EE5332 Project

Kalman filter:

Process model:

Evolution of state from "k-1" instance to "k"

$$x_k = F x_{k-1} + B u_{k-1} + w_{k-1}$$

F: state transition matrix

B: control input matrix

process noise vector

$w_{k-1} \sim N(0, Q)$

covariance: Q

Measurement model:

relation between state & measurement

$$z_k = H x_k + v_k$$

measurement vector

measurement matrix

measurement noise $v_k \sim N(0, R)$

Function: Estimate x_k at time k. given initial estimate of x_0 based on series of measurement $z_0, z_1, z_2 \dots z_k$ & system information: F, B, H, Q, R

Kalman filter algorithm:

Prediction / Propagation:

$\hat{x}_k^- = F \hat{x}_{k-1}^+ + B u_{k-1}$: Predicted state estimate

$P_k^- = F P_{k-1}^+ F^T + Q$: Predicted error covariance

Update / Correction:

$\tilde{y}_k = z_k - H \hat{x}_k^-$: Measurement residual

$K_k = P_k^- H^T (R + H P_k^- H^T)^{-1}$: Kalman gain

$\hat{x}_k^+ = \hat{x}_k^- + K_k \tilde{y}$: Updated state estimate

$P_k^+ = (1 - K_k H) P_k^-$: Updated error covariance.

hat operator $\hat{}$: Estimate of variable

$+$ & $-$ denote updated (posterior) & predicted (prior) estimate

Note:

Predicted error covariance: $P_k^- = F P_{k-1}^+ F^T + Q$

Updated error covariance: $P_k^+ = (I - K_k H) P_k^-$

Error covariance is larger in prediction stage because of summation with Q

means filter is more uncertain of state estimate after prediction

$\tilde{y}_k = z_k - H\hat{x}_k^-$: residual difference

z_k: true measurement

$H\hat{x}_k^-$: estimated measurement

$K_k \tilde{y}_k$: the correction to be added to the predicted estimate $\hat{x}_k^-$ to get the update state estimate

error covariance P_k^+ is updated. the value is less than predicted.

indicating filter is more certain of state estimate after the measurement is utilized.

6 State variables:

$\Rightarrow \quad \hat{x}_k^- = F\hat{x}_{k-1}^+ + Bu_k$ $\qquad \hat{x}_{k-1}^+, \hat{x}_k^-: 6\times1 \qquad u_k: 6\times1$

$F: 6\times6 \qquad B: 6\times6$

$F\hat{x}_{k-1}^+$ 36 multiplys, 30 adds

Bu_k 36 multiplys, 30 adds

$\hat{x}_k^-$ 6 adds.

$\Rightarrow \quad P_k^- = FP_{k-1}^+F^T + Q$ $\qquad P_{k-1}^+, P_k^-: 6\times6$

6×6: FP_{k-1}^+ : 216 mult 180 adds

$FP_{k-1}^+F^T$: 216 mul 180 adds

P_k^- : 36 adds.

$\Rightarrow \quad \hat{y}_k = z_k - H\hat{x}_k^-$ $\qquad H: 6\times6$

$H\hat{x}_k^-$: 36 mult, 30 adds

$\hat{y}_k$: 6 adds

$\Rightarrow \quad K_k = P_k^-H^T(R + HP_k^-H^T)^{-1}$

$P_k^-H^T$: 216 mult 180 adds

$HP_k^-H^T$ 216 mult 180 adds

$HP_k^-H^T + R$: 36 adds.

$(R + HP_k^-H^T)^{-1}$: Gauss Jordan Elimination

~~5 × [1 div 6 mul 6 sub]~~

~~4 × [1 div 5 mul 5 sub]~~

1 × [1 div 5 mul 5 add]

2 × [1 div 4 mul 4 add]

3 × [1 div 3 mul 3 add]

4 × [1 div 2 mul 2 add] +

5 × [1 div 1 mul 1 add]

[1 div 1 mul 1 add] × [5+4+3+2+1]

K_k : 216 mul 180 add

$\Rightarrow \quad \hat{x}_k^+ = \hat{x}_k^- + K_k \hat{y}$

$K_k \hat{y}$: 36 mul 30 add

$\hat{x}_k^+$: 6 add

$\Rightarrow \quad P_k^+ = (1 - K_k H) P_k^-$

$K_k H$: 216 mul 180 add

$1 - K_k H$: 36 add

$(1 - K_k H) P_k^-$: 216 mul 180 add.

Totally approx: 3300 operations.

n state variables

$$\Rightarrow \underbrace{\hat{x}_k^-}_{n\times 1} = \underbrace{F}_{n\times n}\underbrace{\hat{x}_{k-1}^+}_{n\times 1} + \underbrace{B}_{n\times n}\underbrace{u_k}_{n\times 1}$$

$F\hat{x}_{k-1}^+$: n^2 mul $n(n-1)$ add
Bu_k : n^2 mul $n(n-1)$ add
$\hat{x}_k^-$: n add.

$$\Rightarrow P_k^- = \underbrace{F}_{n\times n}\underbrace{P_{k-1}^+}_{n\times n}F^T + \underbrace{Q}_{n\times n}$$

FP_{k-1}^+ : n^3 mul $n^2(n-1)$ add
$FP_{k-1}^+F^T$: n^3 mul $n^2(n-1)$ add
P_k^- : n^2 add.

$$\Rightarrow \underbrace{\tilde{y}_k}_{n\times 1} = \underbrace{z_k}_{n\times 1} - \underbrace{H}_{n\times n}\hat{x}_k^-$$

$H\hat{x}_k^-$: n^2 mul $n(n-1)$ add
$\tilde{y}_k$: n add

$$\Rightarrow K_k = P_k^- H^T (R + HP_k^- H^T)^{-1}$$

$P_k^- H^T$: n^3 mul $n^2(n-1)$ add
$HP_k^- H^T$: n^3 mul $n^2(n-1)$ add
$HP_k^- H^T + R$: n^2 add.

approx n^3 operations
$$(R + HP_k^- H^T)^{-1} : \sum_{m=1}^{n-1} [1 \text{ div } (n-m) \text{ mul } (n-m) \text{ add}] + [1 \text{ div } 1 \text{ mul } 1 \text{ add}]\frac{n(n+1)}{2}$$

K_k : n^3 mul $n^2(n-1)$ add

$$\Rightarrow \hat{x}_k^+ = \hat{x}_k^- + K_k\tilde{y}$$

n^2 mul + $n(n-1)$ add : $K_k\tilde{y}$
n add : $\hat{x}_k^+$

$$\Rightarrow P_k^+ = (1 - K_k H)P_k^-$$

K_kH : n^3 mul $n^2(n-1)$ add
$1 - K_kH$: n^2 add
$(1 - K_kH)P_k^-$: n^3 mul $n^2(n-1)$ add.

Total operations : $\approx 13n^3 + 7n^2 - n \approx 13n^3$