# §2.2 矩阵的运算

数学系 梁卓滨

2016 - 2017 学年 I 暑修班

定义 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则定义$$

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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$$\stackrel{\text{def}}{=} \begin{pmatrix} \\ \\ \\ \\ \end{bmatrix}$$

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$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & & & & \\ & & & & & \\ & & & & & \\ \end{pmatrix}$$

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§2.2 矩阵的运算 2/32 < ▷ △ ▽

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$$= (a_{ij} + b_{ij})_{m \times n}$$

§2.2 矩阵的运算 2/32  $extstyle extstyle extstyle 2 \ 2 \ \textstyle extstyle \ \textstyle \ \ \textstyle \ \textstyle$ 

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$$= (a_{ij} + b_{ij})_{m \times n}$$

称为矩阵 A. B 的和。

§2.2 矩阵的运算 2/32 < ▶ A ▼

矩阵 A, B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ \end{pmatrix}$$

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§2.2 矩阵的运算 3/32 ◁ ▷ △ ▼

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§2.2 矩阵的运算 3/32 < ▶ A ▼

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§2.2 矩阵的运算 3/32 ◁ ▷ △ ▼

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§2.2 矩阵的运算 3/32 < ▶ A ▼

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§2.2 矩阵的运算 3/32 < ▶ A ▼

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$$= (a_{ii} - b_{ii})_{m \times n}$$

§2.2 矩阵的运算 3/32 < ▷ △ ▽

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

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 $\S 2.2$  矩阵的运算  $4/32 \triangleleft \triangleright \vartriangle 
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 $\S 2.2$  矩阵的运算  $4/32 \triangleleft \triangleright \triangle \triangledown$ 

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32.2 矩阵的运算 4/32  $\triangleleft$   $\triangleright$   $\triangle$   $\triangledown$ 

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$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
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$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ & & \end{pmatrix}_{2 \times 3}$$

 $\S 2.2$  矩阵的运算  $4/32 \triangleleft \triangleright \vartriangle 
ota$ 

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$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
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32.2 矩阵的运算 4/32  $\triangleleft$   $\triangleright$   $\triangle$   $\triangledown$ 

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$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} =$$

§2.2 矩阵的运算 4/32 ✓  $\triangleright$  △  $\triangledown$ 

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,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$
$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} & & & \\ & & & \end{pmatrix}_{2 \times 3}$$

§2.2 矩阵的运算 4/32 ightarrow 6  $\sqrt{3}$   $\sqrt{3}$ 

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$
$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

§2.2 矩阵的运算 4/32 ightarrow 6  $\sqrt{3}$   $\sqrt{3}$ 

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$
$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ & & \end{pmatrix}_{2 \times 3}$$

§2.2 矩阵的运算 4/32 ightarrow 6  $\sqrt{3}$   $\sqrt{2}$   $\sqrt{2}$ 

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$
$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ & & & \end{pmatrix}_{2 \times 3}$$

§2.2 矩阵的运算 4/32 ightarrow 6 ightarrow 4/32 ightarrow 6 ightarrow

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$ 

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$
$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ -8 & -1 & 3 \end{pmatrix}_{2 \times 3}$$

§2.2 矩阵的运算 4/32 ightarrow 6 T

性质 设 A, B, C 均是  $m \times n$  矩阵, O 是  $m \times n$  零矩阵, 则

§2.2 矩阵的运算 5/32 **▽** ▷ △ ▽

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}$$
,  $B = (b_{ij})_{m \times n}$ , 则 
$$A + B =$$
 
$$B + A =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} =$$

$$B + A =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = ( )_{m \times n},$$

$$B + A =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = ( )_{m \times n}.$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 A + B = B + A。

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$
 则
$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 
$$A + B = B + A$$
。另外

$$A + O =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 
$$A + B = B + A$$
。另外

$$A + O = (a_{ii})_{m \times n} + (0)_{m \times n} =$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$
 则 
$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$
 
$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 
$$A + B = B + A$$
。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = ($$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$
则
$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以
$$A+B=B+A$$
。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n}$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$
 则
$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以
$$A+B=B+A$$
。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n} = (a_{ij})_{m \times n}$$

1. 
$$A + B = B + A$$

2. 
$$(A + B) + C = A + (B + C)$$

3. 
$$A + O = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n},$$
则
$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以
$$A+B=B+A$$
。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n} = (a_{ij})_{m \times n} = A.$$

定义 设 
$$A = (a_{ij})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

定义 设 
$$A = (a_{ij})_{m \times n}$$
,  $k$  为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

定义 设 
$$A = (a_{ij})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

 $=(ka_{ij})_{m\times n}$ 

矩阵的运算  $6/32 \triangleleft \triangleright \Delta \nabla$ 

定义 设  $A = (a_{ii})_{m \times n}, k$  为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

 $=(ka_{ii})_{m\times n}$ 

称为数 k 与矩阵 A 的数乘。

6/32 ⊲ ⊳ ∆ ⊽ 矩阵的运算

定义 设  $A = (a_{ij})_{m \times n}, k$  为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$
$$= (ka_{ii})_{m \times n}$$

称为数 k 与矩阵 A 的数乘。

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,求  $2A$ 

定义 设 
$$A = (a_{ii})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$
$$= (ka_{ij})_{m \times n}$$

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,求  $2A$ 

$$\mathbf{H} \ 2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} =$$

定义 设 
$$A = (a_{ii})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$
$$= (ka_{ij})_{m \times n}$$

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,求  $2A$ 

$$\mathbf{H} \ 2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \end{pmatrix}$$

定义 设 
$$A = (a_{ii})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$
$$= (ka_{ij})_{m \times n}$$

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,求  $2A$ 

$$M 2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ & & & \end{pmatrix}$$

定义 设 
$$A = (a_{ii})_{m \times n}, k$$
 为数,则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$
$$= (ka_{ij})_{m \times n}$$

例 
$$A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$$
,求  $2A$ 

$$\mathbf{H} \ 2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ -2 & 4 & 8 \end{pmatrix}$$

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求  $3A + 2B - 4C$ 

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

2.2 矩阵的运算 7/32 ◁ ▷ △ ▼

练习设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求3A + 2B - 4C

 $\begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$ 

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

矩阵的运算

 $oldsymbol{arphi}$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$(3 \ 4)^{3} (7 \ 6)^{3} (9 \ 3)^{3}$$

$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

$$= \begin{pmatrix} 3 & 20 \\ -13 & 12 \end{pmatrix}$$

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 矩阵的运算

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} \qquad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

练习设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且5A + 3X = B, 求X

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} \qquad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

 $7/32 \triangleleft \triangleright \triangle \nabla$ 

$$= (9 \ 12)^{+}(14 \ 12) (-13 \ 12)$$

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 3 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求  $3A + 2B - 4C$ 

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} \quad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设
$$A = \begin{pmatrix} 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ ,  $X$ 

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

矩阵的运算

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求  $3A + 2B - 4C$ 

$$= \begin{pmatrix} 3 & 6 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 3 & 6 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ 3 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

$$(3 12) (14 12) (30 12) (-13 12)$$

$$(45) 25 4 (1 2) 2 (3 5) = 54 + 25 4 2 (1 2)$$

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且5A + 3X = B, 求X

解 X =

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

矩阵的运算  $7/32 \triangleleft \triangleright \Delta \nabla$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$\mathbf{H}$$

$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$

$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 6 \\ 10 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 10 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 10 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且5A + 3X = B, 求X

 $X = \frac{1}{3}(B - 5A) =$ 

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 2 \\ -13 & 1 \end{pmatrix}$$

 $\begin{pmatrix} -\frac{2}{3} & -\frac{3}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ §2.2 矩阵的运算  $7/32 \triangleleft \triangleright \Delta \nabla$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}, 求 3A + 2B - 4C$$

$$\begin{array}{l}
\mathbf{H} \\
3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\
&= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}
\end{array}$$

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

$$\mathbf{F}$$
  $\mathbf{F}$   $\mathbf{F}$ 

$$\frac{\mathbf{A}}{X} = \frac{1}{3}(B - 5A) = \frac{1}{3} \left( \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right)$$

 $\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求 $3A + 2B - 4C$ 

 $\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

7/32 ▷ ▷ ▷ ▽

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

$$\mathbf{H} X = \frac{1}{3}(B - 5A) = \frac{1}{3} \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

 $=\frac{1}{3}\left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix}\right) =$ 

练习 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求 3A + 2B - 4C

$$\begin{array}{l}
\mathbf{H} \\
3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\
&= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}
\end{array}$$

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

$$= \frac{1}{3} \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -2 & -5 \\ -8 & -14 \end{pmatrix} \qquad \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求 3A + 2B - 4C

$$\begin{array}{l}
\mathsf{PR} \\
\mathsf{3}A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\
&= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

练习 设 
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$ 

练习设
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且 $5A + 3X = B$ , 求 $X$ 

 $X = \frac{1}{3}(B - 5A) = \frac{1}{3} \left( \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right)$  $= \frac{1}{3} \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -2 & -5 \\ -8 & -14 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$ 

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

 $, k\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ 

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix}$$

$$, k\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \qquad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \qquad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

§2.2 **矩阵的运算** 8/32 **▽ ▷ △ ▽** 

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

§2.2 矩阵的运算 8/32 < ▷ △ ▽

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

- $1. \ k(A+B) = kA + kB$
- 2. (k + l)A = kA + lA

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

- $1. \ k(A+B) = kA + kB$
- 2. (k + l)A = kA + lA
- 3. (kl)A = k(lA)

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

2. 
$$(k + l)A = kA + lA$$

3. 
$$(kl)A = k(lA)$$

4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}$$
,  $B = (b_{ij})_{m \times n}$ , 则

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

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4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}$$
,  $B = (b_{ij})_{m \times n}$ , 则
$$k(A+B) = kA + kB =$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

2. 
$$(k + l)A = kA + lA$$

3. 
$$(kl)A = k(lA)$$

4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}$$
,  $B = (b_{ij})_{m \times n}$ , 则  $k(A+B) = k($   $)_{m \times n}$   $kA + kB =$ 

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

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3. 
$$(kl)A = k(lA)$$

4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}$$
,  $B = (b_{ij})_{m \times n}$ , 则 
$$k(A+B) = k(a_{ij}+b_{ij})_{m \times n}$$
$$kA+kB =$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

2. 
$$(k + l)A = kA + lA$$

3. 
$$(kl)A = k(lA)$$

4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$k(A+B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB =$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

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$$(k + l)A = kA + lA$$

3. 
$$(kl)A = k(lA)$$

4. 
$$1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$k(A+B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

$$1. \ k(A+B) = kA + kB$$

2. 
$$(k + l)A = kA + lA$$

3. 
$$(kl)A = k(lA)$$

$$4. 1 \cdot A = A$$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$
 
$$k(A+B) = k(a_{ij}+b_{ij})_{m \times n} = (ka_{ij}+kb_{ij})_{m \times n}$$
 
$$kA+kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij}+kb_{ij})_{m \times n}$$

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是  $m \times n$  矩阵, k, l 是数, 则

- $1. \ k(A+B) = kA + kB$
- 2. (k+l)A = kA + lA
- 3. (kl)A = k(lA)
- $4 \quad 1 \cdot A = A$

证明 设 
$$A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n}, 则$$

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$
$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

所以 k(A+B) = kA + kB。

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 
$$Y$$
 满足  $(2A-Y)-2(B+Y)=O$ , 求  $Y$ 

§2.2 矩阵的运算 9/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 
$$(2A-Y)-2(B+Y)=O$$
, 求 Y

§2.2 矩阵的运算 9/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 
$$(2A-Y)-2(B+Y)=O$$
, 求 Y

$$\mathbf{H} Y = \frac{2}{3}(A - B)$$

§2.2 矩阵的运算 9/32 ightarrow 9/32 ightarrow ightarrow 9/32 ightarrow ightarr

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 
$$(2A-Y)-2(B+Y)=O$$
, 求 Y

解 
$$Y = \frac{2}{3}(A - B)$$
, 所以

$$Y = \frac{2}{3}(A - B) =$$

§2.2 矩阵的运算 9/32 ightarrow 9/32 ightarrow ightarrow 9/32 ightarrow ightarr

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 (2A-Y)-2(B+Y)=O, 求 Y

解 
$$Y = \frac{2}{3}(A - B)$$
,所以

$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

§2.2 矩阵的运算 9/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 (2A-Y)-2(B+Y)=O, 求 Y

$$MY = \frac{2}{3}(A - B), 所以$$

$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

$$=\frac{2}{3}\begin{pmatrix} -3 & -1 & -1 & 1\\ 4 & 0 & 4 & 0\\ 1 & 3 & 3 & 5 \end{pmatrix}$$

§2.2 矩阵的运算 9/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC =$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

§2.2 矩阵的运算 10/32 < ▷ ∆ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} & & \\ & & \end{pmatrix}$$

§2.2 矩阵的运算 10/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c \\ \end{pmatrix}$$

§2.2 矩阵的运算 10/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \end{pmatrix}$$

[2.2 矩阵的运算 10/32 **▽ ▷** △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c \end{pmatrix}$$

§2.2 矩阵的运算 10/32 < ▷ ∆ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix}$$

**〔2.2 矩阵的运算** 10/32 **▽ ▷ △ ▽** 

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

§2.2 矩阵的运算 10/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c=1 \end{cases}$$

§2.2 矩阵的运算 10/32 < ▷ △ ▽

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c=1\\ b=0 \end{cases}$$

§2.2 矩阵的运算

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c=1\\ b=0\\ 2a+3b+c=0 \end{cases}$$

§2.2 矩阵的运算

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a+b-c=1\\ b=0\\ 2a+3b+c=0\\ a-c=1 \end{cases}$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a+b-c=1\\ b=0\\ 2a+3b+c=0\\ a-c=1 \end{cases} \Rightarrow \begin{cases} b=0 \end{cases}$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a+b-c=1\\ b=0\\ 2a+3b+c=0\\ a-c=1 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{3}\\ b=0 \end{cases}$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 aA + bB + cC = I, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} a+b-c=1\\ b=0\\ 2a+3b+c=0\\ a-c=1 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{3}\\ b=0\\ c=-\frac{2}{3} \end{cases}$$

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (\alpha_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

定义 设  $A = (\alpha_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

 $c_{ij} = A$ 第i行与B第j列对应元素乘积的和

定义 设 
$$A = (\alpha_{ik})_{m \times l}$$
,  $B = (b_{kj})_{l \times n}$ , 定义矩阵  $A$ ,  $B$  的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A$$
第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

即

$$a_{i1}$$
  $a_{i2}$   $\cdots$   $a_{il}$ 

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A$$
第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

即

$$a_{i1}b_{1j}$$
  $a_{i2}b_{2j}$   $\cdots$   $a_{il}b_{lj}$ 

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A$$
第 $i$ 行与 $B$ 第 $j$ 列对应元素乘积的和

即

$$a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{il}b_{lj}$$

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A$$
第 $i$ 行与 $B$ 第 $j$ 列对应元素乘积的和

即

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^{l} a_{ik}b_{kj}$$

定义 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵 A, B 的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A$$
第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^{l} a_{ik}b_{kj}$$

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ \cdots & c_{ij} & \cdots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

§2.2 矩阵的运算 12/32 < ▶ ∆ ▼

 $a_{il}$ 

 $a_{i2}$ 

 $a_{i1}$ 

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ \cdots & c_{ij} & \cdots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

§2.2 矩阵的运算 12/32 < ▶ ∆ ▼

 $a_{i1}b_{1j}$   $a_{i2}b_{2j}$  ···

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ \cdots & c_{ij} & \cdots & & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

 $a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{il}b_{lj}$ 

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{l1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ \cdots & c_{ij} & \cdots \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

 $a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^{i} a_{ik}b_{kj}$ 

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

 $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{i=1}^{l} a_{ik}b_{kj}$ 

$$(b) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3}$$

$$\left(\begin{array}{ccc}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
a_{31} & a_{32} \\
a_{41} & a_{42}
\end{array}\right)_{4\times 2}
\cdot
\left(\begin{array}{cccc}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23}
\end{array}\right)_{2\times 3} = \left(\begin{array}{cccc}
\end{array}\right)_{4\times 2}$$

§2.2 矩阵的运算 13/32 ◀ ▷ Δ ▼

§2.2 矩阵的运算 13/32 ◀ ▷ Δ ▼

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

§2.2 矩阵的运算 13/32 ◀ ▷ Δ ▼

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

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例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

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$$(b) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

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$$\left( \begin{array}{cccc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{array} \right)_{4 \times 2} \cdot \left( \begin{array}{ccccc} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{array} \right)_{2 \times 3} = \left( \begin{array}{ccccc} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{array} \right)_{4 \times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} =$$

§2.2 矩阵的运算

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} * \\ * \\ * \end{pmatrix}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 \\ 1 & -2 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & * \\ & & \end{pmatrix}_{3 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 \\ & & & \end{pmatrix}_{3 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & * \\ & & & \end{pmatrix}_{3 \times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & & \end{pmatrix}_{3 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
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$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & & \end{pmatrix}_{3 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \end{pmatrix}_{3 \times 3}$$

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

例设
$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$$
,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求 $AB$ 

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ 5 & -7 & -9 \end{pmatrix}_{3 \times 3}$$

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}$$
  $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ 

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} =$$

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}_{3 \times 3}$$

§2.2 矩阵的运算 14/32 < ▶ ∆ ▼

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2\times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3\times 3} = \begin{pmatrix} 10 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

§2.2 矩阵的运算 14/32 d ▷ ∆ ▼

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 \\ & & & \end{pmatrix}_{2 \times 3}$$

§2.2 矩阵的运算 14/32 ◁ ▷ Δ ▼

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ & & & \end{pmatrix}_{2 \times 3}$$

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & & & \end{pmatrix}_{2 \times 3}$$

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & \end{pmatrix}_{2 \times 3}$$

练习 求 
$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}$$
  $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ 

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & -1 \end{pmatrix}_{2 \times 3}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB$ ,  $BA$ 

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB$ ,  $BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = ( )_{1 \times 1}$$

§2.2 **矩阵的运算** 15/32 **▽ ▶ △ ▽** 

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

矩阵的运算 15/32 ▷ ▷ ▷ ▽

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) =$$

§2.2 矩阵的运算 15/32  $exttt{ o}$   $exttt{ o}$   $exttt{ o}$   $exttt{ o}$   $exttt{ o}$   $exttt{ o}$   $exttt{ o}$ 

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2$$

矩阵的运算 15/32 ▷ ▷ ▷ ▽

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB$ ,  $BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \end{pmatrix}_{3 \times 3}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 3 & 3 \end{pmatrix}$$

矩阵的运算 15/32 ▷ ▷ ▷ ▽

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

矩阵的运算 15/32 ▷ ▷ ▷ ▽

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 2 \\ 3 \end{pmatrix}$$

练习 
$$A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$$
 计算  $AB, BA$ 

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3 \times 1}$$

练习  $A = (1, 2, 3), B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算 AB, BA

及 CB。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

 $CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3.11}$ 

例设
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

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$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} =$$

例设 
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

例设 
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{1 \times 2}$$

§2.2 矩阵的运算 16/32 ightarrow 1

例设
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 0 \end{pmatrix}_{1 \times 2}$$

§2.2 矩阵的运算 16/32 ightarrow ▷  $\Delta$  ightarrow

例设
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求AB, BA$$

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$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

§2.2 矩阵的运算 16/32 ightarrow ▷  $\Delta$  ightarrow

例设
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求AB, BA$$

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$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} (3 \ 1 \ 0)_{1 \times 3}$$

§2.2 矩阵的运算 16/32 ightharpoons 16/32 ightharpoons ightharpoons

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$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

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$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3\times 2} (3 \ 1 \ 0)_{1\times 3}$$
 没有意义!

§2.2 矩阵的运算 16/32 ightharpoons 16/32 ightharpoons ightharpoons

例设
$$A = (3 \ 1 \ 0), B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}, 求 AB, BA$$

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$$
 (3 1 0)<sub>1×3</sub> 没有意义!

注 AB 可以存在,但 BA 不一定有意义

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} =$$

例设
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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix}$$

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & & \\ & & \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 \\ \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

§2.2 矩阵的运算

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} =$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & & \\ & & & \end{pmatrix}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & & \\ & & & \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
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解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 \\ \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & \end{pmatrix}_{2 \times 2}$$

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

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$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
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解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

注

1. 即便 *AB*, *BA* 都有意义,也不一定相等。 矩阵的乘法不满足交换律!

例设
$$A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求 $AB$ ,  $BA$ 

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$
$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

#### 注

- 1. 即便 *AB*, *BA* 都有意义,也不一定相等。 矩阵的乘法不满足交换律!
- 2. BA = 0 不能推出 B = 0 或 A = 0

注 即便假设  $A \neq 0$ , BA = CA 也推不出 B = C。如

[2.2 矩阵的运算 18/32 **▽ ▷ △ ▽** 

注 即便假设  $A \neq 0$ , BA = CA 也推不出 B = C。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A} \quad \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A}$$

注 即便假设  $A \neq 0$ , BA = CA 也推不出 B = C。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A}$$

注 即便假设  $A \neq 0$ , BA = CA 也推不出 B = C。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_{A}$$

#### 总结

- 1. AB 可以存在, 但 BA 不一定有意义
  - 2. 即便 *AB*, *BA* 都有意义,也不一定相等。矩阵的乘法不满足交换律!(矩阵相乘要注意顺序)
  - 3. BA = 0 不能推出 B = 0 或 A = 0
  - 4. 即便假设  $A \neq 0$ , BA = CA 也退不出 B = C。

## 矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义,则

1. 
$$(AB)C = A(BC)$$

2. 
$$(A + B)C = AC + BC$$

3. 
$$C(A + B) = CA + CB$$

$$4. \ k(AB) = (kA)B = A(kB)$$

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^{T}$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^{T}$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \end{pmatrix}_{n \times m}$$

§2.2 矩阵的运算 20/32 ◁ ▷ △ ▽

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ \vdots & \vdots \\ a_{1n} & a_{2n} \end{pmatrix}_{n \times m}$$

§2.2 矩阵的运算 20/32 ◁ ▷ △ ▽

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

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定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

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注

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

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注

定义 将  $m \times n$  矩阵 A 的行与列互换,得到的  $n \times m$  矩阵,称为矩阵 A 的转置矩阵,记为  $A^T$ 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^{T} = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

注

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T =$ 

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$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
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例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$ 

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$ 

例 设 
$$x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$$

$$x^Ty =$$

§2.2 矩阵的运算

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$   
例 设  $x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$ 
$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

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例 设  $x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$   
$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \ (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} =$$

§2.2 矩阵的运算 21/32 ightarrow 21/32

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$   
例 设  $x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$ 

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \ (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

§2.2 矩阵的运算 21/32 ightarrow 21/32

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
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例设
$$x = (x_1 \ x_2 \ \cdots \ x_n), y = (y_1 \ y_2 \ \cdots \ y_n),$$
则

$$x^{T}y = \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix} (y_{1} \quad y_{2} \quad \cdots \quad y_{n})_{1 \times n} = \begin{pmatrix} x_{1} \\ \vdots \\ x_{n} \end{pmatrix}$$



例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
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例 设  $x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$   
 $x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \ (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 \\ \vdots \\ x_n \end{pmatrix}$ 

§2.2 矩阵的运算 21/32 < ▶ ∆ ▼

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
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例 设  $x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$ 

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 \\ \vdots \\ x_n \end{pmatrix}$$

§2.2 矩阵的运算 21/32 ◀ ▶ Δ ▼

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$   
例 设  $x = (x_1 \ x_2 \ \cdots \ x_n)$ ,  $y = (y_1 \ y_2 \ \cdots \ y_n)$ ,则 
$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots \\ \vdots & \vdots & \vdots \\ x_n & \vdots & \vdots \\ x_n & \vdots & \vdots \end{pmatrix}$$

§2.2 矩阵的运算 21/32 ◁ ▷ △ ▼

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$ 

例 设 
$$x = (x_1 \ x_2 \ \cdots \ x_n), \ y = (y_1 \ y_2 \ \cdots \ y_n), \ 则$$

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$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \end{pmatrix} (y_1 \ y_2 \ \cdots \ y_n), = \begin{pmatrix} x_1 y_1 \ x_1 y_2 \ \cdots \\ \end{pmatrix}$$

21/32 ▷ ▷ ▷ ▽

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$ 

例设
$$x = (x_1 \ x_2 \ \cdots \ x_n), y = (y_1 \ y_2 \ \cdots \ y_n),$$
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$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 \ x_1 y_2 \ \cdots \ x_2 y_n \\ x_2 y_1 \ x_2 y_2 \ \cdots \ x_2 y_n \end{pmatrix}_{n \times n}$$

21/32 ▷ ▷ ▷ ▽ 矩阵的运算

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3\times 2}$ 

例设
$$x = (x_1 \ x_2 \ \cdots \ x_n), y = (y_1 \ y_2 \ \cdots \ y_n),$$
则

$$x^{T}y = \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \end{pmatrix} \begin{pmatrix} y_{1} & y_{2} & \cdots & y_{n} \end{pmatrix}_{1 \times n} = \begin{pmatrix} x_{1}y_{1} & x_{1}y_{2} & \cdots & x_{1}y_{n} \\ x_{2}y_{1} & x_{2}y_{2} & \cdots & x_{2}y_{n} \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$x^{T}y = \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}_{n \times 1} (y_{1} \quad y_{2} \quad \cdots \quad y_{n})_{1 \times n} = \begin{pmatrix} x_{1}y_{1} & x_{1}y_{2} & \cdots & x_{1}y_{n} \\ x_{2}y_{1} & x_{2}y_{2} & \cdots & x_{2}y_{n} \\ \vdots & \vdots & & \vdots \end{pmatrix}_{n \times n}$$

矩阵的运算

例 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
,则  $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3\times 3}$ 

例设
$$x = (x_1 \ x_2 \ \cdots \ x_n), y = (y_1 \ y_2 \ \cdots \ y_n),$$
则

$$x^{T}y = \begin{pmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{pmatrix}_{n \times 1} (y_{1} \quad y_{2} \quad \cdots \quad y_{n})_{1 \times n} = \begin{pmatrix} x_{1}y_{1} & x_{1}y_{2} & \cdots & x_{1}y_{n} \\ x_{2}y_{1} & x_{2}y_{2} & \cdots & x_{2}y_{n} \\ \vdots & \vdots & & \vdots \\ x_{n}y_{1} & x_{n}y_{2} & \cdots & x_{n}y_{n} \end{pmatrix}_{n \times n}$$

§2.2 矩阵的运算

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

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$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix}$$

§2.2 矩阵的运算

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ \end{pmatrix}_{2 \times 2}$$

§2.2 矩阵的运算 22/32 ◀ ▷ Δ ▼

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ .

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 2 \end{pmatrix}$$

§2.2 矩阵的运算

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ .

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ & & \end{pmatrix}_{2 \times 2}$$

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & \end{pmatrix}_{2 \times 2}$$

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

§2.2 矩阵的运算

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ .

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$
$$A^{T}A = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$
$$A^{T}A = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$

§2.2 矩阵的运算  $22/32 \triangleleft \triangleright \Delta$   $\triangledown$ 

练习 设 
$$A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$
, 计算  $AA^T$  及  $A^TA$ 。

$$AA^{T} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$
$$A^{T}A = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 2 & 2 \\ 2 & 1 & 4 \\ 2 & 4 & 20 \end{pmatrix}_{3 \times 3}$$

§2.2 矩阵的运算 22/32 ightarrow  $2 ext{ } 
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1.  $(A^T)^T = A$ 

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- 2.  $(A + B)^T = A^T + B^T$ ,  $(kA)^T = kA^T$

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- 2.  $(A + B)^T = A^T + B^T$ ,  $(kA)^T = kA^T$
- 3.  $(AB)^T = B^T A^T$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ 

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

AB	$(AB)^T$	$A^T$	$B^T$	$B^TA^T$

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$A^T$	$B^T$	$B^TA^T$
<b>阶数</b>	m × n				

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$A^T$	$B^T$	$B^TA^T$
阶数	$m \times n$	$n \times m$			

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
<b></b>	$m \times n$	n × m	l× m		

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

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证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
阶数	m × n	n × m	l× m	n×l	

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
<b>阶数</b>	m × n	n × m	l× m	n×l	n × m

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

并且

§2.2 矩阵的运算 23/32 < ▶ A ▼

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
 $m \times n$	n × m	l× m	n×l	n × m

并且

B<sup>T</sup>A<sup>T</sup> (i, j)元素

§2.2 矩阵的运算

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$A^T$	$B^T$	$B^TA^T$
阶数	$m \times n$	n × m	l× m	n×l	n × m

并且

$$(AB)^T$$
 =  $AB$  =  $(j, i)$ 元素 =

B<sup>T</sup>A<sup>T</sup> (i, i)元素

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
 $m \times n$	n × m	l× m	n×l	n × m

并且

$$(AB)^T = AB (i, j)元素 = a_{j1} a_{j2} \cdots a_{jl} B^TA^T (i, j)元素$$

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^{T} = B^{T}A^{T}$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
 $m \times n$	n × m	l× m	n×l	n × m

并且

$$(AB)^T = AB (i, j)元素 = a_{j1}b_{1i} \quad a_{j2}b_{2i} \quad \cdots \quad a_{jl}b_{li} \quad B^TA^T (i, j)元素$$

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

AB	$(AB)^T$	$\mathcal{A}^{\mathcal{T}}$	$B^T$	$B^TA^T$
 $m \times n$	n × m	l× m	n×l	n × m

并且

$$(AB)^T = AB = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li}$$
  $B^TA^T$   $(i, j)$ 元素

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
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3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$\mathcal{A}^T$	$B^T$	$B^TA^T$	
<u></u> 阶数	$m \times n$	n × m	l× m	n×l	n × m	_

并且

$$(AB)^T$$
 =  $AB$  =  $a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li}$   $B^TA^T$  ( $i, j$ )元素

 $A^T$ 第j列元素

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	AB	$(AB)^T$	$\mathcal{A}^T$	$B^T$	$B^TA^T$	
阶数	$m \times n$	$n \times m$	l× m	n×l	$n \times m$	_

并且

$$(AB)^T$$
  $=$   $AB$   $(i, j)$ 元素  $=$   $a_{i1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li}$   $A^T$ 第 $j$ 列元素  $B^T$ 第 $i$ 行元素

B<sup>T</sup>A<sup>T</sup> (i, j)元素

1. 
$$(A^T)^T = A$$

2. 
$$(A + B)^T = A^T + B^T$$
,  $(kA)^T = kA^T$ 

3. 
$$(AB)^T = B^T A^T$$

证明 设 
$$A = A_{m \times l}$$
,  $B = B_{l \times n}$ , 则

	$(AB)^T$	$\mathcal{A}^T$	$B^T$	$B^TA^T$
 $m \times n$	n × m	l× m	n×l	n × m

并且

$$(AB)^{T}$$
 =  $AB$  =  $a_{i1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} = B^{T}A^{T}$  ( $i, j$ )元素  $A^{T}$ 第 $i$ 列元素  $B^{T}$ 第 $i$ 行元素

设  $A = (a_{ij})_{n \times n}$  为 n 阶方阵,  $k \in \mathbb{N}$  为自然数,定义

$$A^k = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k \uparrow}$$

称为方阵 A 的 k 次幂

设  $A = (a_{ii})_{n \times n}$  为 n 阶方阵,  $k \in \mathbb{N}$  为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k \uparrow}$$

称为方阵 A 的 k 次幂

方阵的幂的性质  $A^{k_1}A^{k_2} = A^{k_1+k_2}$ ,  $(A^{k_1})^{k_2} = A^{k_1k_2}$ 

设  $A = (a_{ii})_{n \times n}$  为 n 阶方阵,  $k \in \mathbb{N}$  为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k \uparrow}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 
$$A^{k_1}A^{k_2} = A^{k_1+k_2}$$
,  $(A^{k_1})^{k_2} = A^{k_1k_2}$ 

这是因为:

$$A^{k_1}A^{k_2} =$$

设  $A = (a_{ii})_{n \times n}$  为 n 阶方阵, $k \in \mathbb{N}$  为自然数,定义

$$A^k = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k \uparrow}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 
$$A^{k_1}A^{k_2} = A^{k_1+k_2}$$
,  $(A^{k_1})^{k_2} = A^{k_1k_2}$ 

这是因为:

$$A^{k_1}A^{k_2} = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k_1 \uparrow} \cdot$$

设  $A = (a_{ii})_{n \times n}$  为 n 阶方阵, $k \in \mathbb{N}$  为自然数,定义

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§2.2 矩阵的运算 24/32 ▷ ▷ ▷ ▽

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练习 设 
$$A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$$
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§2.2 矩阵的运算

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§2.2 矩阵的运算

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§2.2 矩阵的运算

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$$A^{n} = A^{n} = A^{n} = A^{n} = A^{n} = A^{n}$$

§2.2 矩阵的运算 25/32 ◀ ▷ △ ▼

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$$\vdots$$

$$A^{n} = \begin{pmatrix} 1 & 0 \\ n\lambda & 1 \end{pmatrix}$$

§2.2 矩阵的运算

注 设 A, B 为 n 阶方阵, 一般地

 $(AB)^k \neq A^k B^k$ 

注设 
$$A, B$$
 为  $n$  阶方阵, 一般地

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这是, 例如 
$$k=2$$
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$$(AB)^2 =$$

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$$(AB)^2 = (AB) \cdot (AB) =$$

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§2.2 矩阵的运算 26/32 ◁ ▷ ∆ ▼

注设 
$$A$$
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$$(AB)^k \neq A^k B^k$$

这是, 例如 
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但一般地,  $AB \neq BA$ , 所以  $(AB)^2 \neq A^2B^2$ 

回忆:对n阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

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其行列式规定为

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设 A, B 均是 n 阶方阵, k 为数,则

- 1.  $|A^T| = |A|$
- 2.  $|kA| = k^n |A|$
- 3.  $|AB| = |A| \cdot |B|$
- 4. |AB| = |BA|

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例如

$$|kA| =$$

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例如
$$|kA| = \begin{vmatrix} k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \end{vmatrix} =$$

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§2.2 矩阵的运算 28/32 **▽** ▶ ∆ ▼

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$$= k \cdot k \cdot \cdots \cdot k \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

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$$= k \cdot k \cdot \dots \cdot k \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = k^n |A|$$

例设 
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
,求  $|4A|$ 

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例设
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
,求 $|4A|$ 

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} =$$

例设 
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$$\begin{vmatrix} 4A \end{vmatrix} = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} =$$

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例设 
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
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解

$$\begin{vmatrix} 4A \end{vmatrix} = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

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练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T$ 

例设 
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解

$$\begin{vmatrix} 4A \end{vmatrix} = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T|$ 

解

$$\left| |A|A^2A^T \right| = |A|^3 \left| A^2A^T \right|$$

例设 
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
,求  $|4A|$ 

解

$$|4A| = 4^{3}|A| = 64$$
  $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64$   $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$ 

练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T|$ 

解

$$||A|A^2A^T| = |A|^3 |A^2A^T|$$
  
=  $|A|^3 |A^2| |A^T|$ 

例设
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
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解

$$|4A| = 4^{3}|A| = 64$$
  $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64$   $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$ 

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解

$$||A|A^2A^T| = |A|^3 |A^2A^T|$$
$$= |A|^3 |A^2| |A^T|$$
$$= |A|^3 |A|^2 |A|$$

例设
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
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解

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练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T$ 

$$\begin{aligned} \left| |A|A^2A^T \right| &= |A|^3 \left| A^2A^T \right| \\ &= |A|^3 \left| A^2 \right| \left| A^T \right| \\ &= |A|^3 |A|^2 |A| \\ &= |A|^6 \end{aligned}$$

例设
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
,求 $|4A|$ 

解

$$\begin{vmatrix} 4A \end{vmatrix} = 4^{3} \begin{vmatrix} A \end{vmatrix} = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

 $= |A|^6 = (-2)^6$ 

练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T$ 

$$||A|A^{2}A^{T}| = |A|^{3} |A^{2}A^{T}|$$

$$= |A|^{3} |A^{2}| |A^{T}|$$

$$= |A|^{3} |A|^{2} |A|$$

例设
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$$
,求 $|4A|$ 

解

$$\begin{vmatrix} 4A \end{vmatrix} = 4^{3} \begin{vmatrix} A \end{vmatrix} = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

 $= |A|^6 = (-2)^6 = 64$ 

练习 设 A 为三阶方阵,且 |A| = -2,求  $|A|A^2A^T$ 

$$||A|A^2A^T| = |A|^3 |A^2A^T|$$
$$= |A|^3 |A^2| |A^T|$$
$$= |A|^3 |A|^2 |A|$$

```
\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}
```

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

### 等价于

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

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$$= \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n$$

§2.2 矩阵的运算

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

### 等价于

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \end{pmatrix}$$

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### 等价于

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\parallel$$

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{pmatrix}$$

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### 等价于

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

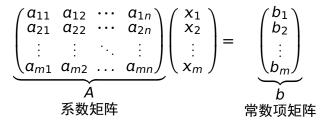
等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$
系数矩阵

§2.2 矩阵的运算 31/32 ◁ ▶ ∆ ▼

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

### 等价于



$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{b}$$
常数矩阵

进一步改写成

Ax = b

#### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 = -2 \\ x_1 +x_2 -2x_3 +3x_4 = 3 \\ 3x_1 -x_2 +8x_3 +x_4 = 7 \end{cases}$$

#### 例方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 Ax = b 是

$$\left( \begin{array}{c} \\ \\ \end{array} \right) \left( \begin{array}{c} \\ \\ \end{array} \right)$$

§2.2 矩阵的运算 32/32 ◀ ▷ Δ ▼

### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 = -2 \\ x_1 +x_2 -2x_3 +3x_4 = 3 \\ 3x_1 -x_2 +8x_3 +x_4 = 7 \end{cases}$$

的矩阵表示 Ax = b 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

§2.2 矩阵的运算 32/32 ◀ ▷ Δ ▼

### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 = -2 \\ x_1 +x_2 -2x_3 +3x_4 = 3 \\ 3x_1 -x_2 +8x_3 +x_4 = 7 \end{cases}$$

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§2.2 矩阵的运算 32/32 **▽** ▶ ∆ ▼

#### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 = -2 \\ x_1 +x_2 -2x_3 +3x_4 = 3 \\ 3x_1 -x_2 +8x_3 +x_4 = 7 \end{cases}$$

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### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 =-2 \\ x_1 +x_2 -2x_3 +3x_4 =3 \\ 3x_1 -x_2 +8x_3 +x_4 =7 \end{cases}$$

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$$\begin{pmatrix} x_1 - x_2 + 5x_3 - x_4 \\ x_1 + x_2 - 2x_3 + 3x_4 \\ 3x_1 - x_2 + 8x_3 + x_4 \end{pmatrix}$$

#### 例方程组

$$\begin{cases} x_1 -x_2 +5x_3 -x_4 =-2 \\ x_1 +x_2 -2x_3 +3x_4 =3 \\ 3x_1 -x_2 +8x_3 +x_4 =7 \end{cases}$$

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