Josh Brown

~~4~~, ~~5~~, 6

1.5

4) $\sqrt{2}\,|z| \ge |\mathrm{Re}\, z| + |\mathrm{Im}\, z|$

square both sides

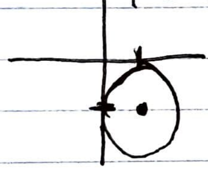
$2|z|^2 \ge |x|^2 + |y|^2 + 2|x||y|$

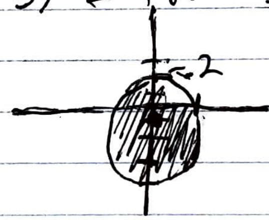
$2(|x|^2 + |y|^2) \ge |x|^2 + 2|x||y| + |y|^2$

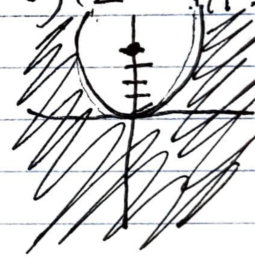
$|x|^2 - 2|x||y| + |y|^2 \ge 0$

$(|x| - |y|)^2 \ge 0$ ✓

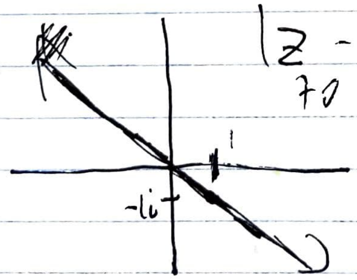
5) a) $|z-(1-i)|=1$ b) $|z+i| \le 3$ c) $|z-4i| \ge 4$







6 $|z-1| = |z+i|$



$|z-1|$ is a point set shifted over to the right, and with $|z+i|$ is same but down.
∴ when set to eachother, form a line through $(0,0)$ as well as $(-1, 1i)$.

X, 3, ~~4~~, ~~3~~

1.6 pg 16

1) a) $\overline{\bar{z}+3i} \stackrel{?}{=} z-3i$

b) $\overline{iz} = -i\bar{z}$

$= \bar{i}\bar{z} = -i\bar{z}$ ✓

$\bar{\bar{z}} + \overline{3i} = z-3i$

$z-3i = z-3i$ ✓

c) $\overline{(2+i)^2} = 3-4i$

$\overline{4+4i-1} = 3-4i$

$\overline{3+4i} = 3-4i$ ✓

d) $|(2\bar{z}+5)(\sqrt{2}-i)|$

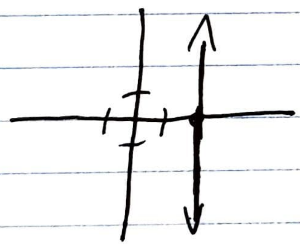
$= \sqrt{3}\,|2z+5|$

$|2\bar{z}+5|\sqrt{\sqrt{2}^2-i^2}$

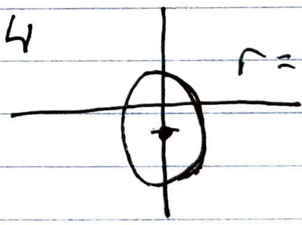
$|2\bar{z}+5|\sqrt{2+1}$

$|2\bar{z}+5|\sqrt{3} = |2\bar{z}+5|\sqrt{3}$

2) a)



b) $|2\bar{z}+i| = 4$



$r=2$

9) $\left|\dfrac{1}{z^4-4z^2+3}\right| \le \dfrac{1}{3}$

$\left|\dfrac{1}{(z^2-3)(z^2-1)}\right| \le \dfrac{1}{3}$ given $|z| = 2^2 = \sqrt{z^2}^2$

$4 = z^2$

$\left|\dfrac{1}{(4-3)(4-1)}\right| \le \dfrac{1}{3}$

$\left|\dfrac{1}{3}\right| \le \dfrac{1}{3}$ ✓

13) $|z - z_0| = R \rightarrow |z|^2 - 2R_e(z\bar{z}_0) + |z_0|^2 = R^2$

$(|z - z_0|)^2 = R^2 \rightarrow (\sqrt{|z|^2 - |z_0|^2})^2 = R^2$

$(|z| - |z_0|)(|z| - |z_0|) = |z|^2 - |2zz_0| + |z_0|^2 = R^2$

$= |z|^2 - 2R_e(z\bar{z}_0) + |z_0|^2 = R^2$

~~1~~, ~~2~~, ~~6~~, 8, 9

Pg 23 ch 1.4

1) a) $\text{Arg}(z) = \text{Arg}(-2) - \text{Arg}(1+\sqrt{3}i)$

$\pi - \text{Arg}(1+\sqrt{3}i) = \pi - \frac{\pi}{3}$

$\frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

b) $\text{Arg}(z) = 6\,\text{Arg}(-\sqrt{3}-i)$

$= 6\frac{\pi}{6} = \pi$

5) $(i)(1-\sqrt{3}i)(\sqrt{3}+i)$

a) $e^{i\frac{\pi}{2}}\, 2e^{i\frac{5\pi}{3}}\, 2e^{i\frac{\pi}{6}}$ $\quad 4e^{i\left(\frac{3\pi}{6}+\frac{10\pi}{6}+\frac{\pi}{6}\right)} = 4e^{i\frac{14\pi}{6}}$

$= 4e^{i\frac{\pi}{3}}$

to rect

$x = r\cos x$

$y = r\sin y$

$x = 2$

$y = 2\sqrt{3}$

$2+2\sqrt{3}i$

$2(1+\sqrt{3}i) = i(1-\sqrt{3}i)(\sqrt{3}+i)$

b) $5i/(2+i) = 1+2i$

$5e^{i\frac{\pi}{2}} \div (\sqrt{5}\, e^{i\tan^{-1}(\frac{1}{2})}) = \frac{5}{\sqrt{5}} e^{i\frac{\pi}{2} - \tan^{-1}(\frac{1}{2})}$

let $\tan^{-1}(\frac{1}{2}) = \alpha$ $\quad = \frac{5}{\sqrt{5}}\left(\cos(\frac{\pi}{2} - \tan^{-1}(\frac{1}{2})) + i\sin(\frac{\pi}{2} - \tan^{-1}(\frac{1}{2})\right)$

$\cos(\frac{\pi}{2}-\alpha) = \cos(\frac{\pi}{2})\cos(\alpha) + \sin(\frac{\pi}{2})\sin(\alpha)$ $\quad \sin\frac{\pi}{2} = 1$

$\sin(\frac{\pi}{2}-\alpha) = \sin(\frac{\pi}{2})\cos(\alpha) - \sin(\alpha)\cos(\frac{\pi}{2})$ $\quad \cos\frac{\pi}{2} = 0$

$\therefore \frac{5}{\sqrt{5}}\left[\sin(\alpha) + i\cos(\alpha)\right] =$

$\frac{5\sin(\alpha)}{\sqrt{5}} + \frac{i5\cos\alpha}{\sqrt{5}} = 1+2i$ ✓

5 c) $(\sqrt{3}+i)^6 = -64 \rightarrow \left(2e^{i\frac{\pi}{6}}\right)^6 = 2^6 e^{i\pi} = -64$

d) $(1+\sqrt{3}i)^{-10} = 2^{-11}(-1+\sqrt{3}i) \rightarrow \left(2e^{\frac{i\pi}{3}}\right)^{-10} = 2^{-10} e^{i\frac{(-10)\pi}{3}}$

$= 2^{-10} e^{i\frac{2}{3}\pi} = -2^{-11} + 2^{-11}\sqrt{3}i = 2^{-11}(-1+\sqrt{3}i)$

6) if $\operatorname{Re} z_1$ & $\operatorname{Re} z_2$ both >0, then they both must lay in the 1st/4th quadrant. if two numbers in the first/4th quadrant are multiplied then their θ must remain in the principal argument zone.

8) $z_1 = c_1 c_2 \quad z_2 = c_1 \bar{c}_2$

$|z_1| = |c_1||c_2|$

$|z_2| = |c_1||\bar{c}_2| = z_1 = |c_1||c_2|$

bc $|\bar{c}_2| = |c_2|$

$\therefore |z_1| = |z_2|$

$z_1 = re^{i\theta_1}$ & $z_1 = re^{i\theta_2}$

$re^{i\frac{(\theta_1+\theta_2)}{2}} e^{i\left(\frac{\theta_1-\theta_2}{2}\right)} = re^{i\theta_1} = z_1$

$re^{i\frac{(\theta_1+\theta_2)}{2}} e^{i\frac{(-\theta_1+\theta_2)}{2}} = re^{i\theta_2} = z_2$

$c_1 = re^{i\frac{(\theta_1+\theta_2)}{2}}$ & $c_2 = e^{i\frac{(\theta_1-\theta_2)}{2}}$

Then $z_1 = c_1 c_2$

$\bar{c}_2 = e^{i\left(\frac{-\theta_1+\theta_2}{2}\right)}$ Then $z_2 = c_1 \bar{c}_2$

z_1 & z_2 must be the same point to show a modul

9) $1 + z + z^2 \ldots + z^n = \dfrac{1 - z^{n+1}}{1 - z}$

$S = 1 + z + z^2 \ldots + z^n$

$\cancel{S = \frac{1-z^{n+1}}{1-z} - \frac{z - z^{n+2}}{1-z}}$

$\cancel{(z^{n-1})(1-z) = 1 - z^{n+1} - z - z^{n+2}}$

$\cancel{z^{n-1} - z^{n} = 1 - z - z^{n+1} - z^{n+2}}$

$\cancel{\frac{z^n}{z} = 1 - z + z^n - z^{n+1} - z^{n+2}}$

$\cancel{z^n = z - z^2 + z^{n+1} - z^{n+2} - z^{n+3}}$

$zS = z + z^2 + \ldots z^{n+1}$

$zS = S - 1 + z^{n+1}$

$(-1)\ zS - S = -1 + z^{n+1}\ (-1)$

$S - zS = 1 - z^{n+1}$

$S(1 - z) = 1 - z^{n+1}$

$S = \dfrac{1 - z^{n+1}}{1 - z}$ ✓

let $z = e^{i\theta}$ $\quad S = 1 + e^{i\theta} + e^{i2\theta} + \ldots + e^{in\theta}$

$\cancel{e^{i\theta} S = e^{i\theta} + e^{i2\theta} + \ldots e^{in\theta + i}}$

$\cancel{e^{i\theta} S = S - 1 + e^{i(n+1)\theta}}$

$\cancel{1 - e^{i(n+1)\theta} = S - e^{i\theta} S = S(1 - e^{i\theta})}$

$S = 1 + (i\sin\theta + \cos\theta) + (i\sin\theta 2 + \cos\theta 2) + \ldots$

$e^{i\theta} S = i\sin\theta + \cos\theta + i\sin 2\theta + \cos\theta) + \ldots$

$e^{i\theta} S = S - 1 + i\sin(n+1)\theta + \cos(n+1)\theta$

$(\cos\theta + i\sin\theta) S = S - 1 + i\sin\theta(n+1) + \cos\theta(n+1)$

$$\cancel{\frac{1-e^{i\theta(n+1)}}{1-e^{i\theta}} = 1-(\cos(n+1)\theta + i\sin(n+1)\theta)}$$

b) $$1+e^{i\theta}+e^{2i\theta}+\dots+e^{ni\theta} = \frac{1-e^{(n+1)i\theta}}{1-e^{i\theta}}$$

$$\left(\frac{ie^{-\frac{i\theta}{2}}}{ie^{-\frac{i\theta}{2}}}\right)\cdot\frac{1-e^{(n+1)i\theta}}{1-e^{i\theta}} = \frac{ie^{-i\theta/2}-ie^{(2n+1)i\theta/2}}{ie^{i\theta/2}-ie^{i\theta/2}}$$

$$= \frac{ie^{-i\theta/2}-ie^{(2n+1)i\theta/2}}{2\sin(\theta/2)}$$

$$1+\cos\theta+\cos 2\theta+\dots+\cos n\theta = Re(1+e^{i\theta}+e^{2i\theta}+\dots)$$

$$Re\left(\frac{ie^{-i\theta/2}-ie^{(2n+1)i\theta/2}}{2\sin(\theta/2)}\right)$$

$$= \frac{\sin\theta/2+\sin((2n+1)(\theta/2))}{2\sin(\theta/2)} = \frac{1}{2}+\frac{\sin[(2n+1)\theta/2]}{2\sin(\theta/2)}$$