§1.2 - 2, 3, 11

② Show that a) $Re(iz) = -Im\, z$ ⓑ $Im(iz) = Re\, z$

③ Show that $(1+z)^2 = 1 + 2z + z^2$

⑪ Solve $z^2 + z + 1 = 0$ for (x, y) by writing

$$(x,y)(x,y) + (x,y) + (1,0) = (0,0)$$

Reduce to 2 eqns in 2 unknowns & solve Ans: $\left(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$

NOTE - of course you could use the quadratic formula, but we will need this technique later.

§1.3 Reduce each of these quantities to a real no.

1, 4 ① a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ ans. $2/5$

b) $\frac{5i}{(1-i)(2-i)(3-i)}$ answer $-1/2$

c) $(1-i)^4$ ans. -4

④ Prove that if $z_1 z_2 z_3 = 0$, then at least one of these factors is zero. write $(z_1 z_2) z_3 = 0$ & proceed as expression (6), sect 3.

§1.5 - 4, 5, 6

4) verify $\sqrt{2}|z| \geq |\text{Re } z| + |\text{Im } z|$. write $z = x+iy$. Then square both sides to recognize the statement $(|x|-|y|)^2 \geq 0$. Hence the original eqn is true

5) In each case, sketch the set of points determined by the given condition

a) $|z-1+i| = 1$

b) $|z+i| \leq 3$

c) $|z-4i| \geq 4$

6) using the fact that $|z_1 - z_2|$ is the distance between z_1 & z_2, give a geometric argument that $|z-1| = |z+i|$ represents the line through the origin whose slope is 1.

§1.6 1, 2, 9, 13

① Use properties of conjugates & moduli to show that

a) $\overline{z+3i} = z-3i$

b) $\overline{iz} = -1 \cdot \overline{z}$

c) $\overline{(2+i)^3} = 3-4i$

d) $|(2\overline{z}+5)(\sqrt{2}-i)| = \sqrt{3}\,|2z+5|$

② Sketch the pts in the $\mathbb{C}$-plane which satisfy

a) $Re(\overline{z}-i) = 2$

b) $|2\overline{z}+i| = 4$

⑨ By factoring z^4-4z^2+3 into two quadratic factors & using $|z_1+z_2| \geq ||z_1|-|z_2||$ – the reverse Δ-inequality, show that if z lies in the circle $|z|=2$, then

$$\left|\frac{1}{z^4-4z^2+3}\right| \leq \frac{1}{3}.$$