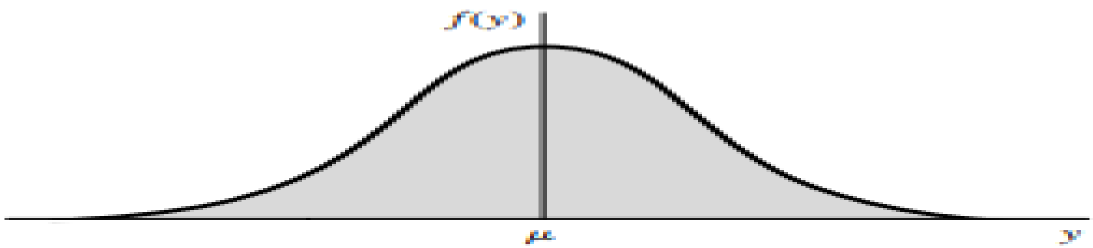
Geno Copertino

Normal, Beta, and Gamma Distributions

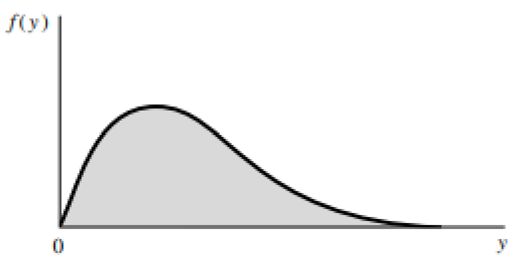
This write-up is going to be about the Normal, Beta, and Gamma Distributions from Chapter 4 of Mathematical Statistics with Applications 7th ed. Firstly, we have the Normal Probability Distribution. This distribution is the most widely used continuous probability distribution. It is a distribution with the bell shape which was relevant with the empirical rule. This is the definition for the normal distribution:

A random variable Y is said to have a *normal probability distribution* if and only if, for σ > 0 and -∞ < μ < ∞, the density function of y is $f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}$ with y being in the range from negative infinity to infinity.

If Y is a normally distributed random variable with parameters μ and σ, then to calculated mean (or expected) you do E(Y) = μ. To calculate Variance you do V(Y) = σ^2. Below is what the normal probability density function looks like visually.



Next, we have the Gamma probability distribution. When some variables are always nonnegative and yield distributions of data that are skewed to the right we get a graph looking like this.



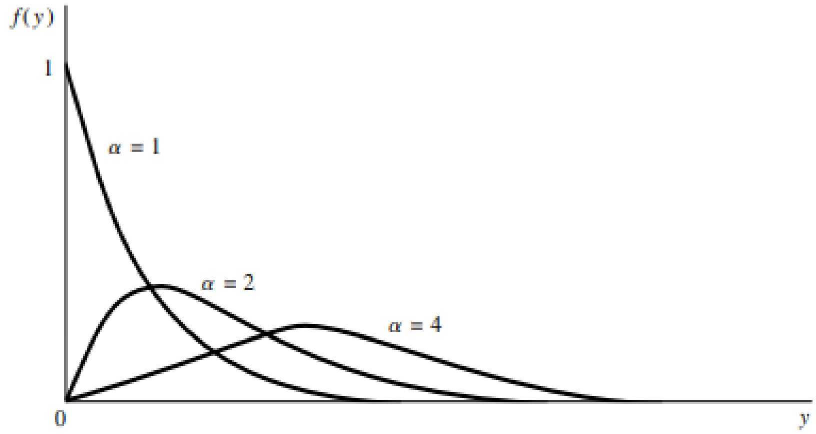
A random variable Y is said to have a *gamma distribution with parameters* α > 0 and β > 0 if and only if the density function of Y is: $f(y) =$

$$\begin{cases} \dfrac{y^{\alpha-1} e^{-\frac{y}{\beta}}}{\beta^{\alpha}\tau} & 0 \le y \le \infty \\ 0, & elsewhere \end{cases}$$

where $\tau(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy$

$\tau(\alpha)$ is known as the *gamma function*. Direct integration verifies that $\tau(1) = 1$. Here is a visual representation of what different α values look like when plugged in.

FIGURE 4.16 Gamma density functions, $\beta = 1$

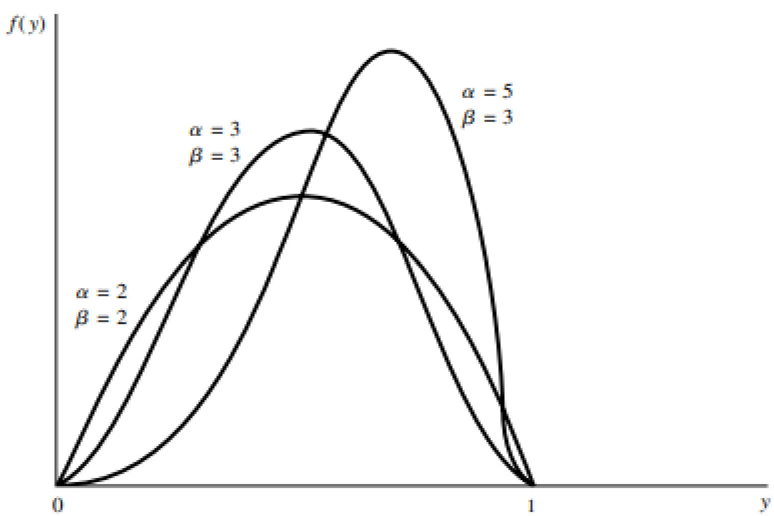


Since the shape of the density changes with each value of α, α is called the shape parameter associated with the gamma distribution. β is called the scale parameter because multiplying a gamma-distributed random variable by a positive constant produces a random variable that also has a gamma distribution with the same value of α but with an altered value of β. The expected (mean) for this distribution is $E(Y) = \alpha\beta$. Variance is $V(Y) = \alpha\beta^2$.

Finally, we have the Beta Probability Distribution. A random variable Y is said to have a *beta probability distribution with parameters* $\alpha > 0$ and $\beta > 0$ if and only if the density function of is:

$$f(y) = \begin{cases} \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, 0 \le y \le 1, \\ 0, \ elsewhere \end{cases} \quad \text{where } B(\alpha, \ \beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy$$

The graphs for beta density functions can assume widely differing shapes depending on the values of α and β. Here is an example of some of them.



Overall, these distributions are not the easiest to implement and solve for, but they are still important in the world of probability and statistics.