# Analysing Networks of Networks*

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**Abstract**

We consider data with multiple observations or reports on a network in the case when these networks themselves are connected through some form of network ties. We could take the example of a cognitive social structure where there is another type of tie connecting the actors that provide the reports; or the study of interpersonal spillover effects from one cultural domain to another facilitated by the social ties. Another example is when the individual semantic structures are represented as semantic networks of a group of actors and connected through these actors' social ties to constitute knowledge of a social group. How to jointly represent the two types of networks is not trivial as the layers and not the nodes of the layers of the reported networks are coupled through a network on the reports. We propose to transform the different multiple networks using line graphs, where actors are affiliated with ties represented as nodes, and represent the totality of the different types of ties as a multilevel network. This affords studying the associations between the social network and the reports as well as the alignment of the reports to a criterion graph. We illustrate how the procedure can be applied to studying the social construction of knowledge in local flood management groups. Here we use multilevel exponential random graph models but the representation also lends itself to stochastic actor-oriented models, multilevel blockmodels, and any model capable of handling multilevel networks. **Keywords:** Multiplex, Multilevel networks, Sociosemantic networks, Multigraphs.

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# 1 Introduction and basic definitions

We consider a set of nodes $\mathcal{N} = \{1, \ldots, N\}$ with a set of possible ties $\mathcal{E} = \binom{\mathcal{N}}{2}$, $e = |\mathcal{E}|$. For a realisation $E \subset \mathcal{E}$ of ties, we let $\mathbf{X} = (X_{ij})$ be the standard adjacency matrix. Here we assume that for index set $V = \{1, \ldots, n\}$, we have adjacency matrices $\mathbf{X}_1, \ldots, \mathbf{X}_n$, representing the networks $\mathcal{H} = \{\mathcal{H}_1, \ldots, \mathcal{H}_n\}$, on the same node set $\mathcal{N}$ and set of pairs $\mathcal{E}$. In addition, we assume that the nodes of $V$ are connected through a network $\mathcal{M}$. We allow for the existence of a fixed and given criterion graph $\mathcal{G}(\mathcal{N}, E)$. We propose to represent the totality of ties as a multilevel network (Lazega et al., 2008) on two node sets, where one node set is $V$ and the other is $\mathcal{E}$ treated as a set of nodes. The latter is key to retaining all information, and by recognising that $\mathcal{E}$, treated as a set of nodes, means that these nodes are not independently defined but connected through the shared, constituent nodes in $\mathcal{N}$. These latter connections are represented through a network $\mathcal{Q}$ on the node set $\mathcal{E}$. The new multilevel network thus represents a *network* $\mathcal{M}$, of *networks* $\mathcal{H}_1, \ldots, \mathcal{H}_n$.

There is a long tradition of analysing multiplex networks (Mitchell, 1974; Davis, 1968; Wasserman and Faust, 1994), with many of the classic network datasets (Sampson, 1969; Kapferer, 1972; Padgett and Ansell, 1993) specifically designed to investigate the interrelation of different types of ties. For a multiplex network, $\mathcal{H}$ represents different relations on $\mathcal{N}$. The networks $\mathcal{H}$ may also represent repeated observations on the same type of tie through time or by different reporters. An example of the latter is when multiple raters report on the same relation, such as the cognitive social structures (CSS) data collection paradigm (Moreno, 1934; Newcomb, 1961; Krackhardt, 1987). The challenges associated with the joint analysis of multiplex networks are well documented (White 1963; White et al., 1976; Pattison 1993; Lazega and Pattison, 1999; Rivero Ostoic, 2020) as are the issues with analysing multiple raters without having a gold standard network on $\mathcal{N}$ for reference in CSS (see e.g., Butts, 2003). Nevertheless, a wealth of research has come out of the study of multiple networks in the form of $\mathcal{H}$. For multiplex analysis $n$ is typically small, enabling the researcher to formulate specific hypotheses about how the networks $\mathcal{H}$ are related (Koehly and Pattison, 2005). Large $n$ leads to a combinatorial explosion that makes investigating theoretically informed multiplexities difficult but large $n$ still permits exploration of dimensions of relations (Vörös and Snijders, 2017); relational algebras (Rivero Ostoic, 2017, 2020) and testing of structure using a multigraph representation (Shafie, 2015). For CSS $n = N$ and the main challenge is typically how to relate $\mathcal{H}$ to an unobserved consensus structure or criterion graph $\mathcal{G}(\mathcal{N}, E)$.

There is a growing number of conceptual frameworks for joint analysis of different types of ties over different types of nodes, such as multilevel networks (Lazega et al., 2008), multilayered networks (Kivelä et al., 2014), sociosemantic networks (Basov, 2020), socioecological networks (Bodin et al., 2016), etc. Many on them have in common that networks on different types of nodes are connected through two-mode affiliation ties. In particular, a network $\mathcal{G}$ is connected to a network $\mathcal{M}$, through affiliations $\mathcal{B}$ linking $V$ to $\mathcal{N}$. This affiliation network consequently connects a social network with another type of network, on a different node set $\mathcal{N}$. If we have multiple networks on $\mathcal{N}$, and that it is these networks that are connected through social ties, the multilevel one-mode by two-mode representation does not apply straightforwardly. For multilevel networks, there are instances where multiple networks on $\mathcal{N}$ may add insight over and above the representation using affiliations of $V$ to nodes in $\mathcal{N}$. For example, Wang et al. (2015), study consumer preferences among products in a product layer, network of similarities among products, and a social network among consumers in a sociomaterial network (Contractor et al., 2011). In the canonical representation of clans, forestry, and ecology in the socioecological network of Bodin and Tengö (2012), the clans are assumed to act on a universally understood network $\mathcal{G}$ of forests. This ecological network $\mathcal{G}$ of forest could be disaggregated and represented as each clan's perception or understanding $\mathcal{H}_1, \dots, \mathcal{H}_n$ of how forests are related. In sociosemantic networks (Roth and Cointet, 2010; Hellsten and Leydesdorff, 2017; Basov, Lee, and Antoniuk, 2016; Basov, 2020), the semantic network may be taken as a normative, exogenously given network $\mathcal{G}$ that social actors relate to through affiliations $\mathcal{B}$ with concepts. If the semantic network is a local semantic network, aggregated across individual meaning structures (the personal 'semantic networks') $\mathcal{H}_1, \dots, \mathcal{H}_n$, the local semantic network may be disaggregated into $n$ versions of how the concepts are related.

Multiplex approaches can handle complex dependencies between and within $\mathcal{H}$ for small $n$. CSS may handle the comparison of $\mathcal{H}$ and a criterion graph $\mathcal{G}$ for large $n$ assuming independence between and within reports $V$, conditional on $\mathcal{G}$. Accounting for complex dependencies within and between $\mathcal{H}$ comparing to a criterion graph $\mathcal{G}$ is thus a considerable challenge. In particular, how do we account for dependence between report, conditionally on $\mathcal{G}$ if these dependencies are induced by a network $\mathcal{M}$ on $V$? Here we propose mapping $\{\mathcal{H}, \mathcal{G}, \mathcal{M}\}$ to a multilevel network (Lazega et al., 2008) on $V$ and $\mathcal{E}$ as nodes, with ties defined by $\mathcal{M}$ and $\mathcal{H}$, and $\mathcal{G}$ represented by node-level covariates. We present how meaningful hypotheses for the original representation of data may be translated into hypotheses expressed in terms of *configurations* (Moreno and Jennings, 1938) in the new, multilevel form. The multilevel representation of the network of networks means that a network of networks lends itself to estimation using any model or statistical package that can model multilevel networks, such as stochastic actor-oriented models (Hollway et al., 2017), exponential random graph models (Wang et al., 2013), and blockmodels (Žibena and Lazega, 2016). We illustrate the application of the representation to test hypotheses about the local production of knowledge for a dataset on flood management. We find dyadic social network effects on knowledge: People who are socially connected also tend to connect the same concepts via meaningful associations, i.e., to generate knowledge jointly. Moreover, the multilevel representation of network of networks approach allows us to show that alignment with a normative semantic network (here, of expert knowledge) moderates local social production of knowledge. This opens new methodological prospects for studying the dispositional effects on local production of culture (Rawlings and Childress, 2019)

# 2 Multilevel representation

When having one-mode networks among a set of actors that are also affiliated with organisations, that, in turn, are connected amongst themselves, Lazega et al. (2008) propose to represent the totality of ties in a multilevel network. A multilevel network thus has two types of nodes and three types of ties. While the multilevel network could be represented as one one-mode network, where the types of ties were identified by their constituent nodes, Lazega et al. (2008) demonstrate the analytical advantages that can be had from retaining a strict distinction between the networks. In particular, the multilevel representation affords specifying different types of dependencies depending on the combination of the types of ties that are used.

## 2.1 Network of networks

In the original representation of data used here $\{\mathcal{H}, \mathcal{G}, \mathcal{M}\}$, there is a clear distinction between the nodes on $\mathcal{G}$ and the $\mathcal{H}_i$'s, on the one hand, and the nodes of $\mathcal{M}$ on the other. The nodes of $V$ have ties amongst themselves but they do not have ties to the nodes of $\mathcal{G}$, something which makes the joint representation less than trivial. To define a joint representation, consider the representations of the original data $\mathcal{H}$ and $\mathcal{G}$ in Figure 1 and $\mathcal{M}$ in Figure 2.

The networks in Figure 1 could represent a multiplex network where $i$, $j$, and $k$, are different relations and $\mathcal{G}$ an additional fourth relation. In the CSS framework the nodeset $\mathcal{N}$ would be the same as $V$, and $\mathcal{G}$ would represent some consensus graph on the same node set. In the case of multiplex networks and $n$ small, we could formulate specific hypotheses for how the networks $\mathcal{H}$ relate pair-wise to each other. For example in the case of generalised exchange, we might ask if a tie $\{s, v\}$ in $\mathcal{H}_k$ closes an open triad $\{\{s, u\}, \{u, v\}\}$ in $\mathcal{H}_i$. For CSS, we may introduce dependencies on $V$ through, for example, asserting that respondents are more accurate when reporting on their own ties (Krackhardt, 1987; Batchelder et al. 1997; Butts, 2003; Koskinen, 2004). If $i = u$ and $j = s$, then affording greater accuracy for self-reports would mean that we would trust the reports by $i$ and $j$ on the tie $\{u, s\} = \{i, j\}$, that the tie is present, than the report of $k$, $k \neq i, j$, that reports the tie as being absent.

For networks where $\mathcal{N}$ is not a set of social actors, there are several examples of multiple, 'parallel' networks. In the framework of Friedkin et al. (2016), $i$, $j$, and $k$ may represent different systems of belief, where beliefs are represented as connections of concepts in $\mathcal{N}$. Friedkin et al. (2016) do no model $\mathcal{H}$ but take these as a small collection of fixed and known belief systems and assume that people may be influenced to change from endorsing one belief system $\mathcal{H}_i$ to another $\mathcal{H}_j$. A related example is when the networks $\mathcal{H}$ are semantic networks representing the local meaning structures (Basov, de Nooy, and Nenko, 2019) of respondents in $V$.

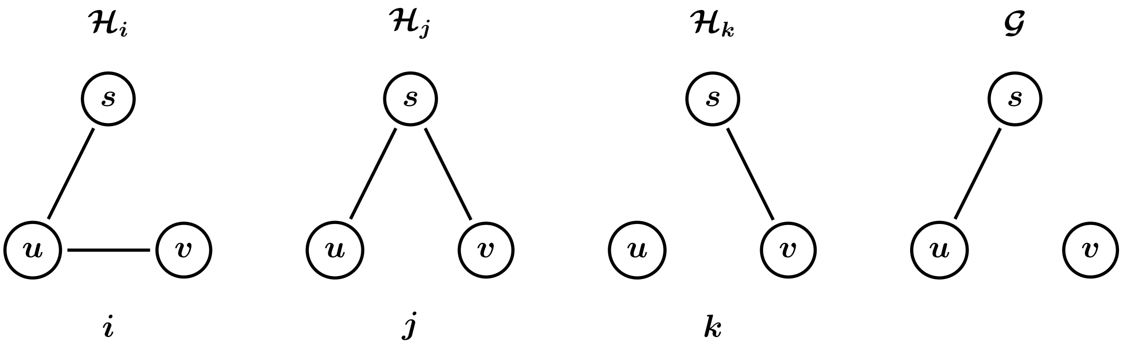



Figure 1: Networks of $i, j, k \in V$ and criterion network $\mathcal{G}$

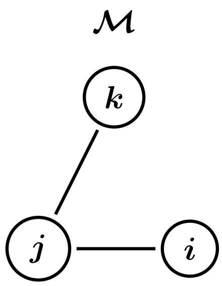



Figure 2: A social network amongst nodes $i$, $j$, and $k$ in $\mathcal{M}$

Assuming a network $\mathcal{M}$ as in Figure 2, means that we have a network amongst the elements of the index set $V$ of $\mathcal{H}$. If $V = \mathcal{N}$, that is, we are only dealing with networks on one type of node, we could represent $\mathcal{M}$ by $\mathcal{G}$ but this does not help us specify how the slices in $\mathcal{H}$ depend on each other. There is nothing in $\mathcal{G}$ in Figure 1 that, for example, connects, say, $\mathcal{H}_i$ and $\mathcal{H}_j$. The relations between the $\mathcal{H}_i$'s as prescribed by $\mathcal{M}$ is in fact a network of networks (NoNs), as represented in Figure 3. This superficially resembles the representation by Friedkin et al. (2016), that models $n$ individuals' $m$ ($n \neq m$) truth statements (a), as a function of a social network (b) among the individuals and a number of belief systems (c). The social network (b) is a weighted version of $\mathcal{M}$ but here we do not have a structure (a) connecting individuals to belief systems. The belief systems (c) are only superficially similar to the $\mathcal{H}_i$'s and are taken to be exogenously defined and not indexed by the actors. The Friedkin et al. (2016) model is not designed to model the network of networks as we conceive of it.

Further examples may include the individual product preferences of consumers linked through social ties (Wang et al., 2015), perceived food webs by fishers (Barnes et al., 2019), intersectional flows in different countries (Leoncini at al., 1996), symptom networks (Borsboom and Cramer, 2013) within individuals, etc, all of which may be connected by a network: consumers through social ties, fishers through communications, countries through trade ties, people through social ties.

## 2.2 Direct modelling of reports, criterion graph, and social network

To motivate the proposed representation of the network of networks as a multilevel graph we briefly consider the challenges associated with modelling $\mathcal{H}$ directly while incorporating the possible dependence through $\mathcal{M}$.

As observed above, we may model $\mathcal{H}$ directly as a multiplex network, conditional or unconditional on $\mathcal{G}$. For example, we can assume that $\mathbf{X}_1, \ldots, \mathbf{X}_n$ follow a multiplex exponential random graph model (Lazega and Pattison, 1999) conditional on $\mathbf{A}$, being the adjacency matrix of $\mathcal{G}$, with parameters $\theta$

$$p_\theta(\mathbf{X}_1, \ldots, \mathbf{X}_n | \mathbf{A}) = \exp\{\theta^\top z(\mathbf{X}_1, \ldots, \mathbf{X}_n; \mathbf{A}) - \psi(\theta)\}$$

for a normalising constant $\psi(\theta)$, and where the vector of statistics $z(\cdot)$ has components

$$z_{k_1,\ldots,k_r}(z(\mathbf{X}_1, \ldots, \mathbf{X}_n; \mathbf{A})) = z_{k_1,\ldots,k_r}(z(\mathbf{X}_{k_1}, \ldots, \mathbf{X}_{k_r}; \mathbf{A}))$$

that are functions of subsets

$$\{k_1, \ldots, k_r\} \in \binom{V}{r}$$

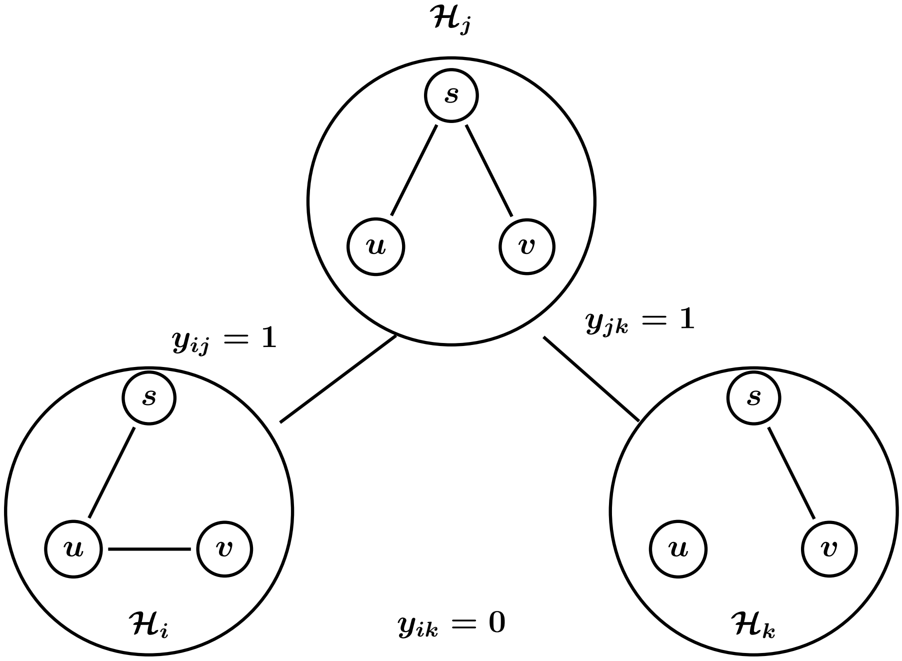



Figure 3: A Network of networks where $\mathcal{M}$ connects the networks on $\mathcal{N}$ of $i, j, k \in V$

For $r = 2$ we can specify statistics corresponding to entrainment and generalised exchange through terms of the type $\sum_{k,h} X_{ikh}X_{jkh}$ and $\sum_{k,h,\ell} X_{ikh}X_{ih\ell}X_{jk\ell}$. Dependence on $\mathcal{M}$ can be introduced through interactions with the variables of $\mathbf{Y}$. For example, assume that there is alignment between ties in $\mathcal{H}_i$ and $\mathcal{H}_j$ only if $Y_{ij} = 1$, in which case the entrainment terms in the multiplex ERGM would be $y_{ij} \sum_{k,h} X_{ikh}X_{jkh}$. Note that when $r = 2$, $Y_{ij}$ is either one or zero, and the interaction $y_{ij} \sum_{k,h} X_{ikh}X_{jkh}$ is either zero or equal to $\sum_{k,h} X_{ikh}X_{jkh}$. The number of possible statistics grows very quickly with $r$ and the types of statistics we can specify are limited in terms of the types of dependencies we may specify. Additionally, the direct multiplex ERGM does not afford simultaneous modelling of $\mathcal{H}$ and $\mathcal{M}$, at least not easily or in a practical way.

Statistical models for CSS (Batchelder et al., 1997; Butts, 2003; Koskinen, 2004) have typically assumed that the variables $X_{ikh}$ and $X_{juv}$ are conditionally independent across $V$ and $\mathcal{E}$, conditional on respondent factors and the criterion graph $\mathcal{G}$. Without loss of generality we can assume that

$$p(\mathbf{X}|g, \mathbf{A}) = \prod_{kh} \prod_{i} \Pr(X_{ikh} = x_{ikh}|A_{kh}, g_i(A_{kh}))$$

where the $g_i(\cdot)$'s may incorporate different actor accuracies. This model is eminently tractable and we cannot relax the independence assumption, introducing dependence among reports through $\mathbf{Y}$, without losing tractability. Let us consider what happens when we introduce dependence through the ties of $\mathbf{Y}$. For example, we may want to allow $X_{ikh}$ to depend on $X_{jkh}$ if $Y_{ij} = 1$. In a modified model

$$p(\mathbf{X}|g, \mathbf{A}) = \prod_{kh} \Pr(\mathbf{X}_{\cdot kh} = (X_{ihk})_{i \in V} | A_{kh}, g_i(A_{kh}))$$

this can be accommodated, for example, through assuming that conditionally

$$\operatorname{logit} \Pr(X_{ikh} = 1 | (X_{ihk})_{j \neq i}, A_{kh} = 1) = \theta_0 + \theta_1 \sum_{j \neq i} Y_{ij} X_{jkh},$$

an expression that we recognise as the conditional form of the auto-logistic actor attribute model (ALAAM) (Daraganova and Robins, 2013; Koskinen and Daraganova, 2020). In other words, for each $\{i, j\} \in \mathcal{E}$ we would end up with an ALAAM for the vector of responses $X_{1kk}, \ldots, X_{nkh}$, in total $e$ different ALAAMs. ALAAMs might be challenging and would restrict the nature of dependencies we can consider. This framework would for example not allow us to consider dependencies within respondents such as how the responses $X_{ikh}$ and $X_{iuv}$ may or may not be dependent.

## 2.3 Representing a network of networks as a multilevel network

We are able to encode the information in $\mathcal{M}$ and $\mathcal{H}$, and $\mathcal{G}$ in one multilevel network, by representing the dyads $\mathcal{E}$ of $\mathcal{H}$ as nodes. Define a mapping from the set of dyads $\mathcal{E}$ to $\mathcal{P} = \{1, \ldots, e\}$, $\pi : \mathcal{E} \to \mathcal{P}$. For nodes $i \in V$, we define an affiliation matrix on $V \times \mathcal{P}$ as an $n \times e$ affiliation matrix $\mathbf{W}$ with elements

$$W_{iu} = \begin{cases} 1, & \text{if } X_{i\pi^{-1}(u)} = 1 \\ 0, & \text{otherwise} \end{cases}$$

Thus, if $i \in V$ has a tie $\{k, \ell\} \in \mathcal{H}_i$, and $\pi(u) = \{k, \ell\}$, then there is an affiliation tie $W_{iu} = 1$. To account for the criterion graph $\mathcal{G}$, the vertices $\mathcal{P}$ are coloured according to whether $\{k, \ell\} \in \mathcal{G}$ or not, for $u$ such that $\pi(\{k, \ell\})$. The binary colouring of $\mathcal{P}$ is a vector

$$D_u = \begin{cases} 1, & \text{if } A_{\pi^{-1}(u)} = 1 \\ 0, & \text{otherwise} \end{cases}$$

Denoting the social network on $V$ by $\mathbf{Y}$, as before, and the networks $\mathbf{X}_1, \dots, \mathbf{X}_n$ expressed as $\mathbf{W}$, we can define a multilevel network $\mathcal{C}$ as a blocked adjacency matrix

$$\underset{(e+n)\times(e+n)}{\mathbf{C}} = \begin{pmatrix} \mathbf{0}_{e\times e} & \mathbf{W}^\top \\ \mathbf{W} & \mathbf{Y} \end{pmatrix}$$

where $\mathbf{0}_{e\times e}$ is a matrix of zeros. The mapping $\pi$ is arbitrary at this point which means that the structure of the network $\mathcal{G}$ is only reflected through $\mathbf{D}$. The representation $\mathbf{C}$ is thus agnostic to whether, for example, $u, v \in \mathcal{P}$ refer to edges $\pi(u)$ and $\pi(v)$ that may share a node or not. To relax this independence, we introduce a top-level network by connecting ties that share nodes. Formally, define a graph $\mathcal{Q}$ as the $e \times e$ matrix $\mathbf{Q}$ with elements

$$Q_{uv} = \begin{cases} 1, & \text{if } \pi^{-1}(u) \cap \pi^{-1}(v) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

The graph $\mathcal{Q}$ is the dependence graph on $\mathcal{E}$ under the Markov dependence assumption for the ties of $\mathcal{N}$ (Frank and Strauss, 1987). The graph $\mathcal{Q}$ is also the complement of the Kneser graph $KG_{N,2}$.[1]This gives us the blocked adjacency matrix of a multilevel network

$$\mathbf{C} = \begin{pmatrix} \mathbf{Q} & \mathbf{W}^\top \\ \mathbf{W} & \mathbf{Y} \end{pmatrix}$$

When analysing $\mathcal{C}$ we need to respect the fact that $\mathcal{Q}$ is a fixed and exogenous graph that is completely determined by the index set.

We now proceed to describe how network configurations (Moreno and Jennings, 1938) in $\mathcal{C}$ correspond to meaningful combinations of ties in the network of networks.

### 2.3.1 Basic configurations for $\mathcal{H}$

To capture the overall number of ties reported across $i \in V$ corresponds to the bipartite density as in Figure 5. The corresponding count, or statistic is simply $\sum_{i,p} W_{ip}$. Centralisation of ties in $\mathcal{E}$ can be further modelled using bipartite 2-stars $\sum_p \binom{W_{+p}}{2}$ or the equivalent alternating stars (Wang et al., 2009). Similarly we may define 2-stars and alternating stars centered on nodes in $V$ (Wang et al., 2009). The majority of configurations we discuss in the sequel have been defined or explored for either two-mode networks (Wang et al., 2009) or multilevel networks (Wang

[1]A Kneser (1955) graph $KG_{n,k}$ has nodes $V = \binom{n}{k}$, and edge set $E = \{(u, v) \in V : u \cap v = \emptyset\}$. The complement of $KG_{n,s}$, is a line-graph where the edge set is $\bar{E} = \{(u, v) \in V : u \cap v \neq \emptyset\}$. The graph on $V$ and $\bar{E}$ is exactly the dependence graph, $D$, of a Markov random graph (Frank and Strauss, 1986; Lusher et al., 2013) on $V = \{1, \dots, n\}$.

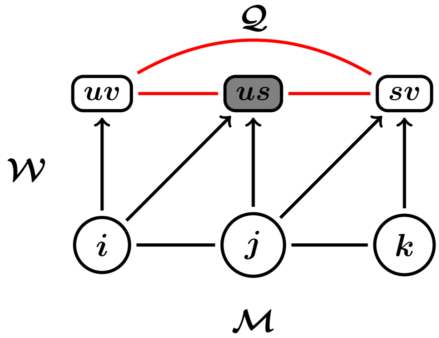



Figure 4: The multilevel network $\mathcal{C}$ representation of the network of networks in Figure 3, consisting of the networks in Figure 1 connected through the network in Figure 2

et al., 2013) and we refer the reader to the literature for mathematical definitions of these configurations in $\mathcal{C}$ and focus here on their interpretation in the pre-image of $\mathcal{H}$, $\mathcal{M}$, and $\mathcal{G}$.

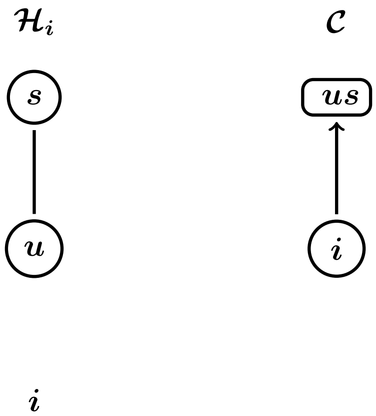



Figure 5: Mapping density in $\mathcal{H}_i$ to $\mathcal{C}$

Centrality of nodes in $\mathcal{N}$ is reflected in multilevel triangles in $\mathcal{C}$ as in Figure 6. If the network on $\mathcal{N}$ is a social network, this centralization reflect the typical heterogeneities that we encounter in social networks, such as preferential attachment. If the nodes on $\mathcal{N}$ are concepts, multilevel closure in $\mathcal{C}$ of the type Figure 6, could reflect differences in saliency or popularity of concepts but also spill-over (Maki et al., 2019). If individuals associate an energy efficient fridge ($s$) with an energy efficient washing machine ($u$), they may also associate the energy efficient fridge with electric cars ($v$).

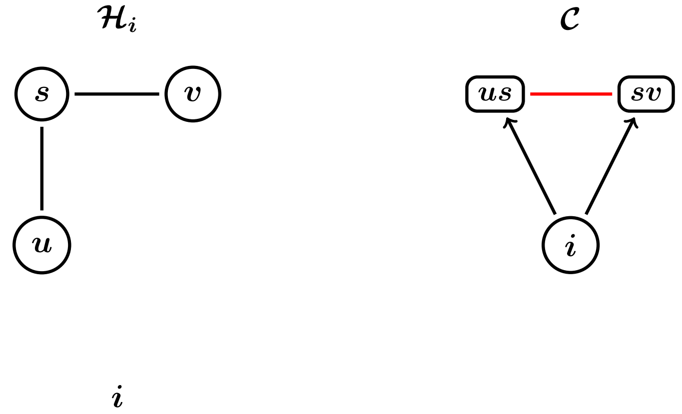



Figure 6: Mapping two-paths in $\mathcal{H}_i$ to multilevel closure in $\mathcal{C}$

We could consider a number of ways in which the reports in $\mathcal{H}$ align with each other. A basic form of entrainment is the agreement on a tie between $i$ and $j$ depicted in Figure 7. In a CSS framework we would think of this as relating to the strength of consensus. More generally, this reflects the cultural consensus (Romney et al., 1986; Batchelder and Romney, 1988). While this basic form of entrainment reflects agreement on individual ties in $\mathcal{E}$, co-nomination of pairs of ties in $\mathcal{E}$, as depicted in Figure 8, reflects a more structural consensus. The four-cycle in Figure 8 represents the most basic form of clustering in a two-mode network (Borgatti and Everett, 1997) and is often taken to represent social processes above simple agreement (Robins and Alexander, 2004; Koskinen and Edling, 2012). Here, in very general terms, the configuration can directly be interpreted as whenever two people $i$ and $j$ agree on one thing, they tend to agree on another.

### 2.3.2 Social dependencies in $\mathcal{H}$

Introducing configurations that include the network $\mathcal{M}$ enable us to investigate social construction and social influence as well as homophily induced by shared beliefs. The agreement in Figure 7 may be the result of social connections, in which case we expect to see high incidence of the social entrainment configuration of Figure 9. For cross-sectional data we cannot tell whether agreement in $\mathcal{W}$ was the result of a social tie in $\mathcal{M}$ or the other way around. The nature of the process explaining configurations as in Figure 9 is context dependent and may reflect a multitude of processes, such as learning, information, influence, etc. Heider's

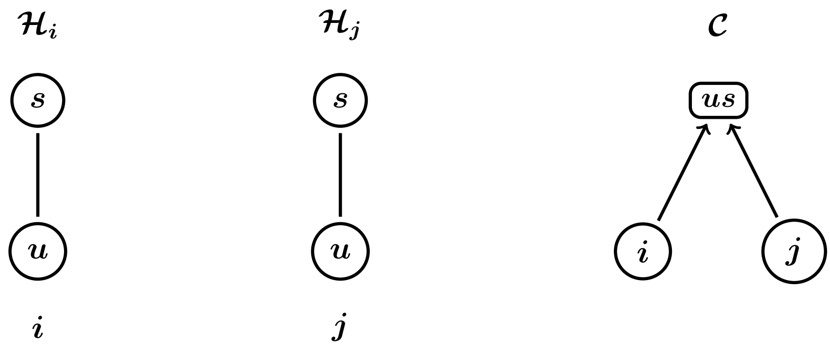



Figure 7: Entrainment of ties of $\mathcal{H}_i$ and $\mathcal{H}_j$ expressed as multilevel agreement in $\mathcal{C}$

(1958) *balance theory* is commonly applied in the networks literature in the triadic form of Cartwright and Harary (1956), where two people with a positive tie are assumed likely to also have a positive tie to the same other. The social alignment of Figure 9, is a direct application of Heider's (1958) $POX$ scheme, where person $P$ (here $i$) seeks to have ties to other $O$ (here $j$) that like the same object $X$ (here $us$).

Moving beyond direct alignment, we may consider the interactions of multiple types of ties and how they relate to each other. The multilevel four-cycle in Figure 10, by itself, represents a form of complementarity. Where $i$ reports that $u$ is connected to $s$, $j$ reports that $u$ is connected to $v$. Considered in combination with the alignment of Figure 9 we can think of two ways in which to interpret the tie $(j, uv)$. If $i$ and $j$ agree on $\{u, s\}$, we would expect that they would also agree on $\{u, v\}$, and we would expect to see few of the configurations in Figure 10. Similarly, under a social process promoting agreement, we would expect the configuration of Figure 10 to be unstable and tend to be *recombined* to the multilevel triangle of Figure 9.

An example of extra-dyadic dependencies in $\mathcal{M}$ and their effect on $\mathcal{H}$ could be that if three nodes $i$, $j$, and $k$ are a clique in $\mathcal{M}$, then they are more likely to agree on $\mathcal{H}$ than what we would see as a result of the dyadic agreement of Figure 9. In Figure 11, three dyads agreeing on $\{u, s\}$ would be the additive effect of a positive tendency for configuration (b). If there is triadic pressure over and above this additive dyadic pressure, then you would expect that there would be a tendency against $j$ to in addition nominate $\{u, v\}$ (Figure 11(a)). Put together, we would expect the tie $(j, us)$ to be more likely than $(j, uv)$ in Figure 11(c).

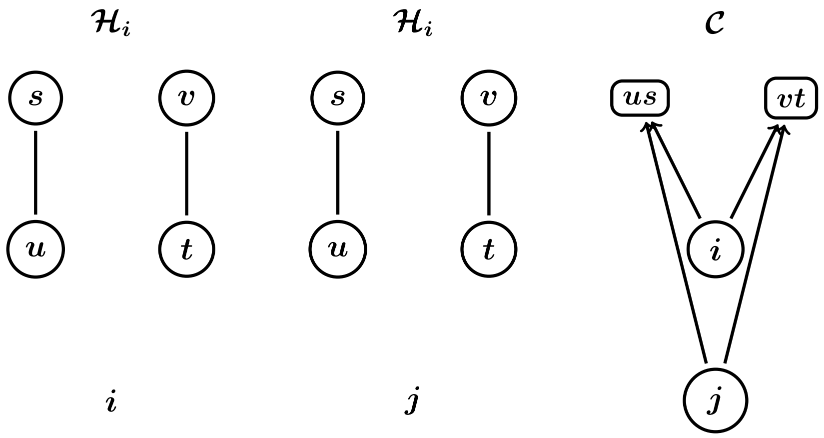



Figure 8: Association of ties in $\mathcal{E}$ that are not necessarily structurally related

### 2.3.3 Alignment with criterion graph

The representation of $\mathcal{H}_1, \ldots, \mathcal{H}_n$ in terms of a multilevel network $\mathcal{C}$ is already a representation of a *massively multiplex* network. This multiplex network may also be modelled jointly with the social network $\mathcal{M}$. In addition we may consider the alignment of $\mathcal{H}_1, \ldots, \mathcal{H}_n$ with a criterion graph $\mathcal{G}$. In the first instance we may consider basic entrainment of $\mathcal{H}$ and $\mathcal{G}$. The simplest form of alignment may be translated as in Figure 12, where the grey node indicates that $A_{uv} = 1$ for $\pi(u, v) = p$ means that $D_p = 1$.

Other multiplex configurations may be expressed in terms of various combinations of ties in $\mathcal{C}$ and attributes $\mathbf{D}$. If there is a tendency for $i$ to directly connect $u$ and $v$ ($u, v \in \mathcal{N}$) that are indirectly connected in $\mathcal{G}$, this is represented as a the triangle-edge configuration in Figure 13. If there is a tendency for $i$ to indirectly connect $u$ and $v$ ($u, v \in \mathcal{N}$) that are directly connected in $\mathcal{G}$, this is represented as a the multilevel four-cycle with a cord in Figure 14.

### 2.3.4 Connecting layers

In sociomaterial networks (Contractor et al., 2011), sociosemantic networks (Basov, 2020), and socioecological networks (Bodin et al., 2016), a social network amongst the nodes of $V$ is connected to a network on $\mathcal{N}$ through some form of two-mode ties in a multilevel network (Lazega et al., 2008). Some of these two-mode ties may lend themselves more readily to be represented as networks $\mathcal{H}_i$, such as for example consumer ($V$) preferences amongst products ($\mathcal{N}$), rather than a consumer by product, two-mode network (Wang et al., 2015). In the multilevel semantic

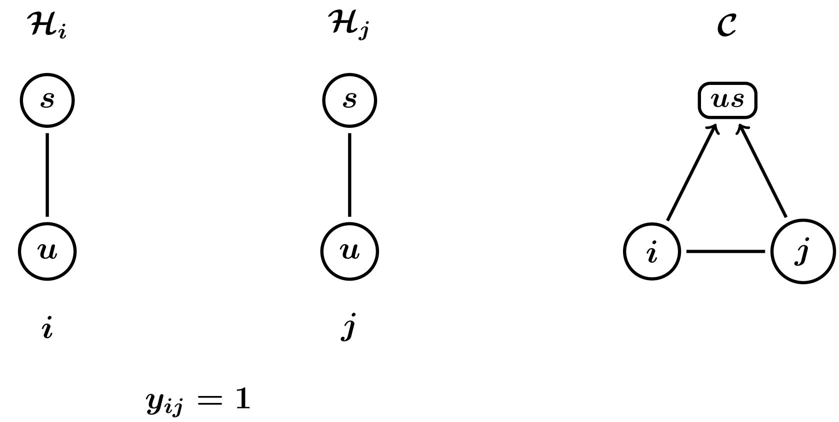



Figure 9: Alignment of ties of $\mathcal{H}_i$ and $\mathcal{H}_j$ for $i$ and $j$ with $\{i, j\} \in \mathcal{M}$, expressed as a multilevel network of networks

network of Basov (2020) people $V$ are connected to concepts $\mathcal{N}$ through usage of concepts. The network amongst $\mathcal{N}$ is however aggregated from the individual semantic networks that are essentially $\mathcal{H}_1, \ldots, \mathcal{N}_n$.

A two-mode $V \times \mathcal{N}$ network may formally be included in $\mathcal{C}$. Denoting this $\mathcal{B}_i$ for each $i \in V$, we may construct a network $\mathcal{K}$ on $V \times \mathcal{P}$, where $(i, p) \in \mathcal{K}$ if $B_{i\pi^{-1}(p)} = 1$. For each $i \in V$, the ties $(i, p)$ in $\mathcal{W}$ are a subset of the ties $(i, p) \in \mathcal{K}$.

### 2.3.5 Computational considerations

Having translated the original data $\mathcal{H}$, $\mathcal{G}$, and $\mathcal{M}$ to a multilevel network, the dependencies between the different types of ties lend themselves to investigation using for example multilevel ERGM (Peng et al., 2013), assuming that the criterion graph $\mathcal{G}$ is fixed and treated as an explanatory network. If, as in CSS, $\mathcal{G}$ is unobserved, the representation applies only either conditionally on an assumed criterion graph or in the absence of an assumed criterion graph. In principle, $\mathcal{G}$ could be treated as a latent, unobserved attributes (e.g. Koskinen, 2009; Schweinberger, 2019) but this would most likely be practically infeasible.

For more general methods for analysing multilevel networks, Hollway, Lomi, Pallotti, and Stadtfeld (2017) redefine the multilevel networks as a blocked one-mode network with structural zeros preventing ties in layers of the network where some ties are not defined (this is further described in Snijders, 2019). If the aim of the analysis is not to model all types of ties but rather focus on one set of ties,

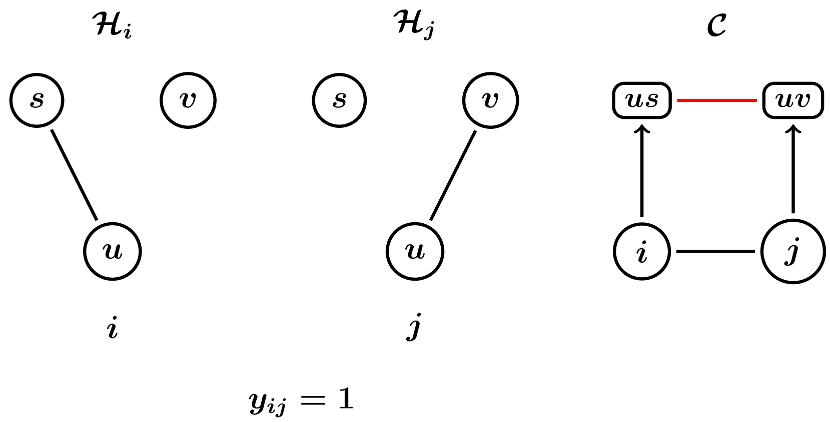



Figure 10: Complementarity of ties of $\mathcal{H}_i$ and $\mathcal{H}_j$ for $i$ and $j$ with $\{i, j\} \in \mathcal{M}$, expressed as a multilevel network of networks

using the others as predictors, multilevel configurations can be projected to dyadic covariates, for example for modelling a social network conditional on affiliations $\mathcal{W}$ and top-level networks $\mathcal{Q}$ (Stys et al., 2020).

Assuming that we are using the entirety of $\mathcal{E}$, the number $e$ of 'top-level' nodes in the multilevel representation will be too large for the translations to be practically feasible for large $N$. Consider for example the size $e$ for a CSS for a standard size network. Careful consideration therefore has to be given to what defines meaningful subsets of $\mathcal{E}$. The subset of $\mathcal{E}$ used for the top-level does not have to be the induced set of possible relations on a subset of $\mathcal{N}$. We may allow for some nodes in $\mathcal{N}$ to be represented in more pairs than others. We will show an example of this next, in our empirical illustration.

# 3 Illustration of social production of knowledge

## 3.1 Data

Our data is obtained from a pilot study of a larger project ('Creation of knowledge on ecological hazards in Russian and European local communities') that aims at investigating how knowledge in local flood-prone communities conforms to knowledge of experts (i.e. flood management agencies and authorities). The set $\mathcal{N}$ consists of concepts or *signs*, relevant for flood management.

The signs chosen by experts is a graph $\mathcal{G}$ on nodes $\mathcal{N} \subset \Omega$ with edge set

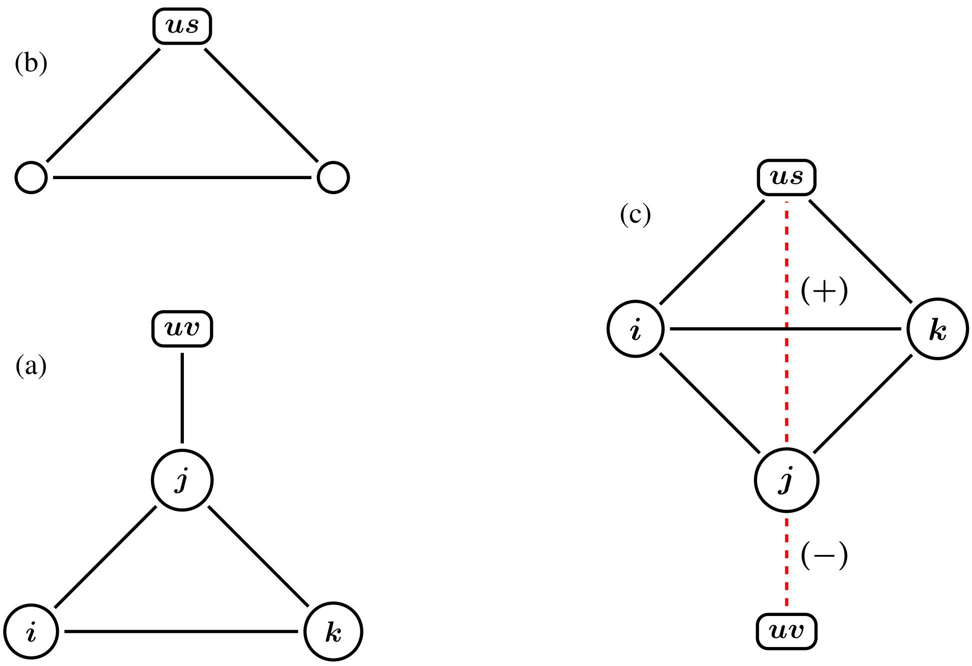



Figure 11: Capturing triadic pressure to conform. If there in addition to the pairwise conformity induced by (b) exists triadic pressure to confirm, we expect (a) to be rare as the combination of (a) and (b) induces extra-dyadic tendency for $j$ to prefer $us$ to $uv$ in (c)

$E \subset \binom{\mathcal{N}}{2}$. The signs $\mathcal{N}$ thus constitute the expert vocabulary. We denote the adjacency matrix of $\mathcal{G}$ by $\mathbf{A}$. Throughout we assume that $\mathbf{A}$ is fixed, exogenous, and unchanging. In terms of elements

$$A_{ik} = \begin{cases} 1, & \text{if sign } i \text{ is connected to sign } k \\ 0, & \text{otherwise} \end{cases}$$

The expert semantic network has been constructed using 'UDPipe' (Wijffels wt al., 2019) applied to expert texts - flood management-related documents issued by flood risk management agencies and authorities.

In the pilot, fieldwork in a site for local flood management has yielded the social network and the individual semantic networks for $n = 15$ individuals - members of two local flood groups voluntarily involved in flood management. For each $i \in V$, the individual semantic structures (Basov and de Nooy, 2019) were constructed using the same software applied to transcripts of semi-structured interviews with the members, as

$$X_{ijk} = \begin{cases} 1, & \text{if } i \text{ nominates a tie between } j \text{ and } k \\ 0, & \text{if } i \text{ does not nominate a tie between } j \text{ and } k \end{cases}$$

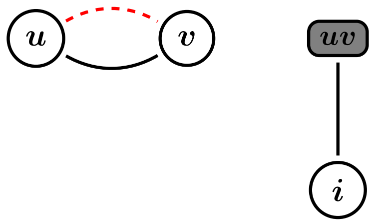



Figure 12: Mapping alignment of $\mathcal{H}_i$ and $\mathcal{G}$ to configurations in $\mathcal{C}$

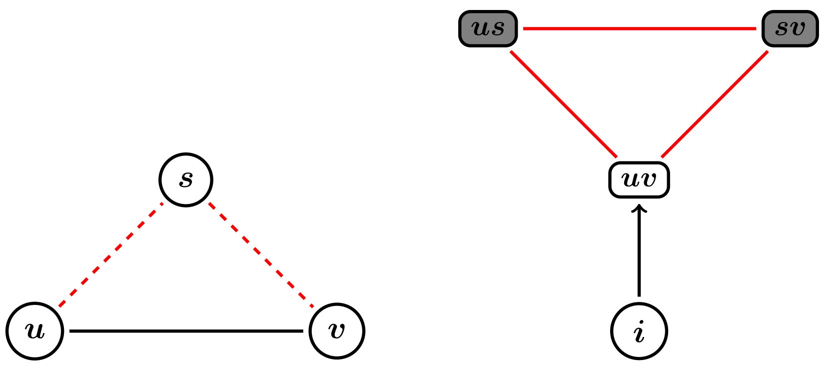



Figure 13: Mapping closure in $\mathcal{H}_i$ of a two-path in $\mathcal{G}$ to multilevel configuration in $\mathbf{C}$

Here we focus the analysis on a subset of $\mathcal{E}$ with $e = 634$ motivated by previous work on multilevel socio-semantic networks where the individual semantic structures are aggregated into a local knowledge network wherein ties exist between concepts that are used together by at least two individuals (Basov, 2020). For the model, this means that we only model tie presence between pairs of concepts that are present in the (implicit) local knowledge network, and thus exclude from the analysis those pairs which are never used together in the local context. This vastly reduces the computational cost as the size of the top-level of the network is reduced from being in the region of tens of thousands to a manageable, and interpretable size. Furthermore, $\mathcal{W}$ does not have any isolate $\mathcal{P}$ nodes.

The social network is an undirected social network among the actors in $V$ with the adjacency matrix $\mathbf{Y}$ defined as having elements

$$Y_{ij} = \begin{cases} 1, & \text{if } i \text{ nominates } j \\ 0, & \text{otherwise} \end{cases}$$

The social network has been derived using visually verified sociometric surveys, triangulated with interview and observational data to guarantee high quality data (for details on the procedure, see Basov, 2020).

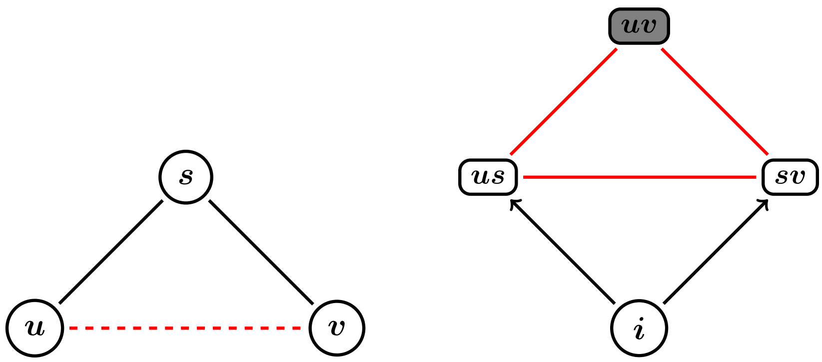



Figure 14: Mapping closure in $\mathcal{G}$ of a two-path in $\mathcal{H}_i$ to multilevel configuration in $\mathcal{C}$

We will analyse the dataset using MPNet and will refer to the corresponding effect names in the description of the configurations we investigate (Wang et al., 2014).

## 3.2 Social construction of knowledge

In the first instance, in the flood management groups studied, we aim to investigate how the actors $V$ negotiate meaning, irrespective of how flood management signs are related in the 'normative' expert network. Symbolic interactionist conception of social behaviour is based on the premise that people collectively construct knowledge about reality rather than passively reproduce images of the world imposed on them: "[h]uman group life on the level of symbolic interaction is a vast process in which people are forming, sustaining, and transforming the objects of their world as they come to give meaning to objects" (Blumer, 1986: 12; Mead, 1934). Interacting ($\mathcal{M}$), individuals use signs and associations of signs ($\mathcal{H}$) that refer to a context where their interaction unfolds and constitute knowledge about reality (Mead, 1934). The interaction context here is flood management in their communities. The act of knowledge creation can be thought of as three interrelated processes that lead to multilevel, social entrainment as in Figure 9 (TriangleXAC). Firstly, to indicate the meaning of an object/action actor A would use signs familiar to B and avoid using signs that are incomprehensible for the group that both A and B belong to (e.g. technical jargon). This entails two levels of entrainment, one is the entrainment in Figure 7 (XASB) and the more structural entrainment in Figure 8 (XACA), neither of which involves defining the interaction in terms of $\mathcal{M}$, rather using membership in the group $V$ as the reference point. Secondly, in interaction between A and B meanings indicated by A are either confirmed or rejected by B. This constitutes the other level of entrainment presented in Figure 9. The shared signs and meaningful associations between them constitute community knowledge. Thirdly, the confirmation of the indicated meanings by actor B becomes a stimulus for individual A to continue interaction. According to these three processes, actors can reproduce and transform local knowledge. Reproduction of knowledge involves actors restating already shared signs and associations between signs . Transformation involves reconfiguration of associations between signs as a result of interaction between actors. Knowledge reproduction and transformation are carried out through several mechanisms working at dyadic and extra-dyadic levels and summarised by Antonyuk et al (2019). The cross-sectional data, however, do not allow distinguishing between knowledge reproduction and transformation. Similarly, for the social ties, they may be created from various forms of social alignment or they may be reconfigured to reduce breach of social alignment. In what follows we offer examples of such mechanisms and their statistical representations.

### 3.2.1 Knowledge reproduction and transformation at dyadic level: Selection and contagion

Knowledge reproduction can take place through the mechanism of 'selection'. This mechanism can be observed in different contexts, for example, when people seek to establish reliable knowledge about an object or an event. In this situation, the identical elements in different indicated meanings are considered reliable, while the divergent parts are dismissed as subjective or inaccurate.

For example, discussing flood relief measures, A and B may disagree on what kind of help, e.g. money or material supplies, needs to be provided to the victims of a recent flood . At the same time, disagreeing on a particular type of aid throughout interaction, they at least agree that some help has to be provided to the victims (both A and B retain links between signs 'provide' and 'victim'). Thus, by dismissing conflicting meanings (i.e. dismissing links 'victim' - 'money' and 'victim' - 'material supplies' previously used by A and B, respectively), the mechanism of selection helps reproduce the common ground necessary for collaboration between the actors.

Knowledge transformation can take place through the mechanism of 'contagion'. Contagion occurs when one group member starts using an association between signs as a result of interacting with another group member using them (Burt 1987; Monge and Contractor 2003; Carley 1986; Coleman 1988). As a result, the association becomes shared and thus becomes part of local group's knowledge. For example, if A introduces the idea of monitoring water levels at the local river to B, who is unfamiliar with this idea, B may adopt the association between existing signs 'water-level' - 'monitoring' from A, that as a result becomes part of group knowledge. Knowledge transformation may also happen when there emerges a new problem not captured by previously existing group knowledge (e.g. flood water cannot be eliminated with a pump) requiring recombination of existing signs (Bolton 1981; Hollander & Gordon 2006; Etzrodt 2008). Reflecting on existing approaches to the problem, actor A can come up with an idea of a better flood protection device combining existing devices in a novel way (e.g., a physical barrier combined with a pump that automatically removes water). The new idea would reveal itself in a recombination of corresponding signs and emergence of a new association 'barrier' - 'pump'. Actor B may support the new idea and adopt the association between signs 'barrier'- 'pump'. Structurally this will be represented as an unstable four-cycle in Figure 10. B may adopt the new association and dismiss the old one - between 'water' and 'pump' (the four-cycle) - in favour of the socially aligned Figure 9. This *recombination* would suggest a positive effect for TriangleXAC and a negative effect for C4AXB.

#### 3.2.2 Knowledge reproduction and transformation at extra-dyadic level

At the extra-dyadic level, transformation of knowledge may occur through the mechanism of preferential attachment, when actors 'frame' an element of shared knowledge by adopting associations with signs that already have general, often emotionally charged meanings (Schultz et al., 2012; Snow et al., 1986), rather than with signs that denote more specific meanings. Signs used for 'framing' usually have many shared associations with other signs because of their generality or abstractness that allows them to enter many different contexts. For instance, actor A may argue that the group should adopt a prevention approach to 'fluvial' floods (that is, floods caused by excessive water in a river). The new association between existing signs used by A, e.g., 'flood prevention', invokes the 'prevention' frame that is known for actors in group $V$ from other contexts like health or road safety. Therefore, the association 'flood'- 'prevention' is likely to be adopted by another group member, B (and hence, become part of group knowledge), unlike a more specialized association 'fluvial floods' that does not involve any frame previously known to member B. A combination of configurations that is consistent with this preferential attachment is not to select isolated ties $\{s, u\}$ (negative XASB), nor are ties $\{s, u\}$ selected merely because there are many pairs with $s$ (negative EXTB). A concept pair $\{s, u\}$ if other ties of $s$ are reported, captured through TriangleXBX (positive), and if many socially tied individuals chose the same $\{s, u\}$ (positive TriangleXAX) not ties $\{s, v\}$ that also include $s$ (negative C4AXB).

In relational patterns involving three individuals, knowledge may be reproduced through the mechanism of triadic pressure. A triad embodies supra-individuality (Simmel, 1950: 257) and downplays individuality, diminishing the power of a single actor to determine the outcome of the whole interaction process. Relations in triads are ‘less free, less independent, more constrained’ than in dyads (Krackhardt, 1999: 185). In addition, actors need ‘social affirmation or reinforcement from multiple sources [...] since contact with a single active neighbour is not enough to trigger adoption’ (Centola and Macy, 2007: 705; see also the elaborate different forms of exposure discussed in e.g. Strang and Tuma, 1993, and related work). Therefore, in triads, reproduction of existing shared knowledge is more efficient than in a dyad (Krackhardt, 1999): if in a triad two interacting individuals A and B share associations between signs e.g., ‘flood’ - ‘management’, the third individual C interacting with both is likely to start sharing these associations as well. This effect is over and above dyadic contagion, so that the propensity of the third individual to share a sign or an association is higher than if he or she was subjected to contagion by two alters who are not socially tied with each other. This triadic pressure translates into the process described in Figure 11)(c) as a positive effect for TriangleXAX (Figure 9) and a negative effect for EXTA Figure 11(a).

### 3.2.3 Entrainment with normative knowledge

The associations between signs in the local knowledge are affected not only by social relationships between actors but also by expert knowledge imposed on the flood management groups by authorities, that is the normative relations between signs indicated by $\mathcal{G}$. The influence of expert knowledge on local knowledge occurs through several mechanisms that reflect structural changes in $\mathcal{Z}$ conditional on the structure of $\mathcal{G}$. Basov and Brennecke (2017) perform a multiplex analysis where an aggregate local knowledge network $\mathcal{Z}$ is compared to the criterion graph $\mathcal{G}$

There are a number of ways in which we can extend the multiplex dependencies of $\mathcal{Z}$ on $\mathcal{G}$, to dependencies of $\mathcal{H}$ on $\mathcal{G}$, and examine theoretically-derived mechanisms of expert knowledge influence on local knowledge, e.g. such as those that result in alignment in Figure 12 and the two forms of closure in Figure 13 and Figure 14. We include a basic entrainment corresponding to a mechanism we call ‘basic reproduction’. It occurs when a local group starts associating preexisting signs in the same way they are associated by the experts. For example, experts can talk about ‘rivers’ as sources of flood risks while locals may not associate ‘river’ with ‘risk’ at all. Locals may realise that the experts’ way of thinking about the river as a source of risks could be useful for them, e.g., to discuss the problem of flooding with authorities.

Hence, following the experts, they start associating ‘river’ and ‘risk’.The mechanism is modelled using the effect Expert XEdgeB corresponding to the configuration in Figure 12.

To control for the possibility that some concept pairs in $\mathcal{G}$ may be salient only because they involve signs $s$ that are part of many other concept pairs, we include the interaction $\sum_{i,p} W_{ip} D_p \sum_{r \neq p} Q_{pr}$ which is the statistic Expert Star2BX. This statistic corresponds to a mechanism we call 'popularity pressure' that describes a situation when a local group starts associating signs that are part of many other associations in the experts' knowledge. For example, speaking about flood risk management, experts may pay significant attention to communities' resilience to flood hazards and highlight the importance of creating flood plans to ensure all stakeholders are prepared for potential floods. Because a local group observes the signs 'resilience' and 'plan' as focal for experts, they also start associating these two signs when speaking about a local document they produce to be prepared for floods - a local group 'resilience plan'.

## 3.3 Results

The results for two models are presented in Tables 1. For Model 1, the social network abides by standard social processes judging by the one-mode effects (Snijders et al., 2006; Lusher et al., 2013), with no heterogeneity in popularity (insignificant ASA) but with evidence for triadic closure (positive ATA or GWESP). The bipartite network is modelled using the three terms XEdge, XASA, and XACA. We will interpret these in relation to the multilevel statistics making up the rest of the table.

In terms of reproduction and transformation of knowledge, the effects go largely in the direction of the predictions. The positive TriangleXAC is consistent with basic contagion or selection. The positive effect, in Model1, for TriangleXAC and a negative effect for C4AXB is consistent with recombination of concept pairs.

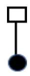
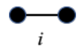
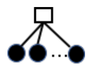
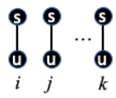
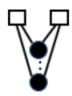
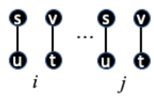
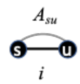
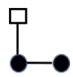
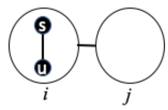
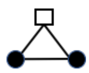
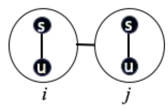
The combination of negative EXTB, positive TriangleXAX, and negative C4AXB, is consistent with the framing operating through preferential attachment.

Amongst the other effects, XACA and TriangleXBX capture a general coherence of the local knowledge structure, connecting concept pairs that are not socially mediated. XEdge, Star2BX, StarAB1X take into account that not all signs are equally represented in $\mathcal{E}$. The statistics Star2BX, L3XBX, and L3AXB also act as lower-order interactions to C4AXB.

Introducing $\mathcal{G}$, we see that there is a significant alignment of the individual semantic networks and the expert network (Expert XEdge). We also see that the dependence on $\mathcal{G}$ completely moderates the effect of C4AXB. This means that some of the recombination and preferential attachment is explained by how concept pairs are related in the expert network. Network-related contagion (TriangleXAX) and local knowledge structure (XACA) are however not affected by the reliance on the expert network.

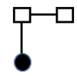
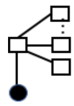
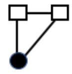
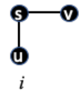
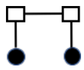
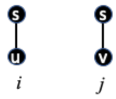
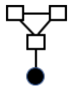
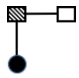
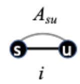
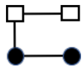
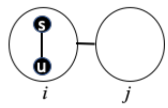
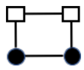
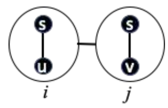
The effect EXTA necessary for inferring triadic pressure is not estimable from this dataset and while the model predicts more configurations EXTA than observed (see GOF in Tables 2 and 3), this difference is not significant. In general, the goodness-of-fit is overall acceptable considering the complexity of the data. Some expert-related configurations in the goodness-of-fit suggest that there is scope for investigating more elaborate multiplex effects. Furthermore, in the goodness-of-fit there are higher-order interactions with $\mathcal{Q}$ that could be investigated pursuant theorising in terms of $\mathcal{H}$, $\mathcal{M}$, and $\mathcal{G}$. Some affiliation configurations are poorly fit by the simple model of Table 1, something which is to be expected for two-mode networks (Wang et al., 2009) and which will be alleviated by incorporating the expert network as a top-level covariate.

Table 1: MERGM results for local meaning structures, social network, and expert semantic network (social and bipartite effects)

| | Model 1 | | Model 2 | | Representation | |
|---|---|---|---|---|---|---|
| Effects | Parameter | Stderr | Parameter | Stderr | Multilevel | NoNs |
| EdgeA | $-0.748$ | 2.019 | 2.052 | 3.245 |  | |
| ASA | $-0.861$ | 0.516 | $-1.966$ | 1.125 |  | |
| ATA | $\mathbf{1.331}$ | 0.48 | $\mathbf{1.314}$ | 0.482 |  | |
| XEdge | $-0.772$ | 0.456 | $-0.457$ | 0.473 |  |  |
| XASB | $\mathbf{-1.075}$ | 0.27 | $\mathbf{-1.298}$ | 0.283 |  |  |
| XACA | $\mathbf{0.01}$ | 0.001 | $\mathbf{0.009}$ | 0.001 |  |  |
| Expert XEdgeB | | | $\mathbf{0.568}$ | 0.13 |  |  |
| Star2AX | $-0.039$ | 0.039 | $-0.062$ | 0.042 |  |  |
| TriangleXAX | $\mathbf{0.354}$ | 0.103 | $\mathbf{0.352}$ | 0.102 |  |  |

Continued on next page

**Table 1 – continued from previous page**

| | Model 1 | | Model 2 | | Representation | |
|---|---|---|---|---|---|---|
| Effects | Parameter | Stderr | Parameter | Stderr | Multilevel | NoNs |
| Star2BX | −0.001 | 0.126 | −0.064 | 0.135 |  |  |
| StarAB1X | 0.058 | 0.064 | 0.081 | 0.07 |  |  |
| TriangleXBX | **0.065** | 0.006 | **0.065** | 0.006 |  |  |
| L3XBX | −0.024 | 0.015 | −0.027 | 0.015 |  |  |
| EXTB | **−0.001** | 0.000 | **−0.001** | 0.000 |  |  |
| Expert Star2BX | | | 0.001 | 0.001 |  |  |
| L3AXB | 0.001 | 0.001 | −0.008 | 0.005 |  |  |
| C4AXB | **−0.008** | 0.004 | 0.017 | 0.014 |  |  |

# 4 Summary

We have proposed a conceptual framework for joint analysis of multiple reports on a network, how these relate to a criterion graph, and how a network among reports induces dependencies. The approach rests on transforming the original data into a multilevel network (Lazega et al., 2008) representation. We have discussed the multilevel representation in terms of meaningful, multilevel network configurations that are amenable to empirical investigation using multilevel ERGM (Wang et al., 2013). The representation is however agnostic to the actual analysis method and in the proposed format, networks of networks are amenable to investigation in any analysis framework for multilevel networks, such as stochastic actor-oriented models (Hollway et al., 2017) and blockmodels (Žibena and Lazega, 2016).

Our analysis of individual semantic structures amongst 15 actors in a local flood management groups provides evidence of a variety of mechanisms for socially constructed local knowledge. Some social mechanisms are moderated by the introduction of an external criterion network representing a semantic network of flood management experts. So, while local actors speak about their reality using their own, partially socially constructed, knowledge, this knowledge is not completely independent of the 'objective' knowledge of experts. There is further heterogeneity to explore. This may be addressed by developing more elaborate multiplex associations between the individual semantic networks and the expert network. We may also elaborate individual-level effects, for example using actor attributes. Socially dependent heterogeneity could also be explored. Here the configuration Star2AX captures the association between social popularity and number of concept pairs nominated, but it does not inform us whether people agree with central people. It is not straightforward to see whether a person who is central in the social network tends to be influential. Future research could study the dependence of the overall structure on particular actors, something that would require multilevel elaborations of outlier diagnostics (Koskinen et al., 2018).

The translation of data from the original domain into the multilevel representation is not dependent on the specific content of the network data and we have suggested how the approach might be applied in other contexts. Examples include sociosemantic networks (Basov, 2020), socioecological networks (Bodin et al., 2016), and sociomaterial networks (Contractor et al., 2011; Basov, 2018), all of which are explicitly multilevel. Other examples include data collection paradigms that assume repeated observations on networks such as multiplex networks and cognitive social structures. Yet another class would be networks that could be repeatedly observed for units (e.g. intersectional flows in different countries, Leoncini et al., 1996; and, symptom networks, Borsboom and Cramer, 2013) where work has already been done on analysing the networks of these units (social networks in psychiatry, Moreno, 1934; and, economic ties between countries, Squartini et al., 2011; Koskinen and Lomi, 2013). All of these different contexts present unique challenges in translating data from the original domain into the multilevel network of networks presented here, some of which we have discussed.

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# A Goodness of fit

Table 2: Goodness-of-fit for Model 1

| Statistics | Observed | Mean | StdDev | t-ratio |
|---|---|---|---|---|
| EdgeA | 14 | 15.119 | 4.352 | −0.257 |
| Star2A | 27 | 32.059 | 21.424 | −0.236 |
| Star3A | 17 | 23.294 | 30.204 | −0.208 |
| Star4A | 6 | 13.501 | 33.548 | −0.224 |
| Star5A | 1 | 6.919 | 31.578 | −0.187 |
| TriangleA | 5 | 5.551 | 3.947 | −0.14 |
| Cycle4A | 5 | 5.299 | 7.802 | −0.038 |
| IsolatesA | 2 | 2.366 | 1.574 | −0.233 |
| IsolateEdgesA | 1 | 0.764 | 0.87 | 0.271 |
| ASA | 19.875 | 23.0891 | 12.139 | −0.265 |
| ATA | 11.25 | 13.4626 | 7.878 | −0.281 |
| A2PA | 22.25 | 27.26 | 15.835 | −0.316 |
| AETA | 20 | 19.893 | 19.989 | 0.005 |
| XEdge | 934 | 937.467 | 17.057 | −0.203 |
| XStar2A | 34071 | 33956.201 | 2422.932 | 0.047 |
| XStar2B | 1361 | 1188.361 | 50.438 | 3.423 |
| XStar3A | 897090 | 1013834.002 | 224473.748 | −0.52 |
| XStar3B | 1821 | 905.179 | 81.721 | 11.207 |
| X3Path | 194167 | 169720.898 | 12961.535 | 1.886 |
| X4Cycle | 11894 | 8735.213 | 1028.4 | 3.072 |
| XECA | 1814656 | 1463003.616 | 316605.014 | 1.111 |
| XECB | 89002 | 38742.874 | 5298.652 | 9.485 |
| XASA | 1808.0156 | 1814.934 | 34.114 | −0.203 |
| XASB | 830.2759 | 835.8853 | 27.296 | −0.206 |
| XACA | 29078.7461 | 29889.6492 | 1990.761 | −0.407 |
| XACB | 195.2369 | 206.8379 | 1.395 | −8.318 |
| XAECA | 47575.4062 | 34940.852 | 4113.602 | 3.071 |
| XAECB | 34082.3115 | 23951.0315 | 2965.459 | 3.416 |
| Star2AX | 1742 | 1910.544 | 575.855 | −0.293 |
| StarAA1X | 1302.625 | 1469.0705 | 805.719 | −0.207 |
| StarAX1A | 3372.0156 | 3700.136 | 1118.421 | −0.293 |
| StarAXAA | 1864.0078 | 1875.41 | 40.86 | −0.279 |
| TriangleXAX | 185 | 205.084 | 66.178 | −0.303 |
| L3XAX | 51989 | 59569.904 | 19784.367 | −0.383 |
| ATXAX | 27.5154 | 30.077 | 8.665 | −0.296 |
| EXTA | 935 | 1065.106 | 785.434 | −0.166 |
| Star2BX | 24924 | 25017.347 | 460.361 | −0.203 |
| StarAB1X | 46413.8962 | 46587.6369 | 875.177 | −0.199 |
| StarAX1B | 25128.7515 | 24606.9828 | 801.756 | 0.651 |
| StarAXAB | 15412.29 | 15423.0444 | 33.081 | −0.325 |
| TriangleXBX | 3740 | 3875.745 | 318.218 | −0.427 |
| L3XBX | 42069 | 42389.237 | 1577.281 | −0.203 |
| ATXBX | 2899.0625 | 3209.6011 | 241.658 | −1.285 |
| EXTB | 438305 | 440039.401 | 9601.51 | −0.181 |
| L3AXB | 45980 | 50570.331 | 15295.115 | −0.3 |
| C4AXB | 4900 | 5455.271 | 1852.387 | −0.3 |
| ASAXASB | 47716.5212 | 48056.7074 | 1231.73 | −0.276 |
| AC4AXB | 6372.4708 | 6902.4475 | 148.779 | −3.562 |
| stddev degreeA | 1.4573 | 1.3706 | 0.382 | 0.227 |
| skew degreeA | 0.6445 | 0.2692 | 0.417 | 0.901 |
| clusteringA | 0.5556 | 0.5144 | 0.156 | 0.264 |
| stddev degreeX A | 70.2424 | 70.0838 | 2.472 | 0.064 |
| skew degreeX A | −1.103 | −0.9459 | 0.044 | −3.588 |
| stddev degreeX B | 3.4014 | 3.238 | 0.057 | 2.875 |
| skew degreeX B | −0.9758 | −1.1618 | 0.01 | 18.911 |
| clusteringX | 0.245 | 0.2053 | 0.009 | 4.227 |
| stddev degreeB | 20.9149 | 20.9149 | 0 | −1 |
| skew degreeB | 0.7521 | 0.7521 | 0 | −1 |
| clusteringB | 0.8341 | 0.8341 | 0 | 1 |

Table 3: Goodness-of-fit for Model 2

| Statistics | Observed | Mean | StdDev | t-ratio |
|---|---|---|---|---|
| EdgeA | 14 | 15.552 | 4.926 | -0.315 |
| Star2A | 27 | 34.914 | 27.413 | −0.289 |
| Star3A | 17 | 31.167 | 50.992 | −0.278 |
| Star4A | 6 | 28.788 | 91.743 | −0.248 |
| Star5A | 1 | 28.722 | 151.577 | −0.183 |
| TriangleA | 5 | 5.594 | 4.37 | −0.136 |
| Cycle4A | 5 | 5.46 | 8.827 | −0.052 |
| IsolatesA | 2 | 2.183 | 1.45 | −0.126 |
| IsolateEdgesA | 1 | 0.719 | 0.868 | 0.324 |
| ASA | 19.875 | 24.1388 | 14.163 | −0.301 |
| ASA2 | 19.875 | 24.1388 | 14.163 | −0.301 |
| ATA | 11.25 | 13.6882 | 8.792 | −0.277 |
| A2PA | 22.25 | 29.9897 | 20.809 | −0.372 |
| AETA | 20 | 20.1698 | 22.405 | −0.008 |
| XEdge | 934 | 937.624 | 16.823 | −0.215 |
| XStar2A | 34071 | 33432.353 | 2609.927 | 0.245 |
| XStar2B | 1361 | 1194.897 | 50.717 | 3.275 |
| XStar3A | 897090 | 962121.188 | 234759.128 | −0.277 |
| XStar3B | 1821 | 929.591 | 86.868 | 10.262 |
| X3Path | 194167 | 168319.628 | 14393.637 | 1.796 |
| X4Cycle | 11894 | 8630.116 | 1170.167 | 2.789 |
| XECA | 1814656 | 1410244.755 | 353314.367 | 1.145 |
| XECB | 89002 | 39200.372 | 6229.365 | 7.995 |
| IsolatesXA | 0 | 0 | 0 | $NaN$ |
| IsolatesXB | 0 | 0 | 0 | $NaN$ |
| XASA | 1808.0156 | 1815.248 | 33.646 | −0.215 |
| XASB | 830.2759 | 836.58 | 26.974 | −0.234 |
| XACA | 29078.7461 | 29424.9486 | 2115.042 | −0.164 |
| XACB | 195.2369 | 207.2175 | 1.278 | −9.375 |
| XAECA | 47575.4062 | 34520.464 | 4680.669 | 2.789 |
| XAECB | 34082.3115 | 23788.9969 | 3385.866 | 3.04 |
| Expert XEdgeA | 0 | 0 | 0 | $NaN$ |
| Expert XEdgeB | 359 | 361.437 | 10.688 | −0.228 |
| Expert X2StarA010 | 0 | 0 | 0 | $NaN$ |
| Expert X2StarB010 | 709 | 560.37 | 38.883 | 3.822 |
| Expert X2StarA100 | 0 | 0 | 0 | $NaN$ |
| Expert X2StarB100 | 21247 | 21035.368 | 1660.846 | 0.127 |
| Expert X2StarA101 | 0 | 0 | 0 | $NaN$ |
| Expert X2StarB101 | 5084 | 5115.007 | 463.06 | −0.067 |
| Expert X4CycleA1 | 0 | 0 | 0 | $NaN$ |
| Expert X4CycleB1 | 8943 | 6275.175 | 876.511 | 3.044 |
| Expert X4CycleA2 | 0 | 0 | 0 | $NaN$ |
| Expert X4CycleB2 | 2997 | 1988.218 | 337.739 | 2.987 |
| Expert XAlt4CycleA1 | 0 | 0 | 0 | $NaN$ |
| Expert XAlt4CycleB1 | 21543.7305 | 22368.1028 | 1605.882 | −0.513 |
| Expert XAlt4CycleA2 | 0 | 0 | 0 | $NaN$ |
| Expert XAlt4CycleB2 | 3948.6875 | 4220.5817 | 329.981 | −0.824 |
| Expert XEdgeAB | 0 | 0 | 0 | $NaN$ |
| Star2AX | 1742 | 1999.047 | 826.083 | −0.311 |
| StarAA1X | 1302.625 | 1666.4692 | 1244.166 | −0.292 |
| StarAX1A | 3372.0156 | 3873.678 | 1614.366 | −0.311 |
| StarAXAA | 1864.0078 | 1877.456 | 42.567 | −0.316 |
| TriangleXAX | 185 | 217.22 | 104.594 | −0.308 |
| L3XAX | 51989 | 63034.626 | 31728.236 | −0.348 |
| ATXAX | 27.5154 | 30.9576 | 9.829 | −0.35 |
| EXTA | 935 | 1144.756 | 1065.915 | −0.197 |
| Star2BX | 24924 | 25079.4 | 426.565 | −0.364 |
| StarAB1X | 46413.8962 | 46709.5707 | 810.195 | −0.365 |
| StarAX1B | 25128.7515 | 24736.3993 | 751.933 | 0.522 |
| StarAXAB | 15412.29 | 15423.3979 | 32.453 | −0.342 |
| TriangleXBX | 3740 | 3814.606 | 353.514 | −0.211 |
| L3XBX | 42069 | 42598.521 | 1462.718 | −0.362 |
| ATXBX | 2899.0625 | 3153.7942 | 265.147 | −0.961 |
| EXTB | 438305 | 441502.275 | 8880.053 | −0.36 |
| Expert Star2BX | 45980 | 53366.561 | 23225.159 | −0.318 |
| L3AXB | 4900 | 5874.284 | 3164.443 | −0.308 |
| C4AXB | 47716.5212 | 48376.0399 | 1509.633 | −0.437 |
| stddev degreeA | 1.4573 | 1.3937 | 0.445 | 0.143 |
| skew degreeA | 0.6445 | 0.3555 | 0.538 | 0.537 |
| clusteringA | 0.5556 | 0.4813 | 0.15 | 0.494 |
| stddev degreeX A | 70.2424 | 69.5409 | 2.663 | 0.263 |
| skew degreeX A | −1.103 | −0.9516 | 0.04 | −3.787 |
| stddev degreeX B | 3.4014 | 3.2445 | 0.057 | 2.761 |
| skew degreeX B | −0.9758 | −1.1588 | 0.011 | 17.01 |
| clusteringX | 0.245 | 0.2043 | 0.011 | 3.77 |
| stddev degreeB | 20.9149 | 20.9149 | 0 | −1 |
| skew degreeB | 0.7521 | 0.7521 | 0 | −1 |
| clusteringB | 0.8341 | 0.8341 | 0 | 1 |