sec 20 $\cancel{2}, \cancel{4}, 8, 9$

2) a) $f'(z) = 6z - 2$ b) $\cancel{20z}$ $20z(2z^2+i)^4$

c) $f'(z) = \dfrac{(2z+1)1 - (z-1)2}{(2z+1)^2} = \dfrac{3}{4z^2+4z+1}$

d) $\dfrac{z^2(8z(1+z^2)^3) - 2z(1+z^2)^4}{z^4}$

$\dfrac{8z^2(1+z^2)^3 - 2\cancel{z}(1+z^2)^4}{z^3} = f'(z)$

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4) $\lim\limits_{z \to z_0} \dfrac{f'(z)}{g'(z)} = \dfrac{\dfrac{f(z)-f(z_0)}{z-z_0}}{\dfrac{g(z)-g(z_0)}{z-z_0}} = \dfrac{f(z)-f(z_0)}{g(z)-g(z_0)}$

Note $f(z_0) = g(z_0) = 0$,

$\dfrac{f(z)-0}{g(z)-0} = \dfrac{f(z)}{g(z)}$ ✓

$\cancel{8}$, 9

8) a) $f(z) = \operatorname{Re} z = \frac{\Delta w}{\Delta z} = \frac{\operatorname{Re}\Delta z}{\Delta z}$

When z is real, $\frac{\operatorname{Re}\Delta z}{\Delta z} = 1$,

when imaginary, $\frac{\operatorname{Re}\Delta z}{\Delta z} = 0$.

Therefore the limit DNE

b) $f(z) = \operatorname{Im}(z) = \frac{\Delta w}{\Delta z} = \frac{\operatorname{Im} z\Delta z}{\Delta z}$

when z is real, $\frac{\operatorname{Im} z\Delta z}{\Delta z} = 0$

when z is imaginary, $\frac{\operatorname{Im} z\Delta z}{\Delta z} = 1$.

Therefore the limit DNE

9) $\frac{\Delta w}{\Delta z} = \cancel{\frac{(\overline{z+\Delta z})^2 - \bar z^2}{z\Delta z}} = \cancel{\frac{(\overline{z+\Delta z})^2 - \bar z^2}{z\cdot\Delta z}} = \cancel{\frac{\bar z^2 + 2\bar z\overline{\Delta z} + \overline{\Delta z}^2}{z\cdot\Delta z}}$

$\frac{1}{\Delta z}\cdot\left(\frac{(\overline{z+\Delta z})^2}{z+\Delta z} - \frac{\bar z^2}{z}\right) \qquad \cancel{= \frac{z + \Delta z}{z}}$

$\frac{z\left(\bar z^2 + 2\bar z\overline{\Delta z} + \overline{\Delta z}^2\right)}{z(z+\Delta z)} - \frac{\bar z^2(z+\Delta z)}{z(z+\Delta z)}$

$= \frac{\cancel{z\bar z^2} + 2\bar z\overline{\Delta z}z + z\overline{\Delta z}^2 \cancel{- \bar z^2 z} - \bar z^2\Delta z}{z(z+\Delta z)}$

$$\frac{2z\,\bar{z}\,\overline{\Delta z} + z\,\overline{\Delta z}^2 - \bar{z}^2\Delta z}{z(z+\Delta z)}$$

$$= \frac{2\bar{z}\,\overline{\Delta z} + \overline{\Delta z}^2}{z+\Delta z} - \frac{\bar{z}^2\Delta z}{z(z+\Delta z)}$$

when $\Delta z = (\Delta x, 0)$

$$\left(\frac{2\bar{z}\Delta x + \Delta x^2}{z+\Delta x} - \frac{\bar{z}^2\Delta x}{z(z+\Delta x)}\right)\frac{1}{\Delta x}$$

$$\frac{2\bar{z}\cancel{z} + \Delta x\cancel{z} - \bar{z}^{\cancel{2}}}{(z+\Delta x)\cancel{z}} = \frac{2\bar{z} + \Delta x + z}{z+\Delta x}$$ as $\Delta x \to 0$

yeilds $\frac{\Delta x}{\Delta x}$ when $z = 0$

likewise when $\Delta z = (0, \Delta y)$

$$\left(\frac{-2z\bar{z}\cancel{\Delta y} + \Delta y^2 z + \bar{z}^2\cancel{\Delta y}}{z(z+\Delta y)}\right)\left(\frac{1}{\cancel{\Delta y}}\right)$$

$$= \frac{-2\cancel{z}\bar{z} + \Delta y\cancel{z} + \bar{z}^{\cancel{2}}}{\cancel{z}(z+\Delta y)} = \frac{-2\bar{z} + \Delta y + z}{z+\Delta y}$$

when $z = 0$ $\quad \frac{\Delta y}{\Delta y} = 1$

now let $\Delta y = \Delta x$

$$\frac{-2z\bar{z}\,\cancel{(\Delta x + i\Delta x)} + \left(\overline{\Delta x \mp i\Delta x}\right)^{2} z + \bar{z}^{2}\,\cancel{(\Delta x + i\Delta x)}}{z(z+2\Delta x)} \cdot \frac{1}{\cancel{2\Delta x}}$$

$$\frac{-2\bar{z}\cancel{z} \mp 2\Delta x\cancel{z} + \bar{z}^{2}}{\cancel{z}(z+2\Delta x)} = \frac{-2\bar{z} - 2\Delta x + \bar{z}}{z+2\Delta x}$$

as $z = 0$

$$\frac{-2\Delta x}{2\Delta x} = -1.$$ Therefore, limit DNE