SET THEORY

✅ 1. DEFINITIONS

- **Set: A collection of distinct, well-defined objects.**
  - **Example: A = {1, 2, 3}**
- **Element: An object in a set. (e ∈ A means e is in A)**
- **Roster Form: Listing elements → A = {2, 4, 6}**
- **Set-Builder Form: Describes rule → A = {x | x is even, x ≤ 6}**

✅ 2. SET TYPES + EXAMPLES

- **Empty Set (∅): No elements**
  → A = {x | x < 1, x ∈ N} ⇒ A = ∅
- **Singleton Set: One element**
  → A = {7}
- **Finite Set: Countable elements**
  → A = {1, 2, 3}
- **Infinite Set: Uncountable**
  → A = {x | x ∈ N}
- **Subset (⊆): A ⊆ B ⇒ All A's elements in B**
  → A = {1, 2}, B = {1, 2, 3} ⇒ A ⊆ B
- **Proper Subset (⊂): A ⊆ B but A ≠ B**
- **Universal Set (U): Set containing all elements under discussion**
- **Power Set (P(A)): Set of all subsets**
  → If A has *n* elements, then |P(A)| = 2^n
- **Disjoint Sets: A ∩ B = ∅ (No common elements)**
- **Equal Sets: A = B ⇔ same elements**

✅ 3. IMPORTANT FORMULAS

- **Total Subsets of A (n elements):**
  → 2^n
- **Proper Subsets:**
  → $2^n - 1$
- **Union Formula:**
  → $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- **Three Sets Union:**
  → $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

✅ **4. SET OPERATIONS WITH EXAMPLES**

- **Union ($A \cup B$): All elements in A or B or both**
  → $A = \{1, 2\}, B = \{2, 3\} \Rightarrow A \cup B = \{1, 2, 3\}$
- **Intersection ($A \cap B$): Common elements**
  → $A \cap B = \{2\}$
- **Difference ($A - B$): Elements in A not in B**
  → $A - B = \{1\}$
- **Complement (A'): Elements in U but not in A**
  → $U = \{1, 2, 3, 4\}, A = \{2, 4\} \Rightarrow A' = \{1, 3\}$
- **Symmetric Difference ($A \triangle B$):**
  → $A \triangle B = (A - B) \cup (B - A)$

✅ **5. VENN DIAGRAM RULES**

- **Use 2-set or 3-set diagrams for word problems**
- **Only in A = $A - (A \cap B)$**
- **Exactly in one = Elements that belong to only one set**
- **None of the sets = Total − ($A \cup B \cup C$)**

✅ **6. DE MORGAN'S + OTHER LAWS**

- **De Morgan's Laws:**
  → $(A \cup B)' = A' \cap B'$
  → $(A \cap B)' = A' \cup B'$

- **Complement Laws:**
  → $A \cup A' = U$
  → $A \cap A' = \varnothing$
- **Idempotent Laws:**
  → $A \cup A = A$, $A \cap A = A$
- **Domination Laws:**
  → $A \cup U = U$, $A \cap \varnothing = \varnothing$
- **Double Complement:**
  → $(A')' = A$

✅ **7. SHORTCUTS FOR COUNT PROBLEMS**

- **Exactly in 1 of 3 Sets:**
  $= n(A) + n(B) + n(C) - 2[n(A \cap B) + n(B \cap C) + n(C \cap A)] + 3n(A \cap B \cap C)$
- **Only in A:**
  $= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
- **Exactly in 2 Sets:**
  $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- **None of the Sets:**
  $= \text{Total} - n(A \cup B \cup C)$

✅ **8. PYQ SAMPLE + THEIR TYPES**

Type 1 – Direct formula:

50 like A, 30 like B, 10 like both. Find only A?
→ Only A = 50 − 10 = 40

Type 2 – Use of 3-set formula:

$A = 40$, $B = 30$, $C = 50$
$A \cap B = 10$, $B \cap C = 15$, $A \cap C = 20$, $A \cap B \cap C = 5$
→ Use 3-set union formula to find total or "none"

Type 3 – Venn Diagram Count

In a group of 60, 30 like tea, 25 like coffee, 10 both. Find number of people who like neither.
→ n(Tea ∪ Coffee) = 30 + 25 − 10 = 45
→ Neither = 60 − 45 = 15

✅ 9. YOUR OWN PRACTICE QUESTIONS (IMPORTANT)

Make sure to solve and review:

1. **A = {1, 2, 3}, B = {2, 3, 4}, find A ∪ B, A ∩ B, A − B, B − A**
2. **In a school, 100 students, 60 like maths, 45 like physics, 25 like both. How many like only one subject?**
3. **If U = {1 to 10}, A = {2, 4, 6, 8}, then find A′**
4. **Find the number of elements exactly in two sets:**
    - **A = 50, B = 60, C = 40**
    - **A∩B = 20, B∩C = 25, A∩C = 15, A∩B∩C = 10**

Cardinality, ABSD & Elementary Counting

📌 1. Cardinality of a Set

Cardinality = Number of elements in a set

- Denoted by $n(A)$ for a set A
- Example: $A = \{2, 4, 6\}$, then $n(A) = 3$

📌 2. Important Cardinality Formulas

➤ Two sets:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(Use when there's overlap between sets)

➤ Three sets:

$n(A \cup B \cup C) =$
$n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(A \cap C)$

- $n(A \cap B \cap C)$

📌 3. ABSD Principle (Shortcut Name)

A – Addition Principle

If event A can occur in m ways, and B in n ways (non-overlapping):

Total ways = $m + n$

B – Both (Multiplication Principle)

If task A can be done in m ways and task B in n ways, then:

Total ways = $m \times n$

S – Subtraction Principle

To find "at least" or "not this", subtract unwanted from total:
$n(\text{total}) - n(\text{unwanted})$

D – Division Principle

When arrangements are repeated or identical objects,

Divide by the number of repetitions
Example: BANANA → 6! / (3!×2!) = 60

📌 4. Elementary Counting Principles

➤ **Fundamental Principle of Counting (FPC):**

If something can be done in m ways and another in n ways,

Total = m × n

➤ **Permutations (Order matters)**

- **Without repetition:**
  nPr = n! / (n − r)!
- **With repetition:**
  n^r (like passwords, digits)

➤ **Combinations (Order doesn't matter)**

- **nCr = n! / [r!(n − r)!]**

➤ **Circular Permutations**

- **(n − 1)! (around a circle)**
- **If clockwise & anticlockwise same: (n − 1)! / 2**

➤ **With Repetition (like digits or letters):**

- **Digits: total = 10,**
  e.g., 3-digit numbers with repetition = 10 × 10 × 10 = 1000

📌 5. Common PYQ-Based Formulas

Problem Type	Formula
"At least one"	= 1 – P(none)
"Exactly one"	Use combination × ways
Words from letters	Total = n! / (repeating letters factorial)
Arrangements where A and B together	Treat A+B as block ⇒ (n−1)!
Cards / Dice	Total outcomes = 52 / 6 etc.
Divide n people into 2 groups of r, s	= n! / (r! × s!)
Arrange with restrictions	Use total – unwanted
No repetition allowed	Decrease values step-by-step

📌 6. Practice-Style PYQ Questions

1. Find number of 3-digit numbers using 1–9 without repetition.
2. From the word "TATTOO", how many distinct arrangements are possible?
3. In how many ways can 5 girls and 3 boys sit in a row such that boys sit together?

4. How many numbers less than 1000 can be formed using digits 2, 3, 4, 5 without repetition?
5. How many ways can a team of 3 be selected from 6 boys and 5 girls including at least 1 girl?

📌 7. Tricks to Remember

Tip	Usage
Use complement for “at least”	Faster
Check if order matters	Use Permutation
Watch for repetition	Use Division Principle
Overlap in sets?	Use inclusion-exclusion
Grouping people?	Use n! / r!s!...

PNC

✅ 1. BASIC DEFINITIONS

- Permutation: Arrangement of objects.
  Formula: $nPr = n! / (n - r)!$

- Combination: Selection of objects.
  Formula: $nCr = n! / (r!(n - r)!)$

✅ 2. FACTORIAL BASICS

- $n! = n \times (n-1) \times \dots \times 2 \times 1$
- $0! = 1$

✅ 3. RELATIONSHIPS

- $nCr = nC(n-r)$
- $nCr + nC(r-1) = (n+1)Cr$

✅ 4. PERMUTATION CASES

1. **All different:**
   - $nPr = n! / (n - r)!$
2. **All arranged:** $nPn = n!$
3. **Circular Permutations:**
   - Clockwise: $(n-1)!$
   - Clockwise + anti-clockwise (necklace): $(n-1)! / 2$
4. **With Repetition (Alike Objects):**
   - $n! / (p! \times q! \times r! \dots)$

✅ 5. COMBINATION CASES

1. **Simple selection:** nCr
2. **With repetition allowed:** $n+r-1Cr$
3. **Team/committee:** Use combinations (nCr)

✅ 6. SHORTCUTS

- Words with repeated letters:
  e.g. BANANA → $6! / (3! \times 2!)$
- All letters used: $n!$ / repetition!
- Vowels together: Treat vowels as one block
- Vowels not together: total − together cases

✅ 7. IMPORTANT FORMULAS

- **nPr = nCr × r!**
- **Total selections from n items = 2^n**
- **Total non-empty subsets = 2^n – 1**
- **Number of ways to divide into groups depends on equality**

✅ 8. PYQ TYPES

- **Arrangement of letters with/without repetition**
- **Selection of groups or teams**
- **Identical objects**
- **Starts with vowel / vowels together / conditional cases**

✅ 9. PRACTICE QUESTIONS

1. **In how many ways can the letters of the word "MISSISSIPPI" be arranged?**
2. **Find total 5-letter words using "DELHI" without repetition.**
3. **From 7 men and 5 women, form a committee of 4 people with at least 1 woman.**
4. **Find number of ways to arrange "SUCCESS".**
5. **How many 4-digit numbers from digits 1 to 5 if repetition not allowed?**

Calculation Booster

First calculations before move to algebra

📚 *Memory + Speed = God-Level Accuracy*

📘 1. Squares and cube (1 to 30)

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

16	256	4096
17	289	4913
18	324	5832
19	361	6859
20	400	8000
21	441	9261
22	484	10648
23	529	12167
24	576	13824
25	625	15625
26	676	17576
27	729	19683
28	784	21952
29	841	24389
30	900	27000

For perfect root just use reverse technic

eg : If 2 is square of 4 , Then √4 is 2.

same with all cube and square roots

📘 2. Tables from 11 to 30

Table	Trick
11	Double digits: 11×1 = 11, 22, 33…
12	10n + 2n
13	10n + 3n
14	7n × 2
15	10n + 5n
16	8n × 2
17	10n + 7n
18	9n × 2
19	20n − n
20	n × 2 × 10
21	7n × 3
22	11n × 2
23–30	Best to write and revise regularly

📌 **Pro Tip: Learn 11–20 by patterns, 21–30 by double multiples or breakdowns.**

📘 3. Factorials (0! to 20!)

n	n!
0!	1
1!	1
2!	2
3!	6
4!	24
5!	120
6!	720
7!	5040
8!	40320
9!	362880
10!	3628800
11!	39916800
12!	479001600
13!	6227020800
14!	87178291200

15!	1307674368000
16!	20922789888000
17!	355687428096000
18!	6402373705728000
19!	121645100408832000
20!	2432902008176640000

🧠 Memorize till 8! – often asked in P&C, Probability, and Logarithms.

📘 7. Bonus Speed Tricks

🧠 a. Multiplying by 5:

→ Multiply by 10 and divide by 2

🧠 b. Squaring Numbers ending in 5:

→ $(n5)^2 = n(n + 1) + "25"$
E.g., $35^2 = 3\times4 = 12 \rightarrow 1225$

🧠 c. Multiply 2-digit numbers close to 100:

→ Use: $(100 - a)(100 - b) = 10000 - 100(a + b) + ab$
E.g., $98\times97 = 10000 - 100\times(98 + 97) + 98\times97$

🧠 d. Approx √ or ∛ values:

Use binomial expansion or nearest perfect square/cube.

✅ Final Tip Sheet for Practice:

- Memorize $\sqrt{1}$ to $\sqrt{30}$, $\sqrt[3]{1}$ to $\sqrt[3]{30}$
- Table drill daily from 11–30
- Learn factorials till 8! by heart
- Write 1–30 squares and cubes 3x per week
-

Algebra

📌 1. Types of Algebra Questions in NIMCET

- Identities & Simplifications
- Quadratic Equations
- Indices & Surds
- Inequalities
- Polynomials & Remainder Theorem
- Word problems (age, speed, time)
- Functional Equations
- Algebraic Fractions & Rational Expressions

📌 2. Most Important Algebraic Identities

🔹 Standard Identities

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a + b)(a - b)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a) - 3abc$
7. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
8. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

📌 3. Factorization Shortcuts

Expression	Trick
$x^2 + (a + b)x + ab$	$= (x + a)(x + b)$
$x^3 + y^3 + z^3 - 3xyz$	$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$x^4 - y^4$	$= (x^2 + y^2)(x + y)(x - y)$

Number-Based Algebraic Tricks

Question Type	Shortcut
$A + 1/A = x$	$A^2 + 1/A^2 = x^2 - 2$
$A - 1/A = x$	$A^2 + 1/A^2 = x^2 + 2$
$A^3 + 1/A^3 = ?$	Use: $(x^3 - 3x)$ where $x = A + 1/A$

Formation of New Quadratics from Roots

- If roots are α and β, then:
New Roots	Equation
α^2, β^2	$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2$
$1/\alpha, 1/\beta$	$x^2 - (1/\alpha + 1/\beta)x + 1/(\alpha\beta)$
$\alpha + k, \beta + k$	$x^2 - (S + 2k)x + (P + kS + k^2)$

📌 4. Special Fractions Tricks

If:
$x + 1/x = k$
Then:

- $x^2 + 1/x^2 = k^2 - 2$
- $x^3 + 1/x^3 = k^3 - 3k$
- $x^4 + 1/x^4 = k^4 - 4k^2 + 2$
- $x^n + 1/x^n$ = (k × previous term) − previous to previous term

📌 5. Quadratic Equations

Standard form: $ax^2 + bx + c = 0$

- Roots:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

- Sum of roots = $-b/a$
- Product of roots = c/a
- Nature of roots from $D = b^2 - 4ac$

Discriminant D	Nature of Roots
> 0 and perfect square	Real, rational, distinct
> 0 and not perfect square	Real, irrational
= 0	Real & equal
< 0	Imaginary

🔶 Trick:

- If α and β are roots of a quadratic, then:
  - $\alpha + \beta = -b/a$

- $\alpha\beta = c/a$
- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

📌 6. Inequalities (Basics)

If $a < b$, then:

- $a + c < b + c$
- $ac < bc$ (only if $c > 0$)
- $ac > bc$ (if $c < 0$)
- $a^2 < b^2$ (only if both a and b > 0)

Always solve by checking critical points, and use sign-table method if needed.

📌 7. Indices & Surds

🔹 **Important Rules**

Rule	Example
$a^m \times a^n = a^{(m+n)}$	$2^2 \times 2^3 = 2^5$
$a^m / a^n = a^{(m-n)}$	$2^5 / 2^2 = 2^3$
$(a^m)^n = a^{(mn)}$	$(2^2)^3 = 2^6$
$(ab)^n = a^n \times b^n$	$(2\times3)^2 = 2^2\times3^2$
$a^0 = 1$	if $a \neq 0$
$a^{(-n)} = 1/a^n$	$2^{-2} = 1/4$

Rule	Formula
Product	$a^m \times a^n = a^{m+n}$
Quotient	$a^m / a^n = a^{m-n}$
Power of Power	$(a^m)^n = a^{mn}$
Power of Product	$(ab)^m = a^m \times b^m$
Zero Power	$a^0 = 1$
Negative Power	$a^{-n} = 1 / a^n$
Fractional	$a\text{\^{}}(m/n) = \sqrt[n]{a^m}$

🔹 Surds Rules

- $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$
- Rationalize: multiply by conjugate if in form $a/\sqrt{b}$
- $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$ ❌

📌 8. Polynomials & Remainder Theorem

- If $f(x)$ is divided by $(x - a)$, then remainder = $f(a)$
- Factor theorem: If $f(a) = 0$, then $(x - a)$ is a factor
- Long division: Check degree pattern
- Sum/Product of roots in cubic:
    - Sum = $-b/a$
    - Product = $-d/a$

📌 9. Functional Equations (Common Type)

Example: If $f(x) + f(1/x) = x$, find $f(x)$
Trick: Replace x with $1/x$
Then solve both equations simultaneously.

📌 10. PYQ-Level Tricks

Question Type	Trick
$x + 1/x = k$ type	Use power identity shortcut
Large roots	Use Vieta's root formula (Sum/Product)
"Exactly divisible"	Use remainder/factor theorem
$x^4 + x^2 + 1$	Multiply/divide by x^2
√ expressions	Use identity like $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
Functional equation	Try replacing variable cleverly
Find value of $x + 1/x$	Convert higher powers using identities
Min/Max value	Use AM ≥ GM or identity trick

📌 11. Golden Practice Questions (PYQ Style)

1. If $x + 1/x = 3$, find $x^2 + 1/x^2$
2. Find value of $x^4 + 1/x^4$ if $x + 1/x = 2$
3. $f(x) + f(1/x) = 5$, find $f(x)$
4. Simplify: $(a + b)^2 - (a - b)^2$
5. Roots of $ax^2 + bx + c = 0$ are equal → condition?
6. Factor: $x^3 + x^2 - x - 1$
7. Find $x^3 + 1/x^3$ when $x + 1/x = -1$
8. If $f(x) = x^2 - 3x + 2$, find remainder when divided by $(x - 2)$

📌 12. Rapid-Recap Sheet (Lightning Memory)

1. $x + 1/x = k \rightarrow x^2 + 1/x^2 = k^2 - 2$

2. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

3. Roots equal → $D = 0$

4. $f(x) \div (x - a)$ → remainder = $f(a)$

5. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(\ldots)$

6. Indices → combine bases, subtract powers

7. Surds → rationalize using conjugate

8. Polynomial sum of roots = $-b/a$

9. Circle equations or simplification → factor + identity!

Logarithm

📘 1. Basics of Logarithms

If: $a^b = c$, then $\log_a(c) = b$
That is, log base a of c = b

🧠 Meaning:

- $\log_a(a) = 1$
- $\log_a(1) = 0$
- $\log_a(0)$ is undefined
- Logarithms are defined only for positive numbers

📘 2. Fundamental Log Identities

Formula	Meaning
$\log_a(m \times n)$	$= \log_a(m) + \log_a(n)$
$\log_a(m/n)$	$= \log_a(m) - \log_a(n)$
$\log_a(m^r)$	$= r \times \log_a(m)$
$\log_a(\sqrt{m})$	$= (1/2) \times \log_a(m)$
$\log_a(1/m)$	$= -\log_a(m)$
$\log_a(m^1/n)$	$= (1/n) \times \log_a(m)$

🧠 PYQ Tip: Use these to break big expressions into sums or subtractions.

📘 3. Base Change Formula

If log base a of b is needed but not given:

➤ $\log_a(b) = \log_x(b) / \log_x(a)$

(You can use base 10 or base e or any convenient base)

🧠 Most-used:

- $\log_a(b) = \log(b) / \log(a)$ (Base 10 used most)

📘 4. Common Logarithms

Log Type	Base
Common log	base 10 ⇒ log(x)
Natural log	base e ⇒ ln(x) ≈ 2.71828

📘 5. Important Log Values (Direct)

Expression	Value
$\log_{10}(1) = 0$	
$\log_{10}(10) = 1$	
$\log_{10}(100) = 2$	
$\log_{10}(1000) = 3$	
$\log_{10}(0.1) = -1$	
$\log_{10}(0.01) = -2$	

🧠 **For fast solving:**

$\log_{10}(10^n) = n$, and $\log_{10}(10^{-n}) = -n$

📘 6. Anti-Logarithm (Exponent Form)

If $\log_a(x) = y$, then $x = a^y$

📘 7. Solving Log Equations – General Approach

➤ **Step-by-Step:**

1. **Use identities to simplify**
2. **Convert to exponential form if needed**
3. **Combine like terms if same base**
4. **Check for domain restriction (argument must be > 0)**
5. **Don't forget to verify solutions**

📘 8. PYQ-Based Log Questions & Tricks

🟡 **PYQ 1**

If $\log_2(x) + \log_2(8) = 5$, find x.

Solution:
$\log_2(x \times 8) = 5 \rightarrow \log_2(8x) = 5$

$\Rightarrow 8x = 2^5 \Rightarrow x = 32 / 8 = 4$

🟡 PYQ 2

If $\log_x(25) = 2$, find x.

Solution:
Convert: $x^2 = 25 \Rightarrow x = \pm 5 \Rightarrow$ But log base must be > 0 and ≠ 1 $\Rightarrow x = 5$

🟡 PYQ 3

$\log(2x - 1) = 1 \Rightarrow$
$10^1 = 2x - 1 \Rightarrow 2x = 11 \Rightarrow x = 5.5$

🟡 PYQ 4

If $\log(x + 2) - \log(x - 2) = 1$, find x.

Solution:
$\log[(x + 2)/(x - 2)] = 1 \Rightarrow (x + 2)/(x - 2) = 10$
Solve: $10(x - 2) = x + 2 \Rightarrow 10x - 20 = x + 2 \Rightarrow x = 22/9$

📘 9. Graphical Behavior (Quick MCQ Tip)

- $\log(x)$ → Increases slowly
- $\log(x)$ defined for $x > 0$ only
- $\log(x) = 0 \Rightarrow x = 1$

📘 10. Logarithmic Inequalities (NIMCET High Level)

Use same base inequality principles

- If base $a > 1$, then:
  $\log_a(x) > \log_a(y) \Rightarrow x > y$
- If base $0 < a < 1$, inequality reverses

📘 11. Misc Formulas & Examples

Expression	Value
$\log_a(b) \times \log_b(a) = 1$	
$\log_a(b) = 1 / \log_b(a)$	
$\log_a(m) + \log_b(m)$	$= \log_a(m) + (\log_a(m) / \log_a(b))$
If $a^x = b$, then $x = \log_a(b)$	

📘 12. Final Revision – Rapid Fire

- $\log_a(xy) = \log_a x + \log_a y$ ✅
- $\log_a(x/y) = \log_a x - \log_a y$ ✅
- $\log_a(x^n) = n \log_a x$ ✅
- $\log_a(1/x) = -\log_a x$ ✅
- $\log_a(\sqrt{x}) = \frac{1}{2} \log_a x$ ✅
- $\log_a(a) = 1$ ✅
- $\log_a(1) = 0$ ✅
- $\log_a(b) = \log(b)/\log(a)$ ✅

🧪 Practice Challenge

1. **Solve: $\log_3(x) + \log_3(x - 2) = 2$**
2. **$\log_x(81) = 4$. Find x**
3. **$\log_{10}(x - 2) + \log_{10}(x - 3) = 1$**
4. **If $\log(x + 1) - \log(x - 1) = \log(3)$, then x = ?**
5. **Evaluate: $\log_9(81)$**

Progressions

📘 1. Arithmetic Progression (AP)

Sequence where difference between terms is constant.

➤ General Form:

$a, a + d, a + 2d, a + 3d, \ldots$

🧠 Formulas:

Concept	Formula
n^{th} term	$a_n = a + (n - 1)d$
Sum of n terms	$S_n = n/2 \times [2a + (n - 1)d]$
Sum (if last term known)	$S_n = n/2 \times (a + l)$
Sum of first n natural numbers	$1 + 2 + \ldots + n = n(n + 1)/2$

🔍 PYQ Trick (AP):

- For finding unknown 'n' or 'd', use $a_n = a + (n - 1)d$
- If three terms in AP: Take as $(a - d), a, (a + d)$
- If four terms in AP: Use $a - 3d, a - d, a + d, a + 3d$

📘 2. Geometric Progression (GP)

A sequence where each term is multiplied by common ratio.

➤ General Form:

$a, ar, ar^2, ar^3, \ldots$

🧠 Formulas:

Concept	Formula
n^{th} term	$a_n = ar^{n-1}$
Sum of n terms ($r \neq 1$)	$S_n = a(r^n - 1)/(r - 1)$
Sum ($r < 1$, infinite GP)	$S_\infty = a / (1 - r)$

🔍 PYQ Trick (GP):

- If three terms in GP: Use a/r, a, ar
- Four terms: a/r^3, a/r, ar, ar^3
- Always simplify using powers & ratios!

📘 3. Harmonic Progression (HP)

A sequence is in HP if reciprocals are in AP

Formula	
HP: a, b, c → 1/a, 1/b, 1/c in AP	
nth term of HP	Take reciprocal of AP term: $1 / [a + (n - 1)d]$

🔍 PYQ Tip:

- Convert HP to AP and solve.
- For 3 terms in HP: take as $1/(a - d)$, $1/a$, $1/(a + d)$

📘 4. Special Results & PYQ-Style Formulas

Expression	Result
$1 + 2 + 3 + ... + n$	$= n(n + 1)/2$
$1^2 + 2^2 + 3^2 + ... + n^2$	$= n(n + 1)(2n + 1)/6$
$1^3 + 2^3 + 3^3 + ... + n^3$	$= [n(n + 1)/2]^2$
Sum of first n odd numbers	$= n^2$
Sum of first n even numbers	$= n(n + 1)$

📘 5. Mixed Series (Trick Questions)

1. **Alternate Series (Even + Odd terms):**
   - **Separate even and odd position terms as two different APs or GPs**
2. **AM = GM = HM Conditions:**
   - **All are equal only when all terms are equal**
3. **Middle Terms (for odd/even n):**
   - **Odd n → Middle term = (n + 1)/2-th term**
   - **Even n → Average of n/2-th and (n/2 + 1)-th terms**

📘 6. PYQ Examples

🟡 **PYQ 1**

If the 5th term of an AP is 18 and 9th term is 30, find the 1st term.

$a_5 = a + 4d = 18$
$a_9 = a + 8d = 30$
$\Rightarrow$ Subtract: $4d = 12 \Rightarrow d = 3$
$\Rightarrow a = 18 - 4 \times 3 = 6$

🟡 **PYQ 2**

Find the sum of first 10 terms of AP: 5, 8, 11...

$a = 5, d = 3$
$S_{10} = 10/2 \times [2 \times 5 + 9 \times 3] = 5 \times (10 + 27) = 185$

🟡 PYQ 3

If three numbers are in GP and their product is 216, and sum is 19, find numbers.

Let terms = a/r, a, ar
Product: $a^3 = 216 \Rightarrow a = 6$
Sum: a/r + a + ar = 19
Put a = 6 ⇒ 6(1/r + 1 + r) = 19 ⇒ Solve for r
r = 1/2 ⇒ Numbers: 12, 6, 3 ✅

🟡 PYQ 4

Find nth term of HP: 1/2, 1/3, 1/4...

This is reciprocal of AP: 2, 3, 4...
AP: a = 2, d = 1 ⇒ $a_n = 2 + (n - 1)$
HP Term = 1 / [n + 1] ✅

📘 7. Fast Revision – Lightning Recap

🔹 AP nth term = a + (n − 1)d
🔹 AP sum = n/2 × (2a + (n − 1)d)

🔹 GP nth term = ar^{n-1}
🔹 GP sum = $a(r^n - 1)/(r - 1)$
🔹 GP infinite = a / (1 − r)

🔹 HP: Convert to AP by taking reciprocals

🔹 Sum of n natural = n(n + 1)/2
🔹 Sum of n squares = n(n + 1)(2n + 1)/6
🔹 Sum of n cubes = $[n(n + 1)/2]^2$

🧪 Practice Drill

1. If the 1st term is 3 and 10th term is 30 in AP, find 'd'.
2. Find the sum of the first 15 natural numbers.
3. If a, b, c are in AP and a + c = 10, b = 7, find a, b, c.
4. Find 10th term of a GP: 3, 6, 12...
5. If 1/x, 1/y, 1/z are in AP, prove x, y, z are in HP.

TRIGONOMETRY

✅ 1. BASIC DEFINITIONS

In a right-angled triangle:

- **sin θ = Opposite / Hypotenuse**
- **cos θ = Adjacent / Hypotenuse**
- **tan θ = Opposite / Adjacent**
- **cot θ = 1 / tan θ = Adjacent / Opposite**
- **sec θ = 1 / cos θ = Hypotenuse / Adjacent**
- **cosec θ = 1 / sin θ = Hypotenuse / Opposite**

✅ 2. TRIGONOMETRIC RATIOS TABLE (0° to 90°)

θ	0°	30°	45°	60°	90°
sin θ	0	1/√2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞
cot θ	∞	√3	1	1/√3	0
sec θ	1	2/√3	√2	2	∞
cosec θ	∞	2	√2	2/√3	1

✔ **Trick to Remember (for sin): √0/2, √1/2, √2/2, √3/2, √4/2**

✅ **3. TRIGONOMETRIC IDENTITIES**

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

→ **Learn and use these in algebra-based questions and equation-solving.**

✅ **4. COMPLEMENTARY ANGLES**

sin(90°−θ)=cosθcos(90°−θ)=sinθtan(90°−θ)=cotθ\sin(90° − θ) = \cos θ \quad \cos(90° − θ) = \sin θ \quad \tan(90° − θ) = \cot θ cot(90°−θ)=tanθsec(90°−θ)=cosecθcosec(90° −θ)=secθ\cot(90° − θ) = \tan θ \quad \sec(90° − θ) = \cosec θ \quad \cosec(90° − θ) = \sec θ

✅ **5. TRICKS: SIGNS IN QUADRANTS (ASTC RULE)**

📌 Use A S T C to remember signs of trig functions:

- 1st Quadrant (0°–90°): All positive
- 2nd Quadrant (90°–180°): Sin & Cosec positive
- 3rd Quadrant (180°–270°): Tan & Cot positive
- 4th Quadrant (270°–360°): Cos & Sec positive

✅ 6. TRIG VALUES FOR ALL ANGLES

- $\sin(180° + \theta) = -\sin\theta$
- $\cos(180° + \theta) = -\cos\theta$
- $\tan(180° + \theta) = \tan\theta$
- $\sin(90° + \theta) = \cos\theta$
- $\cos(90° + \theta) = -\sin\theta$

✔️ Key Trick: Use quadrant + ASTC rule + known value.

✅ 7. ANGLE CONVERSION

- Degrees → Radians:
- x∘=π180×x radiansx^\circ = \frac{π}{180} × x \, \text{radians}
- Radians → Degrees:
- x rad=180π×x∘x \, \text{rad} = \frac{180}{π} × x^\circ

✅ 8. IMPORTANT FORMULAS

📌 Product Identities:

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- $\tan 2\theta = 2\tan\theta / (1 - \tan^2\theta)$

📌 Sum and Difference:

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) = (\tan A \pm \tan B) / (1 \mp \tan A\tan B)$

📌 Trig Equations:

- $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$
- $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$
- $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$

✅ 9. PYQ TYPES + EXAMPLES

🔹 **Type 1: Value-Based**

Find: tan 45° + sec 60° – sin 30°
→ tan 45° = 1, sec 60° = 2, sin 30° = 1/2
→ Answer = 1 + 2 – 0.5 = 2.5

🔹 **Type 2: Equation Solving**

If $\sin^2\theta + \cos^2\theta = x$, find x
→ Identity ⇒ x = 1

🔹 **Type 3: Quadrant Signs**

If $\tan\theta = -1$, in which quadrant does θ lie?
→ tan is negative in 2nd and 4th

🔹 **Type 4: Find angle**

If $\sin\theta = \cos\theta$, then θ = ?
→ $\tan\theta = 1 \Rightarrow \theta = 45°$

✅ 10. YOUR PRACTICE QUESTIONS

1. **Evaluate: $\sin^2 30° + \cos^2 30°$**
2. **Convert 120° into radians**
3. **Find sin(150°) using ASTC and identities**
4. **Solve: $\tan^2\theta - \sec^2\theta + 1 = 0$**
5. **If cot A = √3, find sin A and cos A**
6. **Simplify: $\sin 45° \cos 45° + \tan^2 30°$**

COORDINATE GEOMETRY

✅ 1. BASIC DEFINITIONS

- **Coordinate Plane: X-axis and Y-axis divide the plane into 4 quadrants.**
- **Origin (O): Point (0, 0)**
- **Abscissa = x-coordinate, Ordinate = y-coordinate**
- **Distance Between Two Points:**
  (x2−x1)2+(y2−y1)2\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
- **Midpoint Formula:**
  (x1+x22,y1+y22)\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
- **Section Formula (Internal Division):**
  (mx2+nx1m+n,my2+ny1m+n)\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)

✅ 2. AREA OF TRIANGLE

- **Given vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$:**
- **Area=12 ∣ x1(y2−y3)+x2(y3−y1)+x3(y1−y2) ∣ \text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|**
- **Area = 0 ⇒ Points are collinear**

✅ 3. CENTROID, CIRCUMCENTER, IN-CENTER

- **Centroid (G):**
  G=(x1+x2+x33,y1+y2+y33)G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right)
- **Circumcenter:** Point equidistant from all vertices (perpendicular bisectors meet)
- **Incenter:** Intersection of angle bisectors

✅ 4. SLOPE OF A LINE

- Slope = m=y2−y1x2−x1m = \frac{y_2 - y_1}{x_2 - x_1}
- Line parallel to x-axis ⇒ slope = 0
- Line parallel to y-axis ⇒ slope = undefined
- Lines:
  - Parallel ⇒ same slope
  - Perpendicular ⇒ product of slopes = −1

✅ 5. EQUATIONS OF LINES

- Point-slope form: y−y1=m(x−x1)y - y_1 = m(x - x_1)
- Slope-intercept form: y=mx+cy = mx + c
- Two-point form:
  y−y1=y2−y1x2−x1(x−x1)y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)
- Intercept form: xa+yb=1\frac{x}{a} + \frac{y}{b} = 1
- General form: Ax + By + C = 0
  - Slope = −A/B, Intercept = −C/B

✅ 6. DISTANCE FROM POINT TO LINE

- For line Ax + By + C = 0 and point (x_1, y_1):
- Distance= ∣ Ax1+By1+C ∣ A2+B2\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}

✅ 7. CIRCLE

- **Standard Equation:**
  (x−h)2+(y−k)2=r2(x − h)^2 + (y − k)^2 = r^2
  Center: (h, k), Radius: r
- **General form:**
  x2+y2+2gx+2fy+c=0x^2 + y^2 + 2gx + 2fy + c = 0
  Center: (−g, −f), Radius = g2+f2−c\sqrt{g^2 + f^2 − c}

✅ 8. PARABOLA (Standard Focus at Origin)

- **Right/Left Open: y2=4axy^2 = 4ax**
- **Up/Down Open: x2=4ayx^2 = 4ay**
- **Vertex: (0, 0), Focus at (a, 0) or (0, a)**
- **Axis: x-axis or y-axis**
- **Directrix: line opposite focus**

✅ 9. ELLIPSE

- **Standard: x2a2+y2b2=1\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b**
- **Horizontal axis: major = 2a, minor = 2b**
- **Foci: (±c, 0), where c=a2−b2c = \sqrt{a^2 - b^2}**

✅ 10. HYPERBOLA

- **Standard: x2a2−y2b2=1\frac{x^2}{a^2} − \frac{y^2}{b^2} = 1**
- **Transverse axis: along x-axis**
- **Foci: (±c, 0), where c=a2+b2c = \sqrt{a^2 + b^2}**

✅ 11. STRAIGHT LINE COMBINATIONS

- **Angle Between Two Lines (m_1 and m_2):**
- **tanθ= ∣ m1−m21+m1m2 ∣ \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|**
- **Bisectors of angle between lines: use sign of constants after converting both to same form.**

✅ 12. LOCUS PROBLEMS

- **Follow the definition strictly**
- **Example: "Locus of points equidistant from (x_1, y_1) and (x_2, y_2)" ⇒ Perpendicular bisector of the segment.**

✅ 13. PYQ TYPES

- **Distance and midpoint formula**
- **Triangle area and collinearity**
- **Line slope + parallel/perpendicular line**
- **Point on line or not**
- **Circle equation/center/radius from general form**
- **Tangents, intersection of lines, shortest distance**
- **Locus formation**

✅ 14. PRACTICE PROBLEMS

1. **Find the coordinates dividing line joining A(2, 3) and B(8, 7) in 2:3 ratio.**
2. **What is the equation of the line passing through (1, 2) with slope 3?**
3. **Find the radius of the circle: x2+y2−4x+6y−12=0x^2 + y^2 − 4x + 6y − 12 = 0**
4. **Find area of triangle with vertices (0, 0), (4, 0), (4, 3).**
5. **Find the distance between the point (3, 4) and line $3x - 4y + 5 = 0$.**

CALCULUS

✅ 1. LIMITS

1.1 Basic Concept:

limx→af(x)=Lmeans f(x) approaches L as x→a\lim_{x \to a} f(x) = L \quad \text{means } f(x) \text{ approaches } L \text{ as } x \to a

1.2 Standard Limits:

- limx→0sinxx=1\lim_{x \to 0} \frac{\sin x}{x} = 1
- limx→01−cosxx2=12\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}
- limx→∞(1+1x)x=e\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x = e
- limx→0ex−1x=1\lim_{x \to 0} \frac{e^x - 1}{x} = 1
- limx→0log(1+x)x=1\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1

1.3 Indeterminate Forms:

- 00,∞∞,0×∞,∞−∞,1∞,∞0,00\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, 1^\infty, \infty^0, 0^0

✅ 2. DERIVATIVES

2.1 Basic Formula:

f′(x)=limh→0f(x+h)−f(x)hf'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}

2.2 Common Derivatives:

- d/dx(xn)=nxn−1d/dx(x^n) = nx^{n-1}
- d/dx(sinx)=cosxd/dx(\sin x) = \cos x
- d/dx(cosx)=−sinxd/dx(\cos x) = -\sin x

- d/dx(tanx)=sec2xd/dx(\tan x) = \sec^2 x
- d/dx(cotx)=−csc2xd/dx(\cot x) = -\csc^2 x
- d/dx(secx)=secxtanxd/dx(\sec x) = \sec x \tan x
- d/dx(cscx)=−cscxcotxd/dx(\csc x) = -\csc x \cot x
- d/dx(ex)=exd/dx(e^x) = e^x
- d/dx(lnx)=1/xd/dx(\ln x) = 1/x
- d/dx(ax)=axlnad/dx(a^x) = a^x \ln a

2.3 Rules:

- Sum: (f+g)′=f′+g′(f + g)' = f' + g'
- Product: (fg)′=f′g+fg′(fg)' = f'g + fg'
- Quotient: (fg)′=f′g−fg′g2\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}
- Chain Rule: ddx[f(g(x))]=f′(g(x)) ⋅ g′(x)\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)

✅ 3. APPLICATIONS OF DERIVATIVES

3.1 Monotonicity:

- If f′(x)>0f'(x) > 0, function is increasing
- If f′(x)<0f'(x) < 0, function is decreasing

3.2 Maxima and Minima:

- If f′(x)=0f'(x) = 0 and f′′(x)<0f''(x) < 0, local maxima
- If f′(x)=0f'(x) = 0 and f′′(x)>0f''(x) > 0, local minima

✅ 4. INTEGRATION

4.1 Basic Formulas:

- ∫xndx=xn+1n+1+C\int x^n dx = \frac{x^{n+1}}{n+1} + C (n ≠ −1)
- ∫1xdx=ln ∣ x ∣ +C\int \frac{1}{x} dx = \ln |x| + C

- ∫exdx=ex+C\int e^x dx = e^x + C
- ∫axdx=axlna+C\int a^x dx = \frac{a^x}{\ln a} + C
- ∫sinxdx=−cosx+C\int \sin x dx = -\cos x + C
- ∫cosxdx=sinx+C\int \cos x dx = \sin x + C
- ∫sec2xdx=tanx+C\int \sec^2 x dx = \tan x + C
- ∫csc2xdx=−cotx+C\int \csc^2 x dx = -\cot x + C
- ∫secxtanxdx=secx+C\int \sec x \tan x dx = \sec x + C
- ∫cscxcotxdx=−cscx+C\int \csc x \cot x dx = -\csc x + C

4.2 Integration by Parts:

∫uvdx=u∫vdx−∫(dudx⋅∫vdx)dx\int u v dx = u \int v dx - \int \left(\frac{du}{dx} \cdot \int v dx \right) dx

ILATE Order: Inverse, Log, Algebraic, Trig, Exponential

✅ 5. DEFINITE INTEGRALS

Properties:

- ∫aaf(x)dx=0\int_a^a f(x) dx = 0
- ∫abf(x)dx=−∫baf(x)dx\int_a^b f(x) dx = -\int_b^a f(x) dx
- ∫abf(x)dx=∫abf(a+b−x)dx\int_a^b f(x) dx = \int_a^b f(a + b - x) dx

Fundamental Theorem:

- If F′(x)=f(x)F'(x) = f(x), then ∫abf(x)dx=F(b)−F(a)\int_a^b f(x) dx = F(b) - F(a)

✅ 6. OTHER TECHNIQUES

Implicit Differentiation:
Used when yy is not isolated. Example:
x2+y2=1⇒dydx=−xyx^2 + y^2 = 1 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}

Parametric Differentiation:
If x=f(t),y=g(t)x = f(t), y = g(t), then
dydx=dy/dtdx/dt\frac{dy}{dx} = \frac{dy/dt}{dx/dt}

Logarithmic Differentiation:
Used when y = f(x)^g(x). Take log on both sides then differentiate.

✅ 7. TANGENTS & NORMALS

- Tangent Slope: dydx\frac{dy}{dx}
- Tangent Equation: y−y1=m(x−x1)y - y_1 = m(x - x_1)
- Normal Slope: −1/m-1/m

✅ 8. PYQ-TYPE QUESTIONS

- Limits: Standard identities, rational functions
- Derivatives: Algebraic, trigonometric, exponential
- Applications: Maxima/minima, tangent/normal
- Integrals: Substitution, by parts, definite integrals using properties

✅ 9. PRACTICE EXAMPLES

1. Evaluate: limx→0sin3xx\lim_{x \to 0} \frac{\sin 3x}{x}
2. Differentiate y=xxy = x^x
3. Maximize: f(x)=−x2+4x+5f(x) = -x^2 + 4x + 5
4. Find ∫xx2+1dx\int \frac{x}{x^2 + 1} dx
5. Evaluate ∫0πsin2xdx\int_0^{\pi} \sin^2 x dx

STATISTICS

✅ 1. MEASURES OF CENTRAL TENDENCY

➤ **Mean (Average)**

- **Ungrouped data:**
- **x¯=∑xin\bar{x} = \frac{\sum x_i}{n}**
- **Grouped data (discrete):**
- **x¯=∑fixi∑fi\bar{x} = \frac{\sum f_i x_i}{\sum f_i}**
- **Continuous data (class intervals):**
- **x¯=∑fixi∑fi,xi=midpoint\bar{x} = \frac{\sum f_i x_i}{\sum f_i}, \quad x_i = \text{midpoint}**

➤ **Median**

- **Odd n: Middle value**
- **Even n: Mean of two middle values**
- **Grouped Data:**
- **Median=l+(n2−Ff)⋅h\text{Median} = l + \left(\frac{\frac{n}{2} - F}{f}\right) \cdot h**
- **Where:**
    - **ll: lower boundary of median class**
    - **FF: cumulative frequency before median class**
    - **ff: frequency of median class**
    - **hh: class width**

➤ **Mode**

- **Grouped Data (mode formula):**
- **Mode=l+(f1−f02f1−f0−f2)⋅h\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \cdot h**
- **Where:**
    - **f1f_1: modal class frequency**

- f0f_0: frequency before modal class
- f2f_2: frequency after modal class

✅ 2. MEASURES OF DISPERSION

➤ Range

Range=Maximum−Minimum\text{Range} = \text{Maximum} - \text{Minimum}

➤ Mean Deviation (about mean):

MD=∑ ∣ xi−x¯ ∣ n\text{MD} = \frac{\sum |x_i - \bar{x}|}{n}

➤ Variance and Standard Deviation

- Ungrouped:
- σ2=∑(xi−x¯)2n,σ=σ2\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}, \quad \sigma = \sqrt{\sigma^2}
- Shortcut:
- σ2=∑xi2n−x¯2\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2
- Grouped:
- σ2=∑fixi2∑fi−x¯2\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2

✅ 3. COEFFICIENT OF VARIATION (C.V.)

C.V.=σx¯×100\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100

→ Used to compare variability of different data sets

✅ 4. CORRELATION

➤ Karl Pearson's Coefficient:

r=∑(x−x¯)(y−y¯)n⋅σx⋅σyr = \frac{\sum (x - \bar{x})(y - \bar{y})}{n \cdot \sigma_x \cdot \sigma_y}

- Range of rr: [−1, +1]
- If r = 1 → perfect positive correlation

- If r = −1 → perfect negative correlation
- If r = 0 → no correlation

✅ 5. REGRESSION

➤ Regression Line of Y on X:

Y=a+bX,b=r⋅σyσxY = a + bX, \quad b = r \cdot \frac{\sigma_y}{\sigma_x}

➤ Regression Line of X on Y:

X=a+bY,b=r⋅σxσyX = a + bY, \quad b = r \cdot \frac{\sigma_x}{\sigma_y}

✅ 6. PROBABILITY BASICS (Review – often asked in Statistics)

- Addition Rule:
- P(A∪B)=P(A)+P(B)−P(A∩B)P(A \cup B) = P(A) + P(B) − P(A \cap B)
- Multiplication Rule:
- P(A∩B)=P(A)⋅P(B ∣ A)P(A \cap B) = P(A) \cdot P(B|A)

✅ 7. PYQ TYPE QUESTIONS

1. Find the mean of grouped data
2. Find the mode using the formula
3. Calculate standard deviation using shortcut formula
4. Compare two data sets using C.V.
5. Find correlation coefficient given paired data
6. Regression equation given data and correlation

✅ 8. SHORTCUT TIPS FOR NIMCET

- In mean deviation, always take modulus (|value|)
- For variance shortcut: use ∑x2n−x¯2\frac{\sum x^2}{n} - \bar{x}^2

- If asked: “which distribution is more consistent?” → Lower C.V. wins
- Use mode formula only for grouped data
- Use mid-point of class as xix_i for grouped calculations
- In standard deviation, if values are very large, use assumed mean method

✅ **9. PRACTICE QUESTIONS (Must Attempt)**

1. **Find the median of:**
   Class: 0–10, 10–20, 20–30, 30–40, 40–50
   Frequency: 5, 8, 15, 16, 6
2. **Calculate mode for:**
   Class: 0–5, 5–10, 10–15, 15–20
   Frequency: 2, 10, 25, 5
3. **Compute mean and standard deviation of:**
   4, 6, 8, 10, 12
4. **For x and y:**
   x: 1, 2, 3, 4
   y: 2, 4, 6, 8
   Find correlation and regression line

Probability

📌 1. Basics of Probability

➤ **Classical Definition**

P(E) = (Number of favourable outcomes) / (Total number of outcomes)
Example: Getting a 3 on a die: P = 1/6

📌 2. Types of Events

Type	Explanation
Simple Event	Single outcome (e.g., getting 5 on die)
Compound Event	Combination of outcomes (e.g., even number on die)
Sure Event	Probability = 1
Impossible Event	Probability = 0
Mutually Exclusive	No overlap ($A \cap B = \emptyset$)
Exhaustive Events	Together cover the whole sample space
Equally Likely	All outcomes have equal chance

📌 3. Rules & Formulas

➤ **Addition Rule**

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

➤ **If A and B are Mutually Exclusive:**

- $P(A \cup B) = P(A) + P(B)$

➤ **Multiplication Rule**

- $P(A \cap B) = P(A) \times P(B|A)$
  - If A and B are independent:
    $P(A \cap B) = P(A) \times P(B)$

📌 4. Complementary Rule

$P(A') = 1 - P(A)$
Example: Not drawing an ace from a deck → $1 - (4/52) = 12/13$

📌 5. Conditional Probability

$P(A \mid B) = P(A \cap B) / P(B)$
Used when event B has already occurred.

📌 6. Independent Events

- Events A and B are independent if:

$P(A \cap B) = P(A) \times P(B)$

📌 7. Bayes' Theorem (Advanced, but asked rarely)

$P(A|B) = [P(B|A) \times P(A)] / P(B)$
Useful in probability reversal questions.

📌 8. Shortcut Techniques

Problem Type	Trick
At least 1	Use complement: 1 – P(none)
"Exactly k" in n trials	Use combinations: $nCk \times P^k \times (1-P)^{(n-k)}$
Drawing cards	Use 52 cards logic: 13 of each suit, 4 of each type
Coin flips	Total outcomes = 2^n
Dice rolls	Total outcomes = 6^n
Without replacement	Outcomes decrease after each pick
With replacement	Probability stays constant

📌 9. Common PYQ Concepts

Concept	Example
Two dice rolled – sum 7	(1,6), (2,5), ... = 6 outcomes ⇒ P = 6/36 = 1/6
One card drawn – face card	12/52 ⇒ P = 3/13
3 people – probability that all are born in different months	$12 \times 11 \times 10 / 12^3$
3 bulbs – at least one defective	1 – P(all non-defective)
Circular table – probability of same gender sitting together	Use P-grouping + total arrangements

📌 10. Practice Questions (Self-Made with PYQ Style)

1. **A card is drawn from a pack. What is the probability it is red or a king?**
2. **Two dice are thrown. What is the probability the sum is greater than 10?**
3. **In a box with 6 red, 4 blue balls, what is the probability of drawing 2 red without replacement?**
4. **From 5 men and 4 women, a team of 3 is formed. What is the probability at least 1 woman is included?**
5. **A coin is tossed 3 times. What is the probability of getting at most 2 heads?**

VECTORS

📌 1. Basics of Vectors

- A vector has both magnitude and direction.
- Represented as:
  $\overrightarrow{AB} = B - A = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

📌 2. Types of Vectors

Type	Definition
Zero Vector	Magnitude = 0
Unit Vector	Magnitude = 1
Equal Vectors	Same magnitude and direction
Negative Vector	Opposite direction: −A = A reversed
Collinear Vectors	One is scalar multiple of other

📌 3. Unit Vector Formula

If $A = a\hat{i} + b\hat{j} + c_k$
Then Unit Vector $\hat{a} = A / |A|$
→ $|A| = \sqrt{(a^2 + b^2 + c^2)}$

📌 4. Vector Algebra Rules

- Addition:
  $A + B = (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$
- Scalar Multiplication:
  $\lambda A = (\lambda a_1)i + (\lambda a_2)j + (\lambda a_3)k$

📌 5. Dot Product (Scalar Product)

$A \cdot B = |A||B|\cos\theta$
Also: $A \cdot B = a_1b_1 + a_2b_2 + a_3b_3$

- Orthogonal vectors → $A \cdot B = 0$
- Gives scalar result

📌 6. Cross Product (Vector Product)

$A \times B = |A||B|\sin\theta\ \hat{n}$
Direction → Right-hand rule

- Result is a vector perpendicular to both A and B
- If $A = a_1\hat{i} + a_2\hat{j} + a_{3k}$
  $B = b_1\hat{i} + b_2\hat{j} + b_{3k}$
  Then:

$A \times B = |\ i\ \ j\ \ k\ |$

$|a_1\ a_2\ a_3|$

$|b_1\ b_2\ b_3|$

📌 7. Properties of Dot and Cross Products

- $A \cdot B = B \cdot A$ (commutative)

- $A \times B = -(B \times A)$ (anti-commutative)
- $A \cdot (B \times C)$ = Scalar Triple Product

📌 8. Scalar Triple Product

$[A\ B\ C] = A \cdot (B \times C)$

- Value = Volume of parallelepiped
- If $[A\ B\ C] = 0 \Rightarrow$ Vectors are coplanar

📌 9. Vector Triple Product (Advanced)

$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

- Useful in identity-type vector questions

📌 10. Section Formula (Vector Form)

If point P divides A and B in m:n internally:

$P = (nA + mB) / (m + n)$

For external division:

$P = (nA - mB) / (n - m)$

📌 11. Collinearity Condition

Vectors A and B are collinear if:

$A = kB$ (for some scalar k)

📌 12. Angle Between Vectors

$\cos\theta = (A \cdot B) / (|A||B|)$
If $\cos\theta = 0 \rightarrow \theta = 90°$ (perpendicular)
If $\cos\theta = 1 \rightarrow \theta = 0°$ (parallel, same direction)
If $\cos\theta = -1 \rightarrow \theta = 180°$ (opposite direction)

📌 13. Coplanarity Condition

Vectors A, B, C are coplanar ⇔

[A B C] = 0 (Scalar triple product is zero)

📌 14. Applications in Geometry

- Area of Triangle (using vectors)
  = ½ × |A × B|
- Volume of Tetrahedron
  = (1/6) × |[A B C]|

📌 15. Quick Tricks for PYQs

PYQ Style Question	Trick
Collinear check	$A = \lambda B$
Perpendicular check	$A \cdot B = 0$
Area questions	Use cross product
Coplanarity	Scalar triple product = 0
Find unit vector	Divide vector by its magnitude
Angle between	Use $\cos\theta = A \cdot B$ /