10 Jul 2020

YRM71, YS591 – Mathematical Neuroscience
Project 2: Associative memory

A short oral presentation is required in addition to turn in a short writeup that describes the problem you investigated, why it is interesting, and results.

Assignment

1. Perform computer simulation of associative memory in a neural network of $n = 1000$ neurons, $m = 80$ and 200 memory items. Reproduce the Figure 1 and Figure 2 shown in Amari & Maginu (1988).

   If you want to perform computer simulation of a huge number of neurons (e.g. $n = 10000$), you don't need to use two dimensional array `w[i][j]` since

$$
\begin{aligned}
x_i(t+1) &= \operatorname{sgn}\left(\sum_{j=1}^{n} w_{ij} x_j(t)\right) \\
&= \operatorname{sgn}\left(\sum_{j=1}^{n} \frac{1}{n} \sum_{\alpha=1}^{m} x_i^\alpha x_j^\alpha x_j(t)\right) \\
&= \operatorname{sgn}\left(\sum_{j=1}^{n} \sum_{\alpha=1}^{m} x_i^\alpha x_j^\alpha x_j(t)\right).
\end{aligned}
$$

Computer Experiment Procedure

1. Generate a set of random memories, $\boldsymbol{x}^\alpha, \alpha = 1, 2, \cdots, m$ (e.g. $m = 80, 200$).
2. Set weight connections $w_{ij}, i, j = 1, 2, \cdots, n$ (e.g. $n = 1000$).
3. Set an initial state $\boldsymbol{x}(0)$ which resembles somewhat state $\boldsymbol{x}^1$ by taking first a elements flipped $(\overbrace{x_1^1, \cdots, x_a^1}^{a}, x_{a+1}^1, \cdots, x_n^1)$. (e.g. $a = 0, 25, 50, 75, \cdots$)

   Remark: When $a = \dfrac{n}{2}$, direction cosine $s(\boldsymbol{x}(0), \boldsymbol{x}^1)$ is 0.
4. Updates the network state, $\boldsymbol{x}(t+1) = T_W \boldsymbol{x}(t)$, 20 times, for example.
5. Draw a graph with t in horizontal axis and the direction cosine $s(\boldsymbol{x}(t), \boldsymbol{x}^1)$ in vertical axis.
6. Repeat these from 3. with different a.

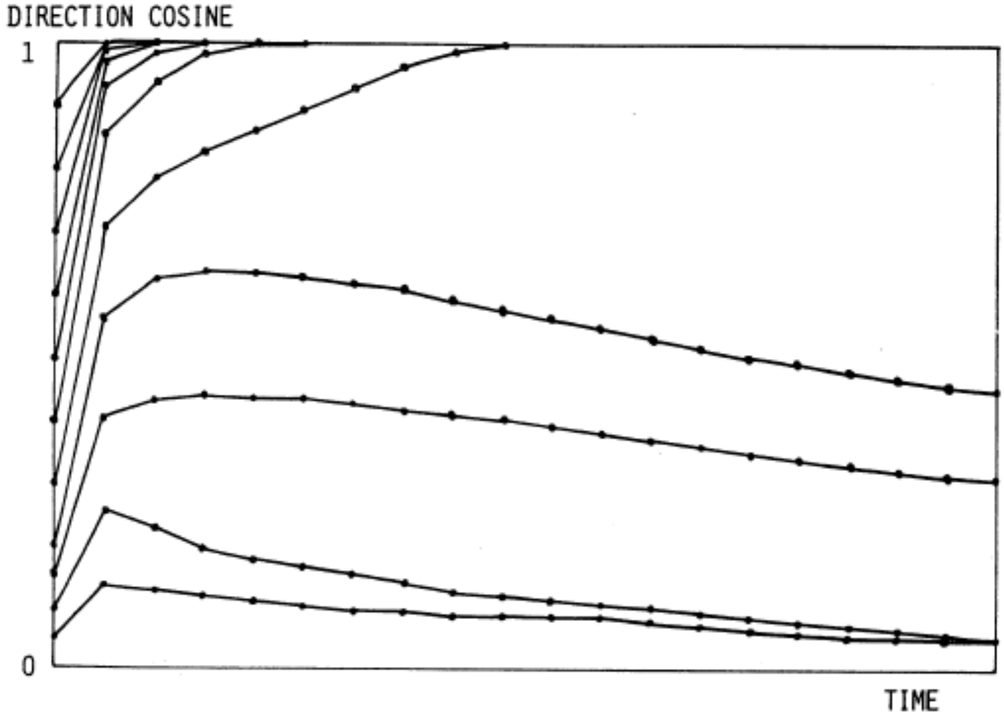

FIGURE 1. Dynamic behaviors of recalling processes; simulation with n = 5000, m = 400, r = 0.08.

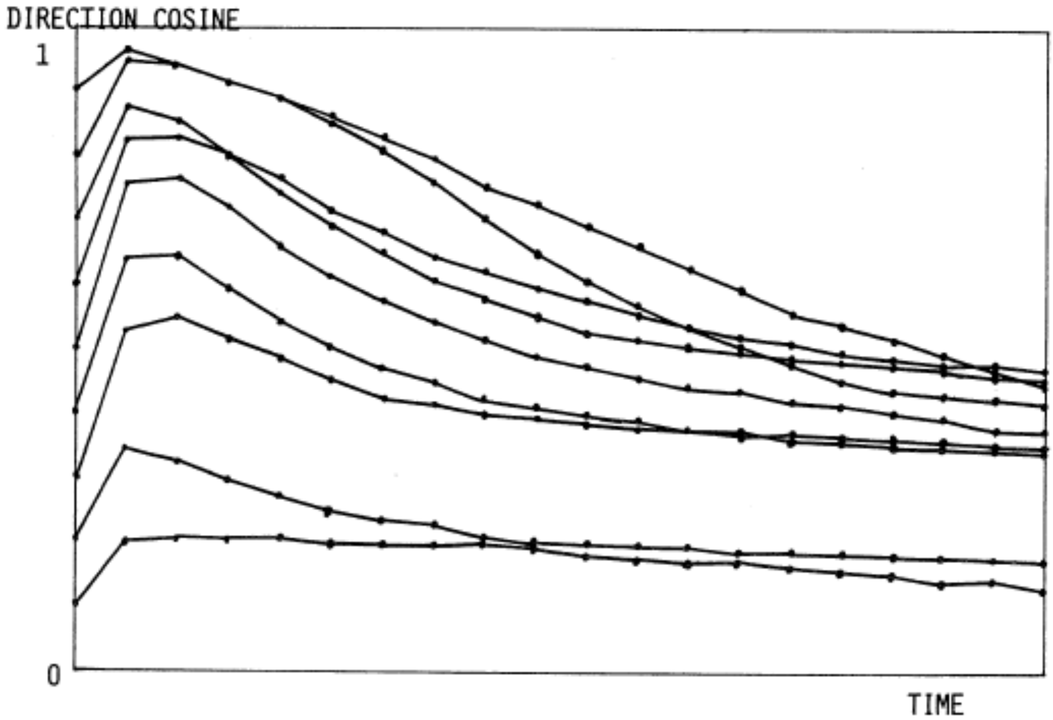

FIGURE 2. Dynamic behaviors of recalling processes; simulation with n = 3000, m = 600, r = 0.2.

Amari, S. & Maginu, K. Statistical neurodynamics of associative memory. *Neural Networks* **1**, 63–73 (1988).

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Associative Memory

Key words: parallel search, distributed/superimposed storage, content-addressable memory

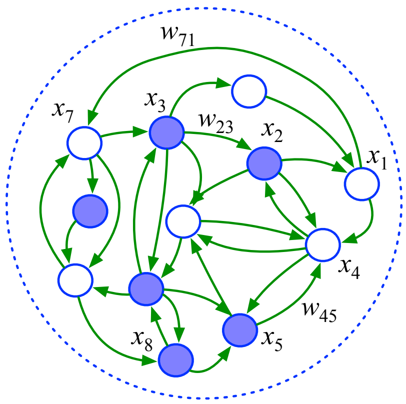
1. **Set up**. Network state: $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$. $x_i \in \{-1, 1\},\ i = 1, \cdots, n$

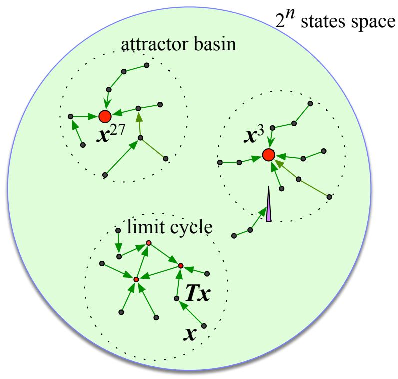
   A set of memories: $\{\boldsymbol{x}^1, \boldsymbol{x}^2, \cdots, \boldsymbol{x}^m\}$, x_i^α takes -1,1 with probability $\frac{1}{2}$ independently.

   Connection weights: $W = \{w_{ij}\}$, $w_{ij} = \frac{1}{n}\sum_{\alpha=1}^{m} x_i^\alpha x_j^\alpha$, $w_{ii} = 0$. $(w_{ij} = w_{ji})$

   Activity Dynamics: $\boldsymbol{x}(t+1) = T_W \boldsymbol{x}(t)$

$$x_i(t+1) = \operatorname{sgn}\left(\sum_{j=1}^{n} w_{ij} x_j(t)\right),\ \operatorname{sgn}(u) = \begin{cases} 1, & u > 0 \\ -1, & u \leqq 0 \end{cases}$$





2. **Hebb learning.** $\Delta w_{ij} = c x_i x_j$ ($c > 0$ is constant). Learning is *local.*

3. **Equilibrium states: $\boldsymbol{x} = T_W \boldsymbol{x}$ and Stable states.** Starting from any arbitrary initial state, the system reaches a stable state and ceases to evolve (or converge to cyclic states with period 2).

4. **Associative memory.** If started from an initial state which resembles somewhat state $\boldsymbol{x}^\beta$ and which resembles other $\boldsymbol{x}^\alpha (\alpha \neq \beta)$ very little, the state will evolve to the state $\boldsymbol{x}^\beta$. The system correctly reconstructs an entire memory from any initial partial information, as long as the partial information was sufficient to identify a single memory.

5. **Direction Cosine** (Similarity between two states): $-1 \leqq s \leqq 1$

$$s(\boldsymbol{x}^\alpha, \boldsymbol{x}^\beta) \quad = \quad \cos\theta = \frac{\boldsymbol{x}^\alpha \cdot \boldsymbol{x}^\beta}{||\boldsymbol{x}^\alpha|| ||\boldsymbol{x}^\beta||} = \frac{1}{n}\sum_{i=1}^{n} x_i^\alpha x_i^\beta$$

6. **Spurious memory.** The storage of a set of assigned memories in $\{w_{ij}\}$ also produces a set of spurious stable states which were not inserted as memory states.

7. **Memory capacity:** $m \approx 0.14n$