1a) $N_0(t)' + N_1(t)\varepsilon + \dots = a(1 + \varepsilon\cos(t) - [N_0(t) + N_1(t)\varepsilon + \dots])$
$\cdot (N_0(t) + N_1(t)\varepsilon + \dots)$

$= (a + a\varepsilon\cos(t) - aN_0(t) - aN_1(t)\varepsilon - \dots)(N_0(t) + N_1(t)\varepsilon + \dots)$

$= N_0(t)a + a\varepsilon\cos t\, N_0(t) - aN_0(t)^2 - N_1(t)\varepsilon N_0(t)$
$+ N_1(t)\varepsilon a + N_1(t)\varepsilon^2 a\cos(t) - N_0(t)N_1(t)\varepsilon - \dots$

b) yeilding
$N_0(t)' = aN_0(t) - aN_0(t)^2$
&
$N_1(t)'\varepsilon = N_0(t)a\cos(t)\varepsilon - 2N_0(t)N_1(t)\varepsilon a + N_1(t)\varepsilon a$

$N_0(0) = 1 \qquad N_1(0) = b$

$N_0(t)' - N_0(t)a = -N_0(t)^2 a$

$N_0(t)^{-2}N_0(t)' - N_0(t)^{-1}a = -a$; let $v = N_0(t)^{-1}$
$v' = -N_0(t)^{-2}N_0(t)'$

$-v' - va = -a$
$v' + va = a \qquad N = e^{\int a\,dt} = e^{at}$

$e^{at}v = \int ae^{at}\,dt \rightarrow v = \dfrac{e^{at} + c}{e^{at}} = 1 + \dfrac{c}{e^{at}}$

$N_0(t)^{-1} = 1 + \dfrac{c}{e^{at}} \rightarrow N_0(t) = \dfrac{e^{at}}{e^{at} + c}$

plug in 0 for t from IV's

$$N_0(0) = \frac{e^0}{e^0 + c} = \frac{1}{1+c} = 1 \qquad c = 0$$

yeilding; $N_0(t) = 1$

c) $N_1(t)' = a\cos(t) - 2a N_1(t) + N_1(t)a$

$= a\cos(t) - a N_1(t)$

$$N_1'(t) + a N_1(t) = a\cos(t) \qquad N = e^{at}$$

$$e^{at} N_1(t) = \int a\cos(t) e^{at}\, dt$$

$$e^{at} N_1(t) = \left(\frac{e^{at}\sin t}{1+a^2} + \frac{a e^{at}\cos t}{1+a^2}\right) a + c$$

$$N_1(t) = \frac{a\sin t}{1+a^2} + \frac{a^2\cos t}{1+a^2} + \frac{c}{e^{at}} \qquad N_1(0) = b$$

$$N_1(0) = \frac{a^2}{1+a^2} + c = b \rightarrow c = b - \frac{a^2}{1+a^2}$$

let $b = \frac{a^2}{1+a^2}$, $c \therefore = 0$, yeiding

$$N_1(t) = \frac{a\sin t}{1+a^2} + \frac{a^2\cos t}{1+a^2}$$

d) $N_1(t) = \dfrac{a\sin t}{1+a^2} + \dfrac{a^2\cos t}{1+a^2}$

$$C_1 = \frac{a^2}{1+a^2} \qquad C_2 = \frac{a}{1+a^2}$$

$$A = \frac{\sqrt{a^4 + a^2}}{\sqrt{a^4+2a^2+1}} = \frac{a\sqrt{a^2+1}}{1+a^2} = A$$

$$\phi = \arctan\left(\frac{\frac{a}{1+a^2}}{\frac{a^2}{1+a^2}}\right) = \arctan\left(\frac{1}{a}\right)$$

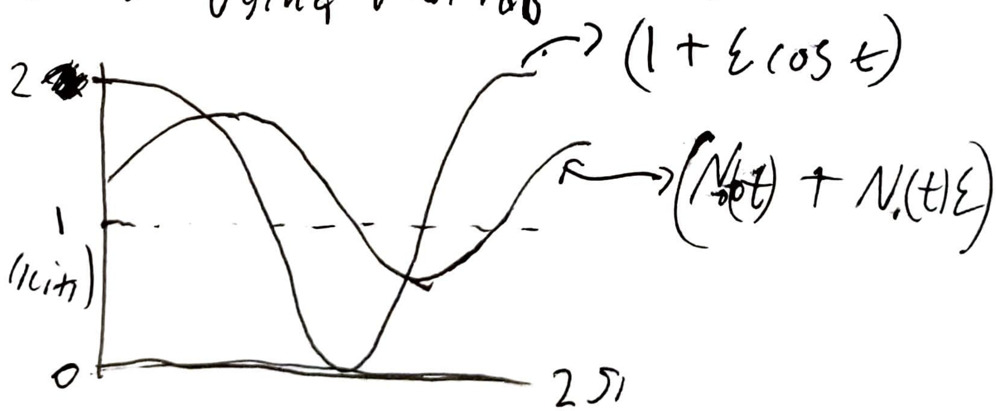
yeilding

$$N_1(t) = \frac{a\sqrt{a^2+1}}{a^2+1}\cos\left(t - \arctan\left(\frac{1}{a}\right)\right)$$

e) $N_0(t) + N_0(t)\varepsilon = 1 + \varepsilon\left(\dfrac{a\sqrt{a^2+1}}{a^2+1}\cos\left(t - \arctan\left(\frac{1}{a}\right)\right)\right)$

plotted using matlab

$a = 1$
$\varepsilon = 1$
(for simplicity)



f) Based on the graph, we can see that as time goes on, the population decreases, but only $\arctan(\frac{1}{a})$ units of time after the carrying capacity starts to decrease. likewise, it takes the same delay for the population to increase after the carrying capacity starts to increase.

f pt2) (I'm NOT taking this on the 500 level, but I would like to see if my intuitions are correct) as $a \to 0$, the graph of the population becomes much more stable. this makes sense because a is the intrinsic groth rate. if the growth is extreemly small, then regardless of the carrying capacity, the population will not change much. on the other hand, as $a \to \infty$, we see that the population becomes the carrying capacity. if the growth rate is very fast, then we expect ~~them~~ there to be very little delay in the time it takes for the change in population to reverse After the population carrying capacity has been reached.