Josh Brown

Hw #1

each problem has seperate page

1a) $ty' + (t+2)y = e^t/t$ $\qquad p(t) = 1 + \frac{2}{t}$

$y' + \left(1 + \frac{2}{t}\right)y = e^t/t^2$ $\qquad \mu(t) = e^{\int 1 + \frac{2}{t}\,dt} = e^{t + \ln t}$

$= t^2 e^t$

$$\frac{d}{dx}\mu(t) = t^2e^t\left(1 + \frac{2}{t}\right)$$

$$\mu(t)\,p(t) = \mu'(t) \to t^2e^t \cdot \left(1 + \frac{2}{t}\right) \checkmark$$

$$(t^2e^t y)' = \frac{e^t}{t^2} \cdot e^t t^2 = e^{2t}$$

$$\int (t^2e^ty)'\,dt = \int e^{2t}\,dt$$

$$t^2e^ty = \frac{1}{2}e^{2t} + C$$

$$y = \frac{\frac{1}{2}e^{2t} + C}{e^t t^2}$$

1b) $y'' + y' - 6y = 0$

$r^2 + r \cancel{x} - 6$

$r = 2, -3$

let $y = e^{2t}$

$y' = 2e^{2t}$

$y'' = 4e^{2t}$

$4e^{2t} + 2e^{2t} - 6e^{2t} = 0$

let $y = e^{-3t}$

$y' = -3e^{-3t}$

$y'' = 9e^{-3t}$

$9e^{-3t} - 3e^{-3t} - 6e^{-3t} = 0$

yeilding $y = C_1 e^{2t} + C_2 e^{-3t}$

1c) $y'' + y' - 20y = xe^{3x} + e^{4x}$ | $r^2 + r - 20 \to r = 4, -5$

$Y_{p_1}(x) = (Ax+B)e^{3x}$ $\quad Y_p'(x) = Ae^{3x} + 3e^{3x}(Ax+B)$

$Y_{p_1}''(x) = 3Ae^{3x} + 3Ae^{3x} + 9e^{3x}(Ax+B) = 6Ae^{3x} + 9e^{3x}Ax + 9e^{3x}B$

$y'' + y' \to 6Ae^{3x} + 9e^{3x}Ax + 9e^{3x}B + Ae^{3x} + 3e^{3x}Ax + 3e^{3x}B$

$(-20y)$

$= 7e^{3x}A + 12e^{3x}Ax + 12e^{3x}B - 20Axe^{3x} - 20Be^{3x}$

$= 7A - 8Ax - 8B = x \quad A = \frac{-1}{8} \quad B = \frac{-7}{8} \Rightarrow Y_{p_1}(x) = \left(\frac{-1}{8}x + \frac{-7}{8}\right)e^{3x}$

$Axe^{4x} = Y_{p_2}(x) \quad y' = Ae^{4x} + 4xAe^{4x}$

$y'' = A4e^{4x} + 4Ae^{4x} + 16A\cancel{x}e^{4x} = 8Ae^{4x} + 16xAe^{4x}$

$y'' - y' - 20y$

$8Ae^{4x} + \underline{16xAe^{4x}} + Ae^{4x} + \underline{4xAe^{4x}} - \underline{20Axe^{4x}}$

$9Ae^{4x} = e^{4x} \quad 9A = 1 \quad A = \frac{1}{9} \quad Y_{p_2}(x) = \frac{xe^{4x}}{9}$

yeilding

$y(x) = Y_{p_1}(x) + Y_{p_2}(x) \to y = \left(\frac{-1}{8}x + \frac{-7}{8}\right)e^{3x} + \frac{xe^{4x}}{9}$

2a) $y' = y(1+e^t y) = y' - y = e^t y^2$ $\quad y(1)=2$

$y'y^{-2} - y^{-1} = e^t$ let $v = y^{-1}$ $\quad v' = y^{-2}y'$

yeilding

$v' - v = e^t$ $\qquad N(t) = e^{\int -1\,dt} = e^{-t}$

$e^{-t}v' - e^{-t}v = 1 = e^{-t}\cdot e^t$

$\int (e^{-t}v)'\,dt = \int 1\,dt = e^{-t}v = t$

$v = \frac{t+c}{e^{-t}} = te^t + e^t c$

$y^{-1} = te^t + ce^t$

$2^{-1} = e + ce \to c = \frac{1-2e}{2e}$

$y^{-1} = te^t + \left(\frac{1-2e}{2e}\right)e^t \to y = \frac{1}{te^t + \frac{e^t - 2e^{t+1}}{2e}}$

$y = \frac{2}{2te^t + e^{t-1} - 2e^t}$.

$y(1) = 2$

2b) $x(1+y^2)\,dx = (1+x^2)^{\frac{1}{2}}\,dy \qquad y(0)=1$

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{1}{1+y^2}\,dy$$

$$\sqrt{1+x^2} + C_1 = \arctan(y) + C_2$$

$$C + \sqrt{1+x^2} = \arctan(y)$$

$$1 + C = \pi/4 \rightarrow C = \frac{\pi}{4} - 1$$

$$y = \tan\left(\sqrt{1+x^2} + \frac{\pi}{4} - 1\right)$$

$$y = \tan\left(\sqrt{1+x^2} - \frac{3\pi}{4}\right)$$

3) $m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} = \dfrac{x+c}{d^2}$ $[x] = L$ $[t] = T$ $[m] = M$

$[c] = L$ because you can only add similar dimensions. $\therefore [x] == [c]$

$[m] = M$

$\rightarrow M\left[\dfrac{d^2x}{dt^2}\right] = M\dfrac{\frac{[dx]}{[dt]}}{[t]} = \dfrac{ML}{T^2} = MLT^{-2}$

as before, $\left[m\dfrac{d^2x}{dt^2}\right] == \left[b\dfrac{dx}{dt}\right] = MLT^{-2}$

$MLT^{-2} = [b]\dfrac{[x]}{[t]} = [b]\dfrac{L}{T}$

$[b]\dfrac{\cancel{L}}{\cancel{T}} = M\cancel{L}T^{\cancel{-2}\,-1} \rightarrow [b] = \dfrac{M}{T}$

$\dfrac{\cancel{L}}{[d^2]} = M\cancel{L}T^{-2} \rightarrow \dfrac{1}{[d^2]} = \dfrac{M}{T^2}$

$[d] = \dfrac{T}{M^{\frac{1}{2}}}$