§5.1, 5.2 二次型与对称矩阵

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$$f(x_1, x_2) = 6x_1^2 + 4x_1x_2 - 2x_2^2$$

$$f(x_1, x_2) = 6x_1^2 + 4x_1x_2 - 2x_2^2 = (x_1, x_2) \begin{pmatrix} 6 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$f(x_1, x_2) = 6x_1^2 + 4x_1x_2 - 2x_2^2 = (x_1, x_2) \begin{pmatrix} 6 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$= (6x_1 + 2x_2, 2x_1 - 2x_2)$$

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二元二次齐次多项式

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$$f(x_1, x_2) = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$$

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二元二次齐次多项式

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$$f(x_1, x_2) = -3x_1^2 + 2x_1x_2 + 5x_2^2 =$$



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$$f(x_1, x_2) = -3x_1^2 + 2x_1x_2 + 5x_2^2 = (x_1, x_2) \left(\begin{array}{c} x_1 \\ x_2 \end{array} \right)$$



二元二次齐次多项式

$$f(x_1, x_2) = 6x_1^2 + 4x_1x_2 - 2x_2^2 = (x_1, x_2) \begin{pmatrix} 6 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

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$$f(x_1, x_2) = -3x_1^2 + 2x_1x_2 + 5x_2^2 = (x_1, x_2)\begin{pmatrix} -3 \\ x_2 \end{pmatrix}$$



二元二次齐次多项式

$$f(x_1, x_2) = 6x_1^2 + 4x_1x_2 - 2x_2^2 = (x_1, x_2) \begin{pmatrix} 6 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

一般地,

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$$f(x_1, x_2) = -3x_1^2 + 2x_1x_2 + 5x_2^2 = (x_1, x_2) \begin{pmatrix} -3 \\ 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

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$$f(x_1, x_2) = -3x_1^2 + 2x_1x_2 + 5x_2^2 = (x_1, x_2)\begin{pmatrix} -3 & 1 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$



$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

+ $2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$

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三元二次齐次多项式

$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$
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$$f(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + 2x_3^2 - 2x_2x_3$$

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$$= (x_1, x_2, x_3) \begin{pmatrix} 0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\underbrace{ \begin{cases} x_1 \\ x_2 \\ x_3 \end{cases}}_{\text{ a.s. } \text{ b.s. } \text{ f.s. } \text{ f.s$$



$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$
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$$f(x_1, x_2, x_3) = x_1x_2 + x_1x_3 + 2x_3^2 - 2x_2x_3$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 0 & & \\ & 0 & \\ & & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$
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$$f(x_1, x_2, x_3) = x_1 x_2 + x_1 x_3 + 2x_3^2 - 2x_2 x_3$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$
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$$(2) \frac{1}{2} (x_1 + x_2) \frac{1}{2}$$

三元二次齐次多项式

$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$
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$$f(x_1, x_2, x_3) = x_1 x_2 + x_1 x_3 + 2x_3^2 - 2x_2 x_3$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -1 \\ \frac{1}{2} & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定二次型, 写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1x_2 + 3x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1x_2 + 3x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

M 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$
$$= (x_1, x_2, x_3) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$
$$= (x_1, x_2, x_3) \begin{pmatrix} 1 \\ x_2 \\ x_3 \end{pmatrix}$$

 \overline{M} 给定对称矩阵 \overline{A} ,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$
$$= (x_1, x_2, x_3) \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

_

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$
$$= (x_1, x_2, x_3) \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 2 \\ & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 \\ \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

例 给定对称矩阵 A,写出相应二次型:

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

=

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2$$



例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2$$



例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2 + 0x_3^2$$

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2 + 0x_3^2 + 2 \cdot 1 \cdot x_1 x_2$$

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2 + 0x_3^2 + 2 \cdot 1 \cdot x_1 x_2 + 2 \cdot \frac{1}{2} \cdot x_1 x_3$$



例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2 + 0x_3^2 + 2 \cdot 1 \cdot x_1 x_2 + 2 \cdot \frac{1}{2} \cdot x_1 x_3 + 2 \cdot 0 \cdot x_2 x_3$$

例 给定二次型,写出对称矩阵 A:

$$f(x_1, x_2, x_3) = x_1^2 + x_1 x_2 + 3x_1 x_3 + 2x_2^2 + 4x_2 x_3 + x_3^2$$

$$= (x_1, x_2, x_3) \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$f(x_1, x_2, x_3) = (x_1, x_2, x_3) \begin{pmatrix} -1 & 1 & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= -x_1^2 + 2x_2^2 + 0x_3^2 + 2 \cdot 1 \cdot x_1 x_2 + 2 \cdot \frac{1}{2} \cdot x_1 x_3 + 2 \cdot 0 \cdot x_2 x_3$$
$$= -x_1^2 + 2x_2^2 + 2x_1 x_3 + x_1 x_3$$

$$f(x_1, x_2, ..., x_n) = a_{11}x_1^2 + 2a_{12}x_1x_2 + ... + 2a_{1n}x_1x_n$$

$$+ a_{22}x_2^2 + ... + 2a_{2n}x_2x_n$$

$$+$$

$$+ a_{nn}x_n^2$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= (x_{1}, x_{2}, ..., x_{n}) \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= (x_{1}, x_{2}, ..., x_{n}) \begin{pmatrix} a_{11} & a_{22} & \\ & \ddots & \\ & & a_{nn} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= (x_{1}, x_{2}, ..., x_{n}) \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \\ \vdots \\ x_{n} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= (x_{1}, x_{2}, ..., x_{n}) \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{12} & a_{22} & ... & a_{nn} \\ \vdots & ... & a_{nn} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= (x_{1}, x_{2}, ..., x_{n}) \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{12} & a_{22} & ... & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & ... & a_{nn} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= \underbrace{(x_{1}, x_{2}, ..., x_{n})}_{x^{T}} \underbrace{\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{12} & a_{22} & ... & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & ... & a_{nn} \end{pmatrix}}_{x_{n}} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

定义 n 元二次型

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + ... + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + ... + 2a_{2n}x_{2}x_{n}$$

$$+$$

$$+ a_{nn}x_{n}^{2}$$

$$= \underbrace{(x_{1}, x_{2}, ..., x_{n})}_{x^{T}} \underbrace{\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{12} & a_{22} & ... & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & ... & a_{nn} \end{pmatrix}}_{x} \underbrace{\begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}}_{x}$$

 $= x^T A x$



定义 n 元二次型

$$f(x_{1}, x_{2}, ..., x_{n}) = a_{11}x_{1}^{2} + 2a_{12}x_{1}x_{2} + \cdots + 2a_{1n}x_{1}x_{n}$$

$$+ a_{22}x_{2}^{2} + \cdots + 2a_{2n}x_{2}x_{n}$$

$$+ \cdots \cdots$$

$$+ a_{nn}x_{n}^{2}$$

$$= \underbrace{(x_{1}, x_{2}, ..., x_{n})}_{x^{T}} \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}$$

 $= x^T A x$

注 n 元二次型与对称矩阵, 是一一对应



$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$



$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

变量 ↓ 代换

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

₩

$$f =$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

变量 ↓ 代换

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

₩

$$f = (y_1 - 2y_2)^2$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

$$f = (y_1 - 2y_2)^2$$

$$+4(y_1-2y_2)y_2-3y_2^2$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2$$

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

$$f = (y_1 - 2y_2)^2$$

$$+4(y_1-2y_2)y_2-3y_2^2$$
$$=y_1^2-7y_2^2$$

$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \Leftrightarrow f(x_1, x_2) = (x_1, x_2) \begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
变量 \(\text{ 代换} \)
$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$
\(\psi \)
$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2 - 3y_2^2 = y_1^2 - 7y_2^2$$



$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 + f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$$

$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$

$$+4(y_1-2y_2)y_2-3y_2^2$$

$$=y_1^2-7y_2^2$$

 $f = (y_1 - 2y_2)^2$

女性受視: 51例
$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \leftrightarrow f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x^T A x$$

变量 ↓ 代换
$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$$
↓
$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2 - 3y_2^2$$

$$= y_1^2 - 7y_2^2$$



线性变换: 引例
$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \leftrightarrow f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x^T A x$$

变量 \(\frac{X_1}{X_2} = \frac{Y_1 - 2y_2}{Y_2} \\
\frac{X_1}{X_2} = \frac{Y_2}{Y_2} \\
\frac{X_1}{X_2} = \frac{1-2}{0} \left(\frac{Y_1}{Y_2} \right) \\
\frac{f = (y_1 - 2y_2)^2}{4} \\
\frac{f = (y_2 - 2y_2)^2}{4} \\
\f

$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2 - 3y_2^2$$

 $= y_1^2 - 7y_2^2$

$$f(x_{1}, x_{2}) = x_{1}^{2} + 4x_{1}x_{2} - 3x_{2}^{2} \Leftrightarrow f(x_{1}, x_{2}) = (x_{1}, x_{2}) \underbrace{\begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix}}_{A} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = x^{T}Ax$$

变量 \(\psi \text{ 代换} \)
$$\begin{cases} x_{1} = y_{1} - 2y_{2} \\ x_{2} = y_{2} \end{cases} \Leftrightarrow \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} \xrightarrow{\mathbf{x} = Cy}$$

\(\mathbf{x} = (y_{1} - 2y_{2})^{2} + 4(y_{1} - 2y_{2})y_{2} - 3y_{2}^{2} = y_{1}^{2} - 7y_{2}^{2} \end{cases}



$$f(x_{1}, x_{2}) = x_{1}^{2} + 4x_{1}x_{2} - 3x_{2}^{2} \Leftrightarrow f(x_{1}, x_{2}) = (x_{1}, x_{2}) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = x^{T} A x$$

变量 \(\psi \text{ 代换} \)
$$\begin{cases} x_{1} = y_{1} - 2y_{2} \\ x_{2} = y_{2} \end{cases} \Leftrightarrow \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - 2 \\ 0 \\ 1 \end{pmatrix}}_{C} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} \xrightarrow{x = Cy}_{x^{T} = y^{T} C^{T}}$$
\(\psi \)

$$f = (y_{1} - 2y_{2})^{2} + 4(y_{1} - 2y_{2})y_{2} - 3y_{2}^{2} = y_{1}^{2} - 7y_{2}^{2}$$



また。 カドリ
$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \leftrightarrow f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix}}_{A} (x_1^2) = x^T A x$$

变量 ψ 代换
$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases} \leftrightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1-2 \\ 0 \\ 1 \end{pmatrix}}_{C} (y_1^2) \xrightarrow{x=Cy}_{x^T = y^T C^T}$$

$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2 - 3y_2^2$$
$$= y_1^2 - 7y_2^2$$

1

$$f = (y_1, y_2)C^T A C \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$



注 1.
$$C^TAC = \begin{pmatrix} 1 & 0 \\ -21 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix} \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix} =$$



§5.1, 5.2

$$f(x_{1}, x_{2}) = x_{1}^{2} + 4x_{1}x_{2} - 3x_{2}^{2} \Leftrightarrow f(x_{1}, x_{2}) = (x_{1}, x_{2}) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = x^{T} A x$$

变量 \(\psi \tau_{\text{th}}\)
$$\begin{cases} x_{1} = y_{1} - 2y_{2} \\ x_{2} = y_{2} \end{cases} \Leftrightarrow \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - 2 \\ 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} \xrightarrow{x = Cy}_{x^{T} = y^{T}C^{T}}$$

\(\psi \)
$$f = (y_{1} - 2y_{2})^{2} \qquad \qquad f = (y_{1}, y_{2})C^{T}AC\begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix}$$

$$= y_{1}^{2} - 7y_{2}^{2}$$

$$\stackrel{:}{\succeq} 1. C^T AC = \begin{pmatrix} 1 & 0 \\ -21 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix} \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0-7 \end{pmatrix}$$



$$f(x_{1}, x_{2}) = x_{1}^{2} + 4x_{1}x_{2} - 3x_{2}^{2} \Leftrightarrow f(x_{1}, x_{2}) = (x_{1}, x_{2}) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = x^{T}Ax$$

变量 \(\text{ 代换} \)

$$\begin{cases} x_{1} = y_{1} - 2y_{2} \\ x_{2} = y_{2} \end{cases} \Leftrightarrow \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - 2 \\ 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} \xrightarrow{x = Cy}_{x^{T} = y^{T}C^{T}}$$

\(\text{ } \\ f = (y_{1}, y_{2})C^{T}AC \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} = (y_{1}, y_{2}) \begin{pmatrix} 1 & 0 \\ 0 - 7 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix} = (y_{1}, y_{2}) \begin{pmatrix} 1 & 0 \\ 0 - 7 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \end{pmatrix}

$$= y_{1}^{2} - 7y_{2}^{2}$$

注 1.
$$C^T A C = \begin{pmatrix} 1 & 0 \\ -21 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix} \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0-7 \end{pmatrix}$$



$$\stackrel{\text{?}}{=} 1. C^T A C = \begin{pmatrix} 1 & 0 \\ -21 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix} \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0-7 \end{pmatrix}$$

2. 变量代换可逆:



支性受快: 5 1 例
$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \Leftrightarrow f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} (x_1) = x^T A x$$

变量 単代换
$$\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases} \Leftrightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - 2 \\ 0 & 1 \end{pmatrix}}_{C} (y_1) \times \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{X^T = y^T C^T}$$

↓
$$f = (y_1, y_2)C^T A C \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

 $=(y_1, y_2)\begin{pmatrix} 1 & 0 \\ 0 - 7 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

 $+4(y_1-2y_2)y_2-3y_2^2$

2. 变量代换可逆:
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 0 \end{pmatrix}^{-1} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 0 \end{pmatrix}^{-1}$$

 $= y_1^2 - 7y_2^2$



$$f(x_1, x_2) = x_1^2 + 4x_1x_2 - 3x_2^2 \Leftrightarrow f(x_1, x_2) = (x_1, x_2) \underbrace{\begin{pmatrix} 1 & 2 \\ 2 - 3 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} = x^T A x$$

变量
$$\downarrow$$
 代换 $\begin{cases} x_1 = y_1 - 2y_2 \\ x_2 = y_2 \end{cases}$

变量
$$\downarrow$$
 代换
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1-2 \\ 0 \\ 1 \end{pmatrix}}_{x_2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \begin{array}{c} x = Cy \\ x^T = y^T C^T \end{array}$$

$$f = (y_1 - 2y_2)^2 + 4(y_1 - 2y_2)y_2 - 3y_2^2$$
$$= y_1^2 - 7y_2^2$$

$$f = (y_1, y_2)C^T A C \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$
$$= (y_1, y_2) \begin{pmatrix} 1 & 0 \\ 0 - 7 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

注 1.
$$C^T A C = \begin{pmatrix} 1 & 0 \\ -21 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2-3 \end{pmatrix} \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0-7 \end{pmatrix}$$

2. 变量代换可逆: $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 01 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad \text{标准形}$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad \text{标准形}$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad \text{标准形}$$

$$f = \mathbf{X}^T \mathbf{A} \mathbf{X}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad 标准形$$

$$f = \mathbf{X}^T \mathbf{A} \mathbf{X}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy$$



$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad 标准形$$
其中 $d_1, \dots, d_r \neq 0$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy$$
, $x^T = y^T C^T$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad 标准形$$

其中
$$d_1, \cdots, d_r \neq 0$$

$$f = x^T A x$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x^T = y^T C^T$$

$$f =$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

二次型
$$f$$
 后,可化为 $x = Cy$, $x^T = y^T C^T$ $f = d_1 y_1^2 + \dots + d_r y_r^2$ 标准形 $f = y^T C^T$

$$f = x^T A x$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x^T = y^T C^T$$

$$f = y^T C^T$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

二次型
$$f$$
 后,可化为 $x = Cy$, $x^T = y^T C^T$ $f = d_1 y_1^2 + \dots + d_r y_r^2$ 标准形 $f = y^T C^T A C y$

$$f = x^T A x$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x' = y'$$

$$f = y^T C^T A C y$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2$$
 标准形 $f = y^T C^T A C y$

$$f = x^T A x$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x^T = y^T C^T$$

$$f = y^T C^T A C y$$



$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T Ax$

一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型f后,可化为

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad \text{标准形}$$

$$f = x^T A x$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy$$
, $x^T = y^T C^T$

$$x = Cy$$
, $x^{T} = y^{T}C^{T}$
 $f = y^{T}C^{T}ACy = y^{T}By$

§5.1, 5.2

定理 对任意n元二次型

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

$$f = d_1 v^2 + \cdots +$$

$$f = d_1 y_1^2 + \dots + d_r y_r^2 \quad \text{标准形}$$

$$f = a_1 y_1 + \cdots + a_n \neq 0$$

其中
$$d_1, \cdots, d_r \neq 0$$

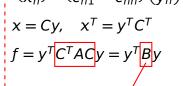
其中
$$d_1, \cdots, d_r \neq 0$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n$$

+ \dots \dots

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x^T = y^T C^T$$



$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$
 $f = x^T A x$ 一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \cdots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \cdots + c_{nn}y_n \end{cases}$$

代入二次型 f 后,可化为
$$f = d_1y_1^2 + \cdots + d_ry_r^2 \quad 标准形$$

其中 $d_1, \cdots, d_r \neq 0$

注

§5.1, 5.2

$$x = Cy$$
, $x^T = y^T C^T$

$$X = y \in I$$

$$^{T}ACy = y^{T}By$$

$$f = y^T C^T A C y = y^T B y$$

 $f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$ $f = x^T A x$ 一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$
代入二次型 f 后,可化为

定理 对任意n元二次型

 $f = d_1 y_1^2 + \cdots + d_r y_r^2$ 标准形 其中 $d_1, \dots, d_r \neq 0$ 注

§5.1, 5.2

 \bullet r = r(B)

$$f = y^{T} C^{T} A C y = y^{T} B y$$

$$\begin{pmatrix} d_{1} & & \\ & \ddots & \\ & & d_{r} & \end{pmatrix}$$

 $\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

x = Cy, $x^T = y^T C^T$

一定存在非退化线性变换
$$f_{X_1} = c_{11}y_1 + \cdots$$

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$
代入二次型 f 后,可化为

 $f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$

 $f = d_1 y_1^2 + \cdots + d_r y_r^2$ 标准形

其中
$$d_1, \dots, d_r \neq 0$$

§5.1, 5.2

$$r = r(B) = r(A);$$

 $\begin{pmatrix} \chi_1 \\ \vdots \\ \chi_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

$$x = Cy, \quad x^T = y^T C^T$$

$$\int ACy = y^T By$$

 $f = y^T C^T A C y = y^T B y$



$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型 f 后,可化为
$$f = d_1y_1^2 + \dots + d_ry_r^2 \quad 标准形$$

其中
$$d_1, \cdots, d_r \neq 0$$

§5.1, 5.2

r = r(B) = r(A); d; 具体取值不唯一

 $f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$

 $f = y^T C^T A C y = y^T B y$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x^T = y^T C^T$$

$$f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$$

一定存在非退化线性变换
$$\begin{cases} X_1 = C_{11}y_1 + \cdots + C_{1n}y_n \\ \vdots \\ X_n = C_{n1}y_1 + \cdots + C_{nn}y_n \end{cases}$$

$$x_n = c_{n1}y_1 + \cdots$$

代入二次型 f 后,可化为

代入二次型
$$f$$
 后,可化为
$$f = d_1 y_1^2 + \cdots + d_r y_r^2 \quad 标准形$$

其中
$$d_1, \cdots, d_r \neq 0$$

注

- r = r(B) = r(A); d_i 具体取值不唯一
 - 可以证明 d_1, \ldots, d_r 中正、负数的个数唯一:

x = Cy, $x^T = y^T C^T$

 $\begin{pmatrix} \chi_1 \\ \vdots \\ \chi_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

$$x^T = y^T C^T$$

 $f = y^T C^T A C y = y^T B y$

其中 $d_1, \dots, d_r \neq 0$ 注

 $f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$ 一定存在非退化线性变换

$$\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$$

代入二次型 f 后,可化为
$$f = d_1y_1^2 + \dots + d_ry_r^2 \quad 标准形$$

- r = r(B) = r(A); d; 具体取值不唯一
- 可以证明 d₁,..., dr 中正、负数的个数唯一: 1. 正惯性指标: *d*₁,..., *d*_r 中正数的个数

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$x = Cy, \quad x^T = y^T C^T$$

 $f = y^T C^T A C y = y^T B y$

定理 对任意n元二次型

定理 对任意n元二次型 $f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + 2a_{1n}x_1x_n + \dots + a_{nn}x_n^2$ 一定存在非退化线性变换 $\begin{cases} x_1 = c_{11}y_1 + \dots + c_{1n}y_n \\ \vdots \\ x_n = c_{n1}y_1 + \dots + c_{nn}y_n \end{cases}$ $\begin{pmatrix} \chi_1 \\ \vdots \\ \chi_n \end{pmatrix} = \begin{pmatrix} c_{11} \cdots c_{1n} \\ \vdots \\ c_{n1} \cdots c_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ x = Cy, $x^T = y^T C^T$ 代入二次型f后,可化为 $f = y^T C^T A C y = y^T B y$ $f = d_1 y_1^2 + \dots + d_r y_r^2$ 标准形 其中 $d_1, \dots, d_r \neq 0$ 注 r = r(B) = r(A); d; 具体取值不唯一 可以证明 d₁,..., dr 中正、负数的个数唯一: 1. 正惯性指标: d_1, \ldots, d_r 中正数的个数 2. 负惯性指标: d_1, \ldots, d_r 中负数的个数 二次型与对称矩阵

想法: a² + 2ab =

• 想法: $a^2 + 2ab = a^2 + 2ab + b^2 - b^2 =$

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 $a^2 + 2ab + 2ac =$

• 想法:
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 $a^2 + 2ab + 2ac = a^2 + 2a(b+c)$
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= $x_1^2 + 2x_1(x_2 + x_3) + (x_2 + x_3)^2 - (x_2 + x_3)^2 + 2x_2^2 + 4x_2x_3 + x_3^2$
= $(x_1 + x_2 + x_3)^2 +$

• 想法:
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例 配方法化二次型为标准形

 $=(x_1+x_2+x_3)^2+(x_2+x_3)^2-x_3^2$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= x_1^2 + 2x_1(x_2 + x_3) + (x_2 + x_3)^2 - (x_2 + x_3)^2 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3 + x_3^2 - x_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

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$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$

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$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases}$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3 + x_3^2 - x_3^2$$

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$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases}$$

则
$$f = y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$f = y_1^2 + y_2^2 - y_3^2$$

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$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \\ x_3 = y_3 \end{cases}$$

$$\emptyset \qquad f = y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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作线性变量代换

$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = (x_1 + y_2) \\ x_3 = x_3 \end{cases}$$

则 $f = y_1^2 + y_2^2 - y_3^2$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = \begin{pmatrix} 1 - 1 & 0 \\ x_3 = y_3 \end{pmatrix}$$

$$\begin{cases} f = y_1^2 + y_2^2 - y_3^2 \end{cases} \Rightarrow \begin{cases} f = y_1^2 + y_1^2 - y_2^2 - y_1^2 - y_1^2$$

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$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3 + x_3^2 - x_3^2$$

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$$\begin{cases} f = y_1^2 + y_2^2 - y_3^2 \end{cases} \Rightarrow \begin{cases} f = y_1^2 + y_1^2 - y_2^2 - y_2^2 \end{cases} \Rightarrow \begin{cases} f = y_1^2 + y_1^2 - y_1$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

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$$\begin{cases} f = y_1^2 + y_2^2 - y_3^2 \end{cases}$$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

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作线性变量代换

$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \ \Pi^{\frac{1}{2}}} y$$

则 $f = y_1^2 + y_2^2 - y_3^2$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= x_1^2 + 2x_1(x_2 + x_3) + (x_2 + x_3)^2 - (x_2 + x_3)^2 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

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作线性变量代换

$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \vec{\eta} \not\equiv} y$$

则 $f = y_1^2 + y_2^2 - y_3^2$

MA UNIVERSE

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= x_1^2 + 2x_1(x_2 + x_3) + (x_2 + x_3)^2 - (x_2 + x_3)^2 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3 + x_3^2 - x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \exists j \not\equiv} y$$

则 $f = y_1^2 + y_2^2 - y_3^2$

注 正惯性指标 = 2; 负惯性指标 =

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= x_1^2 + 2x_1(x_2 + x_3) + (x_2 + x_3)^2 - (x_2 + x_3)^2 + 2x_2^2 + 4x_2x_3 + x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3$$

$$= (x_1 + x_2 + x_3)^2 + x_2^2 + 2x_2x_3 + x_3^2 - x_3^2$$

$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + x_2 + x_3 \\ y_2 = x_2 + x_3 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \exists j \not\equiv} y$$

则 $f = y_1^2 + y_2^2 - y_3^2$

注 正惯性指标 = 2; 负惯性指标 = 1



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

配方法
= $(x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$

配方法
= $(x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$

$$= y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x$

$$= y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$

变量代换
$$y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$$

$$= y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$= y_1^2 + y_2^2 - y_3^2$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2 = x^T A x$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$= y_1^2 + y_2^2 - y_3^2$$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2 = x^T A x$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 + y_2^2 - y_2^2 = y^T C^T A C y$$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2 = x^T A x$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 + y_2^2 - y_3^2 = y^T C^T A x$$

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2 = x^T A x$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 + y_2^2 - y_3^2$$

$$= y^T C^T A C y$$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$



が結

$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2$$
 = $x^T A x$
配方法
= $(x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$
变量代換 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$
= $y_1^2 + y_2^2 - y_3^2$ = $y^T C^T A C y$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}}_{C^T} \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 &$$



$$f = x_1^2 + 2x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + x_3^2 = x^T A x$$
配方法
$$= (x_1 + x_2 + x_3)^2 + (x_2 + x_3)^2 - x_3^2$$
变量代换 $y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 + y_2^2 - y_3^2 = y^T C^T A x$$

特别地,找到了可逆阵 C,使得

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}}_{C^T} \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 &$$



 $= y^T C^T A C y$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

= $x_1^2 + 2x_1 \cdot (2x_2 + 2x_3)$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

= $x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

= $x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$
+ $2x_2^2 + 8x_2x_3 + 4x_3^2$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \end{cases}$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_3 = y_1 \\ x_3 = y_2 \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_2 = y_2 \\ x_3 = y_2 \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - 2y_2 - 2y_3 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases}$$

$$f = y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - 2y_2 - 2y_3 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases} \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 2 - 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \ \square \mid \Psi} y$$

$$f = y_1^2 - 2y_2^2$$



$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - 2y_2 - 2y_3 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases} \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 2 - 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \ \vec{\eta} \not\equiv} y$$

则

$$f = y_1^2 - 2y_2^2$$

注 正惯性指标 = ; 负惯性指标 =

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - 2y_2 - 2y_3 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases} \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 2 - 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \ \overline{\eta} \not E} y$$

则

$$f = y_1^2 - 2y_2^2$$

注 正惯性指标 = 1; 负惯性指标 =



$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= x_1^2 + 2x_1 \cdot (2x_2 + 2x_3) + (2x_2 + 2x_3)^2 - (2x_2 + 2x_3)^2$$

$$+ 2x_2^2 + 8x_2x_3 + 4x_3^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

作线性变量代换

$$\begin{cases} y_1 = x_1 + 2x_2 + 2x_3 \\ y_2 = x_2 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 - 2y_2 - 2y_3 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases} \Rightarrow x = \underbrace{\begin{pmatrix} 1 - 2 - 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C: \ \exists \not \sqsubseteq} y$$

则

$$f = y_1^2 - 2y_2^2$$

注 正惯性指标 = 1; 负惯性指标 = 1



$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

配方法
 $= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

$$=y_1^2-2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x$

$$= y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$= y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$= y_1^2 - 2y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T Ax$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$= y_1^2 - 2y_2^2$$



$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T Ax$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

$$y^T C^T A C y$$



 $=y_1^2-2y_2^2$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T A x$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 - 2y_2^2 = y^T C^T A x$$

$$^{T}C^{T}ACy$$



第

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T A x$$

配方法
 $= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$
 $= y_1^2 - 2y_2^2$ $= y^T C^T A C y$

$$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$



が结

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T A x$$

配方法
 $= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$
 $= y_1^2 - 2y_2^2$ $= y^T C^T A C y$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}}_{C^{T}} \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \\ 2 & 4 & 4 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$



$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2 = x^T A x$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= y_1^2 - 2y_2^2$$

$$= y^T C^T A C y$$

特别地,找到了可逆阵 C,使得

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}}_{C^{T}} \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \\ 2 & 4 & 4 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$



$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$=x_1x_2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

= $x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - (-\frac{1}{2} x_1 + x_2)^2 + (-\frac{1}{2} x_1 + x_2)^2 + 2x_3 (-\frac{1}{2} x_1 + x_2) + x_3^2$$

$$= + (-\frac{1}{2} x_1 + x_2 + x_3)^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

 $= -\frac{1}{4}x_1^2 + +2x_1x_2 - x_2^2$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

= $x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$

 $= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -(x_1-x_2)^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$=x_1x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$=\frac{3}{4}x_1^2-(x_1-x_2)^2$$

 $= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1 x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

= $x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

作线性变量代换
$$\begin{cases} y_1 = x_1 \\ y_2 = x_1 - x_2 \\ y_3 = -\frac{1}{2}x_1 + x_2 + x_3 \end{cases}$$



$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$=x_1x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 + x_2\right)^2 + 2x_3\left(-\frac{1}{2}x_1 + x_2\right) + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + \left(-\frac{1}{2}x_1 + x_2 + x_3\right)^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$\text{作线性变量代换}$$

$$\begin{cases} y_1 = x_1 \\ y_2 = x_1 - x_2 \\ y_3 = -\frac{1}{2}x_1 + x_2 + x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 \\ \end{cases}$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

= $x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$(1 - x_2)^2 + (-$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

作线性变量代换
$$\begin{cases} y_1 = x_1 \\ y_2 = x_1 - x_2 \\ y_3 = -\frac{1}{2}x_1 + x_2 + x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 \\ x_2 = y_1 - y_2 \end{cases}$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_1$$
$$= x_1 x_2 - \left(-\frac{1}{2}x_1 + x_2\right)^2 + \left(-\frac{1}{2}x_1 +$$

$$= x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$$

$$-(-\frac{1}{2}x_1+x_2)^2+(-$$

$$\frac{1}{2}x_1 + x_2)^2 + (-$$

$$(2x_3 + x_3)^2 + (--1)^2$$

$$(x_1 + x_2)^2 + 2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$\begin{cases} y_1 = x_1 \\ y_2 = x_1 - x_2 \\ y_3 = -\frac{1}{2}x_1 + x_2 + x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 \\ x_2 = y_1 - y_2 \\ x_3 = -\frac{1}{2}y_1 + y_2 + y_3 \end{cases}$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_1$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

= $x_1 x_2 - (-\frac{1}{2}x_1 + x_2)^2 + (-\frac{1}{2}x_1 + x_2)^2 + 2x_3(-\frac{1}{2}x_1 + x_2) + x_3^2$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$(2x_2x_3 + x_3^2 + x_2)^2 + (-$$

$$x_2x_3 + x$$

$$\frac{1}{2}x_1 + x_2$$

$$= -\frac{1}{4}x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= -\frac{1}{4}x_1^2 + x_1^2 - x_1^2 + 2x_1x_2 - x_2^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$



$$\frac{1}{2}$$

$$^{2} = \frac{3}{4}y_{1}^{2} -$$

$$y^2 = \frac{3}{4}y_1^2 - \frac{3}{4}y_1^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 = \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$(x_1 = y_1) \qquad (1)$$

作线性变量代换

$$f = x_1x_2 - x_1x_3 + 2x_2x_3 + x_3^2$$

配方法

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$f = x_1x_2 - x_1x_3 + 2x_2x_3 + x_3^2$$

配方法

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$f = x_1x_2 - x_1x_3 + 2x_2x_3 + x_3^2$$

配方法

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$
变量代换 $y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$
配方法
$$= \frac{3}{4} x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2} x_1 + x_2 + x_3)^2$$
变量代换 $y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 - 2 - 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} y$

$$= \frac{3}{4} y_1^2 - y_2^2 + y_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$
变量代换 $y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{C} y$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

 $f = x_1x_2 - x_1x_3 + 2x_2x_3 + x_3^2$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$
变量代换 $y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{C} y$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$\mathfrak{E} = (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$\mathfrak{E} = (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$\mathfrak{E} = (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$



 $= x^T A x$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

变量代换
$$y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{y} y$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

 $= y^T C^T A C y$

 $= x^T A x$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

变量代换
$$y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{C} y$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$



 $= x^T A x$

 $= y^T C^T A C y$

小结
$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$-y_2^2 + y_3^2$$

变量代换
$$y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} y$$

$$\Rightarrow x = \left(\begin{array}{cc} 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{array}\right)$$

$$\Rightarrow x = \underbrace{\begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{C} y$$

$$= y^{\mathsf{T}} C^{\mathsf{T}} A C y$$

$$\begin{pmatrix} \frac{3}{4} \\ -1 \\ 1 \end{pmatrix}$$

 $= x^T A x$



§5.1, 5.2

小结 $f = x_1x_2 - x_1x_3 + 2x_2x_3 + x_2^2$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$\text{genth} y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} y$$

$$= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$$

$$\underbrace{\begin{pmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}_{\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & 0 & 1 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}} = \begin{pmatrix} \frac{3}{4} \\ -1 \\ 1 \end{pmatrix}$$

 $= y^T C^T A C y$

 $= x^T A x$

$$\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{4} & 1 \end{pmatrix}$$



小结 $f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

变量代换
$$y = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{C} y$$

特别地,找到了可逆阵
$$C$$
,使得

$$\underbrace{\begin{pmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{0}} \underbrace{\begin{pmatrix} 0 & \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{\mathbf{0}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{\mathbf{0}} = \begin{pmatrix} \frac{3}{4} \\ -1 \\ 1 \end{pmatrix}$$



 $= \frac{3}{4}y_1^2 - y_2^2 + y_3^2$

 $= x^T A x$

 $= y^T C^T A C y$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

配方法
= $(x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

$$(\sqrt{2}x_2)^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - (\sqrt{2}x_2)^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
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$$= (x_1 + 2x_2 + 2x_3)^2 - (\sqrt{2}x_2)^2$$

$$=y_1^2-y_2^2$$

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$
配方法
$$= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$$

$$= (x_1 + 2x_2 + 2x_3)^2 - (\sqrt{2}x_2)^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} x$

$$= y_1^2 - y_2^2$$

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$$= y_1^2 - y_2^2$$

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$$=y_1^2-y_2^2$$

 $\begin{pmatrix} 1 & -1 & \\ & 0 & \end{pmatrix}$



重用があったが

$$f = x_1^2 + 4x_1x_2 + 4x_1x_3 + 2x_2^2 + 8x_2x_3 + 4x_3^2$$

配方法
 $= (x_1 + 2x_2 + 2x_3)^2 - 2x_2^2$

$$= (x_1 + 2x_2 + 2x_3)^2 - (\sqrt{2}x_2)^2$$
变量代换 $y = \begin{pmatrix} 1 & 2 & 2 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} x \Rightarrow x = \underbrace{\begin{pmatrix} 1 & -\sqrt{2} & -2 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} y$

特别地,找到了可逆阵 C,使得

 $= y_1^2 - y_2^2$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -\sqrt{2} & 1/\sqrt{2} & 0 \\ -2 & 0 & 1 \end{pmatrix}}_{C^{T}} \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \\ 2 & 4 & 4 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & -\sqrt{2} & -2 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

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$$= (\frac{\sqrt{3}}{2}x_1)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 - (x_1 - x_2)^2$$

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$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= (\frac{\sqrt{3}}{2}x_1)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 - (x_1 - x_2)^2 = y_1^2 + y_2^2 - y_3^2$$

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变量代换
$$y = \begin{pmatrix} \sqrt{3}/2 & 0 & 0 \\ -1/2 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix} x$$

$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

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配方法

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

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変量代換
$$y = \begin{pmatrix} \sqrt{3}/2 & 0 & 0 \\ -1/2 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 2/\sqrt{3} & 0 & 0 \\ 1/\sqrt{3} & -1 & 1 \\ 2/\sqrt{3} & -1 & 0 \end{pmatrix} y$$

 $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$



$$f = x_1 x_2 - x_1 x_3 + 2x_2 x_3 + x_3^2$$

配方法

$$= \frac{3}{4}x_1^2 - (x_1 - x_2)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2$$

$$= (\frac{\sqrt{3}}{2}x_1)^2 + (-\frac{1}{2}x_1 + x_2 + x_3)^2 - (x_1 - x_2)^2 = y_1^2 + y_2^2 - y_3^2$$

变量代换
$$y = \begin{pmatrix} \sqrt{3}/2 & 0 & 0 \\ -1/2 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix} x \Rightarrow x = \begin{pmatrix} 2/\sqrt{3} & 0 & 0 \\ 1/\sqrt{3} & -1 & 1 \\ 2/\sqrt{3} & -1 & 0 \end{pmatrix} y$$

特别地,找到了可逆阵 C,使得

$$\underbrace{\begin{pmatrix} 2/\sqrt{3} \ 1/\sqrt{3} \ 2/\sqrt{3} \ 0 & -1 & -1 \ 0 & 1 & 0 \end{pmatrix}}_{C^T} \underbrace{\begin{pmatrix} 0 & \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ -\frac{1}{2} & 1 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 2/\sqrt{3} & 0 & 0 \\ 1/\sqrt{3} - 1 & 1 \\ 2/\sqrt{3} - 1 & 0 \end{pmatrix}}_{C} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

定理 任意二次型 $f(x_1, \ldots, x_n)$ 都可以通过非退化线性变换

$$x = Cy$$

$$f = y_1^2 + \dots + y_p^2 - y_{p+1}^2 - \dots - y_r^2$$

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$$\left(\begin{array}{cc}I_p&&\\&-I_{r-p}&\\&&O\end{array}\right)$$

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$$A \qquad \left(\begin{array}{cc} I_p & & \\ & -I_{r-p} & \\ & & O \end{array}\right)$$

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$$C^{\mathsf{T}} A C = \left(\begin{array}{cc} I_{p} & & \\ & -I_{r-p} & \\ & & O \end{array} \right)$$

定理 任意二次型 $f(x_1, \ldots, x_n)$ 都可以通过非退化线性变换

$$x = Cy$$

化为

$$f = y_1^2 + \dots + y_p^2 - y_{p+1}^2 - \dots - y_r^2$$

也就是,任意对称矩阵 A,都存在可逆矩阵 C,使得

$$C^{\mathsf{T}}AC = \left(\begin{array}{cc} I_{p} & & \\ & -I_{r-p} & \\ & & O \end{array}\right)$$

定理 任意二次型 $f(x_1, \ldots, x_n)$ 都可以通过非退化线性变换

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也就是,任意对称矩阵 A,都存在可逆矩阵 C,使得

$$C^{\mathsf{T}}AC = \left(\begin{array}{cc} I_{p} & & \\ & -I_{r-p} & \\ & & O \end{array}\right)$$

注
•
$$r = r(A)$$
, $p =$ 正惯性指标, $r - p =$ 负惯性指标



定理 任意二次型 $f(x_1, \ldots, x_n)$ 都可以通过非退化线性变换

$$x = Cy$$

化为

$$f = y_1^2 + \dots + y_p^2 - y_{p+1}^2 - \dots - y_r^2$$

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$$C^{\mathsf{T}}AC = \left(\begin{array}{cc} I_{p} & & \\ & -I_{r-p} & \\ & & O \end{array}\right)$$

注

● p 是由 A 唯一确定的



合同,合同的等价条件

定义 设 A, B 为两个 n 阶方阵,若存在可逆 n 阶方阵 C,使得 $C^TAC = B$

则称 A合同于B, 记为 $A \simeq B$

合同, 合同的等价条件

定义 设 *A, B* 为两个 *n* 阶方阵,若存在可逆 *n* 阶方阵 *C*,使得

$$C^TAC = B$$

则称 A合同于B, 记为 $A \simeq B$

定理 任意对称矩阵 A, 都成立

$$A \simeq \left(\begin{array}{cc} I_p & & \\ & -I_{r-p} & \\ & & O \end{array} \right)$$

合同, 合同的等价条件

定义 设 A, B 为两个 n 阶方阵,若存在可逆 n 阶方阵 C,使得

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定理 任意对称矩阵 A,都成立

$$A \simeq \left(\begin{array}{cc} I_{\rho} & & \\ & -I_{r-\rho} & \\ & & O \end{array} \right)$$

定理 设 A, B 为对称矩阵,则 $A \simeq B$ 的充分必要条件是 A, B 具有相同的规范形(也就是,秩、正惯性指标都相等)