§1.1 二阶三阶行列式

数学系　梁卓滨

2016 - 2017 学年 I 暑修班

教学要求

掌握求解：

- ◇ 二阶行列式计算
- ♣ 三阶行列式计算

- 行列式的概念来源于线性方程组的求解问题
- 17 世纪末由日本数学家关孝和及德国数学家莱布尼茨引入

$$
\begin{array}{ccc}
\text{二阶行列式} & \longleftrightarrow & \text{二元线性方程组} \\
\text{三阶行列式} & \longleftrightarrow & \text{三元线性方程组} \\
\vdots & & \vdots \\
n\ \text{阶行列式} & \longleftrightarrow & n\ \text{元线性方程组} \\
\vdots & & \vdots
\end{array}
$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法可解：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

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$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

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$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为二阶行列式

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$, $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式:

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2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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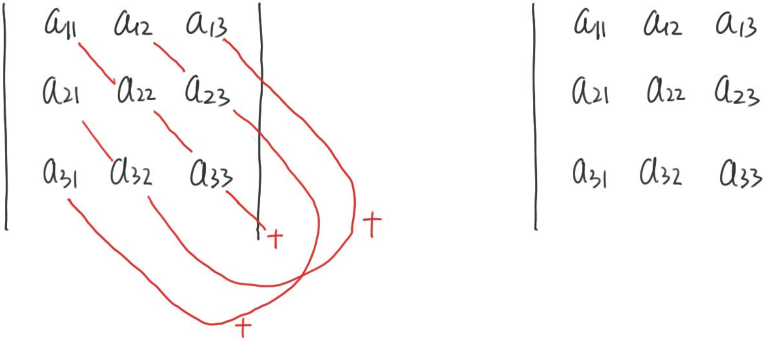

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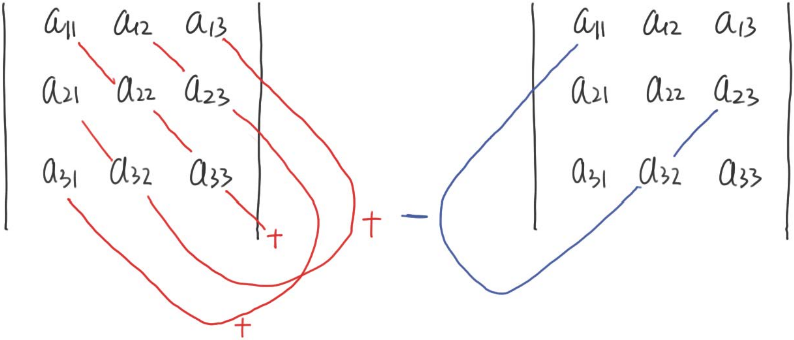

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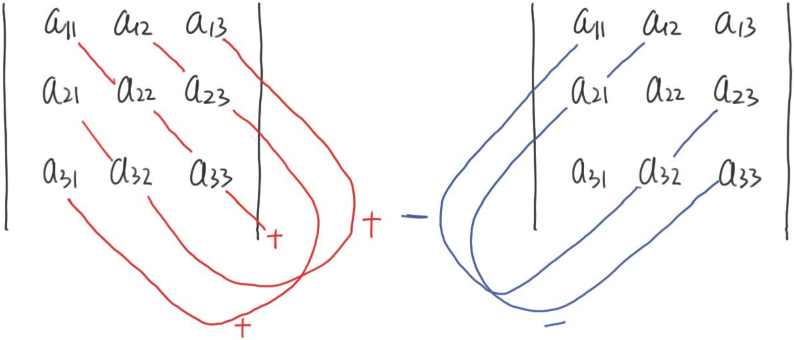

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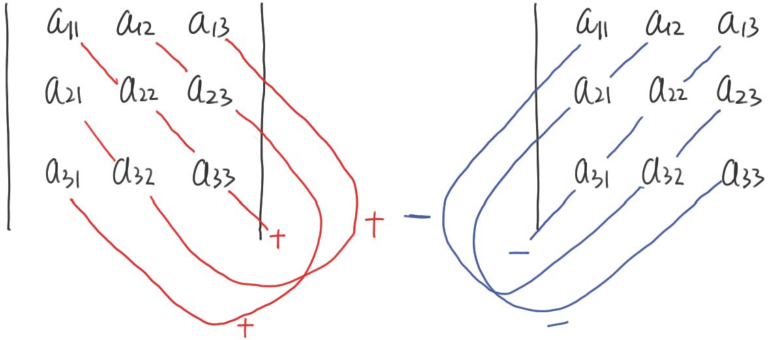

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$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
的解可以表示为：

$$x = \frac{\begin{matrix} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{matrix}}{\begin{matrix} \color{red}{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ \color{blue}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{matrix}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

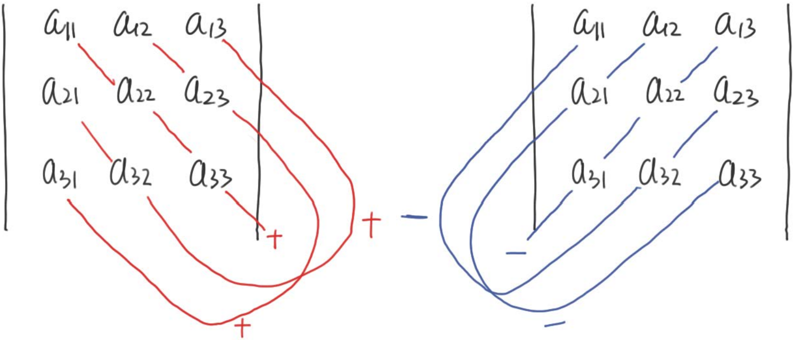
$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}},$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} {\color{red}a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ {\color{blue}-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

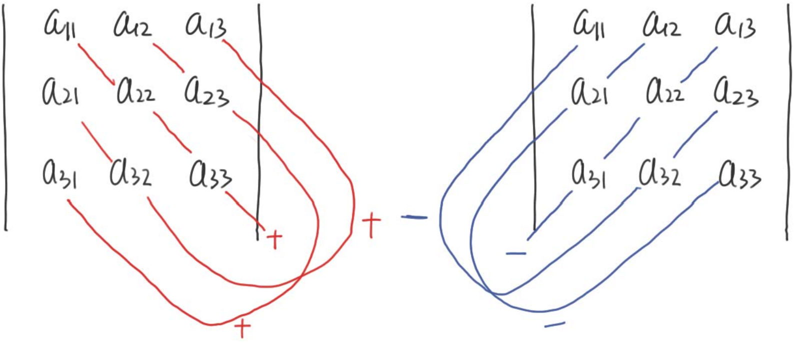
$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

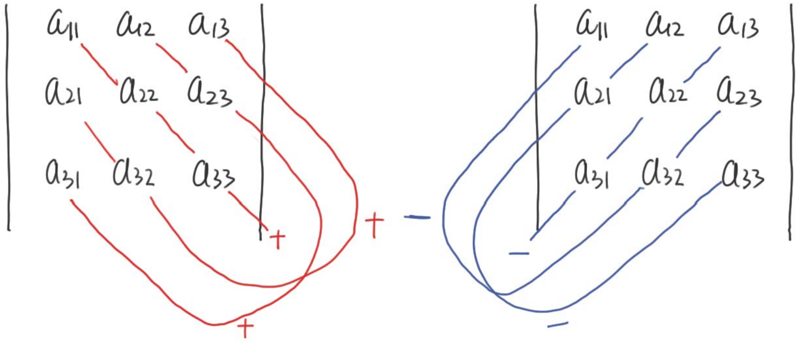
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \qquad 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0$$

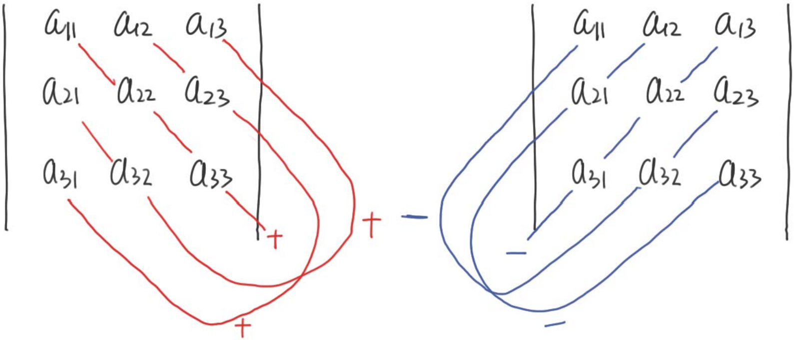
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array}$$

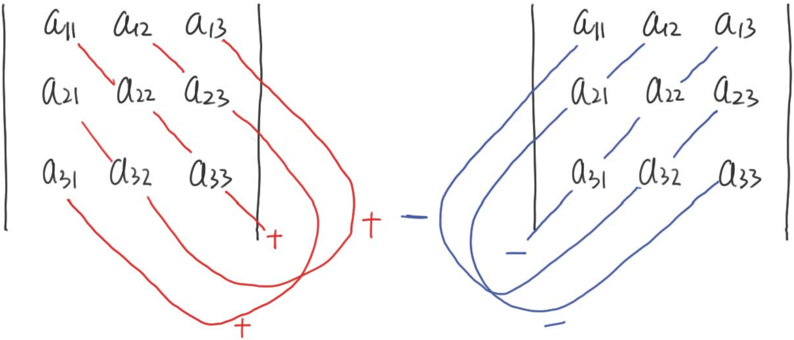
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

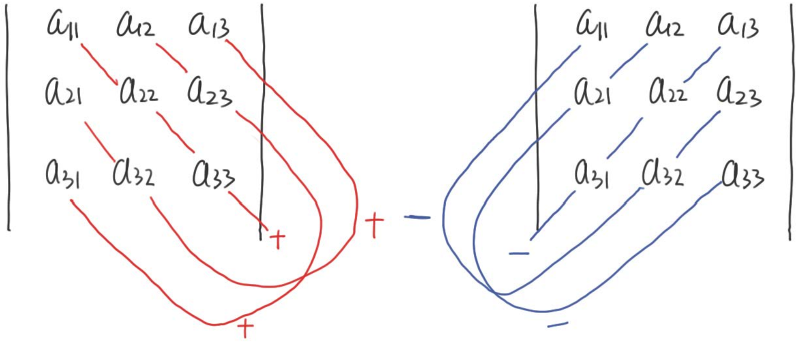
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

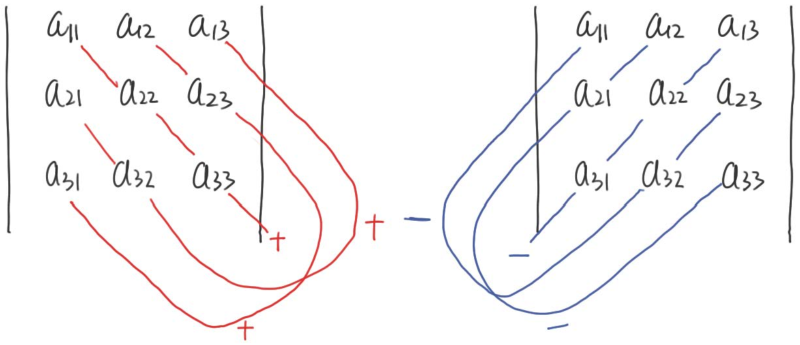
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{array}{c} 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4 \\ -1\times 0\times 4 - 0\times 3\times 1 - (-1)\times 5\times 1 \end{array}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{matrix} 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4 \\ -1\times 0\times 4 - 0\times 3\times 1 - (-1)\times 5\times 1 \end{matrix} = -2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ______________

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ____________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ____________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \quad a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{array}{c} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{array}$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

所以 $a \neq 0$ 且 $b \neq 0$。