§5.3 换元积分法

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教学要求

◇ 熟练掌握换元积分法：“凑微分”，“变量代换”

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Outline of §5.2

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

We are here now...

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

第一类换元积分法——“凑微分”法，能干啥？

能够计算如下的不定积分：

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos x dx$$

$$\int \frac{1}{\cos x}dx$$

……

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{"Ì¡żğÆćÈ"} \, dx$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{"Ì¡żğÆćÈ"}\ dx \xlongequal{\quad} \int f(\varphi(x))\varphi'(x)dx$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{"Ì¡żğÆćÈ"}\; dx \xlongequal{\quad\quad} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{"Ì¡żğÆćÈ"}\; dx = \!=\!= \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du$$

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$$= F(u) + C$$

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$$= F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

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- 验证：$F(\varphi(x))$ 确是 "Ì¡żğÆćÈ" 的原函数！

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$$\frac{d}{dx}F(\varphi(x)) =$$

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$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

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$$\int \text{"Ì¡żğÆćÈ"}\, dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

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$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = \text{"Ì¡żğÆćÈ"}$$

总之 $\displaystyle\int \text{"Ì¡żğÆćÈ"}\, dx \xlongequal{\text{凑微分}} \int f(\varphi(x))d\varphi(x)$

$$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 I：热身

例子 求不定积分

$$\int \frac{1}{1+2x}dx, \qquad \int \frac{1}{2-3x}dx, \qquad \int \sqrt{3x-1}dx,$$

$$\int \frac{1}{\sqrt{1-5x}}dx, \qquad \int \cos(\tfrac{3}{2}x)dx, \qquad \int e^{-\frac{1}{2}x+4}dx.$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I：热身

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热身

$$dx = \underline{\qquad\qquad} d(2x+1), \qquad dx = \underline{\qquad\qquad\qquad} d(2-3x),$$

$$dx = \underline{\qquad\qquad} d(\frac{3}{2}x), \qquad dx = \underline{\qquad\qquad\qquad} d(-\frac{1}{2}x+4).$$

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凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I：热身

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$$\int \frac{1}{\sqrt{1-5x}}dx, \qquad \int \cos(\tfrac{3}{2}x)dx, \qquad \int e^{-\frac{1}{2}x+4}dx.$$

热身

$$dx = \underline{\quad \frac{1}{2} \quad} d(2x+1), \qquad dx = \underline{\quad -\frac{1}{3} \quad} d(2-3x),$$

$$dx = \underline{\qquad\qquad} d(\frac{3}{2}x), \qquad dx = \underline{\qquad\qquad} d(-\frac{1}{2}x+4).$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 I：热身

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热身

$$dx = \underline{\quad \frac{1}{2} \quad} d(2x+1), \qquad dx = \underline{\quad -\frac{1}{3} \quad} d(2-3x),$$

$$dx = \underline{\quad \frac{2}{3} \quad} d(\frac{3}{2}x), \qquad dx = \underline{\qquad\qquad} d(-\frac{1}{2}x+4).$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I：热身

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$$\int \frac{1}{\sqrt{1-5x}}dx, \qquad \int \cos(\tfrac{3}{2}x)dx, \qquad \int e^{-\frac{1}{2}x+4}dx.$$

热身

$$dx = \underline{\quad \frac{1}{2} \quad} d(2x+1), \qquad dx = \underline{\quad -\frac{1}{3} \quad} d(2-3x),$$

$$dx = \underline{\quad \frac{2}{3} \quad} d(\frac{3}{2}x), \qquad dx = \underline{\quad -2 \quad} d(-\frac{1}{2}x+4).$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$”例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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解 $\displaystyle\int \frac{1}{1+2x}dx =$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 I

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解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \qquad d(1+2x)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x)$$

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例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$

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$$= \frac{1}{2}\ln|u| + C$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$
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凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解 $$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$

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$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \qquad d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

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$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

凑微分 "∫ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

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$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$\int \sqrt{3x-1}dx =$$

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$$= -\frac{1}{3}\ln|u| + C$$

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例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

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$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx =$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$
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$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \qquad d(3x-1)$$

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$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1)$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$
$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int\frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int\sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int\sqrt{u}du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$u^{3/2}$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int\sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$\frac{2}{3}u^{3/2}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$$
$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$
$$= \frac{1}{3} \cdot \frac{2}{3}u^{3/2} + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$，$\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$
$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int\frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int\sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int\sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$
$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{9}(3x-1)^{3/2} + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \qquad\qquad d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$”例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$”例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \qquad d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u)+C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}$$

凑微分 "∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \qquad d(-\frac{1}{2}x+4)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^{u}du$$

$$= -2e^{u}+C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

$$= -2e^u+C = -2e^{-\frac{1}{2}x+4}+C$$

例子 求不定积分

(1) $\int xe^{x^2}dx,\qquad \int x\sin(x^2)dx,$

(2) $\int x\sqrt{1-x^2}dx,\qquad \int \frac{x}{\sqrt{3-x^2}}dx,\qquad \int \frac{x}{1+3x^2}dx$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$”例 II：热身

例子 求不定积分

(1) $\int xe^{x^2}dx,\qquad \int x\sin(x^2)dx,$

(2) $\int x\sqrt{1-x^2}dx,\qquad \int \frac{x}{\sqrt{3-x^2}}dx,\qquad \int \frac{x}{1+3x^2}dx$

热身

$xdx=$ ______ $d(x^2)$ $\qquad$ $xdx=$ ________ $d(1-x^2)$

$xdx=$ ________ $d(3-x^2)$ $\qquad$ $xdx=$ ______ $d(1+3x^2)$

凑微分"$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$"例 II：热身

例子 求不定积分

(1) $\int xe^{x^2}dx,\qquad \int x\sin(x^2)dx,$

(2) $\int x\sqrt{1-x^2}dx,\qquad \int \frac{x}{\sqrt{3-x^2}}dx,\qquad \int \frac{x}{1+3x^2}dx$

热身

$$xdx = \underline{\ \ \frac{1}{2}\ \ }\,d(x^2) \qquad xdx = \underline{\qquad\qquad}\,d(1-x^2)$$

$$xdx = \underline{\qquad\qquad}\,d(3-x^2) \qquad xdx = \underline{\qquad\quad}\,d(1+3x^2)$$

例子 求不定积分

(1) $\int xe^{x^2}dx,\qquad \int x\sin(x^2)dx,$

(2) $\int x\sqrt{1-x^2}dx,\qquad \int \frac{x}{\sqrt{3-x^2}}dx,\qquad \int \frac{x}{1+3x^2}dx$

热身

$xdx = \underline{\quad \frac{1}{2} \quad} d(x^2)\qquad xdx = \underline{\quad -\frac{1}{2} \quad} d(1-x^2)$

$xdx = \underline{\qquad\qquad} d(3-x^2)\qquad xdx = \underline{\qquad\quad} d(1+3x^2)$

凑微分"$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$"例 II：热身

例子 求不定积分

(1) $\int xe^{x^2}dx,\qquad \int x\sin(x^2)dx,$

(2) $\int x\sqrt{1-x^2}dx,\qquad \int \frac{x}{\sqrt{3-x^2}}dx,\qquad \int \frac{x}{1+3x^2}dx$

热身

$xdx = \underline{\quad \frac{1}{2} \quad} d(x^2)\qquad xdx = \underline{\quad -\frac{1}{2} \quad} d(1-x^2)$

$xdx = \underline{\quad -\frac{1}{2} \quad} d(3-x^2)\qquad xdx = \underline{\qquad\quad} d(1+3x^2)$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$”例 II：热身

例子 求不定积分

(1) $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$,

(2) $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

热身

$xdx = \underline{\quad \frac{1}{2} \quad} d(x^2)$ $xdx = \underline{\quad -\frac{1}{2} \quad} d(1-x^2)$

$xdx = \underline{\quad -\frac{1}{2} \quad} d(3-x^2)$ $xdx = \underline{\quad \frac{1}{6} \quad} d(1+3x^2)$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$”例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 III

例子 求 $\int xe^{x^2}dx$， $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2} \quad d(x^2)$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 III

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du$$

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) \qquad d(x^2)$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 III

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2)$$

例子 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u du$$

例子 求 $\int xe^{x^2}dx$， $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx=\int e^{x^2}\frac{1}{2}d(x^2)=\frac{1}{2}\int e^u du=\frac{1}{2}e^u+C=\frac{1}{2}e^{x^2}+C$$

$$\int x\sin(x^2)dx=\int\sin(x^2)\cdot\frac{1}{2}d(x^2)=\frac{1}{2}\int\sin u du$$

$$=-\frac{1}{2}\cos u+C$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u du$$

$$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx =$$

凑微分 “$\int$ "Ì¡żğÆEćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \qquad d(1-x^2)$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$u^{3/2}$$

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$\frac{2}{3}u^{3/2}$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx=\int(1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2)=-\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$=-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C=-\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int\frac{x}{\sqrt{3-x^2}}dx=\int(3-x^2)^{-\frac{1}{2}}\qquad d(3-x^2)$$

$$\int\frac{x}{1+3x^2}dx=$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}\cdot(-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx=\int(1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2)=-\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$=-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C=-\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int\frac{x}{\sqrt{3-x^2}}dx=\int(3-x^2)^{-\frac{1}{2}}\cdot(-\frac{1}{2})d(3-x^2)$$

$$=-\frac{1}{2}\int u^{-1/2}du \qquad 2u^{1/2}$$

$$\int\frac{x}{1+3x^2}dx=$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}\cdot(-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2}+C$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx =$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}$$

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \qquad d(1+3x^2)$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}\cdot(-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2)$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx=\int(1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2)=-\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$=-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C=-\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int\frac{x}{\sqrt{3-x^2}}dx=\int(3-x^2)^{-\frac{1}{2}}\cdot(-\frac{1}{2})d(3-x^2)$$

$$=-\frac{1}{2}\int u^{-1/2}du=-\frac{1}{2}\cdot 2u^{1/2}+C=-(3-x^2)^{\frac{1}{2}}+C$$

$$\int\frac{x}{1+3x^2}dx=\int\frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2)=\frac{1}{6}\int\frac{1}{u}du$$

$$=\frac{1}{6}\ln|u|+C$$

例子 求 $\int x\sqrt{1-x^2}dx$，$\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2}+C = -(3-x^2)^{\frac{1}{2}}+C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u|+C = \frac{1}{6}\ln|1+3x^2|+C$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 V

例子 求 $\int e^x \sin(e^x)dx$， $\int \frac{e^x}{1+e^x}dx$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx =$$

凑微分 “$\int$ "Ìįżğ ÆEćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)$$

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\begin{aligned}\int e^x \sin(e^x)dx &= \int \sin(e^x)de^x \\ &= \int \sin u du = -\cos u + C = -\cos(e^x) + C\end{aligned}$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x$$

例子 求 $\int e^x\sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x\sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

凑微分 "∫ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du$$

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$”例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C = \ln(e^x+1) + C$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx =$$

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

$$= \frac{1}{2}u^2 + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\begin{aligned}\int \frac{1}{x}\ln x dx &= \int \ln x d\ln x = \int u du \\ &= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C\end{aligned}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\begin{aligned}\int \frac{1}{x}\ln x dx &= \int \ln x d\ln x = \int u du \\ &= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C\end{aligned}$$

$$\int \frac{1}{x\ln x}dx =$$

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\begin{aligned}\int \frac{1}{x}\ln x dx &= \int \ln x d\ln x = \int u du \\ &= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C\end{aligned}$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x$$

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

$$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 VI

例子 求 $\int \frac{1}{x}\ln x dx$，$\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

$$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du$$

$$= \ln|u| + C$$

例子 求 $\int \frac{1}{x}\ln x dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

$$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du$$

$$= \ln|u| + C = \ln|\ln x| + C$$

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$\sin x dx = d$____________ ，　$\cos x dx = d$____________

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad\quad}, \qquad \cos x dx = d\underline{\qquad\qquad}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\qquad\qquad}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解 $$\int e^{\cos x}\sin x dx =$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解 $$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x$$

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解 $$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$
$$= -e^u + C$$

凑微分 “$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$， $\int \frac{\sin x}{1+\cos^2 x}dx$， $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^{u}du$$

$$= -e^{u} + C = -e^{\cos x} + C$$

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解
$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx =$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分 "$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 VII

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$”例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^{u}du$$

$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分 "∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 VII

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分 “∫ "Ì¡żğÆćÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$” 例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^{u}du$$

$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x$$

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du$$

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du = \ln|u| + C$$

凑微分“$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$”例 VII

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

热身

$$\sin x dx = d\underline{\quad -\cos x \quad} = -d\cos x, \qquad \cos x dx = d\underline{\quad \sin x \quad}$$

解

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凑微分法 “$\int$ "Ì¡żğÆćÈ" $dx = \int f(\varphi(x))d\varphi(x)$”：例子总结

$$\int \frac{1}{1-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

$$\int xe^{x^2}dx =$$

$$\int x\sqrt{1-x^2}dx =$$

$$\int \frac{\ln x}{x}dx =$$

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$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = \int \frac{1}{u}du = \cdots$$

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We are here now...

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

能够计算如下的不定积分：

$$\int x\sqrt{3x-1}dx,\quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx,\quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

$$\cdots\cdots$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

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$$= G(t) + C$$

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- 计算步骤：

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第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

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第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \overset{x=\varphi(t)}{=\!=\!=} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

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- 关键是：如何选取函数 $x = \varphi(t)$？

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

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- 关键是：如何选取函数 $x = \varphi(t)$？

在后面的例子中，选取函数 $x = \varphi(t)$ 的方法：

> 把被积函数 $f(x)$ 中复杂的部分整个设为 t，
>
> 从而得到 x 与 t 的函数关系！

例子 求不定积分 $\int \sqrt{1-x^2}dx$

例子 求不定积分 $\int \sqrt{1-x^2}dx$

解

$\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$,

例子 求不定积分 $\int \sqrt{1-x^2}dx$

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例子 求不定积分 $\int \sqrt{1-x^2}dx$

解

$$\because -1 \leq x \leq 1，设 x = \sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]，\ \cos t \geq 0$$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t$$

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$$\because -1 \le x \le 1，设 x = \sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]，\ \cos t \ge 0$$

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$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1 dt$$

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变量代换 “$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$” 例 I

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$$= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x + C$$

例子 求不定积分 $\int \sqrt{1-x^2}dx$

解

$\because -1 \le x \le 1$，设 $x = \sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$，$\cos t \ge 0$

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注 可见选取合适 $x = \varphi(t)$ 很关键！

变量代换 "$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$" 例 I

例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

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(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$,

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例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$， $\therefore x=\frac{1}{3}(t^2+1)$， $dx=\frac{2}{3}tdt$

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例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t = (3x-1)^{\frac{1}{2}}$, $\quad \therefore x = \frac{1}{3}(t^2+1)$, $\quad dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)$$

例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

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例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

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(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

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(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2$, $\quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx =$$

变量代换 “$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$” 例 I

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例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

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例子 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

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(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

$$= \frac{2}{45}t^5+\frac{2}{27}t^3+C = \frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2$, $\quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx = \int \frac{t^2+2}{t}\cdot 2tdt = 2\int t^2+2dt = \frac{2}{3}t^3+4t+C$$

$$= \frac{2}{3}(x-2)^{\frac{3}{2}}+4(x-2)^{\frac{1}{2}}+C$$

变量代换 "$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$" 例 II

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

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例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$,

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例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

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解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$,

$$\therefore \int \frac{1}{1+\sqrt{x}}dx =$$

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例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx =$$

,

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$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt$$

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, ,

变量代换 “$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$” 例 II

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

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$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$,

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx =$$

变量代换 “$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$” 例 II

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$, $\quad dx = 3(t-1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx =$$

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

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$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt =$$

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$, $\quad dx = 3(t-1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3\int t - 2 + \frac{1}{t}dt$$

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

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$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$, $\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

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$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3\int t - 2 + \frac{1}{t}dt$$

$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

$$= \frac{3}{2}(1+(1+x)^{\frac{1}{3}})^2 - 6(1+(1+x)^{\frac{1}{3}}) + 3\ln|1+(1+x)^{\frac{1}{3}}| + C$$

变量代换“$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$”例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

变量代换“$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$”例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

设 $t = \sqrt{1+e^x}$,

变量代换“$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$”例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1),$$

变量代换“$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$”例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$设\ t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1), \quad dx = \frac{2t}{t^2-1}dt$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$设\ t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1), \quad dx = \frac{2t}{t^2-1}dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

设 $t=\sqrt{1+e^x}$, $\quad \therefore x=\ln(t^2-1), \quad dx=\dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1), \quad dx = \frac{2t}{t^2-1}dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$\text{设 } t=\sqrt{1+e^x}, \quad \therefore x=\ln(t^2-1), \quad dx=\frac{2t}{t^2-1}dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt = \int \frac{1}{t-1}-\frac{1}{t+1}dt$$

$$= \ln|t-1|-\ln|t+1|+C = \ln\left|\frac{t-1}{t+1}\right|+C$$

变量代换"$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$"例 IV

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解

设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1), \quad dx = \frac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

变量代换“$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$”例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1)$, $\quad dx = \dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

$$= 2\ln(\sqrt{1+e^x}-1) - x + C$$

变量代换法 “$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt$”：例子总结

$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法 "$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt$"：例子总结

$$\int x\sqrt{3x-1}dx \xlongequal{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

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$$\int \frac{1}{\sqrt{1+e^x}}dx \xlongequal{t=\sqrt{1+e^x}} \cdots$$