§1.1 二阶三阶行列式

2015 - 2016 学年 I

教学要求

掌握求解：

- ♢ 二阶行列式计算
- ♣ 三阶行列式计算

Outline of §1.1

- 行列式的概念来源于线性方程组的求解问题
- 17 世纪末由日本数学家关孝和及德国数学家莱布尼茨引入

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二阶行列式	$\longleftrightarrow$	二元线性方程组
三阶行列式	$\longleftrightarrow$	三元线性方程组
$\vdots$		$\vdots$
n 阶行列式	$\longleftrightarrow$	n 元线性方程组
$\vdots$		$\vdots$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为

二元线性方程组

$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = a_{22}b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

二元线性方程组

$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = \ a_{22}b_1 & (1)\times a_{22} \\ a_{21}\,a_{12}x + a_{22}\,a_{12}y = \ a_{12}b_2 & (2)\times a_{12} \end{cases}$$

用消元法解：

$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x=\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}$$

$(2)\times a_{11}-(1)\times a_{21}$，消去 x，得：

$$y=\frac{a_{11}b_2-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22}-a_{12}a_{21}$

二元线性方程组

$$\begin{cases} a_{11}\,{\color{blue}a_{22}}x + a_{12}\,{\color{blue}a_{22}}y = {\color{blue}a_{22}}b_1 & (1)\times {\color{blue}a_{22}} \\ a_{21}\,{\color{blue}a_{12}}x + a_{22}\,{\color{blue}a_{12}}y = {\color{blue}a_{12}}b_2 & (2)\times {\color{blue}a_{12}} \end{cases}$$

用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\quad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}\, a_{11}x + a_{22}\, a_{11}y = \, a_{11}b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

二元线性方程组
$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = \ a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = \ a_{11}b_2 & (2)\times a_{11} \end{cases}$$
用消元法解：

$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x=\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2)\times a_{11}-(1)\times a_{21}$，消去 x，得：

$$y=\frac{a_{11}b_2-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22}-a_{12}a_{21}$

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

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$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

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$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

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- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为二阶行列式

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} =, \quad y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x =, \quad y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

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例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 $\underline{\lambda \neq 0 \text{且} \lambda \neq 3}$

解

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三元线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

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$$x =$$

三元线性方程组

$$\begin{cases} a_{11}\,a_{23}x + a_{12}\,a_{23}y + a_{13}\,a_{23}z = \ a_{23}b_1 & (1) \times a_{23} \\ a_{21}\,a_{13}x + a_{22}\,a_{13}y + a_{23}\,a_{13}z = \ a_{13}b_2 & (2) \times a_{13} \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$\begin{cases} a_{11}\,{\color{blue}a_{23}}x + a_{12}\,{\color{blue}a_{23}}y + a_{13}\,{\color{blue}a_{23}}z = \ {\color{blue}a_{23}}b_1 & (1) \times {\color{blue}a_{23}} \\ a_{21}\,{\color{blue}a_{13}}x + a_{22}\,{\color{blue}a_{13}}y + a_{23}\,{\color{blue}a_{13}}z = \ {\color{blue}a_{13}}b_2 & (2) \times {\color{blue}a_{13}} \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$
\begin{cases}
a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\
a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\
a_{31}x + a_{32}y + a_{33}z = b_3 & (3)
\end{cases}
$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \times a_{33} \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \times a_{13} \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$
\begin{cases}
a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \times a_{33} \\
a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\
a_{31}\,a_{13}x + a_{32}\,a_{13}y + a_{33}\,a_{13}z = \,a_{13}b_3 & (3) \times a_{13}
\end{cases}
$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$\begin{cases} a_{11}\,a_{33}x + a_{12}\,a_{33}y + a_{13}\,a_{33}z = \ a_{33}b_1 & (1) \times a_{33} \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}\,a_{13}x + a_{32}\,a_{13}y + a_{33}\,a_{13}z = \ a_{13}b_3 & (3) \times a_{13} \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$
\begin{cases}
a_{11}\,a_{33}x + a_{12}\,a_{33}y + a_{13}\,a_{33}z = a_{33}b_1 & (1)\times a_{33} \\
a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\
a_{31}\,a_{13}x + a_{32}\,a_{13}y + a_{33}\,a_{13}z = a_{13}b_3 & (3)\times a_{13}
\end{cases}
$$

用消元法解：

$(2)\times a_{13} - (1)\times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3)\times a_{13} - (1)\times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x =$$

三元线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}}$$

三元线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法解：

$(2) \times a_{13} - (1) \times a_{23}$，消去 z，得

$$(a_{21}a_{13} - a_{11}a_{23})x + (a_{22}a_{13} - a_{12}a_{23})y = b_2a_{13} - b_1a_{23}$$

$(3) \times a_{13} - (1) \times a_{33}$，消去 z，得：

$$(a_{31}a_{13} - a_{11}a_{33})x + (a_{32}a_{13} - a_{12}a_{33})y = b_3a_{13} - b_1a_{33}$$

得到

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

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规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \; +$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

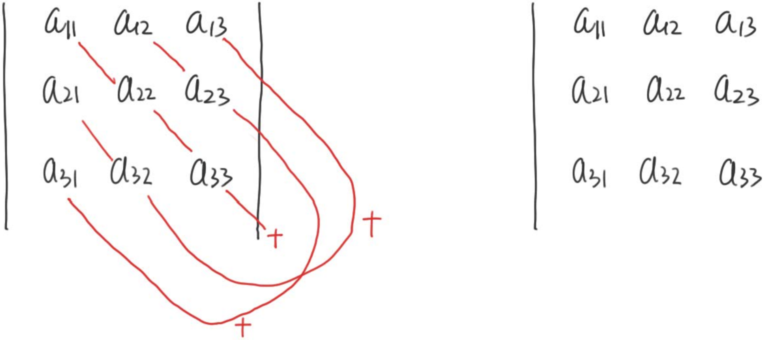
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$+$ $+$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

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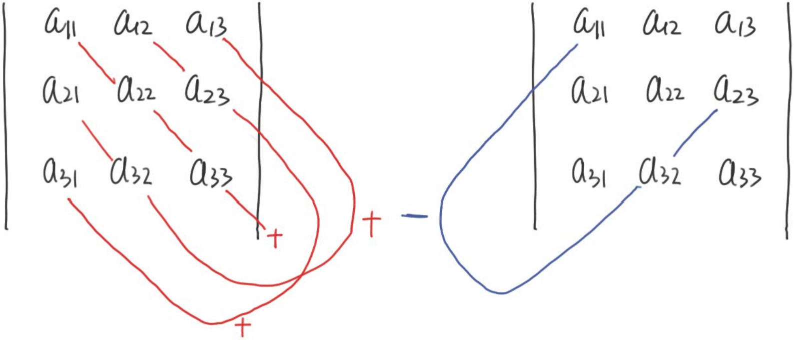
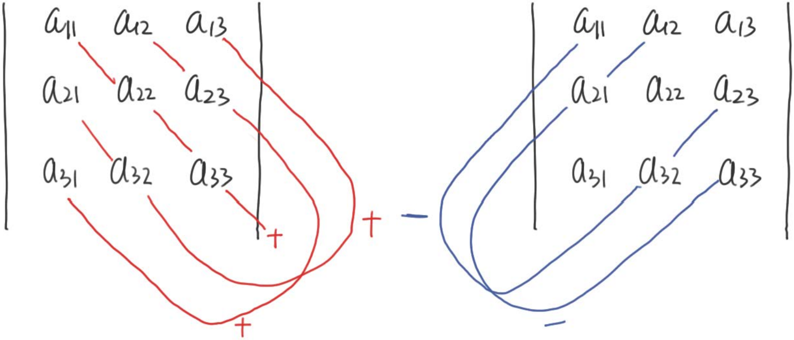


为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

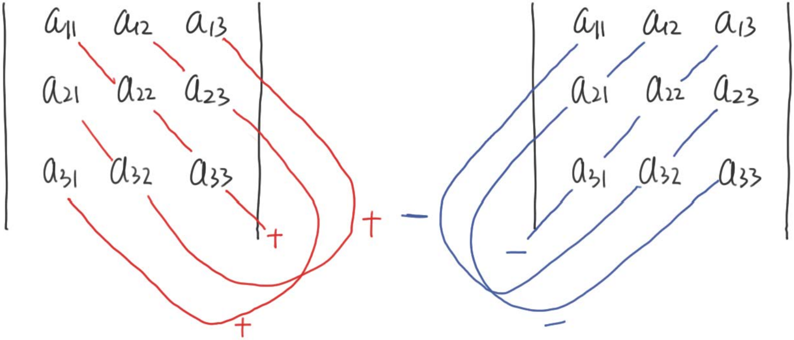
规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：



这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组

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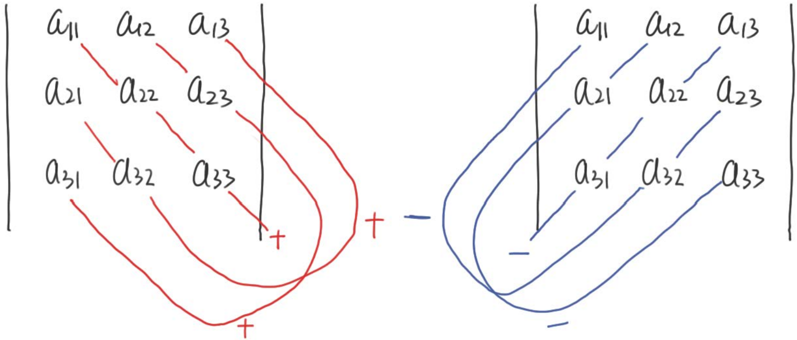

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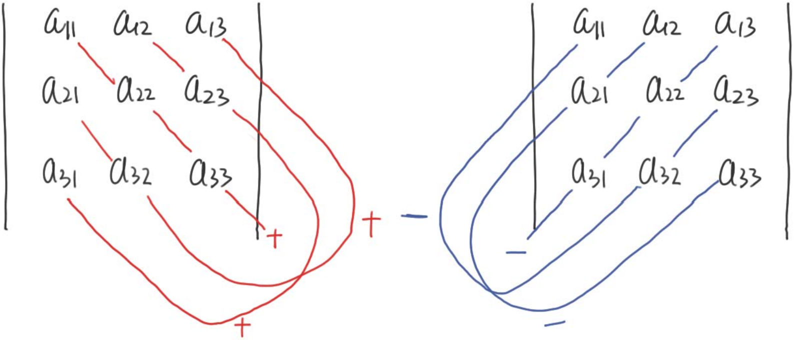

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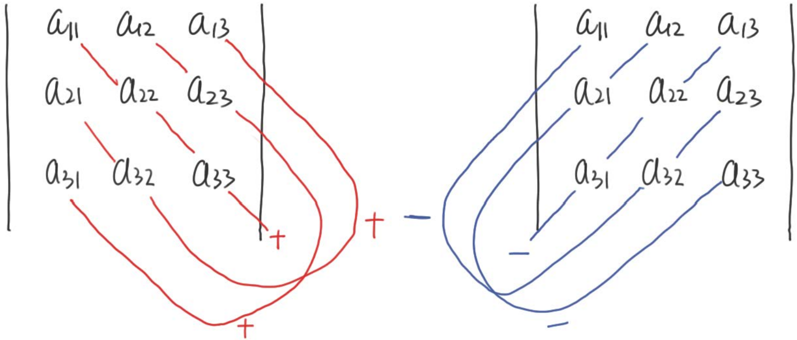

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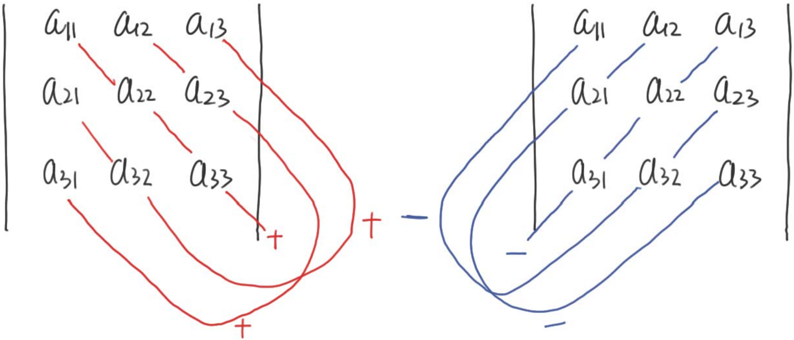

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