~~1, 2, 3~~ 5, 9, 10

2.14, 2.18

1) a) $z \neq \pm i$, b) $z \neq 0$, c) $\mathrm{Re}(z) \neq 0$

d) $|z| \neq 1$

2) $f(z) = z^3 + z + 1 = (x+iy)^3 + (x+iy) + 1$

$(x^2 + 2xiy - y^2)(x+iy) + (x+iy) + 1$

$x^3 + 2x^2iy - y^2x + x^2iy + (-2xy^2) - iy^3 + x + iy + 1$

$(x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)$

b) $\dfrac{\bar{z}^2}{z} \cdot \dfrac{\bar{z}}{\bar{z}} = \dfrac{\bar{z}^3}{|z|^2}$ $|z|^2 = x^2 + y^2$

$\dfrac{(x-yi)^3}{x^2+y^2} = \dfrac{x^3 - 3xy + iy^3 - 3x^2yi}{x^2+y^2}$

$= \dfrac{x^3 - 3xy}{x^2+y^2} + \dfrac{i(y^3 - 3xy)}{x^2+y^2}$

3) $x^2 - y^2 - 2y + i(2x - 2xy)$

$y = \dfrac{z - \bar{z}}{2}$ y

$x^2 - 2xyi - y^2 = \bar{z}^2$

$\bar{z}^2 - 2y + 2xi$

$\bar{z}^2 + 2(-y + xi)$

$\bar{z}^2 + 2i(-x - iy) = \bar{z}^2 - 2iz$

~~3~~, 4, 10

5) $\left(\frac{z}{\bar{z}}\right)^2 = \left(\frac{x+iy}{x-iy}\right)^2$ let $z=(x,0)$

then $\left(\frac{x+i(0)}{x-i(0)}\right)^2 = \left(\frac{x}{x}\right)^2 = 1^2 = 1$

let $z=(0,y) = \left(\frac{0+iy}{0-iy}\right)^2 = \left(\frac{-y}{y}\right)^2 = (-1)^2 = 1$

let $y=x$ & $z = x, x$

$$\left(\frac{x+ix}{x-ix}\right)^2 = \left(\frac{x(1+i)}{x(1-i)}\right)^2 = \left(\frac{1+i}{1-i}\right)^2$$

$$= \frac{(1+i)^2}{(1-i)^2} = \frac{1+2i-1}{1-2i-1} = \frac{2i}{-2i} = -1$$

$\lim_{z\to 0} \left(\frac{z}{\bar{z}}\right)^2$ DNE because $\lim f(z) = 1$

$f(z_0)$ does not exist

$f(z) \neq f(z_0)$

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4) $\lim_{z\to z_0} f(z)g(z) = \left(\lim_{z\to z_0} f(z)\right)\left(\lim_{z\to z_0} g(z)\right)$

let $\lim_{z\to z_0} f(z) = 0$, then $0 \cdot \left(\lim_{z\to z_0} g(z)\right) = 0$

(confused on this part)

let $M = z_0 + 10$, let $\lim_{z\to z_0} |g(z)|$, then $\lim_{z\to z_0} g(z) \leq M$,

10) a) $\lim_{z \to \infty} \frac{4z^2}{(z-1)^2} = 4$ if ~~$\lim_{z \to 0} \frac{(z-1)^2}{4z^2} = 4$~~

$$\lim_{z \to 0} \frac{4/z^2}{(\frac{1}{z}-1)^2} = \frac{\frac{4}{z^2}}{\frac{1}{z^2} - \frac{2}{z} + 1}$$

$$= \frac{4}{1 - 2z + z^2} = \text{as } z \to 0 = \frac{4}{1} = 4 \checkmark$$

b) $\lim_{z \to 1} \frac{1}{(z-1)^3} = \infty$ if $\lim_{z \to z_0} (z-1)^3 = 0$

$$\lim_{z \to 1} (z-1)^3 = (1-1)^3 = 0 \checkmark$$

c) $\lim_{z \to \infty} \frac{z^2+1}{z-1} = \infty$ if $\lim_{z \to 0} \frac{\frac{1}{z}-1}{\frac{1}{z^2}+1} = 0$

$$= \frac{z - z^2}{1 + z^2} = \frac{0}{1} = 0 \checkmark$$