§6.4 微积分基本定理

2015-2016 学年 II

教学要求

- $\diamondsuit$
- $\clubsuit$
- $\heartsuit$
- $\spadesuit$

Outline of §6.4

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

定义 假设 $f(x)$ 是区间 $[a, b]$ 上的连续函数

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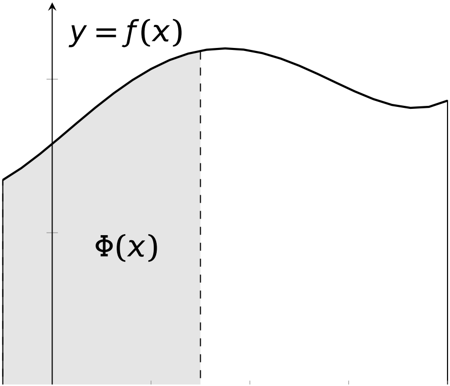

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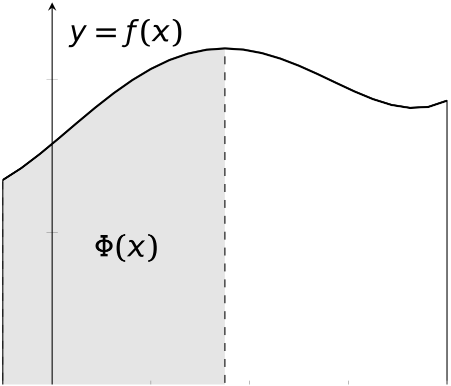

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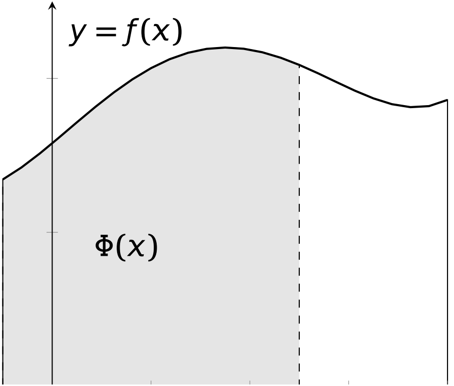

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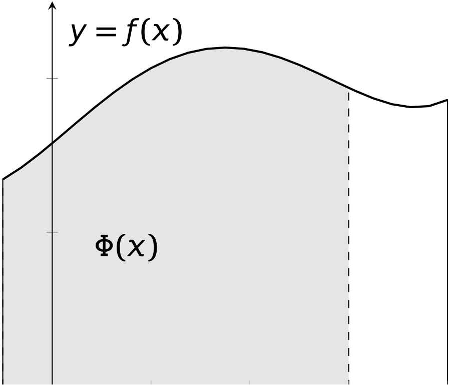

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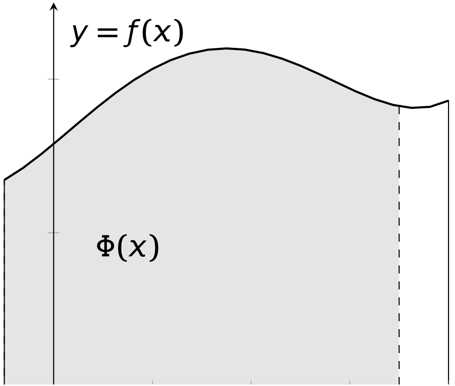

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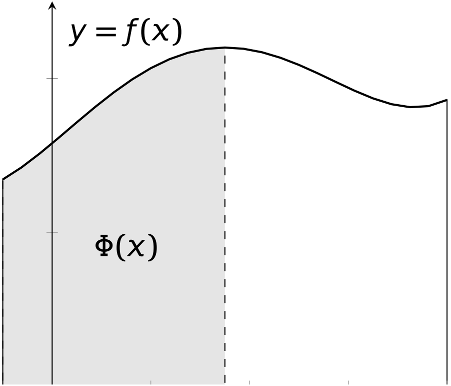

$$\Phi(x) = \int_a^x f(t)dt \qquad \forall x \in [a, b]$$

$$\Phi'(x) = \left[\int_a^x f(t)dt\right]' =? \quad \forall x \in [a, b]$$

$$\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x) \quad \forall x \in [a, b]$$

微积分基本定理 设函数 $f(x)$ 在区间 $[a, b]$ 上连续，那么

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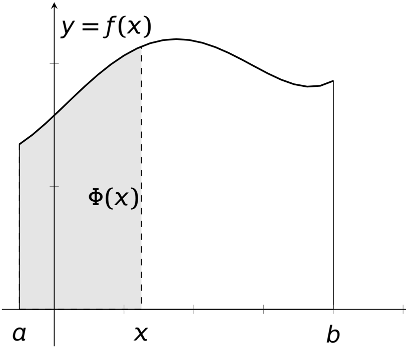

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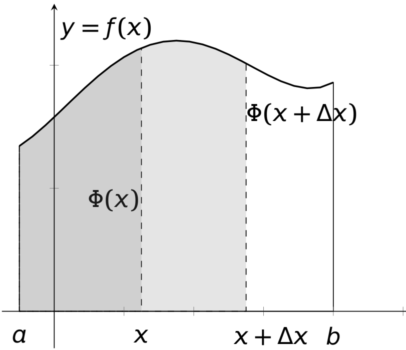

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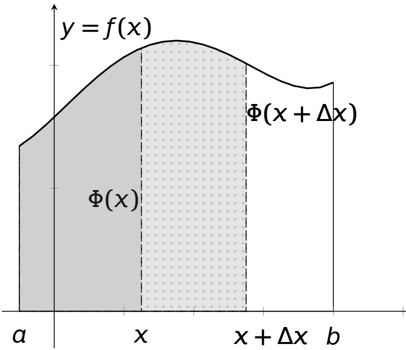

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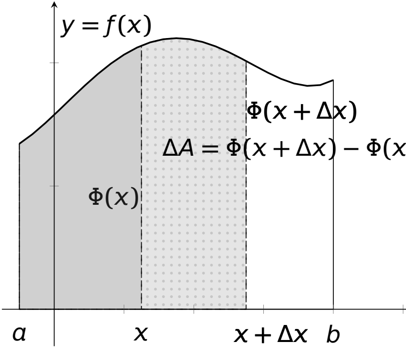

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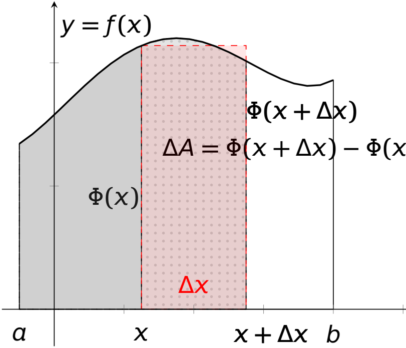

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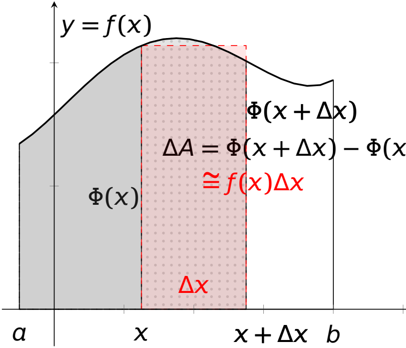

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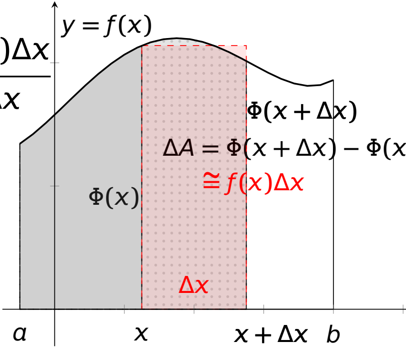

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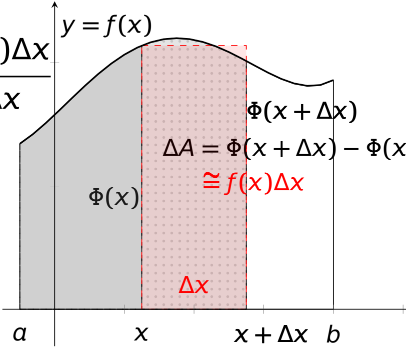

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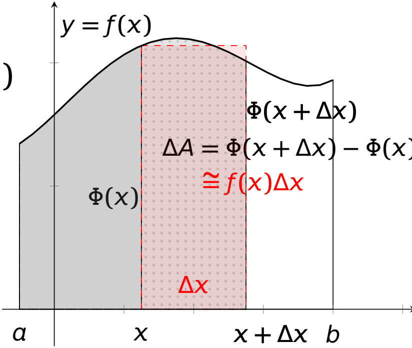

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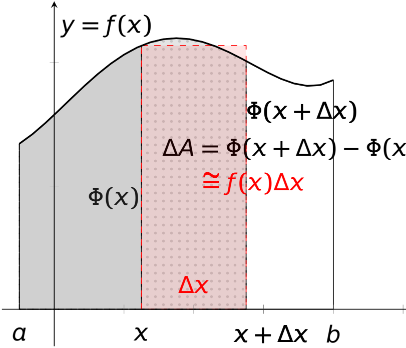

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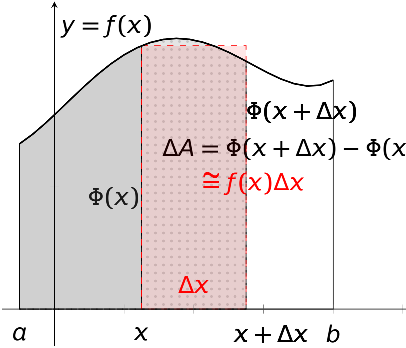

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$$\begin{aligned}
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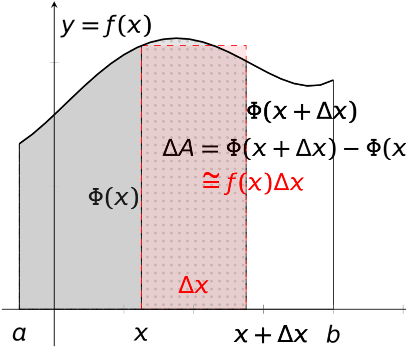
&= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left[\int_a^{x+\Delta x} f(t)dt - \int_a^x f(t)dt\right] \\
&= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\int_x^{x+\Delta x} f(t)dt \\
&= \lim_{\Delta x\to 0} \frac{1}{\Delta x}(f(\xi)\Delta x)
\end{aligned}$$



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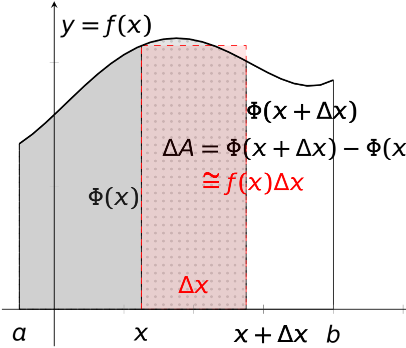

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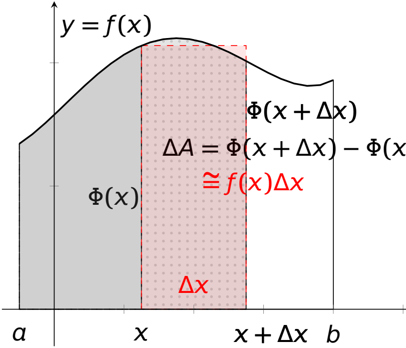

微积分基本定理 $\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x), \quad \forall x \in [a, b]$

例 $\left[\int_2^x e^{-t} \sin(t^2) dt\right]' =$ ____________.

微积分基本定理 $\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x), \quad \forall x \in [a, b]$

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \underline{\qquad\qquad\qquad}$

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \underline{\qquad\qquad\qquad}$

解

$$\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \left[-\int_0^x e^{-t}\sin(t^2)dt\right]'$$

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例 $\left[\int_2^x e^{-t}\sin(t^2)dt\right]'=\underline{e^{-x}\sin(x^2)}.$

例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]'=$ ____________

解

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例 $\left[\int_x^{-2} e^{\sin t}dt\right]'=$ ______

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例 $\left[\int_x^{-2} e^{\sin t}dt\right]' = \underline{-e^{\sin x}}$

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例 $\left[\int_x^{-2} e^{\sin t}dt\right]' = \underline{-e^{\sin x}}$

$$\because \quad \left[\int_x^{-2} e^{\sin t}dt\right]' = \left[-\int_{-2}^x e^{\sin t}dt\right]'$$

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \underline{\qquad\qquad\qquad}$

解

$$\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \left[-\int_0^x e^{-t}\sin(t^2)dt\right]' = -e^{-x}\sin(x^2).$$

例 $\left[\int_x^{-2} e^{\sin t}dt\right]' = \underline{-e^{\sin x}}$

$$\because \quad \left[\int_x^{-2} e^{\sin t}dt\right]' = \left[-\int_{-2}^x e^{\sin t}dt\right]' = - - e^{\sin x}.$$

注：

$$\left[\int_a^{\varphi(x)} f(t)dt\right]' =$$

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$$\left[\int_a^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)]\cdot$$

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$$\left[\int_a^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)]\cdot\varphi'(x).$$

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例

$\left[\int_1^{x^2}\cos t dt\right]' =$ ______________; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ ______________.

注：
$$\left[\int_a^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)]\cdot\varphi'(x).$$

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解 ：

$$\left[\int_1^{x^2}\cos t dt\right]' = \cos(x^2)\cdot$$

注：
$$\left[\int_a^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)]\cdot\varphi'(x).$$

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$\left[\int_1^{x^2}\cos t dt\right]' =$ ____________; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ ____________.

解：
$$\left[\int_1^{x^2}\cos t dt\right]' = \cos(x^2)\cdot(x^2)'$$

注：
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$\left[\int_1^{x^2}\cos t dt\right]' =$ ____________ ; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ ____________ .

解 ：
$$\left[\int_1^{x^2}\cos t dt\right]' = \cos(x^2)\cdot(x^2)' = 2x\cos(x^2)$$

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注：

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例

$$\left[\int_1^{x^2}\cos t dt\right]' = \_\_\_\_\_\_\_\_\_\_;\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' = \_\_\_\_\_\_\_\_\_\_.$$

解 ：

$$\left[\int_1^{x^2}\cos t dt\right]' = \cos(x^2)\cdot(x^2)' = 2x\cos(x^2)$$

$$\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' = -\left[\int_{-1}^{2x}\sqrt{1+t^2}dt\right]' = \sqrt{1+4x^2}\cdot(2x)'$$

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注：
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例 $\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' =$ ____________;

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解 ：

$$\left[\int_{x^3}^{x^2}\ln(1+t)dt\right]' = \left[\int_{x^3}^{0}\ln(1+t)dt + \int_0^{x^2}\ln(1+t)dt\right]'$$

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We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx =$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a)$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

牛顿—莱布尼茨公式

设 $f(x)$ 在区间 $[a, b]$ 上连续，$F(x)$ 是 $f(x)$ 任意一个原函数，则

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

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证明：

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证明：

$$\because\ \Phi(x) = \int_a^x f(t)dt \text{是} f(x) \text{的一个原函数}$$

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证明：

$$\because\ \Phi(x) = \int_a^x f(t)dt \text{是} f(x) \text{的一个原函数}$$

$$\therefore\ F(x) = \Phi(x) + C$$

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$$\because \Phi(x) = \int_a^x f(t)dt \text{是} f(x) \text{的一个原函数}$$

$$\therefore F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) =$$

牛顿—莱布尼茨公式

设 $f(x)$ 在区间 $[a, b]$ 上连续，$F(x)$ 是 $f(x)$ 任意一个原函数，则

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$$\therefore\ F(b) - F(a) = (\Phi(b) + C) -$$

牛顿—莱布尼茨公式

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$$\therefore\ F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$

$$= \Phi(b) - \Phi(a)$$

牛顿—莱布尼茨公式

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$$\begin{aligned}\therefore\ F(b) - F(a) &= (\Phi(b) + C) - (\Phi(a) + C)\\ &= \Phi(b) - \Phi(a)\\ &= \int_a^b f(t)dt - \int_a^a f(t)dt\end{aligned}$$

牛顿—莱布尼茨公式

设 $f(x)$ 在区间 $[a, b]$ 上连续，$F(x)$ 是 $f(x)$ 任意一个原函数，则

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

证明：

$$\because\ \Phi(x) = \int_a^x f(t)dt \text{是} f(x) \text{的一个原函数}$$

$$\therefore\ F(x) = \Phi(x) + C$$

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例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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解

$$\int_0^1 x^2 dx = \frac{1}{3}x^3\Big|_0^1$$

例 计算定积分

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练习 计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}}dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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解

$$\int_0^2 (2x-5)dx =$$

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$$\int_0^2 (2x-5)dx = (x^2-5x)$$

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$$\int_4^9 \frac{1}{\sqrt{x}}dx =$$

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$$\int_0^2 (2x-5)dx = (x^2-5x)\Big|_0^2 = -6-0$$

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例 计算定积分 $\int_0^2 |1-x|dx$.

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解
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例 计算定积分 $\int_0^2 |1-x|dx$.

解 $$\int_0^2 |1-x|dx$$

$$=\int_0^1 |1-x|dx+\int_1^2 |1-x|dx$$

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