Exam 2

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Problem 1:

(a)

If $y_1, \ldots, y_n$ is an independent sample from a distribution with parameters $\theta = (\theta_1, ..., \theta_p)^T$ with $E(y_i) = \mu_i(\theta)$ and $var(y_i) = \sigma_i^2(\theta)$. By fitting expectations, we mean fitting the model $y_i = \mu_i(\theta) + \epsilon_i$ with weights $w_i = \frac{1}{\sigma_i^2(\theta)}$. Thus we want to find the θ that minimizes:

$$\sum_{i=1}^{n} \frac{y_i - \mu_i(\theta)}{\sigma_i^2(\theta)}$$

This is called Iteratively Reweighted Least Squares.

(b)

If $y_1, \ldots, y_n \sim f(y, \theta)$ (iid), where f is from the exponential family, then the estimates obtained from the method of fitting expectations, using IRLS are the maximum likelihood estimates.

(c)

It is equivalent to the Fisher-Scoring algorithm. (Provided $y_1, \ldots, y_n \sim f(y, \theta)$ (iid), where f is from the exponential family).

Problem 2:

(a)

$$l(\theta) = \sum_{i=1}^{n} log[\frac{\Gamma((v+p)/2) \cdot |\Sigma|^{-1/2}}{\pi^{p/2} v^{v/2} \Gamma(v/2)[v + \delta(y_i|\theta)]^{(v+p)/2}}]$$

(b)

Problem 3:

(a)

```r
data = read.table("http://mathfaculty.fullerton.edu/mori/Math534/Examdata/prob2data.txt", header=T)

y = data[,1]
```

```r
EM_a <- function(y, theta, maxit){

    n = length(y)

    for(it in 1:maxit){

        mu = theta[1]
        sigma_sq = theta[2]

        #step 1
        u_star = (5*sigma_sq)/(4*sigma_sq + (y-mu)^2) #vect n*1

        #step 2
        mu_hat = sum(u_star*y) / sum(u_star)

        sigma_sq_hat = (1/n)*sum(u_star*((y - mu_hat)^2))

        theta_hat = c(mu_hat, sigma_sq_hat)

        theta = theta_hat

        print(theta)

    }

}

theta_0 = c(0,1)

maxit=10

EM_a(y, theta_0, maxit)
```

```
## [1] 1.081509 1.732263
## [1] 1.634606 2.486285
## [1] 1.869816 3.043467
## [1] 1.957608 3.375197
## [1] 1.988870 3.548725
## [1] 2.000126 3.633279
## [1] 2.004346 3.672990
## [1] 2.006007 3.691297
## [1] 2.006689 3.699659
## [1] 2.006979 3.703460
```

(b)

```r
EM_b <- function(y, theta, maxit, tolerr){

    n = length(y)

    for(it in 1:maxit){
```

```r
        mu = theta[1]
        sigma_sq = theta[2]

        #step 1
        u_star = (5*sigma_sq)/(4*sigma_sq + (y-mu)^2) #vect n*1

        #step 2
        mu_hat = sum(u_star*y) / sum(u_star)

        sigma_sq_hat = (1/n)*sum(u_star*((y - mu_hat)^2))

        theta_hat = c(mu_hat, sigma_sq_hat)

        mre_mu = abs(mu_hat-mu)/max(1,abs(mu))
        mre_sigma_sq = abs(sigma_sq_hat-sigma_sq)/max(1,abs(sigma_sq))

        if((mre_mu<tolerr) & (mre_sigma_sq<tolerr)) break

        theta = theta_hat

        cat("it=", it,"theta=", theta, "mre= ", mre_mu, mre_sigma_sq,"\n")

    }

}

theta_0 = c(0,1)

maxit=30; tolerr = 1e-3

EM_b(y, theta_0, maxit, tolerr)
```

```
## it= 1 theta= 1.081509 1.732263 mre=  1.081509 0.7322631
## it= 2 theta= 1.634606 2.486285 mre=  0.511412 0.4352811
## it= 3 theta= 1.869816 3.043467 mre=  0.1438942 0.2241024
## it= 4 theta= 1.957608 3.375197 mre=  0.04695204 0.1089976
## it= 5 theta= 1.98887 3.548725 mre=  0.01596933 0.0514127
## it= 6 theta= 2.000126 3.633279 mre=  0.005659562 0.02382649
## it= 7 theta= 2.004346 3.67299 mre=  0.002110235 0.01092977
## it= 8 theta= 2.006007 3.691297 mre=  0.0008285896 0.00498427
## it= 9 theta= 2.006689 3.699659 mre=  0.0003400895 0.002265254
## it= 10 theta= 2.006979 3.70346 mre=  0.000144348 0.001027495
```