3.30 $\cancel{1}, \cancel{3}, \cancel{5}, \cancel{6}, \cancel{8}$

1) a) $e^2 e^{3\pi i} = e^2 e^{2\pi i} e^{\pi i} = e^2(1)(-1) = -e^2$

b) $\left(e^{\frac{1}{2}} e^{\frac{\pi}{4} i}\right)^2 = \left(\sqrt{\frac{e}{2}}\,(1+i)\right)^2$

$$e^1 e^{\frac{\pi i}{2}} = \frac{e}{2}(1+i)^2$$

$$\frac{2e^{\frac{\pi i}{2}+1}}{e} = (1+i)^2$$

$$2e^{\frac{\pi}{2} i} = (1+i)^2 = 1 + 2i - 1 = 2i$$

$$e^{\frac{\pi}{2} i} = i$$

$$\cos\left(\frac{\pi}{2}\right) + i\left(\sin\frac{\pi}{2}\right) = i \quad \checkmark$$

2) $e^z e^{\pi i} = -e^z$

$e^z(-1) = -e^z$ ✓

3) $e^x e^{-yi} = f(x)$

$\cancel{a}$ $e^x \cos(-y) + i\sin(-y)e^x$

$u = e^x \cos(-y)$ $\qquad v = \cancel{i}\,\sin(-y)e^x$

$u_x = e^x \cos(-y)$ $\qquad v_y = -\cos(-y)e^x$

$u_x \neq v_y$ ✓

5) $|e^{2z} e^{i}| = e^{2x}$ $\quad |e^{i(x^2+2xy-y^2)}|$

$= e^{-2xy}$

$|e^{2z+i} + e^{iz^2}| \leq e^{2x} + e^{-2xy}$

$\leq |e^{2x} + e^{-2xy}$

let $v = e^{2z+i}$ & $w = e^{iz^2}$

$|v + w| \leq |v| + |w|$ ✓

6)

$|e^{z^2}| \leq e^{|z|^2}$ $\quad |z|^2 = x^2 + y^2$

$|e^{z^2}| \leq e^{x^2+y^2}$

$|e^{z^2}| = |e^{x^2+2xyi-y^2}| = e^{x^2-y^2}$

$e^{x^2-y^2} \leq e^{x^2+y^2}$

$x^2 - y^2 \leq x^2 + y^2$ ✓

8) $e^z = -2$ $\quad e^x = 2, \quad e^{iy} = -1$

a) $e^x = 2 \Rightarrow \ln 2$

$e^{iy} = -1 \rightarrow (2n+1)\pi i$

$\ln 2 + (2n+1)\pi i$ $\quad n = \pm 1, \pm 2, \pm 3, \ldots$

b) if $e^x = 2 \Rightarrow \ln 2,$

$\frac{1}{2}\ln 2 = 1$ $\quad e^{iy} \rightarrow e^{\frac{1}{4}\pi i} = i,$

cont. b) $(2n+\frac{1}{4})\pi i \quad n = \pm 1, \pm 2, \pm 3, \dots$ (0)

$\frac{1}{2}\ln 2 + (2n+\frac{1}{4})\pi i \quad n = 0, \pm 1, \pm 2, \pm 3 \dots$

c) $e^{2z-1} = 1 \qquad e^{2x-1} e^{iy} \qquad e^{2x-1} = 1, \; x = \frac{1}{2}$

$e^{iy} = 1, \; y = \pi i$

$\frac{1}{2} + n\pi i \; (n = 0, \pm 1, \pm 2.)$

3.33 $\quad \cancel{1}, \cancel{3}, \cancel{4}, 8$

1) a) $\log(z) = \ln r + i\theta$

$\text{Log}(-ei) = 1 - \frac{\pi}{2} i$

$z = -ei$

$r = \sqrt{z^2} = e, \; \ln e = 1 \qquad \theta = -\frac{\pi}{2}$ since $x = 0$ & y is negative

$\therefore \text{Log}(-ei) = 1 - \frac{\pi}{2} i$

b) $\text{Log}(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4} i$

$r = \sqrt{2}, \; 2^{\frac{1}{2}} \to \frac{1}{2}\ln 2$

$\tan\theta = \frac{y}{x}, \quad \tan^{-1}\left(\frac{y}{x}\right) = \theta, \quad \tan^{-1}(-1) = -\frac{\pi}{4}$

$\therefore \text{Log}(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4} i$

3) $\text{Log}\, i^3 \neq 3\text{Log}\, i$

$\text{Log}(i^3) = \text{Log}(i\cdot i\cdot i) = \text{Log}(-i)$

$\text{Log}(-i) \Rightarrow r = -1, \quad \theta = -\frac{\pi}{2}$

$\text{Log}(-i) = \ln(-1) - \frac{\pi}{2} i$

$3\text{Log}(i) \Rightarrow r = -1 \quad \theta = \frac{\pi}{2}$

$\rightarrow 3\left(\ln(i) + \frac{\pi}{2}\right)$

$= 3\ln(i) + \frac{3\pi}{2} \neq \ln(-1) - \frac{\pi}{2} i$

4) $\log(i^2) \neq 2\log(i)$

$\log(-1) \rightarrow r = 1, \quad \theta = 0 + 2\pi n i$

$2\log(i) \rightarrow r = i \quad \theta = \frac{\pi}{2}$

$= 2\ln(i) + (\pi + 2n\pi) i \qquad n = (\pm 1, \pm 2, \pm 3) \ldots$

$2\ln(i) + \pi i \neq \ln(1) + 0 = 0$

while

$i \neq 0$

8) $\log(z) = i\frac{\pi}{2}$

$\log(z) = \ln r + i\theta$

$\ln r = 0 \qquad \theta = \frac{\pi}{2}$

$r = 1 \qquad r = \sqrt{x^2+y^2}$

$1 = x^2 + y^2 \qquad y \neq 0$ (result is imaginary

~~$\tan(\frac{\pi}{2}) = \frac{y}{x}$~~ let $x = 0, \; y = 1$

Then $\log(i) = \ln(1) + i\frac{\pi}{2}$

or $\log(i) = i\frac{\pi}{2}$

$z = i$

3.36 ~~1, 2, 3~~, 7

1) $(1+i)^i \to r = \sqrt{2} \qquad \theta = \frac{\pi}{4}$

a) $(\sqrt{2}\, e^{\frac{\pi}{4}})^i = \sqrt{2}^i e^{-\frac{\pi}{4}\cancel{i} + 2\pi n}$

$\sqrt{2}^i = re^{i\theta} \quad r = 0, \; \theta = \frac{\ln 2}{2}$

$\therefore (1+i)^i = e^{\frac{\ln 2}{2} i} e^{-\frac{\pi}{4}} + 2n\pi \qquad n = \pm 1, \pm$

1) $\frac{1}{i^{2i}} = e^{(4n+1)\pi}$

$i \Rightarrow r = 1,\ \theta = \frac{\pi}{2},$

$\frac{1}{e^{(\frac{\pi}{2}+2n\pi)2i}} = e^{\pi + 4n\pi} = e^{(4n+1)\pi}$ $\quad n = \pm 0, \pm 1, \pm 2 \ldots$

2) a) $(-i)^{i}$ $\quad -i \to r = 1,\ \theta = -\frac{\pi}{2}$

$(e^{-\frac{\pi}{2}i})^{i} = e^{\frac{\pi}{2}}$

b) let $-1 - \sqrt{3}i = z$ $\quad r = 2,\ \theta = \frac{\pi}{3}$

$= \left[(2e^{\frac{\pi}{3}i})\frac{e}{2}\right]^{3\pi i} = (e e^{\frac{\pi i}{3}})^{3\pi i} = e^{2\pi^2 i}$

since x & y negative, $= -e^{2\pi^2 i}$

c) $(1-i)^{4i}$ $\quad 1 - i \Rightarrow r = \sqrt{2}$ $\quad \theta = -\frac{\pi}{4}$

$(\sqrt{2}e^{-\frac{\pi}{4}})^{4i} = 4e^{-\pi i}$

$= 4e^{\pi}e^{-i}$

$= e^{\pi}[\cos(2\ln 2) + i\sin(2\ln 2)]$

3) $(-1+\sqrt{3}i)^{3/2} = \pm 2\sqrt{2}$

~~$-1+\sqrt{3}i$, $r=2$, $\theta=-\frac{\pi}{3}$~~

~~$\left(2e^{-\frac{\pi}{3}i}\right)^{\frac{3}{2}} = 2\sqrt{2}e^{-2\pi i}$~~

$e^{\left(\frac{3}{2}\right)\log(-1+\sqrt{3}i)}$

$r=2 \quad \theta=-\frac{\pi}{3}$

$e^{\frac{3}{2}\left(\ln 2+\frac{\pi i}{3}\right)} = e^{\frac{3}{2}\ln 2+2\pi i}$

$e^{\frac{3}{2}\ln 2} = 2\sqrt{2} \qquad e^{2\pi i} = \pm 1$

$2\sqrt{2}\,e^{2\pi i} = \pm 2\sqrt{2}$

7) $i^c = e^{c\log i} = e^{c\left(i+\frac{\pi}{2}\right)} = e^{ci+\frac{\pi}{2}} = e^{ci}e^{c\frac{\pi}{2}}$

$e^{ci} = \cos c + i\sin c$

$e^{c\frac{\pi}{2}}(\cos c + i\sin c)$

sin c must be gone, therefore

c must be real.