Kalman Filter

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Used to estimate states in <u>linear</u> dynamical systems in state space format.

Evolution of state from k-1 to k

Process model

$$x_k = F x_{k-1} + B u_{k-1} + w_{k-1}$$

- F — state transition matrix
- x_{k-1} — prev ~~state~~ vector
- B — control input matrix
- u_{k-1} — control vector
- w_{k-1} — process noise vector → Assumed gaussian with mean = 0, covariance = Q

$$w_{k-1} \sim N(0, Q)$$

Measurement Model

relationship b/w state & measurement at current time step k.

$$z_k = H x_k + v_k$$

- z_k — measurement vector
- H — measurement matrix
- x_k — state vector
- v_k — measurement noise

$$v_k \sim N(0, R)$$

(0 — mean, R — cov.)

What does a Kalman filter really do?

Provide ~~of~~ estimate of x_k ~~for~~ at time k, given a series of measurements $z_1, z_2, z_3 \ldots z_k$, init state x_0 & F, B, H, Q, R (system params).

Kalman Filter Algorithms

"Predict" stage

predicted state estimate $\longrightarrow \hat{x}_k^- = F\hat{x}_{k-1}^+ + Bu_{k-1}$ (1)

predicted error covariance $\longrightarrow P_k^- = FP_{k-1}^+F^T + Q$ (2)

"Update" stage

measurement residual $\longrightarrow \tilde{y}_k = z_k - H\hat{x}_k^-$ (3)

kalman gain $\longrightarrow K_k = P_k^- H^T(R + HP_k^- H^T)^{-1}$ (4)

updated state estimate $\longrightarrow \hat{x}_k^+ = \hat{x}_k^- + K_k\tilde{y}_k$ (5)

updated error covariance. $\longrightarrow P_k^+ = (I - K_kH)P_k^-$ (6)

$\hat{}$ ← estimated value

$+$ ← just after time step

$-$ ← just before timestep.

Number of computations

432 FLOPs per matmul

258 FLOPs per inversion

~~fo~~Predict

No. of ~~vars~~ = state var = 6

P, F, B, R, Q → 6 x 6 matrices

step (1) → 6 MACs + 6 MACs + 36 adds

= 12 MACs + 36 add

144 + 36 = 180 FLOPs

step (2) → 2 matMul(6x6)
+ 36 adds

2 x 432
+ 36
900 FLOPs

step (3) → 6 MACs + 6 add

42 FLOPs

step (4) → 4 matMul (6x6)
+ 36 add
+ 1 matInv (6x6)

4 x 432
36
258
2022 FLOPs

step (5) → 6 add
+ 6 MACs

6
12
18 FLOPs

step (6) → 2 Matmul(6x6)
+ 36 adds

2 x 432
36
900 FLOPs