# Non-Gaussian correlations in the steady-state of driven-dissipative clouds of two-level atoms

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We report experimental measurements of the second-order coherence function $g^{(2)}(\tau)$ of the light emitted by a laser-driven dense ensemble of $^{87}$Rb atoms. We observe a clear departure from the Siegert relation valid for Gaussian chaotic light. Measuring intensity and first-order coherence, we conclude that the violation is not due to the emergence of a coherent field. This indicates that the light obeys non-Gaussian statistics, stemming from non-Gaussian correlations in the atomic medium. More specifically, the steady-state of this driven-dissipative many-body system sustains high-order correlations in the absence of first-order coherence. These findings call for new theoretical and experimental explorations to uncover their origin and they open new perspectives for the realization of non-Gaussian states of light.



The properties of the light emitted by an ensemble of atoms become collective when they are placed inside a volume with a size smaller than their transition wavelength, or when they share a common electromagnetic mode, e.g. inside an optical cavity or along a waveguide. For example, superradiance is a consequence of a collective coupling to a common mode [1, 2]. This collective coupling of the emitters may induce a modification of the statistics of the emitted light and quantum correlations of the emitters' internal degrees of freedom. Relating the statistical properties of the light to the correlations inside the atomic ensemble remains, in the general case, challenging [3, 4]. In this context an outstanding goal is to stabilize non-trivial correlations in the steady-state of a driven-dissipative many-body system [5–8].

In a recent experiment [9], we observed a modification of the photon emission rate in a mode propagating along the long axis of a cigar-shaped cloud of two-level atoms strongly driven by a resonant laser. This enhancement of intensity observed *during the early dynamics* was due to the spontaneous establishment of interatomic correlations, not imparted by the driving laser but rather resulting from superradiance. The question then arises as to whether the *steady state* also features atomic correlations. Information on them may be provided by measuring intensity correlations [10–13]. In particular, a test for the statistical independence of a large number of emitters is the so-called Siegert relation [14–16]. It relates the second order coherence (intensity correlations) of $N \gg 1$ emitters $g_N^{(2)}(\tau) = \langle \hat{E}^-(t)\hat{E}^-(t+\tau)\hat{E}^+(t+\tau)\hat{E}^+(t)\rangle / \langle \hat{I}(t)\rangle^2$ to the first order coherence (field correlations) $g_N^{(1)}(\tau) = \langle \hat{E}^-(t)\hat{E}^+(t+\tau)\rangle / \langle \hat{I}(t)\rangle$, where $\hat{I}(t) = \hat{E}^-(t)\hat{E}^+(t)$ is the intensity and $\hat{E}^-$ is the field radiated by the ensemble. This relation reads: $g_N^{(2)}(\tau) = 1 + |g_N^{(1)}(\tau)|^2$. Its validity has been tested on different platforms with statistically independent atoms generating chaotic light [15], including few atoms in cavity [17], ions [18] or dilute atomic clouds [16, 19–21]. Its violation is a marker of a correlated medium [22–24].

Here, we measure the second order coherence $g_N^{(2)}(\tau)$ of the light emitted by cigar-shaped atomic clouds laser driven *perpendicularly* to their long axis. We observe a violation of the Siegert relation in steady state, revealing the presence of correlations between atoms. In particular, the violation always appears as a reduction of $g_N^{(2)}(\tau)$ for photons emitted in the mode in which the system features superradiance. Ab-initio numerical calculations for our regime of thousands of emitters are out of reach. However, the Siegert relation can be discussed without knowledge of the microscopic dynamics: It assumes that the connected correlations (or cumulants as defined in [25]) of order higher than 2 cancel, *i.e.* that the field obeys *Gaussian statistics*, and that the radiated field has *zero mean* ($\langle \hat{E}^- \rangle = 0$) [22]. Its experimental violation indicates a failure of one of these two hypotheses. We provide experimental evidence that, in our system, the average field cancels implying that the field is non-Gaussian, and emerges from a non-Gaussian steady-state of the driven atomic medium.

Our experimental platform, detailed in [26], is sketched in figure 1(a). It relies on 4 high-numerical-aperture aspheric lenses. We load up to $\simeq 5000$ $^{87}$Rb atoms in a 3.4 µm-waist optical dipole trap making use of gray molasses. The atomic cloud has a typical temperature of 200 µK, with a (calculated) radial size $\ell_{\rm rad} \simeq 0.6\lambda$ ($1/e^2$ radius), and a measured axial size $\ell_{\rm ax} \simeq 20-25\lambda$, where $\lambda = 2\pi/k = 780.2$ nm is the wavelength of the D2 transition. To isolate two internal states and produce a cloud of two-levels atoms, we perform the experiment in the presence of a magnetic field $B = 96$ G oriented perpendicularly to the atomic cloud. We prepare the atoms in the state $|g\rangle = |5S_{1/2}, F = 2, m_F = 2\rangle$ by hyperfine and Zeeman optical pumping. The cloud is then excited to $|e\rangle = |5P_{3/2}, F = 3, m_F = 3\rangle$ ($D_2$ transition, $\Gamma_0/2\pi = 6$ MHz and $I_{\rm sat} \simeq 1.67\,\mathrm{mW/cm^2}$) using a $\sigma^+$-polarized laser on resonance, propagating along $B$, *i.e.* perpendicularly to the cloud axis. The excitation beam is much larger than the cloud size, so that all atoms experience the same Rabi frequency $\Omega$. We collect the light emitted by the cloud in two different directions: the first one is aligned along the main axis of the cloud ($\hat{u}_z$), the second one is perpendicular to it ($\hat{u}_\perp$) [9, 27].

To measure $g_N^{(2)}(\tau)$, we implement a Hanbury-Brown and Twiss setup: the collected fluorescence is coupled into an optical fiber and then split by a fiber-based 50/50 beam-splitter, whose two outputs are connected to fiber-coupled avalanche photodiodes (APDs) operating in single-photon counting mode. We record the photon arrival times in each arm of the beam-splitter using a time-to-digital converter.

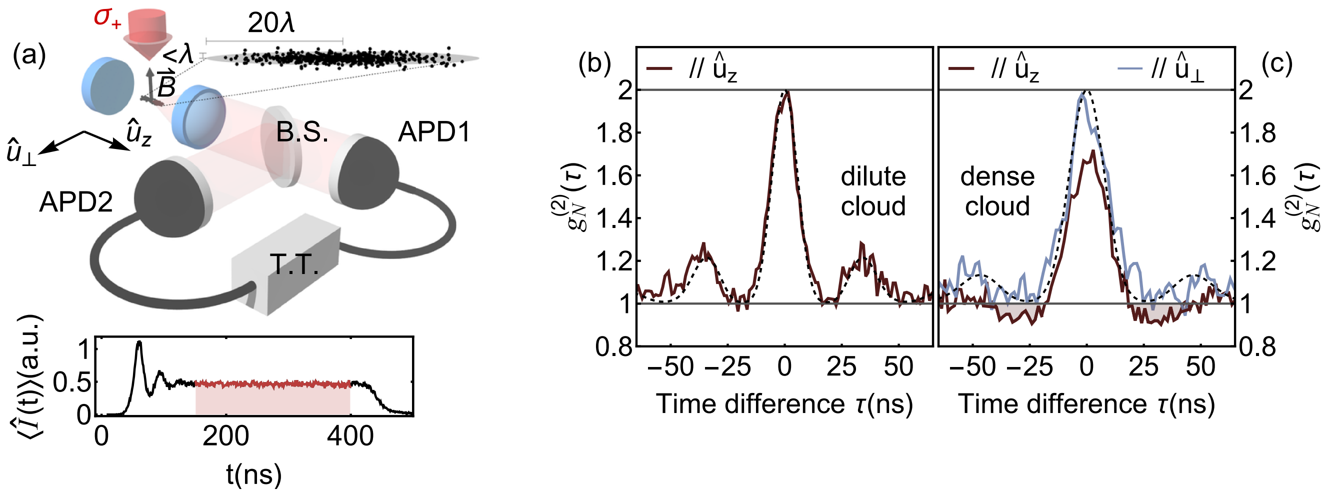


FIG. 1. **Experimental setup and measurements of $g_N^{(2)}(\tau)$:** (a) Schematic representation of the experimental setup. A cigar shaped cloud of $^{87}$Rb atoms is excited by a resonant laser beam propagating along the quantization axis defined by an external magnetic field $B$, perpendicular to the long axis of the cloud. The light emitted by the cloud is collected either along its axis ($\hat{u}_z$) or in a perpendicular direction ($\hat{u}_\perp$). Two avalanche photodiodes (APD1,2) are arranged in a Hanbury-Brown and Twiss configuration. A time-to-digital converter (T.T.) records the photon arrivals times. Inset: example of intensity $\langle \hat{I}(t) \rangle$ emitted along $\hat{u}_z$. Light red: time interval used to calculate $g_N^{(2)}(\tau)$ in steady-state. (b) Measurement of $g_N^{(2)}(\tau)$ along $\hat{u}_z$ for a dilute cloud, compared to the Siegert relation (black dashed line) calculated assuming $g_N^{(1)}(\tau) = g_1^{(1)}(\tau)$ (see text). (c) Measurements of $g_N^{(2)}(\tau)$ in the dense regime. Dark red: light collected along $\hat{u}_z$. The Siegert relation is violated in this direction. Light blue: collection along $\hat{u}_\perp$.

From these, we compute $g_N^{(2)}(t_1,t_2) = n_c(t_1,t_2)/n_1(t_1)n_2(t_2)$ where $n_i(t_i)$ is the photon number detected in arm $i$ at time $t_i$ and $n_c(t_1,t_2)$ the number of coincidences on both arms at times $t_1$ and $t_2$ (see details in [28]). The time bin is 1 ns.

To calibrate our experiment, we first study a case where the Siegert relation is expected to hold, that is a cloud of independent atoms. To reach this regime, we release the cloud from the trap and let it expand in free flight. The radial size increases by a factor $> 10$, up to $\sim 5\,\mu$m. The atoms are excited by a $10\,\mu$s-long pulse of resonant light. This long duration is necessary to detect a large number of correlations. In these conditions, the intensity correlation $g_N^{(2)}(\tau)$ measured along the main axis of the cloud (see figure 1b) is in excellent agreement with the prediction of the Siegert relation without any free parameter: it features oscillations at the Rabi frequency of the laser, as expected [15]. This confirms that the correlations between atoms are negligible. This good agreement also serves as a quantitative calibration of our detection scheme. Indeed, several effects could reduce the value of $g_N^{(2)}(0)$ below 2: First, a too low time-resolution would lead to a reduction of $g_N^{(2)}(0)$ [29]. The resolution time of the detectors is 350 ps, much shorter than atomic dynamics timescales ($\geqslant 5$ ns). Second, collecting multiple spatial modes over a solid angle larger than a coherence area would also reduce $g_N^{(2)}(0)$ [19, 30]. Here we collect the fluorescence light with an aspheric lens and project it on a single-mode optical fiber. We thus do not expect these two systematics to play a role. The fact that we measure a nearly perfect contrast in this dilute case ($g_N^{(2)}(0) = 1.98 \pm 0.03$) demonstrates that this is the case and that no systematic effects could reduce the value of $g_N^{(2)}(\tau)$.

To study the dense regime, we prepare the clouds as presented above, then switch off the trap and immediately shine a 400 ns-long pulse of resonant laser light. We then recapture the atoms in the optical tweezer and repeat this sequence up to 20 times to accumulate statistics, checking that the atom number is reduced by less than 10%. During the laser pulse, the thermal expansion is negligible, and we thus assume that the atomic distribution remains identical to the trapped one. To obtain the steady-state correlation function $g_N^{(2)}(\tau)$, we restrict the times $t$ and $t' = t + \tau$ to a time window of 250 ns when the atomic system has reached steady state, as highlighted in the inset of figure 1(a) (more details in [28]).

Strikingly, as shown in figure 1(c), we observe in this dense regime a violation of the Siegert relation (dotted line) *along* the cloud axis. This is the direction along which the emission is collective and leads to superradiance during the early dynamics [9]. In this axial direction, we measure a reduction of $g_N^{(2)}(\tau)$ with respect to the dilute case for all delays $\tau$. We even obtain $g_N^{(2)}(\tau) < 1$ at around half a period of the oscillation (colored area in figure 1c). Furthermore, we observe that the photon statistics depends on the direction of detection: The statistics of photons emitted perpendicularly to the cloud axis (where emission is not collective and we do not observe superradiance [9]) is well described by the Siegert relation. To plot this relation, we assume $g_N^{(1)}(\tau) = g_1^{(1)}(\tau)$. We have experimentally verified this assumption by measuring the first-order correlation function $g_N^{(1)}(\tau)$ in the dense regime, following the method used in [31, 32] (see details in [28]. As shown in figure 2, we find it to be in agreement with the single atom expectation.

Let us discuss the implication of the observed violation of the Siegert relation along the cloud axis and how it can reveal non-Gaussian statistics of the emitted light. If one first assumes Gaussian light statistics, all connected correlations of more than two operators cancel and the correlation of four operators then reads [25]: $\langle\hat{A}\hat{B}\hat{C}\hat{D}\rangle = \langle\hat{A}\hat{B}\rangle\langle\hat{C}\hat{D}\rangle + \langle\hat{A}\hat{C}\rangle\langle\hat{B}\hat{D}\rangle + \langle\hat{A}\hat{D}\rangle\langle\hat{B}\hat{C}\rangle - 2\langle\hat{A}\rangle\langle\hat{B}\rangle\langle\hat{C}\rangle\langle\hat{D}\rangle$. Applying this to $\hat{A} = \hat{E}^-(t) = \hat{D}^\dagger$, $\hat{B} = \hat{E}^-(t+\tau) = \hat{C}^\dagger$ [33] yields

$$g_N^{(2)}(\tau) = 1 + |g_N^{(1)}(\tau)|^2 - \frac{2|\langle\hat{E}^-\rangle|^4}{\langle\hat{I}\rangle^2} + \frac{|\langle\hat{E}^-(t)\hat{E}^-(t+\tau)\rangle|^2}{\langle\hat{I}\rangle^2}, \tag{1}$$

with $t$ taken in steady state and $\langle\hat{E}^-\rangle$ the average electric field radiated by the cloud in steady state. The last term oscillates fast and is in general neglected [22, 23]. Thus the observed violation of the Siegert relation, $g_N^{(2)}(\tau) \leqslant 1 + |g_N^{(1)}(\tau)|^2$ for all delays $\tau$ can only be explained in two ways. Either the field does not obey Gaussian statistics so that Eq. (1) does not apply, or the average field $\langle\hat{E}^-\rangle$ is non-zero in steady state.

The existence of such an average field in steady state would be non-trivial as it is not externally imposed by the driving laser. This laser imprints a phase factor $e^{-i\boldsymbol{k}_{\rm las}\cdot\boldsymbol{r}_n}$ on atom $n$. The field emitted by the cloud in the direction $\hat{u}_z$ is $\hat{E}^- = \sum_{n=1}^N \hat{\sigma}_n^+ e^{ik\hat{u}_z\cdot\boldsymbol{r}_n}$. Since $\boldsymbol{k}_{\rm las} \perp \hat{u}_z$, the laser does not directly excite atomic dipoles whose radiations constructively interfere along $\hat{u}_z$. A non-zero average field would then result from a many-body dynamics creating a coherence along $\hat{u}_z$. Coherence has been observed in dilute clouds, during the late decay following the extinction of the laser excitation [34]. In order to assess if a coherent field is emitted in steady-state for our strongly driven clouds, we perform two experimental tests.

We obtain the first compelling evidence that $\langle\hat{E}^-\rangle \approx 0$ by measuring the intensity emitted by the cloud along $\hat{u}_z$. Figure 2(a) shows the steady-state intensity $\langle\hat{I}\rangle$ measured along $\hat{u}_z$ as a function of the atom number $N$. Since the field is the sum of the radiation of the individual dipoles, a non-zero average field $\langle\hat{E}^-\rangle$ should be proportional to $N$, leading to an intensity $\langle\hat{I}\rangle = |\langle\hat{E}^-\rangle|^2 \propto N^2$. We however observe in figure 2(a) a clear linear scaling, indicating that the field radiated by the cloud has a negligible average value. From the residuals of a linear fit to the data, we obtain $|\langle\hat{E}^-\rangle|^2/\langle\hat{I}\rangle \leqslant 0.17$ so that the third term in (1) could cause a reduction of $g_N^{(2)}(\tau)$ of at most 0.06. This is much smaller than the reduction of $\simeq 0.3$ that we observe. The second evidence comes from the observation of the decay of the first order coherence to zero at long times, as shown in figure 2(b): $g_N^{(1)}(\tau) \to 0$ for $\tau \gg 1/\Gamma$. In the long-time limit, we expect $\langle\hat{E}^-(t)\hat{E}^+(t+\tau)\rangle = \langle\hat{E}^-(t)\rangle\langle\hat{E}^+(t+\tau)\rangle$ and hence $g_N^{(1)}(\tau) \to |\langle\hat{E}^-\rangle|^2/\langle\hat{I}\rangle$. The data in figure 2(b) again sets a bound of about $|\langle\hat{E}^-\rangle|^2/\langle\hat{I}\rangle \leqslant 0.2$. As a consequence our measurement of first-order coherence further demonstrates the fact that no average coherent field emerges. We thus have strong experimental evidence that our observation of $g_N^{(2)}(\tau) < 1 + |g_N^{(1)}(\tau)|^2$ reveals a non-Gaussian statistics of the light emitted by the cloud along its main axis.

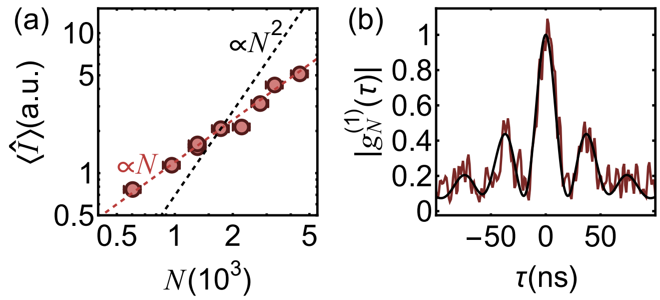


FIG. 2. **Evidence for a negligible average field.** (a) Measured $g_N^{(2)}(0)$ in steady state, as a function of the atom number $N$, measured along $\hat{u}_z$. Error bars are standard error on the mean. Dashed lines: linear and a quadratic power law. (b) Measured —$g_N^{(1)}(\tau)$— with $\Omega \simeq 4.5\Gamma$ (red) and expectation for a single atom (black).

To quantify the departure from Gaussian statistics we measure high-order connected correlations [35–38], which would cancel in the Gaussian case. The measured $g_N^{(2)}(\tau)$ can be related to the normalized two-times connected correlation $\boldsymbol{C}(\tau) = \langle\hat{E}^-(t)\hat{E}^-(t+\tau)\hat{E}^+(t+\tau)\hat{E}^-(t)\rangle_{\rm c}/\langle\hat{I}\rangle^2$, using the equation (derived in [28]):

$$g_N^{(2)}(\tau) = g_{\rm Gauss}^{(2)}(\tau) + \boldsymbol{C}(\tau), \tag{2}$$

where $g_{\rm Gauss}^{(2)}(\tau) = 1 + |g_N^{(1)}(\tau)|^2 + |\langle\hat{E}^-(t)\hat{E}^-(t+\tau)\rangle|^2/\langle\hat{I}\rangle^2$. This expression assumes $\langle E^-\rangle = 0$ as justified above. These connected correlations quantify the lack or excess of photon pairs separated by $\tau$ with respect to the case of a Gaussian light. From Eq. (2), one can indeed show that $\boldsymbol{C}(\tau) = g_{\rm Gauss}^{(2)}(\tau)(f(\tau) - 1)$ where $f(\tau) = n_{\rm c}(\tau)/n_{\rm c\,Gauss}(\tau)$ is the fraction of detected photon pairs separated by $\tau$ with respect to what would have been detected for Gaussian light (with the same average intensity $\langle\hat{I}\rangle$). Since the third term of $g_{\rm Gauss}^{(2)}(\tau)$ is always positive, we get the following lower bound: $|\boldsymbol{C}(\tau)| \geqslant 1 + |g_N^{(1)}(\tau)|^2 - g_N^{(2)}(\tau)$. This quantity can be directly extracted from the data. We show in figure 3(a) the intensity correlation $g_N^{(2)}(\tau)$ as a function of the atom number $N$, for $I_{\rm las} \simeq 50 I_{\rm sat}$ ($\Omega/\Gamma_0 \simeq 5$). Figure 3(b,c) reports the values of $g_N^{(2)}(0)$, and the corresponding connected correlation $\boldsymbol{C}(0)$, as a function of $N$. We do find that at low $N$ the data converge towards the prediction of the Siegert relation. For increasing $N$ the disagreement grows. We also found (not shown) that $g_N^{(2)}(0)$ [$\boldsymbol{C}(0)$] did not vary when changing the drive between $\Omega = 2\Gamma$ and $\Omega = 10\Gamma$. These findings confirm the collective origin of the observed correlations. The inset of figure 3(c) shows how the connected correlations decay in time. We observe a maximum of correlation and a non-monotonic decay towards zero. The correlations observed in figure 3 with

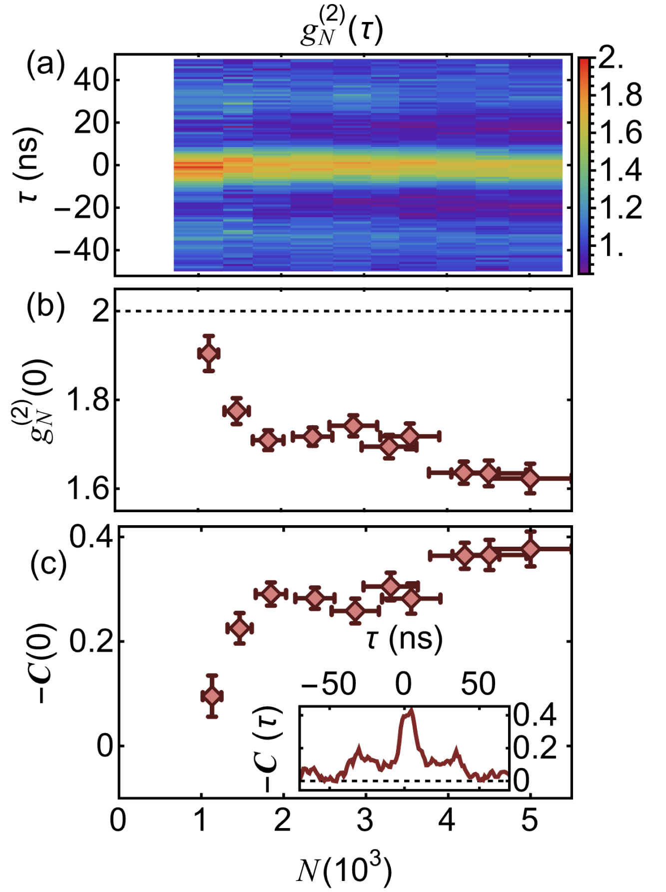


FIG. 3. **Influence of atom number $N$ on the violation of the Siegert relation** (a) Color map of $g_N^{(2)}(\tau)$ as a function of $\tau$ and $N$. (b) Measured values of $g_N^{(2)}(0)$ obtained by averaging $g_N^{(2)}(\tau)$ in the interval $-2\,\mathrm{ns} \leqslant \tau \leqslant 2\,\mathrm{ns}$. (c) Connected correlation $\boldsymbol{C}(0)$ as defined in the main text. Inset: example of $\boldsymbol{C}(\tau)$. The error bars in (b) and (c) are standard errors on the mean.

$\boldsymbol{C}(\tau) \neq 0$ indicate that second-order coherence emerges. The fact that second-order coherence is built in the absence of first-order coherence is a signature of non-Gaussian statistics. A theoretical understanding of the measured $\boldsymbol{C}(\tau)$ is beyond the scope of the present work. It requires a description of the atomic correlations emerging in the cloud.

In this perspective, we relate the statistics of the light field to the one of the atomic state. In ref. [9], we already observed the appearance of beyond-mean-field correlations between the atomic dipoles, *i.e.* $\langle \hat{\sigma}_i^+ \hat{\sigma}_j^- \rangle_{\rm c} \neq 0$, during the early dynamics following the application of the excitation laser. To do so, we measured the intensity emitted along $\hat{u}_z$, whose expression in terms of atomic dipoles is $\langle \hat{I} \rangle = \sum_{ij} \langle \hat{\sigma}_i^+ \hat{\sigma}_j^- \rangle\, e^{-ik\hat{u}_z\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)}$. These correlations resulted from the superradiant emission along the axial direction of the cloud. Contrarily, the measurements of second order coherence presented here probe higher order correlations *in steady state*. In terms of atomic dipoles, the connected correlations of the field measured above read

$$\langle \hat{E}^- \hat{E}^{-\prime} \hat{E}^{+\prime} \hat{E}^+ \rangle_{\rm c} = \sum_{ijkl} \langle \hat{\sigma}_i^+ \hat{\sigma}_j^{+\prime} \hat{\sigma}_k^{-\prime} \hat{\sigma}_l^- \rangle_{\rm c}\, e^{-ik\hat{u}_z\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j + \boldsymbol{r}_k - \boldsymbol{r}_l)}. \tag{3}$$

Here, an operator $\hat{\mathscr{O}}$ is taken at time $t$ in steady state and $\hat{\mathscr{O}}'$ at time $t' = t + \tau$. Hence, the observation of non-zero connected correlations in the field implies that $\langle \hat{\sigma}_i^+ \hat{\sigma}_j^{+\prime} \hat{\sigma}_k^{-\prime} \hat{\sigma}_l^- \rangle_{\rm c} \neq 0$, *i.e.* the atomic medium features high-order correlations that obey non-Gaussian statistics. These high-order correlations are not externally imposed and emerge in steady-state as a result of the competition between driving and collective dissipation. This shows that in our free-space system, despite the absence of spatial order, collective dissipation can stabilize non-trivial correlations.

In conclusion, we have investigated the photon statistics of the light emitted in steady state by a dense superradiant cloud of atoms under strong driving, observing a violation of the Siegert relation. Our data support the fact that this violation is not due to the appearance of a coherent field. They rather indicate that a non-Gaussian field emerges in the steady state of this driven-dissipative system, which originates from non-Gaussian correlations between atoms in the cloud. The appearance of stable non-Gaussian correlations in steady state under strong driving is an unexpected and fascinating observation. Our findings thus call for theoretical investigations to identify the mechanisms at play in this dissipative quantum many-body system, and to elucidate their relationship with superradiance. More generally, this should motivate investigations to determine whether the correlations we observed could be used as a resource to prepare non-trivial states of the field [39–41]. Experimentally, we plan to measure the quadratures of the radiated field to extract its Wigner function and determine if the non-Gaussian character we observed is accompanied by Wigner negativity. Another outlook would be to extend our investigation beyond the steady state, studying for instance the photon statistics during a superradiant burst [42–44].

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# Supplemental Material

### Data analysis and reconstruction of the correlation function $g_N^{(2)}(\tau)$

In this section, we give more details about the measurement of the second-order coherence $g_N^{(2)}(\tau)$. This correlation function is related to measurable quantities by the following expression:

$$g_N^{(2)}(t_1,t_2) = \frac{n_c(t_1,t_2)}{n_1(t_1)n_2(t_2)} \tag{S1}$$

where $n_i(t_i)$ is the photon arrival rate on detector $i$ *at time* $t_i$ and $n_c(t_1,t_2)$ is the rate of two-photon coincidence at times $t=t_1$ and $t=t_2$ in detectors 1 and 2. As explained in the main text, for every excitation pulse, we record all the photons arrival times $t_1$ ($t_2$) on detector 1 (detector 2) with respect to a common trigger. We thus measure the total number of photons $N_i(t_i)$ detected by detector $i$ at time $t_i$. In the same way, we measure the total number of coincidences $N_c(t_1,t_2)$. The rates $n_i$ are related to $N_i$:

$$\begin{aligned} N_1(t_1) &= \varepsilon_1 N_S n_1(t_1) \\ N_2(t_2) &= \varepsilon_2 N_S n_2(t_2) \\ N_c(t_1,t_2) &= \varepsilon_1 \varepsilon_2 N_S n_c(t_1,t_2) \end{aligned} \tag{S2}$$

where $\varepsilon_i$ is the detection efficiency of detector $i$ and $N_S$ is the number of times the experiment is repeated. The two-times correlation function is then obtained as:

$$g_N^{(2)}(t_1,t_2) = N_S \frac{N_c(t_1,t_2)}{N_1(t_1)N_2(t_2)}. \tag{S3}$$

An example of matrix $g_N^{(2)}(t_1,t_2)$ is shown in figure S1, together with the respective time-dependent photon counts recorded by the two APDs. The measurements reported in the main text focus on the steady state, where $g_N^{(2)}(t_1,t_2) = g_N^{(2)}(|t_1-t_2|) = g_N^{(2)}(\tau)$. This function is evaluated selecting the data where the atomic system has reached the steady state: we thus average over the last $\simeq 250\,$ns of the excitation pulse and select $|\tau| \leqslant 50\,$ns, a range where the number of correlations is sufficiently large. An illustrative example is reported in Fig S1.

### Heterodyne measurement of the fluorescence spectrum

To further characterize the light field emitted by the cloud in steady-state, we measure the field correlation function $g_N^{(1)}(\tau)$, and show that it is compatible with $g_1^{(1)}(\tau)$ also in the dense regime. To do so, we use a heterodyne detection scheme [31, 32]. In short, a local oscillator (LO) is derived from the laser light used to excite the atoms. It is then shifted by $\omega_{\mathrm{LO}}/(2\pi) = 110\,$MHz, and coupled into a fiber. We measure the intensity correlation $g_{N,HD}^{(2)}(\tau)$ obtained by combining the LO field and the one scattered by the atoms $E^-$ into a fiber-based beamsplitter. This quantity is related to the first and second order correlations of the light field by [31]:

$$g_{\mathrm{HD}}^{(2)}(\tau) = 1 + \alpha\left(g_N^{(2)}(\tau) - 1\right) - \beta\cos(\omega_{\mathrm{LO}}\tau) g_N^{(1)}(\tau) \tag{S4}$$

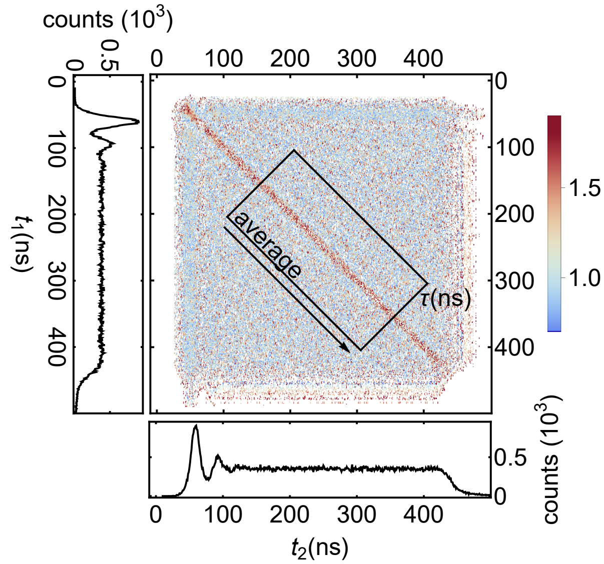


FIG. S1. **Example of the two-time correlation function** $g_N^{(2)}(t_1,t_2)$ detected in the experiment, together with the two corresponding time-dependent photon counts detected by each APD. The black rectangle represents the region where the steady state $g_N^{(2)}(\tau)$ is extracted with $|\tau| \leqslant 50\,\mathrm{ns}$.

Here, we have assumed the probe to be resonant to the atomic transition and thus $\mathrm{Im}[g_N^{(1)}(\tau)] = 0$. The parameters $\alpha$ and $\beta$ In Eq. S4 depend on the intensity of the emitted field ($I_{\mathrm{SC}}$) and of the local oscillator ($I_{\mathrm{LO}}$):

$$\alpha = \frac{\langle I_{\mathrm{SC}}\rangle^2}{(\langle I_{\mathrm{SC}}\rangle + \langle I_{\mathrm{LO}}\rangle)^2}\,, \qquad \beta = 2\frac{\langle I_{\mathrm{SC}}\rangle\langle I_{\mathrm{LO}}\rangle}{(\langle I_{\mathrm{SC}}\rangle + \langle I_{\mathrm{LO}}\rangle)^2}\,. \tag{S5}$$

A typical signal of $g_{\mathrm{HD}}^{(2)}(\tau)$ is shown in figure S2(a), by demodulation of the frequency component at $\omega_{\mathrm{LO}}$ we can obtain $g_N^{(1)}(\tau)$ as shown in the main text. Furthermore the spectrum of the light emitted by the cloud is the Fourier transform of $g_N^{(1)}(\tau)$. We thus extracted it by Fourier transforming $g_{\mathrm{HD}}^{(2)}(\tau)$ and considering the frequency components centered around $\omega_{\mathrm{LO}}$.

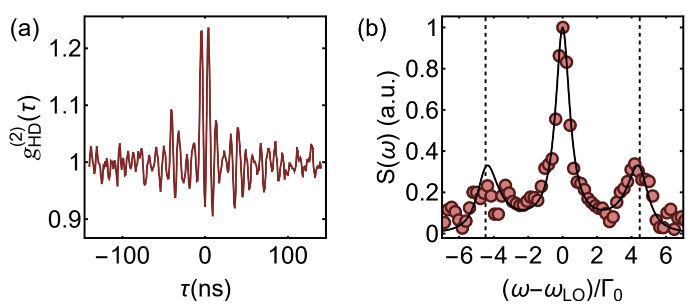


FIG. S2. (a) Intensity correlation function measured by the heterodyning technique used to extract $g_N^{(1)}(\tau)$ shown in the main text. Here $N \simeq 3000$ atoms and $\Omega \simeq 4.5\Gamma_0$. (b) Spectrum obtained by Fourier transforming the data in (a) and selecting the component around $\omega_{\mathrm{LO}}$. Black line: theoretical prediction from the calculation of the resonant fluorescence spectrum of a single atom for the same $\Omega$. Both the experimental data and the theoretical curve are normalized by setting $S(0) = 1$. The dashed lines indicate $\Omega$.

Figure S2 reports a typical example of spectrum $S(\omega)$. The experimental findings exhibit the Mollow triplet, and are always well described by the expression of the resonant fluoresence spectrum of a single atom [15]. Varying the atom number up to $N = 5000$ and scanning the driving strength up to tens of $I_{\mathrm{Sat}}$ we have always observed agreement with the single-atom expectations. This shows that in our experiment $g_N^{(1)}(\tau)$ is *not* modified with respect to single atom case, contrarily to $g_N^{(2)}(\tau)$,

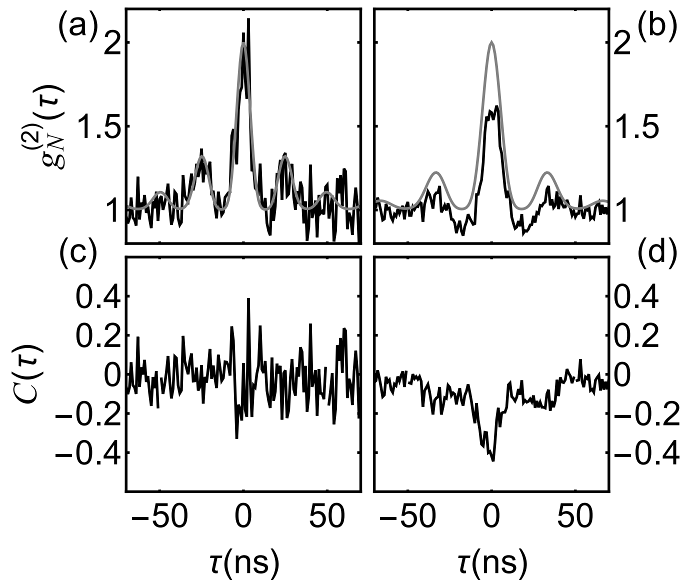


FIG. S3. **Measurement of the connected correlations.** (a,b) Black lines: examples of $g_N^{(2)}(\tau)$ measured for $N \simeq 500$ and $\Omega \simeq 8\Gamma_0$ (a) and $N \simeq 5000$ and $\Omega \simeq 5\Gamma_0$ (b). Gray lines: Siegert relation for the same parameters. (c,d) $\boldsymbol{C}(\tau)$ for the experimental parameters of (a, b) respectively.

as predicted for superradiant ensembles [47]. In any case, as $|g_N^{(1)}(\tau)|^2 > 0$ and $g_N^{(1)}(0) = 1$ by definition, if the Siegert relation applies a modification of $g_N^{(1)}(\tau)$ cannot account for the reduction of $g_N^{(2)}(\tau)$ reported in the main text.

### Connected correlations

Here, following the main text, we assume $\langle \hat{E}^- \rangle = 0$. The connected correlation of order 4 is defined by [25]:

$$\begin{aligned} \langle \hat{A}\hat{B}\hat{C}\hat{D}\rangle_{\mathrm{c}} =& \langle \hat{A}\hat{B}\hat{C}\hat{D}\rangle - \langle \hat{A}\hat{B}\rangle\langle \hat{C}\hat{D}\rangle - \langle \hat{A}\hat{C}\rangle\langle \hat{B}\hat{D}\rangle - \langle \hat{A}\hat{D}\rangle\langle \hat{B}\hat{C}\rangle + 2\langle \hat{A}\rangle\langle \hat{B}\rangle\langle \hat{C}\rangle\langle \hat{D}\rangle \\ & - \langle \hat{A}\hat{B}\hat{C}\rangle_{\mathrm{c}}\langle \hat{D}\rangle - \langle \hat{B}\hat{C}\hat{D}\rangle_{\mathrm{c}}\langle \hat{A}\rangle - \langle \hat{C}\hat{D}\hat{A}\rangle_{\mathrm{c}}\langle \hat{B}\rangle - \langle \hat{D}\hat{A}\hat{B}\rangle_{\mathrm{c}}\langle \hat{C}\rangle \ . \end{aligned} \tag{S6}$$

For $\langle \hat{E}^- \rangle = 0$, the contribution of the third order connected correlations cancels, leading to Eq. (2) of the main text involving the normalized connected correlation $\boldsymbol{C}(\tau)$: $\boldsymbol{C}(\tau) = \langle \hat{E}^-(t)\hat{E}^-(t+\tau)\hat{E}^+(t+\tau)\hat{E}^+(t)\rangle_{\mathrm{c}}/\langle \hat{I}\rangle^2$. Here $\langle \hat{E}^-(t)\hat{E}^-(t+\tau)\hat{E}^+(t+\tau)\hat{E}^+(t)\rangle_{\mathrm{c}}$ is the connected correlation of the field.

Figure S3(c,d) show examples of normalized connected correlation $\boldsymbol{C}(\tau)$. They have been evaluated from the intensity correlation $g_N^{(2)}(\tau)$ reported respectively in (a,b), measured for different $N$ and $\Omega$. In the low atom number regime, $\boldsymbol{C}(\tau)$ averages to zero, while it clearly departs from 0 for large $N$.