7.91

1) $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = \int_C \frac{1}{iz\left(5+4\left(\frac{z-z^{-1}}{2i}\right)\right)}$

$= \int_C \frac{1}{2z^2+5iz-2}\,dz$

$\frac{-5i \pm \sqrt{-25+16}}{4} = -2i, -\frac{1}{2}i$

$-2i$ outside C, $-\frac{1}{2}i$ inside.

$\int_C \frac{1}{2z^2+5iz-2}\,dz = \underset{z=\frac{1}{2}i}{\text{Res}}\, f(z) \cdot 2\pi i$

$\underset{z=i\frac{1}{2}}{\text{Res}} \frac{1}{2z^2+5iz-2} = \frac{P(z)}{Q'(z)} = \frac{1}{4z+5i}$

let $z = z_0$, $\frac{1}{-2i+5i} = \frac{1}{3i}$

$\left(\frac{1}{3i}\right)2\pi i = \frac{2\pi}{3}$

$$2\int_{\rho}^{R} \frac{\cos ar - \cos br}{r^2}\, dr \quad \text{simplifies to}$$

$$-\int_{C_\rho} \frac{e^{iaz} - e^{ibz}}{z^2}\, dz - \int_{C_R} \frac{e^{iaz} - e^{ibz}}{z^2}$$

$$-\int_{C_\rho} \frac{e^{iaz} - e^{ibz}}{z^2}\, dz = \operatorname*{Res}_{z=0} \frac{\phi'(z_0)}{(z-0)^2}$$

$$= -(a-b) = -(-b+a) = (b-a) \cdot \pi$$

$$2\int_{\rho}^{R} \frac{\cos ar - \cos br}{r^2}\, dr = (b-a)\pi$$

divide by 2 & re paramatarize

$$\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2}\, dx = \frac{\pi}{2}(b-a)$$

now let $a=0$ & $b=-2$, $\cos(-2x) = \cos(2x)$

$$\int_0^\infty \frac{\cos(0) - \cos 2x}{x^2} = \frac{\pi}{2}(0+2)$$

$$\int_0^\infty \frac{1-\cos(2x)}{x^2}\, dx = \pi \rightarrow 2\int_0^\infty \frac{\sin^2 x}{x^2} = \pi$$

divide both sides by 2, yeilding

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$$

$$\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2} = \int_0^{2\pi} \frac{d\theta}{1+\sin^2\theta}$$

$$= \int_0^{2\pi} \frac{2}{2+1-\cos 2\theta} = \int_0^{2\pi} \frac{2}{iz\left(3-\left(\frac{e^{i2\theta}+e^{-i2\theta}}{2}\right)\right)}$$

$$= \int_C \frac{2}{iz\left(3-\left(\frac{z^2+z^{-2}}{2}\right)\right)} dz$$

$$= \int_C \frac{2}{3iz - \frac{iz^3 - iz^{-1}}{2}} = \int_C \frac{1}{\frac{3iz}{} - \frac{iz^3}{2} - \frac{i}{2z}}$$

$$\int \frac{z}{3iz^2 - \frac{iz^4}{2} - \frac{i}{2}} = \frac{2}{i}\left(\int_C \frac{z}{\frac{3}{2}z^2 - z^4 - 1}\right.$$

$$\frac{3}{2}z^2 - z^4 = 1$$

$$3z^2 - 2z^4 = 2$$

$$z^2(3-2z^2) = 2$$

$$(\sqrt{3}z + \sqrt{2}z^2)($$

1) $\displaystyle\int_0^{2\pi} \frac{1}{1+\sin^2(x)} = \int_{-\pi}^{\pi} \frac{1}{1+\sin^2(x)}$

$$\int_0^{2\pi} \frac{1}{1+\sin^2(x)} = \int_C \frac{1}{1+\left[\frac{z-z^{-1}}{2i}\right]^2} \frac{dz}{iz} \cdot \frac{-4z}{-4z}$$

$$= \int_C \frac{-4z\,dz}{i(z^4-6z^2+1)}$$

let $v = z^2$

$dv = 2z\,dz$

$\cancel{\text{Res}_{z=3-2\sqrt{2}} \frac{-4z}{4z^3-12z}}$

$\cancel{= \frac{-4}{4i}}$

$$= 2\int_C \frac{-2\,dv}{i(v^2-6v+1)} = 4i\int_C \frac{dv}{v^2-6v+1}$$

$v = 3 \pm 2\sqrt{2}$

$v = 3\cancel{+}-2\sqrt{2}$

$$\text{Res}_{v\to 3-2\sqrt{2}} \frac{1}{v^2-6v+1} = \frac{-1}{4\sqrt{2}} \rightarrow 4i\cdot 2\pi i\cdot\frac{-1}{4\sqrt{2}}$$

$= \sqrt{2}\pi$