Assignment due on 09/03/2022

Remark: This first HW is based on the Taylor ideas discussed in class and should be done on paper. Please show all your work clearly so I get a feeling and assess your preparation for this class. I will open a dropbox folder so you can submit a scan of your work.

1. Find the third-degree Taylor polynomial for $f(x) = (x+1)^{-1}$, around $x_0 = 0$.

2. Find the smallest degree Taylor polynomial that is approximating the function e^x with an absolute error less than or equal to 10^{-6}. Explain and show all steps.

3. Given that

$$R(x) = \frac{|x|^4}{4!}\left(\frac{-1}{1+\xi}\right)$$

for $x \in [-0.5, 0.5]$, where ξ is between x and 0, find an upper bound for $R(x)$, valid for all $x \in [-0.5, 0.5]$, that is independent of x and ξ.

4. Let $p(x)$ be an arbitrary polynomial of degree less than or equal to n. What is its Taylor polynomial of degree n, about an arbitrary x_0? Test for $n = 2, 3$.

5. The Fresnel integrals are defined as

$$C(x) = \int_0^x \cos(\pi t^2/2)\ dt$$

and

$$S(x) = \int_0^x \sin(\pi t^2/2)\ dt$$

Use Taylor expansions appropriately to find approximations to $C(x)$ and $S(x)$ that are 10^{-4} accurate for all x with $|x| < 0.5$. Hint: Substitute $s = \pi t^2/2$ into the Taylor expansions of $\cos(s)$ and $\sin(s)$ and afterwards apply the integral on each term of the expansion.

6. Use the Integral Mean Value Theorem to demonstrate that the "pointwise" form

$$\frac{f^{n+1}(\xi_x)}{(n+1)!}(x-x_0)^{n+1}$$

of the Taylor remainder (usually called the Lagrange form) follows from the "integral" form

$$\int_{x_0}^x \frac{f^{n+1}(t)}{n!}(x-t)^n\ dt$$

(usually called the Cauchy form).