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1)
a)

$$y = \frac{x}{v^2 g^{-1}}, \quad \tau = \frac{t}{v g^{-1}}$$

Var	Dim
x	L
v	L/T
g	L/T²
t	T

$$[y] = \frac{L}{\frac{L^2}{T^2} \cdot \frac{T^2}{L}} \rightarrow \frac{L}{L} = 1,$$ Dimension-less ✓

$$[\tau] = \frac{T}{\frac{L}{T} \cdot \frac{T^2}{L}} = \frac{T}{T} = 1$$ " " ✓

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b) vg^{-1} is the time it takes for the protectile to reach its max height when v is initial velocity and g is the accoleration of gravity.

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c)

$$\frac{d^2 y}{d\tau^2} = \frac{d^2\left(\frac{x}{v^2 g^{-1}}\right)}{d\left(\frac{t}{vg^{-1}}\right)^2} = \frac{\frac{g}{v^2} d^2 x}{\frac{g^2}{v^2} d t^2} = \frac{d^2 x}{dt^2} \cdot \frac{1}{g}$$

$$\frac{d^2 y}{d\tau^2} g = \frac{d^2 x}{dt^2}$$

$$x = \frac{y v^2}{g} \qquad t = \frac{\tau v}{g}$$

$$\frac{d^2 y}{d\tau^2} g = -\frac{gR^2}{\left(\frac{yv^2}{g} + R\right)^2} \Rightarrow \frac{-1}{\left(\frac{yv^2}{gR} + 1\right)^2} = \frac{d^2 y}{d\tau^2}$$

$$= \frac{-1}{(\varepsilon y + 1)^2} \quad ✓ \quad \varepsilon = \frac{v^2}{gR}$$

c cont.)

$$y(0) = \frac{0}{v^2 g^{-1}} = 0$$

$$\frac{d\,\cancel{x}\,y}{d\,\cancel{t}\,\tau} = \frac{d\left(\frac{x}{v^2 g^{-1}}\right)}{d\left(\frac{t}{v g^{-1}}\right)} = \frac{\frac{g}{v^2}\,dx}{\frac{g}{v}\,dt}$$

$$= \frac{1}{v}\frac{dx}{dt} \to \frac{dx}{dt}(0) = v$$

$$\frac{v}{v} = 1 \qquad \frac{dy}{d\tau} = 1$$

✓

d) $\varepsilon = \frac{v^2}{gR}$

Var	Dim
V	L/T
g	L/T^2
R	L

$$[\varepsilon] = \frac{\left(\frac{L}{T}\right)^2}{\frac{L}{T^2}\cdot L} = \frac{\frac{L^2}{T^2}}{\frac{L^2}{T^2}} = 1$$

dimensionless ✓

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e) from C,

$$\frac{d^2y}{d\tau^2}g = \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -g \qquad \frac{d^2y}{d\tau^2} = \frac{-g}{g} = -1 \checkmark$$

$$y(0) = \frac{0}{v^2 g^{-1}} = 0 \checkmark$$

$$\frac{dy}{d\tau} = \frac{1}{V}\frac{dx}{dt} \qquad \frac{dx}{dt}(0) = V$$

$$\frac{dy}{d\tau}V = \frac{dx}{d\tau} \qquad \frac{dy}{d\tau}(0) = \frac{V}{V} = 1 \checkmark$$

f) to start, first we reqognise what ε is

$$\varepsilon = \frac{v^2}{gR}$$ or, in other words it is

The square of the velocity divided by 9.8 m/s times the distance the object is from the center of the earth. When we origionally structured the problem, we used the formula

$$F = G\frac{m_1 m_2}{R^2}$$ where G is grav. const.

m_1 is mass of earth, & m_2 is our object, & R is the distance. In phisics this demonstrates that the force weakens exponentially as distance increases. In our IVP, we expect that the solutions to the two figures will be similar ~~because~~ when ε is close to zero because in order for ε to be zero, R must be large, or v must be very very small, or g could be small. This makes sense because R is very large when dealing with objects thrown standing on the earth. in this case R is very large, & would take an extreme v value to overcome. Similarly, if we barely throw our object, then we

Should expect the ΔR to be extreemly small, which checks out with our force equation, creating again a very small change in force. ε would only be not close to zero when dealing with a system that has a substantial impact on the force of gravity