

Unit I: System of Linear Equation

Rank of Matrix

Let 'A' be any $m \times n$ matrix.

I] Row Echelon form (REF) for finding Rank of Matrix.

NOTE:

Use only Row operation on Matrix.

Rule I: Make all entry (if present) zero below first non-zero entry (if present) in first row.

Rule II: Apply Rule I to remaining row of matrix.

Rule III: If given matrix have zero row or during process of REF if we get zero row then shift that zero row to bottom of the matrix.

Rule IV: Next leading (Pivot) entry must be placed at RHS of previous leading entry.

* Leading Entry: An entry from REF is said to be leading if it is first non-zero entry in a row below that all entries are zero (if present)

* Rank: Number of non-zero Row's in REF called Rank of Matrix and denoted by $\rho(A)$

OR

Rank: Rank of a matrix = No. of leading Entries

Ex. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$R_2 - 3R_1 \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \rightarrow \text{REF}$$

$$\rho(A) = 2$$

$$LE = 1, -2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} = \text{REF}$$

$$R_{1,2} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$3R_2 - R_1 = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix} \rightarrow \text{REF}$$

$$\rho(A) = 2, LE = 3, 2$$

writing from to classmate

Q. find the rank of the given matrix.

① $A = \begin{bmatrix} 9 & 3 \\ 0 & 9 \\ 6 & 8 \\ 6 & 0 \end{bmatrix}$ \Rightarrow first non-zero entry in first row is '9'.
 so, make all the entries zero below '9'.
 $R_3 \leftarrow 9R_3 - 5R_1$, $R_4 \leftarrow 3R_4 - 2R_1$, $R_5 \leftarrow 3R_5 - 2R_1$

$\Rightarrow A = \begin{bmatrix} 9 & 3 \\ 0 & 9 \\ 0 & 16 \\ 0 & 18 \\ 0 & 0 \end{bmatrix}$ (same for) second row. \Rightarrow first non-zero entry in second row is '9'.
 $R_3 \leftarrow 9R_3 - 16R_2$, $R_4 \leftarrow 9R_4 - 18R_2$, $R_5 \leftarrow 9R_5 + 18R_2$
 \Rightarrow rank of A is 2. \Rightarrow rank of matrix A is 2.

② $B = \begin{bmatrix} 6 & 0 & 0 \\ 6 & 1 & 4 \\ 3 & 1 & 3 \end{bmatrix}$ \Rightarrow first non-zero entry in first row is '6'.
 so, make all entries zero below '6'.
 $R_2 \leftarrow 6R_2 - 6R_1$, $R_3 \leftarrow R_3 - R_1$

$\Rightarrow B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 0 & 36 \\ 0 & 1 & 4 \end{bmatrix}$ same for second row. \Rightarrow first non-zero entry in second row is '36'.
 $R_3 \leftarrow 9R_3 - R_2$, $R_4 \leftarrow 6R_4 - R_2$

$\Rightarrow B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 0 & 36 \\ 0 & 1 & 0 \end{bmatrix}$ \Rightarrow rank of B is 2.

$$B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 0 & 36 \\ 0 & 9 & 0 \\ 0 & 12 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc} 6 & 0 & 0 & 1 \\ 0 & 0 & 36 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 12 & 0 & 0 \end{array} \right] \quad \text{for third row, first non-zero entry is '9'.}$$

$$R_4 \leftarrow 3R_4 - 4R_3$$

$$B = \begin{bmatrix} 6 & 0 & 0 & 1 \\ 0 & 0 & 36 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 6 & 0 & 0 & 1 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 36 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 0 & 0 & 1 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 36 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{LE} = 6, 9, 36$$

Q1. find rank of the matrix, find pivot element.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -3 & -6 \\ 0 & 0 & 0 & -9 \end{bmatrix} \xrightarrow{\text{REF}}$$

$$\therefore \text{rank}(A) = 2$$

$$\text{pivot element} = 1, -3$$

$$R_2 \leftarrow R_2 - 4R_1 \quad R_3 \leftarrow R_3 - 7R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -9 \end{bmatrix} \quad \text{rank}(A) = 2$$

$$\text{rank}(A) = 2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -9 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 2 & 2 & -4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$

 $R_2 \rightarrow R_2 - 2R_1$
 $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$
 $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & -2 & -2 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$

 $R_3 \rightarrow 2R_3 + R_2$
 $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -2 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$
 $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -2 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & -2 & -2 & 4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & -2 & 2 \end{bmatrix}$

 $R_4 \rightarrow 2R_4 - R_3$
 $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & -2 & -2 & 4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & -2 & -2 & 4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

 $\xrightarrow{\text{REF}} \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A$

$\text{rref}(A) = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A$

$\text{Leading entries} = 1, 2, -1, -2, 1$

$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Q. $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow R_2 \rightarrow R_2 - 3R_1$

 $R_3 \rightarrow R_3 + R_1$
 $R_5 \rightarrow R_5 - R_1$
 $\begin{bmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\Sigma = 1, 2, 1 \geq \text{leading entries} + \text{pivot}$
 $\therefore \text{rref}(B) = \begin{bmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$R_3 \rightarrow 5R_3 + 3R_2 \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$R_5 \rightarrow 5R_5 - R_2 \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

 $\therefore \text{rref}(B) = \begin{bmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

leading entries = 1, -5

If order is zero and matrix is skew-symmetric
then its determinant is zero

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Q. $C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 5 & 4 \end{bmatrix}$ what will be the answer? $\det C = ?$
 Ans: $2R_1 - R_2 \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 5 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 4 \end{bmatrix}$ now $R_3 - 2R_2 \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix}$
 $R_3 \leftarrow R_3 - 5R_1 \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix}$

$R_3 \leftarrow R_3 - 2R_2$ bcoz $\text{min } \text{entry} = 2$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{\text{f}(C) = 1} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{\text{f}(C) = 1}$

$R_1 \leftrightarrow R_2 \rightarrow \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{\text{f}(C) = 2} \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{\text{leading entries are } 2, 1}$

Q. $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad f(B) = 0$

Q. $E = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad f(E) = 1, \text{ Leading entry} = 1.$

NOTE / Shortcut: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{f}(E) = 1}$

1. Let 'A' be $m \times n$ matrix then $f(A) \leq \min\{m, n\}$

2. $f(A+B) \leq f(A) + f(B)$

3. $f(AB) \leq \min\{f(A), f(B)\}$

4. Let 'A' be any $m \times n$ matrix then $f(A) = f(A^T)$

5. Let 'A' be any $n \times n$ matrix then

(a) $f(A) = n$, iff $|A| \neq 0$

(b) $f(A) < n$, iff $|A| = 0$

* Row Reduced Echelon form (RREF):
 first convert given matrix into REF then make all
 leading entry '1' and entry above and below of all
 Leading entry must be 'zero' (if present) is called
 as RREF

Q. Convert given matrix into RREF and find LE

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\Rightarrow R_2 \rightarrow R_2 - 4R_1 \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{array}{l} R_3 \rightarrow R_3 - 7R_1 \\ \text{swapping entries} \end{array}$$

$$R_3 \rightarrow R_3 - 2R_2 \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} \quad \rightarrow \text{REF}$$

$$f(A) = 2, \quad LE = 1, -3$$

$$L = \sqrt{1+3} \text{ (using } f(A) = 1 = (\lambda)^2 \text{)} \quad \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] = 2$$

$$\frac{R_2}{-3} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \text{swapping entries} \\ \text{REF} \end{array}$$

$$R_1 \rightarrow R_1 - 2R_2 \quad A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \text{swapping entries} \\ \text{REF} \end{array}$$

$$(T_A) f(A) = 2 \quad \text{and} \quad LE = 1, 0 \times m \quad \text{and} \quad f(A) = 2$$

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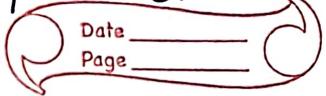
Q. find 'k' such that $f(A) = 3$ and $|f(A)| < 3$

$$A = \begin{bmatrix} k & 1 & 2 \\ 0 & k & 1 \\ 0 & 0 & k \end{bmatrix} \quad K=0$$

$$|A| = k^2 - 1 \quad f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad f(A)=2$$

$m = \text{no. of eqn}$

$n = \text{no. of unknowns/variables}$



Solution of System of Linear equation (SLE). $AX=B$ of order $m \times n$

$$\text{Ex. } x + y = 0 \quad \text{--- (1)}$$

$$x - y = 0 \quad \text{--- (2)}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$A \quad X \quad B$

Where, 'A' is matrix of coeff.

'X' is matrix of unknown

'B' is matrix of constant

* Homogeneous SLE

If RHS of every eqn is zero then it is called as Homogeneous SLE and denoted by $AX=0$

* Non-Homogeneous SLE

If RHS of atleast one eqn is non-zero then it is called as Non-Homogeneous SLE and denoted by $AX=B$ ($B \neq 0$)

* Augmented Matrix of SLE

$$\text{Ex. } 2x + 8y = 3 \quad m=2, n=2$$

$$x + 9y = -2 \quad AX=B$$

$$[A | B] = \left[\begin{array}{cc|c} 2 & 8 & 3 \\ 1 & 9 & -2 \end{array} \right]$$

$A \quad B$

$$AX = B$$

SLE of order $m \times n$,

$m = \text{no. of eqn}$, $n = \text{no. of unknown}$

$\Rightarrow AX = 0$ (Homo)

① If $\mathcal{S}(A) = \mathcal{S}(A | B) = n$
Then $AX = 0$ is consistent
(soln exist) and $AX = 0$
have trivial soln (zero soln)

Note: 1. Trivial soln: value
of all unknown are zero.
2. In this case, No free
variable.

$\Rightarrow AX = B, B \neq 0$ (Non-Homo)

① If $\mathcal{S}(A) = \mathcal{S}(A | B) = n$
Then $AX = B$ is consistent
and $AX = B$ have unique
soln (unique non-zero soln)

Note:

1. In this case No free
variables

② If $\mathcal{S}(A) = \mathcal{S}(A | B) < n$
Then $AX = 0$ is consistent
and $AX = 0$ have infinitely
many non-zero soln + infinitely
(Trivial soln + infinitely
many non-zero soln)

Note: In this case we
get at least one free
variable.

③ If $\mathcal{S}(A) \neq \mathcal{S}(A | B)$
Then $AX = B$ is inconsistent

Homo. have always solns
that's why it cannot
inconsistent

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Q. Solve the system completely if it consistent.

$$2x + 3y = 0$$

$$4x + 5y = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$m=2, n=2$$

$$m=n=2$$

(no. of eqns) (no. of unknowns)

$$A \quad x = 0 \quad AX=0 \text{ system.}$$

corresponding Augmented matrix.

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 4 & 5 & 0 \end{array} \right]$$

convert $[A|B]$ into REF.

$$R_2 - 2R_1, [A|B] \rightarrow \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & -1 & 0 \end{array} \right] \rightarrow \text{REF}$$

$$\delta(A) = 2, \delta(A|B) = 2, n=2$$

$$\therefore \delta(A) = \delta(A|B) = n.$$

\therefore It has trivial soln. ($x=0, y=0$) $\times A$, $s=0, t=n$

Q. solve the system completely if it consistent.

$$\Rightarrow m=2, n=2, AX=0 \text{ system.}$$

corresponding Augmented matrix.

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 4 & 6 & 0 \end{array} \right]$$

convert $[A|B]$ into REF.

$$R_2 - 2R_1, [A|B] \rightarrow \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \text{REF} = [A|B]$$

$$\delta(A) = 1, \delta(A|B) = 1, n=2$$

$\therefore AX=0$ is consistent

$$\delta(A) = \delta(A|B) < n$$

$AX=0$ have infinitely many solns.

from REF 2 is LE and 2 is coeff of 'x'

$\therefore x$ is LE and y is free

Put, $y=t$, $t \in \mathbb{R}$

From REF, $2x + 3y = 0$

$$\frac{x - 3y}{2} = \frac{-3t}{2}$$

$$\Rightarrow x = \frac{-3t}{2} \quad \text{and } y = t$$

but as we say it has infinitely many solns that means, $t \in \mathbb{R}$ and there are infinite values of t i.e. of y .

and therefore infinite values of $x \in [A]$

\therefore It has infinitely many solns. $[B(A)]$

Q.

$$2x + 3y = 1$$

$$AX=B$$

$$4x + 6y = 2$$

$$\Rightarrow m=2, n=2$$

corresponding augmented matrix.

$$[A|B] := \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 6 & 2 \end{array} \right]$$

$$\text{convert } [A|B] \text{ into REF} \rightarrow [A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 0 & 0 \end{array} \right] \rightarrow [E|S] = [B(A)]$$

$$\ell(A)=1, \ell(A|B)=1, n=2$$

$\therefore A\bar{x}=B$ is consistent \Rightarrow $[B(A)]$

$$\ell(A) = \ell(A|B) < n \rightarrow [E|S] = [B(A)]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \end{array} \right]$$

$$S = A$$

$$S = A, I = (B(A))^{-1}, I = (A)^{-1}$$

$$I = (B(A))^{-1} = (A)^{-1}$$

Find your solution and $A = XA$

$$X = (B(A))^{-1} = (A)^{-1}$$

Q. $2x + 3y = 1$

$$4x + 6y = 35$$

$\Rightarrow m=2, n=2, AX=B$ system $\alpha = XA, \beta = B, \gamma = m$
corresponding augmented matrix

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 6 & 35 \end{array} \right] \quad \text{Convert } [A|B] \text{ into REF}$$

$$R_2 - 2R_1, [A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 0 & 33 \end{array} \right] \rightarrow \text{REF}$$

$$A \quad B \quad A|B$$

$$\delta(A) = 1, \delta(A|B) = 12 \neq 0, \gamma = (A|B)\delta^{-1}, \zeta = (A)\delta$$

$\therefore AX=B$ is inconsistent $\delta \neq (A|B)\delta = (A)\delta$

$$\delta(A) \neq \delta(A|B) \quad \delta \neq (A|B)\delta = (A)\delta$$

No solution

Q. $2x + 3y = 1$ from $A|B$ $\alpha = XA, \beta = B, \gamma = m$
 $x + 6y = 35$

$\Rightarrow m=2, n=2, AX=B$ system

corresponding augmented matrix

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & 6 & 35 \end{array} \right] \quad \text{Convert } [A|B] \text{ into REF}$$

convert $[A|B]$ into REF

$$2R_2 - R_1, [A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 9 & 69 \end{array} \right] \rightarrow \text{REF}$$

$$A \quad B \quad A|B$$

$$\delta(A) = 2, \delta(A|B) = 2, n=2, \gamma = 2, \zeta = 1$$

$\therefore AX=B$ is consistent

$AX=B$ have unique solution

from REF, $2x + 3y = 1$

$$9y = 69$$

$$y = \frac{69}{9} = \frac{23}{3} \quad \text{and} \quad x = -11$$

Q.

$$x + 2y + 2z + 2w = 0$$

$$x + 2y + 2z + 14w = 0$$

 \Rightarrow

$m=2, n=4$, $AX=0$ system is $x=A^{-1}B$, $L=0$, $S=n$

Corresponding augmented matrix is

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & 0 \\ 1 & 2 & 2 & 14 & 0 \end{array} \right] \xrightarrow{\text{R}_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 12 & 0 \end{array} \right] \xrightarrow{\text{REF}} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

Convert $[A|B]$ into REF

$$R_2 - R_1, \left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{REF}} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\text{rank}(A) = 2, \text{rank}(A|B) = 2, n = 4 = (\text{rank}(A))n, L = (\text{rank}(A))n$$

$$\text{rank}(A) = \text{rank}(A|B) \neq n$$

$$\text{rank}(A) = \text{rank}(A|B) < n \quad (\text{rank}(A))n \neq (\text{rank}(A))n$$

$\therefore AX=0$ is consistent and has infinitely many non-zero soln.

from REF, 1 and 12 are LE and 11 is coeff of x ,

12 is coeff of w .

$\therefore x$ and w are LES $= XA$, $L=0$, $S=n$

$\therefore y$ and z are free

$$\text{Put, } y = t_1 \text{ and } z = t_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] = [8|A]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\text{from REF, } x + 2y + 2z + 2w = 0 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] = [8|A]$$

$$\therefore 12w = 0 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] = [8|A]$$

$$\boxed{w=0} \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$x + 2t_1 + 2t_2 + 0 = 0 \quad A$$

$$\Rightarrow x = -2t_1 - 2t_2, \quad S = (\text{rank}(A))n, L = (\text{rank}(A))n$$

$$y = t_1 \quad \text{Free variable as } B = XA$$

$$z = t_2 \quad \text{Free variable as } B = XA$$

$$w = 0.$$

$$L = \{x\} \quad \text{rank } A = \text{rank } B$$

$$B = LB$$

$$LB = LB$$

$$LB = LB$$

Q. Find the value of α, β such that given system have

① Unique soln

② Infinite soln

③ No soln

$$x + 2y + 3z = 1$$

$$4x + 5y + 6z = 2$$

$$7x + 8y + \alpha z = \beta$$

\Rightarrow ① Unique soln.

$M = 3, n = 3$ AX = B system.

Corresponding augmented matrix

$$[A | B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & \alpha & \beta \end{array} \right]$$

Convert $[A | B]$ into REF

$$R_2 \rightarrow R_2 - 4R_1$$

$$R_3 \rightarrow R_3 - 7R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & -6 & \alpha-21 & \beta-7 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & \alpha-9 & \beta-3 \end{array} \right]$$

$$\left[\begin{array}{cc|c} A & B & I \end{array} \right] = [B | A]$$

For ① Unique soln. $f(A) = f(A|B) = n$

i.e. $f(A) = f(A|B) = 3$ if $\alpha-9 \neq 0$ otherwise $[B|A] \neq [I|A]$

$f(A) = 3$ if $\alpha-9 \neq 0$ and $B \in R$

② Infinite soln $f(A) = f(A|B) < n$

$f(A) = f(A|B) < 3$

$f(A) < 2$ (if $\alpha-9=0$, $\beta-3=0$)

③ No soln if $f(A) \neq f(A|B)$

if $\alpha-9=0$ and $\beta-3 \neq 0$

$$\left[\begin{array}{cc|c} 0 & B-1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

H.W

Q. ① Find value of 'k' such that given system have (i) trivial soln, (ii) Infinitely many solns.

$$x + 2y + 3z = 0 \quad \text{Ansatz standard} \quad \text{② Infinite soln}$$

$$4x + 5y + 6z = 0$$

$$7x + 8y + kz = 0$$

③

Ansatz

② find k such that $kx + 2y + 3z = 0$ Ansatz suppos. 6. f.

$$2x + ky + 6z = 0 \quad n=3, \quad \varepsilon=0$$

$$x+2y+kz=0 \quad \text{Ansatz prüfung}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & k & 0 \\ 2 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] = [A | B]$$

$$③ \quad x+y+z+w=0$$

$$④ \quad x+2y+3z+4w=0$$

$$y-z+x=0$$

$$3x+4y-z=0$$

$$x-y+z=0$$

$$\Rightarrow ① \quad x+2y+3z=0$$

$$4x+5y+6z=0$$

$$7x+8y+kz=0$$

$$m=3, \quad n=3, \quad AX=0 \quad \text{Ansatz system.}$$

corresponding Augmented matrix is,

$$[A | B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & k & 0 \end{array} \right]$$

$$\text{Ansatz } ① \quad 7x+8y+kz=0$$

$$\text{Ansatz } ② \quad 4x+5y+6z=0$$

$$\text{Ansatz } ③ \quad x+2y+3z=0$$

$$\text{Ansatz } ④ \quad x-y+z=0$$

$$\text{Ansatz } ⑤ \quad 3x+4y-z=0$$

$$\text{Ansatz } ⑥ \quad y-z+x=0$$

$$\text{Ansatz } ⑦ \quad x-y+z=0$$

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$$\mathcal{S}(A) = \mathcal{S}(A|B) = n \quad \text{for trivial soln.}$$

$$n=3 \quad \mathcal{S}(A)=3 \quad \text{then } k-g \neq 0 \Rightarrow k \neq g$$

$$\mathcal{S}(A|B)=3 \quad \text{then } k-g \neq 0$$

①. $\therefore k \neq g$. (for trivial soln)

② for infinite many non-zero soln. $\Rightarrow \mathcal{S}(A) = \mathcal{S}(A|B) < n$

$n=3$ $\mathcal{S}(A)$ should be less than 3.

$\mathcal{S}(A|B)$ should be less than 3.

$$(k-g=0) \quad 0=x_3 \text{ or } x_3 = 0 \quad [0 \mid 0 \mid 0 \mid 0] \quad [10 \mid 0 \mid 0 \mid 0]$$

$\therefore k=g$ (for infinite soln)

$$[0 \mid 0 \mid 1 \mid 1]$$

$$(2) \quad kx + 2y + 3z = 0 \quad m=3, \quad n=3, \quad AX=0 \text{ system.}$$

$$2x + ky + 6z = 0 \quad \text{corresponding Augmented matrix is.}$$

$$x + y + kz = 0. \quad [A|B] = \begin{bmatrix} k & 2 & 3 & 0 \\ 2 & k & 6 & 0 \\ 1 & 1 & k & 0 \end{bmatrix}$$

Convert $[A|B]$ into RREF.

$$R_3 \leftrightarrow R_1 \quad [1 \quad 1 \quad (k-1) \quad 0] \quad (k-1) \neq 0 \Rightarrow k \neq 1$$

$$[2 \quad k \quad 6 \quad 0] \quad \text{Divide by 2.} \quad [1 \quad k/2 \quad 3 \quad 0]$$

$$R_2 \rightarrow R_2 - 2R_1 \quad [1 \quad 1 \quad k-1 \quad 0] \quad \text{Divide by } k-1 \quad [1 \quad 1 \quad 0 \quad 0]$$

$$R_3 \rightarrow R_3 - R_1 \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & k-2 & 6-2k & 0 \\ 0 & \frac{2}{k-1} & \frac{3-k}{k-1} & 0 \end{bmatrix} \quad \text{Initial step.} \quad C \times A$$

$$R_3 \rightarrow R_3 + R_2 \quad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & k-2 & 6-2k & 0 \\ 0 & 0 & -\frac{k^2-2k+9}{k-1} & 0 \end{bmatrix} \quad R=0, \quad C=0 \quad (A)$$

for trivial soln, $\mathcal{S}(A) = \mathcal{S}(A|B) = n$

$$n=3 \quad \mathcal{S}(A)=3.$$

$$-\frac{k^2-2k+9}{k} \neq 0 \quad \therefore k \neq -3 \pm 3\sqrt{5}$$

zero row and zero column does not affect the Rank.

classmate

Date _____

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for infinitely many non-zero soln. $\mathcal{R}(A) = n$

$$f(A) = f(A|B) < n \text{ iff } \mathcal{R}(A) = n$$

$$f(A) < n$$

$$-k^2 - 2k + 9 = 0 \Rightarrow k = \frac{-2 \pm \sqrt{4 + 36}}{2} = \frac{-2 \pm 2\sqrt{10}}{2} = -1 \pm \sqrt{10}$$

k must be a divisor of $\mathcal{R}(A)$. $2 \leq n \leq 3$

OR

$$[A|B] = \left[\begin{array}{ccc|c} k & 2 & 3 & 0 \\ 2 & k & 6 & 0 \\ 1 & 1 & k & 0 \end{array} \right] \quad \text{But in } AX=0 \quad \mathcal{R}(A) = \mathcal{R}(A|B)$$

zero row or a column does not affect the rank of matrix.

$$\text{so, } A = \left[\begin{array}{ccc} k+2 & 3 & 0 \\ 2 & k & 6 \\ 1 & 1 & k \end{array} \right] \quad \mathcal{R}(A) = \left[\begin{array}{ccc} k+2 & 3 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-1 \end{array} \right]$$

we have $\mathcal{R}(A) = n$, iff $|A| \neq 0$

$\mathcal{R}(A) < n$, iff $|A| = 0$. $|A| = k(k-2)(k-1)$

$$|A| = k(k^2 - 3k + 2) = k^3 - 3k^2 + 2k$$

① $AX=0$ have trivial soln.

$$\mathcal{R}(A) = \mathcal{R}(A|B) = n = 3$$

$$\therefore \mathcal{R}(A) = 3 \text{ if } |A| \neq 0 \Rightarrow k^3 - 3k^2 + 2k \neq 0$$

② $AX=0$ have infinite non-zero soln.

$$\mathcal{R}(A) = \mathcal{R}(A|B) < n = 3$$

$$\mathcal{R}(A) < n \text{ iff } |A| = 0. \Rightarrow k^3 - 3k^2 + 2k = 0$$

④ $m=3, n=4, AX=0$ system

corresponding augmented matrix,

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 3 & 4 & -1 & 0 & 0 \end{array} \right]$$

convert $[A|B]$ into REF.

$$R_2 \rightarrow R_2 - R_1 \quad [A|B] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & -4 & -4 & 0 \\ 0 & -2 & -10 & -12 & 0 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_2 \quad [A|B] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & -4 & -4 & 0 \\ 0 & 0 & -2 & -4 & 0 \end{array} \right]$$

$\delta(A) = 3, \delta(A|B) = 3, n = 3.$ trivial soln
 $\therefore \delta(A) = \delta(A|B) = n = 3. \therefore$ trivial soln.

(5)

$$m=4, n=2 \Rightarrow AX=0 \text{ system.}$$

Corresponding augmented matrix, $[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 4 & 3 & 0 \\ 3 & -1 & 0 \\ 1 & 1 & 0 \end{array} \right]$

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 4 & 3 & 0 \\ 3 & -1 & 0 \\ 1 & 1 & 0 \end{array} \right] \xrightarrow{R_4 \leftrightarrow R_1} [A|B] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 4 & 3 & 0 \\ 3 & -1 & 0 \\ 2 & 3 & 0 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 4R_1$$

$$R_3 \rightarrow R_3 - 3R_1 \quad [A|B] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -4 & 0 \end{array} \right] \Rightarrow R_3 \rightarrow R_3 + 4R_2 \quad [A|B] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$R_4 \rightarrow R_4 - 2R_1 \quad [A|B] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad R_4 \rightarrow R_4 + R_2 \quad [A|B] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\delta(A) = 2, \delta(A|B) = 2$$

$$n=2.$$

$$\therefore \delta(A) = \delta(A|B) = n=2. \therefore$$
 trivial soln.

$$(3) m=1, n=4, AX=0 \text{ system. (Homeo)}$$

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \end{array} \right] \rightarrow \text{REF.}$$

$$\delta(A) = 1, \delta(A|B) = 1$$

$$\therefore \delta(A) = \delta(A|B) < 4 = n. \therefore$$
 Infinitely many non-zero soln.

$$LE = 1, \therefore LE = x = ?$$

and y, z, w are free.

$$\text{Put } y=t_1, z=t_2, w=t_3 \Rightarrow \text{from REF}$$

$$x+y+z+w=0$$

$$\Rightarrow x = -(t_1+t_2+t_3), y=t_1, z=t_2, w=t_3$$

Gauss Jordan Elimination method. to find solⁿ of system of linear eqn.

Step I: System is given $\begin{bmatrix} 1 & 2 & 3 & | & 1 \end{bmatrix} = [A|B]$

Step II: Write $[A|B]$ - matrix

Step III: Convert $[A|B]$ into RREF

Step IV: Apply condition from solⁿ of SLE and find value of unknown. $\bar{x}=0, \bar{y}=1, \bar{z}=2$

$$\text{Solⁿ given: } \begin{bmatrix} 1 & 2 & 3 & | & 1 \end{bmatrix} = [A|B]$$

Using Gauss Jordan elimination method find the solⁿ of given system. $x + 2y + 3z = 1$

$$4x + 5y + 6z = 2$$

$$7x + 8y + 9z = 3$$

$m=3, n=3, A\bar{x}=B$ (system is non-Homo)

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

Convert $[A|B]$ into RREF. firstly convert into REF

$$R_2 \rightarrow R_2 - 4R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

$$R_3 \rightarrow R_3 - 7R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

$$R_3 \rightarrow R_3 - 2R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

Convert it into RREF, $\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$

$$R_1 \rightarrow 3R_1 + 2R_2 \rightarrow \left[\begin{array}{ccc|c} 3 & 0 & 3 & 3 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] = [A|B]$$

$$R_1 \rightarrow R_1 - \frac{1}{3}R_2, R_2 \rightarrow R_2 + 3R_1 \quad [A|B] = \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1/3 \\ 0 & 1 & 2 & 2/3 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow RREF$$

$\delta(A) = 2$ and $\delta(A|B) = 2$, $n=3$

$AX=B$ is consistent and infinitely many non-zero soln.
 x and y are LE.

z is free

Put $z=t$, $t \in \mathbb{R}$ from RREF

$$x - z = -\frac{1}{3} \Rightarrow x = -\frac{1}{3} + t$$

$$y + 2z = 2/3 \Rightarrow y = 2/3 - 2t$$

$$x = -\frac{1}{3} + t, y = \frac{2}{3} - 2t, z = t \quad (t \in \mathbb{R})$$

$$y = \frac{2}{3} - 2z = \frac{2}{3} - 2t = 2 - 6t$$

$$\therefore x = -\frac{1}{3} + 3t, y = \frac{2}{3} - 6t, z = t \quad (t \in \mathbb{R})$$

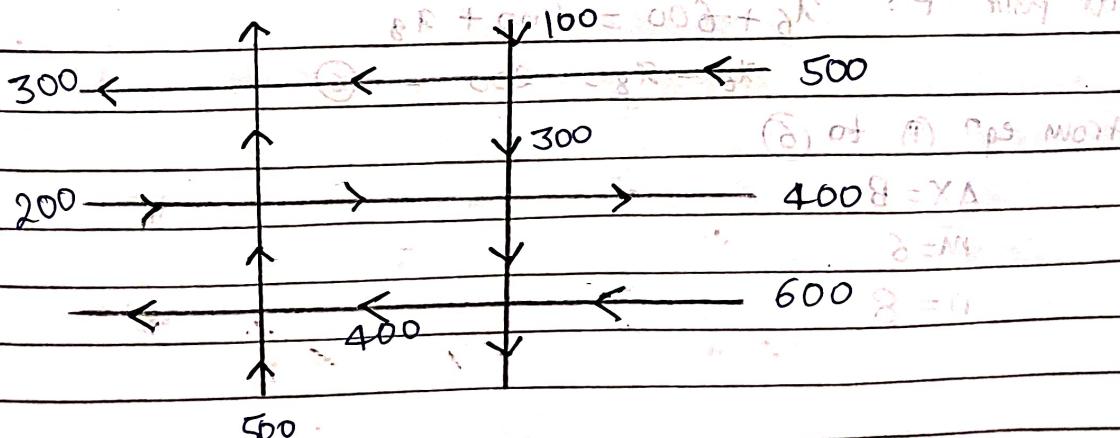
* Application of SLE

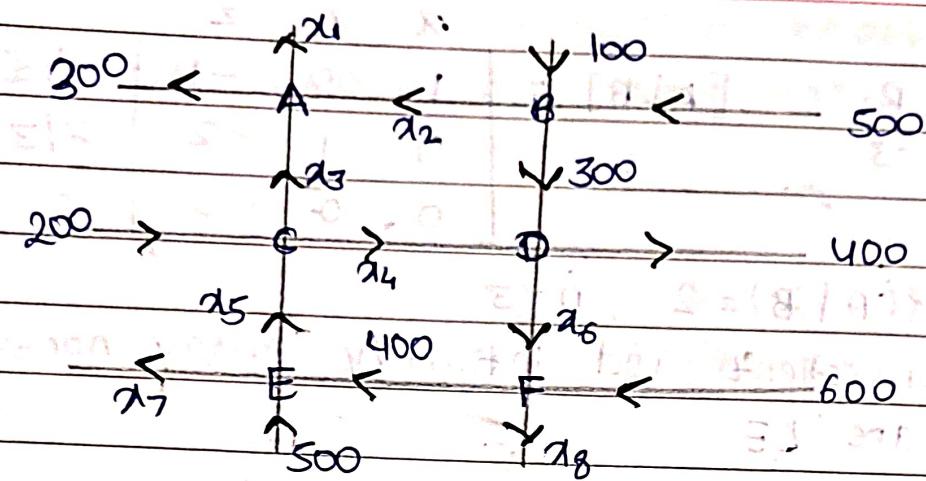
I Network flow

Total inflow = Total outflow : 3rd Q

$$② - 00B = 5R + 3R$$

Ex. solve the Network.





at point A: $x_2 + x_3 = x_1 + 300$ —①

$$x_1 - x_2 - x_3 = -300 \quad \text{---} \quad \textcircled{1}$$

at point B: $100 + 500 = x_2 + 300$

$$x_2 = 300 \quad \text{---} \quad \textcircled{2}$$

at point C: $200 + x_5 = x_3 + x_4$

$$x_3 + x_4 - x_5 = 200 \quad \text{---} \quad \textcircled{3}$$

at point D: $300 + x_4 = 400 + x_6$

$$x_4 - x_6 = 100 \quad \text{---} \quad \textcircled{4}$$

at point E: $500 + 400 = x_5 + x_7$

$$x_5 + x_7 = 900 \quad \text{---} \quad \textcircled{5}$$

at point F: $x_6 + 600 = 400 + x_8$

$$x_6 - x_8 = -200 \quad \text{---} \quad \textcircled{6}$$

From eqn ① to ⑥

$$AX = B$$

$$m = 6$$

$$n = 8$$

$$[A | B] = \left[\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ \hline 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -300 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 300 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 900 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -200 \end{array} \right] \rightarrow \text{REF.}$$

$$f(A) = 6, \quad f(A|B) = 6, \quad n = 8. \quad (\text{System is non-Hamro})$$

$\therefore f(A) = f(A|B) < n. \quad (\text{Infinite many non-zero soln}).$

$$LE = x_1, x_2, x_3, x_4, x_5, x_6$$

$$\text{free} = \alpha_7 + \alpha_8 (\psi_{\text{left}} + \psi_{\text{right}}) = (6+8) \quad \text{Total free}$$

Put, $\alpha_7 = t_1$, $\alpha_8 = t_2$ (and $\alpha_7 + \alpha_8 = t_1 + t_2$) ($t_1, t_2 \in R$)

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$$x_5 + x_7 = 900 \quad x_6 - x_8 = -200 \quad (V-1) \text{ equations. (II)}$$

$$\Rightarrow x_5 + t_1 = 900 \quad (\text{---} \Rightarrow) \quad x_6 - t_2 = -200$$

$$\Rightarrow x_5 = 900 - t_1 \quad \Rightarrow x_6 = t_2 - 200$$

like these find all the values of x_4 to x_8 following steps (II)

$$g \in \partial E_B \quad \text{and} \quad u = g \circ \theta_B$$

所以 $(V, \mathcal{X}) = (V, X) \times \mathbb{C}^n$ 为 \mathbb{C}^n 的一个子集。

• *Widjaja sydibba* 水

$$f_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N f_i(\theta, \phi) = 11$$

$$(0,0) = (0,0) + (0,0)$$

59. $\exists x \exists y \forall z (x < y \rightarrow \neg (x < z \wedge z < y))$

$$\{s \in \Sigma^* \mid (\Sigma^* s) \beta = \text{some suffix of } v\}$$

$$93 \pm 0.05, \quad \theta = 35.6^\circ \quad (\text{N}, \text{E}) \quad \pm 0.05$$

故 $\nabla \phi = V \hat{e}_z$ (E_z, B_{\perp})