7.86 $\cancel{8.4}$

3) $\int_0^\infty \frac{1}{x^4+1} = \frac{\pi}{2\sqrt{2}}$

$\int_0^\infty f(x)_{dx} = \operatorname*{Res}_{z=z_k} f(z) \cdot \pi i \quad x=0$ at $\frac{1+i}{\sqrt{2}}$

$\operatorname*{Res}_{z=z_k} f(z) = \frac{1}{4c_k^3} = \frac{c_k}{4c_k^4} = \frac{-c_k}{4}$

$-\left(\frac{1+i}{4\sqrt{2}}\right) + -\left(\frac{-1+i}{4\sqrt{2}}\right) = \frac{-1-i+1-i}{4\sqrt{2}} = \frac{-2i}{4\sqrt{2}} = \frac{-i}{2\sqrt{2}}$

$\int_0^\infty f(x) = \frac{-i}{2\sqrt{2}} \cdot \pi i = \frac{\pi}{2\sqrt{2}}$

4) $\int_0^\infty \frac{x^2}{x^6+1} dx = \frac{\pi}{6}$ Zeros at $x = i, e^{\frac{i\pi}{6}}, e^{\frac{5\pi i}{6}}$

$\operatorname*{Res}_{z=z_k} f(z) = \frac{x_k^2}{6x_k^5} = \frac{x_k^3}{6x_k^6} = \frac{c_k^3}{-6}$

$= \frac{+i}{6} + \frac{e^{i\frac{\pi}{2}}}{-6} - \frac{e^{-i\frac{\pi}{2}}}{-6}$

$= \frac{+i}{6} + \frac{2i\sin(\frac{\pi}{2})}{-6} = \frac{+i}{6} + \frac{2i}{-6} = \frac{-i}{6}$

$\int_0^\infty \frac{x^2}{x^6+1} dx = \left(\frac{-i}{6}\right)\pi i = \frac{\pi}{6}$

7.88 ~~1~~, ~~4~~, 2

1) $\int_{-\infty}^{\infty} \frac{\cos x \, dx}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{a^2-b^2} \left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a} \right)$

$x = ia, \; ib$

$$\operatorname*{Res}_{z=z_k} = \frac{e^{iz}}{2x(x^2+b^2) + 2x(x^2+a^2)}$$

let $z_k = ia$

$$\frac{e^{iia}}{2ia(-a^2+b^2) + 2ia(-a^2+a^2)}$$

$$= \frac{-e^{-a}}{2ia(a^2-b^2)}$$

let $z_k = ib$

$$\frac{e^{iib}}{2ib(-b^2+b^2) + 2bi(-b^2+a^2)} = \frac{e^{-b}}{2ib(a^2-b^2)}$$

$$\left[\frac{e^{-b}}{2ib(a^2-b^2)} - \frac{e^{-a}}{2ib(a^2-b^2)} \right] 2\pi i = \frac{\pi e^{-b}}{b(a^2-b^2)} - \frac{\pi e^{-a}}{a(a^2-b^2)}$$

$$= \frac{\pi}{a^2-b^2} \left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a} \right)$$

4) $\int_{-\infty}^{\infty} \frac{x \sin ax}{x^4+4}\,dx = \frac{\pi}{\cancel{2}} e^{-a} \sin a$

$x = (1+i), (-1+i)$

$\underset{z=c_k}{\text{Res}} = \frac{c_k \sin(a c_k)}{4 \cancel{z}_k^3} = \frac{-c_k^2 \sin(a c_k)}{\cancel{4} 16}$

let $c_k = (1+i)$ $\quad \frac{-(1+i)^2 \sin(a(1+i))}{\cancel{4} 16}$

$= \frac{-2i \sin(ia+a)}{\cancel{4} 16} = \frac{-i(\sin(ia)\cos(a) + \sin(a)\cos(ia)}{8 \cancel{4}}$

let $c_k = (-1+i)$

$= \frac{i \sin(ai-a)}{\cancel{2} 8} = \frac{i(\sin(ia)\cos(a) - \sin(a)\cos(ia))}{8}$

add Residues yeilds

$i\left[\frac{\cancel{-\sin(ia)\cos a} - \sin(a)\cos(ia) + \cancel{\sin(ia)\cos(a)} - \sin(a)\cos(ia)}{8}\right]$

$= \left[\frac{-i \sin(a)\cos(ia)}{\cancel{8} 4}\right] 2\pi i = \frac{\pi}{2} \cos(ia) \sin(a)$

$= \frac{\pi}{2} e^{-a} \sin(a)$

7) $\int_0^\infty \frac{x^3 \sin x}{(x^2+1)(x^2+9)}\,dx$

$x = i,\ 3i$

$$\underset{z=z_0}{\mathrm{Res}} = \frac{x^3 e^{ix}}{2x(x^2+1)+2x(x^2+9)}$$

let $x = i$ $\qquad \frac{-i e^{-1}}{2i(8)} = \frac{-e^{-1}}{16}$

$x = 3i$ $\qquad \frac{27 i e^{-3}}{6i(-8)} = \frac{27 e^{-3}}{48} = \frac{9e^{-3}}{16}$

$$\int_0^\infty \frac{x^3 \sin x}{(x^2+1)(x^2+9)}\,dx = \frac{9e^{-3} - e^{-1}}{16} \cdot \pi i = \frac{\pi 9 i e^{-3} - \pi i e^{-1}}{16}$$

$$= \pi\left(\frac{9e^{-3} - e^{-1}}{16}\right)$$

from Jordans lemma