

Spring 2024

Homework 3 Numerical Method

Consider the 2-D Laplace equation

$$(u_{xx} + u_{yy}) = 0 \text{ for } 0 \leq x, y \leq 2\pi$$

with the following boundary conditions

$$u(0, y) = 0$$

$$u(2\pi, y) = 0$$

$$u(x, 0) = \sin(2x) + \sin(5x) + \sin(7x)$$

$$u(x, 2\pi) = 0$$

(1) Write a computer code and obtain the numerical solution using the point Jacobi and Gauss-Seidel iterative schemes. Use a mesh with $\Delta x = \Delta y = 2\pi / 20$. Track the convergence by calculating the residual, $r^k = (\delta_x^2 / \Delta x^2 + \delta_y^2 / \Delta y^2) u_{i,j}^k$. Does the convergence behave as expected? Discuss.

(2) Try the above problem with the SOR point Jacobi and SOR point Gauss-Seidel schemes: Investigate and comment on the convergence properties for various values of the under- and over-relaxation parameter for both schemes. Can you find optimal values of the relaxation parameter for these schemes?

NOTE:

1) Submit as a report.

2) Plots should only be as big as they need to be for clear viewing. Plots should be numbered and have clear captions that describe the plot.

3) Attach source code of your programs in the report.