4.57 Grade: 4.57 [ ] 1, 2, 3, 4, 7

4.59 4.59 [ ] 1, 2, 5

$$\int_C \frac{f(z)\,dz}{z-z_0} = 2\pi i\, f(z_0) \quad z_0 \text{ in } C$$

$$\int_C \frac{f(z)\,dz}{(z-z_0)^{n+1}} = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

1a) $\int_C \frac{e^{-z}\,dz}{z-\frac{\pi i}{2}} = 2\pi i\,(e^{-z})\Big|_{\frac{\pi i}{2}} = z_0 \text{ in } C$

2π

b) $\int_C \frac{[\cos(z)/(z^2+8)]\,dz}{z-0} = 2\pi i\left[\frac{\cos(z)}{z^2+8}\right]\Big|_{0} = z_0 \text{ in } C$

$2\pi i \cdot \frac{1}{8} = \frac{\pi i}{4}$

c) $\int_C \frac{z/2\,dz}{z+\frac{1}{2}} = 2\pi i\left(\frac{z}{2}\right)\Big|_{-\frac{1}{2}} = z_0 \text{ in } z$

$\frac{2\pi i}{-4} = -\frac{\pi i}{2}$

d) $\int_C \frac{\cosh z/z^3\,dz}{z-0} = \frac{2\pi i}{3!}\left(\frac{\cosh z}{z^3}\right)\Big|_{z_0=0}$ $\frac{d^3}{dz}$

$= \frac{2\pi i}{6}\sinh(0) = 0$

e) $\int_C \frac{\tan(z/2)\,dz}{(z-\cancel{z}_0)^2} = \frac{2\pi i}{2!}\left[\frac{d}{dz}\tan\left(\frac{z}{2}\right)\right]dz$

$= i\pi \sec^2\left(\frac{x^0}{2}\right)$

2) a) $\int_C \frac{1}{z^2+4}\,dz = \int_C \frac{1/(z+2i)\,dz}{z-2i} = 2\pi i\left(\frac{1}{z+2i}\right)\Big|_{z_0=2i}$ in C.

$\frac{2\pi i}{4i} = \frac{\pi}{2}$

b) $\int_C \frac{1}{(z^2+4)^2}\,dz = \int_C \frac{1/(z^2+4)\,dz}{z^2+4} = \int_C \frac{1/(z^2+4)(z+2i)\,dz}{z-2i}$

$= 2i\pi\left[\frac{1}{z^3+2iz^2+4z+8i}\right]\Big|_{z_0=2i}$

$= 2i\pi\left[\frac{1}{-8i+8i+8i+8i}\right] = \frac{2i\pi}{\cancel{8i}\,\cancel{24}\,32i} = \frac{\pi}{16}$

3) $\int_C \frac{2s^2-s-2}{s-z}\,ds = 2i\pi\,(2s^2-s-2)\Big|_{s_0=2}$

$2i\pi(8-2-2) = 4\cdot 2i\pi = 8\pi i$

4) $\int_C \frac{s^3+2s}{(s-z)^3} = \frac{2\pi i}{2!}\left(\frac{d^2}{dz^2}\left(s^3+2s\right)\right)\Big|_{z \text{ inside}}$

$\frac{2i\pi}{2}(6s) = 6\pi i s$

if z is outside, then $6s = 0$.

7) $\int_C \frac{e^{az}}{z}\,dz = \int_C \frac{e^{az}}{z-0}\,dz = 2\pi i$ iff 0 in C, since unit circle, 0 in C.

$\frac{d}{d\theta} e^{i\theta} = ie^{i\theta}$

$dz = ie^{i\theta}\,d\theta$

in terms of θ

$z = e^{i\theta} = \cos\theta + i\sin\theta$

$$\int_C \frac{e^{az}}{z}\,dz = \int_{-\pi}^{\pi} \frac{e^{a(\cos\theta + i\sin\theta)}}{\cancel{e^{i\theta}}} \cdot i\cancel{e^{i\theta}}\,d\theta$$

$$i\int_{-\pi}^{\pi} e^{a\cos\theta}\left[\cos(a\sin\theta) + i\sin(a\sin\theta)\right] d\theta$$

$$i\int_{-\pi}^{\pi} e^{a\cos\theta}\cos(a\sin\theta)\,d\theta - \int_{-\pi}^{\pi} e^{a\cos\theta}\sin(a\sin\theta))\,d\theta$$

$$= 2i\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta))\,d\theta = 2i\pi$$

divide by $2i$

$= \pi$

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~~if f is entire & bounded, f(z) constant~~

1) if $g(z) = e^{f(z)}$, then $g(z)$ is entire, and

$$|g(z)| = |e^{u+iv}| = |e^{u}| \leq e^{u_0}$$

by Liouville's theorem $g(z)$ is constant,

$\therefore g'(z) = 0$ $\qquad g'(z) = e^{f(z)} f'(z)$, since $e \neq 0$,

$f'(z) = 0 \therefore f(z) = c$

2) let $g(z) = \frac{1}{f(z)}$ if $f(z)$ is continuous on R,

Then g is continuous since $f(z) \neq 0$

by corollary of maximum modulus prinipal,

max of $|g(z)|$ on R is at some point $z_0 \in R$

$$|g(z)| \leq |g\ z_0| \to \frac{1}{|f\ z|} \leq \frac{1}{|f\ z_0|} \to |f(z)| \geq |f(z_0)|$$

for every $z \in R$. this shows f is a min at point $z_0 \in \partial R$

5) let $g(z) = \frac{1}{f(z)}$

$$|g(z)| = \frac{1}{|f(z)|} = \frac{1}{|u(x,y)|} \to \frac{1}{|u(x,y)|} \leq \frac{1}{u_0(x,y)}$$

$$\to |u(x,y)| \geq u_0.$$