Recent Highlights in Dark Matter Physics: Primordial Black Holes at Planck scale as DM candidates

Seminar

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Introductory background

Origin of Primordial Black Holes

- PBH's may arise from adiabatic quantum fluctuations appearing during inflation
- special features in the inflaton potential could produce large scalar perturbations that seed PBHs when scales around the critical point re-enter the Hubble radius after inflation
- what is required? A super slow-roll period, leading a flat plateau features on large scale in the potential of the inflaton *(during which the inflaton quantum fluctuations get amplified and produce a peak in the spatial curvature power spectrum)*
- PBHs then evaporate leading stable **Planck mass relics**, constituting all or a part of DM
- BHs with masses of the order $0.1 M_{\odot}$ can only be of primordial origin
- Planck PBH relics are stables and non interacting *being part of the quantum foam of the space time vacuum* due to the **Generalized Uncertanty Principle**
- PBHs with $M < 5 \cdot 10^{8}$ g evaporate much before the BBN

Higgs potential

The starting point is the Lagrangian of the SM non-minimally coupled to gravity:

SM-Higgs action

$$S = \int d^4x\sqrt{-g}\left[(M^2 + \xi h^2)g^{\mu\nu}R_{\mu\nu} - V(h) - \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h + \mathcal{L}_{SM}\right] \tag{1}$$

Let's make contact with the usual analysis of inflation performing a Weyl transformation from the Jordan frame, in which the scalar field is directly coupled with the Ricci scalar R, to the Einstein frame *and define a new canonical scalar field* χ which decouple from the Ricci scalar:

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \tag{2}$$

where $\Omega^2 = (1 + \xi h^2)$ is the conformal factor. This transformation leads to a non-minimal kinetic term for the Higgs field. So, it is convenient to make the change to the new scalar field χ through the following transformation:

$$\frac{dh}{d\chi} = \frac{1 + \xi h^2}{\sqrt{1 +^2 + 6\xi^2 h^2}} \tag{3}$$

So the action in the Einstein frame becomes:

$$S_{EH} = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\} \tag{4}$$

The Higgs potential becomes:

$$U(\chi) = \frac{1}{\Omega^4(\chi)}\frac{\lambda}{4}(h(\chi)^2 - v^2)^2 \tag{5}$$

For small field values $h \sim \chi$, $\Omega \sim 1$ so the potential for the field χ is the same as that for the initial Higgs field and the conformal transformation becomes an identity tranformation

Things change if $h \gg M_P/\sqrt{\xi}$ $(\chi \gg \sqrt{6}M_P)$:

$$h \simeq \frac{M_P}{\sqrt{\chi}} e^{\left(\frac{\chi}{\sqrt{6}M_P}\right)} \tag{6}$$

This leads to:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 + e^{\left(-\frac{2\chi}{\sqrt{6}M_P}\right)}\right)^{-2} \tag{7}$$

The presence of the non-minimal coupling insures the flatness of the scalar potential in the Einstein frame at large values of the Higgs field. Successful inflation occurs for $\xi \approx 4.7 \cdot 10^4 \sqrt{\lambda}$. from the values of scalar perturbations measured by COBE. After fixing ξ the theory is completely determined. This make chaotic inflation possible [6].

One-loop Quantum corrections

Now we must run the Higgs coupling λ and the non-minimal coupling ξ through the scale of the SM, using the β-functions:

$$\beta_\xi = \frac{(6\xi+1)}{16\pi^2}\left(2\lambda + y_t^2 + \frac{3}{4}g^2 - \frac{1}{4}g'^2\right) \tag{8}$$

$$\beta_\lambda = \frac{1}{16\pi^2}\left(24\lambda^2 - 6y_t^4 + \frac{9}{8}g^4 + \frac{3}{4}g^2 g'^2 + \frac{3}{8}g'^4 + \lambda(12y_t^2 - 9g^2 - 3g'^2)\right) \tag{9}$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right) \tag{10}$$

$$\beta_g = \frac{1}{16\pi^2}(-13/4g^3) \tag{11}$$

$$\beta_{g'} = \frac{1}{16\pi^2}(27/4g'^3) \tag{12}$$

$$\beta_{g_s} = \frac{1}{16\pi^2}(-7g_s^3) \tag{13}$$

$$\tag{14}$$

Effective potential form

The behavior for $U(\chi)$ in equation (5) is reported in Figure 1.

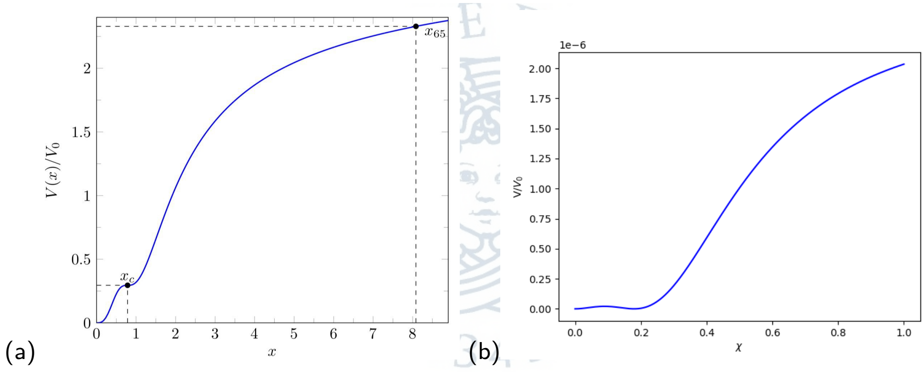


图: In figure a) is reported the effective potential from [1], and in figure b) my experimental result obtained through RGE of the same potential (initial values of the parameters must be tuned)

Slow-roll approximation

The slow-roll parameters can be analytically expressed as functions of $h(\chi)$:

$$\begin{aligned} \epsilon &= \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4M_P^4}{3\xi^2 h^4} \\ \eta &= M_P^2 \frac{d^2U/d\chi^2}{U} \simeq \frac{-4M_P^2}{3\xi^2 h^4} \end{aligned} \tag{15}$$

here we consider the approx. $h^2 \gg M_P^2/\xi \gg v^2$. Slow-roll ends when $\epsilon \sim 1$. The field value and the corresponding e-folds at the end of inflation are:

$$h_{end} \sim (4/3)^{1/4} M_P/\sqrt{\xi} \sim 1.07 M_P/\sqrt{\xi}, \quad N = \int_{h_{end}}^{h} {}_0 \frac{1}{M_P^2} \frac{U}{dU/dh}\left(\frac{d\chi}{dh}\right)^2 dh \sim \frac{3}{4}\frac{h_0^2 - h_{end}^2}{M_P^2/\xi} \tag{16}$$

we can see that for values $\xi \gg 10^{17}$ the SM scale v does not enter in the equations and the inflationary physics is independent on it. Interactions of Higgs with SM particles after the end of inflation are strong, leading to **Reheating** phase at $T_{reh} \simeq (2\lambda/(\pi^2 g^*))^{1/4} M_P/\sqrt{\xi} \simeq 10^{15} GeV$, with $g^* = 106.75$ the d.o.f. of the SM. Reheating is the process through which the energy stored in the oscillations of the inflaton field is transferred to other particles, effectively creating the hot, dense, particle-filled Universe

Critical mass

Quantum effects analysis suggests that Higgs inflation take place if the mass of Higgs boson is g.t. a certain critical lower bound value $M_h > M_{crit}$:

$$M_{crit} = \left[129.6 + \frac{y_t - 0.9361}{0.0058} \cdot 2 - \frac{\alpha_s - 0.1184}{0.0007} \cdot 0.5\right] \quad GeV \tag{17}$$

where $y_t \equiv y_t(\mu_t)$ is the top Yukawa coupling in $\bar{MS}$ scheme.

The critical point

As we seen, Higgs potential develops an inflection point at a certain χ_c, s.t. $U'(\chi_c) = U''(\chi_c) = 0$. As a consequence, the number of *e*-folds has a sharp jump ΔN in that point [7]. On this plateau the field rolls over that point in a **super slow-roll** condition in which $|\dot{\chi}|$ decreases exponentially, while during SR the velocity remains approximately constant .

Background evolution

$U(\chi)$ gives the evolution of the background field $\chi(t)$ and the scale factor $a(t)$ through the Friedmann equations:

$$3H^2 = \frac{1}{2}\dot{\chi} + U(\chi), \qquad \ddot{\chi} + 3H\dot{\chi} + U'(\chi) = 0 \tag{18}$$

for cosmic inflation to take place:

$$\frac{1}{2}\dot{\chi}^2 \ll U(\chi) \Rightarrow 3H^2 \sim U(\chi), \qquad \epsilon \equiv \frac{\dot{\chi}^2}{2H^2} < 1 \tag{19}$$

Super-SR

If $U'(\chi) \sim 0$ in a certain point, then (18) become:

$$3H^2 \sim U(\chi) \sim const, \qquad \ddot{\chi} + 3H\dot{\chi} \sim 0 \tag{20}$$

which leads to the solutions:

$$\dot{\chi}(t) \propto e^{-3Ht} \qquad \Rightarrow \qquad \chi(t) \propto const + e^{-3Ht} \tag{21}$$

Mode equations

We pass in conformal time $d\eta = a^{-1}dt$ and use $z = a\frac{\dot{\chi}}{H}$, obtaining the Klein Gordon equation in Fourier space:

$$\mu_k'' + \left(k^2 - \frac{z''}{z}\right)\mu_k = 0 \tag{22}$$

we use the Bunch-Davies vacuum initial conditions (state of minimum energy invariant under the symmetries of de Sitter space):

$$\mu_k' = -ik\mu_k, \qquad |\mu_k| = \frac{1}{\sqrt{2k}} \tag{23}$$

define a new variable suppressing rapid phase oscillations on sub-Hubble scale, we also return in the comoving time for computational purposes:

$$g_k \equiv \frac{\mu_k e^{ik\eta}}{z} \tag{24}$$

and rewrite the KG equation (22) as:

$$\ddot{g}_k + \left(H + 2\frac{\dot{z}}{z} - \frac{2ik}{a}\right)\dot{g}_k - \frac{2ik}{a}\frac{\dot{z}}{z}g_k = 0 \qquad \frac{\dot{z}}{z} = H(1 + \epsilon_H - \eta_H) \tag{25}$$

In the SSR approximation $\eta_H \approx 3$ and $\epsilon_H \approx 0$ so we have a contracting universe in which $\frac{\dot{z}}{z} = -2H$ which represents the rate of change of the redshift with respect to time. Physically, it corresponds to the rate of change of the scale factor of the universe. The equation (25) in the super-Hubble limit $k \ll aH$, becomes:

$$\ddot{g}_k - 3H\dot{g}_k = 0, \qquad \Rightarrow \qquad \dot{g}_k \propto e^{3Ht}, \quad g_k \propto A + e^{3Ht} \tag{26}$$

these solutions suggest that the spectrum does not freeze on sH scale but it is amplified, creating dense zones which collapse when reenter in the horizon Exact solution are linear combinations of

$$\begin{aligned} g_{kA} &= \left(i + \frac{k}{aH}\right) a^3 \\ g_{kB} &= \left(i - \frac{k}{aH}\right) a^3 e^{-2ik/(aH)} \end{aligned} \tag{27}$$

Power spectrum

Tensor perturbations

These modes follow the equation[8]:

$$h_k'' + \left(k^2 - \frac{a''}{a}\right) h_k = 0 \tag{28}$$

which is the same of (22) with $z = a$ and $\dot{z}/z = H$ in both SR and SSR. The tensor power spectrum is:

$$\mathcal{P}_{\mathcal{T}}(k) = 8 \left(\frac{H}{2\pi}\right)^2 \bigg|_{k=aH} \tag{29}$$

we can deduce that the tensor perturbations are not amplified, in spite of scalar ones.

Scalar perturbations

Keeping in mind the solution (27), we can calculate the time-dependent power spectrum for scalar perturbations during the Super-SR period:

$$\mathcal{P}_{\mathcal{R}}(k,t) = \frac{k^3}{2\pi^2}|g_k(t)|^2 \tag{30}$$

1. during SR, H and $\dot{\chi}$ are constant, $\mathcal{P}_{\mathcal{R}}$ freezes on super-Hubble scales
2. during SSR, scales that exit the Hubble radius during SSR are amplified without limit as long as SSR lasts

Simulations & results

g_k & $\mathcal{P}_\mathcal{R}$

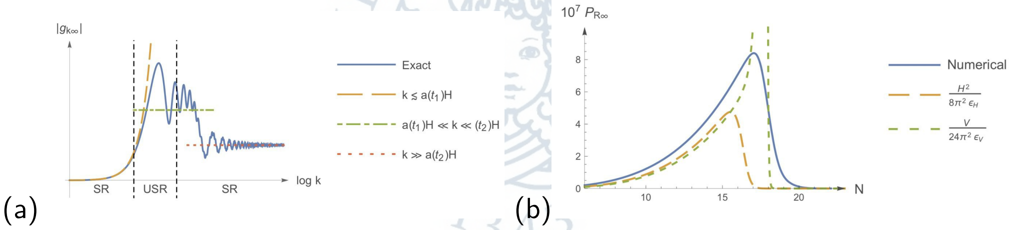
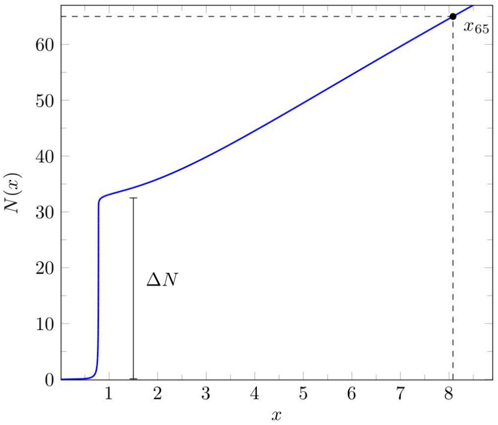


图: In Figure a) is reported the solution for the scalar perturbations $g_{k,\infty} = \lim_{t\to\infty} g_k(t)$,in Figure b) is reported the corresponding Power spectrum, taken from [1]

e-folds

- As we told, at the critical point the number of *e*-folds has a sharp jump $\Delta N \in (30, 35)$ [7], which produce a sufficiently large peak in the powes spectrum at small scales to later give rise to **PBHs** through gravitational collapse upon entry. In Figure 3 is reported N as a function of χ from (16).



Mass distribution

The total mass of the PBH at formation is given by $M_{PBH} \sim \gamma M_P^2 e^{2N}/2H_{inf}$ which is the total mass within the horizon at the time of reentry. At small scale at the peak of $\mathcal{P}_\mathcal{R}$, one finds a lognormal distribution for the possible values of the PBH masses:

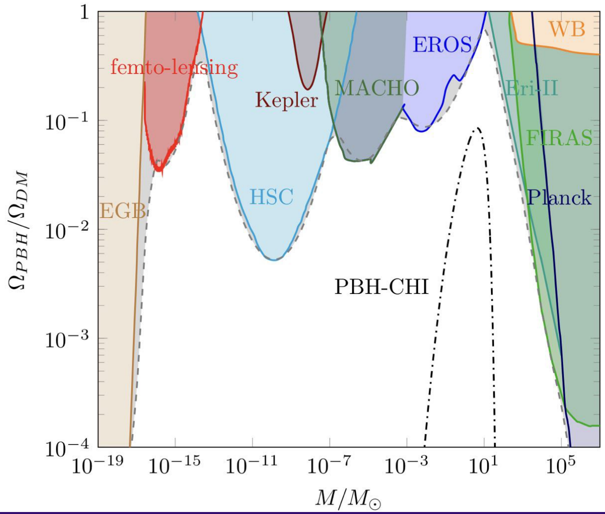
$$P(M) \sim \frac{\mu}{M\sqrt{2\pi\sigma^2}} e^{-\frac{\log^2 M/\mu}{2\sigma^2}} \tag{31}$$

with μ is the value of the field at the critical point, for which $\lambda(\mu) = \beta_\lambda(\mu) = 0$ around the critical scale $\mu = 10^{17} - 10^{18}$. One can evaluate the fraction β of the Universe collapsing into primordial black holes of mass M at the time of formation t_M in the hypotesis of Gaussian fluctuation of the curvature perturbation ζ:

$$\beta_{form} = \frac{\rho_{PBH}(M)}{\rho_{tot}}\bigg|_{t=t_M} = erfc\left(\frac{\zeta_c}{\sqrt{2\mathcal{P}_\mathcal{R}}}\right), \qquad \sigma^2 = \mathcal{P}_\zeta(k_M) \tag{32}$$

where k_M is the wavelength mode reentering inside the Hubble radius at time tM

Constraints from experiments



Hawking radiation & Information Paradox

- BHs evaporate emitting a random pattern radiation with a PDF depending only on spin, charge and temperature
- collapse transforms pure states in mixed state, causing a loss of information called **Information Paradox**
- this paradox violates the unitarity, the Von Neumann entropy increases

Generalized Uncertainty Principle

- Various hypotesis exsist about the lack of total evaporation of BH. *s.a. LQG, non-commutative geometry, gup....*
- GUP may prevent *by dynamics and not symmetry* the total collapse of a black hole.
- *As a result of String theory or other considerations on QM* We can modify the uncertainty principle, adding a term representing the gravitational interaction of the electron with the proton:

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_p^2 \frac{\Delta p}{\hbar}, \qquad L_p = \sqrt{\frac{G\hbar}{c^3}} \tag{33}$$

with this assumption one can calculate the **Hawking temperature** for PBHs[3]:

$$T_{GUP} = \frac{M_{PBH}}{4\pi}\left[1 - \sqrt{1 - \frac{M_p^2}{M_{PBH}^2}}\right] \tag{34}$$

PBH remnants and low frequency gravitational waves

- If evaporation leaves some relics:

$$m_{relic} = rM_P = M_{PBH}(t_{eva}), \qquad r > 1 \tag{35}$$

- We require that PBH remnants occupy *a fraction* $f_{relic} \equiv \frac{\rho_{relic}}{\rho_{DM}}$ of the total DM. Extrapolating backwards from today until evaporation, using $\rho_{DM} \sim a^{-3}$, we have [5]:

$$\Omega_{relic}|_{eva} = 2.5 \cdot 10^{-20} \mathcal{F}(f_{relic}, M_{PBH}, g_{s\star}, g_\star) \tag{36}$$

- if there was a PBH dominated stage in the EU for certain $\beta > \beta_{min}$ *($\beta = \rho_{PBH}/\rho_{total}$ is the initial density fraction)* and if $f_{relic} = 1$ *which means all DM are PBHs*, the initial PBH mass is uniquely determined:

$$M_{PBH} \approx 5 \cdot 10^5 g \cdot \frac{r^{2/5}}{\alpha^{1/5}} \tag{37}$$

- This implies $T_{eva} \approx 80 GeV$ corresponding to a GW signal with peak at

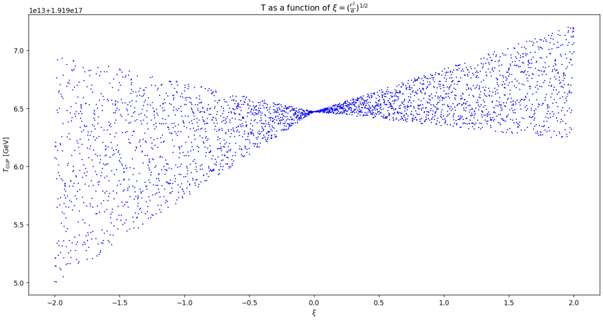
$$f_{peak} = 80 Hz \cdot \alpha^{-1/6} \tag{38}$$

- Keep in mind the results (37) and (34), we can do a plot in function of the two parameters r, α:

$$\mathrm{T}_{GUP} = 5 \cdot 10^5 \xi \frac{1}{4\pi} \cdot$$
$$\cdot \left[1 - \sqrt{1 - \left(\frac{M_p \cdot 10^{-5}}{5\xi}\right)^2}\right], \qquad \xi = \left(\frac{r^2}{\alpha}\right)^{1/5}$$

.

- The max values for T is around GUT scale



Conclusions

- The existence of relics after Hawking evaporation is suggested in some theory of quantum gravity.
- PBHs, if exist, could explain also the baryon asymmetry [10]. They evaporate before BBN
- for some parameter space the PBH remnants could explain both reheating and DM
- a theoretical prediction for the GW background is possible and if a peak around $f \sim 80Hz$ could be a good indication for both PBH DM and reheating scenario.
- however it is impossible to prove

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