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Optimising Real-World Traffic Cycle Programs by Using Evolutionary Computation

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ABSTRACT Traffic congestion, and the consequent loss of time, money, quality of life, and higher pollution, is currently one of the most important problems in cities, and several approaches have been proposed to reduce it. In this paper, we propose a novel formulation of the *traffic light scheduling problem* in order to alleviate it. This novel formulation of the problem allows more realistic scenarios to be modeled, and as a result, it becomes much harder to solve in comparison to previous formulations. The proposal of more advanced and efficient techniques than those applied in past research is thus required. We propose the application of diversity-based multi-objective optimizers, which have shown to provide promising results when addressing single-objective problems. The wide experimental evaluation performed over a set of real-world instances demonstrates the good performance of our proposed diversity-based multi-objective method to tackle traffic at a large scale, especially in comparison to the best-performing single-objective optimizer previously proposed in the literature. Consequently, in this paper, we provide new state-of-the-art algorithmic schemes to address the traffic light scheduling problem that can deal with a whole city, instead of just a few streets and junctions, with a higher level of detail than the one found in present studies due to our micro-analysis of streets.

INDEX TERMS Traffic light scheduling problem, traffic management, diversity preservation, real-world application.

I. INTRODUCTION

Nowadays, cities are growing in the number of inhabitants, many of whom are arriving at the city for the first time [1]. City services are then almost never fine tuned for the dwellers, always lacking behind the needs and requests of citizens. In particular, the number of vehicles in streets is continuously increasing, affecting all aspects of daily life and provoking travelling by car is becoming slower than it used to be. Additionally, it is a common source of delays, economic loss, and stress because of the effect that traffic congestion has on people's leisure time and work [2]. Another consequence is the amount of greenhouse gases emitted to the atmosphere

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since the more people driving at low speeds or even stuck in traffic jams, the greater the emissions from vehicle's motors engines [3].

Several strategies have been proposed to prevent traffic jams and reduce the amount of gases emitted to the atmosphere [4]. Some of them are based on the optimisation of the traffic light configurations [5]. These devices are mainly positioned at road intersections to control conflicting flows of traffic and avoid possible accidents. All traffic lights in an intersection are synchronised to carry out a sequence of valid phases—combination of colour states—periodically. Finding an optimal traffic light plan—duration of each phase—is crucial for reducing the number of stops for red lights, thereby minimising the travel time of vehicles through the road network. Intuitive examples are the

well-known green waves [6], which facilitate a continuous traffic flow in one main direction.

The many obvious benefits of optimal traffic light scheduling have motivated a growing field of research related to automatic traffic control signals. Some industrial solutions have been proposed in the past, such as that described in [7] or in [8], but these solutions focus on the real-time configuration of a single traffic light junction and they require additional infrastructure in order to measure traffic flow features. Current approaches [9] use advanced computational techniques to transition from the local control of a single intersection to a holistic approach considering a wide urban area.

The consideration of real-world and large scenarios is only possible by using new and efficient algorithmic tools due to the complexity of the problem, which is twofold. Firstly, the problem usually offers huge search spaces. For example, a simple intersection with eight traffic light phases represents 55^8 —more than $8.3 \cdot 10^{13}$ —candidate solutions, by considering the classical mathematical model [10]. With our novel formulation, which considers more realistic scenarios, the search space is increased in several orders of magnitude. Secondly, there is no closed mathematical formulation of the problem to assess the quality of candidate traffic lights configurations. Thus, the utilisation of simulators is even more required: since we want to be realistic, no abstract flows are considered, but micro-analysis of cars moving in streets are used. Nevertheless, simulators usually are time-consuming and typically require from seconds up to a few minutes per simulation.

The current paper contributes in various aspects of the *Traffic Light Scheduling Problem* (TLSP):

- We increase the realism of the instances by considering the time offset of each intersection in the traffic light scheduling. This parameter is a key value used by traffic managers, which allows the coordination among traffic lights in adjacent junctions to be performed. The inclusion of this new parameter forces us to change the solution encoding, thus providing a novel mathematical formulation of the TLSP.
- We tackle larger and more realistic instances in comparison to those addressed in previous works. For instance, in [10], where several state-of-the-art algorithms were proposed to deal with the TLSP, instances with only 190 traffic lights and less than 500 vehicles were considered, while in this work we take into account instances with more than 950 traffic lights and more than 2,500 vehicles. Since very large scenarios and new configuration parameters are considered, a very important increment in the size of the search space arises.
- We also propose the utilisation of multi-objective techniques based on diversity preservation in order to deal with the huge search space of our scenarios and to outperform that best-performing technique previously provided to address the TLSP: *Particle Swarm Optimisation* (PSO) [11].

- Finally, we perform a wide experimental evaluation consisting of more than 150,000 computational hours where our approaches are exhaustively compared, among others, to those state-of-the-art schemes previously proposed.

The rest of this paper is structured as follows: Section II reviews several publications related to the contributions of this work. In Section III, we describe the novel mathematical model for the TLSP. Then, we present the considered algorithmic approaches in Section IV. Section V discusses the experimental results. Finally, Section VI concludes the paper and provides guidelines for future work.

II. LITERATURE REVIEW

This section is devoted to a brief description of some of the most relevant works that can be found in the related literature with respect to, first, the optimisation of traffic light systems by using different meta-heuristics (Section II-A), and second, the application of diversity-based *Multi-objective Evolutionary Algorithms* (MOEAs) to solve problems which usually are modelled as single-objective optimisation problems (Section II-B).

A. OPTIMISATION OF TRAFFIC LIGHT SYSTEMS

Meta-heuristics have been widely used to tackle traffic light scheduling problems. Early attempts were mostly based on Genetic Algorithms (GAs). One of the first studies appeared in [12], where a GA was employed to optimise the timing of the traffic light cycles of nine intersections located in the city of Chicago (IL), USA. The authors proposed further investigation of GAs on larger problem instances. In [13], reactions of drivers to changes of the traffic light timings were studied. Their approach used a GA and it was evaluated on a case study of the city of Chester, United Kingdom. A GA was also used in [14] to optimise traffic light cycle programs. In this work, the authors assumed that the traffic lights timing of each intersection works independently to other intersections. They tested their approach on an use case of a commercial area of the city of Santa Cruz de Tenerife, Spain. Another work involving the application of GAs on a traffic light scheduling problem appeared in [15]. The proposed approach tackled the problem of controlling the traffic lights timing for vehicles and pedestrians under a dynamic traffic load situation.

Recently, there has been a significant number of works focusing on the application of the Particle Swarm Optimisation (PSO) algorithm to find optimal traffic light schedules. In [16], PSO was employed to train a fuzzy logic controller installed at each intersection. Specifically, PSO was used to train the membership functions and the rules of the controller, targeting to detect the optimal duration of the green signal for each phase of the traffic lights. In [17], a PSO algorithm to discover isolation niches on a traffic light scheduling problem was proposed. The approach was evaluated on a small problem instance, consisting of a one-way road with two junctions. This work focused on the potential of the algorithm

to keep its diversity, without trying to gain deep insight on the problem.

A multi-objective PSO algorithm that employed a predictive model control strategy to optimise traffic light cycle schedules was studied in [18]. The proposed algorithm was evaluated on an urban network consisting of 16 intersections and 51 links. In other works, PSO was proposed as an attempt to compute the optimal traffic light cycle programs [19], [20]. The main objectives of these works were the maximisation of the number of vehicles that reach their destinations, as well as the minimisation of the total trip time of the vehicles. The evaluation of the cycle programs was based on a popular microscopic traffic simulator. The proposed algorithm was assessed on small urban areas located in the cities of Málaga and Sevilla, Spain, and in Bahía Blanca, Argentina.

More recently, PSO algorithms were used for detecting traffic light cycles programs, aiming at the reduction of fuel consumption and vehicular emissions in metropolitan areas [10], [21]. These approaches followed a traffic emission model standardised by the European Union reference framework. The proposed algorithm achieved significant improvements in the considered objectives compared to traffic light cycle programs designed by experts. Finally in [9], the authors used a parallel platform for solving larger instances based on *Differential Evolution* (DE). Results were slightly better than those provided by PSO, but DE showed to require larger computational times in comparison to PSO.

B. TRANSFORMING SINGLE-OBJECTIVE FORMULATIONS TO MULTI-OBJECTIVE ONES TO IMPROVE PERFORMANCE

One of the goals of most MOEAs is to maintain a proper diversity of individuals in order to minimise the premature convergence problem [22]. Due to this implicit feature that the majority of MOEAs share, their application to solving single-objective optimisation problems might be helpful [23]. Three different types of mechanisms have been proposed for solving single-objective optimisation problems by means of MOEAs [24]: methods that transform a constrained single-objective problem into an unconstrained multi-objective problem [25]; methods that consider diversity in the definition of additional objective functions [26]; and methods known as *multi-objectivisation*, which transform a single-objective problem into a multi-objective one by modifying its fitness landscape [27]. This section is devoted to provide a background for the second type of methods, i.e., diversity-based MOEAs as techniques for solving single-objective problems. The application of MOEAs to induce a proper diversity when tackling single-objective optimisation problems is a promising idea. Multi-objective schemes try to optimise several objective functions simultaneously, and consequently, the use of diversity measures as auxiliary objective functions might provide a suitable balance between the exploration and exploitation abilities of a MOEA. Such auxiliary objective functions are also called *diversity-based objectives* [24].

In keeping with the taxonomy provided in [24], in this work, diversity-based objective functions are defined by considering *genotypic measures*. They are incorporated into a well-known MOEA, as we will describe later in Section IV. Hence, since a diversity-based MOEA is used to solve a single-objective optimisation problem, two different objective functions are simultaneously optimised herein: the original objective function corresponding to the single-objective formulation at hand, and an additional diversity-based objective. Genotypic measures are designed by considering differences among individuals at the genotypic domain. The most frequently used genotypic diversity measures are based on the calculation of distance metrics [28]. To do this, the values of the genes have to be used in order to calculate the distance metrics, and different approaches can be taken into consideration: Hamming distance, Euclidean distance, and edit distance, among others. Thus, these diversity measures are also known as *direct diversity measures*. A large number of diversity measures have been proposed for the genotypic space [28], [29].

One of the first diversity-based objective functions to use a direct measure of diversity was based on a distance metric suitable for tree representations [30]. In this case, the diversity-based objective of an individual was calculated as its mean distance to the remaining individuals in the population. This diversity-based objective was incorporated into a multi-objective *Genetic Programming* algorithm and optimised together with two other objective functions. In [31], a bi-objective formulation of a problem aimed at optimising compliant mechanisms—flexible elastic structures—is addressed by the definition of a diversity-based objective. In this case, individuals are encoded using a binary string. The original objective belonging to the single-objective definition of the problem consists of minimising the weight of the structure. The diversity-based objective involves maximising the Hamming distance between the individual at hand and a reference design. Lastly, other diversity-based objectives have been specifically designed to deal with real-valued encodings of the individuals [22], [27].

In [32], different diversity-based objective functions were integrated into the *Non-dominated Sorting Genetic Algorithm II* (NSGA-II) [33] to solve a single-objective formulation of the antenna positioning problem. Although the diversity-based MOEA had a slower convergence, in the long term it was able to achieve statistically better—or, at least, similar—solutions than those provided by the best-performing single-objective optimisers published in the literature to deal with this problem.

A novel diversity-based multi-objective memetic algorithm to deal with a single-objective variant of a frequency assignment problem was proposed in [34]. The memetic approach was based on NSGA-II and several options for the diversity-based objective function were tested. The previously known best frequency plans for both tested real-world networks were improved by the novel diversity-based multi-objective memetic approach. In another work [35], a fuzzy

logic controller specifically designed to adapt several parameters of the above diversity-based multi-objective memetic scheme was proposed. Finally, several studies regarding the scalability and robustness of a parallel hyper-heuristic applied to adapt the parameters of the aforementioned diversity-based multi-objective memetic method were presented in [36].

Diversity-based multi-objective memetic algorithms were also applied to address different instances of a two-dimensional packing problem in [37]. In this particular case, in addition to NSGA-II, the *Improved Strength Pareto Evolutionary Algorithm* (SPEA2) [38] was also considered to define the memetic approaches. Furthermore, diversity-based objective functions that promote diversity in a much smarter manner were proposed, and a parallel island-based model, combined with a hyper-heuristic, was applied in order to speed up the convergence of the memetic schemes to better solutions. Computational results demonstrated that both diversity-based multi-objective memetic schemes, as well as the parallel island-based model and the hyper-heuristic, were able to provide the best-known solutions for the two-dimensional packing problem.

Finally, we should note that diversity-based MOEAs have not only been applied to solve real-world applications, such as antenna positioning, frequency assignment and two-dimensional packing problems, but also benchmark problems [39], including large-scale ones [40].

III. THE TRAFFIC LIGHT SCHEDULING PROBLEM

Urban traffic planning is a fertile area of *Smart Cities* to improve efficiency, environmental care and safety, since traffic jams and congestion are some of the biggest sources of pollution, noise and health. Traffic lights play an important role in solving these problems as they control the flow of the vehicular network in the city. These devices are positioned at road intersections, pedestrian crossings, and other locations to control conflicting flows of traffic and avoid possible accidents. At each intersection, all traffic lights are synchronised to carry out a sequence of valid phases periodically. Each phase consists of a combination of colour states and a time span for which vehicles are allowed to use the roadway. The assignment of the time span for each phase in the phase sequence of all intersections at an urban area is called a *traffic light plan or schedule*. In this work, we tackle the optimal scheduling for all the traffic lights located in a given urban area. The traffic flow of a particular city is a complex system mainly governed by these traffic light program cycles, and therefore, their configuration has a large influence in all city movements.

The novel formulation of the problem we are presenting herein is based on the one proposed in [19] and [20]. Our solutions, however, consider a more complex model, which allows a more realistic environment to be analysed. The mathematical model presented in [19] and [20] is very straightforward, since it uses a vector of integers as the encoding of individuals, where each element represents the phase duration of one particular state of the traffic lights involved in a given

intersection. Phases of different intersections are successively placed in the solution vector, and therefore, the complete traffic light plan is mapped as a simple array of integers. Despite its simplicity, it allows to represent realistic scenarios since actual traffic lights also employ integer values to specify the duration of phases.

In addition to the phase lengths, our new model also considers the time offset of each intersection. This value is used by traffic managers to allow the synchronisation among near junctions, and at the same time, it is a key parameter to avoid constant traffic flow interruptions in common routes. As a result, we use a similar encoding, but including these time offset values. This apparently small change has large implications. First of all, it allows more realistic scenarios to be modelled and the results obtained can be directly used by traffic managers without any additional processing. However, at the same time, it makes the problem even harder, since the number of decision variables is increased proportionally to the number of traffic light intersections. As the existing techniques already had difficulties to solve medium-sized instances of previous formulations of the problem, we need even more efficient approaches to solve this novel formulation.

Fig. 1 shows how the configuration of two consecutive junctions in the city is encoded as a candidate solution for the problem. We can observe that the first intersection consists of traffic lights with a cycle plan comprising six phases. The durations of these phases are encoded as integer values which are included in the solution vector. Furthermore, another value indicating the time offset of this particular junction is added to the solution vector before the duration of the phases. Afterwards, the encoding considers the configuration of the following junction.

Once a solution is generated by a particular approach, we need to evaluate its quality. For doing this, we consider the software *Simulation for Urban MObility* (SUMO) [41], in order to get the basis data to aggregate and compute the fitness of every solution. SUMO is a cross-platform and open-source traffic micro-simulator.¹ Microscopic traffic simulators implement the highest level of detail in the simulation by involving, not only the vehicles moving through streets, but also traffic lights, pedestrians, buses and bicycles, among others. They need large computational resources, as each single vehicle is modelled and updated at a defined time step. In spite of that, the outputs obtained such as travel times, emissions, queue lengths and distances travelled are very accurate because they are calculated for each vehicle while travelling throughout the road network. Particularly, the problem considered herein will be tackled by defining multiple objectives:

- The first objective is to maximise the number, V_R , of vehicles that reach their destination or, equivalently,

¹For further information, visit <http://sumo.dlr.de/index.html>.

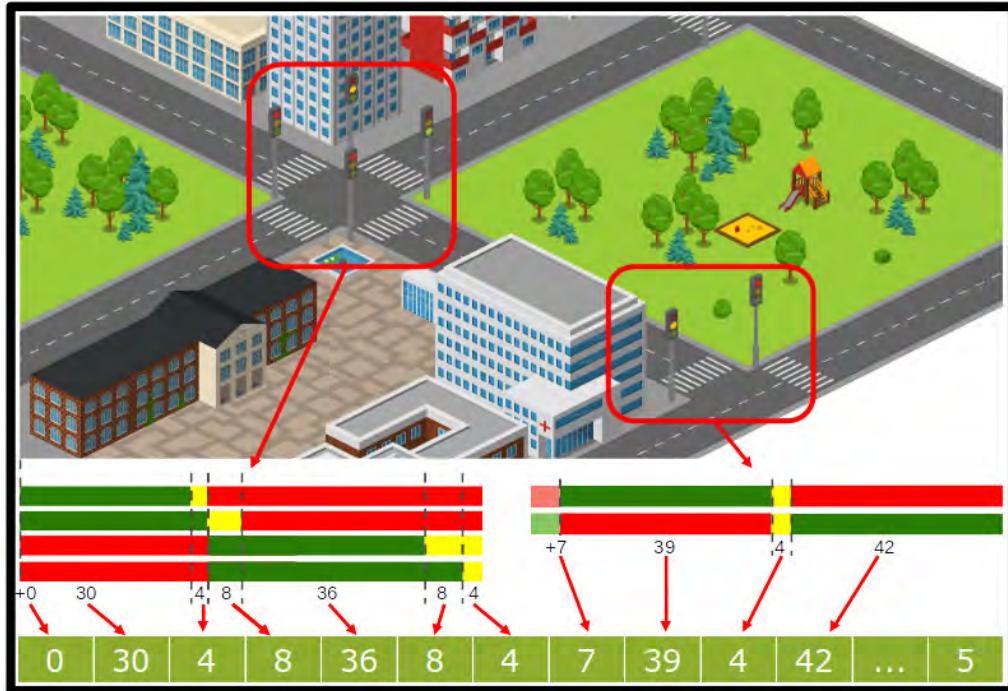


FIGURE 1. Solution encoding for the duration of the phases and offsets of the traffic lights involved in two intersections.

minimise the number V_{NR} of vehicles that do not arrive at their destination, during a given simulation time T_{sim} .

- A second objective is to minimise the total trip time, T_{trip} , of all vehicles, which is equal to the sum of the trip times of all vehicles. The trip time refers to the time individually consumed by each vehicle to arrive at its destination in the time window of study. Evidently, vehicles that fail to reach their destination consume the whole simulation time.
- A third objective is to minimise the sum of stop and wait times of all vehicles, denoted by T_{sw} . The stop and wait time refers to the overall time that each vehicle individually has to stop at those intersections that have traffic lights in red colour, thereby delaying its trip.
- A final objective is to maximise the ratio P of green and red colours in each phase state of all intersections, which is defined as follows:

$$P = \sum_{i=0}^{intr} \sum_{j=0}^{ph} d_{i,j} \frac{g_{i,j}}{r_{i,j}}, \quad (1)$$

where $intr$ denotes the number of all intersections; ph denotes the number of all phases; and $g_{i,j}$, $r_{i,j}$, denote the number of green and red signal colours, respectively, at intersection i and phase state j , with duration $d_{i,j}$. The minimum value of $r_{i,j}$ is set to 1 in order to prevent division by zero. The motivation behind (1) lies in the effort to promote green traffic signals at intersections overburdened by traffic flow, and red traffic signals at intersections where low traffic flow is observed. Traffic

lights with extended times in red colour may overwhelm not only the intersection where they are located, but also neighbouring intersections, thus creating extensive traffic flow problems in the city.

We combine all the above objectives into the single-objective function shown in (2). We should note that this function was proposed and has been considered in previous works related to the TLSP [10], [19], [20].

$$f_{obj} = \frac{T_{trip} + T_{sw} + V_{NR} T_{sim}}{V_R^2 + P}. \quad (2)$$

We should note that the quantities under minimisation are placed in the numerator of (2), whereas the ones under maximisation are placed in the denominator. Therefore, the overall problem is a global minimisation task. The term V_R is squared to prioritise it over all the remaining terms, since it represents the main (first) objective. Additionally, the number of non-arriving vehicles V_{NR} is multiplied by the simulation time T_{sim} to induce a penalisation for this undesired scenario. This is the main objective but in our multi-objective, we will an additional one related with the diversity (it will be described in the next section).

IV. ALGORITHMIC PROPOSALS

This section is devoted to describe the algorithmic schemes selected to carry out comparisons in this work. Particularly, three different population-based approaches are considered: a diversity-based MOEA, a single-objective GA and PSO. The details of these techniques are given in Sections IV-A–IV-C, respectively. Finally, with the aim of

Algorithm 1 Pseudocode of the Non-Dominated Sorting Genetic Algorithm II (NSGA-II)**Require:** n, p_m, p_c

- 1: **Initialisation.** Randomly generate the initial parent population P_0 with n individuals. Assign $t = 0$.
- 2: **Evaluation.** Evaluate all the individuals in the initial parent population by calculating the objective functions.
- 3: **while** (stopping criterion is not satisfied) **do**
- 4: **Fitness assignment.** Calculate the fitness values of individuals in P_t . Use the non-domination rank in the first generation, and the crowded comparison operator in remaining generations.
- 5: **Parent selection.** Perform deterministic binary tournament selection with replacement on P_t in order to fill the mating pool with n parents.
- 6: **Variation.** Apply the crossover and mutation operators with probabilities p_c and p_m , respectively, to the individuals of the mating pool in order to create the offspring population CP with $m = n$ new individuals.
- 7: **Evaluation.** Evaluate offspring in CP by computing the objective functions.
- 8: **Survivor selection.** Select the n fittest individuals from among n parents and m offspring by using the crowded comparison operator so as to constitute P_{t+1} .
- 9: $t = t + 1$
- 10: **end while**

including a trajectory-based meta-heuristic into the comparison, a Variable Neighbourhood Search (VNS) method is also taken into account (see Section IV-D).

A. DIVERSITY-BASED MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM BASED ON THE NSGA-II

The *Non-dominated Sorting Genetic Algorithm II* (NSGA-II) [33] is one of the most widely used MOEAs as of today. One of its most important features is that it uses a fast non-dominated sorting approach with reduced computational complexity. Furthermore, it applies a selection operator which combines previous populations with newly generated ones to ensure elitism in the approach.

The pseudocode of the NSGA-II is shown in Algorithm 1. We note that at step 2, the original objective function of the TLSP, as it is shown in (2), is not only calculated, but also the diversity-based objective function, which will be described at the end of this section. At the same time, the reader should recall that the original objective function is calculated through SUMO. Internally, the algorithm operates with continuous decision variables. Before invoking SUMO, however, a conversion to discrete values is required, which consists of rounding a continuous value to its closest integer.

The following diversity-based objective functions to be maximised, which were selected because they have been

successfully applied when dealing with other real-world problems [35]–[37], are considered in the current work:

- *Average Distance to all Individuals* (ADI) [22]. It is calculated as the mean Euclidean distance in the genotypic space to the remaining individuals in the population.
- *Distance to the Best Individual* (DBI) [27]. It is calculated as the Euclidean distance in the genotypic space to the best individual in the population, with this best individual being determined by its original objective value.
- *Distance to the Closest Neighbour* (DCN) [27]. It is computed as the Euclidean distance in the genotypic space to the closest neighbour in the population.

In order to complete the definition of the approach, we note that the uniform crossover [42] and the polynomial mutation [43] are applied. The polynomial mutation requires the specification of the distribution index η . These variation operators were compared against other choices in a preliminary experiment, in which they presented the best overall performance when dealing with some instances of the TLSP. Finally, after the application of the crossover and/or mutation operators, unfeasible individuals may be obtained, i.e., individuals for which some of their decision variables may have not valid values. A repair method is therefore applied to those individuals obtained from the application of the said variation operators. This repair method consists of assigning feasible values, obtained at random, to the unfeasible genes.

B. SINGLE-OBJECTIVE GENETIC ALGORITHM

Genetic algorithms (GAs) originated as problem-independent adaptive systems [44]. We selected a single-objective GA whose variation operators are the same than those applied by the diversity-based MOEA described in Section IV-A. The idea is to measure the contribution that the diversity-based MOEA may provide in comparison to a single-objective GA, which does not promote diversity in the population explicitly, and whose operation is very similar. Additionally, as we stated in Section II, GAs have been widely applied in order to deal with the optimisation of traffic light systems in previous works. The operation of the said GA is shown in Algorithm 2.

As it can be observed, at steps 2 and 10, the population is evaluated by using the objective function, i.e., that one described in (2) of Section III, to assign a fitness value to every individual. As in the case of the diversity-based MOEA, the GA operates with continuous decision variables. Before invoking SUMO, however, a conversion to discrete values is required. In order to make the conversion, a continuous value is rounded to its closest integer. At this point, we note that once the offspring population is generated, i.e., before step 10, if m is odd, the worst individual from the offspring population is discarded, since $m + 1$ offspring have been produced. As in the case of the diversity-based MOEA, the uniform crossover, the polynomial mutation, and the same repair method are applied together with this GA. These variation

Algorithm 2 Pseudocode of the Genetic Algorithm (GA)**Require:** n, p_m, p_c

- 1: **Initialisation.** Create the initial parent population by filling it with n randomly generated individuals.
- 2: **Evaluation.** Evaluate all individuals in the initial parent population by applying the objective function in order to assign a fitness value to every individual.
- 3: **while** (stopping criterion is not satisfied) **do**
- 4: **Offspring population generation:**
- 5: **while** (offspring population is not filled with $m = n$ individuals) **do**
- 6: **Parent selection.** Apply deterministic binary tournament selection with replacement on the current parent population in order to select two parents.
- 7: **Recombination.** Apply the crossover operator with probability p_c to both parents in order to produce two offspring. If crossover operator is not applied, both parents become the two new offspring.
- 8: **Mutation.** Apply the mutation with prob. p_m to both generated offspring.
- 9: **end while**
- 10: **Evaluation.** Evaluate the m generated offspring by means of the objective function so as to assign a fitness value to every offspring.
- 11: **Survivor selection.** Select individuals from among n parents and m offspring that will constitute the parent population for the next generation.
- 12: **end while**

operators also presented the best overall performance in a preliminary experiment when combined with this GA. Finally, it is important to remark that the survivor selection mechanism applied at step 11 is generational with elitism, i.e., the best individual in the parent population is always selected to survive for the next generation, together with the best $m - 1$ individuals belonging to the offspring population.

C. PARTICLE SWARM OPTIMISATION

Particle Swarm Optimisation (PSO) is a population-based algorithm widely used for numerical optimisation. It was initially proposed in [11]. The inspiration behind the algorithm originates from the collective behaviour of socially organised living organisms. A significant amount of work has been devoted to the theoretical and empirical investigation of PSO [45], [46]. Its pseudocode is shown in Algorithm 3.

PSO employs a population $P = \{x_1, x_2, \dots, x_n\}$, called a *swarm*, of n candidate solutions, where each vector x_i is called a *particle*. Initially, the D -dimensional particles $x_i = (x_{i1}, x_{i2}, \dots, x_{iD})$ are randomly initialised within the search space \mathbb{S} . Then, each particle probes the search space iteratively, retaining in memory the best position it has ever discovered, denoted by $p_i = (p_{i1}, p_{i2}, \dots, p_{iD}) \in \mathbb{S}$. The movement of each particle is conducted by adding to its current position an adjustable position shift, called *velocity*,

Algorithm 3 Pseudocode of Particle Swarm Optimisation (PSO)**Require:** $n, \omega_{\max}, \omega_{\min}, c_1, c_2$

- 1: Generate n particles as the initial swarm through an random initialisation strategy
- 2: **while** (stopping criterion is not satisfied) **do**
- 3: Select of the best particle, p_{best} , from the current swarm
- 4: Calculate inertia weight according $\omega_{\max}, \omega_{\min}$ and the current generation
- 5: **for** ($j = 1 : n$) **do**
- 6: The particle p_j belonging to the current swarm is referred to as the target particle
- 7: Calculate the new velocity, \vec{v}'_j of the selected particle based on its inertia and cognitive and social components weighted according ω, c_1 and c_2
- 8: Update the position, \vec{x}'_j , using the current position of the particle, \vec{x}_j and the calculated velocity, \vec{v}'_j
- 9: **end for**
- 10: **end while**
- 11: **return** the fittest individual in the population

and denoted as $v_i = (v_{i1}, v_{i2}, \dots, v_{iD})$. Thus, at iteration g , each particle updates its position according to:

$$x_{ij}^{[g+1]} = x_{ij}^{[g]} + v_{ij}^{[g+1]}, \quad (3)$$

$$v_{ij}^{[g+1]} = \omega v_{ij}^{[g]} + c_1 R_1 \left(p_{ij}^{[g]} - x_{ij}^{[g]} \right) + c_2 R_2 \left(p_{best}^{[g]} - x_{ij}^{[g]} \right) \quad (4)$$

where, $p_{best}^{[g]}$ is the best particle of the swarm; ω is the inertia weight of the particle; c_1 and c_2 are the cognitive and the social parameters, respectively; and R_1 and R_2 are uniformly distributed random variables in the range $[0, 1]$.

At each iteration, each particle of the swarm also updates its best position as follows:

$$p_i^{[g+1]} = \begin{cases} x_i^{[g+1]}, & \text{if } f(x_i^{[g+1]}) < f(p_i^{[g]}) \\ [0.3cm] p_i^{[g]}, & \text{otherwise} \end{cases} \quad (5)$$

Following the recommendations given in [9], in the present work, the inertia weight changes linearly throughout the optimisation process according to the following rule:

$$\omega = \omega_{\max} - \frac{(\omega_{\max} - \omega_{\min})g}{g_{\max}}, \quad (6)$$

where ω_{\min} and ω_{\max} define its range, g is the iteration counter, and g_{\max} is the maximum number of iterations.² At the beginning of the optimisation process, (6) allows the inertia weight to take high values, thereby promoting exploration, whereas as ω reduces, the balance moves towards exploitation.

²In the case of setting a stopping criterion based on the number of function evaluations performed, rather than using a stopping criterion based on the number of iterations carried out, g and g_{\max} would refer to the number of function evaluations performed and the maximum number of function evaluations, respectively.

Algorithm 4 Pseudocode of Variable Neighbourhood Search (VNS)**Require:** n_i, n_f, n_s, CD

Calculate the number of neighbourhoods k_{max} according to n_i, n_f and n_s

Select the set of neighbourhood structures $N_k, k = 1, \dots, k_{max}$;

Find an initial solution \vec{x} ;

while stopping condition is not met **do**

$k = 1$;

while $k <= k_{max}$ **do**

 Generate randomly $\vec{x}' \in N_k(\vec{x})$;

if \vec{x}' is better than \vec{x} **then**

$\vec{x} = \vec{x}'$;

$k = 1$;

else

if \vec{x} was not updated during CD steps using N_k **then**

$k = k + 1$;

end if

end if

end while

end while

```

1:   instance name
2:   path containing the network and route files
3:   number of junctions (nI)
4:   total number of phases considering all junctions
5:   <junctID_1> <nP_1> <p-1-1, ..., p-1-nP_1>
6:   <junctID_2> <nP_2> <p-2-1, ..., p-2-nP_2>
...
nI + 4: <junctID_nI> <nP_nI> <p-nI-1, ..., p-nI-nP_nI>
nI + 5: number of vehicles
nI + 6: simulation time (seconds)

```

FIGURE 2. File format of an instance.

are modified by adding a value randomly selected from $[-n, n]$. The value of n depends on the neighbourhood selected. The first neighbourhood (N_1) starts with a small value (n_i) in order to intensify the search in the current region. The different neighbourhoods are generated by increasing the value of n by n_s until the value n_f is reached. Since the neighbourhoods increase n , they allow larger areas of the search space to be explored in order to escape from a potential local optimum. Therefore, the algorithm starts by generating solutions using N_1 until a solution is improved or a maximum number of solutions CD is obtained. If it generates CD solutions without getting a better one, it changes to the next neighbourhood. When a better solution is found, it replaces the current one and the algorithm backs to the first neighbourhood.

V. EXPERIMENTAL EVALUATION

This section is aimed to present the computational experiments carried out to assess the different algorithmic schemes described in Section IV through their application to the novel formulation of the TLSP that was proposed at Section III. Particularly, the diversity-based MOEA, as well as the GA, PSO and VNS were applied to optimise the traffic light cycle programs of four different areas of real-world cities: Berlin, Paris, Stockholm, and Malaga. The experimental method, the features of the instances, as well as the parameterisation of the algorithms, will be detailed in the next paragraphs.

a: EXPERIMENTAL METHODOLOGY

All the algorithms, as well as the novel formulation of the TLSP, were implemented through the *Meta-heuristic-based Extensible Tool for Cooperative Optimisation* (METCO) proposed in [48]. Tests were carried out on Teide High Performance Computing facilities, which are composed of 1100 Fujitsu® computer servers, with a total of 17,800 computing cores and 36 tb of memory. With respect to the software, we used the version 0.28.0 of SUMO with 23432 as the seed for the generation of vehicle routes. Since all the approaches are stochastic, with the aim of statistically supporting in a sound manner the conclusions extracted each run was repeated 30 times. In particular, the following statistical testing procedure, which was formerly used in previous work by Segura et al. [49], was applied to conduct comparisons among algorithmic schemes. First, a *Shapiro-Wilk test* was performed to check whether the values of the results followed a normal (Gaussian) distribution. If so,

Also, as suggested in [19] and [20], the update of the velocity can be properly modified to tackle combinatorial problems:

$$v_{ij}^{[g+1]} = \begin{cases} \lfloor v_{ij}^{[g+1]} \rfloor, & \text{if } R \leq \lambda, \\ [0.3cm] \lceil v_{ij}^{[g+1]} \rceil, & \text{otherwise,} \end{cases} \quad (7)$$

where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ are the floor and ceiling functions, respectively, and R is a random number uniformly distributed in the range $[0, 1]$. Parameter λ determines the probability of using the floor or ceiling function in the computation of the velocity. In our study, its value is set to 0.5. A comprehensive presentation of the PSO algorithm can be found in [46].

D. VARIABLE NEIGHBOURHOOD SEARCH

Variable Neighbourhood Search (VNS) is a meta-heuristic presented in [47]. VNS solves optimisation problems by doing systematic changes of neighbourhood within a *Local Search* (LS). VNS is a descendent method explores different predefined neighbourhoods of the current solution using LS. The current solution is changed by a new one if and only if an improvement has been made. The basic idea is to change the neighbourhood structure when the local search is trapped in a local optimum. A neighbourhood structure in a solution space S is a mapping $N : S \rightarrow 2^S, X \rightarrow N(X)$, where $N(X)$ constitutes the neighbourhood of X . Algorithm 4 shows its pseudocode.

The main feature which should be defined is the different neighbourhoods. Since we encode the solutions of the TLSP as a vector of integers, we can apply traditional variation operators for this representation. In particular, we use a variant of the arithmetic mutation in which some positions of the vector

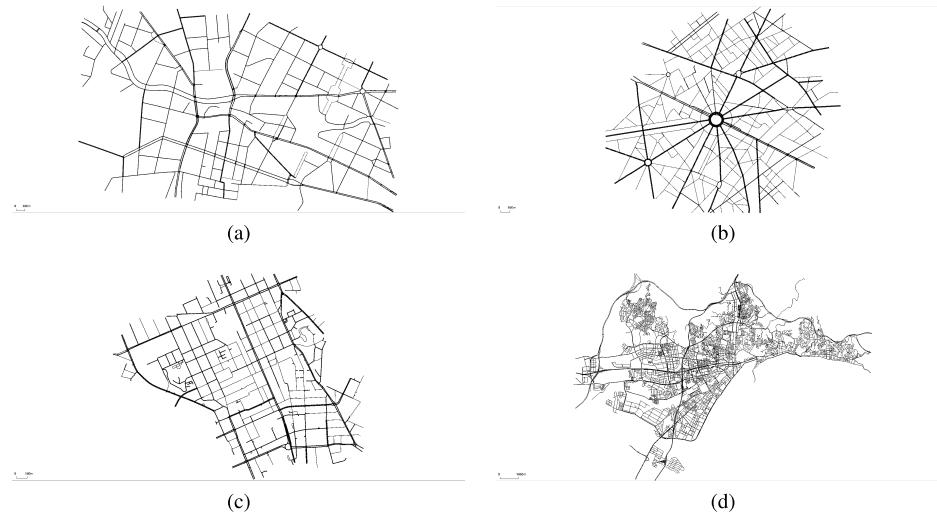


FIGURE 3. Geographical areas of four real cities used in this study. (a) Berlin. (b) Paris. (c) Stockholm. (d) Malaga.

the *Levene test* checked for the homogeneity of the variances. For Gaussian distributions, if the samples had equal variance, an *anova test* was done. Otherwise, a *Welch test* was performed. For non-Gaussian distributions, the non-parametric *Kruskal-Wallis* test was used. For all tests, a significance level $\alpha = 10^{-2}$ was considered.

b: INSTANCES

A particular instance of the problem is given by an input file with the format described in Fig. 2.³ The first line indicates the name of the instance, while the second one specifies the particular path where the files containing information about the network and routes of the vehicles can be found. The third and fourth lines give the total number of intersections, and the total number of phases considering all intersections, respectively. Then, for each intersection, its identifier, the number of different phases of that particular intersection, and the composition of each of those phases, are given in subsequent lines. At this point, it is important to remark that a particular phase of a given intersection indicates the current state of each traffic light belonging to the intersection at hand. For instance, the line `-187462 4~GGGr yyrrG rrY` describes an intersection that consists of three traffic lights (three colour values), which change their state according to four different phases. In the first phase (`GGGr`), for instance, the first two traffic lights are green (G), while the third one is red (r). In the second phase (`yyrr`), however, the first two traffic lights are yellow (y), while the last one is still red (r). Finally, the last two lines of the file detail the number of vehicles involved in the simulation and the simulation time, respectively. At this point, we should note that the path specified in the second line

³The particular instances addressed in the current work, the wrapper to use SUMO as the evaluator of potential solutions, as well as the results and graphics extracted from the studies presented in the current paper, can be found through <https://github.com/esegredo/Optimisation-Real-World-Traffic-Light-Cycle-Programs-EvoComp>.

of the instance file should contain two files with additional information required for the simulation through SUMO:

- `instanceName.net.xml`. It consists of information about the network or map of the particular geographical area where the traffic light system is located.
- `instanceName.rou.xml`. It contains data about the routes of the vehicles.

Bearing the above discussion in mind, in the current work, we have addressed four different real-world instances, whose details are described in Table 1. Furthermore, Fig. 3 shows the particular geographical areas of the four considered cities. First of all, we would like to mention that these particular instances are much harder to solve than those addressed in previous works like [9], [10], and [19]–[21]. In these previous works, instances considered 20–40 intersections, a total number of vehicles lower than 500, and analysis times lower than 500 seconds. While here, we are considering up to 961 intersections, 2,632 vehicles and 9,000 seconds of analysis time. Additionally, since in this work our novel formulation of the TLSP also considers the time offset of each intersection, the decision search space is significantly larger in comparison to previous definitions of the problem. At this point, we should recall that the sum of the total number of intersections and the total number of phases provides the total number of decision variables D of an individual. The evaluation of an individual involves the execution of a simulation through SUMO. We note that a fitness function,

TABLE 1. Information about the different real-world instances.

	Berlin	Paris	Stockholm	Malaga
Total number of intersections	97	70	75	961
Total number of phases	514	378	370	3,800
Number of decision variables (D)	611	448	445	4,761
Total number of vehicles	1,300	1,200	1,400	2,632
Simulation time (sec.)	3,400	3,400	4,000	9,000

TABLE 2. Parameterisation of the different approaches considered.

Diversity-based multi-objective evolutionary algorithm (NSGAII-ADI/DBI/DCN)			
Parameter	Value	Parameter	Value
Population size (n)	100 individuals	Mutation rate (p_m)	$1/D$
Crossover rate (p_c)	1	Poly. mut. dist. index (η)	20
Diversity-based objectives	ADI, DBI, DCN		
Single-objective genetic algorithm (MonoGA)			
Parameter	Value	Parameter	Value
Population size (n)	100 individuals	Mutation rate (p_m)	$1/D$
Crossover rate (p_c)	1	Poly. mut. dist. index (η)	20
Particle swarm optimisation (PSO)			
Parameter	Value	Parameter	Value
Swarm size (n)	50 particles	Min. inertia weight(ω_{min})	0.1
Max. inertia weight (ω_{max})	0.5	Cognitive parameter (c_1)	2.05
Social parameter (c_2)	2.05		
Variable neighbourhood search (VNS)			
Parameter	Value	Parameter	Value
Initial neighbourhood size (n_i)	5	Final neighbourhood size (n_f)	60
Neighbourhood size step (n_s)	2	Convergence detection (CD)	D

which is the most expensive part of the algorithms in terms of computational cost, does not only depend on the number of decision variables we are dealing with, but also on the number of vehicles involved in the simulation and the simulation time. Finally, we have to say that the number of vehicles was estimated by gathering real traffic data from the different areas taken into account herein. Moreover, the analysis time was adjusted such that it was hard to have a situation where most vehicles arrive to their destinations, unless a suitable traffic light scheduling is found.

c: PARAMETERS

The particular parameterisation applied for each of the algorithms depicted in Section IV is detailed in Table 2. In the case of the diversity-based MOEA, it can be observed that the different diversity-based objective functions described in Section IV-A are considered herein, i.e., ADI, DBI and DCN. In the rest of the paper, the diversity-based MOEA combined with each of the functions ADI, DBI and DCN will be termed as NSGAII-ADI, NSGAII-DBI and NSGAII-DCN, respectively. Regarding the diversity-based MOEA and the GA, the mutation rate p_m was fixed such that only one decision variable, on average, is modified by the polynomial mutation operator each time it is applied, and at the same time, the uniform crossover operator is always used ($p_c = 1$), something that is very common in the related literature. Furthermore, the distribution index of the polynomial mutation was set to a typical value ($\eta = 20$). Regarding the population size n , 100 individuals were selected since this value provided the best overall performance in a preliminary experiment. The parameter values of PSO were set by following the corresponding recommendations given in previous works, where the best-known results were provided considering the resolution of the TLSP [19]–[21]. With respect to VNS, since it is the first time we apply this trajectory-based approach to the TLSP, its parameters were set by carrying out a preliminary parameter setting experiment where different values were tested. Finally, since different experiments that involve executions with different features are performed, the

particular stopping criterion considered for each of them will be given in the corresponding sections below.

A. FIRST EXPERIMENT: PERFORMANCE OF DIVERSITY-BASED OBJECTIVES

The main goal of this first experiment is to carry out a study of the performance attained by the combination of the NSGA-II together with different diversity-based objective functions: ADI, DBI and DCN. Since it is the first time that a diversity-based MOEA is applied to solve this particular problem, it would be interesting to study the performance that a diversity-based MOEA provides when it is combined with different diversity-based objectives. For doing that, schemes NSGAII-ADI, NSGAII-DBI and NSGAII-DCN were applied to solve the four aforementioned instances. In the case of Berlin, Paris and Stockholm, a stopping criterion equal to 10^4 function evaluations was considered, while in the case of Malaga, only 10^3 function evaluations were performed, since the latter is a much bigger instance in comparison to the remaining ones (see Table 1), and as a result, the evaluation of one individual through SUMO takes significantly longer.

TABLE 3. Statistics obtained by NSGAII-ADI, NSGAII-DBI and NSGAII-DCN at the end of 30 independent runs.

NSGAII-ADI			
Instance	Mean	Median	SD
Berlin	8.360e-01	8.418e-01	2.090e-02
Paris	7.009e-01	7.011e-01	1.078e-02
Stockholm	6.895e-01	6.895e-01	1.342e-02
Malaga	9.391e-01	9.648e-01	1.183e-01
NSGAII-DBI			
Instance	Mean	Median	SD
Berlin	8.049e-01	8.021e-01	1.557e-02
Paris	6.718e-01	6.717e-01	8.296e-03
Stockholm	6.730e-01	6.711e-01	1.301e-02
Malaga	8.902e-01	8.999e-01	1.189e-01
NSGAII-DCN			
Instance	Mean	Median	SD
Berlin	7.797e-01	7.742e-01	1.960e-02
Paris	6.593e-01	6.587e-01	1.251e-02
Stockholm	6.564e-01	6.544e-01	1.550e-02
Malaga	9.044e-01	9.269e-01	9.686e-02

TABLE 4. P-values resulting from the pairwise statistical comparison among NSGAII-ADI, NSGAII-DBI and NSGAII-DCN, summarising 30 independent runs. The particular statistical test giving the corresponding p-value for each case is also shown.

Instance	DCN vs. DBI			DCN vs. ADI			DBI vs. ADI		
	p-value	w	Test	p-value	w	Test	p-value	w	Test
Berlin	2.583e-06	↑	Kruskal-Wallis	7.733e-10	↑	Kruskal-Wallis	1.723e-08	↑	ANOVA
Paris	3.492e-05	↑	Welch	6.224e-20	↑	ANOVA	6.668e-17	↑	ANOVA
Stockholm	3.265e-05	↑	ANOVA	2.313e-12	↑	ANOVA	1.023e-05	↑	ANOVA
Malaga	6.139e-01	↔	ANOVA	2.188e-01	↔	ANOVA	1.157e-01	↔	ANOVA

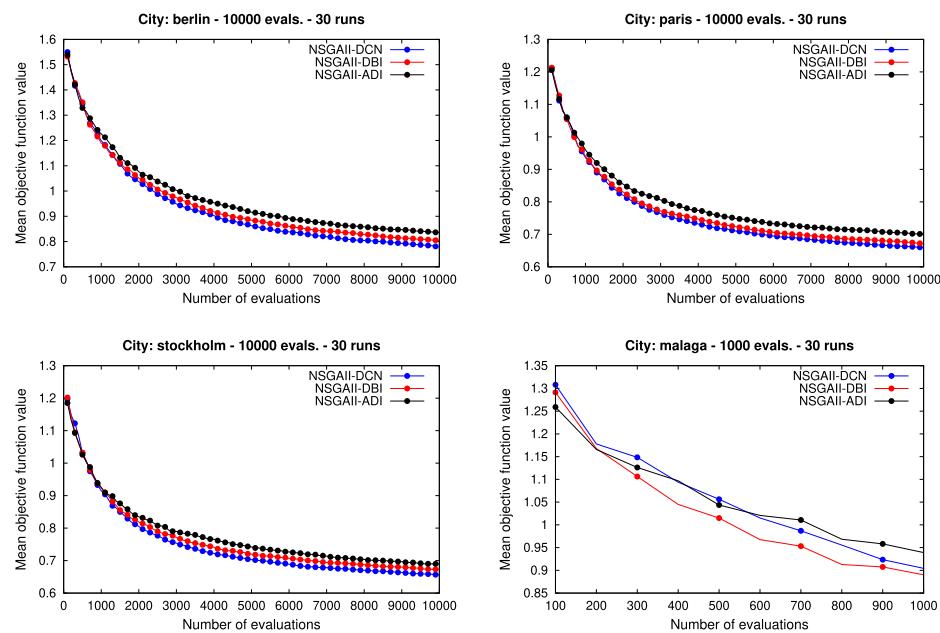


FIGURE 4. Evolution of the mean objective function value achieved by NSGAII-ADI, NSGAII-DBI and NSGAII-DCN.

Finally, we note that the runs performed in this particular experiment involved almost 10,000 computational hours.

Table 3 shows the mean, median and standard deviation (SD) provided by the schemes NSGAII-ADI, NSGAII-DBI and NSGAII-DCN at the end of the executions for the four instances considered. For each instance, data corresponding to the lowest mean and median objective function value are shown in boldface. We can observe how algorithm NSGAII-DCN was able to attain the lowest mean and median of the objective function value at the end of the runs for Berlin, Paris and Stockholm instances. In the case of the Malaga instance, the lowest mean and median of the objective function value was provided by the scheme NSGAII-DBI.

In order to statistically support the above results, Table 4 shows the p-values and information about the winner (w) approaches resulting from all possible statistical pairwise comparisons among schemes NSGAII-ADI, NSGAII-DBI and NSGAII-DCN. Particularly, it shows if the first scheme of a pairwise comparison statistically outperformed the second approach (↑), if the first scheme was statistically worse than the second one (↓), and if both approaches did not present statistically significant differences (↔). Scheme A statistically outperforms scheme B if the p-value obtained

from the statistical comparison procedure explained at the beginning of this section is lower than the significance level α , and if at the same time, A provides a lower mean and median of the objective function value in comparison to the mean and median of the objective function value attained by B. Finally, as it was explained at the beginning of this section, the statistical procedure designed to compare the different approaches involves the application of several statistical tests. For each case, the particular statistical test providing the corresponding p-value is also shown in Table 4. It can be observed that NSGAII-DCN statistically outperformed schemes NSGAII-DBI and NSGAII-ADI considering Berlin, Paris and Stockholm instances, while the former was not statistically outperformed by any other approach. As a result, we can conclude that the best-performing scheme at the end of the runs for those three instances was NSGAII-DCN. At the same time, the worst-performing scheme was NSGAII-ADI, since it was not only statistically outperformed by the approach NSGAII-DCN, but also by the method NSGAII-DBI, considering the same three instances. In the case of the Malaga instance, statistically significant differences did not arise among the three approaches. It has been shown that very long executions are required by approaches that manages diversity in an explicit way [50], such as the diversity-based

MOEAs considered herein. As a result, even longer executions may be performed to notice differences among the diversity-based approaches, particularly, when dealing with large instances of the TLSP.

The previous analysis considered the behaviour of the three variants of the diversity-based MOEA at the end of their executions. At this point, however, it would also be interesting to study their run-time behaviour. Fig. 4 shows, for each of the instances considered, the evolution of the mean objective function value attained by each of the approaches NSGAII-ADI, NSGAII-DBI and NSGAII-DCN during the runs. First of all, it can be observed how the approach NSGAII-DCN was able to provide the lowest mean objective function value for almost all the execution, in the case of Berlin, Paris and Stockholm instances. In the case of the Malaga instance, however, the best mean objective function value was provided by the scheme NSGAII-DBI during almost the entire run.

Bearing the above discussion in mind, we can confirm that, in this first experiment, the best-performing diversity-based objective function was DCN in the case of dealing with smaller instances of the problem—Berlin, Paris and Stockholm—and not only considering the results attained at the end of the executions, but also along them. However, when tackling larger instances—Malaga—a clear conclusion cannot be extracted. Although the diversity-based objective DBI provided the lowest mean objective function value for almost the whole run, at the end of the executions, the different approaches did not present statistically significant differences among them for this particular instance. Consequently, and as we said before, the three schemes considered herein may be executed for longer, specially in the case of the Malaga instance, so as to shed more light on the above fact. The reader should recall that due to time restrictions, executions performed with the Malaga instance considered 10^3 function evaluations, rather than 10^4 evaluations, which was the stopping criterion fixed for the remaining instances.

B. SECOND EXPERIMENT: COMPARISON OF THE DIVERSITY-BASED MOEA AGAINST OTHER OPTIMISERS

The main aim of the second experiment is to compare the diversity-based MOEA to the remaining approaches described in Section IV, in terms of the performance they are able to attain for the instances considered herein. The approach NSGAII-DCN was selected for this second experiment, since in the previous one it was the scheme providing the best overall performance. Additionally, the single-objective GA (MonoGA), PSO and VNS were also executed by considering the parameterisation shown in Table 2. For this particular experiment, executions were repeated 30 times. Finally, note that a sufficiently long stopping criterion was fixed, consisting of 7 days. The idea was to study whether approaches selected converged prematurely or not, and thus determining a proper value for the stopping criterion to be used in subsequent experiments. We note that all the executions carried out in this second experiment involved 100,800 computational hours.

TABLE 5. Statistics obtained by the different approaches at the end of 30 independent runs of 7 days.

Berlin							
	Best	Q1	Median	Mean	Q3	Worst	SD
NSGAII-DCN	0.640	0.658	0.668	0.667	0.676	0.691	0.015
MonoGA	0.669	0.688	0.696	0.698	0.706	0.734	0.016
PSO	1.478	1.539	1.610	1.622	1.714	1.772	0.098
VNS	1.199	1.384	1.521	1.570	1.706	2.002	0.231
Paris							
	Best	Q1	Median	Mean	Q3	Worst	SD
NSGAII-DCN	0.566	0.584	0.589	0.588	0.593	0.601	0.007
MonoGA	0.593	0.598	0.604	0.607	0.613	0.646	0.012
PSO	1.081	1.168	1.240	1.233	1.284	1.442	0.084
VNS	0.937	0.990	1.041	1.064	1.090	1.341	0.105
Stockholm							
	Best	Q1	Median	Mean	Q3	Worst	SD
NSGAII-DCN	0.531	0.557	0.560	0.561	0.568	0.580	0.011
MonoGA	0.562	0.588	0.594	0.595	0.606	0.627	0.015
PSO	1.019	1.138	1.186	1.189	1.249	1.383	0.106
VNS	0.939	0.980	1.046	1.076	1.125	1.336	0.119
Malaga							
	Best	Q1	Median	Mean	Q3	Worst	SD
NSGAII-DCN	0.506	0.625	0.654	0.650	0.699	0.771	0.066
MonoGA	0.466	0.496	0.560	0.560	0.625	0.658	0.065
PSO	0.802	1.073	1.143	1.181	1.315	1.540	0.180
VNS	0.515	0.799	1.261	1.172	1.479	1.738	0.385

TABLE 6. P-values resulting from the pairwise statistical comparison between NSGAII-DCN and each of the remaining approaches summarising 30 independent runs of 7 days. The particular statistical test giving the corresponding p-value for each case is also shown.

Berlin					Paris		
	p-value	w	Test	p-value	w	Test	
MonoGA	2.0655e-10	↑	ANOVA	6.4042e-10	↑	Kruskal-Wallis	
PSO	1.9924e-31	↑	Welch	7.8751e-28	↑	Welch	
VNS	2.2234e-19	↑	Welch	2.8396e-11	↑	Kruskal-Wallis	
Stockholm							
	p-value	w	Test	p-value	w	Test	
MonoGA	2.3731e-14	↑	ANOVA	5.2529e-06	↓	Kruskal-Wallis	
PSO	1.1511e-24	↑	Welch	1.9293e-17	↑	Welch	
VNS	2.8307e-11	↑	Kruskal-Wallis	6.2037e-08	↑	Kruskal-Wallis	
Malaga							
	p-value	w	Test	p-value	w	Test	
MonoGA	2.3731e-14	↑	ANOVA	5.2529e-06	↓	Kruskal-Wallis	
PSO	1.1511e-24	↑	Welch	1.9293e-17	↑	Welch	
VNS	2.8307e-11	↑	Kruskal-Wallis	6.2037e-08	↑	Kruskal-Wallis	

Table 5 shows, for each instance, statistical information about the performance of the different optimisers at the end of the runs. Furthermore, for each instance, the lowest mean and median of the objective function value are shown in boldface. In this second experiment, it can be observed how the diversity-based MOEA (NSGAII-DCN) was able to provide the best results in the case of Berlin, Paris and Stockholm instances. In the case of the Malaga instance, however, the best performance was attained by the single-objective GA (MonoGA). We should note that PSO was not able to attain good results in comparison to both aforementioned schemes, in spite of being the best-performing approach previously proposed to deal with this problem in past research. Finally, in the case of VNS, results were not promising either, in comparison to the NSGAII-DCN and MonoGA.

In order to statistically support the above statements, Table 6 shows information about the p -values and winner (w) schemes resulting from the pairwise statistical comparison between the NSGAII-DCN and each of the remaining optimisation schemes. The same statistical comparison procedure used in the first experiment was also applied herein. As a result, the particular statistical test providing the corresponding p -value is also shown in this table. The reader should recall that, in this case, an \uparrow is shown when

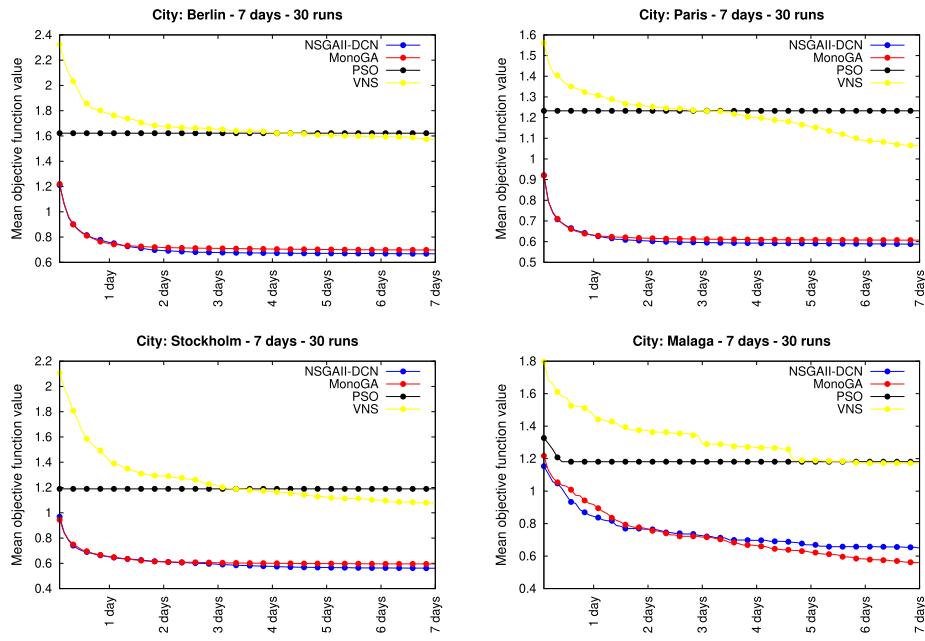


FIGURE 5. Evolution of the mean objective function value considering 30 independent runs of 7 days.

the scheme NSGAII-DCN was statistically better than the approach shown in the corresponding row. Cases for which the NSGAII-DCN was statistically worse than the approach shown in the corresponding row, a ↓ is shown. As it can be observed, considering Berlin, Paris and Stockholm instances, the approach NSGAII-DCN was able to statistically outperform all the remaining schemes. In the case of Malaga, the NSGAII-DCN was also able to statistically outperform PSO and VNS, but it was statistically outperformed by the method MonoGA. In fact, for this particular instance, the scheme MonoGA was statistically superior to any other approach, thus demonstrating its good performance for this particular test case.

Regarding the run-time behaviour of the techniques considered, Fig. 5 shows the evolution of the mean objective function value attained by each of the schemes during the runs performed in this second experiment for each instance considered. With respect to PSO, it converged prematurely at early stages of the optimisation procedure. At the same time, although VNS did not converge prematurely to local optima, it was not able to provide competitive results. We should note, however, that it was able to provide better results than those attained by PSO at the end of the executions. The reader should recall that VNS is a trajectory-based metaheuristic, rather than a population-based approach, such as the remaining ones, so it is a wonder that it is not stuck in as many local optima as PSO for this complex problem.

With regard to NSGAII-DCN and MonoGA, we can conclude that, in the case of Berlin, Paris and Stockholm instances, NSGAII-DCN did not only provide the best results at the end of the runs, but also throughout them. Nevertheless, the best performance along the whole set of executions was

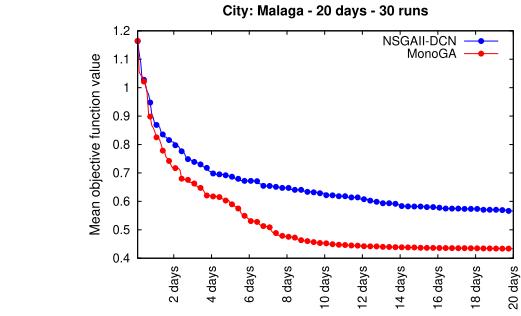


FIGURE 6. Evolution of the mean objective function value considering 30 independent runs of 20 days length for the Malaga instance.

attained by MonoGA, in the case of the Malaga instance. Anyway, both schemes were able to outperform, for all test cases considered, the state-of-the-art approach previously applied to solve this particular problem: PSO.

We note that, in past research, the authors demonstrated that diversity-based MOEAs usually required longer executions than those performed by their equivalent single-objective methods to outperform the latter when dealing with some instances of the problem at hand [50]. Furthermore, any of the approaches NSGAII-DCN and MonoGA did not converge prematurely even after 7 days of execution, as it is shown in Fig. 5, and consequently, longer runs may be performed in order to analyse the behaviour of both methods in the long term. The above fact was even more noticeable in the case of the Malaga instance.

Bearing the above in mind, methods NSGAII-DCN and MonoGA were run during 20 days to solve the Malaga instance. Executions were repeated 30 times, which involved

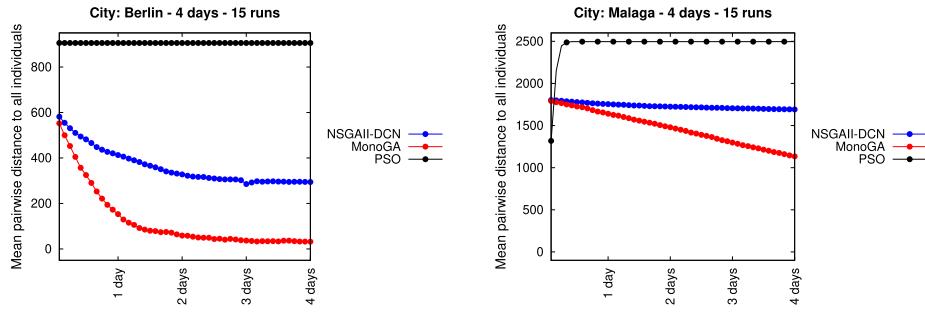


FIGURE 7. Evolution of the mean pairwise distance to all individuals considering 15 independent runs of 4 days length for Berlin and Malaga instances.

28,800 computational hours. Fig. 6 shows the evolution of the mean objective function value provided by NSGAII-DCN and MonoGA. The superiority of the single-objective GA in comparison to the diversity-based MOEA for this particular instance was clear. For some reason, the diversity-based MOEA is not working properly when addressing this instance, which in fact, is the largest one considered. It is likely that, for some instances, maintaining a large diversity in a population of solutions may be counterproductive. As a result, more advanced diversity preservation approaches, that manage diversity in a smarter way than those applied herein, may be required in order to tackle these types of instances. The above, which is out of the scope of the current work, will be addressed as further research.

C. THIRD EXPERIMENT: BEHAVIOUR OF THE DIFFERENT APPROACHES IN TERMS OF DIVERSITY MANAGEMENT

Previous experiments demonstrated that the diversity-based MOEA was appropriate for smaller instances, while for larger ones, the single-objective GA performed significantly better, even in the long term. Our hypothesis is that the diversity-based MOEA may be failing because it does not manage diversity in a suitable manner for some test cases. In addition to the above, in the current work we are studying the performance of a diversity-based MOEA in comparison to other optimisation schemes. It would be interesting, therefore, to analyse its behaviour in terms of its ability to manage diversity. The metric selected for measuring the diversity in a set of solutions was the mean pairwise Euclidean distance to all individuals. The higher its value, the larger the amount of diversity in a population of individuals. The different schemes considered in the current work were applied to a small instance (Berlin) and a large instance (Malaga) during 4 days. Since VNS is a trajectory-based meta-heuristic, its inclusion in this experiment did not make any sense. Finally, runs were repeated 15 times, and therefore, this study involved 11,520 computational hours.

Fig. 7 shows the evolution of the mean pairwise distance to all individuals attained by each of the optimisation methods, and for each of both instances considered. For both test cases, it can be observed how PSO was able to keep the largest diversity during the whole run. The reader should recall that

PSO did not provide the best results for Berlin and Malaga instances, as we demonstrated in previous experiments. As a result, we can conclude that keeping a very large diversity in a set of solutions may not be suitable so as to provide good performance.

In the case of the diversity-based MOEA and the single-objective GA, it can be observed how both schemes started the runs with a diverse population, and as the execution progressed, the amount of diversity decreased. In the case of the Malaga instance, the amount of diversity preserved by NSGAII-DCN did not decrease significantly. In addition to the above fact, NSGAII-DCN was able to keep a larger diversity in the population in comparison to MonoGA for both instances, something that was expected, since NSGAII-DCN is a method that explicitly promotes diversity. In conclusion, for small test cases (Berlin), keeping a large enough diversity allowed a better performance to be achieved, while for large instances (Malaga), maintaining a large diversity was counterproductive, so reducing the diversity existing in the set of solutions to some extent would be a better choice.

D. FOURTH EXPERIMENT: QUANTITATIVE ANALYSIS OF SOLUTIONS

In previous sections, we have analysed the algorithms from different standpoints, mainly based on the fitness of solutions, but it would also be very important to analyse the solutions from a quantitative point of view, i.e., based on their features. Therefore, we dedicate this last section to study the characteristic of the solutions from the point of view of a citizen or the city manager. We consider two types of attributes. Firstly, some ones related to the travel of vehicles, such as how many vehicles finalise their journey in a given time, the mean travel time, the mean number of times a vehicle stops due to traffic jams or traffic lights—red colour—and the mean amount of fuel consumed during their trip. We note here that most of these values are already considered by the objective function of the TLS, among other parameters. The second group of attributes are the amount of greenhouse gases emitted to the atmosphere. In this case, we took into account some of the main types of emissions: Carbon Dioxide (CO_2), Carbon Monoxide (CO), Particulate Matter (PM_x), Nitrogen Oxides (NO_x) and Hydrocarbons (HC). These values are not directly

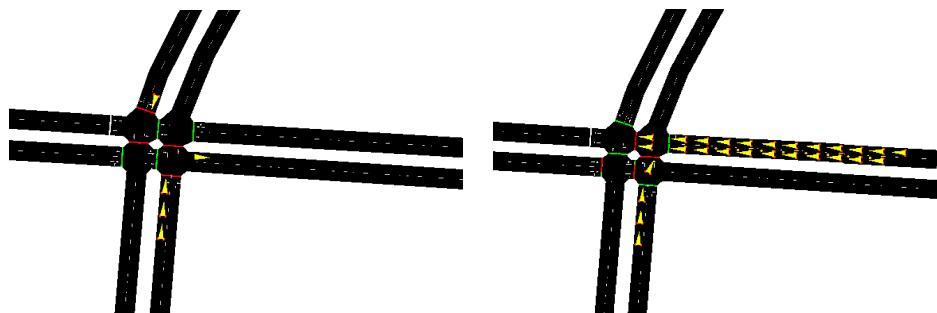


FIGURE 8. Traffic flow on a small region of the Malaga instance generated by MonoGA (left-hand side—no jam) and the expert solution (right-hand side—jam).

TABLE 7. Quantitative analysis for the Berlin instance considering 7-days-length runs.

Journey attribute	Expert	NSGAII-DCN	MonoGA
Vehicles arriving	1,296	1,300	1,300
Vehicles not arriving	4	0	0
Mean travel time (s)	927.66	628.04	649.34
Mean number of stops	442.58	208.73	225.67
Mean fuel (ml)	409.29	349.37	365.60
Emission attribute	Expert	NSGAII-DCN	MonoGA
Mean CO_2 (mg)	1,029,290.12	877,523.08	918,323.08
Mean CO (mg)	9,299.31	7,371.31	7,441.35
Mean HC (mg)	1,465.83	1,180.55	1,192.95
Mean NO_x (mg)	6,795.39	5,652.76	5,995.59
Mean PM_x (mg)	154.57	121.45	129.69

considered in the current formulation of the TLSP, and as a result, it would be interesting to know if our algorithms are also able to reduce those types of emissions aimed to obtain greener solutions. At this point, we note that, since SUMO only returns results considering those vehicles arriving at their destination, mean values are calculated regarding that number of vehicles.

For this analysis, we selected the smallest and largest instances, i.e., Berlin and Malaga, respectively, and the two best-performing algorithms in previous analyses: NSGAII-DCN and MonoGA. They were compared against the expert solution provided by SUMO for both aforementioned scenarios. Expert solutions are generated by applying some common rules used by traffic managers. Another important fact is that expert solutions do not contemplate the use of time offsets, and therefore, this comparison also helps us to assess the effect of adding these parameters in our novel formulation.

Table 7 shows the attribute values associated to the best solutions attained by the different algorithmic schemes for Berlin city. Best attribute values are shown in boldface only for those solutions where all vehicles arrived at their destination. In this small scenario, both approaches NSGAII-DCN and MonoGA provided a traffic light scheduling that allowed all vehicles to arrive at their destination in the established simulation time. However, in the case of the scheduling associated to the expert solution, the above did not happen. Taking the expert solution as reference, the solutions achieved by both the NSGAII-DCN and MonoGA not only

TABLE 8. Quantitative analysis for the Malaga instance considering 20-days-length runs.

Journey attribute	Expert	NSGAII-DCN	MonoGA
Vehicles arriving	1,805	2,610	2,632
Vehicles not arriving	827	22	0
Mean travel time (s)	976	845.71	758.68
Mean number of stops	521.77	360.96	282.60
Mean fuel (ml)	853.04	876.30	863.65
Emission attribute	Expert	NSGAII-DCN	MonoGA
Mean CO_2 (mg)	2,139,634.35	2,197,977.01	2,166,238.60
Mean CO (mg)	37,549.31	39,106.51	38,760.64
Mean HC (mg)	758.81	763.78	745.90
Mean NO_x (mg)	4,643.81	4,833.03	4,786.09
Mean PM_x (mg)	242.89	257.16	255.52

improved the quality of the journey—by shortening the mean travel time, the mean number of stops, and the mean fuel consumption—but also reduced the amount of gases emitted by vehicles. The values of this table also line up with some conclusions we stated previously. The good performance of the diversity-based NSGAII-DCN when dealing with small instances, which was showed in previous sections, is demonstrated herein once more. As it can be observed, NSGAII-DCN provided the best values for all attributes in comparison to the method MonoGA.

Differences were even more noticeable considering the Malaga scenario, as it is shown in Table 8. The method MonoGA was the unique providing a solution where all vehicles could reach their destination. It can be observed that the expert solution provided the best mean values for the fuel consumption, as well as for almost all the different types of emissions. Nevertheless, we note that mean values were calculated by considering the number of vehicles arriving at their destination, since SUMO only provides output data for those cases. Only 1,805 vehicles reached their destination regarding the expert solution. That is the reason why we show data corresponding to the approach MonoGA in boldface, since its traffic light planning allowed all vehicles to reach their destination. At this point, the reader should recall that the approach MonoGA attained the best results for the Malaga instance in previous experiments. The above fact is also reflected in this analysis. This particular technique outperformed NSGAII-DCN in terms of all the attributes analysed.

Finally, we would also want to illustrate the effect of having included the time offset of the intersections in our novel formulation of the TLSP. Fig. 8 shows the traffic flow for a very small sub-region of the Malaga instance considering, on the one hand, the best solution provided by the approach MonoGA, and on the other hand, the expert solution provided by SUMO, at the same simulation step. We can observe that the solution obtained by the method MonoGA was able to synchronise the phases of consecutive traffic lights, i.e., *green waves*, by means of time offsets, thus enabling a very fluid and smooth traffic. In real life, engineers work hard to get this desired effect, while our algorithms are working them out as a natural way of having a smoother traffic. At the same time, the expert solution implemented by SUMO assigns different phases in adjacent intersections, since it can not change the offset, obtaining as a result a very dense traffic flow. The above is only an illustrative example of the effect of using time offsets, but similar behaviour could be observed for all the instances considered. This is one of the main reasons behind the accurate results achieved by our proposals.

VI. CONCLUSIONS AND FURTHER RESEARCH

In this work, we have presented a novel formulation of the *Traffic Light Scheduling Problem* (TLSP). Previous versions of this problem had only considered the duration of the phases of a set of traffic lights located in the different intersections of the urban area to be analysed. In addition to the above, we have also introduced the time offset of each intersection in our novel formulation of the problem. Time offsets allow the synchronisation among near intersections to be considered, which is a key parameter in order to avoid continuous interruptions in the traffic flow at common routes. This novel formulation has two main implications. First, more realistic scenarios can be modelled, and the results attained can be directly used by traffic light system managers with no additional processing. Second, the TLSP becomes even harder, since the search space increases proportionally as the number of traffic light intersections rises.

In past research the proposed methods for the TLSP have presented some difficulties when addressing medium-size instances. As a result, even more efficient approaches are required to solve this new formulation. That is the reason why, in the current work, we have not only selected algorithmic schemes which have been applied previously to this problem, i.e., PSO, but also novel techniques, such as diversity-based MOEAs (NSGAII-ADI, NSGAII-DBI and NSGAII-DCN), a single-objective GA (MonoGA) and a trajectory-based method (VNS), for comparison.

The wide experimental evaluation performed, which has involved more than 150,000 computational hours, and considered four real-world scenarios based on traffic light systems located at the cities of Berlin, Paris, Stockholm and Malaga, has revealed the following conclusions.

During the first experiment (analysing different diversity mechanisms), the diversity-based objective function providing the best performance when it was embedded into the

diversity-based MOEA was the approach DCN, particularly in the case of addressing smaller instances of the TLSP (Berlin, Paris and Stockholm). Nevertheless, when dealing with larger instances (Malaga), a clear conclusion could not be extracted, since the three diversity-based objective functions considered (ADI, DBI and DCN) did not present statistically significant differences.

In the second experiment, the diversity-based MOEA using the scheme DCN (NSGAII-DCN), as well as the single-objective GA (MonoGA), were compared against the remaining methods (PSO and VNS). The approach NSGAII-DCN attained the best results for smaller instances (Berlin, Paris and Stockholm), and not only at the end of the runs, but also along them. In the case of larger instances (Malaga), however, the best performance was provided by the scheme MonoGA, even considering very long executions. The remaining approaches were not able to obtain competitive results in comparison to those given by both aforementioned techniques. We note that the above fact was even more important considering that PSO was the state-of-the-art approach proposed in previous research in order to deal with the TLSP. Therefore, we can conclude that this paper provides new state-of-the-art optimisers to tackle the TLSP.

Regarding the third experiment, we concluded that keeping a very large diversity, as in the case of PSO, did not provide good performance. For small test cases (Berlin), keeping a large enough diversity (NSGAII-DCN) allowed a better performance to be attained, while for larger instances (Malaga), reducing the diversity in the population (MonoGA), was a better choice in terms of performance.

Finally, in the fourth experiment, the good performance of the methods NSGAII-DCN and MonoGA was demonstrated once more. The best solutions obtained by both algorithmic schemes were compared against the expert solutions provided by SUMO. NSGAII-DCN and MonoGA were able to attain the best values for a set of journey and emission attributes for Berlin and Malaga instances, respectively, in comparison to the attribute values of the expert solutions. Furthermore, the benefits of adding time offsets to our novel formulation of the TLSP were also demonstrated.

For some reason, the diversity-based MOEA failed when addressing the largest instance considered in the current work. Our hypothesis is that keeping a significantly large diversity in a set of solutions could even be harmful when dealing with some instances. Hence, more advanced diversity preservation approaches, which manage diversity in a smarter way than those applied herein, could be used to tackle larger instances. The above might be an interesting research line worth being explored in the near future. In addition, much stronger conclusions could be given if more real-world instances are addressed. In this sense, it would be good to know in the future of other novel instances of this problem having real data. Finally, we should note that the definition of the TLSP provided herein takes into consideration different objectives which are combined into a unique function to

be optimised. In addition to consider additional features in this single-objective formulation, it would also be interesting to propose a multi-objective formulation of the TLSP for which multi-objective optimisers could be applied. The main goal would be to analyse if better solutions are attained by multi-objective approaches in comparison to those provided by single-objective methods.

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