

Spring 2024

Homework 4

Numerical Method

Solve the given partial differential equation using both the *Conjugate Gradient Method* and *the Multigrid Method*:

Consider the following Poisson equation defined on a unit square domain $[0, 1] \times [0, 1]$:

$$\Delta u(x, y) = f(x, y) \quad \text{for } (x, y) \in (0, 1) \times (0, 1)$$

with boundary conditions $u(x, y) = 0$ on the boundary of the domain. Here, Δ denotes the Laplacian operator, and $f(x, y) = -2\pi^2 \sin(\pi x) \sin(\pi y)$.

Discretize the domain using a uniform grid with spacing $h = 1/n$ for some n . Convert the continuous problem into a discrete system of linear equations ($A u = b$).

Compare the solutions obtained from both methods in terms of accuracy and computational efficiency. Discuss any observations or insights gained from the comparison.

You should submit:

1. A *detailed report* including:

- A brief explanation of the discretization process.
- The implementation details of both the Conjugate Gradient Method and the Multigrid Method.
- Numerical results and solution plots.
- A comparison and analysis of the results.

2. Source code for both the Conjugate Gradient and Multigrid implementations.