Spring 2024

<div align="center">

Homework 2
Numerical Method

</div>

Consider the unsteady conduction problem

$$u_t = u_{xx} \text{ for } 0 \le x \le 2\pi \text{ given } u(0,t) = u(2\pi,t) = 0 \text{ and } u(x,0) = \sin(mx).$$

(a) Derive the exact solution to this problem.

(b) Write computer programs to obtain the numerical solutions for the above equation using:
    (i) Forward Euler method
    (ii) Backward Euler method*
    (iii) Crank-Nicolson method*
    *Write your own <u>TDMA</u> solver and use it for the BE and CN methods.

(c) For *m*=2, obtain numerical solutions using a mesh with $\Delta x = 2\pi/20$ and different time step sizes Δt (use at least three different time step sizes).
(i) Make some representative plots comparing the numerical and exact solutions for three methods.
(ii) Based on your numerical solutions, discuss the accuracy and stability of the three methods; i.e. do the error and stability characteristics for each method behave as expected? Why/not?

NOTE:
1) Submit as a report.

2) Plots should only be as big as they need to be for clear viewing. Plots should be numbered and have clear captions that describe the plot.

3) Attach source code of your programs in the report.