

Math Models, Spring 2021

Midterm Exam, Version A

2.. Second Attempt

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Directions: Please show all your work. I will regrade this question and use this score to replace the score you received on question 2. on the exam. If you pick up your exam between now and when this is due, I need you to scan the first page of question 2. on your original exam and upload that along with your new solution.

Consider the scalar differential equation

$$u' = (u + 1)(h - 1 - u^2), \quad (1)$$

where h is a parameter.

(a)[5pts.] On the pair of axes above, make an initial sketch of the bifurcation diagram for (1). Sketch just the branches. In this diagram, you do NOT need to indicate which branches are stable and which branches are unstable.

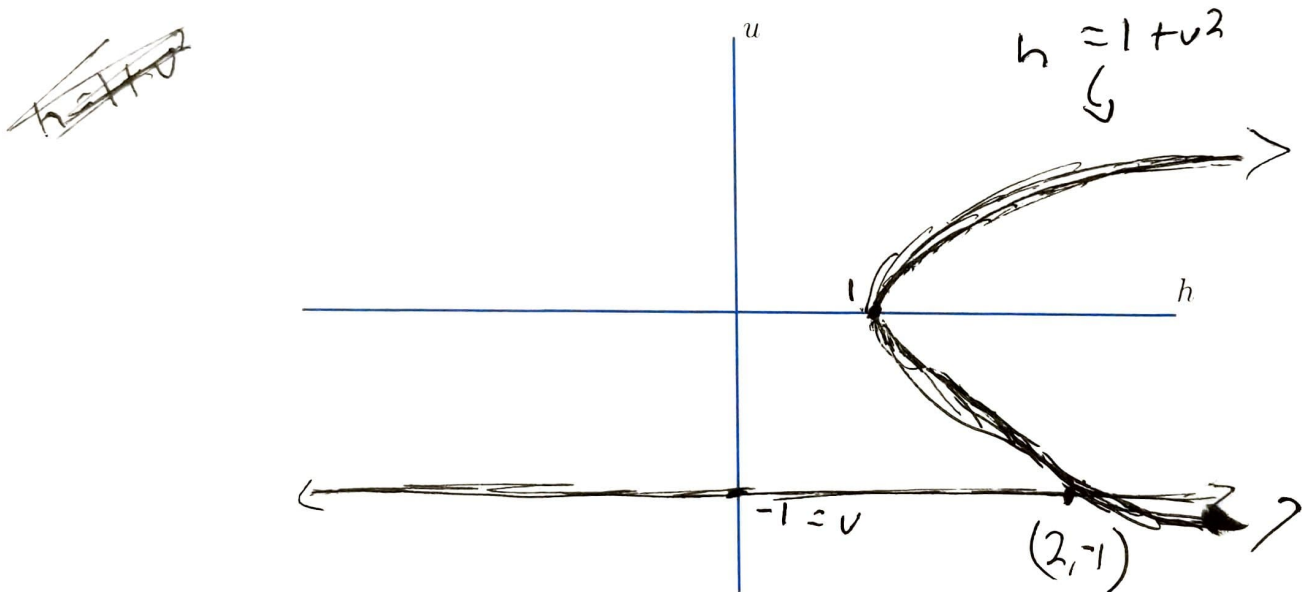


Figure 1: Initial Bifurcation Diagram

(b)[4pts.] Using the diagram from part (a), you should discover that there are 2 bifurcation points, h_1 and h_2 (assume $h_1 < h_2$).

$$h_1 = \underline{1}$$

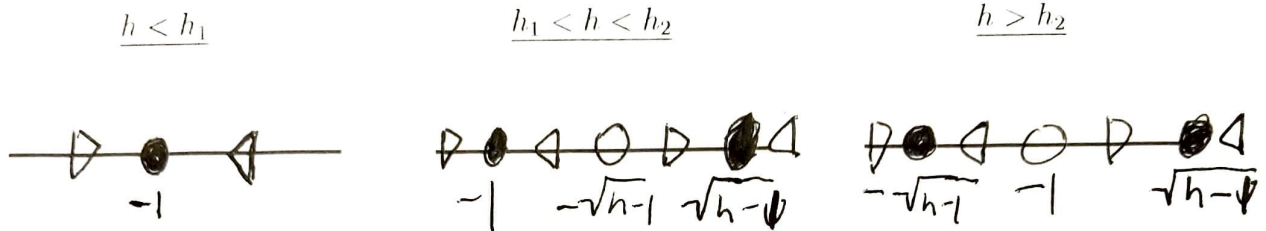
$$h_2 = \underline{2}$$

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** Question 1 Continued **

$$u' = (u + 1)(h - 1 - u^2) \quad (1)$$

(c)[9pts.] Construct the phase lines for (1) for the following cases. Use circles to mark fixed points. Below each fixed point put the corresponding value of u . Shade in circles corresponding to stable fixed points and leave hollow circles corresponding to unstable fixed points.



(d)[6pts.] On the pair of axes below, sketch the bifurcation diagram for (1) again. Add your phase lines from part (c), oriented vertically, at appropriate places in the bifurcation diagram. Then indicate which branches in the diagram are unstable by using a different linestyle for the unstable branches.

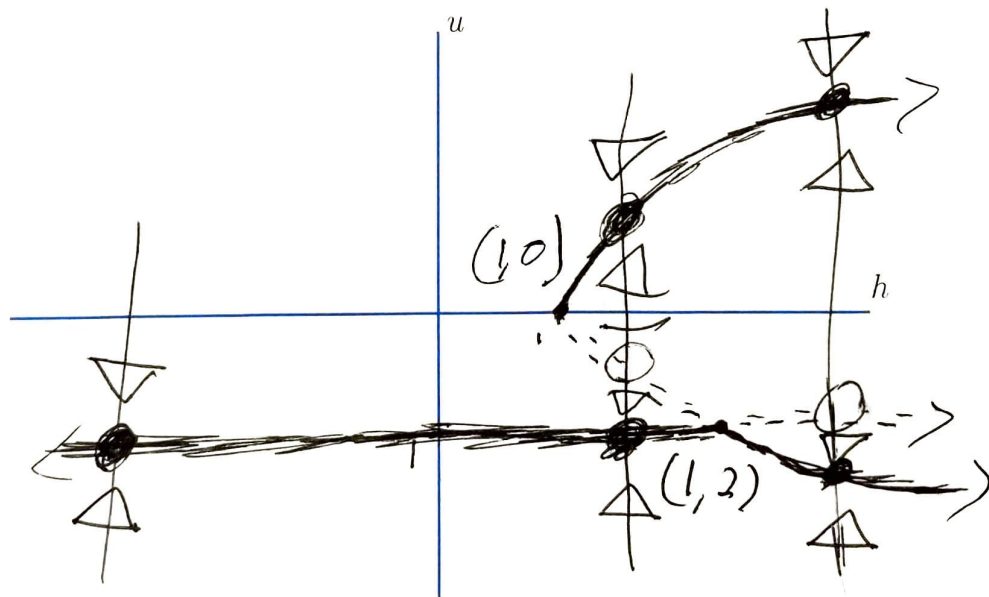


Figure 2: Final Bifurcation Diagram

(e)[6pts.] Classify each bifurcation point as one of the 3 types we discussed in class.

$h = 1$: Saddle point / Saddle node
 $h = 2$: transcritical bifurcation