

¹ Appendix 1: Unconstrained Product Indicator Using All Pairs of Product Indicators

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3 Marsh et al. (2004) demonstrated that all-pair UPI was not preferred due to its lack
4 of substantial improvement, compared to matched-pair UPI, in latent interaction
5 estimation but more complex model specification. In our preliminary study, we
6 investigated the performance of all-pair UPI on congeneric items, as Marsh et al. (2004)
7 only examined parallel items without varying factor loadings and error variances.

8 We found that while standardized bias (SB) of interaction estimates produced by
9 all-pair UPI were negligible for parallel items, they were more remarkable for congeneric
10 factor items (with varied factor loadings but consistent error variances) and fully
11 congeneric items (with varied factor loadings and error variances). It was shown that SB
12 increased as sample size increased when $\gamma_{xm} = 0.3$ for all-pair UPI, which was
13 contradictory to the findings of other latent interaction methods in our main study.

14 Moreover, the relative standard error (SE) biases for all-pair UPI unacceptable exceeded
15 the threshold range of [-10% to 10%], which were consistently downward. The results
16 indicated that all-pair UPI, though similar to matched-pair UPI, tended to underestimate
17 standard error of interaction effects particularly under the conditions of low and medium
18 reliability, which were consistent with Marsh et al. (2004). Considering the pattern of SB
19 and relative SE bias, a likely explanation for the increased SB could be that, although raw
20 biases (RB) were very small across all the conditions, the corresponding SE estimates
21 systematically decreased as sample size increased and thus amplified the calculation of SB.

22 Another issue we found with all-pair UPI was that its coverage rates, especially for
23 low and medium item reliability, did not reach to the threshold of 91%, which was
24 unsatisfactory. The low coverage rates indicated that all-pair UPI, potentially due to
25 downward biased SE estimates, could not capture the true effects of latent interaction
26 when observed items had certain amount of measurement errors, while the performance
27 improved substantially as the reliability level increased.

28 These results suggested that all-pair UPI did not perform as well as matched-pair
29 UPI on congeneric items, particularly when the item reliability level was not high enough,
30 in addition to model complexity.

Table 1

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Evaluation Criteria of Zero Latent Interaction Effect ($\gamma_{xm} = 0$) for All-Pair UPI Across 2,000 Replications.

N	$Corr(\xi_x, \xi_m)$	Bias		Relative Standard Error Bias		Coverage Rate		RMSE		Type I Error Rate						
		$\rho = .70$	$\rho = .80$	$\rho = .90$	$\rho = .80$	$\rho = .90$	$\rho = .70$	$\rho = .80$	$\rho = .90$	$\rho = .70$	$\rho = .80$					
100	0	-0.01 (-0.01)	-0.01 (-0.00)	0.07 (0.01)	-10.71 (9.35)	-9.01 (6.40)	-4.41 (3.20)	97.75	96.70	95.00	0.67	0.16	0.11	0.05	0.05	0.06
	0.3	-0.01 (-0.00)	0.02 (0.00)	0.05 (0.01)	-12.28 (10.65)	-12.07 (6.55)	-4.86 (1.85)	97.15	96.75	95.45	0.39	0.15	0.10	0.05	0.05	0.06
	0.6	-0.02 (-0.01)	0.01 (0.00)	0.04 (0.00)	-11.31 (9.10)	-2.66 (5.20)	-3.57 (1.55)	96.40	96.25	95.65	0.36	0.12	0.08	0.06	0.06	0.05
250	0	0.01 (0.00)	0.00 (0.00)	-0.00 (-0.00)	-4.98 (6.85)	-7.37 (4.25)	-5.83 (1.50)	95.80	95.00	94.15	0.12	0.08	0.07	0.05	0.06	0.06
	0.3	0.06 (0.01)	0.00 (0.00)	-0.01 (-0.00)	-5.29 (6.40)	-6.43 (3.25)	-2.56 (1.25)	96.20	94.50	94.00	0.13	0.07	0.06	0.05	0.06	0.06
	0.6	0.02 (0.00)	0.00 (0.00)	-0.01 (-0.00)	-4.71 (5.90)	-2.06 (3.30)	-2.36 (1.40)	95.60	94.45	94.00	0.08	0.06	0.05	0.06	0.07	0.07
500	0	0.00 (0.00)	-0.01 (-0.00)	-0.01 (-0.00)	-3.98 (4.65)	-2.89 (1.95)	-1.04 (1.00)	95.80	94.90	94.70	0.06	0.05	0.04	0.05	0.05	0.05
	0.3	0.00 (0.00)	-0.00 (-0.00)	-0.01 (-0.00)	-4.21 (3.80)	-1.32 (1.85)	-1.63 (1.05)	95.30	94.90	95.20	0.06	0.05	0.04	0.05	0.05	0.05
	0.6	0.00 (0.00)	-0.00 (-0.00)	-0.01 (-0.00)	-4.14 (3.05)	-2.03 (1.50)	2.65 (1.20)	95.30	95.00	94.55	0.05	0.04	0.04	0.05	0.06	0.06

Note. N = sample size; $Corr(\xi_x, \xi_m)$ = correlation between ξ_x and ξ_m ; ρ = reliability level; Bias = standardized bias of latent interaction effect (with raw bias shown in parentheses); Relative standard error bias = relative standard error bias of estimated standard errors (with outlier proportions of standard errors shown in parentheses); Coverage rate = coverage rate of 95% confidence interval (CI) of latent interaction effect; RMSE = root mean square error of latent interaction effect; Type I error rate = empirical error rate of falsely rejecting the null hypothesis of zero latent interaction effect. All numerical values are rounded to two decimal places for consistency. Note that values close to zero are displayed as 0.00, with negative signs maintained to indicate the direction of bias. Besides, values exceeding the recommended threshold (0.40) are bolded.

Table 2

Evaluation Criteria of Non-Zero Latent Interaction Effect ($\gamma_{xm} = 0$) for All-Pair UPI Across 2,000 Replications.

N	$Corr(\xi_x, \xi_m)$	Bias		Relative Standard Error Bias		Coverage Rate		RMSE		Statistical Power							
		$\rho = .70$	$\rho = .80$	$\rho = .90$	$\rho = .80$	$\rho = .90$	$\rho = .70$	$\rho = .80$	$\rho = .90$	$\rho = .70$	$\rho = .80$						
100	0	-0.03 (-0.01)	-0.10 (-0.02)	-0.09 (-0.01)	-25.16 (9.45)	-18.52 (6.00)	-3.74 (3.00)	75.35	85.15	91.95	0.43	0.12	0.38	0.62	0.82		
	0.3	0.04 (0.03)	-0.01 (-0.00)	-0.10 (-0.01)	-25.96 (10.20)	-14.40 (6.30)	-5.81 (2.35)	77.05	86.95	92.80	0.86	0.34	0.11	0.45	0.71	0.87	
	0.6	0.01 (0.00)	-0.03 (-0.01)	-0.11 (-0.01)	-22.18 (10.35)	-10.76 (5.30)	-1.69 (1.45)	80.85	87.60	92.50	0.42	0.23	0.09	0.58	0.83	0.93	
	250	0	-0.12 (-0.02)	-0.25 (-0.02)	-0.21 (-0.02)	-21.97 (7.40)	-11.64 (4.25)	-9.20 (1.45)	79.75	86.20	91.05	0.19	0.10	0.07	0.83	0.96	0.99
	0.3	-0.09 (-0.02)	-0.25 (-0.02)	-0.21 (-0.01)	-20.09 (6.75)	-9.27 (3.40)	-7.33 (1.15)	79.55	87.20	92.00	0.23	0.09	0.07	0.88	0.98	1.00	
	0.6	-0.16 (-0.02)	-0.23 (-0.02)	-0.19 (-0.01)	-12.95 (5.45)	-4.74 (2.90)	-5.38 (1.20)	83.85	89.35	92.40	0.12	0.08	0.06	0.96	1.00	1.00	
500	0	-0.40 (-0.04)	-0.43 (-0.03)	-0.32 (-0.02)	-19.01 (4.25)	-9.74 (2.25)	-3.09 (0.90)	79.00	85.70	91.25	0.10	0.07	0.05	0.99	1.00	1.00	
	0.3	-0.39 (-0.03)	-0.40 (-0.02)	-0.30 (-0.01)	-19.43 (4.30)	-10.84 (2.50)	-3.60 (1.15)	79.60	86.20	92.15	0.09	0.06	0.05	1.00	1.00	1.00	
	0.6	-0.36 (-0.03)	-0.35 (-0.02)	-0.26 (-0.01)	-19.25 (3.55)	-11.94 (1.95)	-4.90 (0.90)	82.80	88.10	92.00	0.08	0.06	0.04	1.00	1.00	1.00	

Note. N = sample size; $Corr(\xi_x, \xi_m)$ = correlation between ξ_x and ξ_m ; ρ = reliability level; Bias = standardized bias of latent interaction effect (with raw bias shown in parentheses); Relative standard error bias = relative standard error bias of estimated standard errors (with outlier proportions of standard errors shown in parentheses); Coverage rate = coverage rate of 95% confidence interval (CI) of latent interaction effect; RMSE = root mean square error of latent interaction effect; Statistical power = rate of correctly rejecting the null hypothesis of zero latent interaction effect. All numerical values are rounded to two decimal places for consistency. Note that values close to zero are displayed as 0.00, with negative signs maintained to indicate the direction of bias. Besides, values exceeding the recommended threshold (0.40) are bolded.