3.38, 3.39, 3.40

~~14~~, ~~15~~, ~~16~~ / ~~7~~, ~~16~~, ~~17~~/ ~~8~~, ~~13~~

14) $\overline{\cos(i z)} = \cos(i \bar{z})$ for all z.

a)

Theorem $\cos(z) = \cos x \cosh y - i \sin x \sinh y$

$\overline{\cos(iz)} = \cos y \cosh x + i \sin y \sinh x$

$\cos(i\bar{z}) = \cos y \cosh x + i \sin y \sinh x$

$\cos(i\bar{z}) = \overline{\cos(iz)}$

b) $\overline{\sin(i z)} = \sin(i\bar{z})$ if $z = n\pi i$ $n = \pm 1, \pm 2, \pm 3$

$\overline{\sin(iz)} = \sin y \cosh x - i \cos y \sinh x$

$\sin(i\bar{z}) = \sin y \cosh x + i \cos y \sinh x$

$\overset{-\sin y \cosh x}{}$ $\overset{-\sin y \cosh x}{}$

$\sin y \cosh x - i \cos y \sinh x = \sin y \cosh x + i \cos y \sinh x$

$-i \cos y \sinh x = i \cos y \sinh x$

$2 i \cos y \sinh x = 0$

iff $z = n\pi i$ $n = 0, \pm 1, \pm 2$ etc

15) $\sin z = \cosh 4$

$\sin z = \sin x \cosh y + i \cos x \sinh y$

$\cosh 4 = \sin x \cosh y + i \cos x \sinh y$

$\sin x \cosh y = \cosh 4$

$i \cos x \sinh y = 0$

$\sin x = 1, \quad x = \frac{\pi}{2} \pm 2\pi n$

$\cosh y = \cosh 4$

$y = 4$

$\therefore z = \left(\frac{\pi}{2} + 2\pi n\right) \pm 4i \qquad n = 0, \pm 1, \pm 2, \pm 3$

16) $\cos z = 2, \qquad \cos z = \cos x \cosh y - i \sin x \sinh y$

$\cos x \cosh y = 2$

$-\sin x \sinh y = 0$

let $\sin x = 0, \quad x = 2\pi n$

$\therefore \cos(2\pi n) = 1, \quad$ so $\cosh y = 2 \qquad y = \cos^{-1}(2)$

$z = 2\pi n + i \cosh^{-1} 2 \qquad n \in \mathbb{Z}$

16 cont) $2\pi n + i\cosh^{-1} 2$

$$\cosh^{-1} 2 = \ln\left(2 + \sqrt{2^2 - 1}\right) = \ln\left(2 + \sqrt{3}\right)$$

$$z = 2n\pi \pm i\ln\left(2 + \sqrt{3}\right) \quad n \in \mathbb{Z}$$

3.3 9

7) a) $\sinh(z + \pi i) = -\sinh z$

$$\sinh(z + \pi i) = \frac{e^{z+\pi i} - e^{-z-\pi i}}{2}$$

$$\frac{e^{z}e^{\pi i} - e^{-z}e^{-\pi i}}{2}$$ note. eulers identity. $e^{\pi i} = -1, e^{-\pi i} = -1$

$$\frac{-e^{z} + e^{-z}}{2} = -\left(\frac{e^{z} - e^{-z}}{2}\right) = -\sinh z$$ ✓

b) $\cosh(z + \pi i) = -\cosh z$

$$\cosh(z + \pi i) = \frac{e^{z+\pi i} + e^{-z-\pi i}}{2} = \frac{-e^{z} - e^{-z}}{2} = -\left(\frac{e^{z} + e^{-z}}{2}\right)$$

$$= -\cosh(z)$$ ✓

c) $\tanh(z + i\pi) = \tanh z$

$$\tanh(z+\pi i) = \frac{\sinh(z+\pi i)}{\cos(z+\pi i)} = \frac{-\sinh(z)}{-\cosh(z)} = \tanh(z)$$

✓

16) a) $\sinh z = i$

$\sinh z = \sinh x \cos y + i \cosh x \sin y$

$i = \sinh x \cos y + i \cosh x \sin y$

$\sinh x \cos y = 0, \quad i \cosh x \sin y = i$

$\cosh x = 1, \ \sin y = 1 \qquad y = \left(\frac{1}{2} + 2n\right)\pi i$

$x = 0$

$\sinh 0 = 0$ ✓

$z = \left(2n + \frac{1}{2}\right)\pi i \qquad n \in \mathbb{Z}$

b) $\cosh z = \frac{1}{2} \qquad \frac{1}{2} = \cosh x \cos y + i \sinh x \sin y$

$\cosh x \cos y = \frac{1}{2} \qquad i \sinh x \sin y = 0$

let $x = 0$, $\quad \sinh(0) = 0$,

$\cosh(0) = 1 \qquad \cos y = \frac{1}{2} \qquad y = \frac{\pi}{3} + 2n\pi$

$z = \left(2n \pm \frac{1}{3}\right)\pi i \qquad n \in \mathbb{Z}$

17) $\cosh z = -2$

$\cosh z = \cosh x \cos y + i \sinh x \sin y$

$-2 = \cosh x \cos y$

$0 = i \sinh x \sin y$

$\cosh x = 2, \quad \cos y = -1$

$\cosh x = 2 = \cosh^{-1} 2 = x = \pm \ln(2+\sqrt{3})$

$\cos y = -1 \qquad y = 3\pi, \qquad y = \pi + 2\pi n \qquad n \in \mathbb{Z}$

$z = \pm \ln(2+\sqrt{3}) + (2n+1)\pi i \qquad n \in \mathbb{Z}$

3.40

1) a) $\tan^{-1}(2i) \Rightarrow \frac{i}{2} \log \frac{3i}{-i} = \frac{i}{2} \log(-3)$

$\frac{i}{2} \operatorname{Log}(-3) = \frac{\pi}{2} + \frac{i}{2} \ln(3) \rightarrow (n+\frac{1}{2})\pi + \frac{i}{2} \ln 3$

b) $\tan^{-1}(1+i) = \frac{i}{2} \log\left(\frac{1+2i}{-1}\right) = \frac{i}{2} \log(-1-2i)$

$\frac{i}{2} \log(-1-2i) = \frac{i}{2} \ln r + i\theta,$

or $\quad \frac{i}{2} \ln(\sqrt{5}) + i \tan^{-1}(2) + 2\pi n$

$n \in \mathbb{Z}$

c) $\cosh^{-1}(-1) = \ln(\cancel{x} + \sqrt{x^2-1}) = \ln(-1 + \sqrt{1-1})$

$= \ln(-1) = \pi i$

d) $\tanh^{-1}(0) = \dfrac{\sinh^{-1}(0)}{\cosh^{-1}(0)}$

$\tanh(\cancel{z}) = 0 \to \dfrac{\sinh \cancel{z}}{\cosh z} = 0 \qquad \sinh(z) = 0$

$\sinh = \dfrac{e^{x} - e^{-x}}{2} = 0$

$e^{x} - e^{-x} = 0$

$e^{x} = e^{-x}$

$x = \pi i, \quad x = n\pi i$

$\tanh^{-1}(0) = n\pi i \quad n \in \mathbb{Z}$

3) $\cos z = \sqrt{2} \qquad z = \cos^{-1}(\sqrt{2})$

$\cos^{-1}(\sqrt{2}) = -i \log[\sqrt{2} + i(1-2)^{\frac{1}{2}}]$

$= -i \log(\sqrt{2} \pm 1)$

$= \pm(2n\pi - i\ln(\sqrt{2}+1)$ $=$ ~~[illegible]~~

$=$ ~~[illegible]~~