Josh Brown

1. Suppose that $y = y_h + O(\beta(h))$ & $z = z_h + O(\beta(h))$ for h ~~large~~ small. does it follow that

$y - z = y_h - z_h$ for h small?

~~let~~ substitute y & z yeilds

$y_h + O(\beta(h)) - z_h - O(\beta(h))$

if h is sufficiently small, it follows that $O(\beta(h)) \to 0$, and likewise $O(\beta(h)) - O(\beta(h)) = 0$.

$y_h - z_h = y_h - z_h = y - z$ ✓

~~2) Show that~~

$$\cancel{f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h)^2}$$

$\cancel{f''(x) =}$

$$\cancel{f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + O(h)^2}$$

$$\cancel{f(x-h) = f(x) - hf'(x) + O(h)^2}$$

$$\cancel{f(x+h) + f(x-h) = f(x) + hf'(x) + f(x) - hf'(x) + O}$$

$$\cancel{= 2f(x) + O(h)^2}$$

2). Show that

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h)^2$$

$$f(x+h) + f(x-h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + O(h)^2$$
$$+ f(x) - hf'(x) + \frac{h^2}{2}f''(x)$$

$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + O(h)^2$$

$$\Rightarrow \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x) + O(h)^2$$

$O(h)^2$ is remainder, which is absolute, $\therefore$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h)^2$$

3.) write the following in polynomial nested form

$$5x^6 + x^5 + 3x^4 + 3x^3 + x^2 + 1$$

$$1 + x^2(1 + x(3 + x(3 + x(1 + 5x)$$

4) write the polynomial in nested form

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$$

$$1 + x^2\left(-\frac{1}{2} + \frac{x^2}{24}\right)$$

6) Write in nested form

$$p(x) = 1 + (x-1) + 4(x-1)(x-2) + 12(x-1)(x-2)(x-4)$$

$$= 4(x^2 - 2x - x + 2) + 12(x^2 - 2x - 6x + 12)(x-4)$$

$$= 4x^2 - 8x - 4x + 8 + 12(x^3 - 2x^2 - 6x^2 + 12x$$
$$- 4x^2 + 8x + 24x - 48)$$

$$= 4x^2 - 12x + 8 + 12(x^3 - 12x^2 + 44x - 48)$$

$$= 4x^2 - 12x + 8 + 12(x^3 - 12x^2 + 44x - 48)$$

$$= 4x^2 - 12x + 8 + 12x^3 - 144x^2 + 528x - 576$$

$$= 12x^3 - 140x^2 + 516x - 568$$

$$-568 + x(516 + x(-140 + 12x))$$

7.) let
$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

Describe an algorithm/method that you can apply Horners algorithm to calculate efficiently the taylor expansion of the polynomial $p(x)$ around any point $x_0 \neq 0$.

let $p(x) = \sum_{k=0}^{n} c_k (x - x_0)^k$, $c_k = \frac{p^k(x_0)}{k!}$

$$p(x_0) = \sum_{k=0}^{n} c_k (x_0 - x_0)^k$$

$$p(x_0) = c_0 (0)^0 + 0 \dots$$

$$p(x_0) = c_0 = \frac{p(x_0)}{1} = p(x_0)$$

~~p(x_0)~~

$$p(x) \approx p(x_0) + p'(x_0)(x - x_0)$$

$$p(x) - p(x_0) = p'(x_0)(x - x_0)$$

$$p'(x_0) = \frac{p x - p(x_0)}{x - x_0} = q(x_0) \text{ yeids } p'(x_0).$$