ENGNX627 Lab Tutorial: Simple Observer

In this lab tutorial, we will implement a simple observer for the pose of our robot as it drives around a circle and observes a few landmark. We use a simple line following solution to drive in a circle.

The landmarks are at known positions $p_i = {}^{A}_{A}p_i$, where $i = 1, ..., n$. If $P = {}^{A}_{A}P_B$ denotes the pose of the robot (as a homogeneous matrix), then we measure the position of the landmarks as seen from the robot frame,

$$\begin{aligned}
\overline{y}_i &:= {}^{B}_{B}\overline{p}_i, \\
&= {}^{B}_{B}P_A {}^{A}_{A}\overline{p}_i, \\
&= {}^{A}_{A}P_B^{-1} {}^{A}_{A}\overline{p}_i, \\
&= P^{-1}\overline{p}_i.
\end{aligned}$$

Let $\hat{P}$ denote our estimate of the robot pose, obtained by combining integration of kinematics and applying an innovation based on the measurements. That is, the dynamics of $\hat{P}$ are

$$\dot{\hat{P}} = \hat{P}W - \Delta\hat{P},$$

where W is the homogeneous velocity input, and Δ is a correction term based on the measurements.

Define the measurement error $\overline{e}_i := \hat{P}\overline{y}_i$. Then the innovation Δ given by

$$\Delta := \mathbb{P}\left(\sum_{i=1}^{n}(\overline{e}_i - \overline{p}_i)\overline{e}_i^{\top}\right),$$

where $\mathbb{P}$ is the $\mathfrak{se}(2)$ projector (coming up in the lectures!), will ensure $\hat{P} \to P$ over time.

During the lab tutorial, we will look at how to set up the simulator with landmarks, and then how to integrate differential equations involving homogeneous velocities. Many of the things covered in this lab will help in implementing the visual odometry, so keep that in mind while you solve this problem!