8.

ON THE LIGHT FROM THE SKY, ITS POLARIZATION AND COLOUR.

[*Phil. Mag.* XLI. pp. 107—120, 274—279; 1871.]

It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course. On this point the experiments of Tyndall with precipitated clouds seem quite decisive. Whenever the particles of the foreign matter are sufficiently fine, the light emitted laterally is blue in colour, and, in a direction perpendicular to that of the incident beam, is *completely polarized.*

About the colour there is no *prima facie* difficulty; for as soon as the question is raised, it is seen that the standard of linear dimension, with reference to which the particles are called small, is the wave-length of light, and that a given set of particles would (on any conceivable view as to their mode of action) produce a continually increasing disturbance as we pass along the spectrum towards the more refrangible end; and there seems no reason why the colour of the compound light thus scattered laterally should not agree with that of the sky.

On the other hand, the direction of polarization (perpendicular to the path of the primary light) seems to have been felt as a difficulty. Tyndall says, "......the polarization of the beam by the incipient cloud has thus far proved itself to be *absolutely independent of the polarizing-angle.* The law of Brewster does not apply to matter in this condition; and it rests with the undulatory theory to explain why. Whenever the precipitated particles are sufficiently fine, no matter what the substance forming the particles may be, the direction of maximum polarization is at right angles to the illuminating beam, the polarizing angle for matter in this condition being invariably 45°. This I consider to be a point of capital importance with reference to the present question" *. As to the importance there will not be two opinions; but I venture to think that the difficulty is imaginary, and is caused mainly by misuse of the word reflection. Of course there is nothing in the etymology of reflection or refraction to forbid their application in this sense; but the words have acquired technical meanings, and become associated with certain well-known laws called after them. Now a moment's consideration of the principles according to which reflection and refraction are explained in the wave theory is sufficient to show that they have no application unless the surface of the disturbing body is larger than many square wave-lengths; whereas the particles to which the sky is supposed to owe its illumination must be *smaller* than the wave-length, or else the explanation of the colour breaks down. The idea of polarization by reflection is therefore out of place; and that "the law of Brewster does not apply to matter in this condition" (of extreme fineness) is only what might have been inferred from the principles of the wave theory.

Nor is there any difficulty in foreseeing what, according to the wave theory, the direction of polarization ought to be. Conceive a beam of plane-polarized light to move among a number of particles, all small compared with any of the wave-lengths. The foreign matter, if optically denser than air, may be supposed to *load* the æther so as to increase its *inertia* without altering its resistance to distortion, provided that we agree to neglect effects analogous to chromatic dispersion. If the particles were away, the wave would pass on unbroken and no light would be emitted laterally. Even with the particles retarding the motion of the æther, the same will be true if, to counterbalance the increased inertia, suitable forces are caused to act on the æther at all points where the inertia is altered. These forces have the same period and *direction* as the undisturbed luminous vibrations themselves. The light actually emitted laterally is thus the same as would be caused by forces exactly the opposite of these acting on the medium otherwise free from disturbance; and it only remains to see what the effect of such forces would be.

On account of the smallness of the particles, the forces acting throughout the volume of any one are all of the same intensity and direction, and may be considered as a whole. The determination of the motion in the æther, due to the action of a periodic force at a given point, requires, of course, the aid of mathematical analysis; but very simple considerations will lead us to a conclusion on the particular point now under discussion. In the first place there is a complete symmetry round the direction of the force. The disturbance, consisting of transverse vibrations, is propagated outwards in all directions from the centre; and in consequence of the symmetry, the

* *Phil. Mag.* S. 4, vol. XXXVII. p. 388.

direction of vibration in any ray lies in the plane containing the ray and the axis; that is to say, the direction of vibration in the scattered or diffracted ray makes with the direction of vibration in the incident or primary ray the least possible angle. The symmetry also requires that the intensity of the scattered light should vanish for the ray which would be propagated along the axis; for there is nothing to distinguish one direction transverse to the ray from another. We have now got what we want. Suppose, for distinctness of statement, that the primary ray is vertical, and that the plane of vibration is that of the meridian. The intensity of the light scattered by a small particle is constant, and a maximum for rays which lie in the vertical plane running east and west, *while there is no scattered ray along the north and south line.* If the primary ray is unpolarized, the light scattered north and south is entirely due to that component which vibrates east and west, and is therefore *perfectly polarized*, the direction of its vibration being also east and west. Similarly any other ray scattered horizontally is perfectly polarized, and the vibration is performed in the horizontal plane. In other directions the polarization becomes less and less complete as we approach the vertical, and in the vertical direction itself altogether disappears.

So far, then, as disturbance by very small particles is concerned, theory appears to be in complete accordance with the experiments of Tyndall and others. At the same time, if the above reasoning be valid, the question as to the direction of the vibrations in polarized light is decided in accordance with the view of Fresnel. Indeed the observation on the plane of polarization of the scattered light is virtually only another form of Professor Stokes's original test with the diffraction-grating. In its present shape, however, it is free from certain difficulties both of theory and experiment, which have led different physicists who have used the other method to contradictory conclusions. I confess I cannot see any room for doubt as to the result it leads to*.

The argument used is apparently open to a serious objection, which I ought to notice. It seems to prove too much. For if one disturbing particle is unable to send out a scattered ray in the direction of original vibration, it would appear that no combination of them (such as a small body may be supposed to be) could do so, at least at such a distance that the

* I only mean that *if* light, as is generally supposed, consists of transversal vibrations similar to those which take place in an elastic solid, the vibration must be normal to the plane of polarization. There is unquestionably a formal analogy between the two sets of phenomena extending over a very wide range; but it is another thing to assert that the vibrations of light are really and truly to-and-fro motions of a medium having mechanical properties (with reference to small vibrations) like those of ordinary solids. The fact that the theory of elastic solids led Green to Fresnel's formulæ for the reflection and refraction of polarized light seems amply sufficient to warrant its employment here, while the question whether the analogy is more than formal is still left open.

body subtends only a small solid angle. Now we know that when light vibrating in the plane of incidence falls on a reflecting surface at an angle of 45°, light *is* sent out according to the law of ordinary reflection, whose direction of vibration is perpendicular to that in the incident ray. And not only is this so in experiment, but it has been proved by Green* to be a consequence of the very same view as to the nature of the difference between media of various refrangibilities as has been adopted in this paper. The apparent contradiction, however, is easily explained. It is true that the disturbance due to a foreign body of any size is the same as would be caused by forces acting through the space it fills in a direction parallel to that in which the primary light vibrates; *but these forces must be supposed to act on the medium as it actually is—that is, with the variable density.* Only on the supposition of complete uniformity would it follow that no ray could be emitted parallel to the line in which the forces act. When, however, the sphere of disturbance is small compared with the wave-length, the want of uniformity is of little account, and cannot alter the law regulating the intensity of the vibration propagated in different directions.

Having disposed of the polarization, let us now consider how the intensity of the scattered light varies from one part of the spectrum to another, still supposing that all the particles are many times smaller than the wave-length even of violet light. The whole question admits of analytical treatment; but before entering upon that, it may be worth while to show how the principal result may be anticipated from a consideration of the *dimensions* of the quantities concerned.

The object is to compare the intensities of the incident and scattered rays; for these will clearly be proportional. The number (i) expressing the ratio of the two amplitudes is a function of the following quantities:—T, the volume of the disturbing particle; r, the distance of the point under consideration from it; λ, the wave-length; b, the velocity of propagation of light; D and D', the original and altered densities: of which the first three depend only on space, the fourth on space and time, while the fifth and sixth introduce the consideration of mass. Other elements of the problem there are none, except mere numbers and angles, which do not depend on the fundamental measurements of space, time, and mass. Since the ratio i, whose expression we seek, is of no dimensions in mass, it follows at once that D and D' only occur under the form $D : D'$, which is a simple number and may therefore be omitted. It remains to find how i varies with T, r, λ, b.

Now, of these quantities, b is the only one depending on time; and therefore, as i is of no dimensions in time, b cannot occur in its expression. We are left, then, with T, r, and λ; and from what we know of the dynamics

* *Camb. Phil. Trans.* vol. VII. 1837.

of the question, we may be sure that i varies directly as T and inversely as r, and must therefore be proportional to $T \div \lambda^2 r$, T being of three dimensions in space. In passing from one part of the spectrum to another λ is the only quantity which varies, and we have the important law:—

When light is scattered by particles which are very small compared with any of the wave-lengths, the ratio of the amplitudes of the vibrations of the scattered and incident light varies inversely as the square of the wave-length, and the intensity of the lights themselves as the inverse fourth power.

I will now investigate the mathematical expression for the disturbance propagated in any direction from a small particle which a beam of light strikes.

Let the vibration corresponding to the incident light be expressed by $A\cos(2\pi bt/\lambda)$. The acceleration is

$$-A\left(\frac{2\pi}{\lambda}b\right)^2\cos\frac{2\pi}{\lambda}bt;$$

so that the force which would have to be applied to the parts where the density is D', in order that the wave might pass on undisturbed, is, per unit of volume,

$$-(D'-D)A\left(\frac{2\pi b}{\lambda}\right)^2\cos\frac{2\pi}{\lambda}bt.$$

To obtain the total force which must be supposed to act over the space occupied by the particle, the factor T must be introduced. The opposite of this conceived to act at O (the position of the particle) gives the same disturbance in the medium as is actually caused by the presence of the particle. Suppose, now, that the ray is incident along OY, and that the direction of vibration makes an angle α with the axis of x, which is the line of the scattered ray under consideration—a supposition which involves no loss of generality, because of the symmetry which we have shown to exist round the line of action of the force. The question is now entirely reduced to the discovery of the disturbance produced in the æther by a given periodic force acting at a fixed point in it. In his valuable paper "On the Dynamical Theory of Diffraction"*, Professor Stokes has given a complete investigation of this problem; and I might assume the result at once. The method there used is, however, for this particular purpose very indirect, and accordingly I have thought it advisable to give a comparatively short cut to the result, which will be found at the end of the present paper. It is proved that if the total force acting at O in the manner supposed be $F\cos(2\pi bt/\lambda)$, the resulting disturbance in the ray propagated along OX is

$$\zeta = \frac{F\sin\alpha}{4\pi b^2 Dr}\cos\frac{2\pi}{\lambda}(bt-r).$$

* *Camb. Phil. Trans.* vol. IX. p. 1, 1849.

Substituting for F its value, we have

$$\zeta = A \frac{D' - D}{D} \frac{\pi T}{r\lambda^2} \sin\alpha \cos\frac{2\pi}{\lambda}(bt - r)^*,$$

an equation which includes all our previous results and more.

One reservation, however, must not be omitted. Since we have supposed the medium uniform throughout, whereas it really has a different density at the place where the force acts, our investigation does not absolutely correspond to the actual circumstances of the case. As before remarked, no error is on that account to be feared in the law determining the intensity of the vibration in different directions; but it is probable that the coefficient, so far as it depends on $D : D'$, may be changed†, and there may be a change in the phase comparable with $(2\pi/\lambda) \times$ the linear dimension of the particle, which is of importance when the scattered and primary waves have to be compounded.

So much for a single particle. In actual experiments, as, for instance, with Professor Tyndall's "clouds," we have to deal with an immense number of such particles; and the question now is to deduce what their effect must be from the results already obtained. Were the particles absolutely motionless, the partial waves sent out in any direction from them would have permanent relations as to phase, and the total disturbance would have to be found by compounding the *vibrations* due to all the particles. Such a supposition, however, would be very wide of the mark; for, in consequence of the extreme smallness of λ, the slightest motion of any particle will cause an alteration of phase passing through many periods in a less time than the eye could appreciate. Our particles are, then, to be treated as so many *unconnected* sources of light; and instead of adding the *vibrations*, we must take the *intensities* represented by their squares. Only in one direction is a different treatment necessary, namely along the course of the primary light. I mention this because it would not otherwise appear how the reduction in the intensity of the transmitted light is effected; but we do not require to follow the details of the process, because, when once we know the intensity of the light emitted laterally, the principle of energy will tell us what the primary wave has lost.

The intensity of the light scattered from a cloud is thus equal to

$$A^2 \frac{(D' - D)^2}{D^2} \sin^2\alpha \frac{\pi^2 . \Sigma T^2}{\lambda^4 r^2},$$

* [1898. The factor π was omitted in the original paper.]

† I find that no alteration of any kind is needed.—Jan. 20. [1899. See *Phil. Mag.* XLI. p. 452, 1871; This Collection Art. 9 below.]

where ΣT^2 is the sum of all the squares of T. If T^2 be understood to denote the mean square of T (*not* the square of the mean value of T), and m be the number of particles,

$$\Sigma T^2 = m \,.\, T^2.$$

If the primary light be unpolarized, the intensity in a direction making an angle β with its course becomes

$$A^2 \frac{(D' - D)^2}{D^2} (1 + \cos^2 \beta) \frac{m\pi^2 T^2}{\lambda^4 r^2}.$$

Backwards from the cloud the light is thus twice as bright as normally. To the light scattered nearly in the direction of the primary ray our expression does not apply.

Fig. 1.

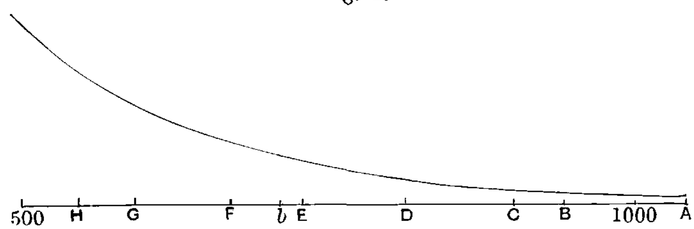


Fig. 1 shows the curve representing the intensity of the scattered light for each part of the spectrum, referred to the intensity in the primary light as a standard. The abscissa being proportional to λ, the base line represents the diffraction-spectrum with the principal fixed lines. Over the brighter portion of the spectrum from B to G the curve differs but little from a straight line, while the small curvature is turned downwards, indicating a deficiency in the green and yellow.

Before making out the theory, I had endeavoured to ascertain by observation the actual prismatic composition of the blue of the sky, and had obtained preliminary results. The experimental method (the description of which I must reserve for another opportunity) was fully adequate to the comparison of two given lights; but the difficulty was to find something to compare the blue light with. In the only complete set of observations that I have hitherto been able to make, the blue of the sky (apparently a very good one) taken from the neighbourhood of the zenith was compared with sunlight diffused through white paper. About thirty consistent comparisons were made, ranging over the spectrum from C to beyond F, and a curve drawn on the plan of fig. 1. I do not give the complete curve, because I hope before long to complete and confirm the observations; but the following numbers will give an idea of the results:—

C.	D.	b_3.	F.	
25	40	63	80	from fig. 1.
25	41	71	90	observed.

The upper line gives the theoretical intensities for the fixed lines C, D, b_3, F, while the lower gives the observed ratios between the lights (sky and diffused sunlight), the two sets of numbers being made to agree at C. Considering the difficulties and uncertainties of the case, the two curves agree very well; and it should be noticed that the sky compared with diffused light was even bluer than theory makes it, on the supposition that the diffused light through the paper may be taken as similar to that whose scattering illuminates the sky. It is possible that the paper was slightly yellow; or the cause may lie in the yellowness of sunlight as it reaches us compared with the colour it possesses in the upper regions of the atmosphere. It would be a mistake to lay any great stress on the observations in their present incomplete form; but at any rate they show that a colour more or less like that of the sky would result from taking the elements of white light in quantities proportional to λ^{-4}. I do not know how it may strike others; but individually I was not prepared for so great a difference as the observations show, the ratio for F being more than three times as great as for C.

There is one point in which our calculations do not exactly meet the case of the sky. In the experiments with precipitated clouds the total quantity of light scattered is quite insignificant compared with the incident beam; but it is by no means so clear that the same is the case with the sky. Each particle is thus struck, not only by the direct light of the sun, but also by that scattered from others. It does not seem that the chromatic effects would be much affected by this consideration; but it is worth notice that the conclusion as to complete polarization perpendicular to the incident ray would have to be modified. To see this, imagine, as before, the light (unpolarized) incident along OY upon a particle O; we have seen that the ray diffracted along OX contains no vibration parallel to OY. By the aid, however, of another particle P in the xy plane such a vibration may be communicated to it; for in the ray diffracted from P to O there is a component vibration in the xy plane perpendicular to PO, which, when again diffracted along OX, will give a component parallel to OY. This is perhaps the explanation of the incomplete polarization of sky-light at right angles to the solar beams; but it must be remembered that an insufficient fineness in some of the particles of foreign matter would have a like result.

By many physicists, from Newton downwards, the light of the sky has been supposed to be reflected from thin plates, and the colour to be the blue of the first order in Newton's scale. Such a view is fundamentally different from that adopted in this paper, though it might not at first seem so. In support of this assertion, it may be sufficient to notice that the two theories are at variance as to the law connecting the intensity with wave-length. By an argument from dimensions similar to that already used, it is easy to find how the intensity of the light reflected from a thin plate (thin, that is, compared with *any* of the wave-lengths) varies with λ. Instead of our former quantities, T, r, λ, we now have merely λ, and δ the thickness of the plate. Since the reflected vibration necessarily varies as δ, it must also be proportional to λ^{-1}, and so the *intensity* of the reflected light $\propto \lambda^{-2}$ instead of λ^{-4}. The ordinary analytical expression for the reflected light leads readily to the same conclusion (Airy's *Tracts*, p. 297). There can, I think, be no question that the composition of the light of the sky agrees more nearly with the latter than with the former law.

The principle of energy makes it clear that the light emitted laterally is not a new creation, but only diverted from the main stream. If I represent the intensity of the primary light after traversing a thickness x of the turbid medium, we have

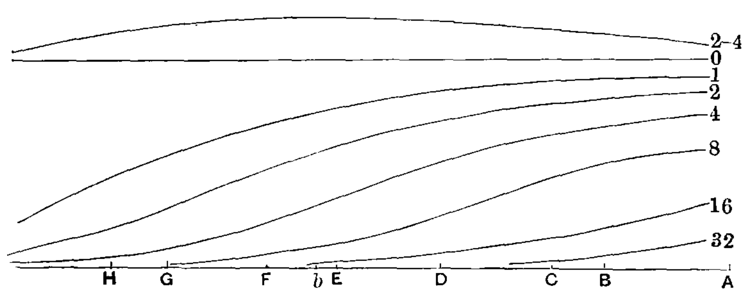
$$dI = -kI\lambda^{-4}\,dx,$$

where k is a constant independent of λ. On integration,

$$I = I_0 \epsilon^{-k\lambda^{-4}x},$$

if I_0 correspond to $x = 0$,—a law altogether similar to that of absorption, and showing how the light tends to become yellow and finally red as the thickness of the medium increases. Fig. 2 shows a series of curves representing the composition of the originally white light after passing through thicknesses in the ratio of 1, 2, 4, 8, 16, 32. The reader will observe how little of the violet light remains when the red is still in nearly its original force. I cannot but think that this rapid diversion of the rays of short

Fig. 2.



wave-length has a good deal to do with the absence of light of the highest refrangibility from the direct rays of the sun. For the line A at the extreme red and R near the upper limit of the photographic spectrum the wave-lengths are 7617 and 3108. The ratio of the fourth powers is about 36 : 1; so that, whatever the fraction representing the transmission of A may be, its 36th power will give the transmission of R. To take an instance, if ·9 of the ray A gets through, only ·018 of R would be able to penetrate. For the rays of still higher refrangibility, which Professor Stokes found abundant in the electric light but missing in the solar rays, the fraction would be smaller still; but I am not aware of any measurements of smaller wave-length on which to found a calculation.

We have hitherto supposed that the light scattered by the finely divided matter reaches the eye without modification, and we have taken no account of any change in the composition of the primary light before diffraction. If x be the total length of the path of the ray through the turbid medium, we may express the quality of the light in terms of x; for it makes no difference whether the lateral leakage takes place before diffraction or after. In fact

$$I \propto \lambda^{-4} \epsilon^{-k\lambda^{-4}x},$$

an expression which shows that I vanishes for very small as well as for very large values of λ, while for some definite value (say Λ) it rises to a maximum (I_0). Expressing I in terms of I_0 and Λ, we have

$$\frac{I}{I_0} = \frac{\Lambda^4}{\lambda^4} \epsilon^{1-\Lambda^2/\lambda^2};$$

from which we may fall back on our original law by supposing Λ indefinitely small, and replacing $\Lambda^4 I_0$ by a finite constant. An approximate idea of the character of these lights may be obtained by subtracting the successive curves of Fig. 2. Thus the difference of the curves marked 2 and 4 represents a light having its maximum brightness (of course relatively to the primary light) in the blue-green portion of the spectrum. I find by calculation that, if the maximum intensity be at b and be taken as unity, the intensities at G and C are given by the numbers ·713, ·710 respectively. The colour would be greenish; but whether the green of the sky is to be accounted for in this way I am not able to say. Some, I believe, consider it to be entirely a contrast effect.

Appendix.

Within a space T, small in all its dimensions against λ, and situated at the origin of coordinates, let a force parallel to OZ, and, so far as it depends upon the time, expressed by a simple circular function, act on the medium. If ξ, η, ζ denote the displacements parallel to the axes at the point x, y, z, and

$$\delta = \frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz},$$

$$\left.\begin{aligned}\frac{d^2\xi}{dt^2} &= b^2\nabla^2\xi + (a^2 - b^2)\frac{d\delta}{dx},\\ \frac{d^2\eta}{dt^2} &= b^2\nabla^2\eta + (a^2 - b^2)\frac{d\delta}{dy},\\ \frac{d^2\zeta}{dt^2} &= b^2\nabla^2\zeta + (a^2 - b^2)\frac{d\delta}{dz} + Z,\end{aligned}\right\}\quad\text{..............(A)*}$$

where ∇^2 stands for $d^2/dx^2 + d^2/dy^2 + d^2/dz^2$; a^2 and b^2 are constants depending on the nature of the medium supposed to be isotropic. For the luminiferous æther, Green has shown that a is to be regarded as indefinitely great †.

To represent the periodic force, write for Z, $Z\epsilon^{int}$. Similar transformations will then apply to ξ, η, ζ, and δ; so that on substitution in (A) and dividing out the common factor ϵ^{int}, there results

$$\left.\begin{aligned}(b^2\nabla^2 + n^2)\,\xi + (a^2 - b^2)\,d\delta/dx &= 0,\\ (b^2\nabla^2 + n^2)\,\eta + (a^2 - b^2)\,d\delta/dy &= 0,\\ (b^2\nabla^2 + n^2)\,\zeta + (a^2 - b^2)\,d\delta/dz &= -Z.\end{aligned}\right\}\quad\text{............(B)}$$

Writing

$$\frac{d\xi}{dy} - \frac{d\eta}{dx} = \varpi_3 \text{ \&c.,}$$

we obtain from (B) by differentiation and subtraction,

$$\left.\begin{aligned}(b^2\nabla^2 + n^2)\,\varpi_3 &= 0,\\ (b^2\nabla^2 + n^2)\,\varpi_1 &= dZ/dy,\\ (b^2\nabla^2 + n^2)\,\varpi_2 &= -dZ/dx.\end{aligned}\right\}\quad\text{..................(C)}$$

ϖ_1, ϖ_2, ϖ_3 are the *rotations* of the elements of the medium round axes parallel to those of coordinates.

The disturbance which we are investigating is that caused and maintained by the force Z acting within the space T. Accordingly ‡

$$\varpi_3 = 0,$$

$$\varpi_1 = -\frac{1}{4\pi b^2}\iiint \frac{dZ}{dy}\frac{\epsilon^{\pm ikr}}{r}\,dx\,dy\,dz,$$

r being the distance between the element $dx\,dy\,dz$ and the point where ϖ_1 is estimated, and

$$k = 2\pi/\lambda = n/b. \quad\text{..........................(D)}$$

* Thomson and Tait's *Natural Philosophy*, § 698.

† *Camb. Phil. Trans.* vol. VII.

‡ Helmholtz, *Crelle's Journal*, 1860.

Since $\epsilon^{\pm ikr}$ will be finally multiplied by ϵ^{int}, and the disturbance which we are dealing with is propagated *outwards* from T, it is evident that the *lower* sign is to be employed. Now

$$\int \frac{\epsilon^{-ikr}}{r} \frac{dZ}{dy} dy = \left[Z \frac{\epsilon^{-ikr}}{r} \right] - \int Z \frac{d}{dy} \left(\frac{\epsilon^{-ikr}}{r} \right) dy,$$

of which the term within brackets vanishes, because the value of Z is only finite within the space T. Thus

$$\varpi_1 = \frac{1}{4\pi b^2} \iiint Z \frac{d}{dy} \left(\frac{\epsilon^{-ikr}}{r} \right) dx\, dy\, dz.$$

The factor $\frac{d}{dy}\left(\frac{\epsilon^{-ikr}}{r}\right)$ within the space T is sensibly constant, so that, if Z stand for the mean value of Z over the volume T,

$$\varpi_1 = \frac{TZ}{4\pi b^2} \frac{d}{dy} \left(\frac{\epsilon^{-ikr}}{r} \right), \qquad \varpi_2 = - \frac{TZ}{4\pi b^2} \frac{d}{dx} \left(\frac{\epsilon^{-ikr}}{r} \right),$$

$$x\varpi_1 + y\varpi_2 \propto \left(x \frac{d}{dy} - y \frac{d}{dx} \right) . \frac{\epsilon^{-ikr}}{r} = 0.$$

And if $R = \sqrt{x^2 + y^2}$,

$$\varpi = \frac{x\varpi_2 - y\varpi_1}{R} = - \frac{TZ}{4\pi b^2 R} \left(x \frac{d}{dx} + y \frac{d}{dy} \right) . \frac{\epsilon^{-ikr}}{r}$$

$$= - \frac{TZ}{4\pi b^2} \frac{d}{dR} . \frac{\epsilon^{-ikr}}{r} = \frac{TZ \sin \alpha}{4\pi b^2} \frac{d}{dr} . \frac{\epsilon^{-ikr}}{r},$$

where α denotes the angle between r and z.

The resultant rotation at any point is thus about an axis perpendicular to the plane passing through the point and the axis of Z; and its magnitude is given by ϖ. In differentiating $(r^{-1} e^{-ikr})$ with respect to r, we may neglect the term divided by r^2 as altogether insensible, kr being an exceedingly great quantity at any ordinary distance from the origin of disturbance. Thus

$$\varpi = \frac{- ik . TZ \sin \alpha}{4\pi b^2} . \frac{\epsilon^{-ikr}}{r}, \quad \text{..................(E)}$$

which completely determines the rotation at any point. For a given disturbance it is seen to be everywhere about an axis perpendicular to r and the direction of the force, and in magnitude dependent only on the angle between these two directions (α) and on the distance (r).

The intensity of the light, however, is more usually expressed in terms of the actual displacement in the plane of the wave. In order to find the connexion between the two quantities, it will be more convenient to suppose the scattered ray parallel to x, and that the force F (for Z is no longer appropriate) acts in the plane of zx at an angle α with Ox. ϖ becomes identical with ϖ_2; that is, with $d\zeta/dx$; for ξ as well as η is zero; so that

$$\zeta = \int \varpi \, dr = \frac{TF \sin \alpha}{4\pi b^2} \cdot \frac{\epsilon^{-ikr}}{r}.$$

Restoring the factor ϵ^{int}, we have

$$\zeta = \frac{TF \sin \alpha}{4\pi b^2} \cdot \frac{\epsilon^{i(nt-kr)}}{r};$$

or throwing away the imaginary part,

$$\zeta = \frac{TF \sin \alpha}{4\pi b^2 r} \cdot \cos \frac{2\pi}{\lambda}(bt - r). \quad \text{.................(F)}$$

This corresponds to a total accelerating force equal to $FT \cos(2\pi bt/\lambda)$; a result which agrees with that of Professor Stokes's more complete investigation, with the exception of a slight difference of notation.

In the February Number of the *Philosophical Magazine* I have propounded a theory of the scattering of light by particles which are small in *all* their dimensions compared with the wave-length of light, and have applied the results to explain the phenomena presented by the sky. Another theory has been given by Clausius, who attributes the light of the sky to reflection from water-bubbles, and has developed his views at length in a series of papers in Poggendorff's *Annalen* and Crelle's *Journal**.

Starting from the ordinary laws of reflection and refraction, he has no difficulty in showing that, were the atmosphere charged with globes of water in sufficient quantity to send us the light which we actually receive, a star instead of appearing as a point would be dilated into a disk of considerable magnitude. But the requirements of the case are satisfied if we suppose the spheres hollow, like bubbles; for then, on account of the parallelism of the surfaces, but little effect is produced by refraction on a wave of light. At the same time, if the film be sufficiently thin, the light reflected from it will be the blue of the first order, and so the colour of the sky is apparently accounted for.

Apart from the difficulty of seeing how such bubbles could be formed, there is a formidable objection to this theory, mentioned by Brücke (Pogg. *Ann.* vol. LXXXVIII. p. 363)—that the blue of the sky is a much better colour than the blue of the first order. That it is so appears clearly from the

* Pogg. *Ann.* vols. LXXII. LXXVI. LXXXVIII. *Crelle*, vols. XXXIV. XXXVI.

measurements quoted in the February Number, and from the theoretical composition of the blue of the first order*. Nor can we escape from this difficulty by supposing, with Brücke, that the greater part of the light from the sky has been reflected more than once.

Brücke also brings forward an experiment of great importance when he shows that mastic precipitated from an alcoholic solution scatters light of a blue tint. He remarks that it is impossible to suppose that the particles of mastic are in the form of bubbles.

In his last utterance on this subject†, Clausius replies to the objections urged by Brücke and others against his theory, and shows that, if the illumination of the sky is due to thin plates at all, those thin plates *must* be in the form of bubbles. While admitting that if the particles are very small the ordinary laws of reflection and refraction no longer apply‡, and that therefore this case is not excluded by his argument, he still holds to his original view as to the nature of the reflecting matter in the sky, considering that the polarization of the light indicates that it has undergone *regular* reflection. His concluding paragraph so well sums up the case that I cannot do better than quote it. "Das Resultat der vorstehenden Betrachtungen kann ich hiernach kurz so zusammenfassen. Soweit man die gewöhnlichen Brechungs- und Reflexionsgesetze als gültig anerkennt, glaube ich auch meine früheren Schlüsse festhalten zu müssen, nämlich, dass in der Atmosphäre Dampfbläschen vorhanden seyen, und dass sie die Hauptursache der in ihr stattfindenden Lichtreflexion und ihrer Farben bilden. Nimmt man aber an, die in der Atmosphäre wirksamen Körperchen seyen so klein, dass jene Gesetze auf sie keine Anwendung mehr finden, dann sind auch diese Schlüsse ungültig. Auf diesen Fall ist aber auch die Theorie der Farben dünner Blättchen nicht mehr anwendbar, und er bedarf vielmehr einer neuen Entwickelung, bei welcher noch besonders berücksichtigt werden muss, in wiefern diese Annahme mit der Polarisation des vom Himmel kommenden Lichtes und mit der angenähert bekannten Grösse der in den Wolken vorhandenen Wassertheilchen vereinbar ist."

* I find that I omitted to explain why it is that the light dispersed from small particles is of so much richer a hue than that reflected from very thin films. In the latter case the reflected wave may be regarded as the sum of the disturbances originating in the elementary parts of the film, and these elementary parts may be assimilated to the small particles of the former supposition. The integration is best effected by dividing the surface into the *zones of Huyghens*; and it is proved in works on physical optics that the total effect is just half of that due to the first zone. Now the zones of Huyghens vary as the wave-length; and thus it appears that in the integration the long waves gain an advantage which diminishes the original preponderance of their quicker-timed rivals.

† Pogg. *Ann.* vol. LXXXVIII. p. 543.

‡ In many departments of science a tendency may be observed to extend the field of familiar laws beyond their proper limits. Thus the properties of gross matter are often assumed to hold equally good for molecules. An example more analogous to that which suggests this remark is to be found in the common explanation of the mode of action of the speaking-trumpet.

Clausius does not seem to have followed up the line of research here indicated. My investigation (written, it so happens, before seeing Clausius's papers) shows in the clearest manner the connexion between the smallness of the particles and the polarization of the light scattered from them. Indeed I must remark that in this respect there is an advantage over the theory of thin plates, according to which the direction of complete polarization would be about 76° from the sun. It would be a singular coincidence if the action of secondary causes were to augment this angle to 90°—its observed magnitude. It seems, therefore, not too much to say that, if the illumination of the sky were due to suspended water-bubbles, neither its colour nor its polarization would agree with what is actually observed.

In his celebrated paper on Fluorescence*, Professor Stokes makes the following significant remark:—"Now this result appears to me to have no remote bearing on the question of the direction of the vibrations in polarized light. So long as the suspended particles are large compared with the waves of light, reflection takes place as it would from a portion of the surface of a large solid immersed in the fluid, and no conclusion can be drawn either way. But if the diameter of the particles be small compared with the length of a wave of light, it seems plain that the vibrations in a reflected ray cannot be perpendicular to the vibrations in the incident ray." This is the only passage that I have met with in which the theory of the reflection of light from very small particles is touched upon.

If it be assumed, as in the theories of Green and Cauchy of reflection at plane surfaces, that the effect of dense matter is merely to load the ether, it follows rigorously that the direction of vibration cannot be turned through a right angle when light is scattered from small particles. But all we know in the first instance is that the velocity of propagation of luminous waves is less in ordinary transparent matter than in vacuum; and this may be accounted for as well by a diminished rigidity as by an increased density. In the first case a scattered ray *might* be composed of vibrations perpendicular to those of the incident beam; so that the matter is not quite so clear as it would seem from the argument of Professor Stokes. I believe, however, that good reasons may be given for rejecting the view that the difference between media of varying refrangibility is one of rigidity. The point is an important one, and I propose to recur to it later.

The experiments of Professor Tyndall† with precipitated clouds exhibit more clearly than had been done by Brücke the relation between the size of the particles and the nature of the dispersed light. The observation that the polarization is complete perpendicular to the track of the incident light is in itself sufficient to disprove the theory of bubbles. As the particles increase

* *Phil. Trans.* 1852, p. 526.

† *Phil. Mag.* vol. XXXVII. p. 385. *Phil. Trans.* 1870.

in magnitude, the azure and polarization are gradually lost. During the transition a different and more complicated set of phenomena present themselves, which will furnish a test for the theory when it is extended so as to include the consideration of particles which are no longer very small in comparison with the waves of light.

All who have written on this subject seem to have taken for granted that the foreign matter in the atmosphere is water or ice. Even Tyndall, who expressly says that any particles, if small enough, will do, still believes in the presence of water-particles. But this view is encumbered with considerable difficulty; for even if, in virtue of its transparency to radiant heat, the air in the higher regions of our atmosphere is at a very low temperature, it would still be capable of absorbing the very small quantity of water which is sufficient to explain the blue of the sky. At any rate it is difficult to imagine particles of water smaller than the wave-length endowed with any stability. These difficulties might perhaps be got over if there were any strong argument in favour of the water-particles; but of the existence of such I am not aware. Every one knows that a blue haze evidently akin to the azure of the sky obliterates the details and modifies the colour of a distant mountain; and this, when it occurs on a hot day, cannot possibly be attributed to aqueous particles. On the face of it, there is no reason for supposing that near the earth's surface the foreign matter is of one kind and at a great altitude another. If it were at all probable that the particles are all of one kind, it seems to me that a strong case might be made out for common salt. Be this as it may, the optical phenomena can give us no clue.

The apparatus by means of which the comparison was made between sky light and that of the sun diffused through white paper, was originally arranged for measurements of the absolute absorption of coloured fluids for the various rays of the spectrum, and had been applied rather extensively in experiments having that object. In the shutter of a darkened room were placed two slits in the same vertical line, each about three inches long, and a foot apart. At the other end of the room was an arrangement of prisms and lenses for producing a pure spectrum on a screen in the ordinary way. At first only one prism was used; but I soon introduced another, and the number might probably be further increased with advantage. It is even more important to have a great dispersion in these experiments than in the ordinary spectroscope. Two spectra would thus be thrown on the screen one over the other, but by means of a very obtuse-angled prism situated in front of the dispersion-prisms they are brought together so as exactly to overlap. The double spectrum thus formed passes through a horizontal slit in the screen placed so as to receive it. Close behind is an opaque card carrying a small vertical slit, which can be slid along so as to allow any desired part of the spectrum to pass through. At the beginning and end of a set of experiments the card is removed, and the principal fixed lines are observed through an eyepiece and referred to a scale situated just over the horizontal aperture.

When the experimenter looks through the eye-slit in the direction of the lens, he sees the two parts of the obtuse prism illuminated with light, in each case homogeneous, and, if the adjustments are properly made, belonging to the same part of the spectrum. By varying the breadths of the original slits, the two parts of the field may be made equally bright; and when the match is attained, the breadths are inversely proportional to the richness of the lights behind them in the homogeneous ray under consideration. But if the object be to make a complete comparison between two lights, it is often more convenient to leave the widths of the slits arbitrary, and then, by sliding the card, to seek that part of the spectrum which allows a match. It was in this way that the observations on the light of the sky were made. To give an idea of the degree of accuracy to which the comparisons may be made, I may mention that in my experiments on absorption, the means of six observations were usually correct to about one in 50 or 60. In the less-luminous parts of the spectrum the error might be somewhat greater.

The difficulty, however, of getting a satisfactory result with the blue of the sky does not lie in the inaccuracy of the measurements, but in the arbitrary character of the light with which it is compared. In order to test the theory in a strict manner, the second light ought to be similar in composition to that which lights up the sky. Now the sky is lit not only by the direct rays of the sun, but also by itself and by the bright surface of the earth. It is evident, therefore, that the requirements of the case are very imperfectly met by taking as the second light that of the sun as received by us, even if the translucent material through which we diffuse it effects no change in the quality. A nearer approximation to what we want would probably be found in the diffused light of a thoroughly cloudy day. But here we meet with an experimental difficulty; for the method described is only available to compare two lights both given at once. A suitable artificial light might no doubt be used as a middle term to be afterwards eliminated; but a candle or a lamp would hardly be available, on account of the yellowness of their light. On the other hand, the bluer radiation from burning magnesium would probably be inconvenient, and difficult to keep constant in quality from day to day. I am, however, in hopes that, by a method founded on a different principle, I may be able to compare the blue of one day's clear sky with the white light from the clouds on another.