INSTITUTE OF MICROELECTRONICS

NSSC2 - Exercise 4

Group 9

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Contents

1 Variation 0 - Basic setup

Setup:

- Domain is a square with length L,
- regular mesh based on a grid of 100 nodes.

Boundary conditions:

- N91 - N100: Dirichlet BCs (depending on the "load case") T =... ,
- N1 - N10: Neumann BCs (depending on the "load case") P =... ,
- otherwise: Neumann BCs, P = 0,

where P is the load ("force") vector entry in the sense of a point-wise source of heat per time. Simulation parameters (constant across all variations):

- Length in x- and y-direction: L = 0.01,
- Thickness (z-direction): hz = 0.001,
- Thermal conductivity ($k = k_{xx} = k_{yy}, k_{xy} = 0.$): k = 236,
- boundary conditions: $q(y = 0) = 3000000$, $T(y = L) = 293$.

Unless otherwise stated, the following SI-units are used:

- T in K
- L in m
- k in $W/(mK)$
- q in W/m^2 - ad Neumann
- P in W - ad nodal forces.

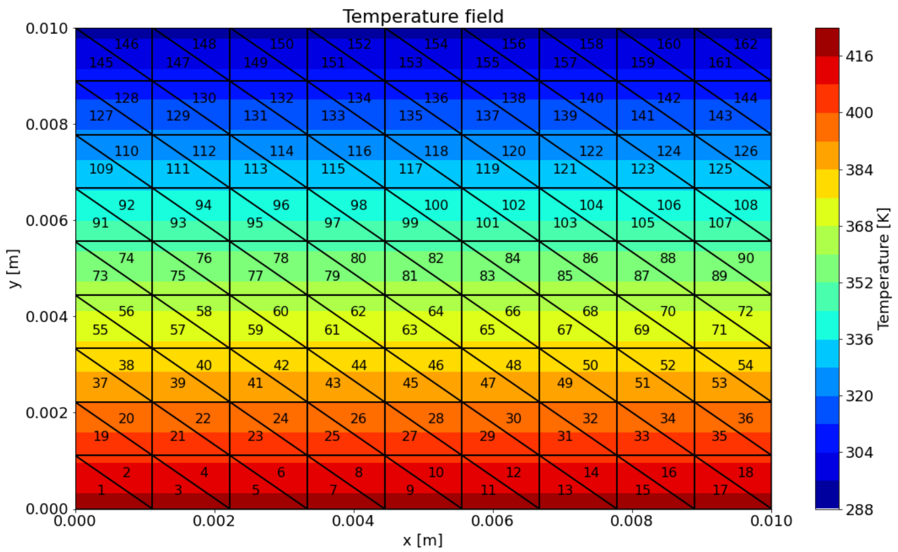

Fig. 1. V0: Temperature field.

Fig.1 shows the resulting temperature field for this basic setup. As expected, the resulting temperature field is of the form $T(x,y) \equiv T(y) \propto y$. The resulting gradient and heat flux is shown in Fig.2 and Fig.3 respectively. As expected, the heat flux shows the flow of heat energy going from the bottom, where the heat source is applied (Neumann BC), to the top. No heat flows through either the left or right side of the domain (homogenous Neumann BCs). The color of the arrows indicate the constant flux/gradient across the domain being equal to the appplied Neumann BCs. The following relations hold,

$$q(y=0) = 3 \cdot 10^6 \frac{W}{m^2} = k\frac{\Delta T}{\Delta y} = 236\frac{W}{mK}\frac{127.118K}{0.01m} \tag{1}$$

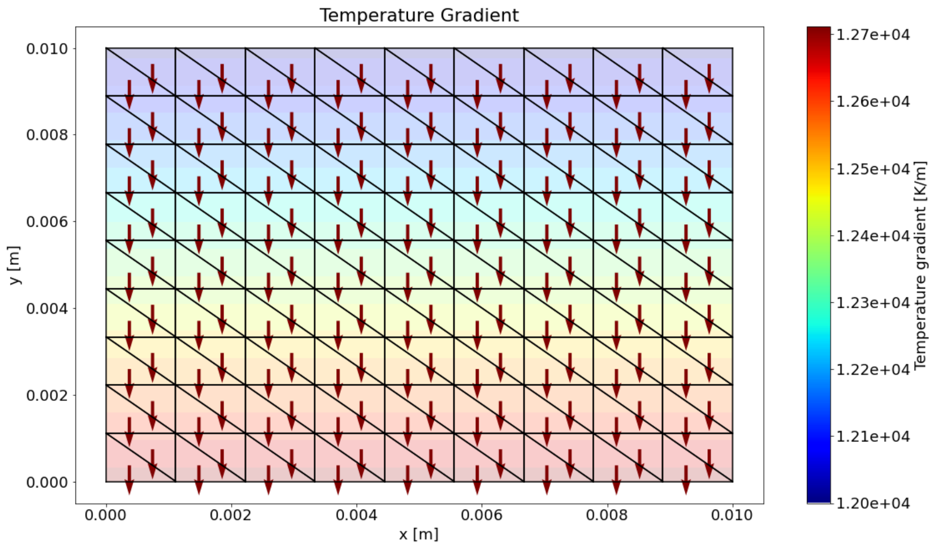

Fig. 2. V0: Temperature gradient.

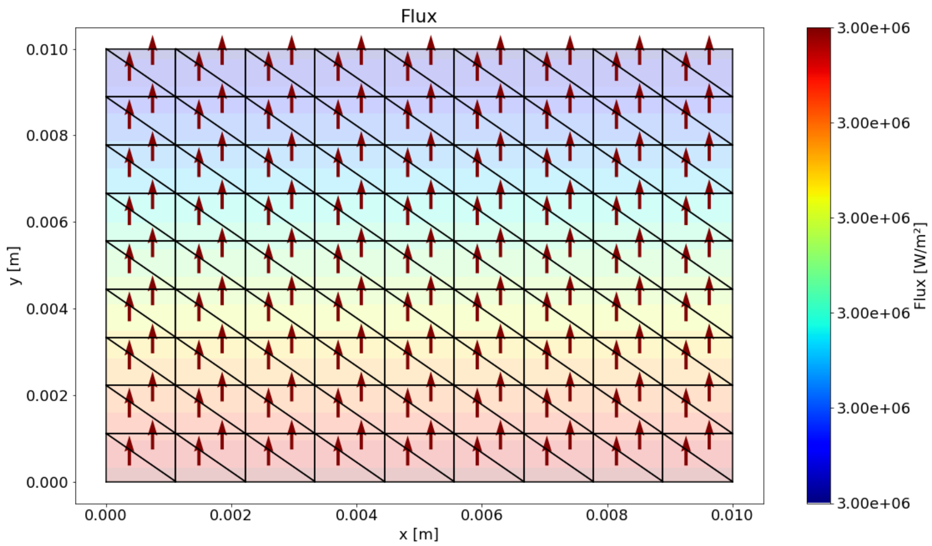

Fig. 3. V0: Temperature flux.

2 Variation 1

Setup:

- Trapezoidal shape of the model,
- set N100 to the coordinates $(\frac{L}{2}, L)$

In Fig.4, we can see the resulting temperature field, which overall looks as expected. When compared to Variation 0, the reduced length of the upper edge, where Dirichlet BCs are applied, means that the incoming heat from the bottom is able to build up more than before, resulting in the higher temperature of close to $500K$ on the bottom right (the part furthest away from any outlet part on the top).

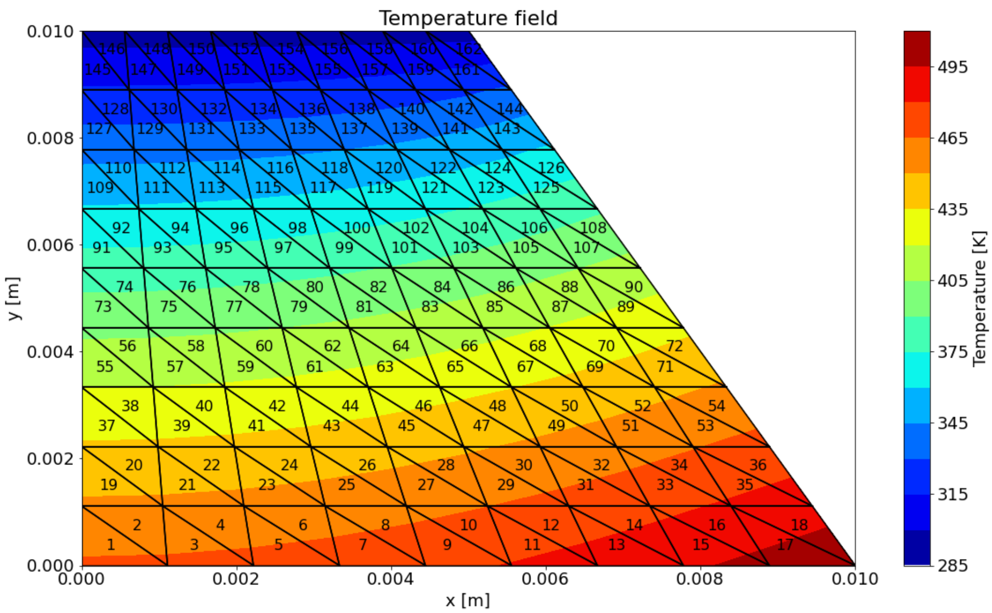

Fig. 4. V1: Temperature field.

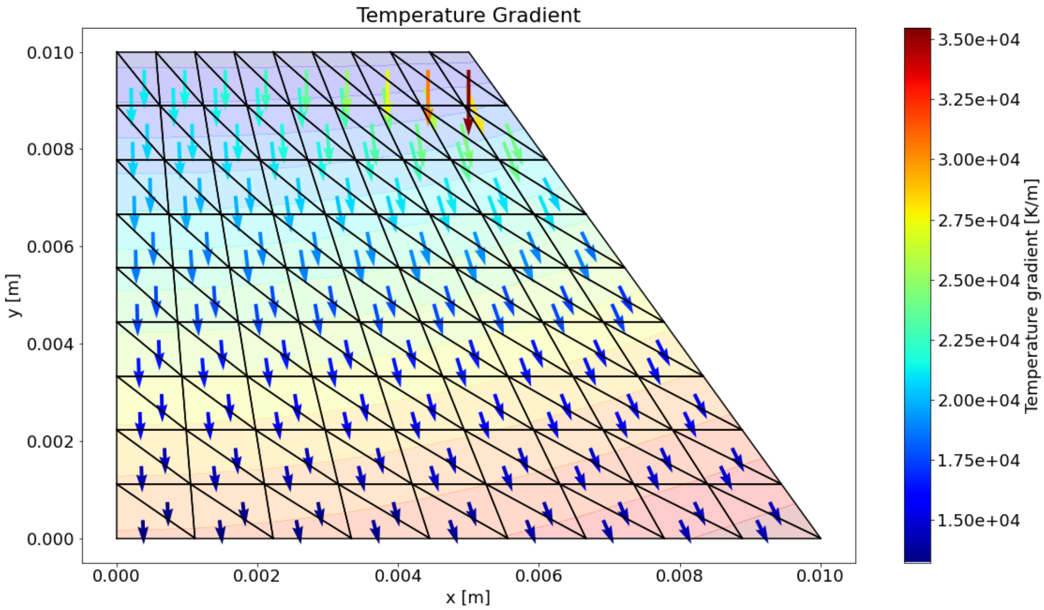

Fig. 5. V1: Temperature gradient.

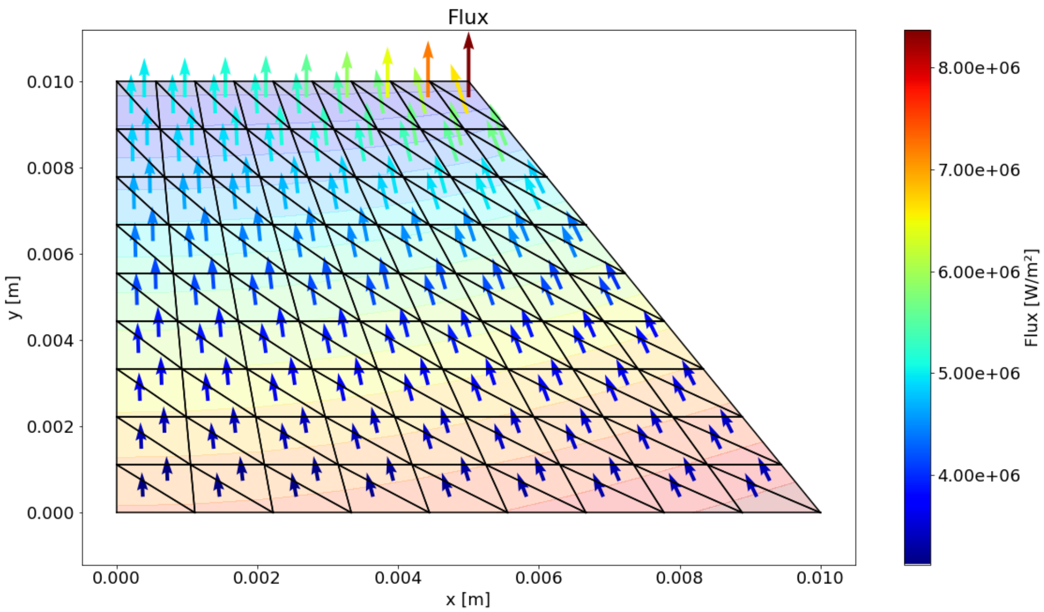

Fig. 6. V1: Temperature flux.

3 Variation 2

Setup:

- quadratic shape,
- but with a biased mesh, see eq. 2.

Apply a modification of the x-coordinates,

$$x_{bias} = x^{regular}[\frac{B}{L}x^{regular} - B + 1] \; with B = \frac{1}{2L}y, \tag{2}$$

to keep the mesh at y = 0 regular (for easy application of nodal"forces") and for a gradual increase to y = L.
As can be seen in Fig.7 - Fig.9, the resulting solution is the same as for Variation 0.

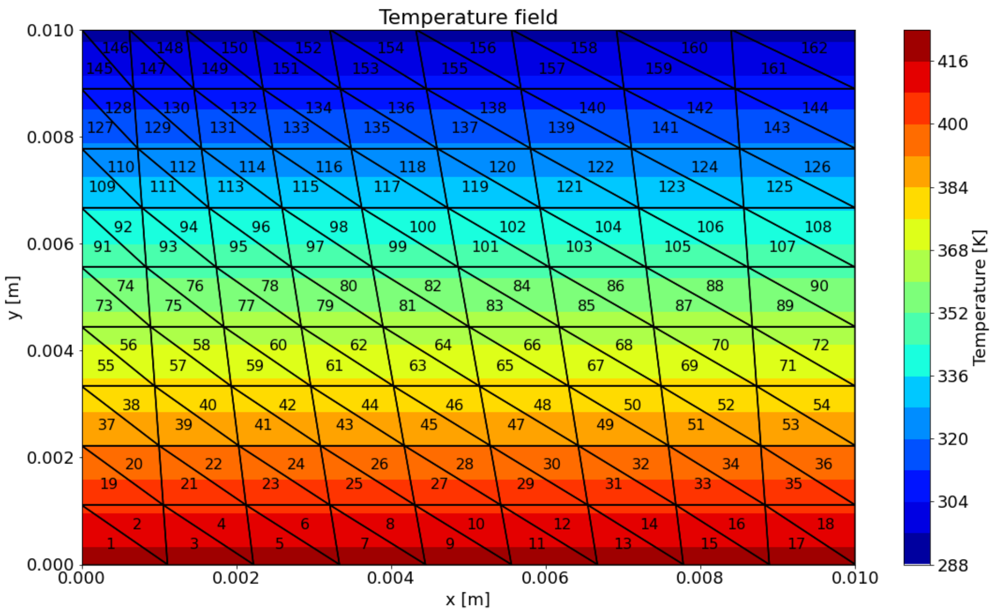

Fig. 7. V2: Temperature field.

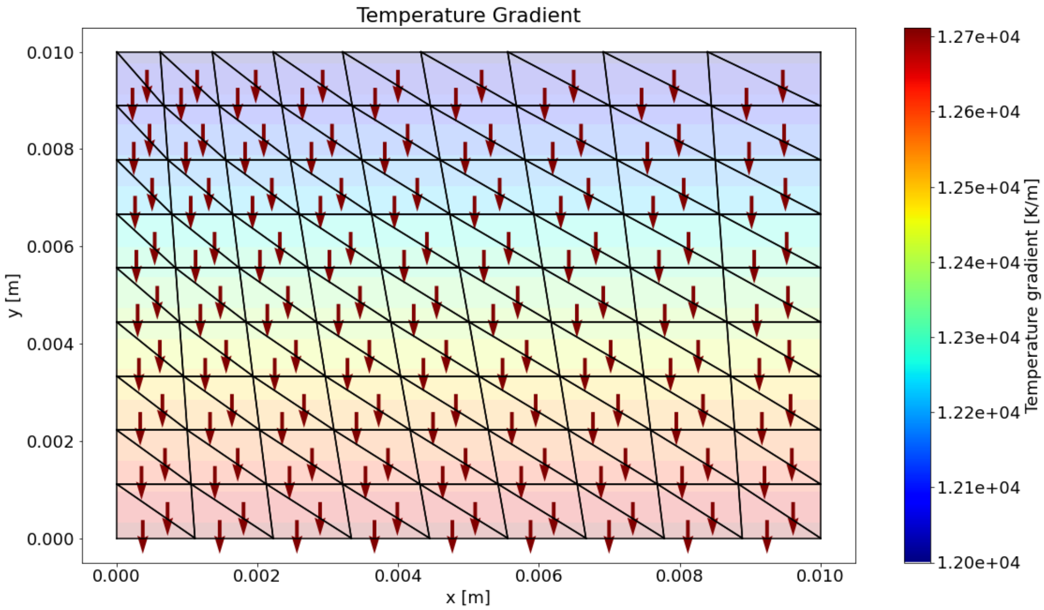

Fig. 8. V2: Temperature gradient.

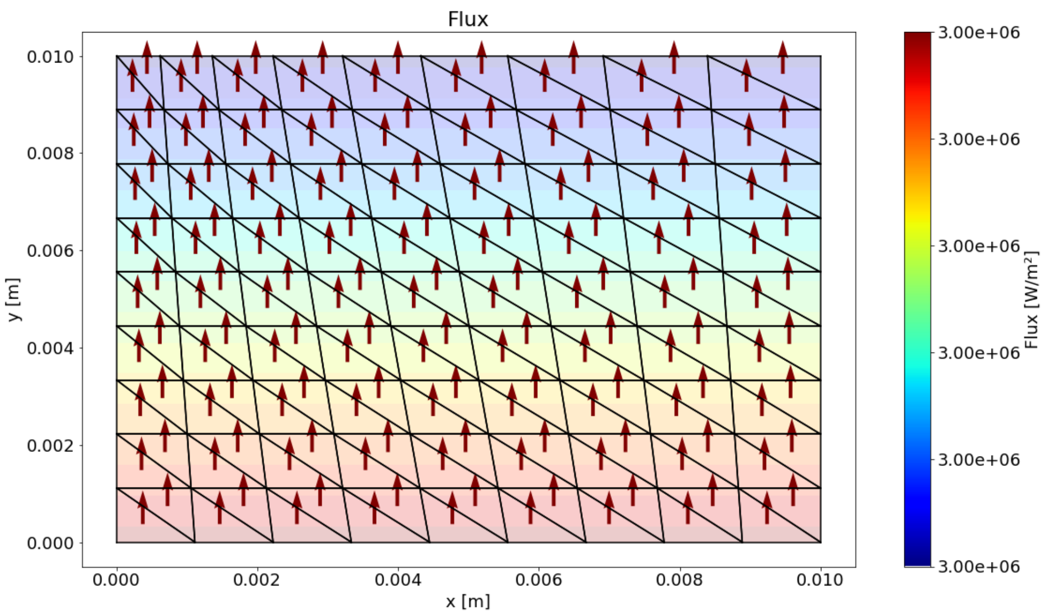

Fig. 9. V2: Temperature flux.

4 Variation 3

Setup:

- Annulus section (ring segment) of $\frac{\pi}{4}$,
- center $(2L, 0)$, outer radius $2L$ and inner radius L.

In Fig.10, we can see the resulting temperature field. Note, that the annulus section appears squished due the different scaling of the x- and y-axis. The correctnes of the domain can be checked via sin-/cos-relations, e.g. when checking the coordinates of the apex point

$$(x, y) = \big(2L \cdot (1 - cos(\pi/4)),\ 2L \cdot sin(\pi/4)\big) \approx \big(0.006,\ 0.014\big). \tag{3}$$

Again, we note a similar behaviour as in Variation 1. This time, the bottom left part of the domain becomes the hottest part of the domain since it is the furthest away from any outflow point (the top right edge), see Fig.10. The flux is shown in Fig.12. Here, the flux is not constant across the lower edge (y=0) due to the curved/warped form of the domain. The same applies to the outflow edge on the top right, where Dirichlet BCs are applied.
However, for both the in- and outflow edges, the flux (and the temperature gradient) are normal to respective edge. Along the both the inner and outer perimeter edges the heat flux flows in parallel to the edge.

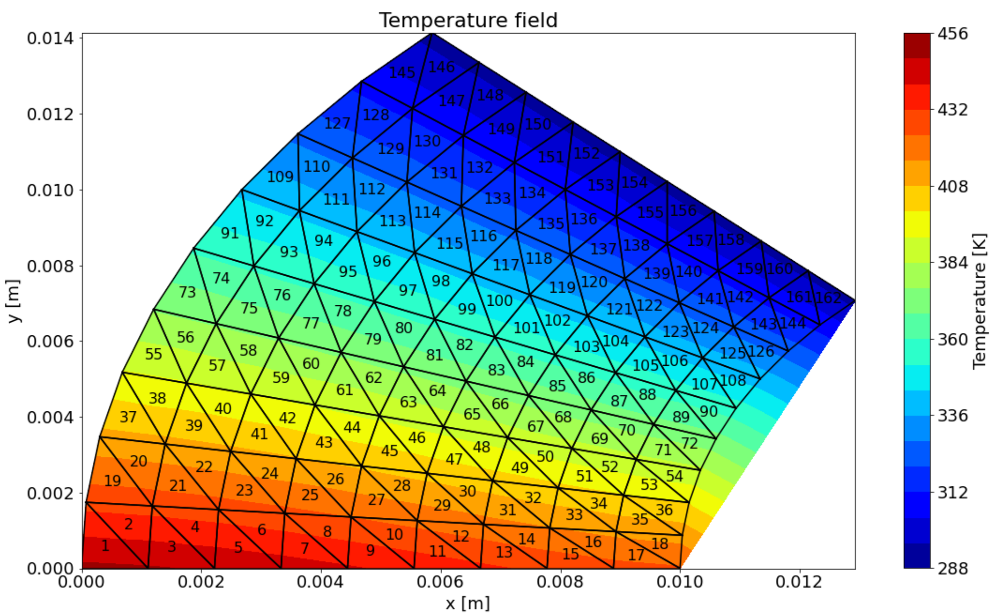

Fig. 10. V3: Temperature field.

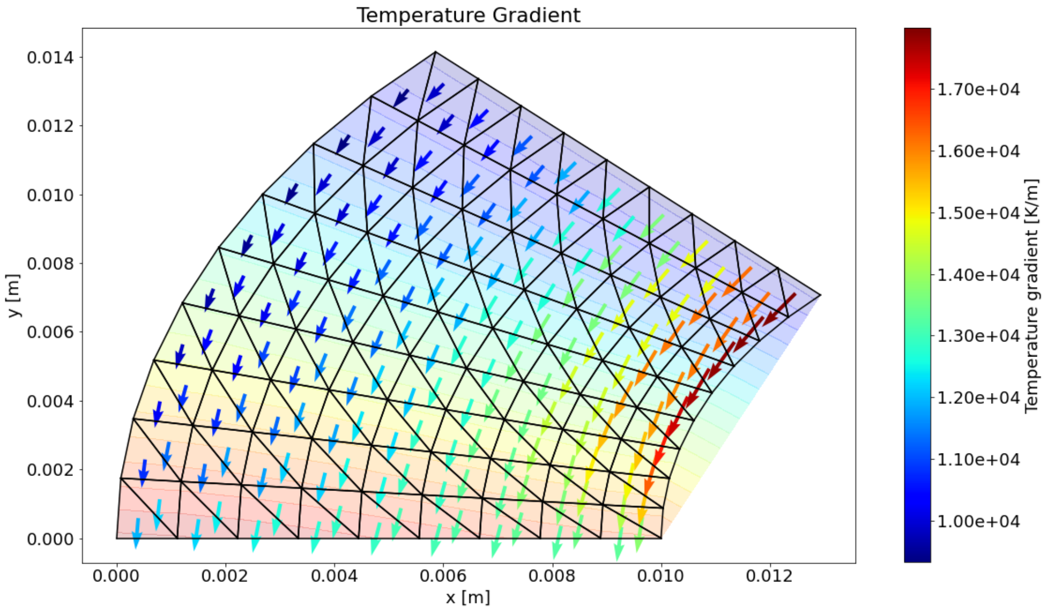

Fig. 11. V3: Temperature gradient.

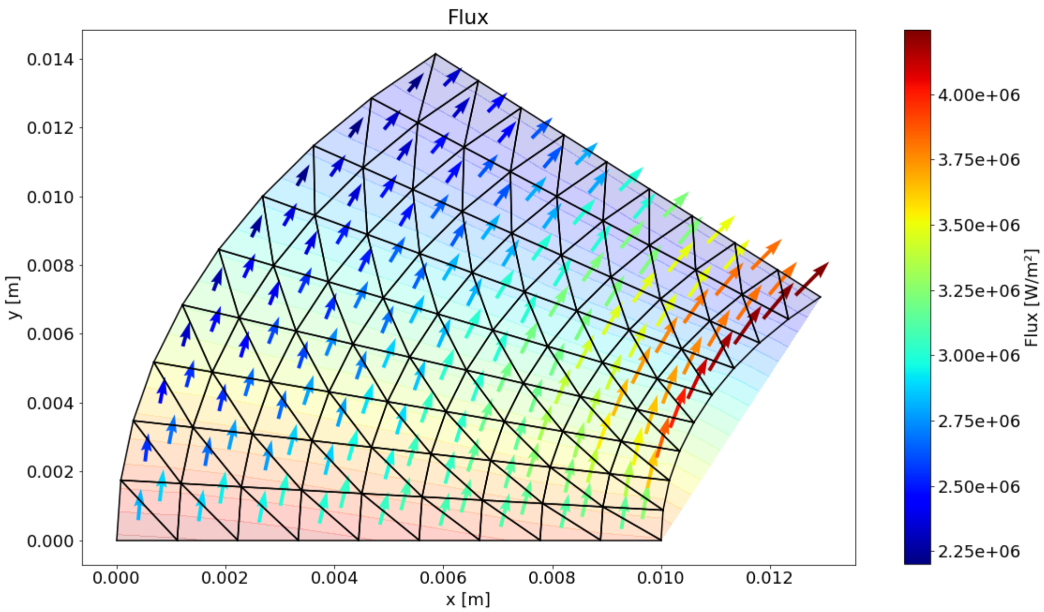

Fig. 12. V3: Temperature flux.

5 Variation 4

5.1 Variation 4.a

Setup:

- quadratic shape,
- modify k in a set of elements to $k^{mod} = ck$, where $c = 10$,
- elements to be modified = [61-68, 79-85, 97-101, 115-118]

Fig.13 shows the resulting temperature field. The increased conductivity in the modified elements (roughly speaking, in the centre) causes them to exhibit higher temperatures than other elements of the domain with similar y-coordinates. The resulting temperature field is not of the same form as it was in Variation 0 ($T(x, y) \equiv T(y) \propto y$) , but instead also depends on x.
The plot for the flux in Fig.15 explains this behavior better. More heat is able to flow through the elements with increased conductivity $k^{mod} > k$. The heat flux is not constant across the domain anymore, but remains normal to the edge at the in- and outflow edges.

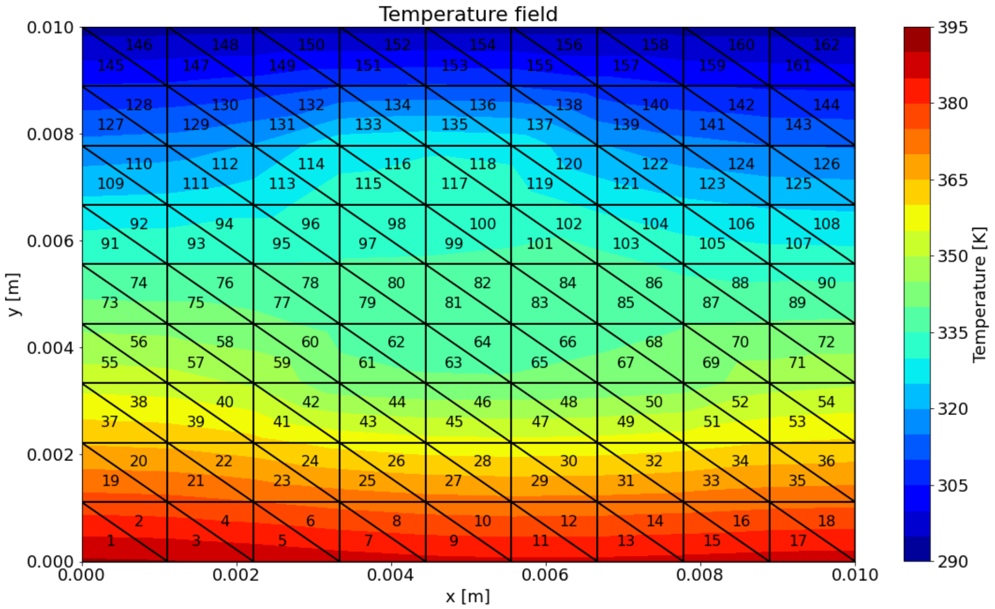

Fig. 13. V41: Temperature field.

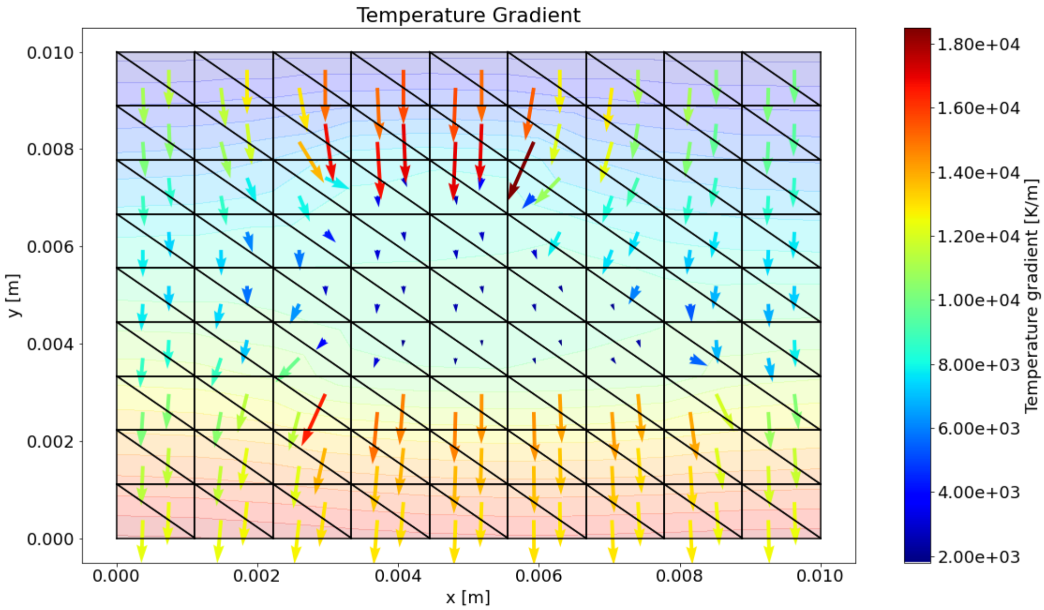

Fig. 14. V41: Temperature gradient.

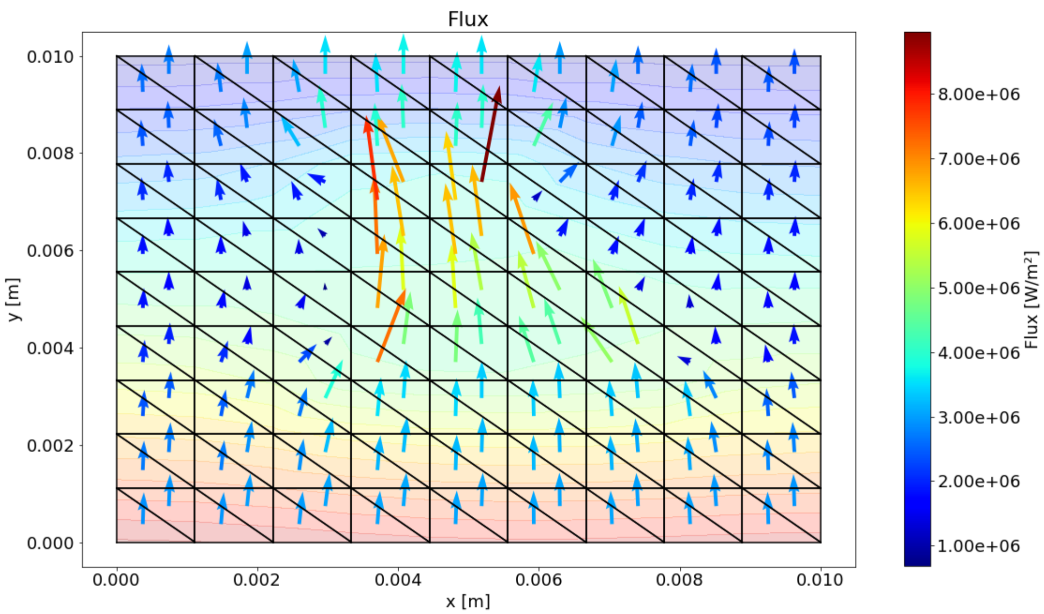

Fig. 15. V41: Temperature flux.

5.2 Variation 4.b

Setup:

- quadratic shape,

- modify k in a set of elements to $k^{mod} = k/c$, where $c = 10$,
- elements to be modified = [61-68, 79-85, 97-101, 115-118]

Fig.16 again shows the resulting temperature field and Fig.18 shows the flux. This setup shows the expected inverse behavior to the setup in Sec. 5.1. The reduced conductivity $k^{mod} = k/c < k$ in the centre of the domain causes the flux to concentrate around the centre.

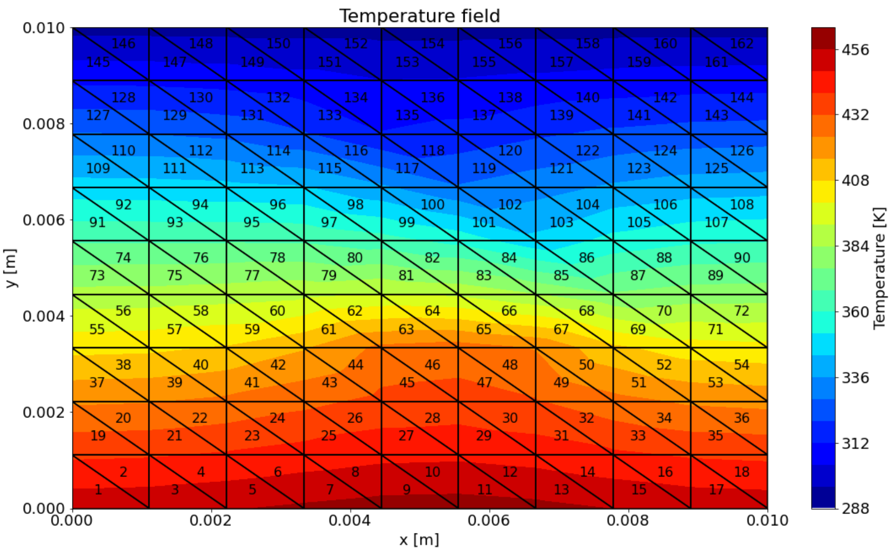

Fig. 16. V42: Temperature field.

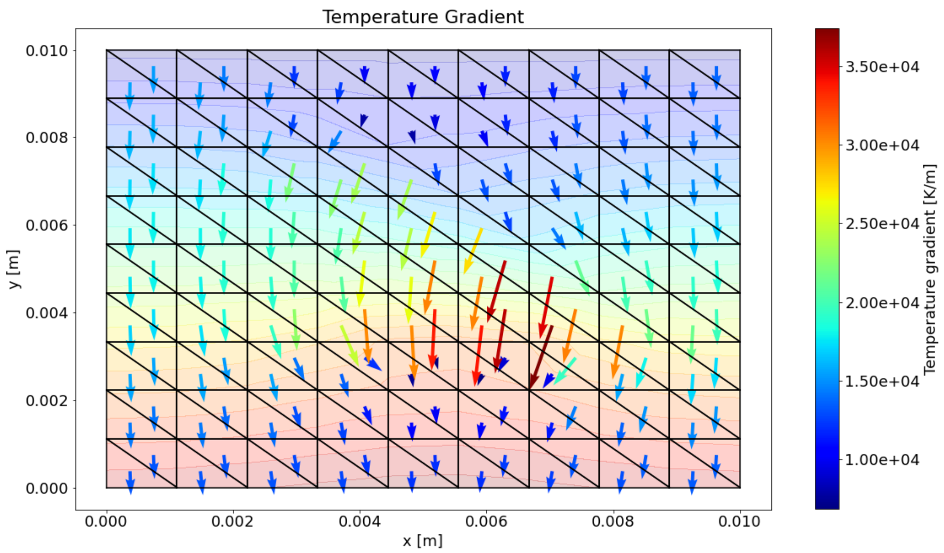

Fig. 17. V42: Temperature gradient.

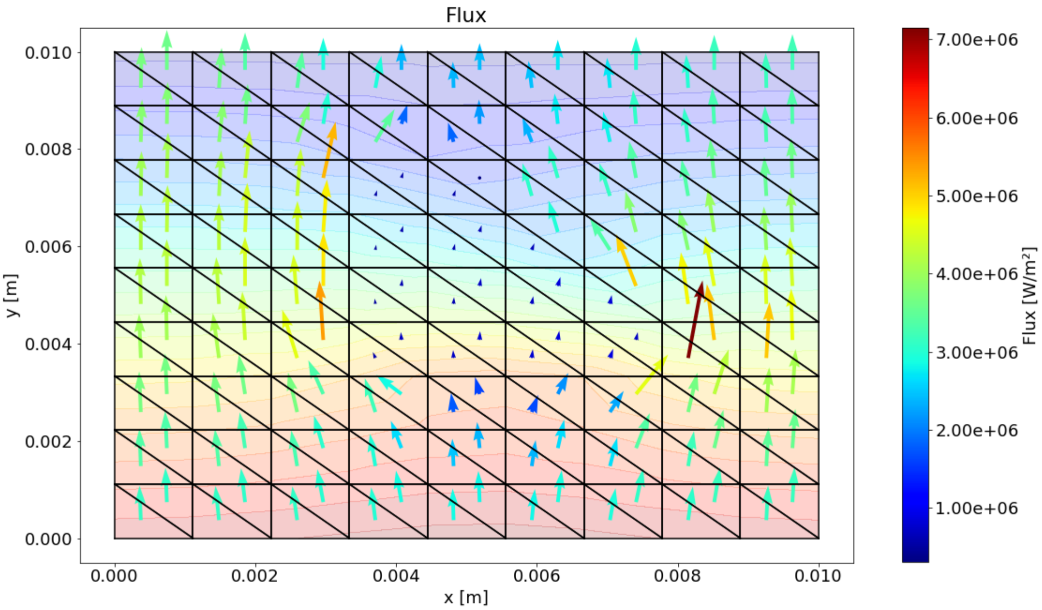

Fig. 18. V42: Temperature flux.