d_2, d_2^*, d_2^s calculation

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1 EX_w, EY_w calculation:

$$
\begin{aligned}
EX_w &= \sum_{i=1}^{M} (\beta_i - k + 1)(p_{Xw} + p_{X\bar{w}}) \\
EY_w &= \sum_{i=1}^{M} (\beta_i - k + 1)(p_{Yw} + p_{Y\bar{w}}) \\
\widetilde{X_w} &= X_w - EX_w \\
\widetilde{Y_w} &= Y_w - EY_w
\end{aligned}
$$

2 D_2, D_2^*, D_2^s calculation:

$$
\begin{aligned}
D_2 &= \sum_{w \in A^k} X_w Y_w \\
D_2^* &= \sum_{w \in A^k} \frac{\widetilde{X_w}\widetilde{Y_w}}{\sqrt{EX_w EY_w}} \\
D_2^s &= \sum_{w \in A^k} \frac{\widetilde{X_w}\widetilde{Y_w}}{\sqrt{\widetilde{X_w}^2 + \widetilde{Y_w}^2}}
\end{aligned}
$$

3 d_2, d_2^*, d_2^s calculation:

$$d_2 = 1 - \frac{D_2}{\sqrt{\sum_{w\in A^k} X_w^2}\sqrt{\sum_{w\in A^k} Y_w^2}}$$

$$d_2^* = \frac{1}{2}\left(1 - \frac{D_2^*}{\sqrt{\sum_{w\in A^k} \frac{\widetilde{X_w}^2}{EX_w}}\sqrt{\sum_{w\in A^k} \frac{\widetilde{Y_w}^2}{EY_w}}}\right)$$

$$d_2^s = \frac{1}{2}\left(1 - \frac{D_2^s}{\sqrt{\sum_{w\in A^k} \frac{\widetilde{X_w}^2}{\sqrt{\widetilde{X_w}^2+\widetilde{Y_w}^2}}}\sqrt{\sum_{w\in A^k} \frac{\widetilde{Y_w}^2}{\sqrt{\widetilde{X_w}^2+\widetilde{Y_w}^2}}}}\right)$$