Anderson Localization in Lattice QCD

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Abstract

Anderson localization was initially described in the context of disordered electronic systems, in which the wave functions, due to interference phenomena, have no diffusive behavior. Localization appears in the low-energy fermionic states when we couple Dirac fermions to the QCD non-Abelian $SU(3)$ gauge field. In the following work, we will use Rational Hybrid Monte Carlo sampling techniques to generate gauge configurations to couple with the staggered Dirac fermion operator and analyze the statistical properties of the spectrum. The localization transition seems to be related to the transition to deconfinement and the restoration of chiral symmetry, spontaneously broken at low temperatures. These phenomena occur at very close temperatures, but it is not well understood if the presence of localized modes affects both chiral symmetry restoration and deconfinement.

Introduction

Physical background: Lattice QCD is currently our best numerical approach to the non-perturbative regime of strong interaction. Asymptotic freedom guarantees that at high energy scales, the theory is weakly interacting and a perturbative treatment is possible. Through Monte Carlo simulations of the Path Integral, it's possible to investigate numerous non-perturbative effects arising from low energy scales. One such effect is Anderson localization, initially described in the context of disordered electronic systems. The *Anderson tight-binding Hamiltonian* [3] with nearest-neighbor hopping, uniform random ϵ_n on-site energy and disorder parameter W is given by (1):

$$\bar{H} = W \sum_n \epsilon_n \hat{c_n}^\dagger \hat{c_n} + \left(\sum_{\langle n,m \rangle} \hat{c_n}^\dagger \hat{c_n} + h.c. \right) \tag{1}$$

The physical meaning of a system is encoded in gauge invariant quantities, as the Wilson loop (2) (traced product of link variables $U_\mu(n)$ in a closed path C), which is an holonomy describing how much of a quantity is parallel transported along a closed path. In non-perturbative approaches to QFT, PI is regularized by discretizing the spacetime on a lattice, allowing us to compute expectation values of physical observables through a functional approach (3). Sampling algorithms should extract new gauge links according to the distribution $e^{-S_{QCD}}/Z$

$$W_C[U] = \mathrm{Tr}\left\{ \prod_{n \in C} U_\mu(n) \right\} \tag{2}$$

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{O}(U) e^{-S_{QCD}[U]}}{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_{QCD}[U]}} \tag{3}$$

$$S_{QCD} = S_G[U] + S_F^{(stag)}[U, \psi, \bar{\psi}] \tag{4}$$

S_{QCD} is given in (4); it consists in a pure gauge part, S_G, (5) and in a fermionic part, reported in (6) for a Staggered Dirac operator:

$$S_G = \frac{6}{g_0}^2 \sum_{\mu,\nu} \left[1 - \frac{1}{6} \mathrm{Tr}\left(P_{\mu\nu} + P_{\mu\nu}^\dagger \right) \right] \tag{5}$$

$$S_F^{(stag)} = \frac{1}{2} \sum_{n,\mu} \eta_\mu(n) \tilde{\chi}(n) [U_\mu(n) \chi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu}) \chi(n-\hat{\mu})] + \hat{M} \sum_n \bar{\chi}(n) \chi(n) \tag{6}$$

Here $P_{\mu\nu}$ are plaquettes, η_μ and $\chi(n)$ are, respectively, staggered phases and pseudofermions, constructed by rearranging the matrices γ of the Clifford algebra in in order to overcome the well known *fermion doubling problem*, $\hat{M}$ is the quark mass (degenerate) [2].

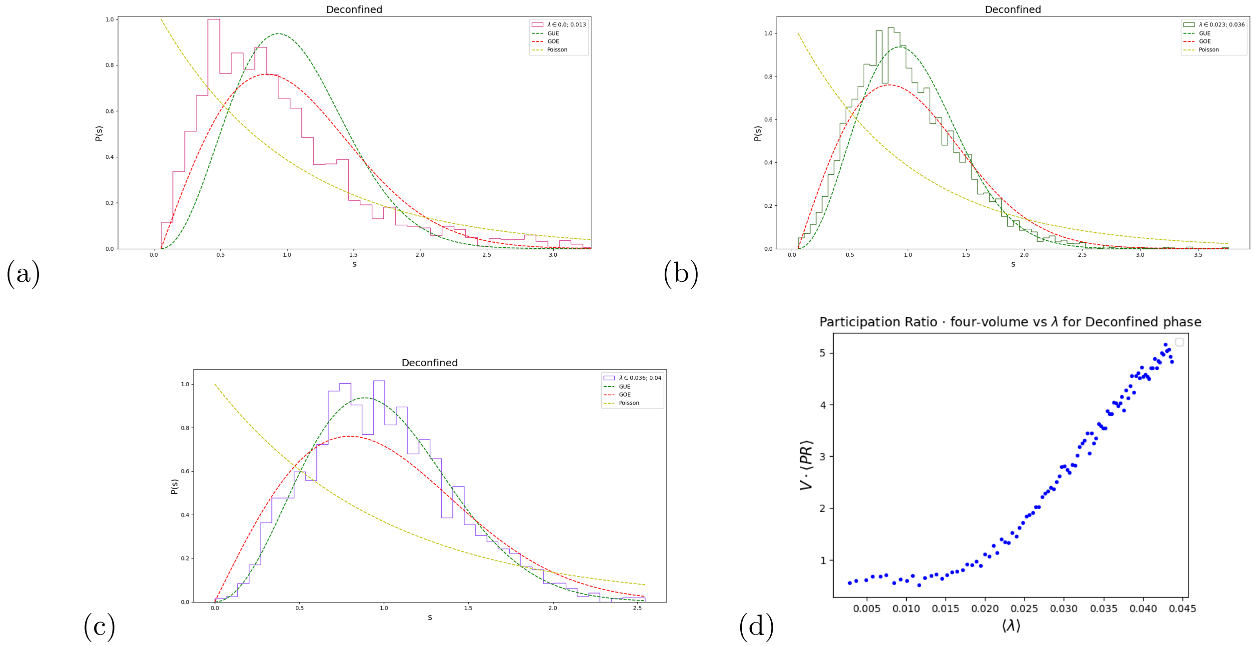

Figure 1: The first 3 figures represent unfolded level spacings distributions in various region of the computed Dirac spectrum. The deviation of the PDF from a Poissonian (which we expect for localization) can be due to edge effects or to the presence in the simulations of an Abelian U(1) phase term representing a magnetic field in a certain direction, coupled with the SU(3) gauge field. The switch between distributions is nevertheless evident. We are still investigating. In figure (b), instead, the $PR \cdot V$ is reported ($V = N_t \cdot N_s^3$); it seems to give us good indications on the presence of the transition, with a λ_e around 0.025 (flex of the curve).

Results and future developments

We will focus our study on the low end of the Dirac spectrum, for a total of 332 configurations and 200 eigenmodes for each of them. A way to locate a localized/delocalized transition is by looking at statistical properties of the *Unfolded Level Spacing Distribution* (ULSD) of the spectrum defined as the distribution of $s = \frac{\lambda_{i+1}-\lambda_i}{\langle\lambda_{i+1}-\lambda_i\rangle}$. After the unfolding of the eigenvalues on all configurations, we compare the spectral statistics for various range of λ. What we expect is a transition in the distribution of s, switching from a *Poissonian* (7) (indicating that the eigenvalues are statistically independent and localized) to a *RMT* distribution, in particular a *Gaussian Unitary Ensemble (GUE)* distribution (8) (see Figure 1).

$$P(s) = exp(-s) \tag{7}$$

$$P(s) = \frac{32}{\pi^2} s^2 \cdot e^{-\frac{4}{\pi}s^2} \tag{8}$$

An order parameter for the Anderson transition is the *mobility edge* λ_e, that can be determined in various way, here using the *Participation Ratio* $PR = \sum_n |\psi(n)|^2$, that is a measure of the fraction of four-volume occupied by the eigenmodes [1]. If PR initially decreases as N_s (spatial extension) increases and in the high end of the spectrum it tends to a constant, we have an evidence that localization took place[1].
For now we are searching to demonstrate that, at fixed lattice spacing a, the Anderson transition in the QCD spectrum belongs to the same universality class as the Anderson model of the corresponding symmetry class. However we should perform the continuum limit varying the temporal extension N_t connected with the temperature via $T = 1/(N_t \cdot a)$, and look how λ_e varies. Future investigations could reveal links between spontaneous chiral symmetry breaking, deconfinement transition, and localization. Furthermore, we are studying potential connections between the near-zero modes of the Dirac operator and topological fluctuations (instantons) of the QCD vacuum.

References

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[3] P. W. Anderson, Physical Review 109, 1492 (1958)