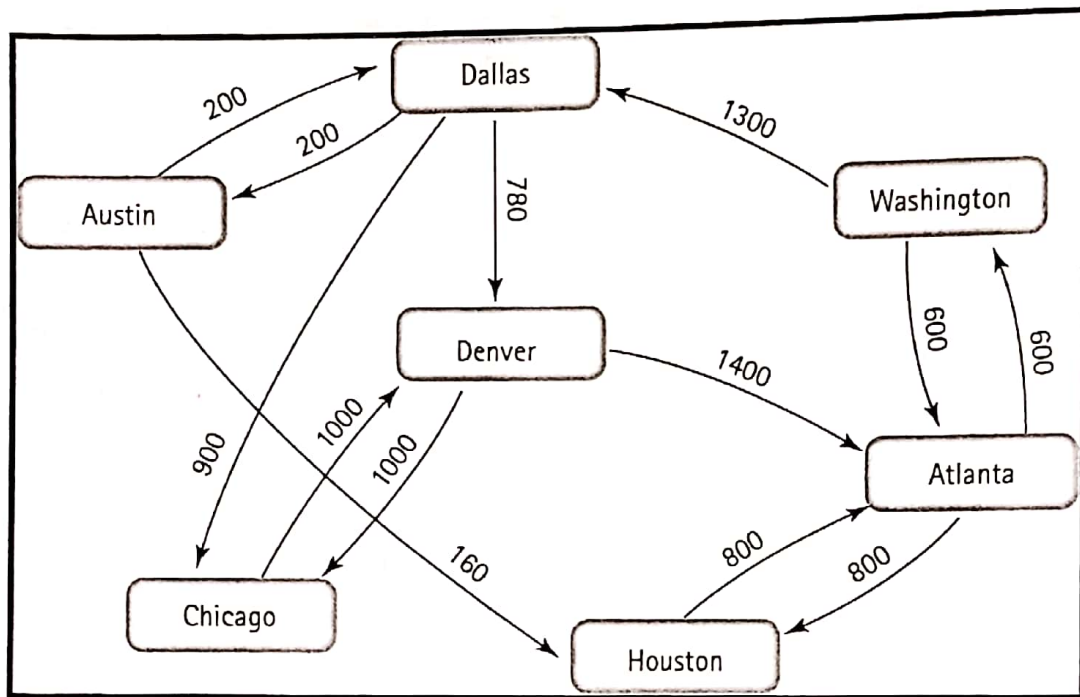


4 a. Which of the following lists the graph nodes in depth first order beginning with E?

- ☐ A) E, G, F, C, D, B, A
- ☐ B) G, A, E, C, B, F, D
- ☒ C) E, G, A, D, F, C, B
- ☐ D) E, C, F, B, A, D, G

4 b. Which of the following lists the graph nodes in breadth first order beginning at F?

- ☒ A) F, C, D, A, B, E, G
- ☐ B) F, D, C, A, B, C, G
- ☐ C) F, C, D, B, G, A, E
- ☐ D) a, b, and c are all breadth first traversals



5. Find the shortest distance from Atlanta to every other city

Specify path

ATL → WSH: ATL-WSH
600

ATL → HOU: ATL-HOU
800

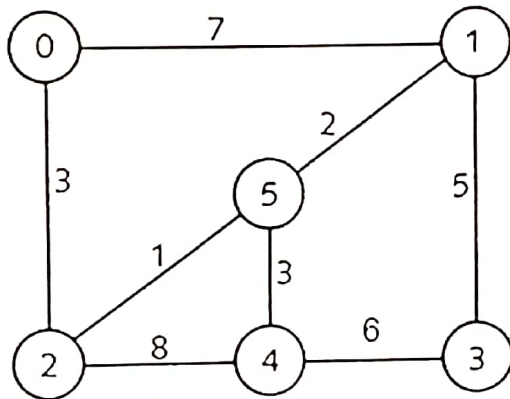
ATL → DAL: ATL-WSH, WSH-DAL
 $600 + 1300 = 1900$

ATL → AST: ATL-WSH, WSH-DAL, DAL-AST
 $600 + 1300 + 200 = 2100$

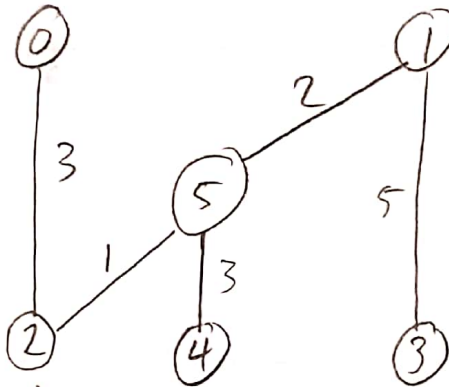
ATL → DEN: ATL-WSH, WSH-DAL, DAL-DEN
 $600 + 1300 + 780 = 2680$

ATL → CHI: ATL-WSH, WSH-DAL, DAL-CHI
 $600 + 1300 + 900 = 2800$

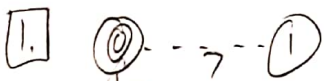
6. Find the minimal spanning tree using Prim's algorithm. Use 0 as the source vertex. Show the steps.



min spanning tree



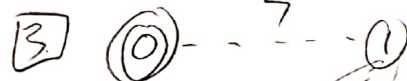
$2w = 14$



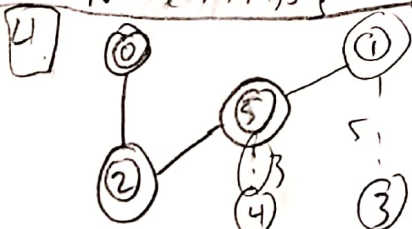
$V = \{0\}$
 $E = \{\}$
 $N = \{1, 2, 3, 4, 5\}$



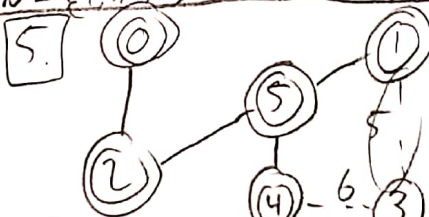
$V = \{0, 2\}$
 $E = \{(0, 2)\}$
 $N = \{1, 3, 4, 5\}$



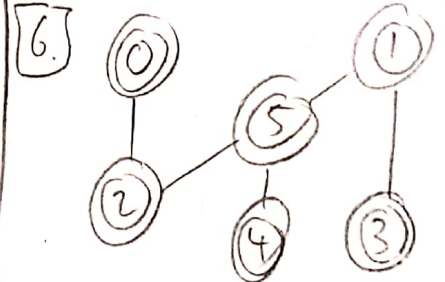
$V = \{0, 2, 5\}$
 $E = \{(0, 2), (2, 5)\}$
 $N = \{1, 3, 4\}$



$V = \{0, 2, 5, 1\}$
 $E = \{(0, 2), (2, 5), (5, 1)\}$
 $N = \{4, 3\}$

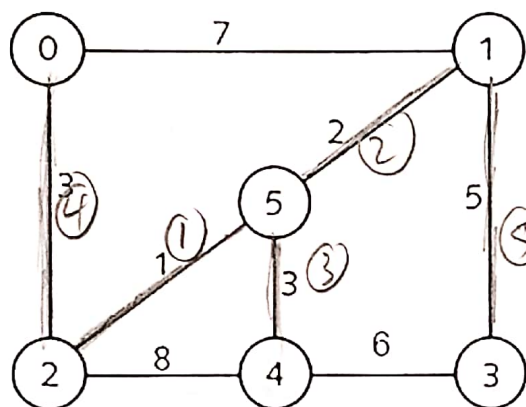


$V = \{0, 2, 5, 1, 4\}$
 $E = \{(0, 2), (2, 5), (5, 1), (5, 4)\}$
 $N = \{3\}$



$V = \{0, 2, 5, 1, 4, 3\}$
 $E = \{(0, 2), (2, 5), (5, 1), (5, 4), (5, 3)\}$
 $N = \{\}$

7. Find the minimal spanning tree using Kruskal's algorithm.
Show the weights in order and the steps.



weights: 1, 2, 3, 5, 6, 7, 8
 $(2,5)$ $(5,1)$ $(5,4)$ $(1,3)$ $(4,3)$
 $(0,2)$ $(0,1)$ $(2,4)$

$$\textcircled{1} E = \{(2,5)\}$$

$$V = \{0, 1, 3, 4\}$$

$$w = \{1\}$$

$$\textcircled{2} E = \{(2,5), (5,1)\}$$

$$V = \{0, 3, 4\}$$

$$w = \{1, 2\}$$

$$\textcircled{3} E = \{(2,5), (5,1), (5,4)\}$$

$$V = \{0, 3\}$$

$$w = \{1, 2, 3\}$$

$$\textcircled{4} E = \{(2,5), (5,1), (5,4), (0,2)\}$$

$$V = \{3\}$$

$$w = \{1, 2, 3, 3\}$$

$$\textcircled{5} E = \{(2,5), (5,1), (5,4), (0,2), (1,3)\}$$

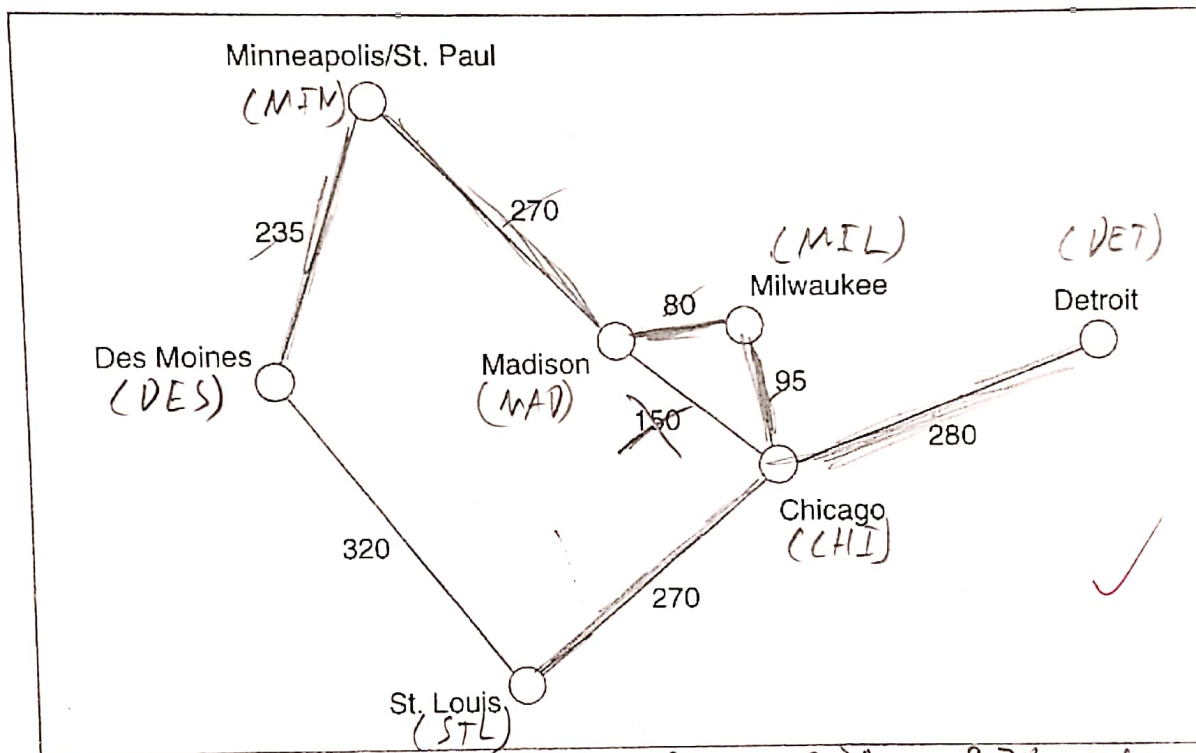
$$V = \{\}$$

$$w = \{1, 2, 3, 3, 5\}$$

$$\underline{\Sigma w = 14}$$

Kruskal's

8. Find the minimal spanning tree using the algorithm you prefer. Use Minneapolis/St. Paul as the source vertex



$$w_A = \{80, 95, 150, 235, 270, 270, 280, 320\}$$

$$E_A = \{(MAD, MIL), (MIL, CHI), (MAD, CHI), (DES, MIN), (STL, CHI), (MIN, MAD), (CHI, DET), (DES, STL)\}$$

① $E = \{(MAD, MIL)\}$
 $U = \{MIN, DES, STL, CHI, DET\}$
 $w = \{80\}$ Span? no. continue

② $E = \{(MAD, MIL), (MIL, CHI)\}$
 $U = \{MIN, DES, STL, DET\}$
 $w = \{80, 95\}$ Span? no. continue

③ $E = \{(MAD, MIL), (MIL, CHI), (MAD, CHI)\}$
 forms cycle

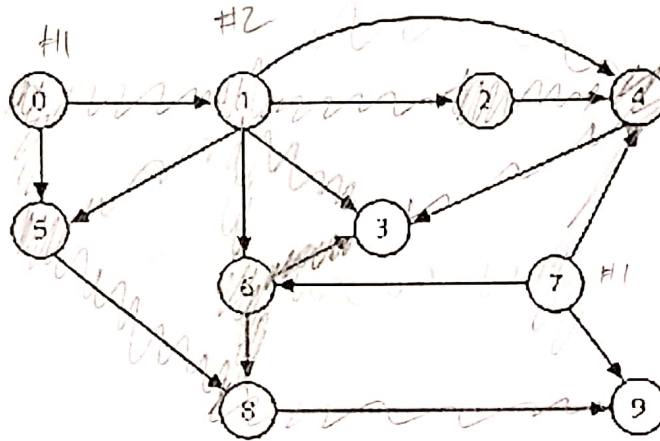
④ $E = \{(MAD, MIL), (MIL, CHI), (DES, MIN)\}$
 $U = \{STL, DET\}$ Span? no. continue
 $w = \{80, 95, 235\}$

⑤ $E = \{(MAD, MIL), (MIL, CHI), (DES, MIN), (STL, CHI)\}$
 $U = \{DET\}$
 $w = \{80, 95, 235, 270\}$

⑥ $E = \{(MAD, MIL), (MIL, CHI), (DES, MIN), (STL, CHI), (MIN, MAD)\}$
 $U = \{DET\}$ Span? no
 $w = \{80, 95, 235, 270, 270\}$

⑦ $U = \{(MAD, MIL), (MIL, CHI), (DES, MIN), (STL, CHI), (CHI, DET)\}$
 $U = \{\}$ Span? yes
 $w = \{80, 95, 235, 270, 270, 280\}$ $\Sigma w = 1,230$

9. List the nodes of the graph in a breadth first topological ordering. Show the steps using arrays predCount, topologicalOrder and a queue



topological Order
0, 7, 1, 2, 5, 6, 4, 8, 3

predCount
index

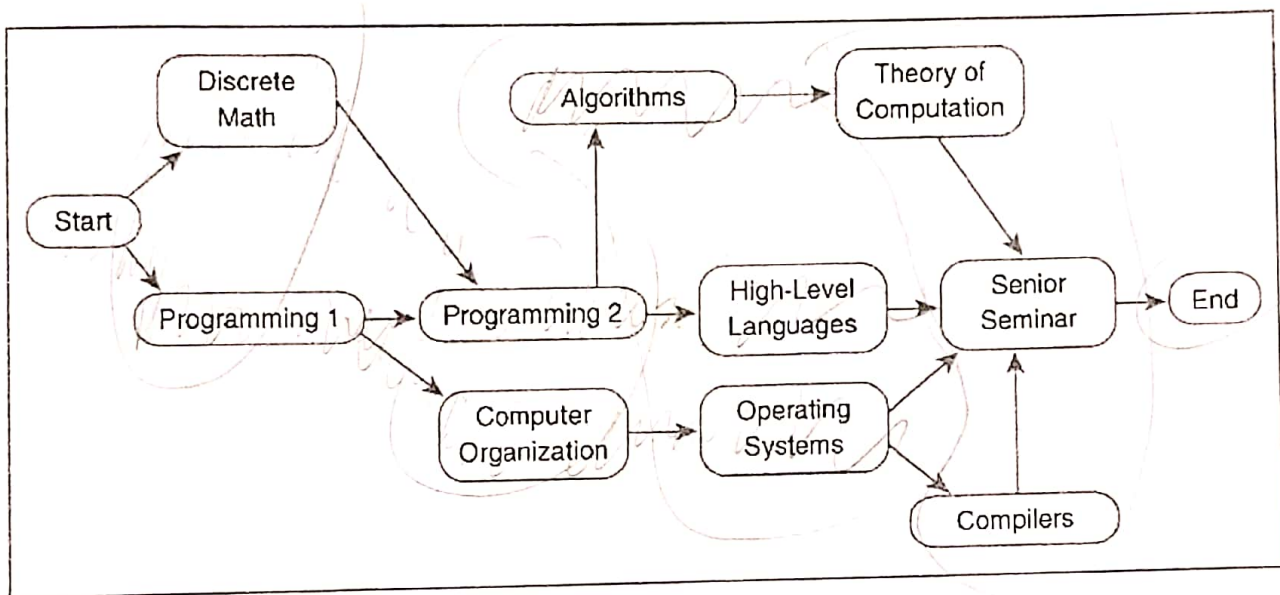
	0	1	2	3	4	5	6	7	8	9
#0	0	1	1	3	3	2	2	0	2	2
#1	/	0	1	3	2	1	1	/	2	1
#2	/	/	0	2	1	0	0	/	2	1
#3	/	/	/	1	0	/	/	/	0	1
#4	/	/	/	/	0	/	/	/	/	0
#5	/	/	/	/	/	/	/	/	/	/

Queue

#0 0, 7
#1 1
#2 2, 5, 6
#3 4, 8
#4 3

In each step, dequeue repeatedly with each element's pred count (and related elements) decreasing by 1. until queue is empty.

10. List the nodes of the graph in a breadth first topological ordering.



Start, Discrete Math, Programming 1, Programming 2, computer organization, Algorithms, High-Level Languages, Operating systems, Theory of Computation, Senior Seminar, END