



# 教学要求

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◇ 熟练运用基本积分公式

♣ 直接积分法



# Outline of §5.2

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1. 基本积分表

2. 直接积分法

# We are here now...

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## 1. 基本积分表

## 2. 直接积分法

# 基本积分表

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基本初等函数的导数公式  $\longrightarrow$  基本的积分公式

# 基本积分表

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基本初等函数的导数公式  $\longrightarrow$  基本的积分公式

$$\diamond \int 0 dx = C$$

$$\diamond \int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1} + C, (\alpha \neq -1)$$

$$\diamond \int \frac{1}{x} dx = \ln |x| + C$$

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$$\spadesuit \int \frac{1}{1+x^2} dx = \arctan x + C$$

# We are here now...

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1. 基本积分表

2. 直接积分法

# 求不定积分 I ——“直接” 利用基本积分公式

例子 求不定积分  $\int \left( \frac{2}{t} - 3 \cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t \right) dt$

解  $\csc t = \frac{1}{\sin t}$

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## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

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$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln |x| + C, & \alpha = -1 \end{cases}$$

## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

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例子 求不定积分  $\int \sqrt{x^5} dx$ ,  $\int \frac{1}{\sqrt{x^3}} dx$

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解

$$\int \sqrt{x^5} dx$$

$$\int \frac{1}{\sqrt{x^3}} dx$$

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例子 求不定积分  $\int \sqrt{x^5} dx$ ,  $\int \frac{1}{\sqrt{x^3}} dx$

解

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx$$

$$\int \frac{1}{\sqrt{x^3}} dx$$



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$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = x^{\frac{5}{2}+1}$$

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$$\int \frac{1}{\sqrt{x^3}} dx$$

## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln |x| + C, & \alpha = -1 \end{cases}$$

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$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx$$

## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

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$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

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## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分  $\int \sqrt{x^5} dx$ ,  $\int \frac{1}{\sqrt{x^3}} dx$

解

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\begin{aligned} \int \frac{1}{\sqrt{x^3}} dx &= \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx \\ &= x^{-\frac{3}{2} + 1} \end{aligned}$$



## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

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$$\begin{aligned} \int \frac{1}{\sqrt{x^3}} dx &= \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx \\ &= \frac{1}{-\frac{3}{2} + 1} x^{-\frac{3}{2} + 1} \end{aligned}$$

## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln |x| + C, & \alpha = -1 \end{cases}$$

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## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

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解

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## 求不定积分 I ——“直接” 利用基本积分公式 (Cont.)

$$\text{熟练计算 } \int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln |x| + C, & \alpha = -1 \end{cases}$$

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解

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$$= \frac{1}{-\frac{3}{2} + 1} x^{-\frac{3}{2} + 1} + C = -2x^{-1/2} + C = -\frac{2}{\sqrt{x}} + C$$

## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数, 化不定积分为  $\int x^\alpha dx$  形式

## 求不定积分 II : 化为 $\int x^\alpha dx$

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提示 整理被积函数, 化不定积分为  $\int x^\alpha dx$  形式

解 
$$\int \frac{x}{\sqrt{x^5}} dx =$$

## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数, 化不定积分为  $\int x^\alpha dx$  形式

解 
$$\int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx =$$



## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数, 化不定积分为  $\int x^\alpha dx$  形式

解 
$$\int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx =$$

## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数, 化不定积分为  $\int x^\alpha dx$  形式

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## 求不定积分 II : 化为 $\int x^\alpha dx$

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## 求不定积分 II : 化为 $\int x^\alpha dx$

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## 求不定积分 II : 化为 $\int x^\alpha dx$

例子 求不定积分  $\int \frac{x}{\sqrt{x^5}} dx$ ,  $\int \frac{(3-x\sqrt{x})^2}{x} dx$ ,  $\int \frac{(1-\sqrt{x})^2}{x} dx$

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=

## 求不定积分 II : 化为 $\int x^\alpha dx$

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## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式



## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

## 求不定积分 III: 形如 $\int \frac{Ax+B}{Cx^2+Dx+E} dx$

**提示** 将分式 “ $\frac{Ax+B}{Cx^2+Dx+E}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解** 
$$\int \frac{x^2}{1+x^2} dx =$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

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**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解** 
$$\int \frac{x^2}{1+x^2} dx = \int \frac{1+x^2-1}{1+x^2} dx =$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

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**解** 
$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx =$$

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**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解** 
$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解** 
$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx$ ,  $\int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx =$$



## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int \frac{1+x^2+2}{1+x^2} dx =$$

## 求不定积分 III: 形如 $\int \frac{Ax+B}{x^2+Cx+D} dx$

**提示** 将分式 “ $\frac{Ax+B}{x^2+Cx+D}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx =$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x +$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

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**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
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$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx =$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx =$$



## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx$ ,  $\int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx$ ,  $\int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x}$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x$$

## 求不定积分 III: 形如 $\int \frac{f(x)}{g(x)} dx$

**提示** 将分式 “ $\frac{f(x)}{g(x)}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
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**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$

## 求不定积分 III: 形如 $\int \frac{Ax^2+Bx+C}{x^2+px+q} dx$

**提示** 将分式 “ $\frac{Ax^2+Bx+C}{x^2+px+q}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx =$$

## 求不定积分 III: 形如 $\int \frac{\text{分子}}{\text{分母}} dx$

**提示** 将分式 “ $\frac{\text{分子}}{\text{分母}}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx = \int \frac{(e^x+1)(e^x-1)}{e^x+1} dx =$$

## 求不定积分 III: 形如 $\int \frac{Ax^2+Bx+C}{x^2+px+q} dx$

**提示** 将分式 “ $\frac{Ax^2+Bx+C}{x^2+px+q}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx = \int (e^x - 1) dx =$$

## 求不定积分 III: 形如 $\int \frac{Ax^2+Bx+C}{x^2+Dx+E} dx$

**提示** 将分式 “ $\frac{Ax^2+Bx+C}{x^2+Dx+E}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx$ ,  $\int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx$ ,  $\int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx = \int (e^x - 1) dx = e^x - x + C$$

## 求不定积分 III: 形如 $\int \frac{\tilde{A}x^2 + Bx + C}{\tilde{D}x^2 + Ex + F} dx$

**提示** 将分式 “ $\frac{\tilde{A}x^2 + Bx + C}{\tilde{D}x^2 + Ex + F}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx = \int e^x - 1 dx = e^x - x$$



## 求不定积分 III: 形如 $\int \frac{\dot{A} \dot{B} \dot{C} \dot{D}}{E \cdot \dot{F} \cdot \tilde{G} \dot{H} \dot{I} \dot{O}} dx$

**提示** 将分式 “ $\frac{\dot{A} \dot{B} \dot{C} \dot{D}}{E \cdot \dot{F} \cdot \tilde{G} \dot{H} \dot{I} \dot{O}}$ ” 拆成两个（或多个）简单的式子/分式

**例子** 求不定积分  $\int \frac{x^2}{1+x^2} dx, \quad \int \frac{3+x^2}{1+x^2} dx$

**解**

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$
$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

**例子** 求不定积分  $\int \frac{1}{x^2(1+x^2)} dx, \quad \int \frac{e^{2x}-1}{e^x+1} dx$

**解**

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$$
$$\int \frac{e^{2x}-1}{e^x+1} dx = \int e^x - 1 dx = e^x - x + C$$

## 求不定积分 IV: 化为 $\int a^x dx$

---

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

---

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

## 求不定积分 IV: 化为 $\int a^x dx$

---

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

---

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx =$$

## 求不定积分 IV: 化为 $\int a^x dx$

---

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

---

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx = \int (3e)^x dx =$$

## 求不定积分 IV: 化为 $\int a^x dx$

---

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

---

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C$$

## 求不定积分 IV: 化为 $\int a^x dx$

---

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

---

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C$$

## 求不定积分 IV: 化为 $\int a^x dx$

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C$$
$$\int 5^{-x} e^x dx =$$

## 求不定积分 IV: 化为 $\int a^x dx$

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\begin{aligned}\int 3^x e^x dx &= \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C \\ \int 5^{-x} e^x dx &= \int \left(\frac{1}{5}\right)^x e^x dx =\end{aligned}$$



## 求不定积分 IV: 化为 $\int a^x dx$

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\begin{aligned}\int 3^x e^x dx &= \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C \\ \int 5^{-x} e^x dx &= \int \left(\frac{1}{5}\right)^x e^x dx = \int \left(\frac{1}{5}e\right)^x dx\end{aligned}$$

## 求不定积分 IV: 化为 $\int a^x dx$

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\begin{aligned}\int 3^x e^x dx &= \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C \\ \int 5^{-x} e^x dx &= \int \left(\frac{1}{5}\right)^x e^x dx = \int \left(\frac{1}{5}e\right)^x dx \\ &= \frac{1}{\ln\left(\frac{1}{5}e\right)} \left(\frac{1}{5}e\right)^x + C\end{aligned}$$

## 求不定积分 IV: 化为 $\int a^x dx$

掌握  $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分  $\int 3^x e^x dx, \int 5^{-x} e^x dx$

解

$$\begin{aligned}\int 3^x e^x dx &= \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C \\ \int 5^{-x} e^x dx &= \int \left(\frac{1}{5}\right)^x e^x dx = \int \left(\frac{1}{5}e\right)^x dx \\ &= \frac{1}{\ln\left(\frac{1}{5}e\right)} \left(\frac{1}{5}e\right)^x + C = \frac{e^x}{(1 - \ln 5)5^x} + C\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$

---

求不定积分

$$(1) \int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2) \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3) \int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$

## 求不定积分 V: $\int$ "triangle functions" $dx$

求不定积分

$$(1) \int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2) \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3) \int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$

提示 利用“三角恒等式”

平方关系  $\sin^2 x + \cos^2 x = 1$

倍角公式  $\cos 2x = \cos^2 x - \sin^2 x$ ;  $\sin 2x = 2 \sin x \cos x$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

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利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$



## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx =$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

---

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx =$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx =$$



## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx =$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx =$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx =$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x - x$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \tan^2 x dx$ ,  $\int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \quad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x - x + C.$$

---

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$



## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{1}{\cos^2 x \sin^2 x} dx$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{1}{\cos^2 x \sin^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x - \cot x\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用:  $\sin^2 x + \cos^2 x = 1$

例子 求不定积分  $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x - \cot x + C\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$



## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2},$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx =$$

$$\int \sin^2 \frac{x}{2} dx =$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx =$$

$$\int \sin^2 \frac{x}{2} dx =$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2}x + \frac{1}{2} \sin x + C$$

$$\int \sin^2 \frac{x}{2} dx =$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2}x + \frac{1}{2} \sin x + C$$

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx =$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分  $\int \cos^2 \frac{x}{2} dx$ ,  $\int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2}x + \frac{1}{2} \sin x + C$$

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2}x - \frac{1}{2} \sin x + C$$



## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

---

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned} \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned} \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\&= -\cot x\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\&= -\cot x - \tan x\end{aligned}$$

## 求不定积分 V: $\int$ "triangle functions" $dx$ (Cont.)

利用 “倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分  $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\&= -\cot x - \tan x + C\end{aligned}$$