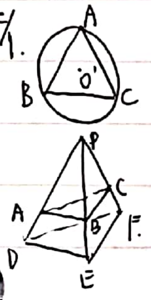
61. 棱柱、棱锥、棱台

一/ 1. $\frac{Ph}{\sin\alpha}$

2. 4

3. 13

4. 104.

二/ 1. 

设正三棱锥被圆纸片截得的图形为△ABC.

由 $AO'=O'C=O'B=\sqrt{3}$. 有 $AC=AB=BC=3$.

设P-ABC的高的最大值为H. 原正三棱锥P-DEF的高为h.

由相似可知. $\frac{H}{h}=\frac{BC}{EF}$ $\therefore H=\frac{3}{4}\times6=\frac{9}{2}$

故能做过这张纸面的棱锥的高的最大值为 $\frac{9}{2}$

2. (1). $BB_1\perp$平面$ABC$. $BC\subset$平面$ABC$ $\therefore BB_1\perp BC$

又$\angle ABC=90^\circ$ 即 $AB\perp BC$

$\because AB\perp BC$ $BB_1\perp BC$. $AB\cap BB_1=B$

$\therefore BC\perp$平面 $ABB_1A_1$

又 $AA_1\subset$平面 $ABB_1A_1$

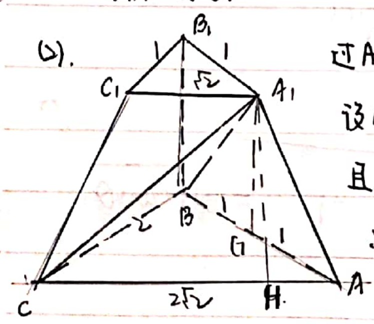
$\therefore BC\perp AA_1$.

$\because BC\perp AA_1$. $A_1C\perp A_1A$. $A_1C\cap BC=C$

$\therefore AA_1\perp$平面 $A_1BC$

又 $A_1B\subset$平面 $AA_1$

$\therefore AA_1\perp A_1B$.

(2). 

过$A_1$作$A_1H\perp AC$. 过$A_1$作$A_1G\perp AB$.

设 $AG=x$ $\because A_1G\perp AB$. $BB_1\perp AB$

且$A_1G$. $BB_1$同在平面 $AA_1B_1B$内

$\therefore A_1G\parallel BB_1$ 又$BB_1\perp$平面$ABC$

$\therefore A_1G\perp$平面$ABC$. 即 $A_1G\perp GA$.

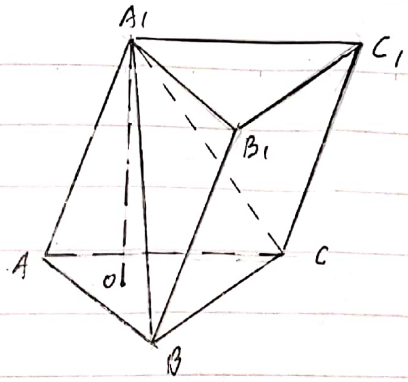
$A_1G\perp GC$. $\therefore A_1C^2=x^2+5$. $AA_1^2=x^2+1$ $\therefore AC^2=2x^2+6=8$.

$\therefore x=1$. 即 $BB_1=1$. $A_1H=\frac{AA_1\cdot A_1C}{AC}=\frac{\sqrt{2}\cdot\sqrt{6}}{2\sqrt{2}}=\frac{\sqrt{6}}{2}$

$S_侧=2S_{A_1B_1BA}+S_{A_1C_1CA}$

$=2\times\frac{1}{2}\times(1+2)\times1+\frac{1}{2}\times(\sqrt{2}+2\sqrt{2})\times\frac{\sqrt{6}}{2}$

$=3+\frac{3\sqrt{2}\cdot\sqrt{6}}{4}=3+\frac{3\sqrt{3}}{2}$

3. 

(1). $\because O$为△ABC中点 $\therefore AO\perp BC$

又$A_1O\perp$平面$ABC$, 且$BC\subset$平面$ABC$. $\therefore A_1O\perp BC$

$\because A_1O\perp BC$. $AO\perp BC$. $A_1O\cap AO=O$ $\therefore BC\perp$平面 $AA_1O$

又 $A_1A\subset$平面$AA_1O$ $\therefore AA_1\perp BC$.

$\because AO=OB$. $\angle A_1OA=\angle A_1OB$ $\therefore \triangle A_1OA\cong\triangle A_1OB$

$\therefore A_1A=A_1B$ 即$\angle A_1AB=\angle A_1BA=45^\circ$

$\therefore\angle AA_1B=90^\circ$. 即$AA_1\perp A_1B$

$\because AA_1\perp A_1B$. $AA_1\perp BC$. $A_1B\cap BC=B$

$\therefore AA_1\perp$平面 $A_1BC$

(2) $\because AA_1\perp$平面$A_1BC$. $A_1C\subset$平面 $A_1CB$ $\therefore AA_1\perp A_1C$

$\therefore S_{AA_1B_1B}=AA_1\cdot A_1B$

$S_{AA_1C_1C}=AA_1\cdot A_1C$

而$\because AA_1\parallel BB_1$. $AA_1\perp BC$ $\therefore BB_1\perp BC$

$\therefore S_{BB_1C_1C}=BB_1\cdot BC$.

又 $AA_1=\cos45^\circ\cdot AB=2\cdot\frac{\sqrt{2}}{2}=\sqrt{2}$.

$\therefore S_侧=AA_1\cdot A_1C+AA_1\cdot A_1B+BB_1\cdot BC$

$=\sqrt{2}\cdot\sqrt{2}\times2+2\times\sqrt{2}$

$=2\sqrt{2}+4$.

三/ 1. $\frac{\sqrt{3}b}{3a}$

2. $12.25\,cm^2$ $60^\circ$

3. 6

4. A.

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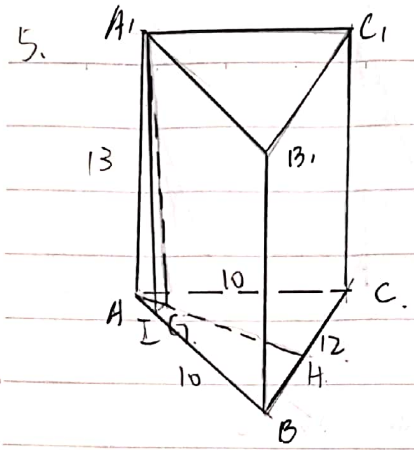
No.

Date

5.




过A作AH⊥BC. 过$A_1$作$A_1G \perp AH$.

∵ $A_1B=A_1C$, $BH=HC$ ∴ $A_1H \perp BC$

∵ $A_1H \perp BC$, $AH \perp BC$, $A_1H \cap AH = H$

∴ $BC \perp$ 平面$A_1AH$ 又 $A_1A \subset$ 平面$A_1AH$

∴ $BC \perp AA_1$

又 $AA_1 // BB_1$ ∴ $BB_1 \perp BC$ 即 $S_{BB_1C_1C} = 12 \times 13 = 156$

∵ $A_1A = A_1B = A_1C$ 即 G为$\triangle ABC$外心.

设$AG = x$ ∴ $GH = AH - x = 8 - x$

∴ $x^2 = (8-x)^2 + 36$

∴ $16x = 100$ 即 $x = \frac{25}{4}$

过G作$GI \perp AB$.

∵ $A_1G \perp AB$, $GI \perp AB$, $A_1G \cap GI = G$ ∴ $AB \perp$ 平面$A_1GI$

又$A_1I \subset$ 平面$A_1GI$ ∴ $A_1I \perp AB$

∴ $\cos\angle A_1AB = \frac{AI}{A_1A} = \frac{AI}{AG} \cdot \frac{AG}{AA_1} = \cos\angle HAB \cdot \cos\angle A_1AG$

$= \frac{4}{5} \times \frac{\frac{25}{4}}{13} = \frac{5}{13}$

∴ $\sin\angle A_1AB = \frac{12}{13}$

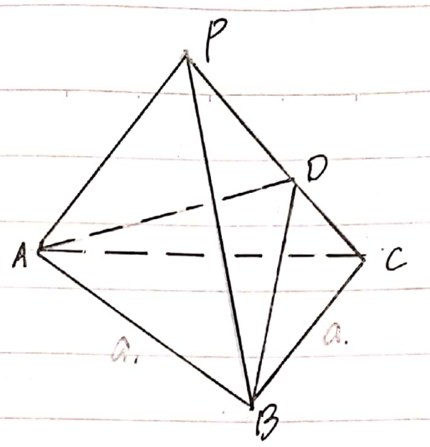
∴ $A_1I = AA_1 \cdot \sin\angle A_1AB = 12$

∴ $S_{A_1ABB_1} = 10 \times 12 = 120$

同理 $S_{A_1ACC_1} = 120$

∴ $S_{侧} = 120 \times 2 + 156 = 396$

6.




在正三棱锥P-ABC中

(1). ∵ $BD \perp PC$. ∴ $AD \perp PC$. 且 $AD = BD$

∴ $\angle ADB$ 为相邻2个侧面所成二面角的平面角, θ.

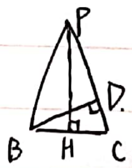
$\cos\angle ADB = \frac{AD^2 + DB^2 - AB^2}{2AD \cdot DB}$ 设 $DB = AD = x$

∴ $\cos\theta = \frac{2x^2 - a^2}{2x^2}$

$x^2 = \frac{a^2}{2(1-\cos\theta)}$  $x = \frac{a\sqrt{2(1-\cos\theta)}}{2(1-\cos\theta)}$

∴ $BD = \frac{a\sqrt{2(1-\cos\theta)}}{2(1-\cos\theta)}$

(2) ∵ 在正三棱锥中, $PB = PC$




设$\angle PCB = \alpha$

$\sin\alpha = \frac{BD}{BC} = \frac{PH}{PC}$

∴ $S_{侧} = 3 S_{\triangle PBC} = 3 \cdot \frac{1}{2} BC \cdot PH$.

$PH = \tan\alpha \cdot HC = \frac{1}{2}\tan\alpha \cdot BC$.

$\cot^2\alpha = \csc^2\alpha - 1 = \frac{1}{\sin^2\alpha} - 1$

∴ $S_{侧} = \frac{3}{2} BC \cdot \frac{1}{2\cot\alpha} \cdot BC$

$= \frac{3BC^2}{4\cot\alpha}$

$\cot\alpha = \sqrt{\frac{1}{\frac{1}{2(1-\cos\theta)}} - 1} = \sqrt{1 - 2\cos\theta}$.

∴ $S_{侧} = \frac{3a^2}{4\sqrt{1-2\cos\theta}}$