BME2126 Spring 2024 Assignment 1

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Question

Consider the function

$$f(x) = \frac{\sin(x)}{x^3}$$

Consider the first-order forward difference, second-order central difference, and fourth-order central-difference approximations to the first derivative. Plot the absolute value of the difference between the computed and exact derivative (i.e. the truncation error) at $x = 4.0$ for different grid sizes (Δx) and show that the error changes with grid size as expected (order of accuracy). Employ at least five different grid sizes.

Answer

The first-order forward difference approximation function of the original function is:

$$\frac{\partial f(x)}{\partial x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

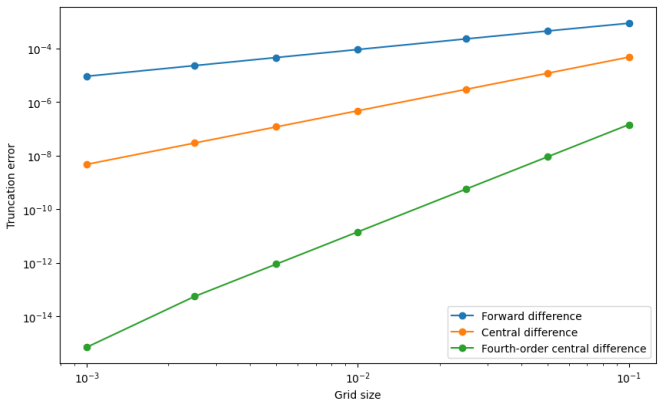
The second-order central difference approximation function of the original function is:

$$\frac{\partial f(x)}{\partial x} = \frac{f(x - \Delta x) - f(x + \Delta x)}{2\Delta x}$$

The fourth-order central difference approximation function of the original function is:

$$\frac{\partial f(x)}{\partial x} = \frac{f(x - 2\Delta x) - 8f(x - \Delta x) + 8f(x + \Delta x) - f(x + 2\Delta x)}{12\Delta x}$$

The grid size is set as $\Delta x = \{0.1, 0.05, 0.025, 0.01, 0.005, 0.0025, 0.001\}$, the truncation error changed with the grid size is shown as follows:



Also the figure of approximation derivate function near the $x = 4$ with the grid size $x = 0.1$ is shown as follows: