MRI Reconstruction

Assignments

November, 2023

Deadline: 24 December 2023 at 12:00 pm.

Your lab write-up should include answers to all the questions. Please comment and clearly justify your answers. The Matlab code and report of this lab need to be **uploaded to the Blackboard** system before the deadline. You need to submit: 1) All the **matlab files** that you have generated and 2) A **report**, in pdf with the obtained figures, answers and detailed comments. Failure to submit code will result in a **40% reduction** of the assignment mark.

1 Iterative SENSE

Provided Material

- **C.mat** 2D coil sensitivity information for eight coils.
- **M.mat** 2D fully sampled MR image data.
- **ktoi.m** This function transforms data from k-space to image space using the inverse Fast Fourier Transform (iFFT). Example: `ImageData = ktoi(KSpaceData);`.
- **itok.m** This function transforms data from image space to k-space using the Fast Fourier Transform (FFT). Example: `KSpaceData = itok(ImageData);`.

Assignment

Consider the fully sampled image m.mat and the coil sensitivity maps C.mat from an 8-channel brain coil (Nc = 8). Apply the coil sensitivity information to the image data creating the coil images m_c obtained by each of the coils.

(a) *(4marks)* Determine and depict m (the 2D fully sampled provided image), C_i, $i = 1,2,...,N_c$ and m_i, $i = 1,2,...,N_c$. Employ the root-sum-of-square approach and the weighted coil sensitivity approach to combine the data from the individual coils. Depict and compare the combined images with respect to the original fully sample image m.

(b) *(6 marks)* Generate 2 uniform undersampling patterns with acceleration factors of 3 and 7 (U_3 and U_7) and 2 random undersampling patterns (U_{R3} and U_{R7}). Each sampling pattern must be a matrix with 1s in the sampled positions and 0s in the remaining entries. Obtain the corresponding point spread functions (PSFs) and comment about the expected aliasing generated by these undersampling patterns.

(c) *(4 marks)* Obtain the aliased images for each coil as a result of undersampling with the generated patterns. For this you should use:

$$b_i = F^{-1}UFC_im$$

where U is the corresponding undersampling pattern, F is the Fourier transform and b_i are the aliased images for each coil $i = 1,...,N_c$. Depict and compare the aliased images for the different undersampling factors and patterns.

(d) *(16 marks)* The SENSE undersampled reconstruction can be written as a linear problem:

$$Em = B$$

where m is the image to be reconstructed and the encoding matrix $E = UFC$ corresponds to the forward sampling operator, with U the undersampling operator, F the Fourier transform operator, C the coil sensitivity maps and B the undersampled k-space data.

Iterative SENSE reconstruction is obtained by solving the linear problem $Em = B$ as a least square minimization $min_m||Em - B||_2^2$. Implement the Gradient Descent method

to solve the above problem.

To do this, you can use the forward E and conjugate transpose E^H SENSE encoding operators you implemented in MRI Recon Lab.

Show and compare your results for the undersampling patterns generated in part b). What can you conclude from them? How many iterations are needed to reconstruct acquisitions with the different sampling patterns?

(e) *(5 marks)* Define the reconstruction error as the difference between the fully sampled image m and your reconstructions $\hat{m}$ as:

$$e = m - \hat{m}$$

and the mean square error (MSE) of the reconstruction as

$$\epsilon = \frac{1}{N}\sum_{i=1}^{N} |m(i) - \hat{m}(i)|^2$$

where $i = 1...N$ indicate each pixel in the image.

For all reconstructions (i.e. uniform and random 3x and 7x) plot the MSE w.r.t the number of iterations for the Gradient descent iterative SENSE method implemented. Comment about the convergence of the method.

2 Compressed Sensing

Provided Material

- **M.mat** 2D image data.
- **itok.m** This function transforms data from the image domain to k-space using the Fast Fourier Transform (FFT). Example: KSpaceData = itok(ImageData);.
- **ktoi.m** This function transforms data from k-space to image domain using the inverse Fast Fourier Transform (iFFT). Example: ImageData = ktoi(KSpaceData);.

Assignment

(a) *(4 marks)* Generate a random undersampling pattern (U_R) and a variable density random undersampling pattern (U_{VDR}), both with acceleration factor of 4. Each sampling pattern must be a matrix with 1s in the sampled positions and 0s in the remaining ones. Obtain the corresponding PSFs and compare them. Obtained the aliased images as a result of undersampling with the generated patterns. For this you should use:

$$b = F^{-1}UFm$$

where m is the fully sampled image, U is the corresponding undersampling pattern, F is the Fourier transform and b is the aliased image. Depict the aliased images for the different undersampling patterns and compare against the fully sampled image.

(b) *(6 marks)* Select one algorithm, among those available on internet (some of these are listed below, and you can use any other algorithms you found), to solve the Compressed Sensing problem with Total Variation regularization. Download the software and briefly describe each of the algorithms, you can use the test examples, usually provided together with these softwares, for your description.

Examples of algorithms:

- **L1-magic** https://statweb.stanford.edu/~candes/software/l1magic/
- **NESTA** https://statweb.stanford.edu/~candes/software/nesta/
- **TWIST** http://www.lx.it.pt/~bioucas/TwIST/TwIST.htm
- **SALSA** http://cascais.lx.it.pt/~mafonso/salsa.html

(c) *(15 marks)* Employ one of the algorithms to solve the undersampled problem in MRI for both undersampling patterns, i.e.:

$$\min_{m} \|Em - b\|_2^2 + \lambda TV(m)$$

where $E = UF$ corresponds to the forward sampling operator, with U the undersampling operator, F the Fourier transform operator, B the undersampled k-space data and TV the spatial finite difference operator. Show and compare your results for different values of the parameter λ, including those with the near optimal parameters.

(d) *(10 marks)* Employ the iterative SENSE reconstruction implemented in Question 1 to reconstruct the undersampled problem in MRI for both undersampling patterns (considering a parallel imaging acquisition with coil maps given in Question 1). Compare the algorithms and performance of iterative SENSE versus the Compressed Sensing (single coil) method you investigated in (c). This comparison could consider e.g. usability, number of iterations, convergence, root mean square error, computation time, etc.

(e) *(15 marks)* Considering now a parallel imaging acquisition with coil maps given in Question 1 for Compressed Sensing, modify the algorithm you investigated in (c) to combine Compressed Sensing and Parallel Imaging, i.e. now $E = UFC$. Compare the performance of this reconstruction with your results obtained in (d).