EXST 7160 — Advanced Statistical Theory I+
Fall 2021/March 19, 2021

Based on Boos and Stefanski (2013)

Contents

I. Introductory Material

- Notation, Definitions, Asymptotics, and Simulations
- Families of Distributions
- Moments and Cumulants
- Quantiles and Percentiles
- Asymptotic Normality and Some Basic Asymptotic Results
- Simulation Methods

II. Likelihood Based Concepts

1. Likelihood Construction and Estimation
   For Univariate Models
   - Discrete IID Random Variables
   - Multinomial Likelihoods
   - Continuous IID Random Variables
   - Mixtures of Discrete and Continuous Components
   - Proportional Likelihoods
   - The Empirical Distribution Function as an MLE
   - Likelihoods from Censored Data

   For Regression Models
   - Linear Model
   - Additive Errors Nonlinear Model
   - Generalized Linear Model
   - Generalized Linear Mixed Model (GLMM)
   - Accelerated Failure Model

2. Marginal and Conditional Likelihoods
   - Marginal Likelihoods
   - Logistic Regression Measurement Error Model
   - General Form for Exponential Families
   - Conditional Logistic Regression

3. The Maximum Likelihood Estimator and the Information Matrix

- Examples of Information Matrix Calculations
- Variance Cost for Adding Parameters to a Model
- The Information Matrix for Transformed and Modeled Parameters

4. Methods for Maximizing the Likelihood or Solving the Likelihood Equations

- Analytical Methods via Profile Likelihoods
- Newton Methods
- EM Algorithm

5. Appendix A – Uniqueness of Maximum Likelihood Estimators

- Definitions
- Main Results
- Application of Theorem 2.2 to the Multinomial
- Uniqueness of the MLE in the Normal location-scale Model
- Application of Theorems 2.1 and 2.3 to the Exponential Threshold Model
- Uniqueness of the MLE for Exponential Families

6. Appendix B –Exponential Family Distributions

- Canonical Representation
- Families with Truncation or Threshold Parameters
- Exponential Family Glossary

7. Confidence Regions

- Confidence Interval for a Binomial Probability
- Confidence Interval for the Difference of Binomial Probabilities: Independent Samples **Likelihood Testing for Regression Models**
- Linear Model
- Additive Errors Nonlinear Model
- Generalized Linear Model

III. Bootstrap

1. Introduction

2. Sample Bootstrap Applications

3. Bootstrap Standard Errors

- Plug-In Interpretation of Bootstrap Variance Estimates

4. Comparison of Variance Estimation Methods

5. Bootstrap Asymptotics

6. Bootstrap Confidence Intervals

- The Percentile Interval

References

Boos, D. D. and L. A. Stefanski (2013). *Essential Statistical Inference.* Springer. 1