



# 教学要求

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掌握求解：

◇ 二阶行列式计算

♣ 三阶行列式计算

- 行列式的概念来源于线性方程组的求解问题
- 17 世纪末由日本数学家关孝和及德国数学家莱布尼茨引入



二阶行列式	$\longleftrightarrow$	二元线性方程组
三阶行列式	$\longleftrightarrow$	三元线性方程组
$\vdots$		$\vdots$
$n$ 阶行列式	$\longleftrightarrow$	$n$ 元线性方程组
$\vdots$		$\vdots$

## 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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用消元法可解：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

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• 定义  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

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$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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**练习** 利用二阶行列式求解下面二元线性方程组

1.  $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$

公式:

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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## 三元线性方程组

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+

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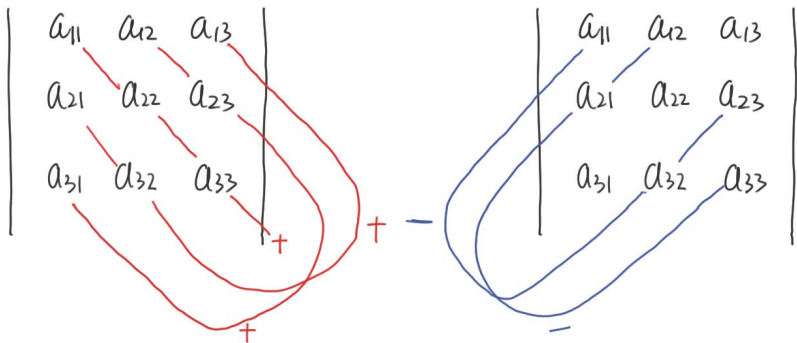
**规律** 不同行不同列的 3 个元素乘积，共  $3! = 6$  个，并且：

The diagram illustrates the expansion of a 3x3 determinant using the rule of Sarrus. It shows two determinants side-by-side, separated by a minus sign. The first determinant is expanded with red lines, showing the sum of three diagonal products (a<sub>11</sub>a<sub>22</sub>a<sub>33</sub>, a<sub>12</sub>a<sub>23</sub>a<sub>31</sub>, a<sub>13</sub>a<sub>21</sub>a<sub>32</sub>) and the subtraction of three anti-diagonal products (a<sub>11</sub>a<sub>23</sub>a<sub>32</sub>, a<sub>12</sub>a<sub>21</sub>a<sub>33</sub>, a<sub>13</sub>a<sub>22</sub>a<sub>31</sub>). The second determinant is shown with blue lines, representing the same structure.

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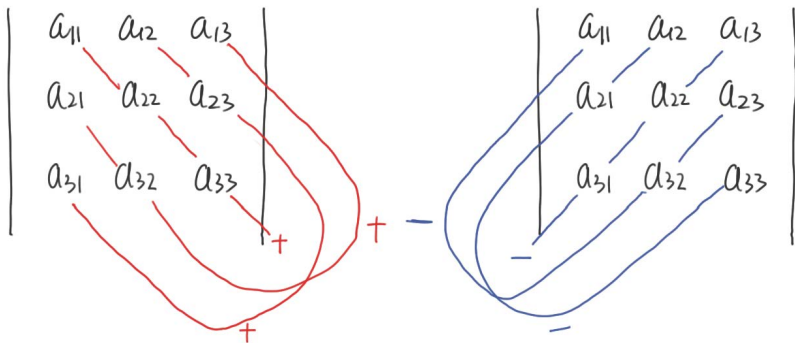
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$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}},$$

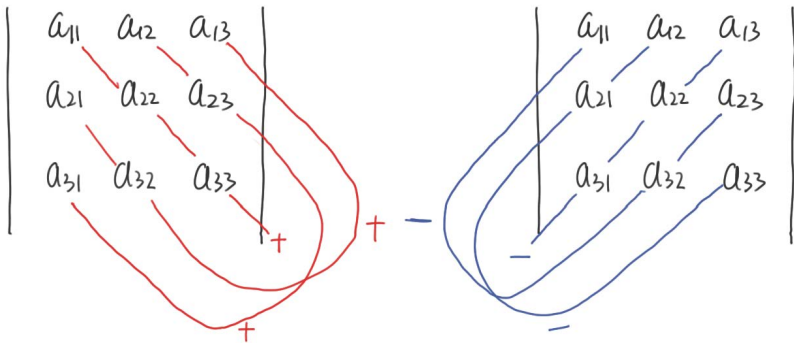
这时方程组

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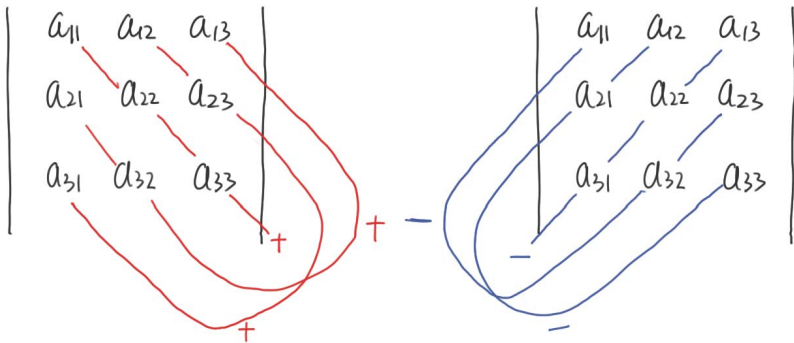
$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

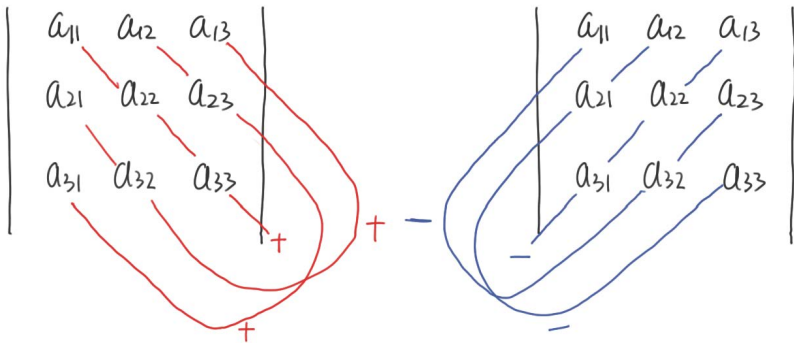
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

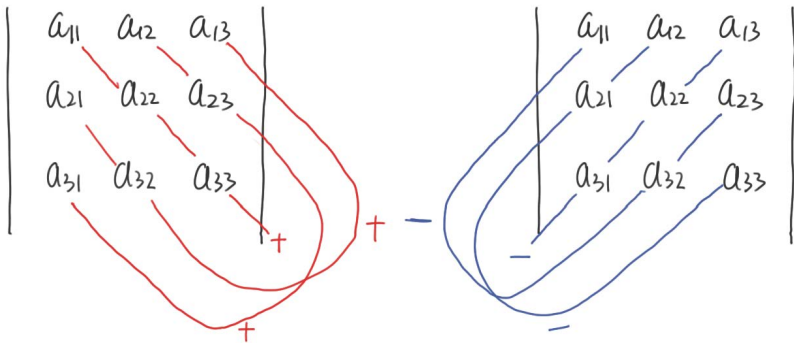


例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &-1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned}$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

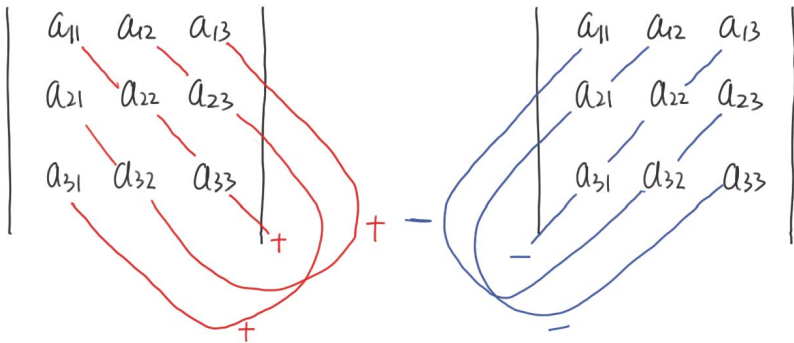




例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{matrix} = -58$$

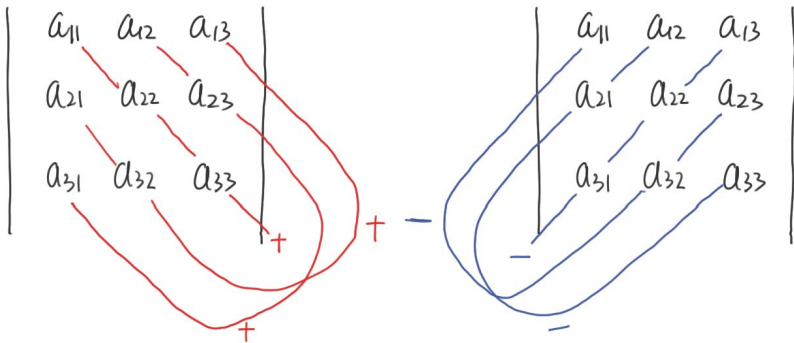
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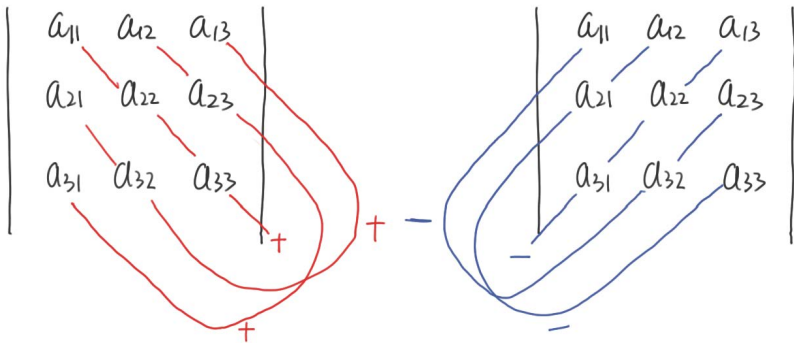
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{matrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{matrix}$$



### 例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{aligned} &1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ &- 1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{aligned} = -2$$

例  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

例  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$



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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

例  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1$$



例  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

解 因为

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例  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

所以  $a \neq 0$  或  $b \neq 0$ 。