HW 5 Part II

1. $p(\beta \mid \Sigma, X, y) = MVN\left((X^T \Sigma^{-1} X)^{-1} X^T \Sigma^{-1} y,\ (X^T \Sigma^{-1} X)^{-1} \right)$

$= MVN\left((X^T (\sigma^2 I)^{-1} X)^{-1} X^T (\sigma^2 I)^{-1} y,\ (X^T (\sigma^2 I)^{-1} X)^{-1} \right)$

$= MVN\left(\sigma^2 \cdot \sigma^{-2} (X^T X)^{-1} X^T y,\ \sigma^2 (X^T X)^{-1} \right)$

$= MVN\left(\underbrace{(X^T X)^{-1} X^T y}_{}, \ \sigma^2 (X^T X)^{-1} \right) \rightarrow \frac{\sigma^2}{X^T X}$

This is $\hat{\beta}$ for simple linear regression

2. $E[\beta \mid \Sigma, X, y] = (X^T X)^{-1} X^T y$, that is the ordinary least squares estimator

3. A uniform (non-informative) prior

4. It would still be a uniform (non-informative) prior

5. $Var(\beta \mid \Sigma, X, y) = \sigma^2 (X^T X)^{-1}$