DAT7304: BUSINESS ANALYTICS

ASSESSMENT 1

PORTFOLIO

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## ABSTRACT

This study focuses on building and comparing five different time series forecasting models namely: Triple Exponential Smoothing (Holt Winters), ARIMA, SARIMA, SARIMA and Vector Autoregression (VAR), choosing the best model through residual evaluation to forecast future estimated quantity demand per transaction for fuel. The dependent variable is the fuel estimated quantity demand per transaction while the exogenous variable is the motor fuels inflation rate. Each model is trained on the merged fuel historical data after appropriate data preprocessing and transformation to ensure stationarity for model suitability. Residual diagnostic check using error measure metrics such as mean absolute error (MAE) , root mean standard error RMSE) and mean absolute percentage error MAPE) is carried out as a measure to select the best fit model for the business problem before further assessments on the model, such as checking for normality ( for normal distribution) and no autocorrelation to ensure the selected model meets the residual requirements for model adequacy and performance. Among the models, the one demonstrating the most accurate and unbiased forecast based on residual evaluation is selected to forecast future fuel quantity demand.

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**Keywords:** Time series analysis, forecasting, Predictive model, Fuel quantity demand, Motor inflation rate.

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# CHAPTER 1

## INTRODUCTION

Accurate demand forecasting lies at the core of strategic business planning and inventory management in a wide range of industries. Time series modeling provides an extended framework for modeling temporal data and forecasting, especially when augmented with the right external variables that condition demand behaviour. This study explores the application of five time series forecasting models, namely: Triple Exponential Smoothing (Holt Winters), Autoregressive Integrated Moving Average (ARIMA), Seasonal ARIMA (SARIMA), Seasonal ARIMA with Exogenous Regressors (SARIMAX), and Vector Autoregression (VAR) on a data set consisting of fuel estimated quantity of demand per transaction, with motor fuel inflation rate as an external variable. The objective is to compare the forecast accuracy of all the models through residual behaviour and prediction errors. The inclusion of an exogenous variable in SARIMAX and VAR models allows for the analysis of its impact on the performance of forecasting. Data preprocessing steps of the time series such as testing for stationarity and any required transformations are performed to satisfy model assumptions. Residual diagnostic tests and statistical metrics such as Mean Absolute Errors (MAE), Mean Absolute Percentage Errors (MAPE) and Root Mean Squared Errors (RMSE) are used to assess the performance of models. The optimal model with the most stable residual behaviour and smallest forecast errors is employed to generate future demand estimates. The comparison provides an estimate of the relative usefulness of univariate and multivariate forecasting techniques in practice.

## 1.1 Background

Time series prediction has been one of the central challenges of data analysis, particularly where demand forecasting has been at the center of significant business and policy decisions (Hamilton, 2020). The single variable techniques such as Autoregressive Integrated Moving Average (ARIMA) have sufficed for stationary time series, while extensions such as Seasonal ARIMA (SARIMA) accommodate cyclic seasonal patterns repeated over a period. However, true demand is regulated by

external economic forces, and this has created the need for building and application of models using exogenous variables such as SARIMAX and Vector Autoregression (VAR). Triple Exponential Smoothing, or Holt-Winters technique, is among the popular techniques that captures level, trend, and seasonality in time series data without imposing strict stationarity. It is very useful in short-run forecasting situations when patterns are non-linear but stationary. SARIMAX, on the other hand, addresses external predictors specifically and thus apt where indicators like inflation, fuel price, or macroeconomic variables have been observed to influence the target variable. Similarly, VAR regards all variables within the system as endogenous and thus one is able to influence another and interrelate with others dynamically in the long run.

Several studies have shown that the incorporation of proper external variables can successfully enhance the reliability of forecasts, especially in unstable or economically weak segments. Changes in fuel prices, for instance, have been seen to directly influence consumption patterns and demand structures in transport, retail, and logistics sectors. This study extends this finding by implementing and comparing five other forecasting techniques both univariate and multivariate with fuel quantity demand data as dependent and motor fuel inflation rate as an exogenous variable. By residuals and forecast errors in each model, the study aims to identify the best method to use in future demand estimation under economically driven conditions.

## 1.2 Description of the Business and Business Stakeholders.

The business involves fuel sales and distribution, motor fuel gas and diesel trading among other petroleum products to clients through a retailing chain of outlets or direct transactional services. The core purpose of the business is ensuring there is continuous and cheap availability of fuel products and precisely fulfilling consumers demand. In this industry, reliable demand forecasting is quite crucial given the fact that business operations are highly susceptible to fluctuations in global oil prices, governmental policies, seasonality of patterns of demand fluctuations and inflation.

The key stakeholders that are affected are:

Business Decision Makers and Managers: Accurate forecasts are required to facilitate top line strategic decisions such as procurement planning, pricing strategy and growth projects.

Operations and Supply Chain Managers: They depend on forecasted demand to optimise stock levels, regulate fuel deliveries and reduce wastage or shortage to the retail outlets.

Financial Analysts and planners: Employ forecasts to project cash flows, budget fuel buying and examine the fiscal impact of market volatility.

Regulators and Policymakers: May base their fuel price regulation, taxation and environmental policy on industry forecasts.

Retail Partners and Distributors: Enjoy better predictability and planning of supply, resulting in enhanced levels of service and customer satisfaction.

End users: Indirect stakeholders that are impacted by levels of fuel prices and supply, especially in times of high volatilities or shortages.

## 1.3 Business problem

The oil distribution business is marred by the failure to make proper predictions of the demand for oil products by customers which is critical in attaining sufficient levels of stock, lowering the cost of doing business and ensuring uninterrupted provision of services. Demand is extremely vulnerable to external factors such as motor fuel inflation rate, which can have the potential to affect customer behaviour, purchasing power and consumption patterns.

Incorrect forecasting leads to:

Stockouts that results in missed sales and customer dissatisfaction.

Overstocking that increases the storage and handling costs, especially of controlled or perishable fuel products.

Poor supply chain choices such as poor delivery or procurement timing.

Inefficient pricing strategies owing to reactive rather than data based planning.

Firms therefore require a stable and dynamic forecasting model that project and react market volatility not only on historical demand but also on economic conditions to enhance forecast accuracy and decision making.

## 1.4 Significance of the problem

Accurate fuel demand forecasting is required to enable operational efficiency, cost management and customer satisfaction for the fuel distribution industry. As fuel

prices are cause and effect related with consumption volumes and are extremely volatile, rudimentary forecasting techniques are proven to fail when applied individually. By not incorporating external economic drivers such as motor fuel inflation, companies have a risk of ending up with making wrong estimates, resulting in lost revenues, supply chain losses and low customer satisfaction. Addressing this problem through advanced forecasting models that take into account history of demand as well as external factors (e.g. inflation) gives several benefits:

improved stock control, avoidance of stockouts and shortages.

Improved purchasing and shipping, cost saving and utilisation of assets.

Improved dynamic pricing and sensitivity to market conditions.

Improved planning strategy founded on evidence based facts on future patterns.

Lastly, this solution enables the oil industry firms to be more customer focused, competitive and resilient in an economically uncertain and dynamic world.

## 1.5 Business Questions

This study aims to address the following questions;

Question 1

What forecasting model performs best in capturing the seasonal behavior of Estimated quantity demand per transaction?

Question 2

What will be the predicted weekly estimated quantity demand per transaction for the next one year?

Question 3

Which week will record the highest estimated quantity demand per transaction in the next 6 months?

Question 4

What is the relationship between Estimated quantity demand per transaction and motor fuel inflation rate in line plot?

Question 5

Which smoothing technique gives a better trend visualisation?

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Related literature

This study examines the effect of some of the characteristics of time series data on the performance of fuel demand forecasting for petrol stations using ARIMA, SARIMA, and Markov chain models. The findings demonstrate that selection of a model from special data characteristics can maximise accuracy. SARIMA models were more sensitive to highly variant time series, whereas Markov chain models were more effective in detecting longer and more complex cycles and less susceptible to forecasting errors. Accurate forecasts enable petrol stations to better plan deliveries, optimise tank capacity utilisation, and reduce fuel shortage risks. Contrary to most studies, which merely focus on comparing forecasting methods, this paper focuses on highlighting the importance of taking into consideration how structure in time series affects model performance. Additional research needs to be done to increase the diversity of the time series studied and to account for other characteristics of the data that could potentially affect forecasting precision. It is also interesting to examine how external factors such as fuel price volatility and macroeconomic conditions impact forecasting performance. Emphasis in these areas would provide a better understanding of forecasting issues in the fuel industry and allow data-driven, informed decision-making for supply planning which are lacking in the study (Wiecek and Kubek, 2024).

In this study, the increasing reliance on imported foreign oil as a result of increased domestic consumption and declining production is examined. It focuses on fuel oil supply planning for the intention of minimising the chances of shortage. Key findings include that widened price differentials between similar products lead to lower demand for the more expensive ones. In addition, the ensemble of forecasting models provides a better result than a single model, and a three year data set provides better forecasts than a five year data set. The study compares forecasting methods and concludes that Holt-Winters additive method is better than the ARIMA model. It also concludes that basic time-series methods such as Holt-Winters make better forecasts than complex methods such as ARIMA. Besides that, the Neural Network (NN) model is more suitable for detecting non-linear patterns among data.

Notwithstanding, the research has identified areas of forecasting accuracy shortcomings where prediction discrepancies are commonplace. Research is proposed in the study to investigate levels of oil supplies and propose a simulation model to handle supply chain volatility. Existence of evidence to support the need for an improved way of dealing with uncertainties of the oil supply chain is present (Mardiana, Saragih and Huseini, 2020).

The ARIMA model was employed in the study to forecast future gasoline consumption by transport sector from past data. The prediction of the findings was that gasoline consumption was going to rise in the future. The research is beneficial to policymakers interested in promoting ethanol as an alternative source of gasoline and indicates the requirement to invest in refineries in order to maximise production efficiency of gasoline. In spite of that, there are several limitations to the research. Firstly, it does not cover more recent data, which can serve to render the forecast more accurate and timely. Secondly, the research does not involve a comparison with other forecasting methods that would validate the use of the ARIMA model and provide a more accurate forecast of trends in fuel consumption. These gaps reflect areas where further research is required to enable further understanding and projection of fuel demand in the transport sector (Waheed, et al., 2017).

Because of the limited word count of this study, only three among the literatures review are stated in the session with the remaining cited in the reference session.

## 2.2 Study contribution to existing knowledge.

My study contributes to the body of the knowledge through an extension of comparative examination of many time series forecasting techniques such as Triple Exponential Smoothing, ARIMA, SARIMA, SARIMAX and VAR into driving demand forecast with real data with the inclusion of an external variable (motor fuel inflation rate) that has been ignored by most studies. In contrast to earlier research that failed to consider the application of one or a double method of forecasting or did not even involve economic factors, this study highlights the importance of diagnostic testing of the model, including residual tests and normality tests and autocorrelation tests, to obtain statistical consistency and prediction accuracy. Moreover, by choosing the

most accurate model on the basis of complete performance measures (MAE, RMSE, MAPE), this research contributes methodological rigor and provides a reproducible method for fuel demand forecasting on the basis of internal data characteristics and external macroeconomic variables. This multi dimensional approach fills gaps in previous research and provides actionable insights for supply chain planning as well as policy making in the energy sector.

# CHAPTER 3

# METHODOLOGY

## 3.1 Description of dataset

### Index

The dataset is a fuel time series data which uses a datetime index spanning from January 31, 2021 to January 5, 2025 consisting of 206 weekly time points spaced at regular intervals.

### Columns

The dataset contains two float-type variables, namely:

i) Estimated_quantity_demand_per_transaction which represents an estimated quantity of fuel demanded per transaction and is indicative of customer purchasing behavior over time.

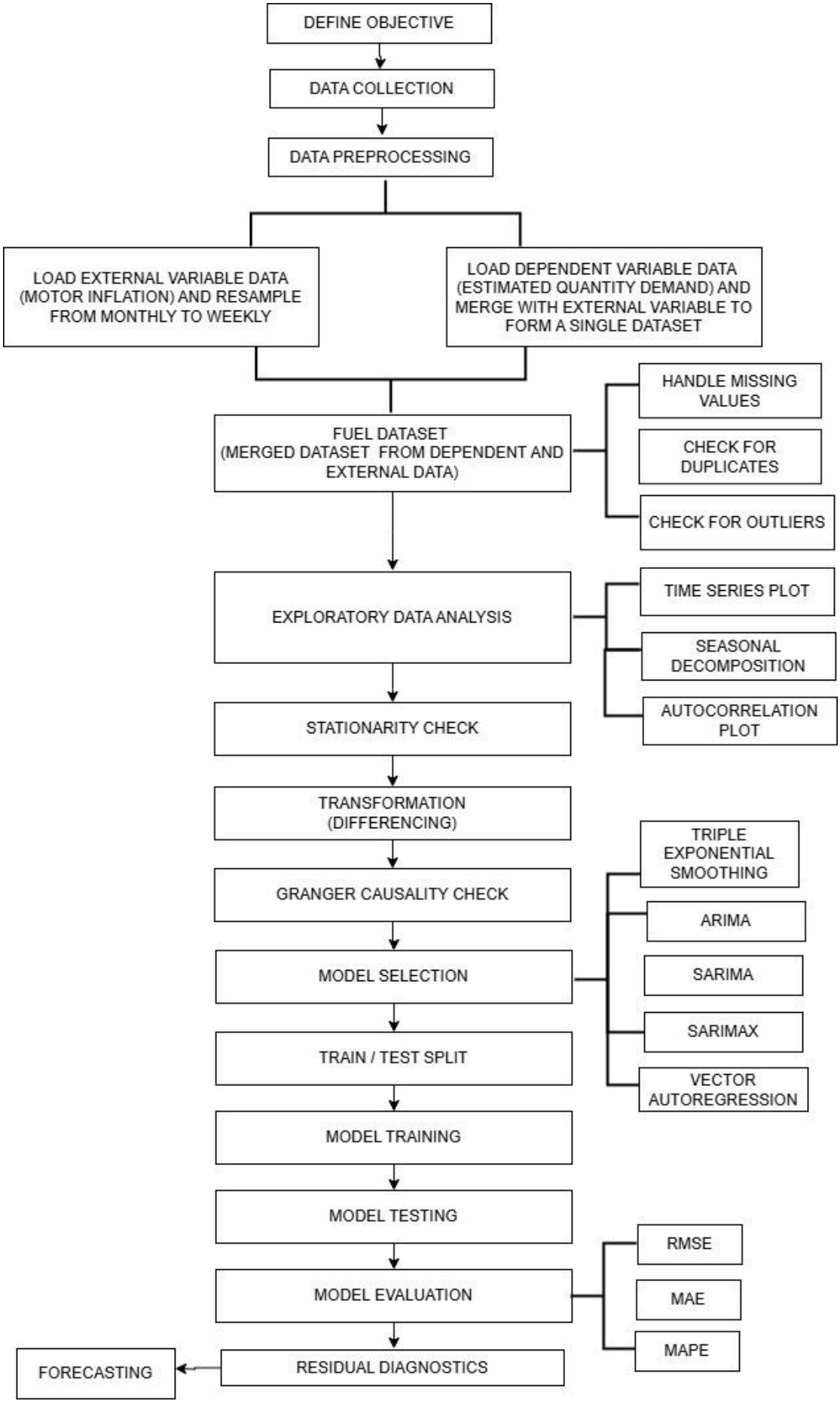
ii) Motor_fuels_inflation rate, which represents the inflation rate for motor fuels at each time point, potentially impacting purchasing power or consumer demand.

### Data type

Both columns are of float.

## 3.2 Methodology Flowchart

Below is a flowchart of the methodology process;


DEFINE OBJECTIVE
DATA COLLECTION
DATA PREPROCESSING
LOAD EXTERNAL VARIABLE DATA (MOTOR INFLATION) AND RESAMPLE FROM MONTHLY TO WEEKLY
LOAD DEPENDENT VARIABLE DATA (ESTIMATED QUANTITY DEMAND) AND MERGE WITH EXTERNAL VARIABLE TO FORM A SINGLE DATASET
FUEL DATASET (MERGED DATASET FROM DEPENDENT AND EXTERNAL DATA)
HANDLE MISSING VALUES
CHECK FOR DUPLICATES
CHECK FOR OUTLIERS
EXPLORATORY DATA ANALYSIS
TIME SERIES PLOT
SEASONAL DECOMPOSITION
AUTOCORRELATION PLOT
STATIONARITY CHECK
TRANSFORMATION (DIFFERENCING)
GRANGER CAUSALITY CHECK
MODEL SELECTION
TRIPLE EXPONENTIAL SMOOTHING
ARIMA
SARIMA
SARIMAX
VECTOR AUTOREGRESSION
TRAIN / TEST SPLIT
MODEL TRAINING
MODEL TESTING
MODEL EVALUATION
RMSE
MAE
MAPE
RESIDUAL DIAGNOSTICS
FORECASTING

### 3.3 Steps required for time series analysis

#### 1. Problem Definition

The initial step is defining a clear cut goal of the analysis. The objective can involve forecasting future values, identifying outliers, examining cyclical patterns, seasonality, or forecasting relationships between variables. A proper problem description determines the choice of analytical method, model performance measure, and overall approach. This analysis aims to build predictive models and identify the most suitable one to forecast fuel quantity demand.

#### 2. Data Collection

The quality and pertinence of the time-stamped data are of utmost importance. Data need to be collected at frequent intervals and at a frequency appropriate to the analytical purpose. Data completeness and proper formatting must be ensured at this stage. The data applied in this study are secondary data of fuel estimated quantity demand per transaction and motor fuel inflation rate.

#### 3. Data Preprocessing

Raw time series data may need to be reprocessed so that it becomes amenable to analysis. This is where missing values are addressed, perhaps through interpolation or imputation techniques. There is also the need to eliminate duplicates to avoid any form of data manipulation. Outliers that would distort the analysis must be identified and handled accordingly. In some cases, resampling of the data to a different frequency is necessary such as resampling the monthly fuel inflation data to weekly data in order to align with the quantity demand data already in weekly frequency.

#### 4. Exploratory Data Analysis (EDA)

EDA helps in uncovering the underlying structure and pattern within the data.

Visual representations such as time series plots, seasonal decomposition, autocorrelation (ACF) and partial autocorrelation (PACF) plots and lagged plot are usually used. The visualizations help to determine trends, seasonal cycles, and any apparent randomness, thus informing the model selection process.

#### 5 Stationarity and causality check

Stationarity testing, often by means of tests like the Augmented Dickey-Fuller (ADF) test, is crucial since most time series models assume constant statistical properties

over time. Granger cause testing is also important when there is more than one variable involved since this helps inform the impact one variable has on another.

### 6. Model Selection

The model selection is founded on the characteristics of the time series and the aim of the analysis. ARIMA, SARIMA, and exponential smoothing methods are ideal for univariate time series with linear trends and evident seasonality while SARIMAX and vector autoregression (VAR) are ideal for multivariate time series as they take into consideration the exogenous variable. In this study, model selection is limited to the five models mentioned.

### 7. Model Training and Validation

Models must be trained using time aware data splits to preserve temporal order. Although there is no standard split ratio between the train data and test data but in this study, 78% of the dataset is considered to train the mode while the remaining 22% is assigned to the test data.

### 8. Model Evaluation

Model performance is carried out to examine the accuracy and reliability of the forecast. The performance metrics like Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE) are calculated. These compare predicted and observed values.

### 9. Forecasting

After model validation is done, future values are estimated. Point forecasts or prediction intervals can be generated based on the application to quantify uncertainty. The forecasting horizon have to match the original aim and purpose.

## 3.4 Model description

Below is a concise summary of the models description.

### 1) Triple Exponential Smoothing (Holt-Winter)

This method extend exponential smoothing to capture seasonality, trend and level in time series data. It consists of three components:

Level: the base value

Trend: the rate of change

Seasonality: recurring patterns over fixed periods.

Good for seasonal data forecasting.

### 2) ARIMA (Autoregressive Integrated Moving Average) :

This is a statistical model that is a blend of

AR (AutoRegression): regression of a variable on its past values.

I (integrated) : differencing to make the time series stationary.

MA (Moving Average) : error term is a linear combination of past error terms. Appropriate for non seasonal stationary time series.

### 3) SARIMA (Seasonal ARIMA) :

Extension of ARIMA with addition of seasonal terms:

Adds the non seasonal and the seasonal trend.

Additional seasonal AR, I, MA terms. Appropriate for seasonal time series with fixed periodicity.

### 4) SARIMAX (Seasonal ARIMA with eXogenous variables) :

Further generalise SARIMA to incorporate external variables (exogenous variables) which may affect the target series.

### 5) Vector Autoregression (VAR):

Multivariate time series model of a time series with linear interdependencies across several time series.

Represents all variables as linear functions of their own lagged variables and the lagged variables of all other variables in the system.

# CHAPTER 4

# IMPLEMENTATION AND RESULTS

## 4.1 Implementation

Below is the breakdown of the implementation process alongside the corresponding code snippets and visualisation;

### Data collection

The dataset for the time series analysis in this study is a merged fuel dataset from estimated fuel quantity demand per transaction being the dependent variable as shown in fig. 2 and motor fuel inflation rate being the external variable as shown in fig. 1a and fig. 1b below, sourced from https://www.ons.gov.uk/economy/inflationandpriceindices/bulletins/consumerpriceinflation/january2025

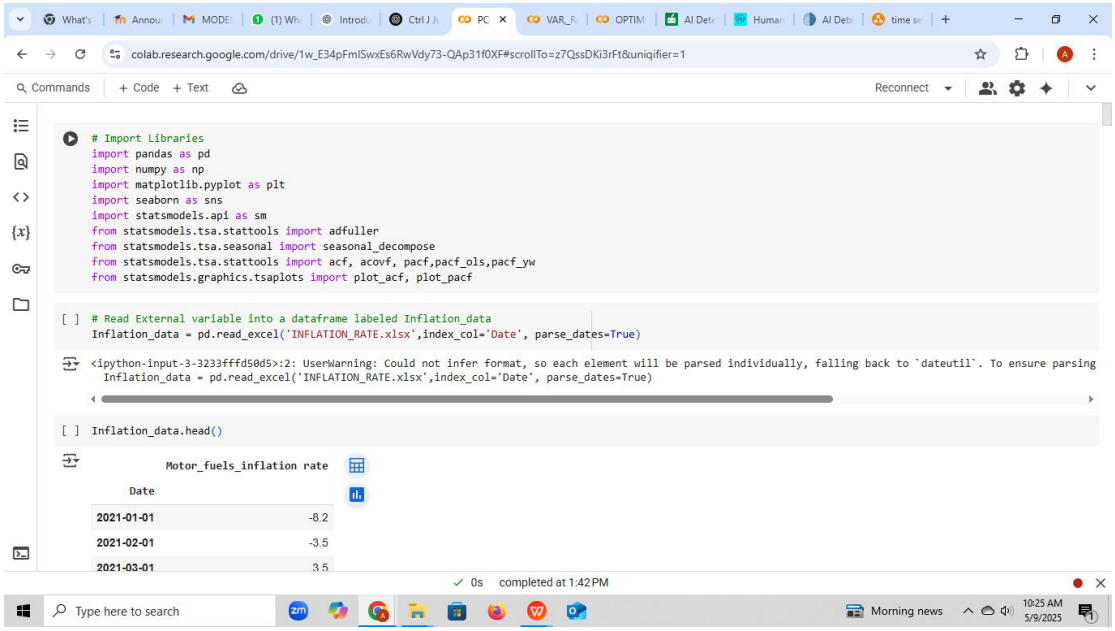



Fig. 1a: motor fuel inflation rate data

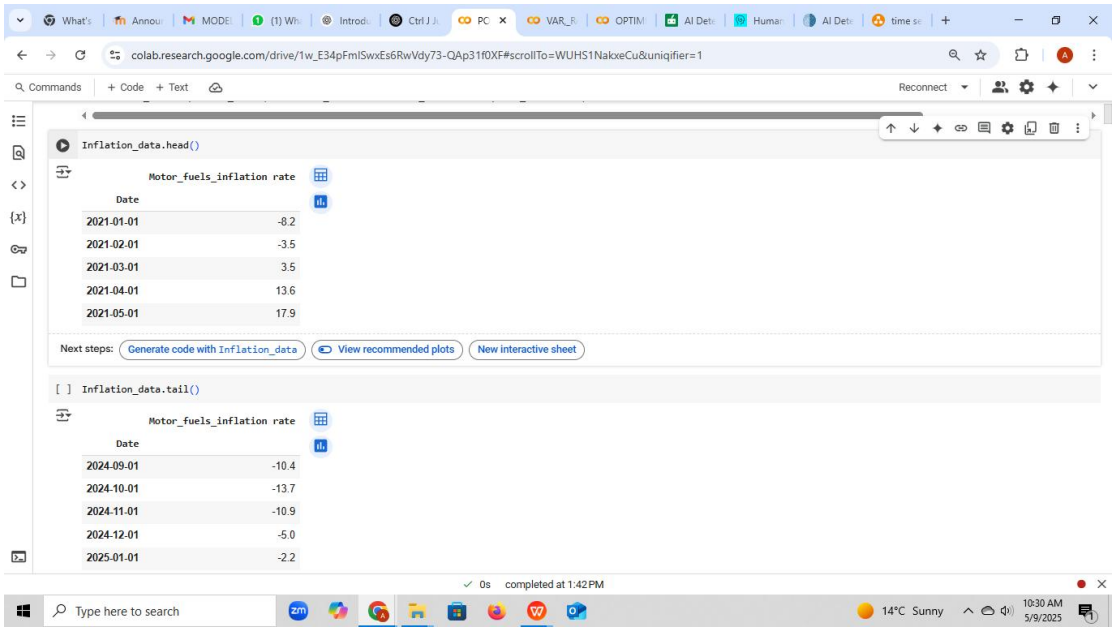



Fig. 1b: motor fuel inflation rate data

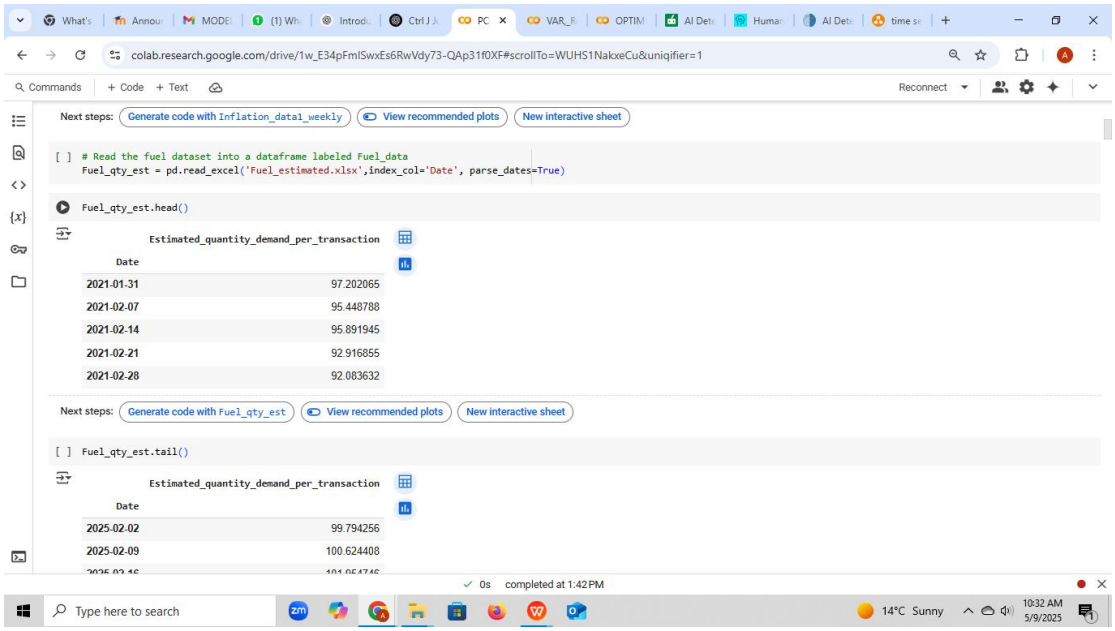



Fig. 2: Estimated quantity demand per transaction data

## Data preprocessing

The data preprocessing step starts with creating a copy and resampling the motor fuel inflation rate data from monthly to weekly frequency using the forward fill function as seen in fig. 3 below in order to standardise the frequency of both data since the dependent variable data is given in weekly frequency.

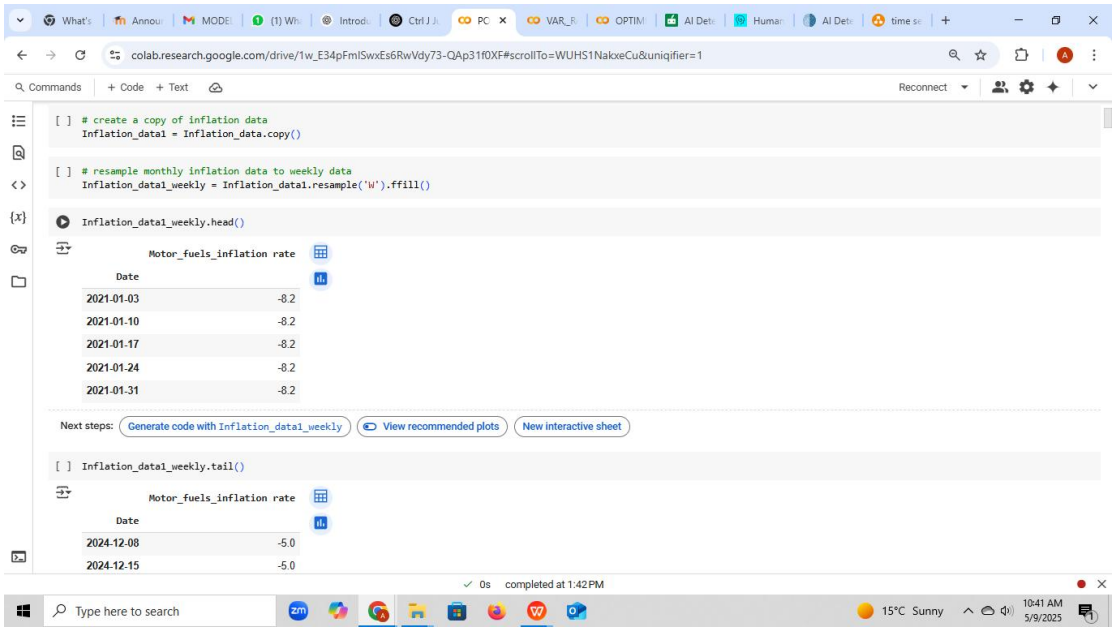



Fig. 3: resampled motor fuel inflation rate data.

After resampling, the estimated quantity demand per transaction data is uploaded and merged with the resampled motor fuel inflation rate data to form the fuel dataset for the time series analysis as shown in fig. 4 below;

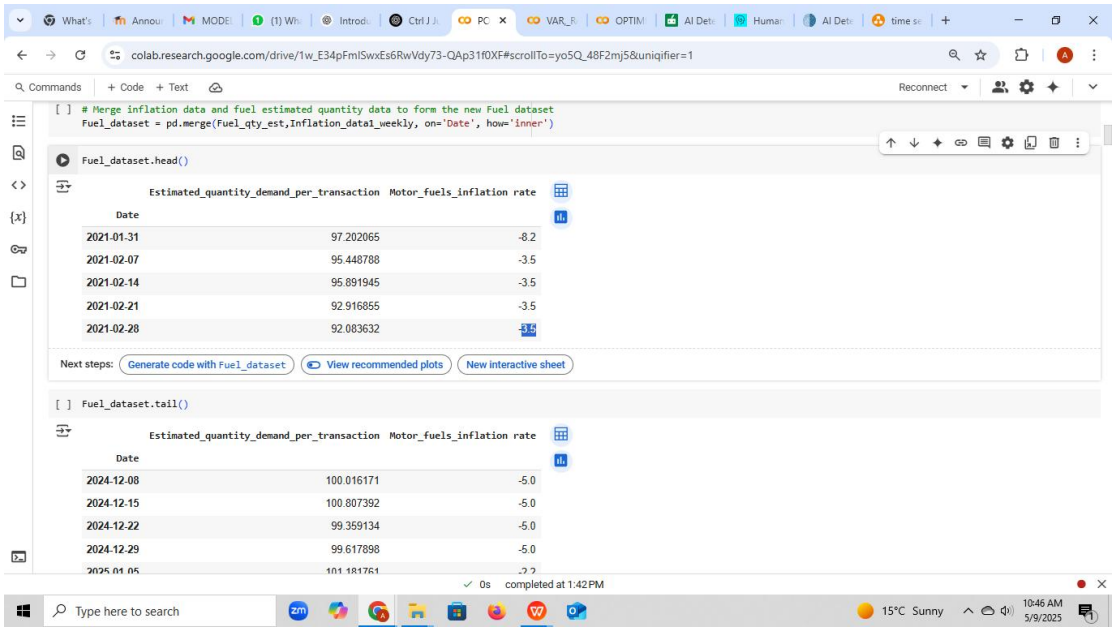



Fig. 4: merged fuel dataset

A check for missing values on the merged fuel data shows that two values are missing as shown in fig. 5a. The interpolation technique is used to handle the missing values and check for duplicates as shown in fig. 5b which shows that there is no duplicate;

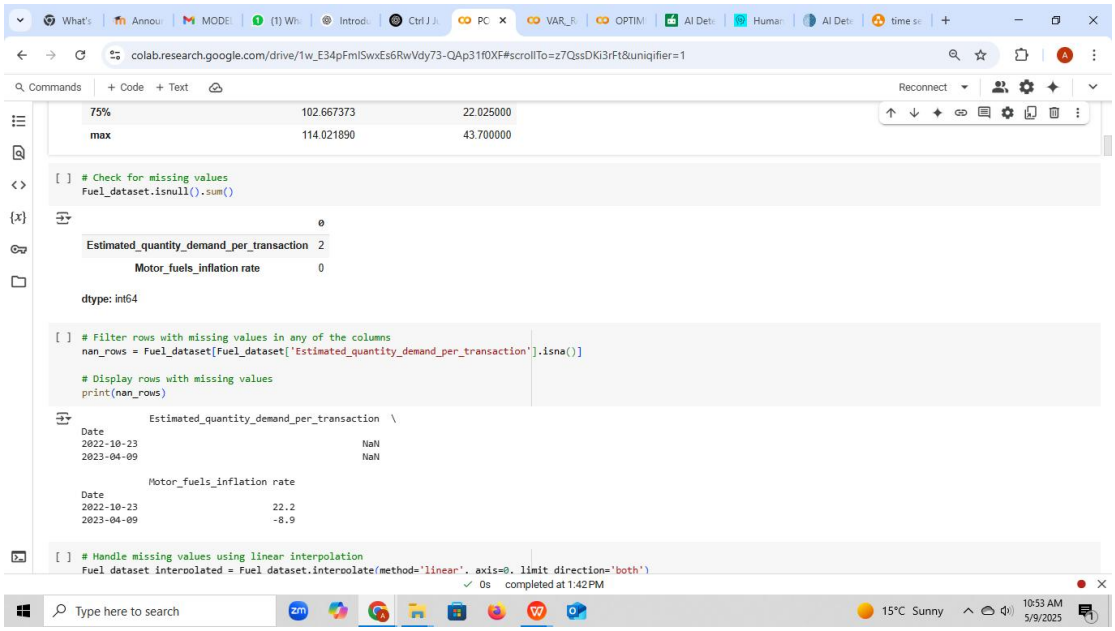



Fig. 5a: identifying missing values

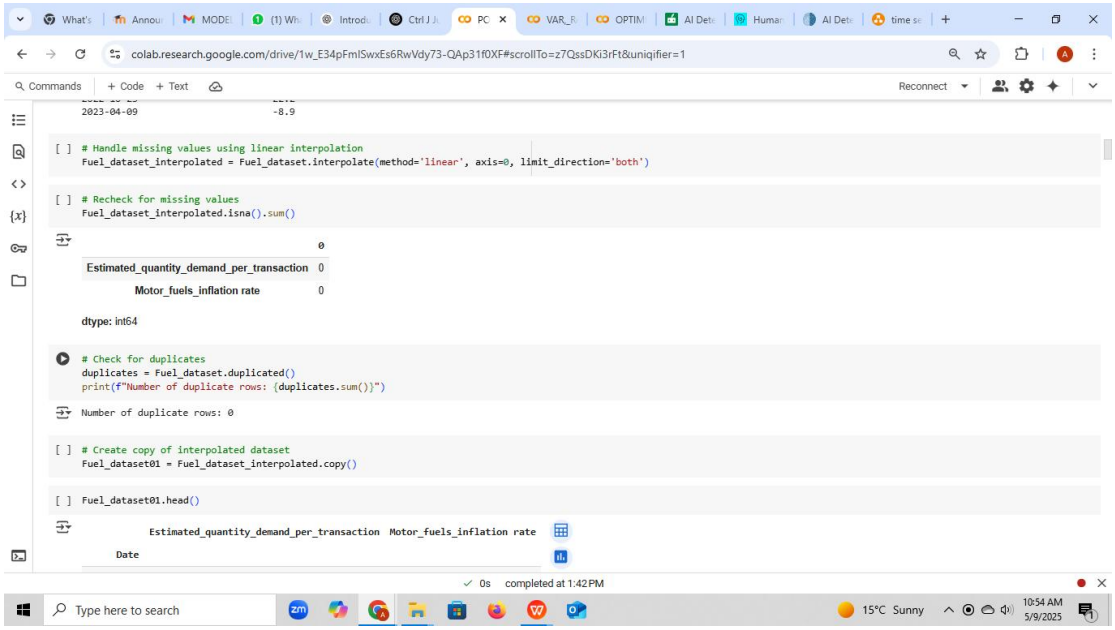



Fig. 5b: handle missing values with interpolation and check for duplicates.

An overview of the fuel data is carried out with respect to the data index, info, description and shape as seen in fig. 6a and fig. 6b respectively;

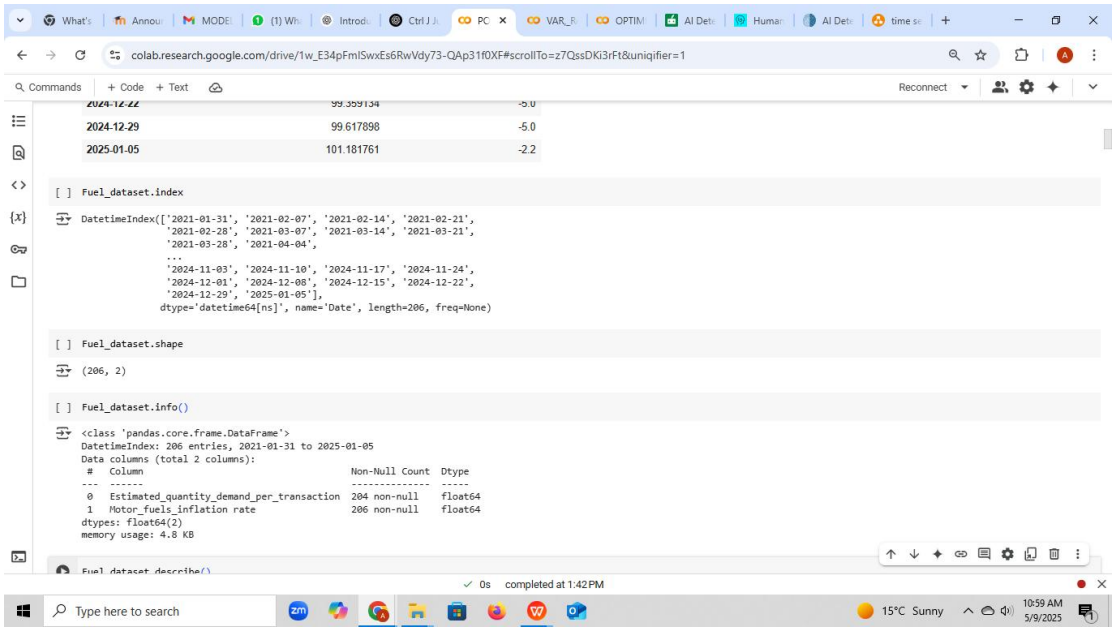



Fig. 6a: Fuel data overview

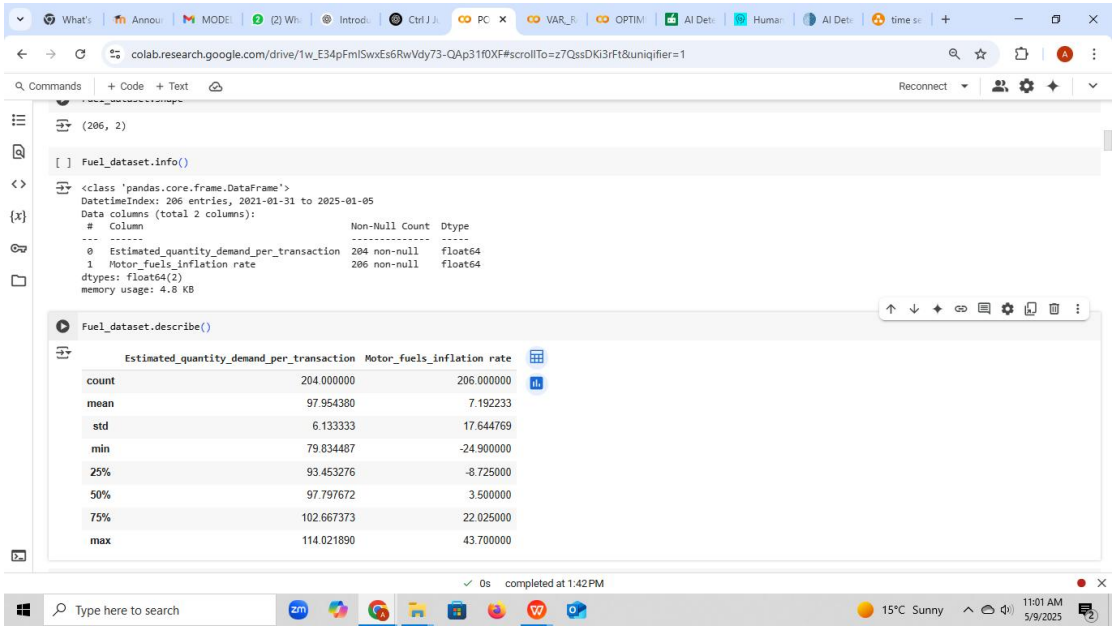



Fig. 6b: Fuel data overview.

In order to understand the data distribution of the fuel data for each column, histogram plot is carried out which gives a visual insight that the dependent variable is likely to follow a normal distribution as shown in fig.7a. Shapiro test is carried out to verify that as shown in fig. 7b and from the result of the test, the p-value is 0.80 which is greater than 0.05 indicating that the estimated quantity demand per transaction data is normally distributed. The check is just to have idea of the distribution of the data.

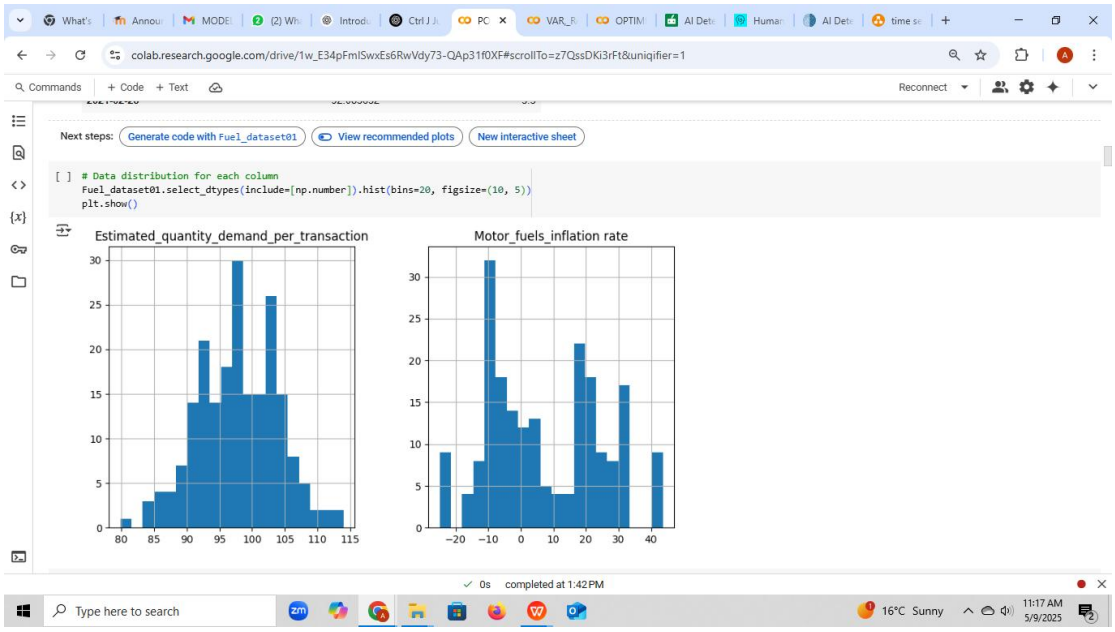



Fig. 7a: Fuel data distribution plot

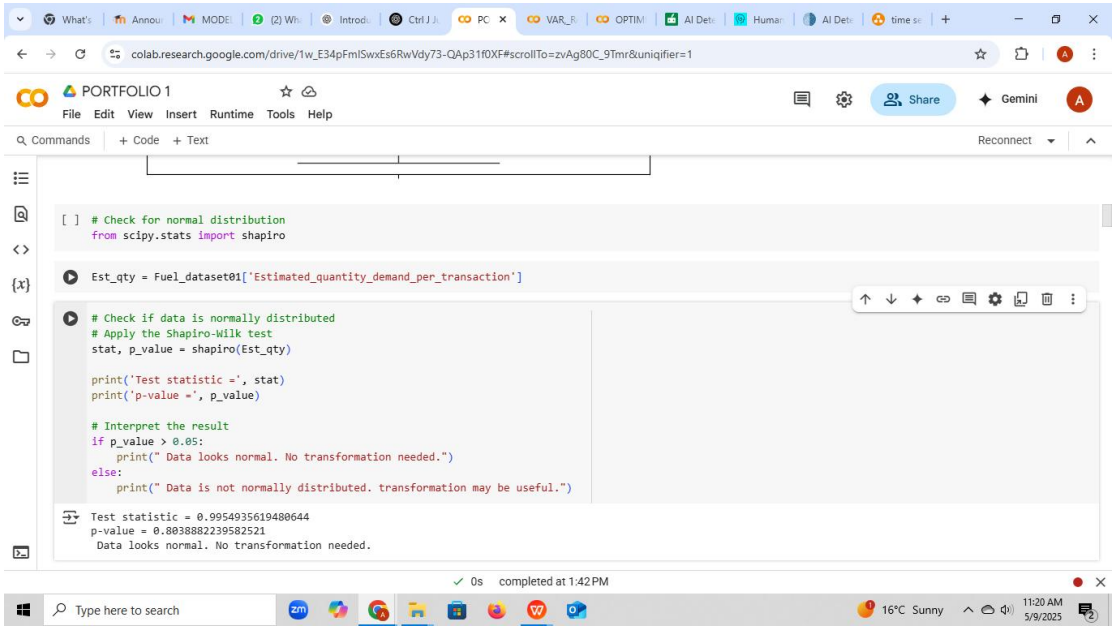



Fig. 7b: check for normal distribution.

Furthermore, a check for outliers is carried out using box plot and as shown in fig. 8a and fig. 8b for the estimated quantity demand per transaction and the motor fuel inflation rate respectively which shows that there is no outlier in the fuel data;

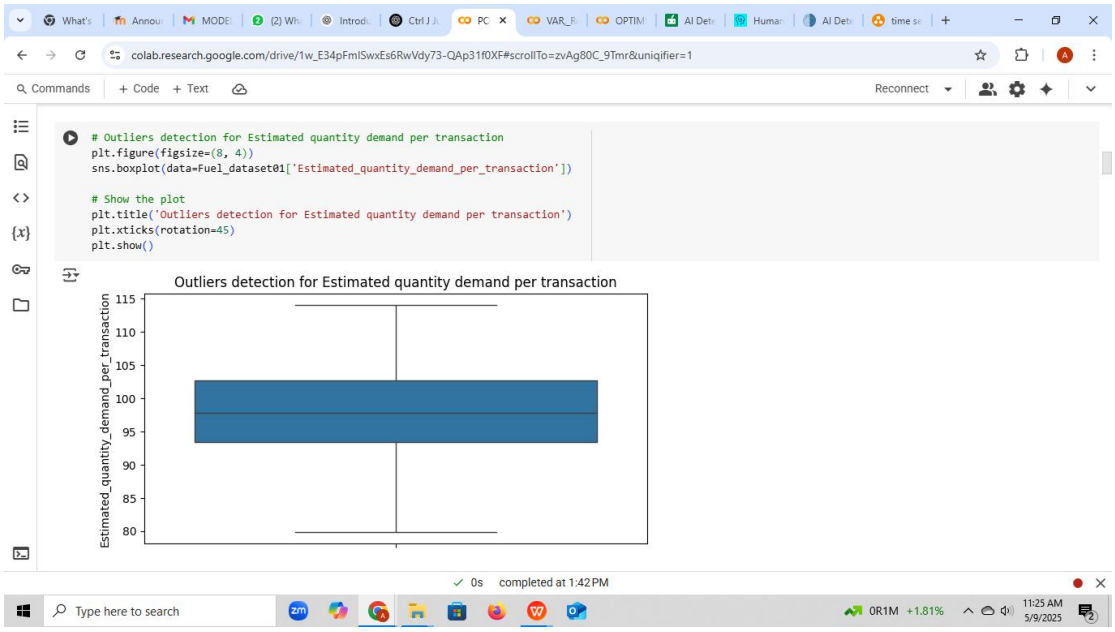



Fig. 8a: outliers detection for estimated quantity demand per transaction

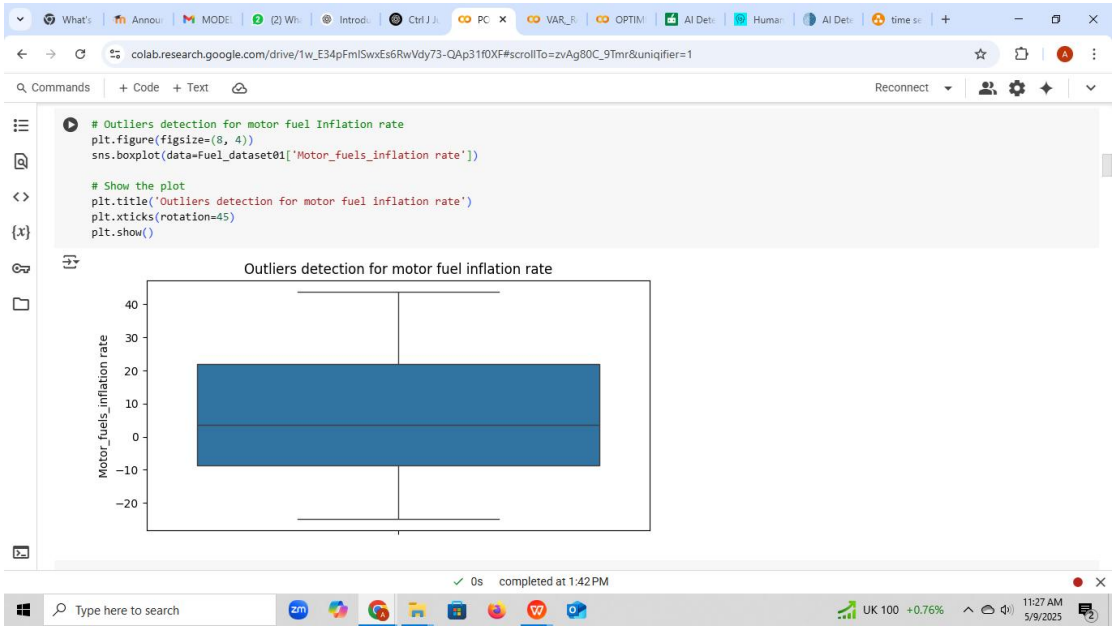



Fig. 8b: outliers detection for motor fuel inflation rate.

## Exploratory data analysis (EDA)

In this study, the EDA starts with a trend plot as shown in fig. 9a to give insight on the patterns of the dependent and independent variables simultaneously as this helps to understand the possible relationship between the two variables by examining their trends;


Fig. 9a: fuel data trend plot

To have a clearly visualisation of the trend plot for both variables together, the rolling smoothing technique is used at window 7 and 14 respectively as shown in fig. 9b and fig. 9c below;

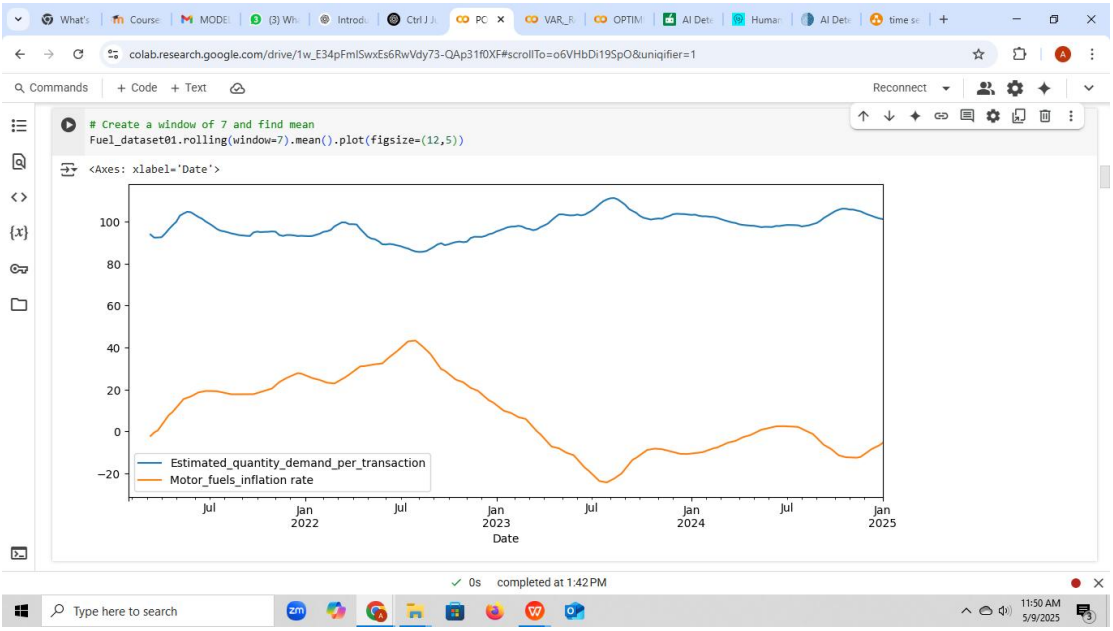



Fig. 9b: fuel data trend plot with rolling window 7

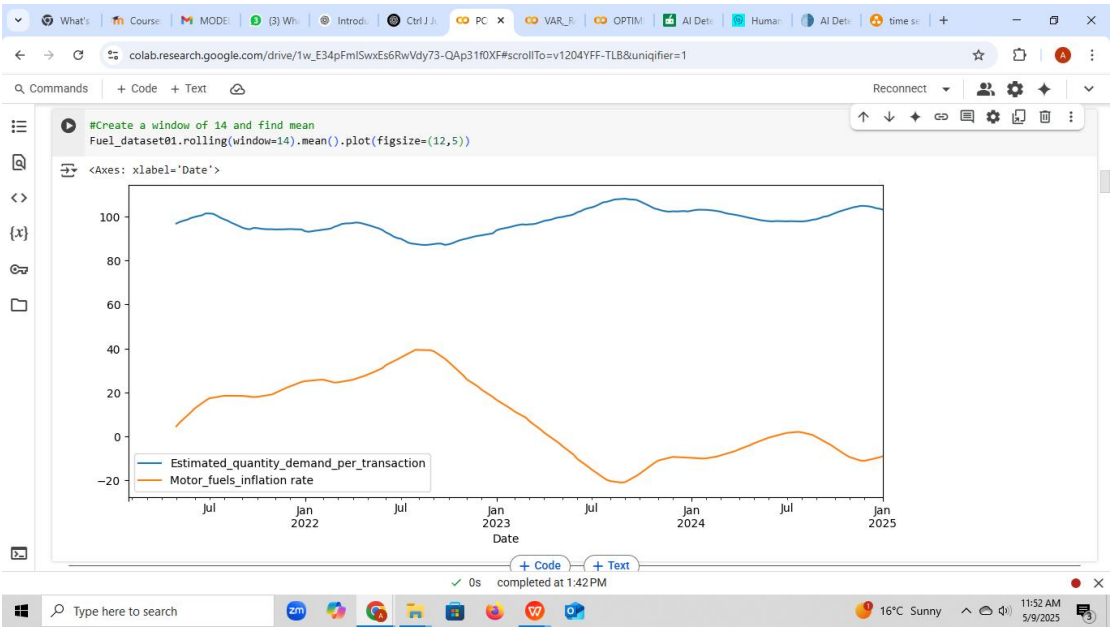



Fig. 9c: fuel data trend plot with rolling window 14.

Furthermore, trend plot is carried out individually on the variables alongside the smoothing plot using the rolling function at window 7 and 14 respectively to have a visual comparison of the original trend plot and the smoothing trend for each variable as shown in fig. 9d and fig. 9e respectively;


Fig. 9d: trend plot comparison for estimated quantity demand per transaction

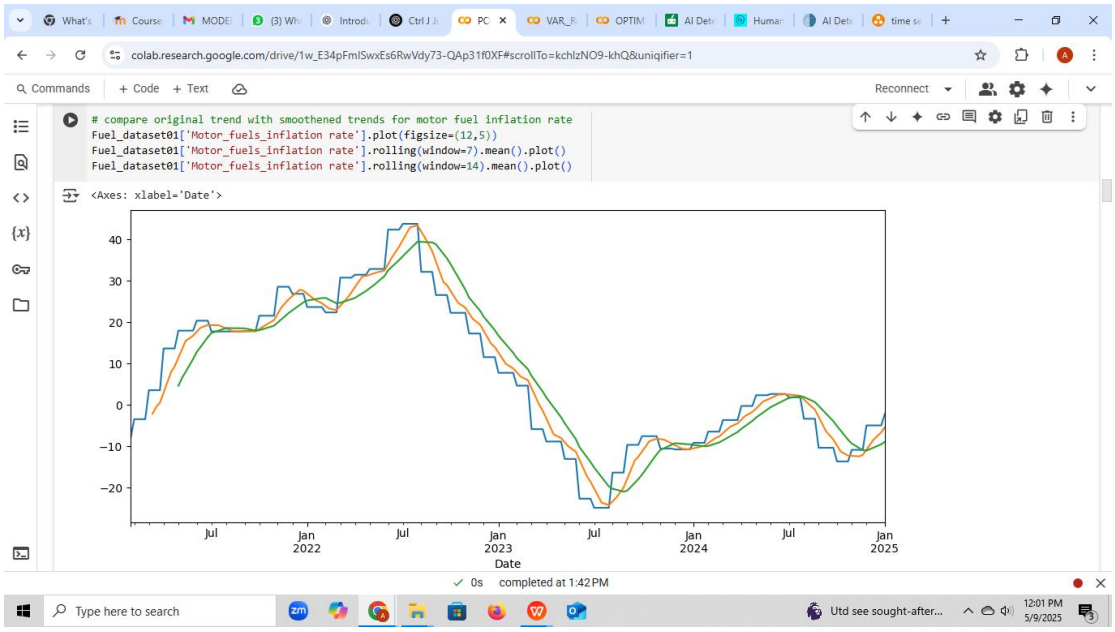



Fig. 9e: trend plot comparison for motor fuel inflation rate.

To have a robust trend plot analysis, the hpfilter technique is employed to check for trend and cycle components as shown below in fig. 9f(1&2), fig. 9g, fig. 9h and fig, 9i for estimated quantity demand per transaction trend and cycle component, and for motor fuel inflation rate trend and cycle components respectively;

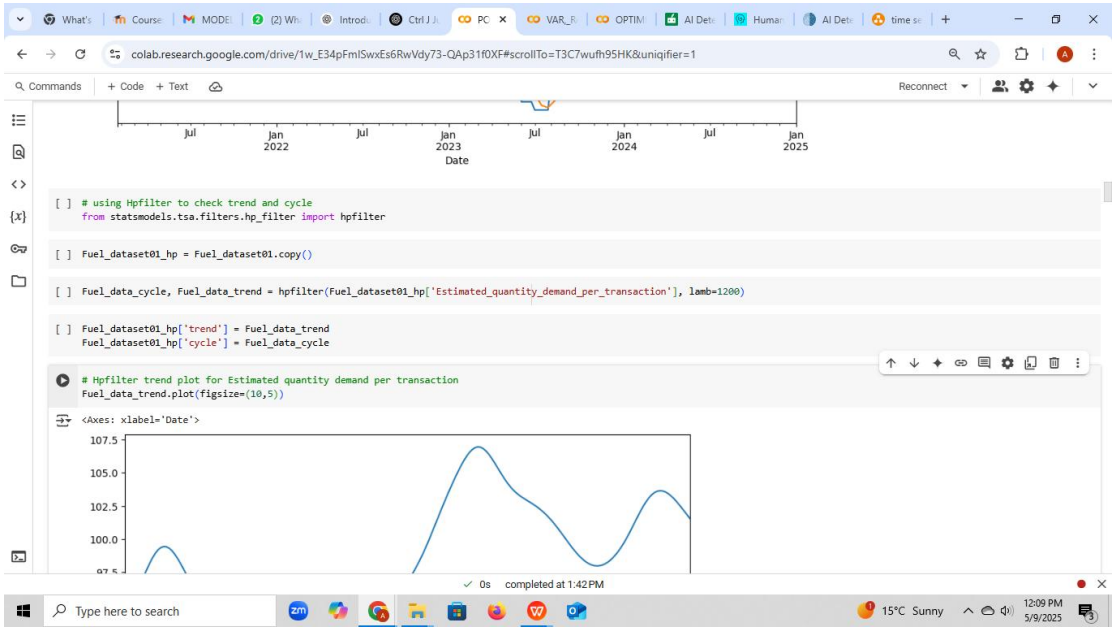



Fig. 9f(1): hpfilter trend plot for estimated quantity demand per transaction

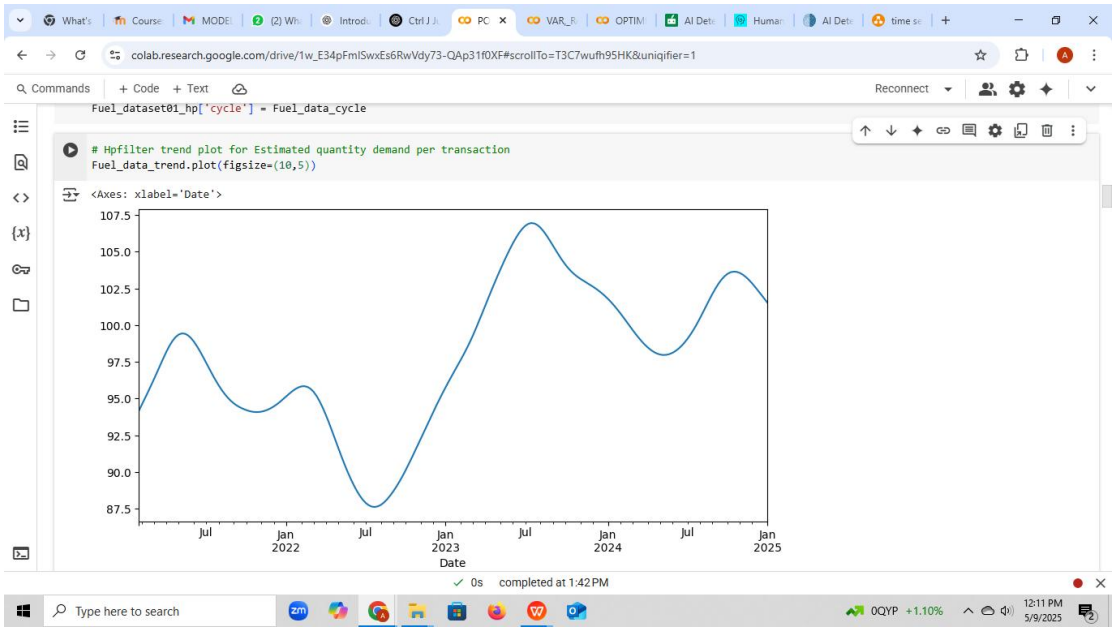



Fig. 9f(2): hpfilter trend plot for estimated quantity demand per transaction


Fig. 9g: hpfilter cycle plot for estimated quantity demand per transaction

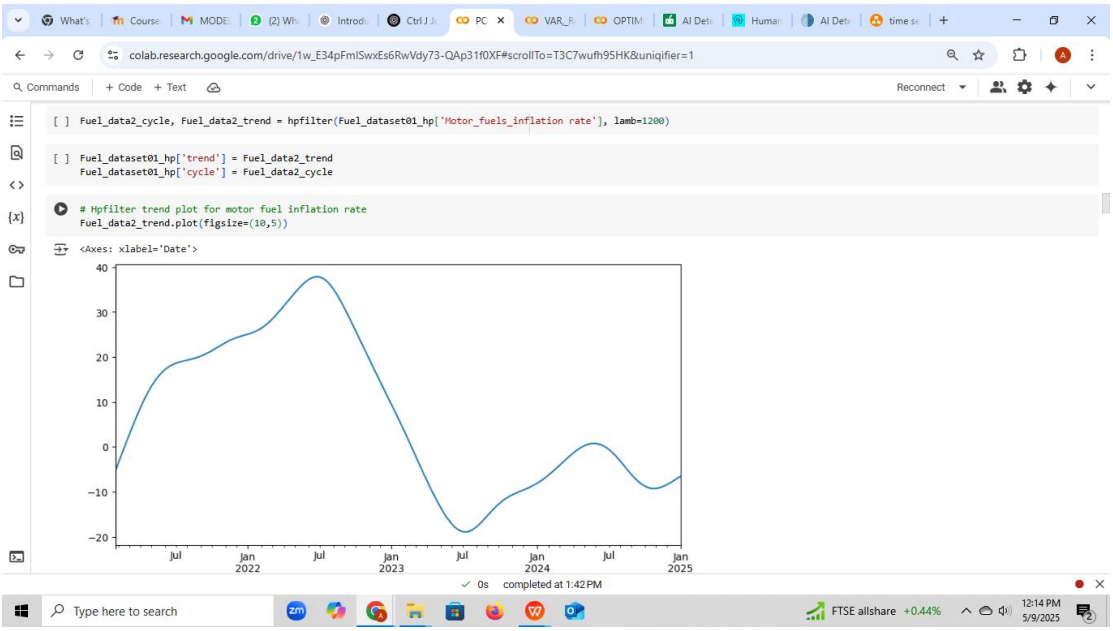



Fig. 9h: hpfilter trend plot for motor fuel inflation rate


Fig. 9i: hpfilter cycle plot for motor fuel inflation rate.

## Trend plot summary

From the trend plot comparison, the hpfilter trend plot gives a smoother and clearly trend visualisation and insight compare to the other trend plots.

The next visual analysis under EDA is the seasonal decomposition. Fig. 10a shows the multiplicative seasonal decomposition of the fuel dataset and fig.10b shows the additive seasonal decomposition. A review of the two seasonal decomposition shows

that the additive model is the ideal model for the fuel data seasonal decomposition as its seasonal effect is constant in magnitude over time ranging from +2 to -2 which implies the seasonal impact adds or subtracts up to 2 units from the baseline. In addition, the residual looks like noise, indicating the model has successfully captured the trend and seasonality. What remains are random, unexplained fluctuations, exactly what a residual should be in a good decomposition.

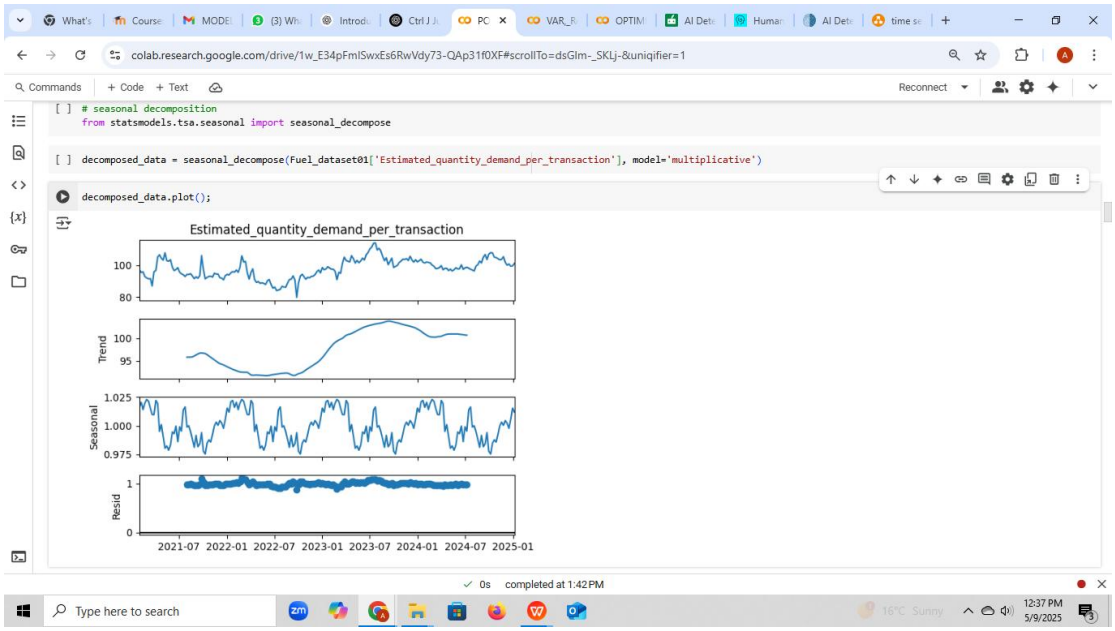



Fig. 10a: seasonal decomposition with multiplicative model

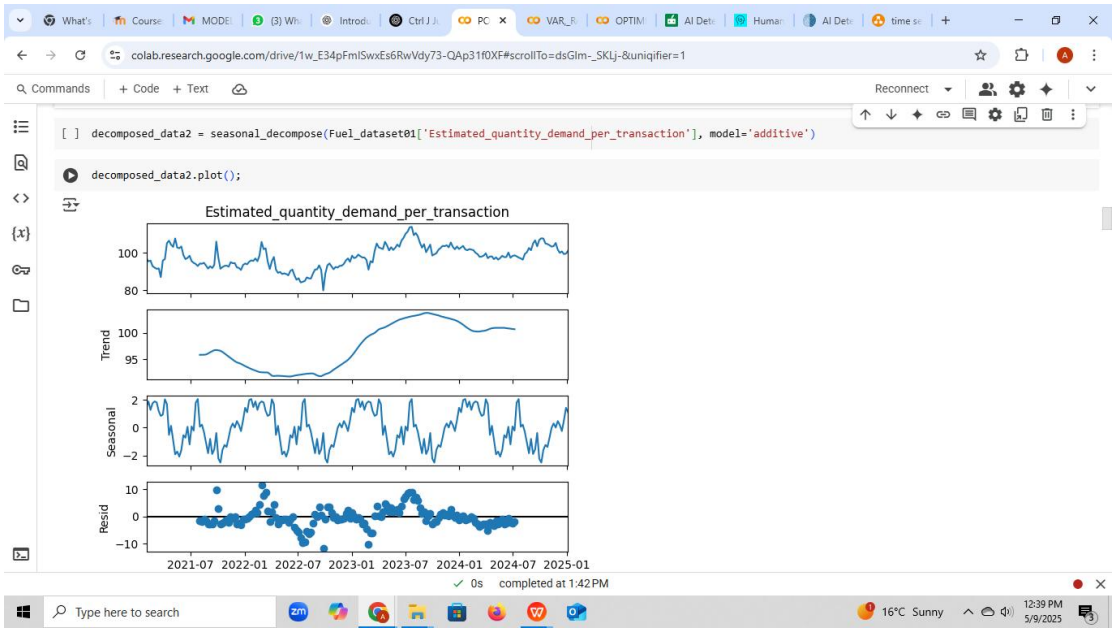



Fig. 10b: seasonal decomposition with additive model

The next step is to check for autocorrelation in the dependent variable using the lagged plot, ACF plot and PACF plot. It can be seen there is autocorrelation from the plots with the lag plot exhibiting a pattern, the ACF plot decaying and the PACF plot showing autocorrelation at lag 1 as shown in fig. 11a, fig. 11b and fig. 11c respectively;

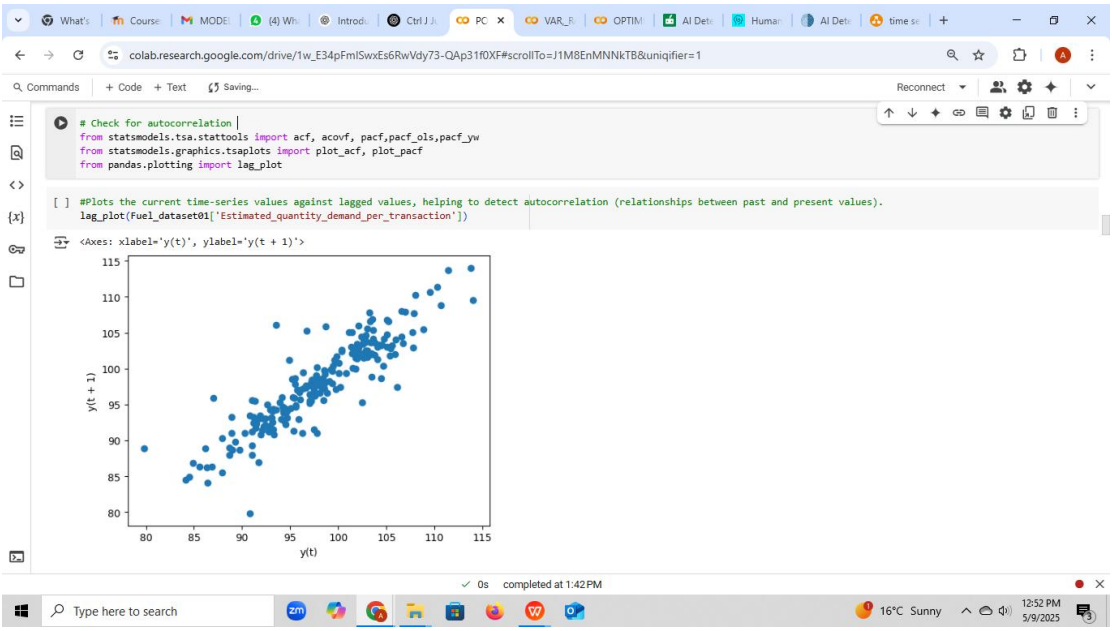



Fig. 11a: lag plot

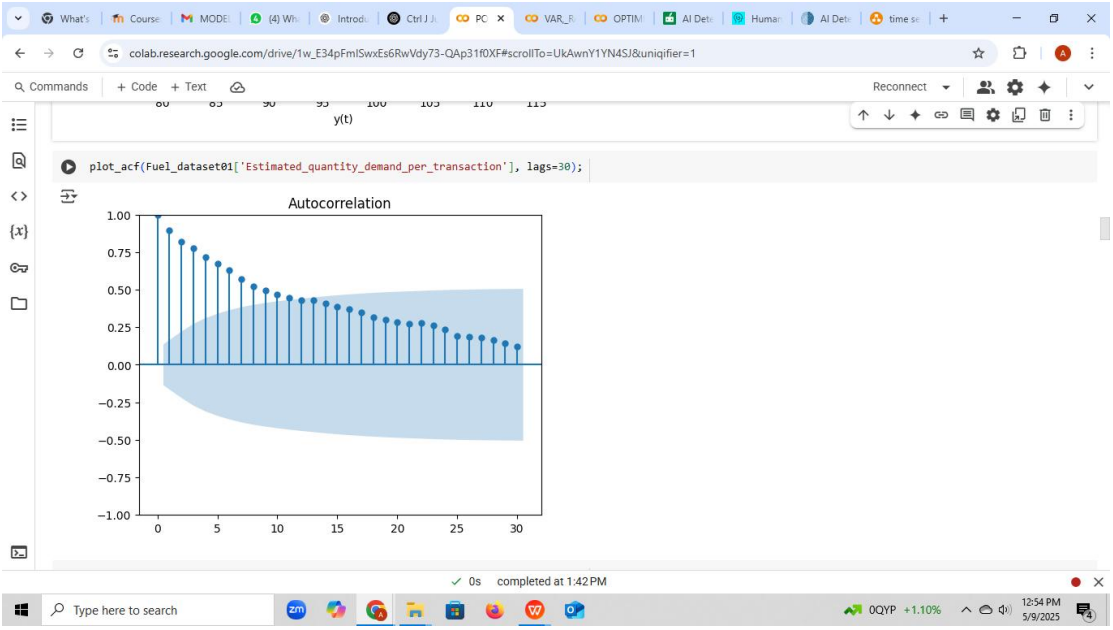



Fig. 11b: ACF plot

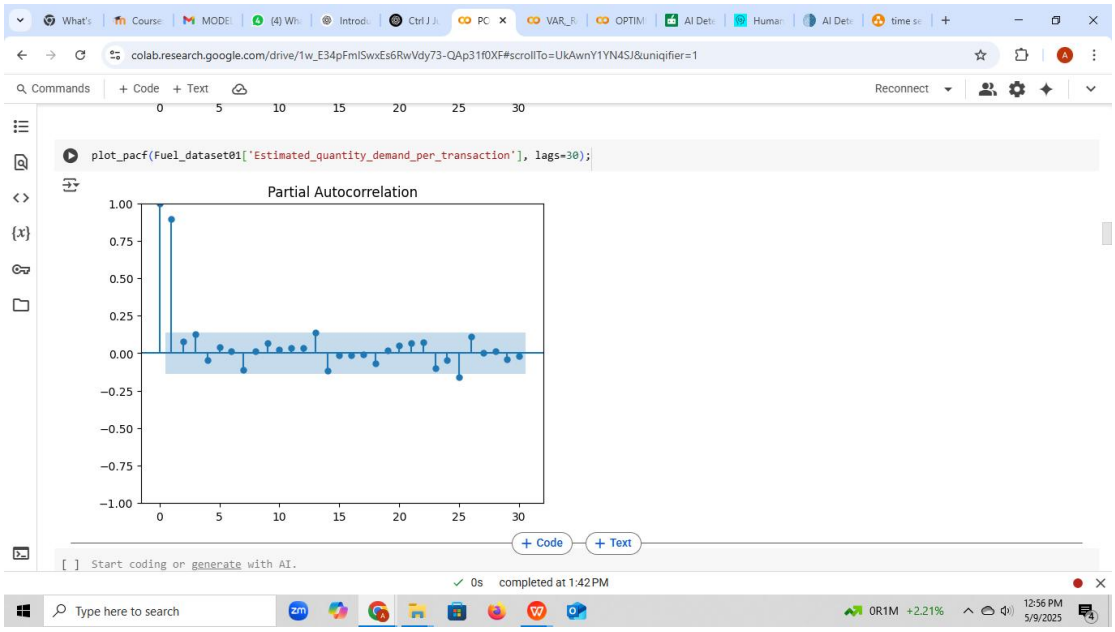



Fig. 11c: PACF plot.

## Stationarity and Granger causality check

The stationarity testing using the Augmented Dickey-Fuller (ADF) test is carried out on the dependent and independent variables to check if the data are stationary before commencing with the building of the selected models as stationarity is a key requirement for any data to meet before modeling. From the stationarity test in fig. 12a and fig. 12b for the dependent and independent variables, it is seen that the p-value for the dependent variable is 0.11 and the p-value for the independent variable is 0.45, both greater than 0.05 indicating that the two variables data are non stationary;

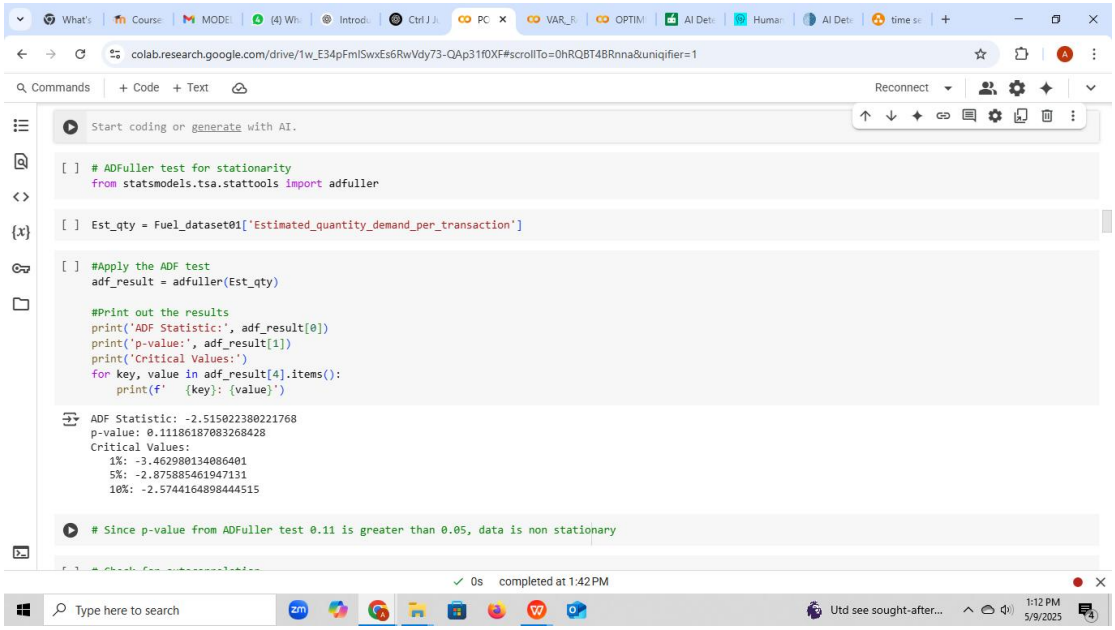


Fig. 12a: ADF test for estimated quantity demand per transaction

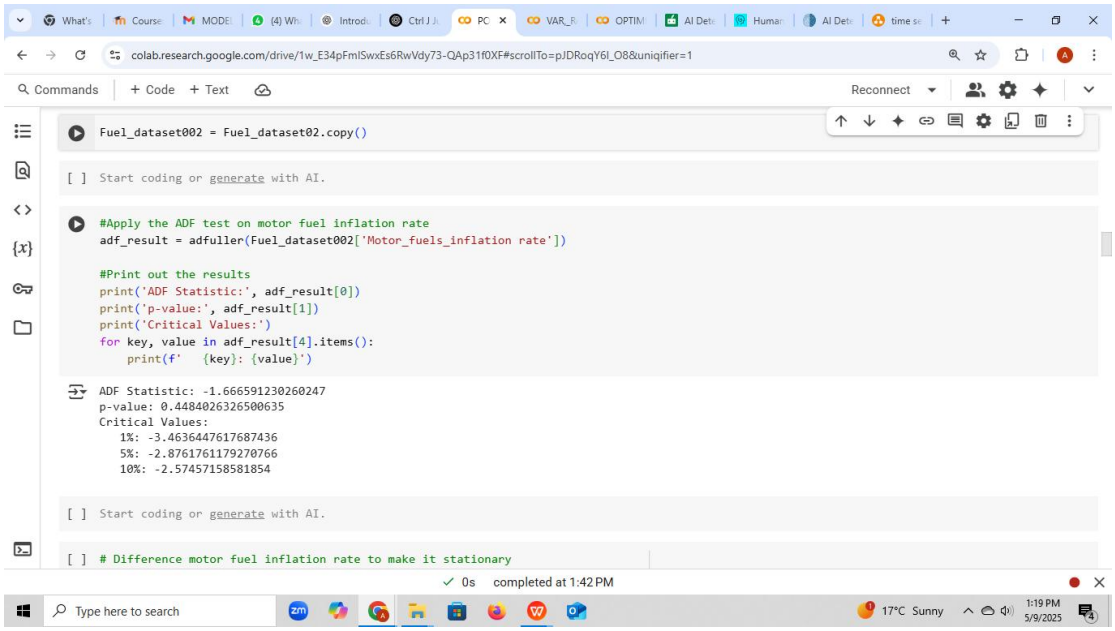



Fig. 12b: ADF test for motor fuel inflation rate.

The next step is to transform the data by differencing the two variables to make them stationary. The ADF test is repeated on the differenced data and it can be seen in fig. 12c and fig. 12d that the data are now stationary with 2.96e-23 as the p-value for the dependent variable and 0.002 as the p-value for the independent variable, both less than 0.05 and can be used for building the model. Fig. 12e displays the fuel data with the differenced columns;

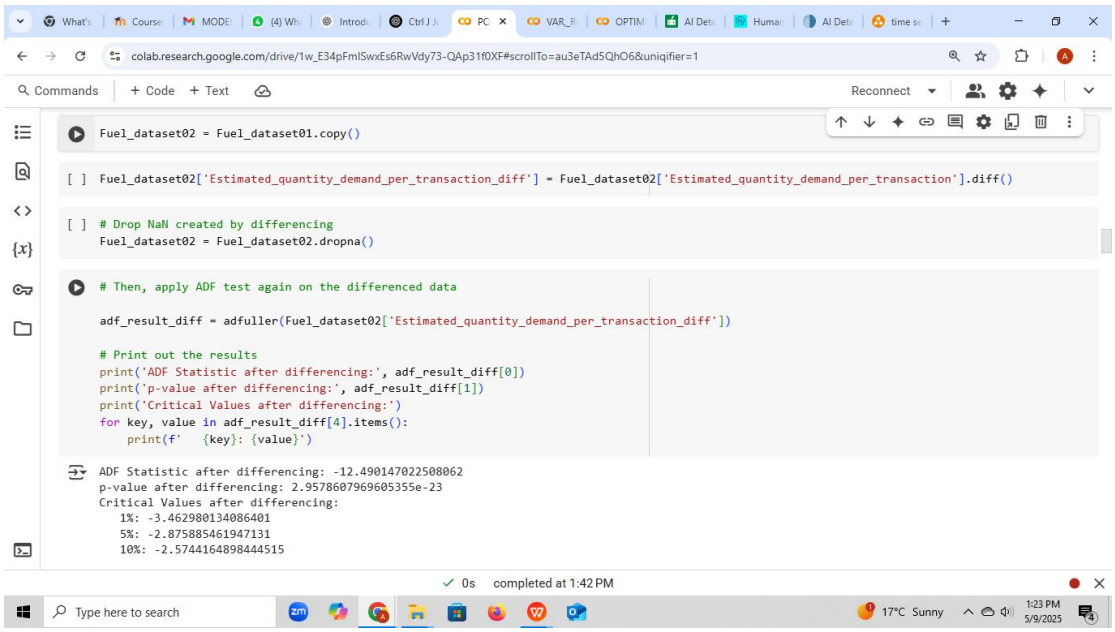



Fig. 12c: repeated ADF test for quantity demand per transaction diff

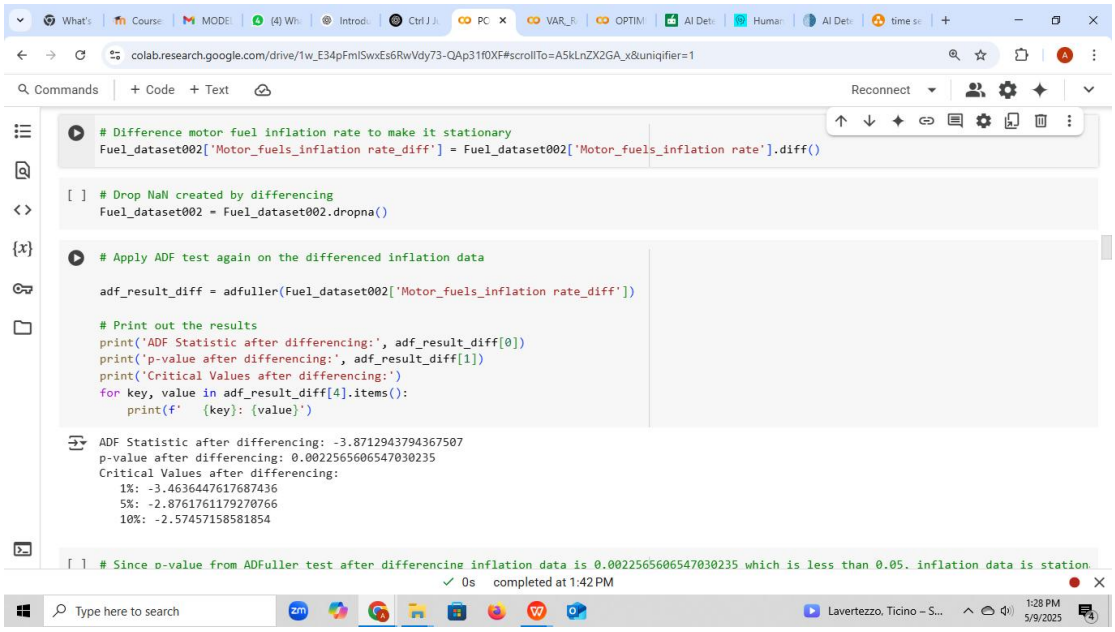



Fig. 12d: repeated ADF test for motor fuel inflation rate diff

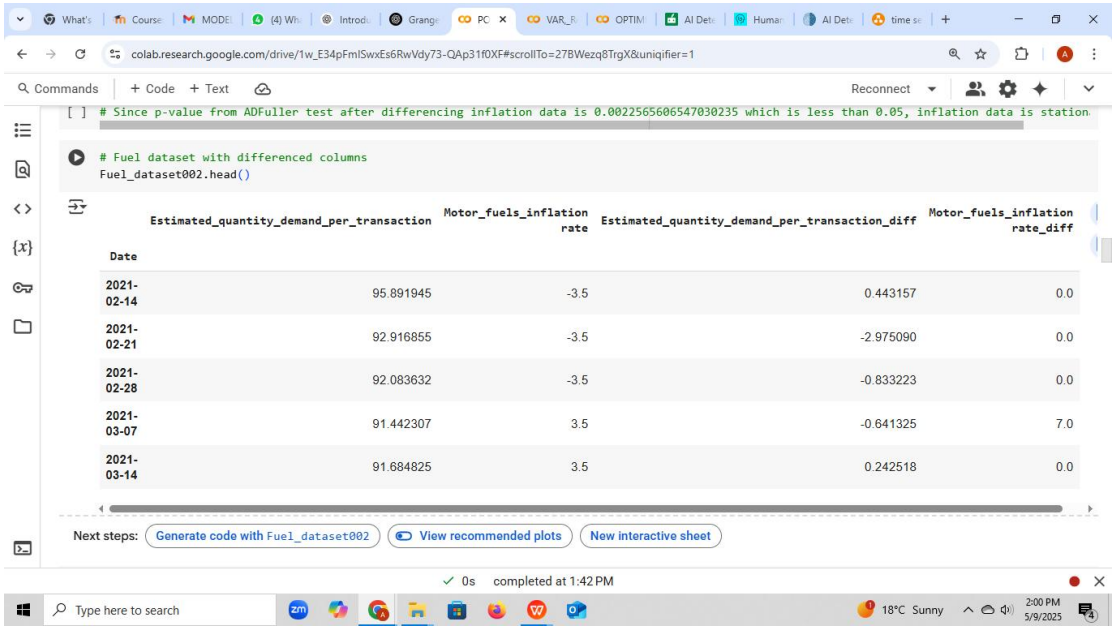



Fig. 12e: fuel data with differenced columns

## Granger causality

To check for causation of one variable on the other, the Granger causality test is carried out on the differenced data using maxlag at 5. Based on p-values across all tests, Granger causality test shows no significance at lag 1 as the p-value 0.74 is greater than 0.05. Lag 4 provides the strongest evidence of Granger causality, with the lowest p-values ( p = 0.0016 to 0.0030). While lags 2, 3, and 5 also show

significance, lag 4 has the most consistently low p-values, making it the best lag in terms of statistical strength as shown in fig. 13a and fig, 13b;

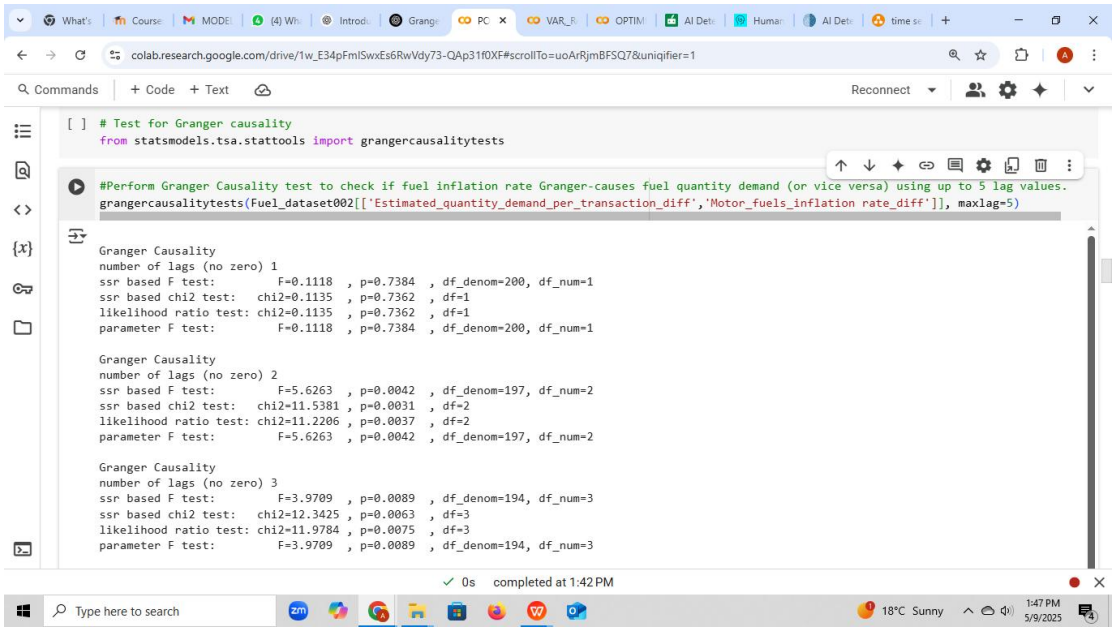



Fig. 13a: Granger causality test output

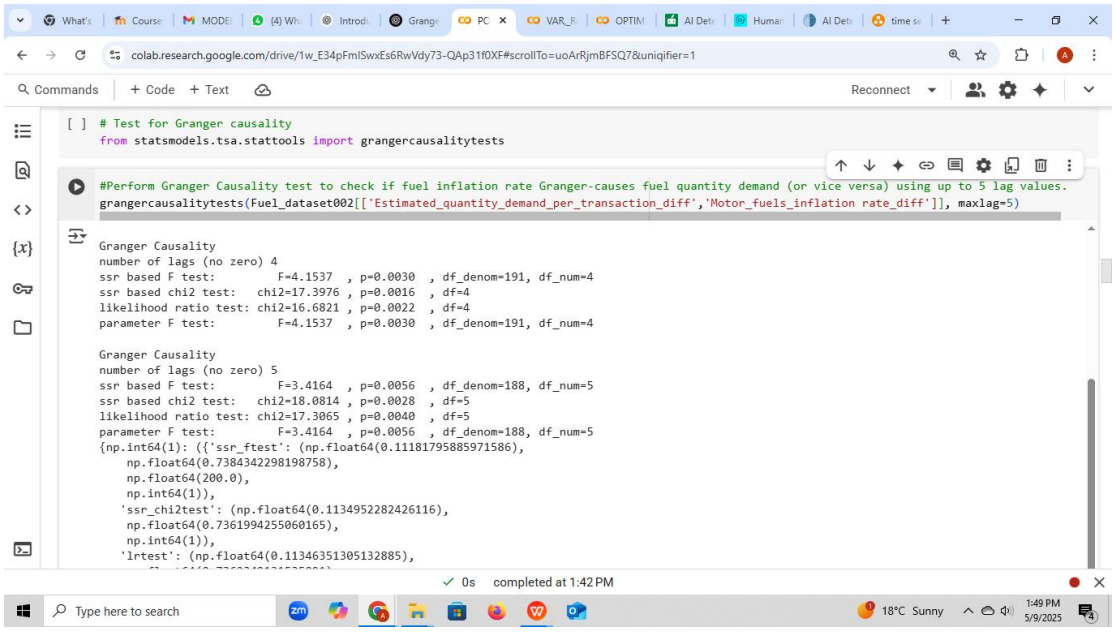



Fig. 13b: Granger causality test output.

Check for autocorrelation after differencing using the autocorrelation plots as shown in fig.14a, fig.14b and fig.14c for the lag plot, ACF plot and PACF plot , also with the Durbin -Watson test with the DW statistic = 2.23 in fig. 14d show insignificant autocorrelation;

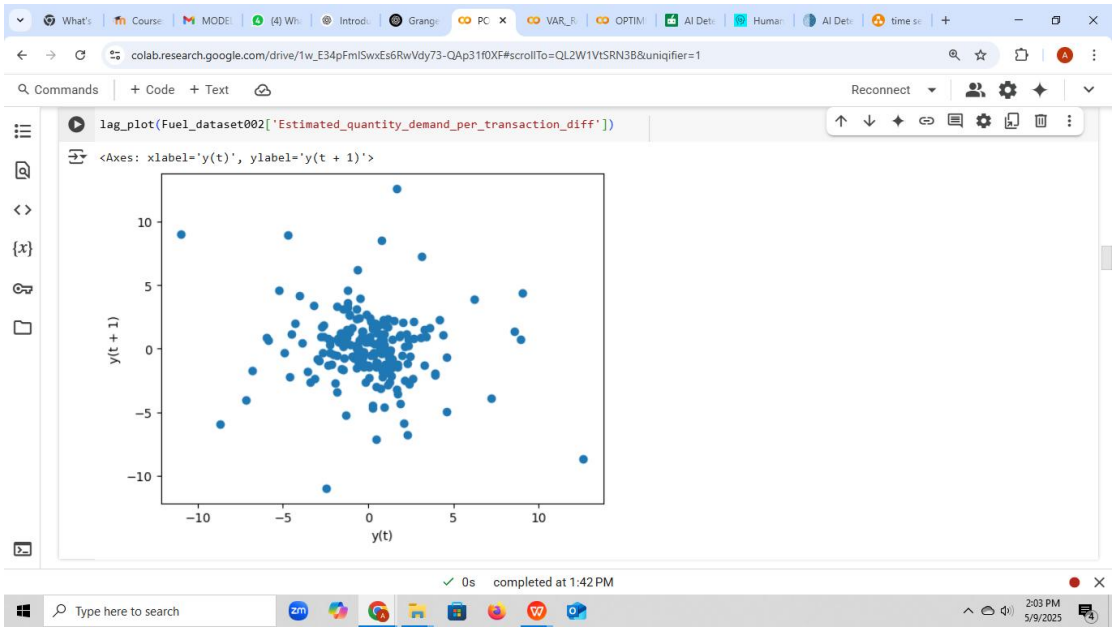



Fig. 14a: lag plot after differencing

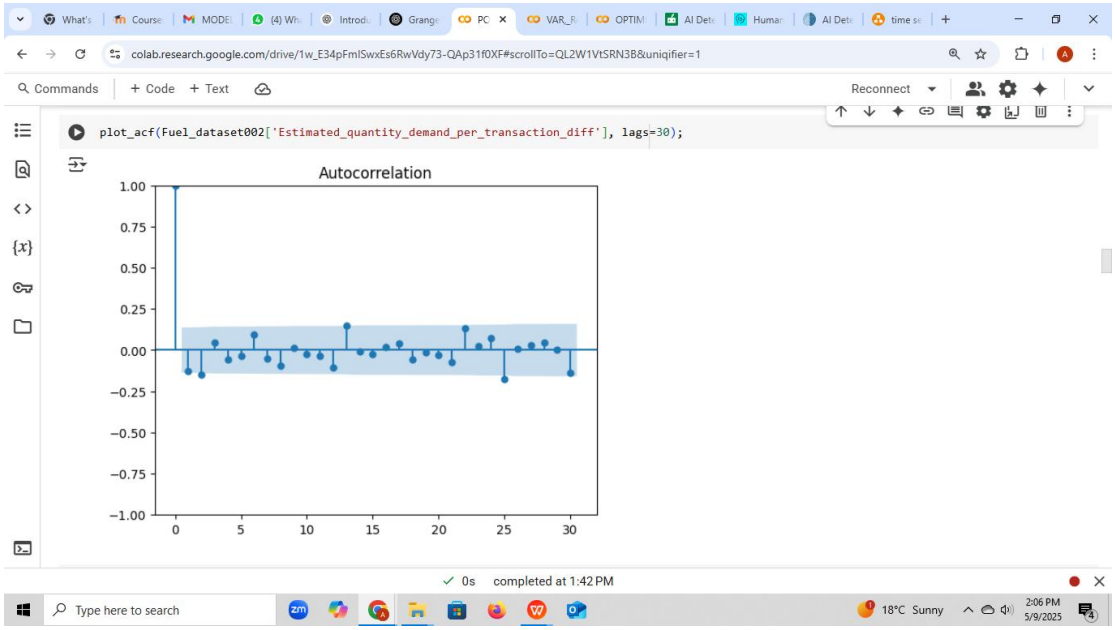



Fig. 14b: ACF plot after differencing

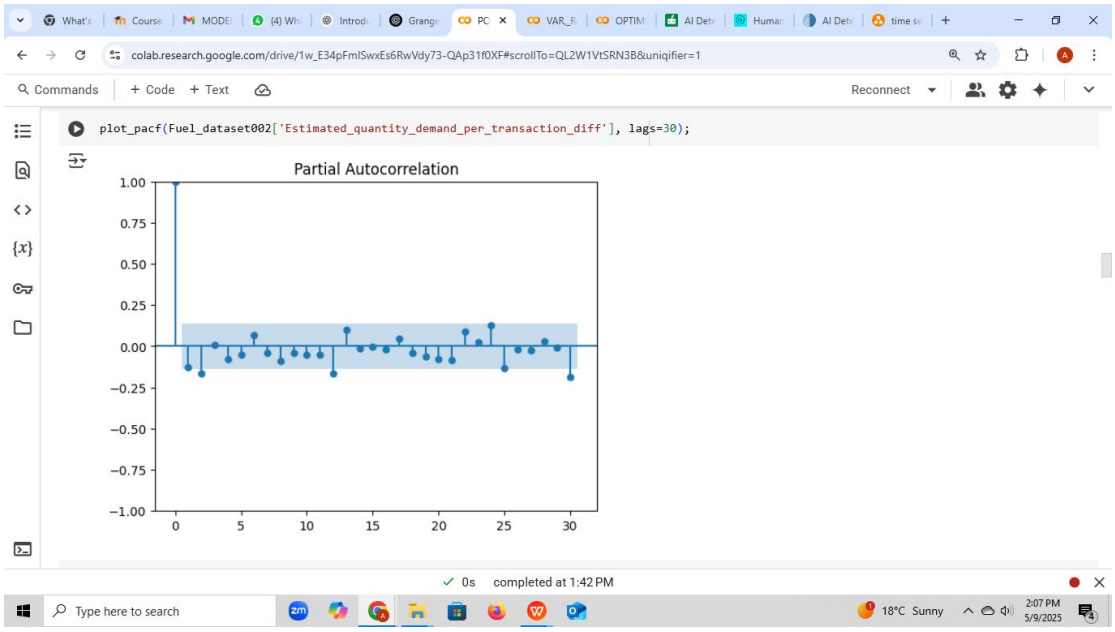



Fig. 14c: PACF plot after differencing

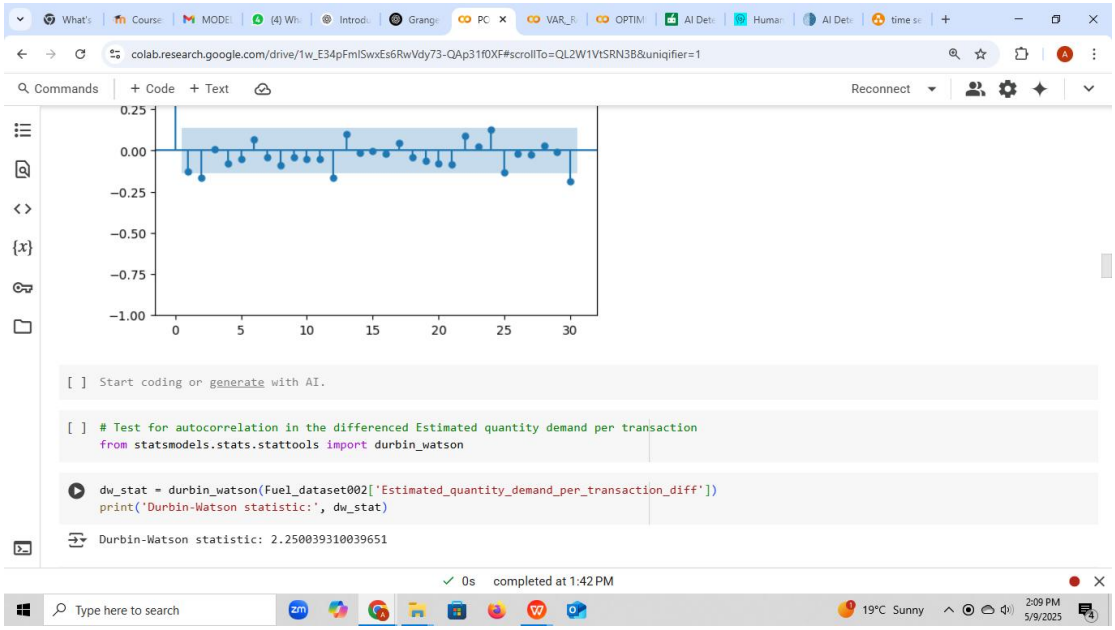



Fig.14d: Durbin watson test

## Model selection, training, validation and evaluation

In this study, the following models; Exponential smoothing, ARIMA, SARIMA, SARIMAX and Vector Autoregression (VAR) are considered with the aim of selecting the best fit model for forecasting via error evaluation metrics.

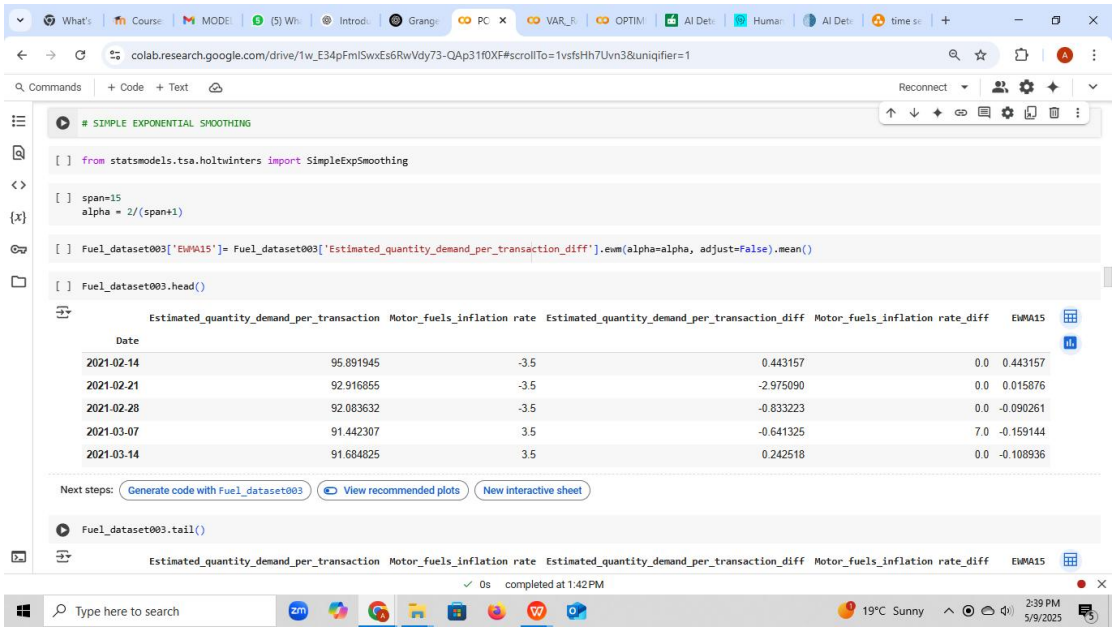
## Exponential Smoothing

The following models are evaluated under Exponential smoothing; Simple exponential

smoothing, Double exponential smoothing and Triple exponential smoothing (Holt-Winters).

## Simple Exponential Smoothing (SES).

Below is a concise visual summary of the modeling process for SES with a span of 15



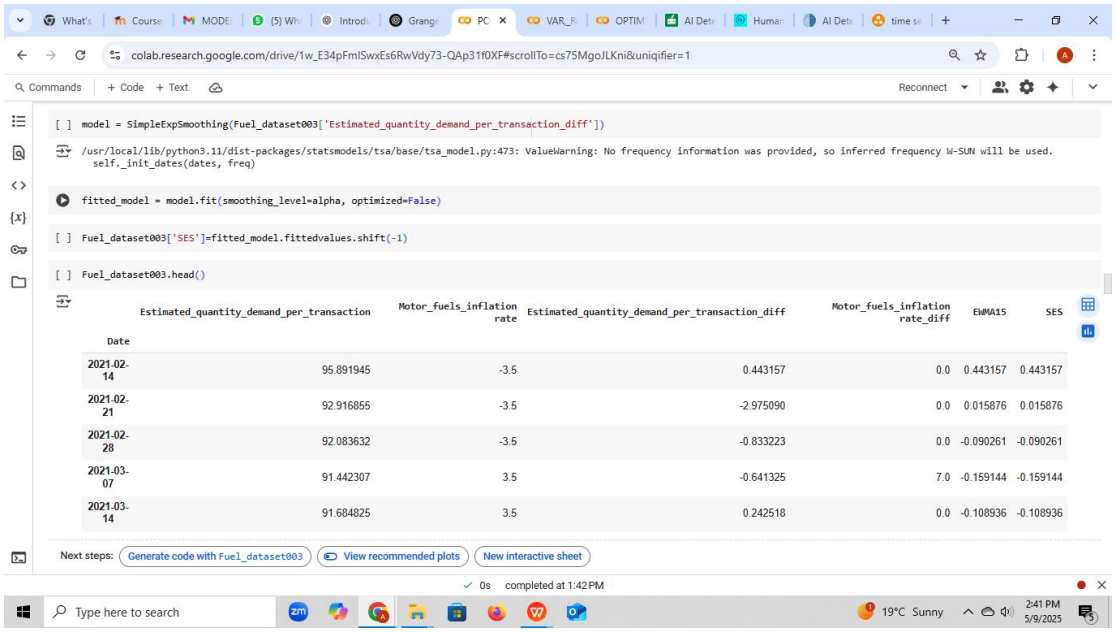
```
# SIMPLE EXPONENTIAL SMOOTHING

from statsmodels.tsa.holtwinters import SimpleExpSmoothing

span=15
alpha = 2/(span+1)

Fuel_dataset003['EWMA15']= Fuel_dataset003['Estimated_quantity_demand_per_transaction_diff'].ewm(alpha=alpha, adjust=False).mean()

Fuel_dataset003.head()
```

| Date | Estimated_quantity_demand_per_transaction | Motor_fuels_inflation rate | Estimated_quantity_demand_per_transaction_diff | Motor_fuels_inflation rate_diff | EWMA15 |
|---|---|---|---|---|---|
| 2021-02-14 | 95.891945 | -3.5 | 0.443157 | 0.0 | 0.443157 |
| 2021-02-21 | 92.916855 | -3.5 | -2.975090 | 0.0 | 0.015876 |
| 2021-02-28 | 92.083632 | -3.5 | -0.833223 | 0.0 | -0.090261 |
| 2021-03-07 | 91.442307 | 3.5 | -0.641325 | 7.0 | -0.159144 |
| 2021-03-14 | 91.684825 | 3.5 | 0.242518 | 0.0 | -0.108936 |

Next steps: Generate code with Fuel_dataset003 | View recommended plots | New interactive sheet

```
Fuel_dataset003.tail()
```

Fig. 15a: Exponential weighted moving average



```
model = SimpleExpSmoothing(Fuel_dataset003['Estimated_quantity_demand_per_transaction_diff'])

/usr/local/lib/python3.11/dist-packages/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency W-SUN will be used.
  self._init_dates(dates, freq)

fitted_model = model.fit(smoothing_level=alpha, optimized=False)

Fuel_dataset003['SES']=fitted_model.fittedvalues.shift(-1)

Fuel_dataset003.head()
```

| Date | Estimated_quantity_demand_per_transaction | Motor_fuels_inflation rate | Estimated_quantity_demand_per_transaction_diff | Motor_fuels_inflation rate_diff | EWMA15 | SES |
|---|---|---|---|---|---|---|
| 2021-02-14 | 95.891945 | -3.5 | 0.443157 | 0.0 | 0.443157 | 0.443157 |
| 2021-02-21 | 92.916855 | -3.5 | -2.975090 | 0.0 | 0.015876 | 0.015876 |
| 2021-02-28 | 92.083632 | -3.5 | -0.833223 | 0.0 | -0.090261 | -0.090261 |
| 2021-03-07 | 91.442307 | 3.5 | -0.641325 | 7.0 | -0.159144 | -0.159144 |
| 2021-03-14 | 91.684825 | 3.5 | 0.242518 | 0.0 | -0.108936 | -0.108936 |

Next steps: Generate code with Fuel_dataset003 | View recommended plots | New interactive sheet

Fig. 15b: Simple exponential smoothing (SES) model


Fig. 15c: SES and estimated quantity demand plot

## Summary

From the plot, the SES model captures only the level with no trend and seasonality.

## Double Exponential Smoothing

Below is a concise visual summary of the modeling process for DES

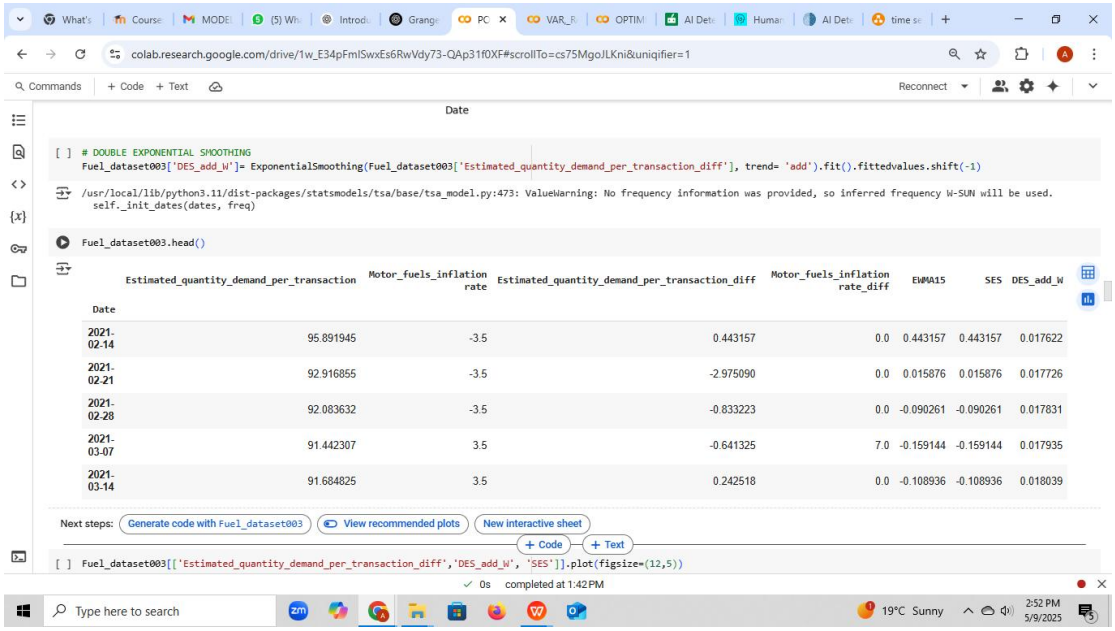



Fig. 16a: DES with linear additive trend

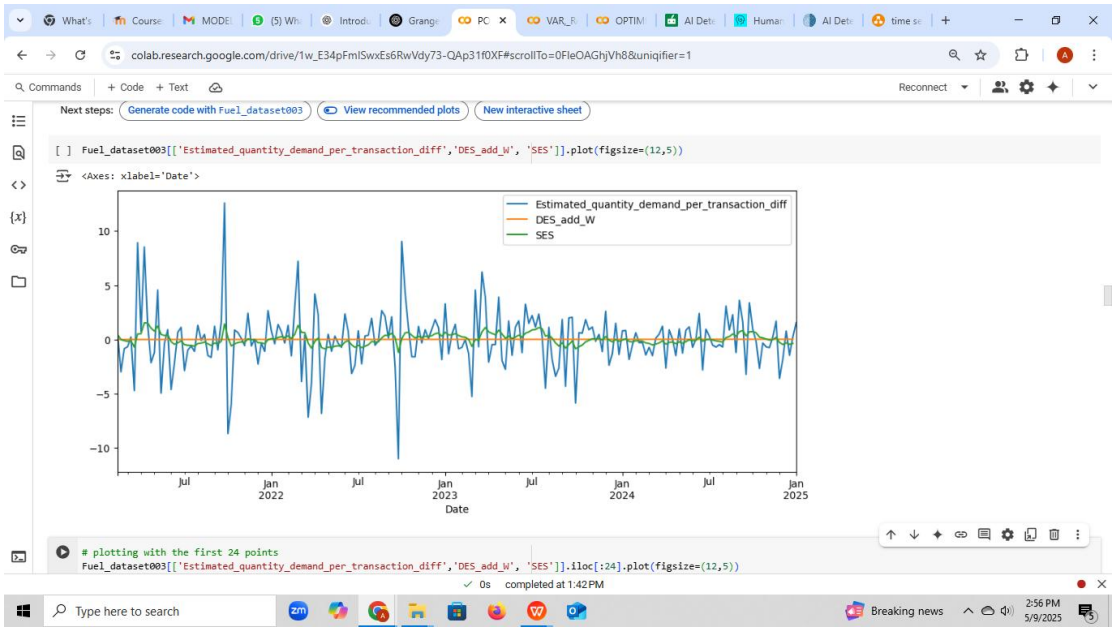



Fig. 16b: DES, SES and estimated quantity demand plot

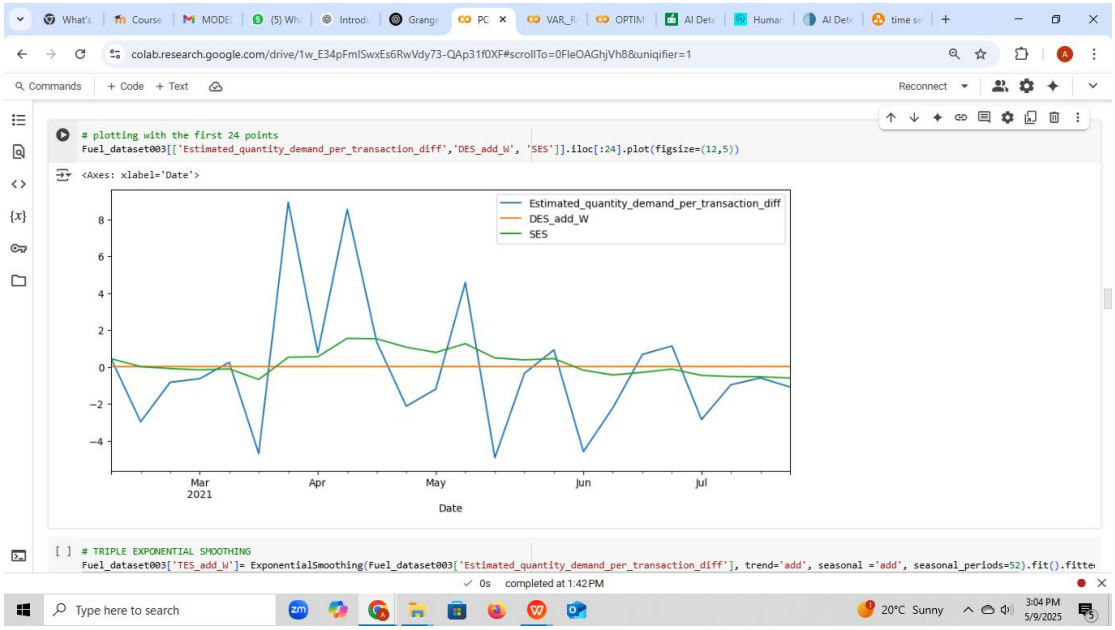



Fig. 16c: DES, SES and estimated quantity demand plot with first 24 points

## Summary

From the visual insight, the model captures only the level and trend without the seasonality resulting to a straight line on the plot.

## Triple Exponential Smoothing (Holt Winters)

Below is a concise visual summary of the modeling process for TES;

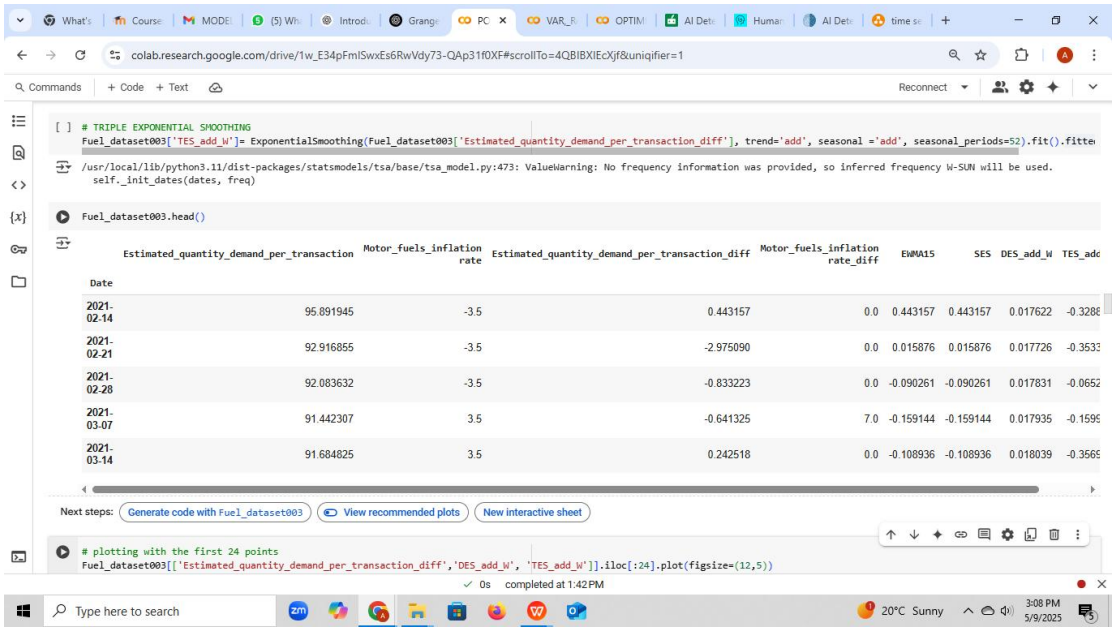



Fig. 17a: TES with linear additive trend and seasonality

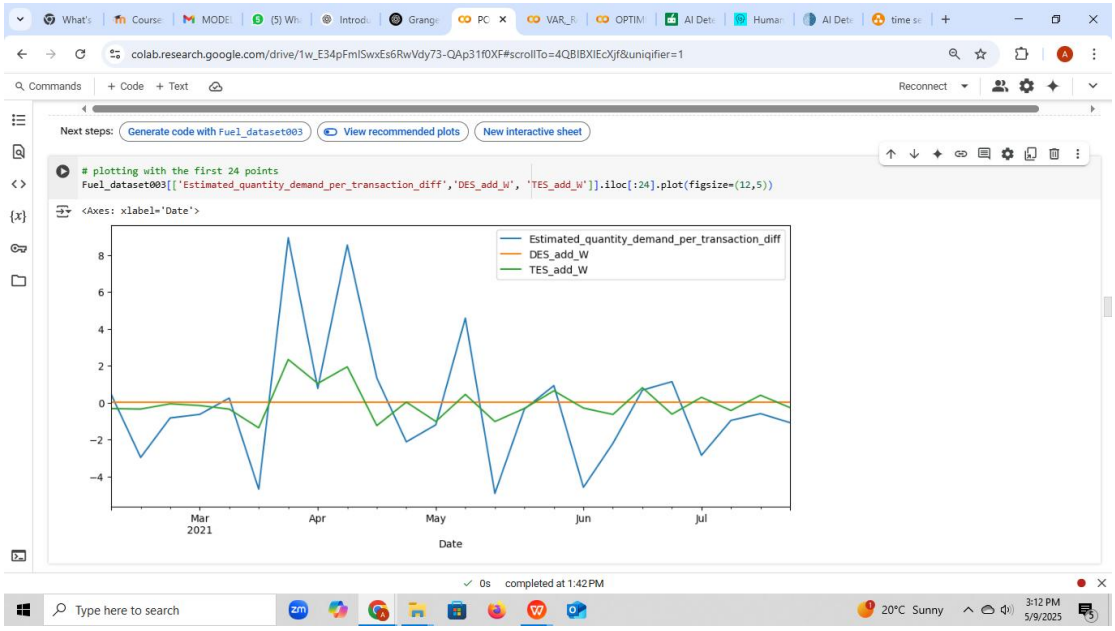



Fig. 17b: TES, DES and estimated quantity demand plot with first 24 points

From the plot, the triple exponential smoothing captures the seasonality thereby making it a better fit for exponential smoothing model. The next step captures the model training, validation and evaluation as shown in the figures below. The fuel data is partitioned into train data and test data with 78% (159 data points) for training and 22% (45 data points) for testing and validation of the TES model;

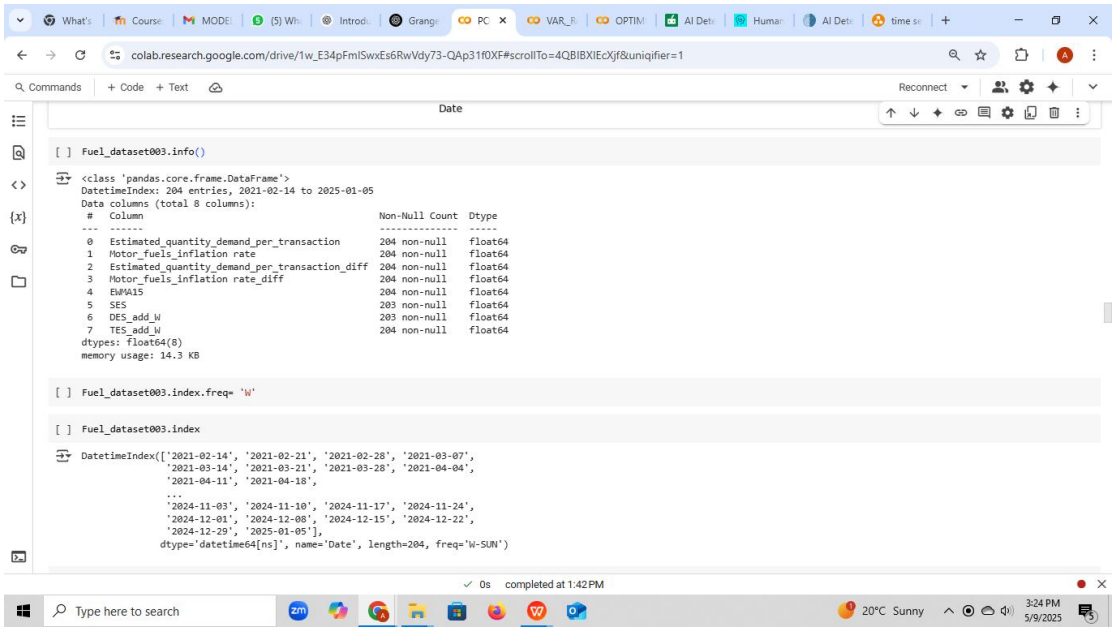



Fig. 17c: defining fuel data index frequency

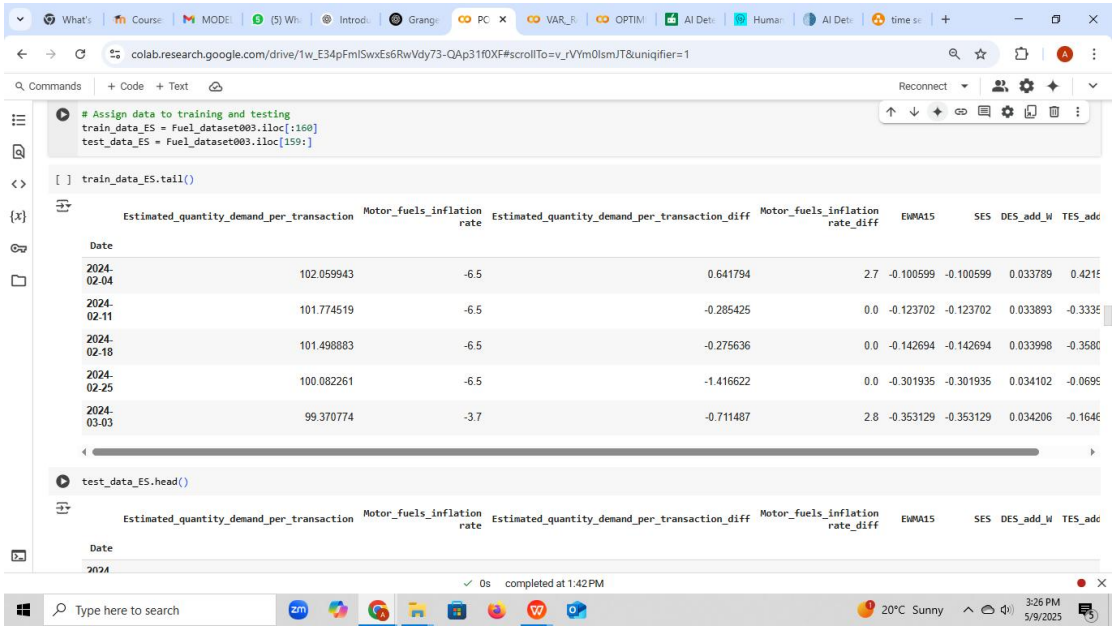



Fig. 17d(1): split data to train and test set


Fig. 17d(2): split data to train and test set

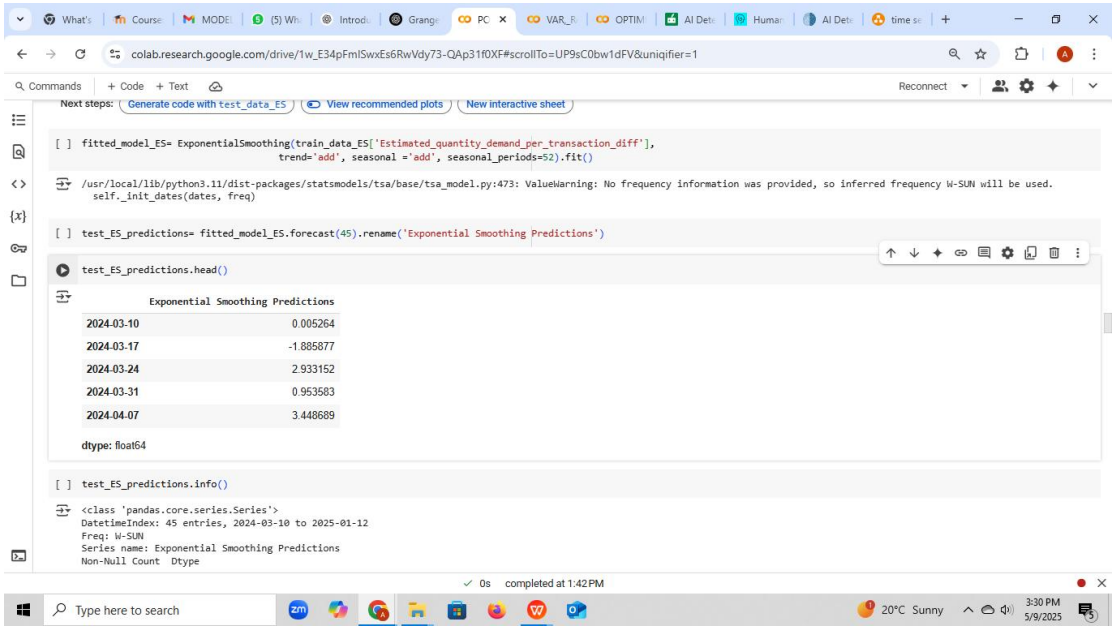



Fig. 17e: TES fit model and test prediction


Fig. 17f: TES train, test and prediction data plot

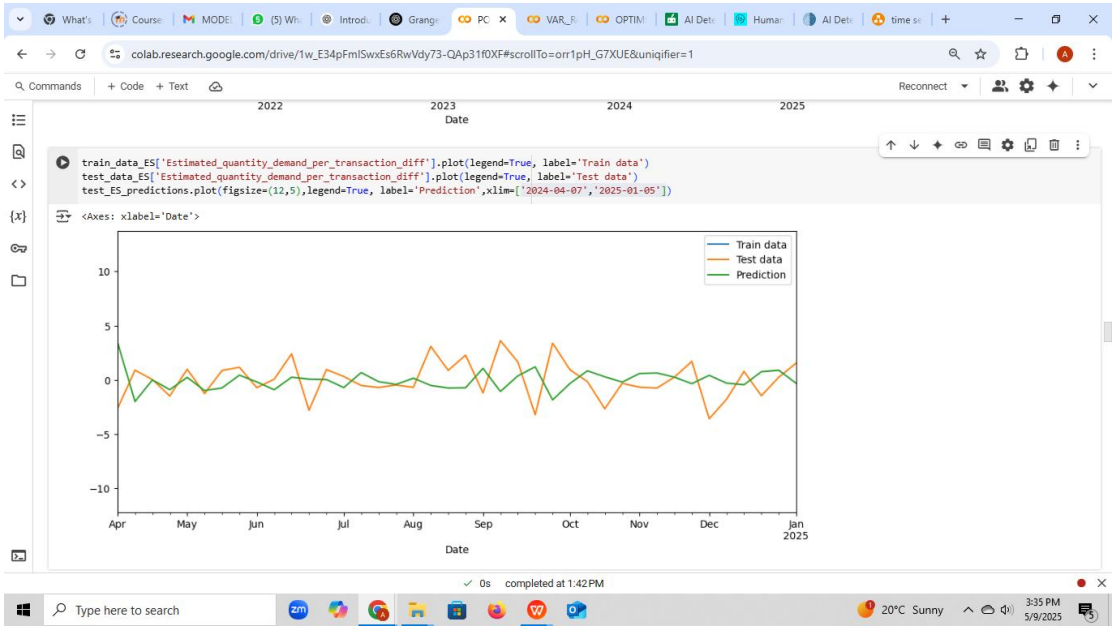



Fig. 17g: TES test and prediction data plot

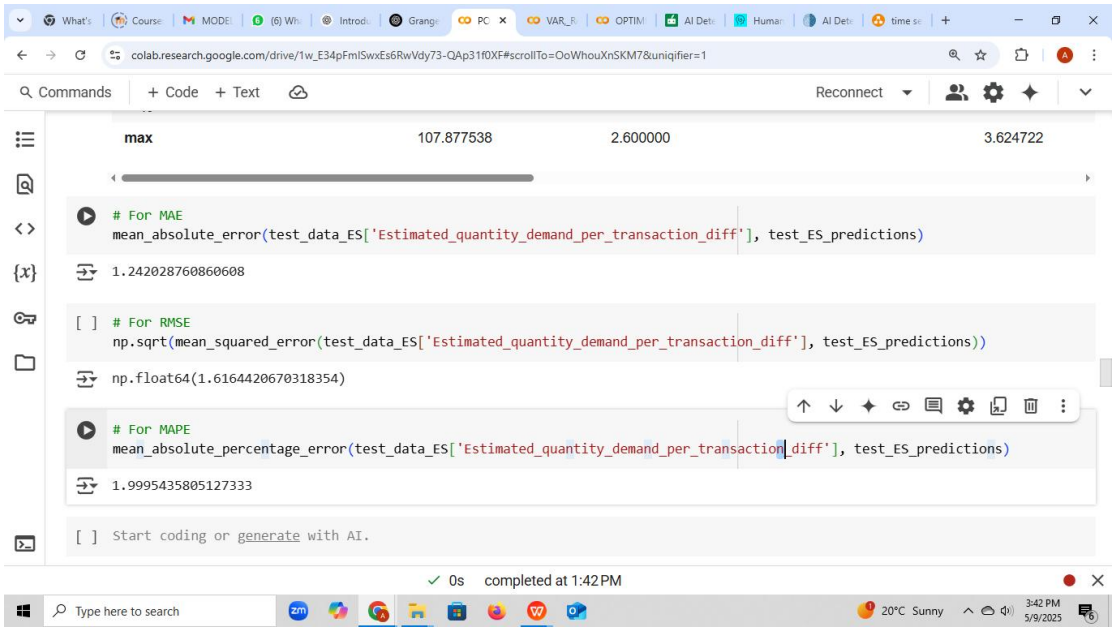



Fig. 17h: TES error evaluation

## ARIMA

Below is a concise visual summary of the modeling process for ARIMA;

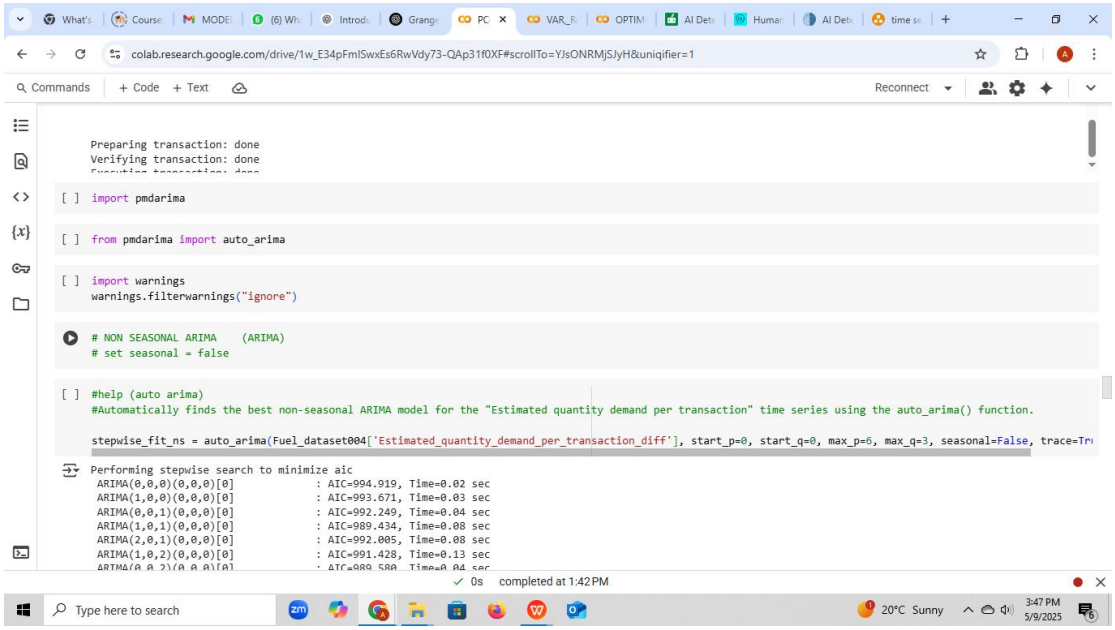



Fig. 18a(1): find best non seasonal ARIMA model

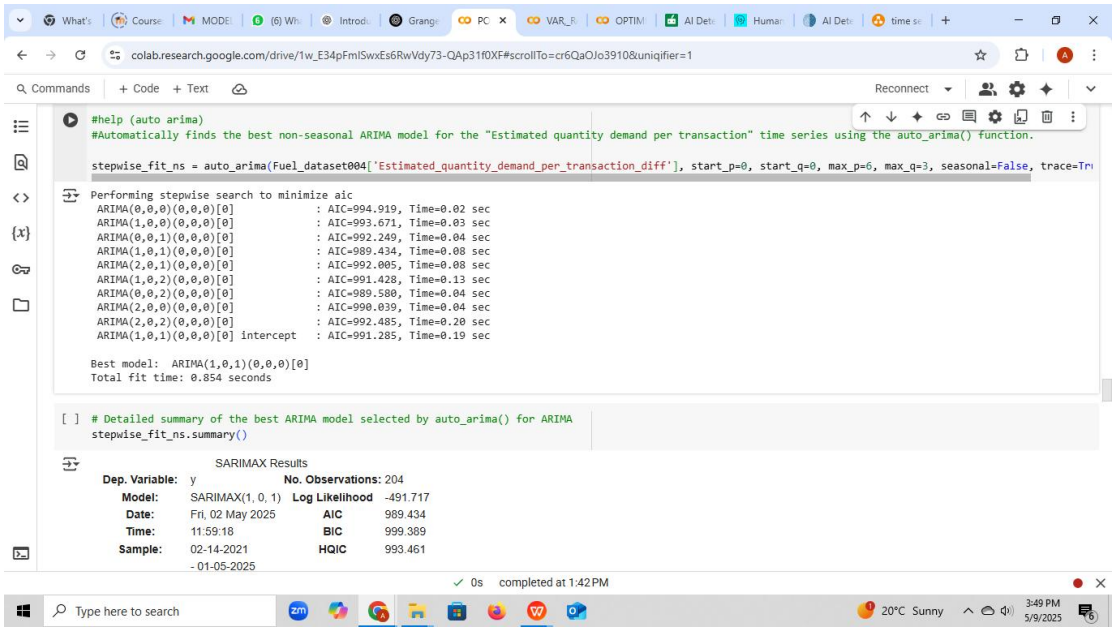



Fig. 18a(2): find best non seasonal ARIMA model

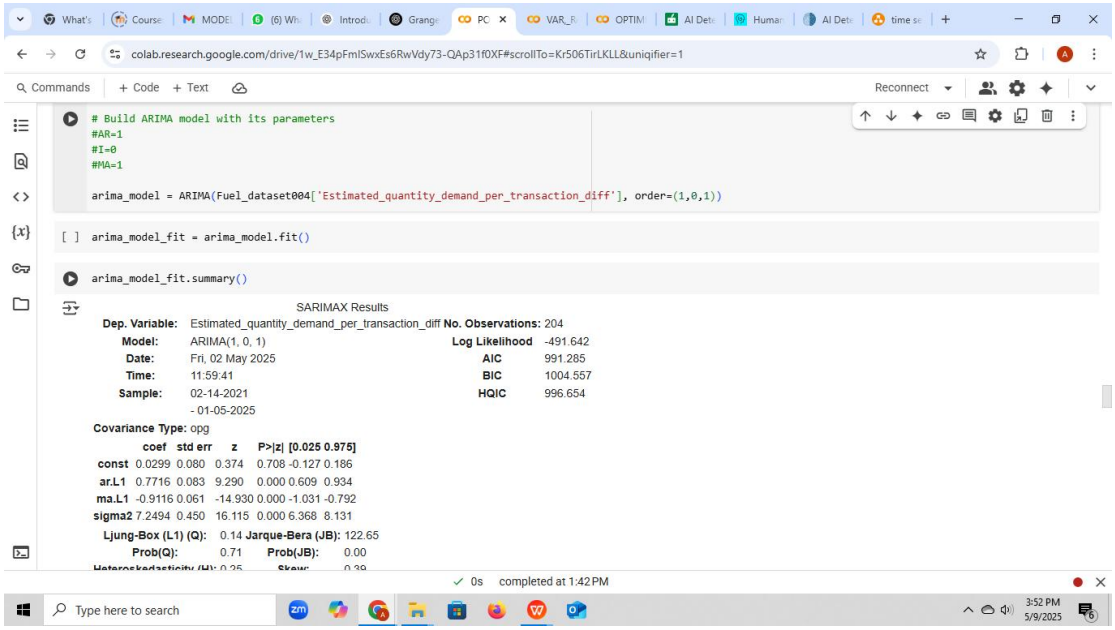



Fig.18b: Build ARIMA model

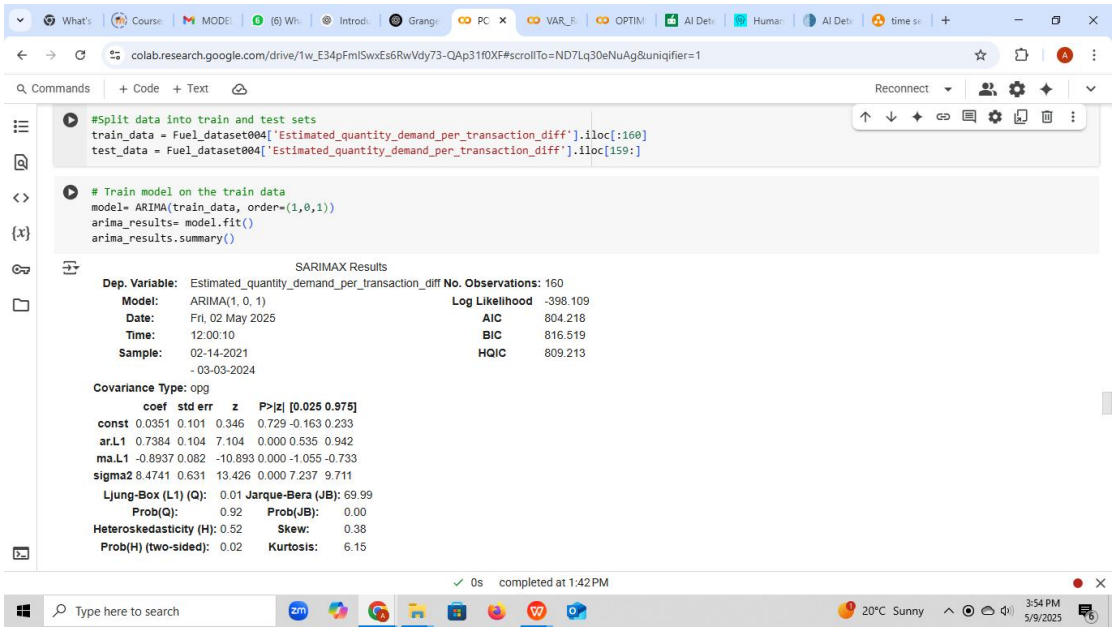



Fig. 18c: split data and train ARIMA model

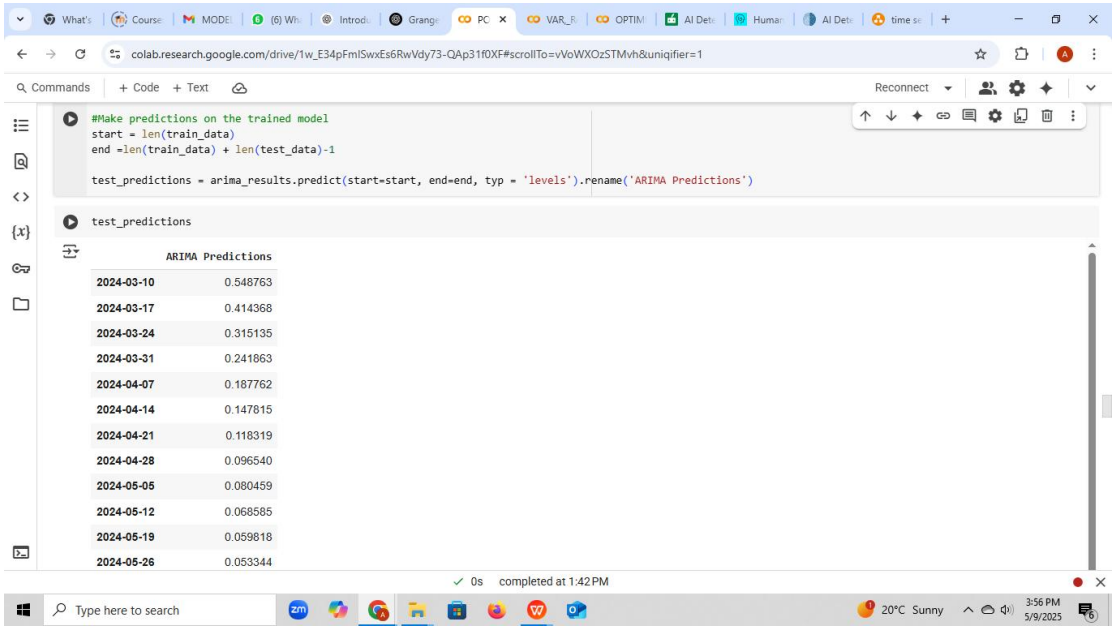



Fig. 18d: test prediction on trained ARIMA model

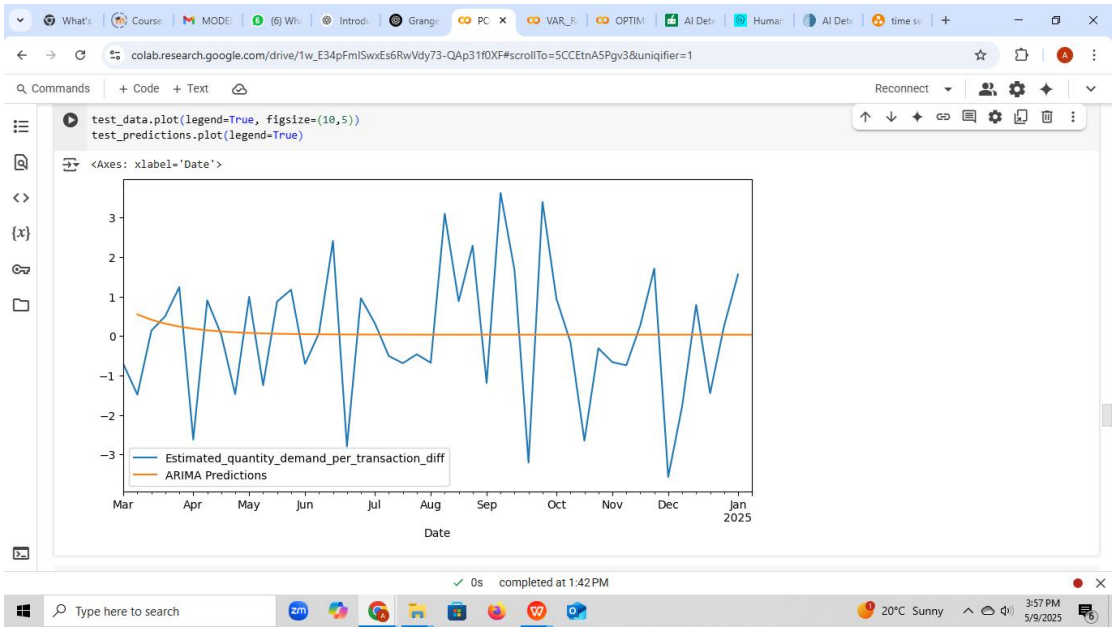



Fig. 18e: ARIMA test data and test prediction plot

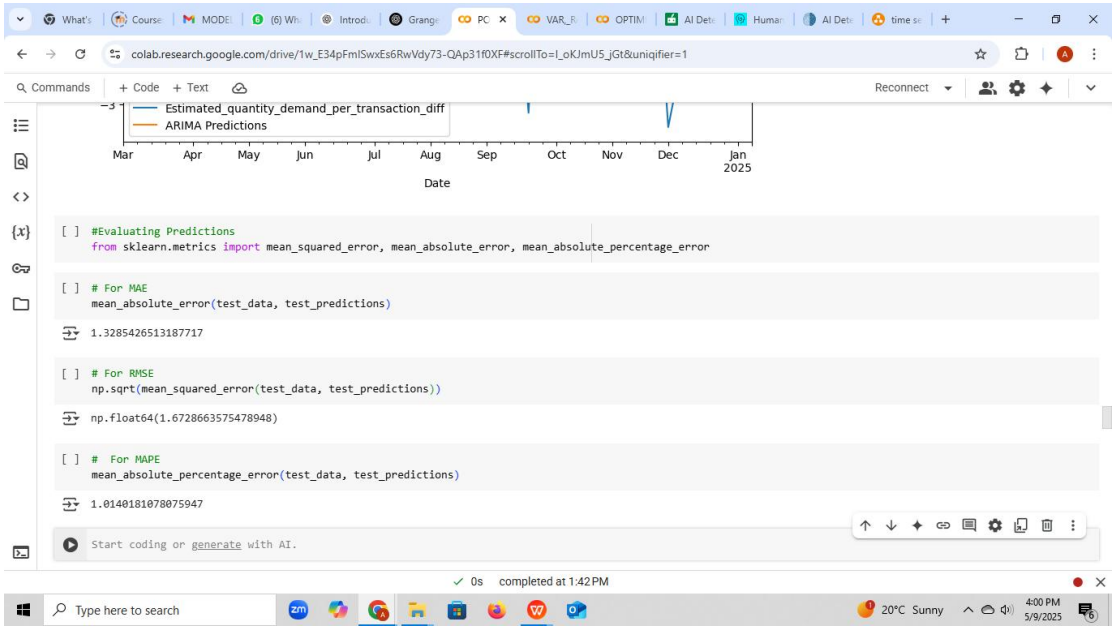



Fig. 18f: ARIMA error evaluation

## Summary

From the ARIMA visual insight,

1) The best model fit is (1,0,1) where the AIC value 804.218 is lowest.

2) The model fails to capture the seasonality in the data .

## SARIMA

Below is a concise visual summary of the modeling process for SARIMA;

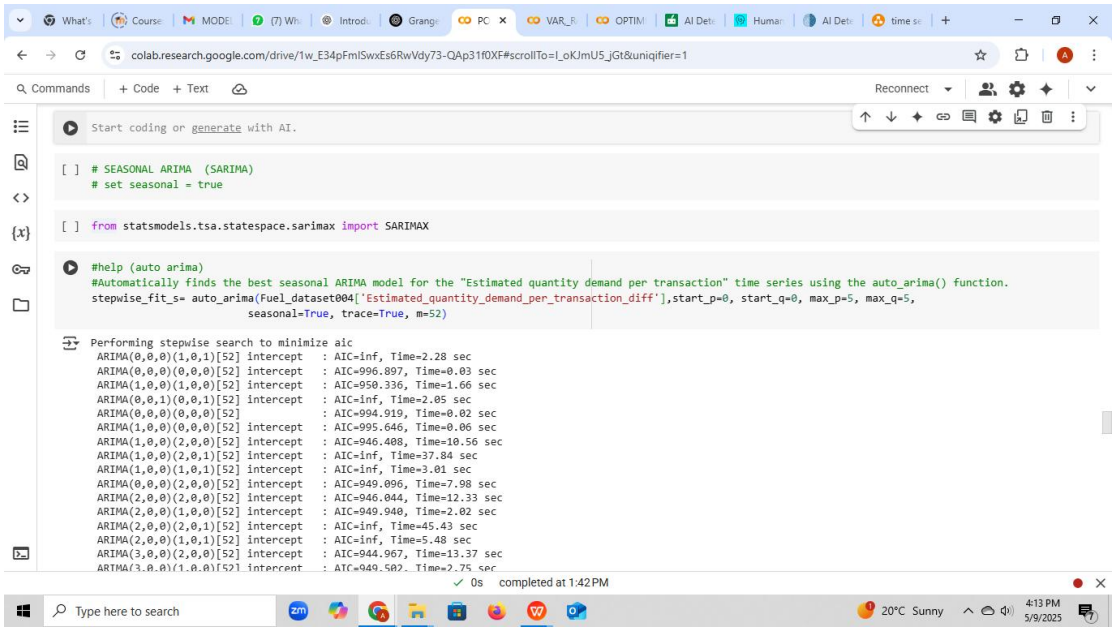



Fig. 19a: find best seasonal ARIMA model

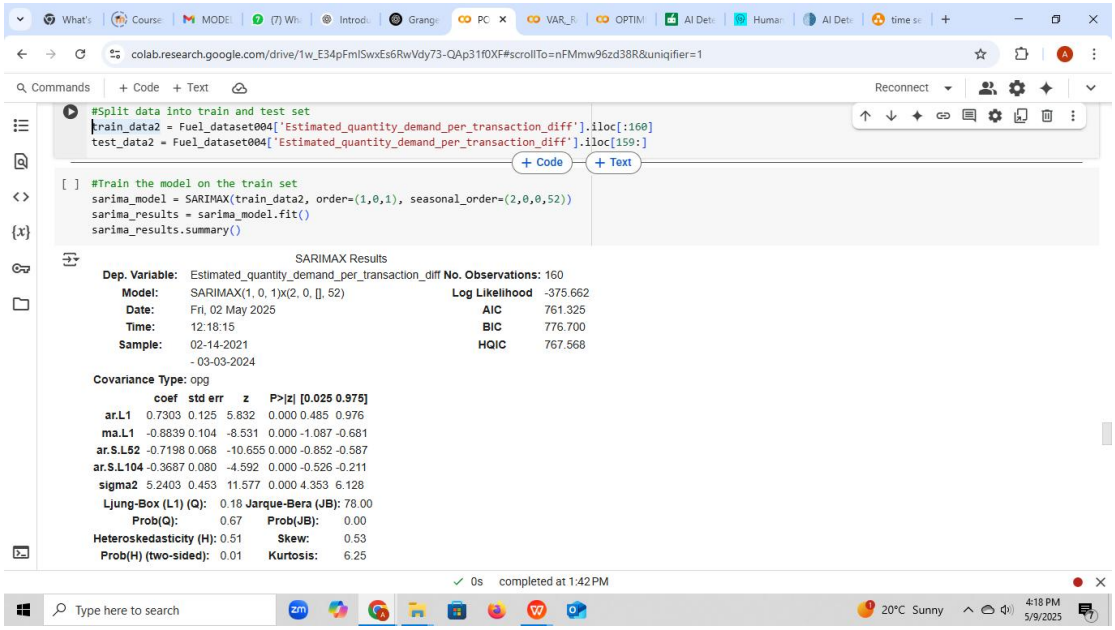



Fig. 19b: split data and train SARIMA on train data

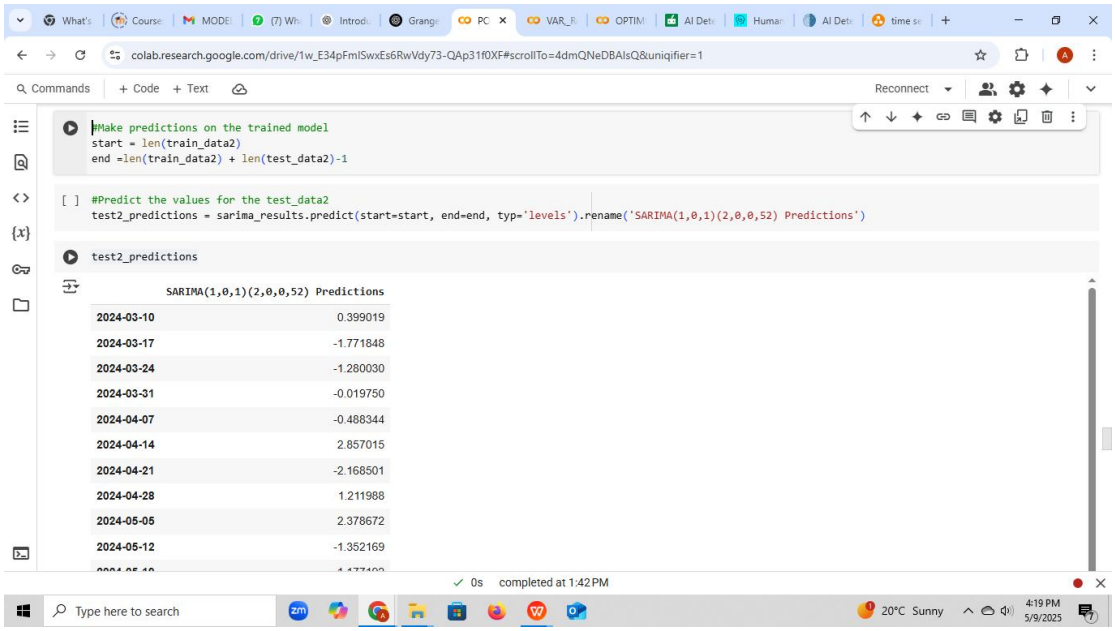



Fig. 19c: test prediction on SARIMA model

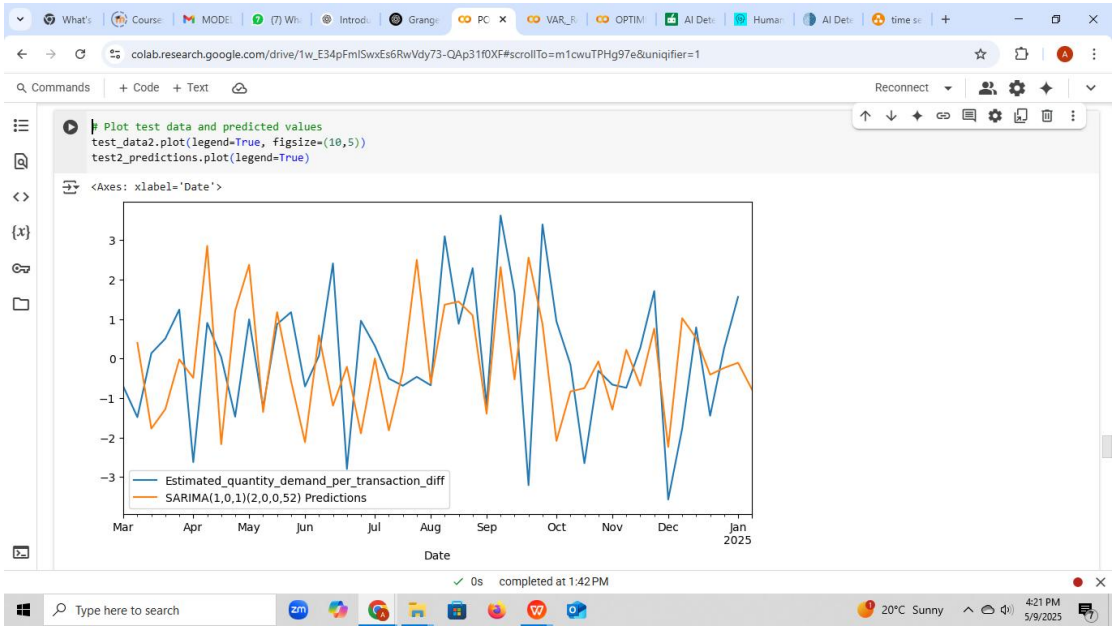



Fig. 19d: SARIMA test data and test prediction plot

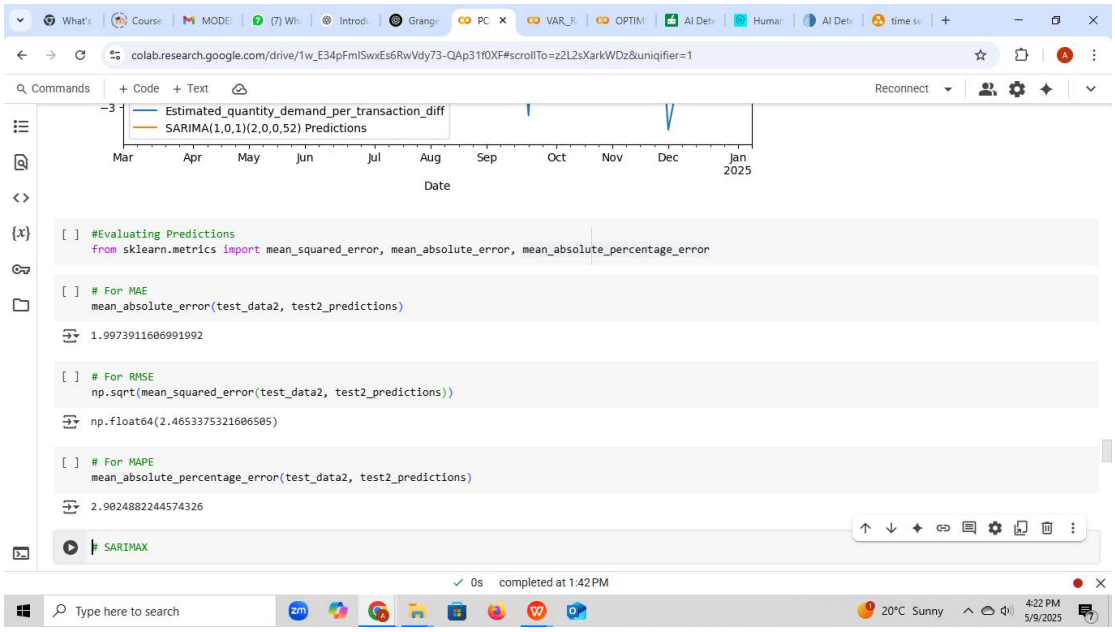



Fig. 19e: SARIMA error evaluation

## Summary

The SARIMA model best model fit is evaluated at (1,0,1)(2,0,0,52) where the AIC value is 761.325 being the least. Unlike the ARIMA model, the SARIMA model captures the seasonality of the time series data.

## SARIMAX

Below is a concise visual summary of the modeling process for SARIMAX;

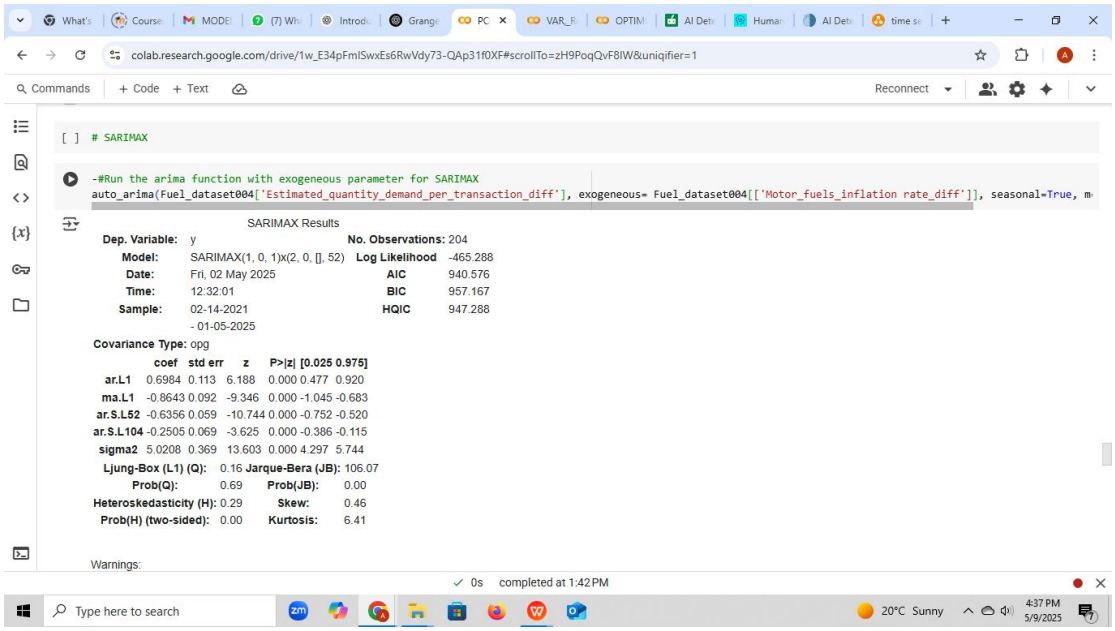
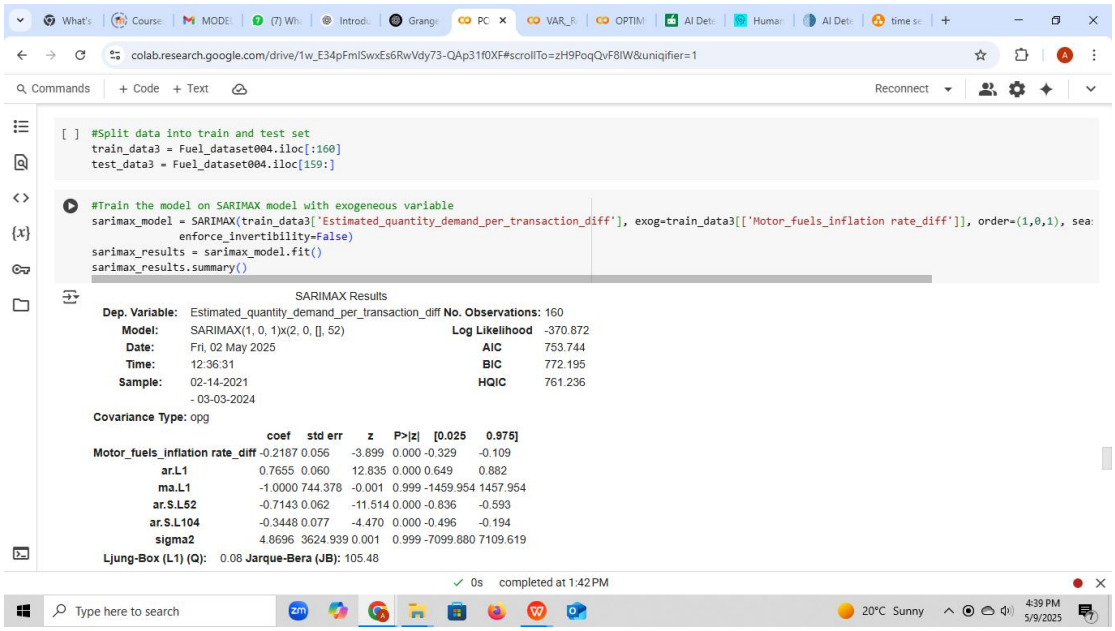



Fig. 20a: ARIMA function with exogenous parameter for SARIMAX



```
#Split data into train and test set
train_data3 = Fuel_dataset004.iloc[:160]
test_data3 = Fuel_dataset004.iloc[159:]
```

```
#Train the model on SARIMAX model with exogeneous variable
sarimax_model = SARIMAX(train_data3['Estimated_quantity_demand_per_transaction_diff'], exog=train_data3[['Motor_fuels_inflation rate_diff']], order=(1,0,1), sea
                enforce_invertibility=False)
sarimax_results = sarimax_model.fit()
sarimax_results.summary()
```

SARIMAX Results

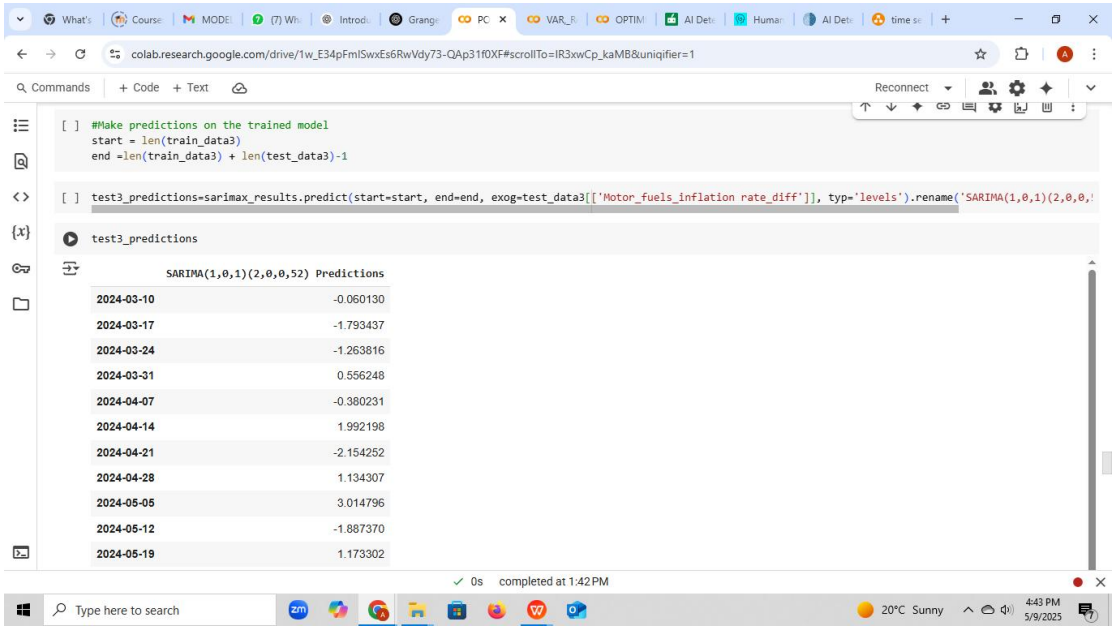
| | | | |
|---|---|---|---|
| **Dep. Variable:** | Estimated_quantity_demand_per_transaction_diff | **No. Observations:** | 160 |
| **Model:** | SARIMAX(1, 0, 1)x(2, 0, [], 52) | **Log Likelihood** | -370.872 |
| **Date:** | Fri, 02 May 2025 | **AIC** | 753.744 |
| **Time:** | 12:36:31 | **BIC** | 772.195 |
| **Sample:** | 02-14-2021 - 03-03-2024 | **HQIC** | 761.236 |

**Covariance Type:** opg

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **Motor_fuels_inflation rate_diff** | -0.2187 | 0.056 | -3.899 | 0.000 | -0.329 | -0.109 |
| **ar.L1** | 0.7655 | 0.060 | 12.835 | 0.000 | 0.649 | 0.882 |
| **ma.L1** | -1.0000 | 744.378 | -0.001 | 0.999 | -1459.954 | 1457.954 |
| **ar.S.L52** | -0.7143 | 0.062 | -11.514 | 0.000 | -0.836 | -0.593 |
| **ar.S.L104** | -0.3448 | 0.077 | -4.470 | 0.000 | -0.496 | -0.194 |
| **sigma2** | 4.8696 | 3624.939 | 0.001 | 0.999 | -7099.880 | 7109.619 |

**Ljung-Box (L1) (Q):** 0.08 **Jarque-Bera (JB):** 105.48

Fig. 20b: split data and train SARIMAX with exogenous variable



```
#Make predictions on the trained model
start = len(train_data3)
end =len(train_data3) + len(test_data3)-1
```

```
test3_predictions=sarimax_results.predict(start=start, end=end, exog=test_data3[['Motor_fuels_inflation rate_diff']], typ='levels').rename('SARIMA(1,0,1)(2,0,0,
```

```
test3_predictions
```

| | SARIMA(1,0,1)(2,0,0,52) Predictions |
|---|---|
| **2024-03-10** | -0.060130 |
| **2024-03-17** | -1.793437 |
| **2024-03-24** | -1.263816 |
| **2024-03-31** | 0.556248 |
| **2024-04-07** | -0.380231 |
| **2024-04-14** | 1.992198 |
| **2024-04-21** | -2.154252 |
| **2024-04-28** | 1.134307 |
| **2024-05-05** | 3.014796 |
| **2024-05-12** | -1.887370 |
| **2024-05-19** | 1.173302 |

Fig. 20c: test prediction on SARIMAX model

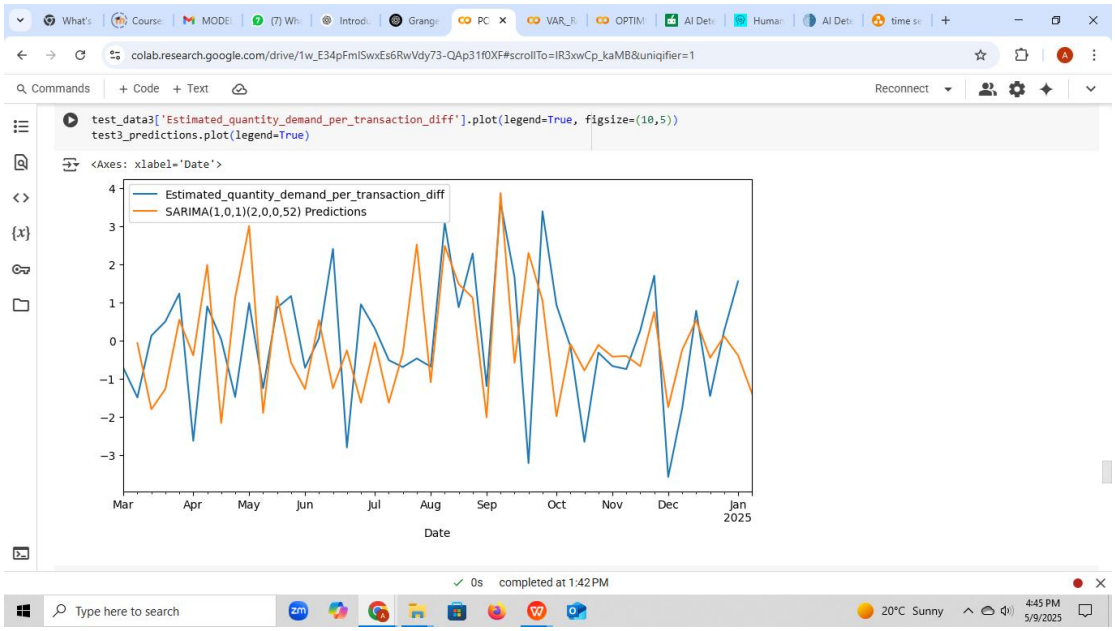



Fig. 20d: SARIMAX test data and test prediction plot

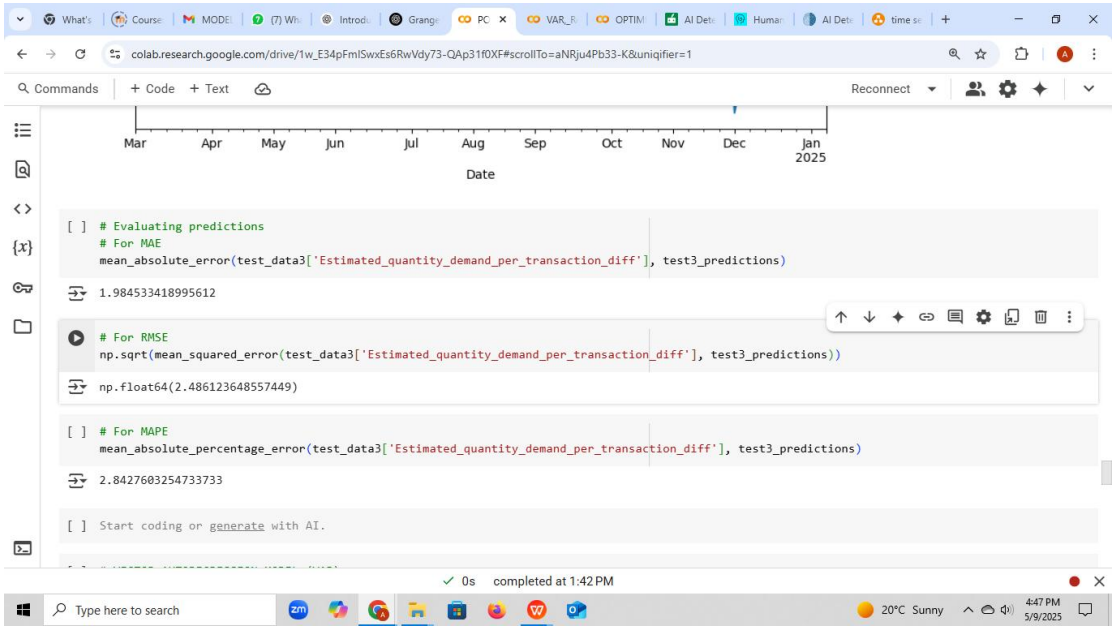



Fig.20e: SARIMAX error evaluation

## Summary

Similar to the SARIMA model, the SARIMAX best model fit is captured at (1,0,1)(2,0,0,52) where the AIC value is 753.744 being the least. The model captures the seasonality of the time series data taking the exogenous variable into consideration.

## Vector Autoregression (VAR)

Below is a concise visual summary of the modeling process for VAR;

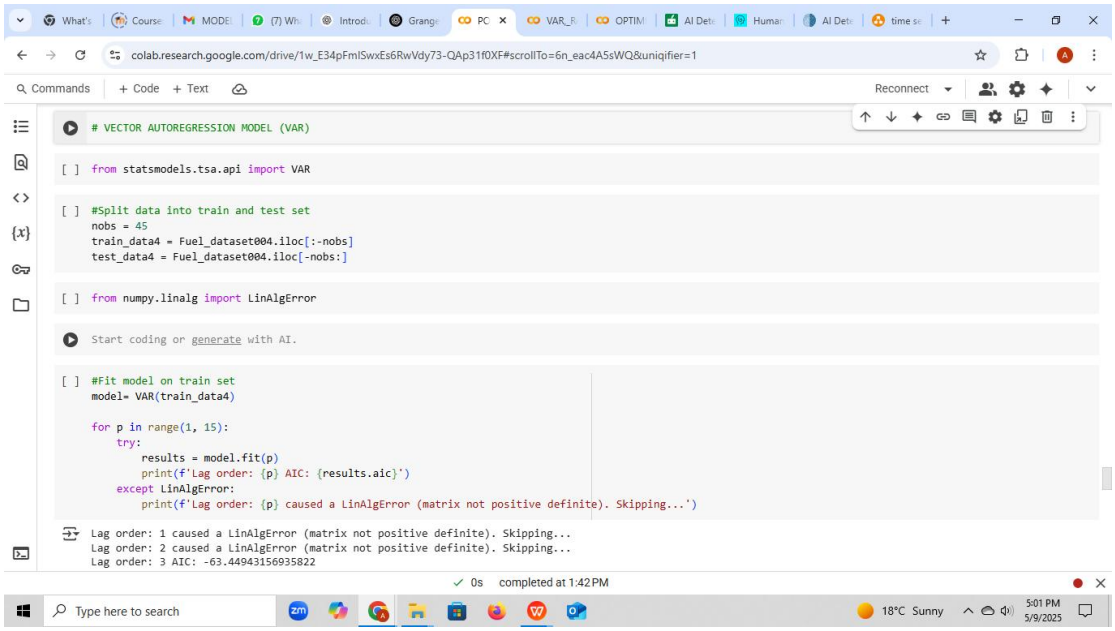



Fig. 21a: split data and fit model on train data

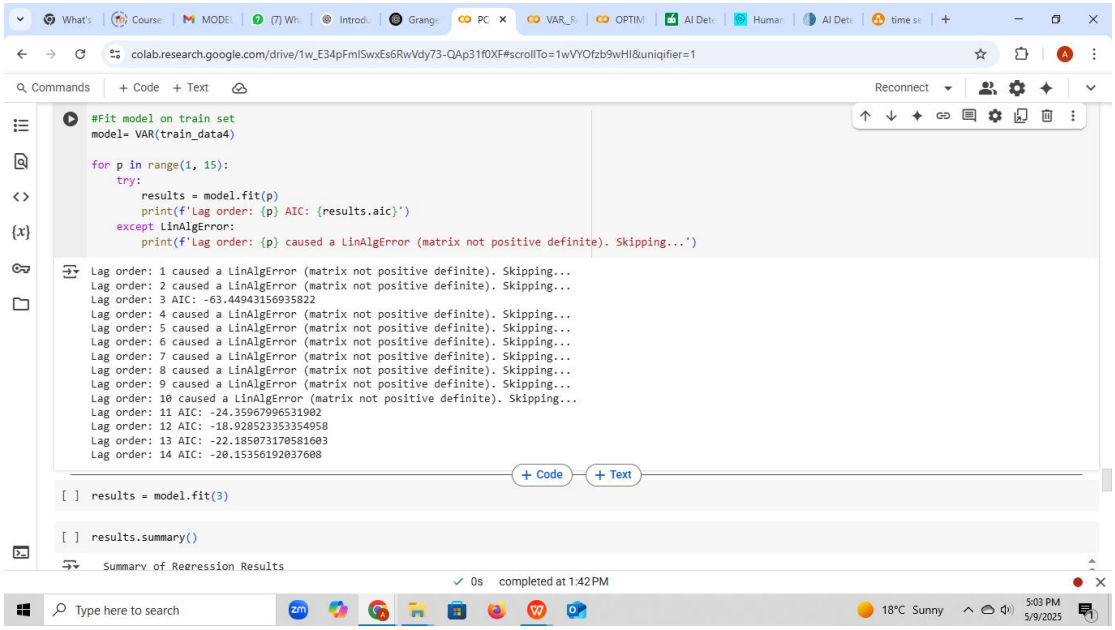



Fig. 21b: split data and fit model on train data

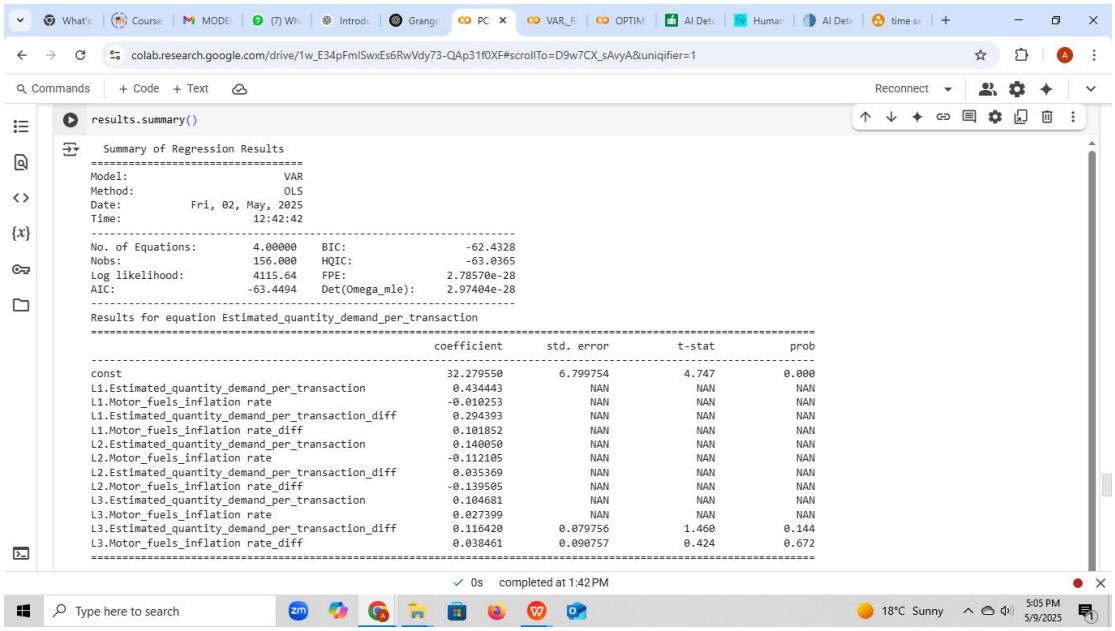



Fig. 21c: summary of regression results

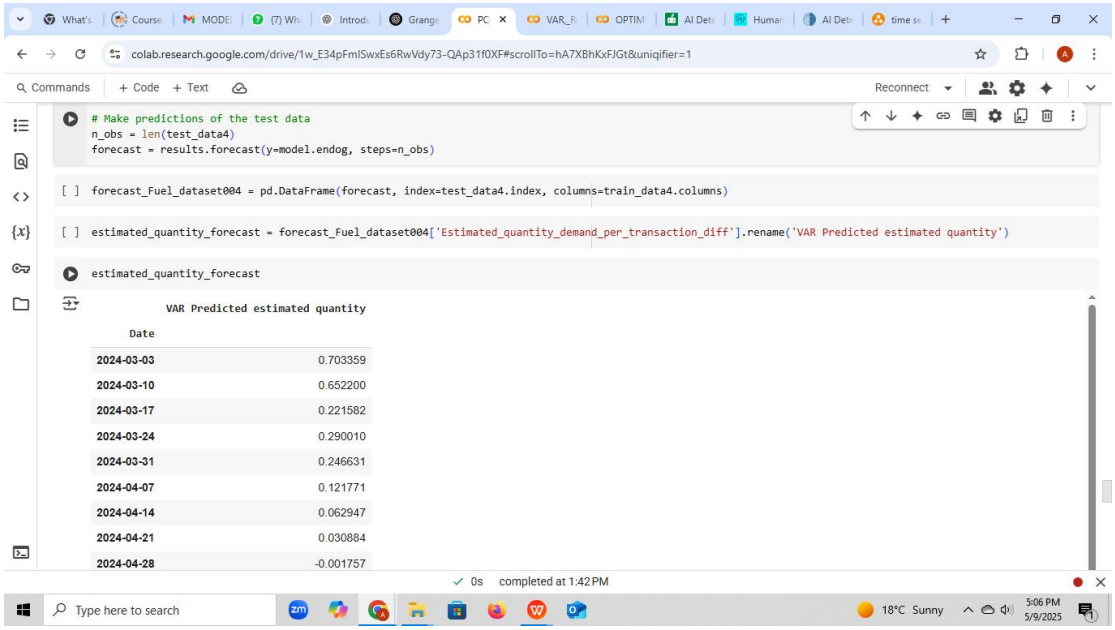



Fig. 21d: prediction of test data with VAR model

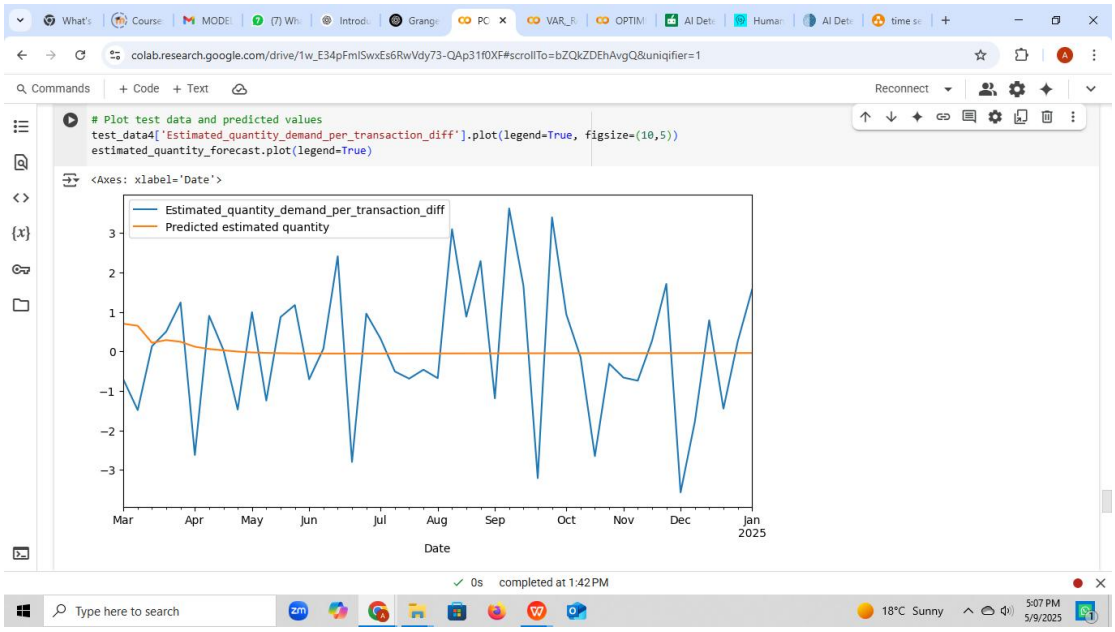



Fig. 21e: VAR test data and test prediction plot

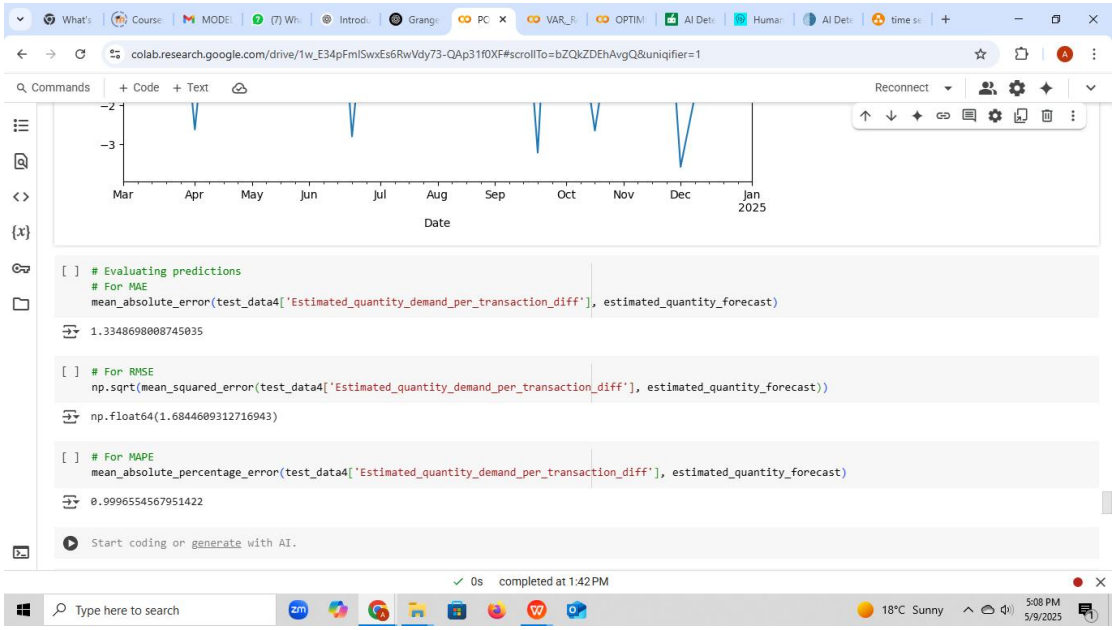



Fig. 21f: VAR error evaluation.

## Summary

From the plot, the VAR model fails to capture seasonality. Although the model appears to fit the data well as indicated by strong statistical measures such as low AIC and BIC values. However, some results show NaN values, which may point to estimation issues from the differenced data as observed when fitting model on train data from the LinAlgError from none positive values.

Summary of prediction error metrics

SUMMARY OF PREDICTION ERROR METRIC

| MODELS | Mean Absolute Error | Root Mean Squared Error | MAPE |
|---|---|---|---|
| TRIPLE EXPONENTIAL SMOOTHING | 1.242028761 | 1.616442067 | 1.999543581 |
| ARIMA | 1.328542651 | 1.672866358 | 1.014018108 |
| SARIMA | 1.997391161 | 2.465337532 | 2.902488224 |
| SARIMAX | 1.984533419 | 2.486123649 | 2.842760325 |
| VECTOR AUTOREGRESSION | 1.334869801 | 1.684460931 | 0.999655457 |

Table 1.1: prediction error metrics table

The above error metric evaluation table gives a summary of the model error measured using MAE, RMSE and MAPE metrics. From the table, Triple exponential smoothing perform best under MAE and RMSE while VAR perform best under MAPE. ARIMA stands a good chance under all metrics but going by the plot analysis, Triple Exponential Smoothing captures the seasonality compare to VAR and ARIMA that fails to capture the seasonality thereby giving TES a better advantage over them. In a nutshell, the Triple Exponential Smoothing is the best fit model for this study.

The next step is to evaluate the residuals of TES ensuring it meet the assumptions for residual evaluation such as normal distributed by using the shapiro test and Q-Q plot for visualisation in addition to histogram plot, and also check for no autocorrelation using the ACF plot.

Residuals evaluation

The figures below give a summary of the residuals evaluation. Fig. 22a gives the residuals calculation and fig. 22b the residuals plot. From fig. 22c, the residuals follow a normal distribution as the p-value from Shapiro test is 0.97 indicating model is a good fit, also give the Ljung Box test value as 0.45 validating no autocorrelation. Fig. 22d shows no autocorrelation in the ACF plot while fig. 22e and fig. 22f show a normally distributed data visualisation with the histogram having a visible bell shape and the Q-Q plot showing no major deviation from normality.

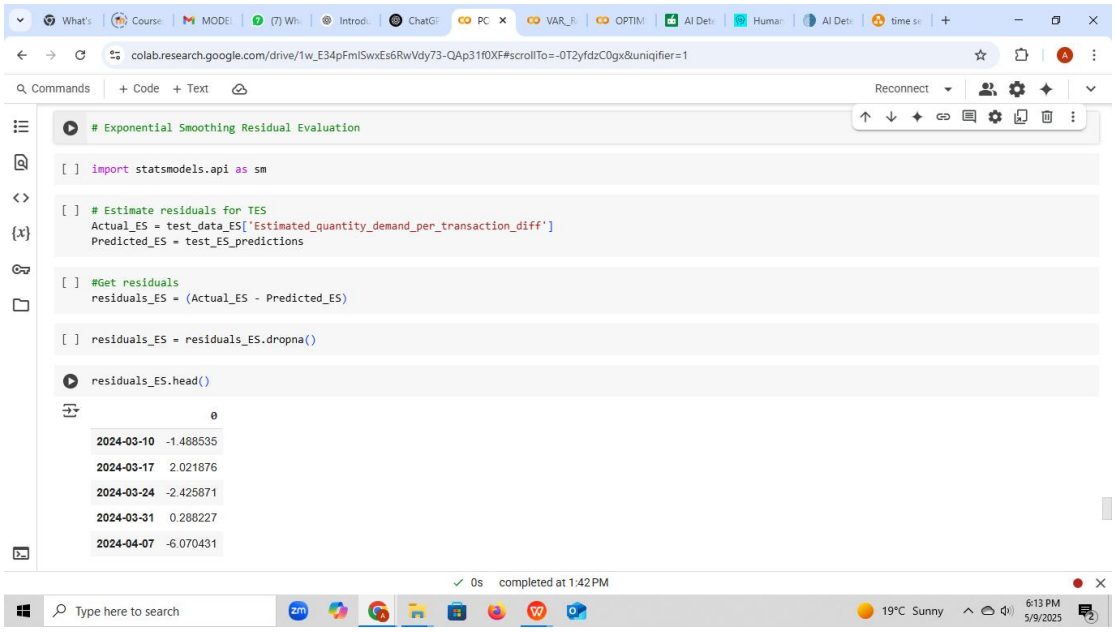



Fig. 22a: residuals calculation

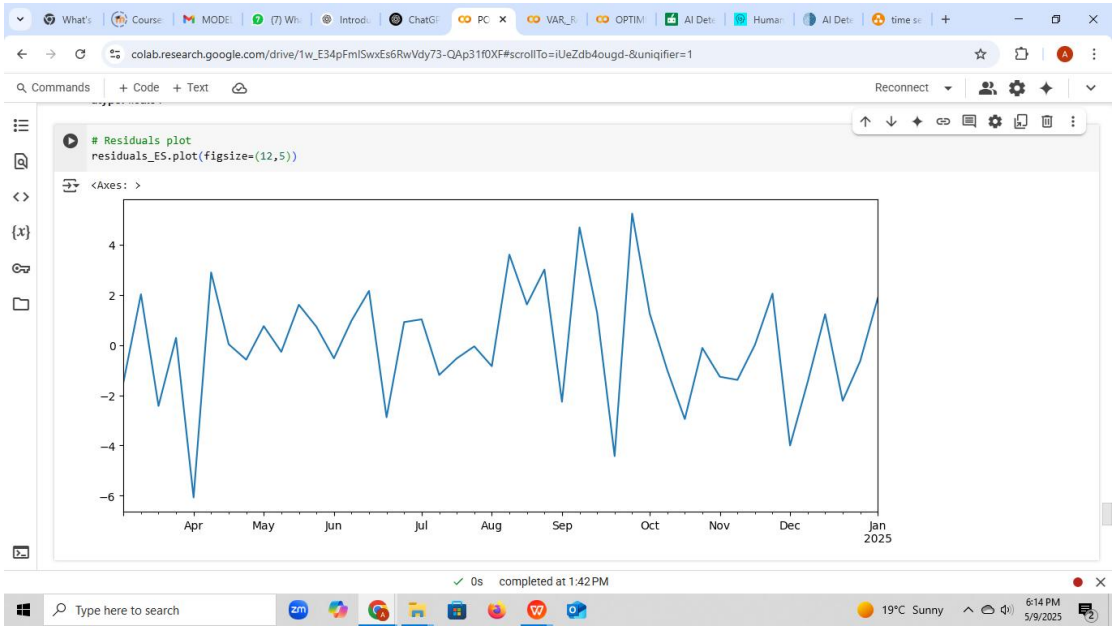



Fig. 22b: residual plot

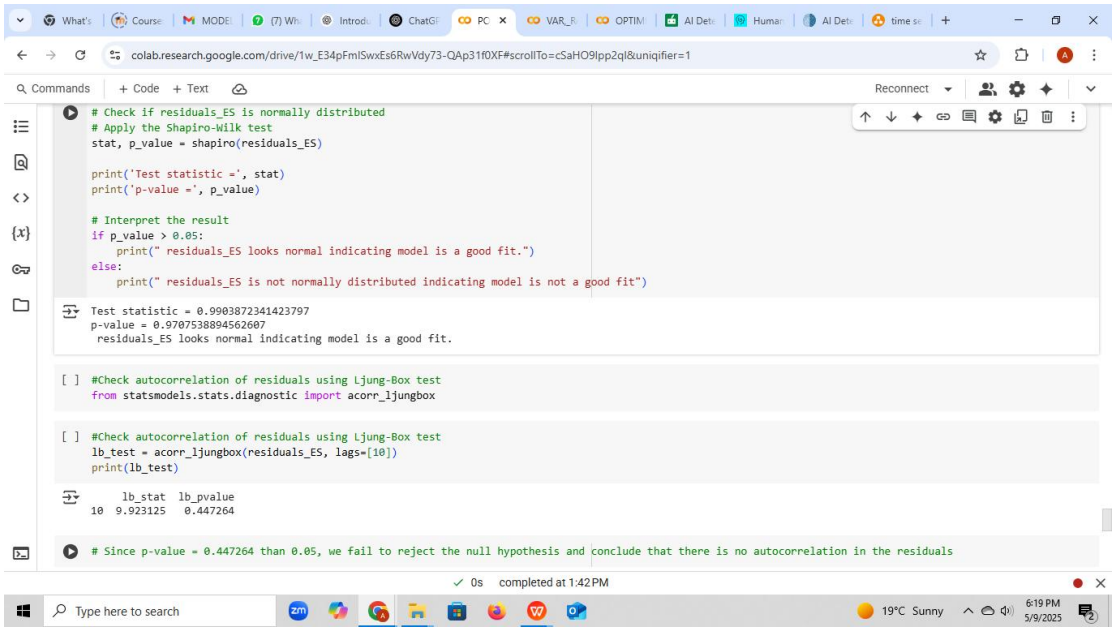



Fig. 22c: Shapiro and Ljung Box test

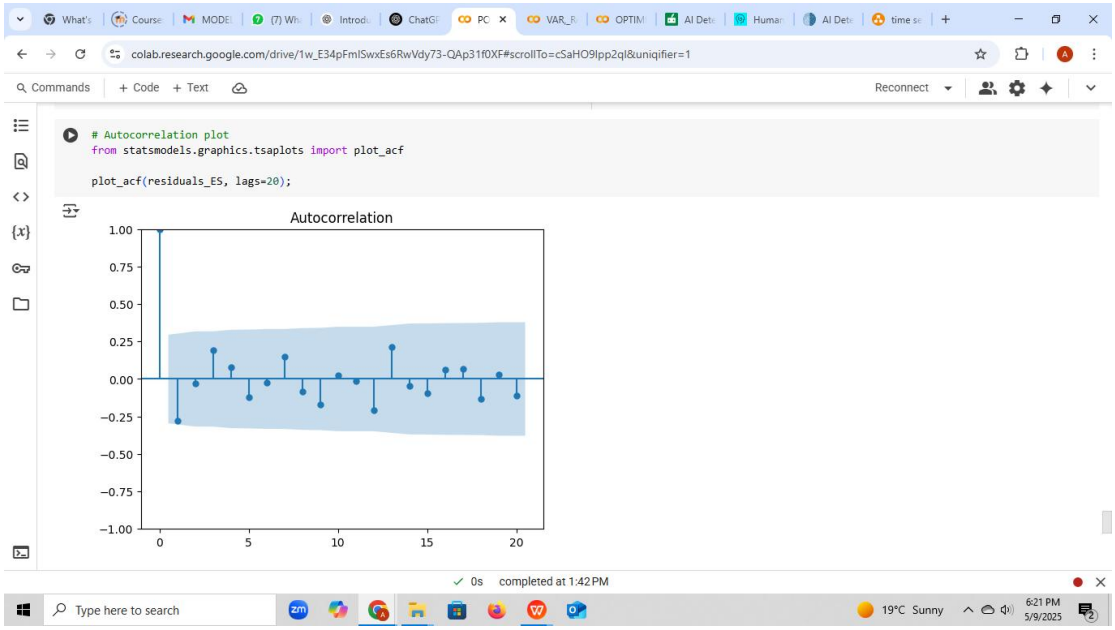



Fig. 22d: ACF plot

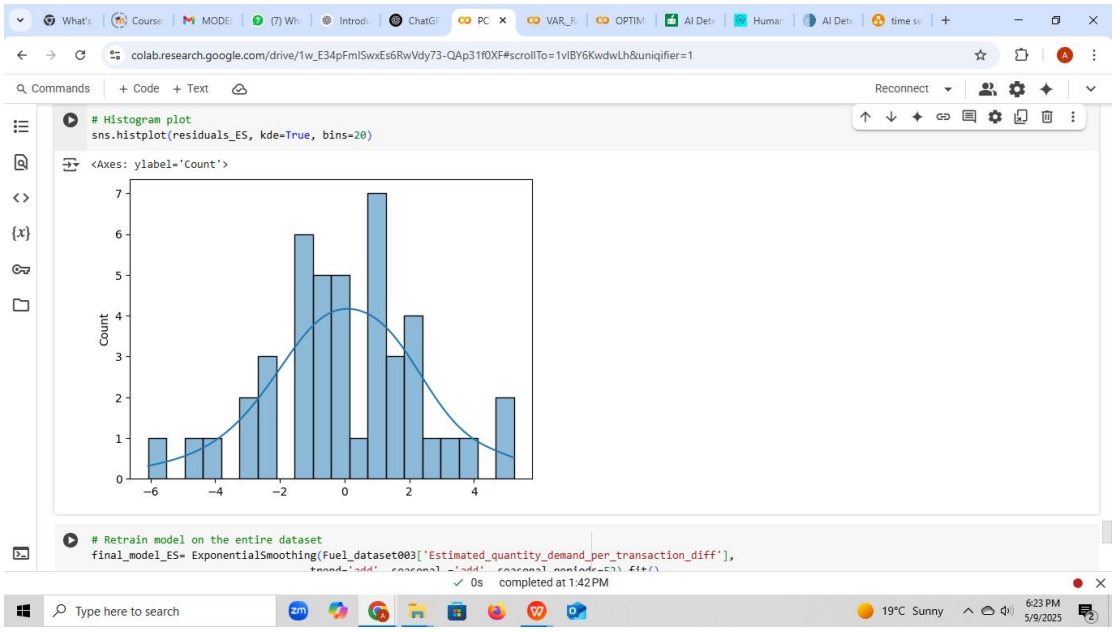



Fig. 22e: residuals histogram plot

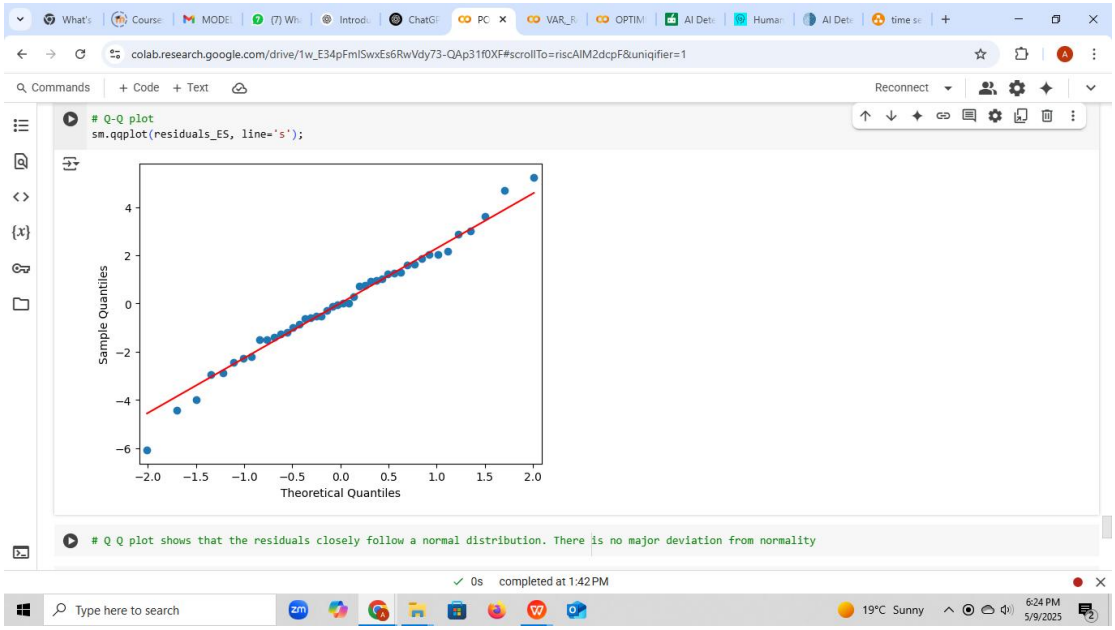
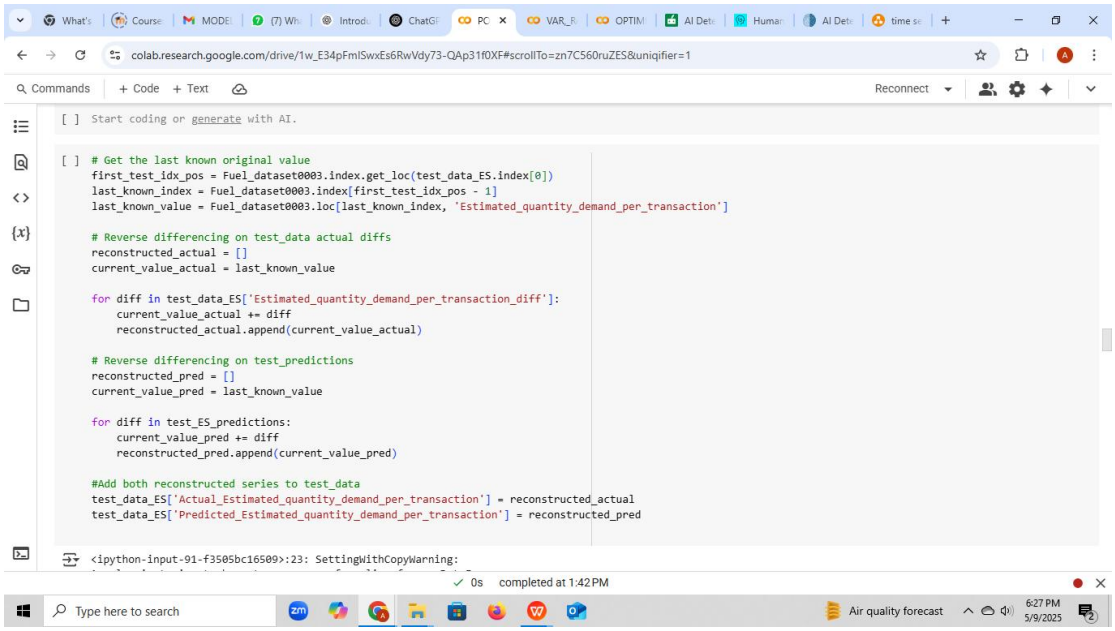


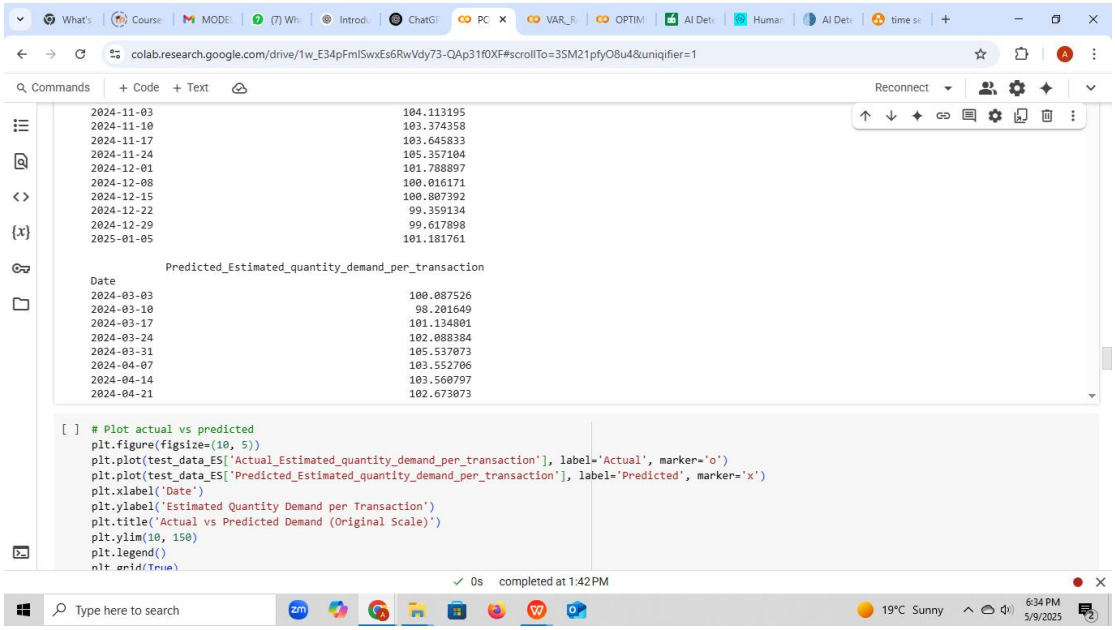

Fig. 22f: Q-Q plot

Reverse differenced values for TES test and predicted data to original scale and plot for better insight into model prediction via trend observation.



```
[ ] Start coding or generate with AI.

[ ] # Get the last known original value
    first_test_idx_pos = Fuel_dataset0003.index.get_loc(test_data_ES.index[0])
    last_known_index = Fuel_dataset0003.index[first_test_idx_pos - 1]
    last_known_value = Fuel_dataset0003.loc[last_known_index, 'Estimated_quantity_demand_per_transaction']

    # Reverse differencing on test_data actual diffs
    reconstructed_actual = []
    current_value_actual = last_known_value

    for diff in test_data_ES['Estimated_quantity_demand_per_transaction_diff']:
        current_value_actual += diff
        reconstructed_actual.append(current_value_actual)

    # Reverse differencing on test_predictions
    reconstructed_pred = []
    current_value_pred = last_known_value

    for diff in test_ES_predictions:
        current_value_pred += diff
        reconstructed_pred.append(current_value_pred)

    #Add both reconstructed series to test_data
    test_data_ES['Actual_Estimated_quantity_demand_per_transaction'] = reconstructed_actual
    test_data_ES['Predicted_Estimated_quantity_demand_per_transaction'] = reconstructed_pred

<ipython-input-91-f3505bc16509>:23: SettingWithCopyWarning:
```

Fig. 23a: reverse TES test and predicted data



```
2024-11-03                                      104.113195
2024-11-10                                      103.374358
2024-11-17                                      103.645833
2024-11-24                                      105.357104
2024-12-01                                      101.788897
2024-12-08                                      100.016171
2024-12-15                                      100.807392
2024-12-22                                       99.359134
2024-12-29                                       99.617898
2025-01-05                                      101.181761

            Predicted_Estimated_quantity_demand_per_transaction
Date
2024-03-03                                       100.087526
2024-03-10                                        98.201649
2024-03-17                                       101.134801
2024-03-24                                       102.088384
2024-03-31                                       105.537073
2024-04-07                                       103.552706
2024-04-14                                       103.560797
2024-04-21                                       102.673073

[ ] # Plot actual vs predicted
    plt.figure(figsize=(10, 5))
    plt.plot(test_data_ES['Actual_Estimated_quantity_demand_per_transaction'], label='Actual', marker='o')
    plt.plot(test_data_ES['Predicted_Estimated_quantity_demand_per_transaction'], label='Predicted', marker='x')
    plt.xlabel('Date')
    plt.ylabel('Estimated Quantity Demand per Transaction')
    plt.title('Actual vs Predicted Demand (Original Scale)')
    plt.ylim(10, 150)
    plt.legend()
    plt.grid(True)
```

Fig. 23b: reverse TES test and predicted data

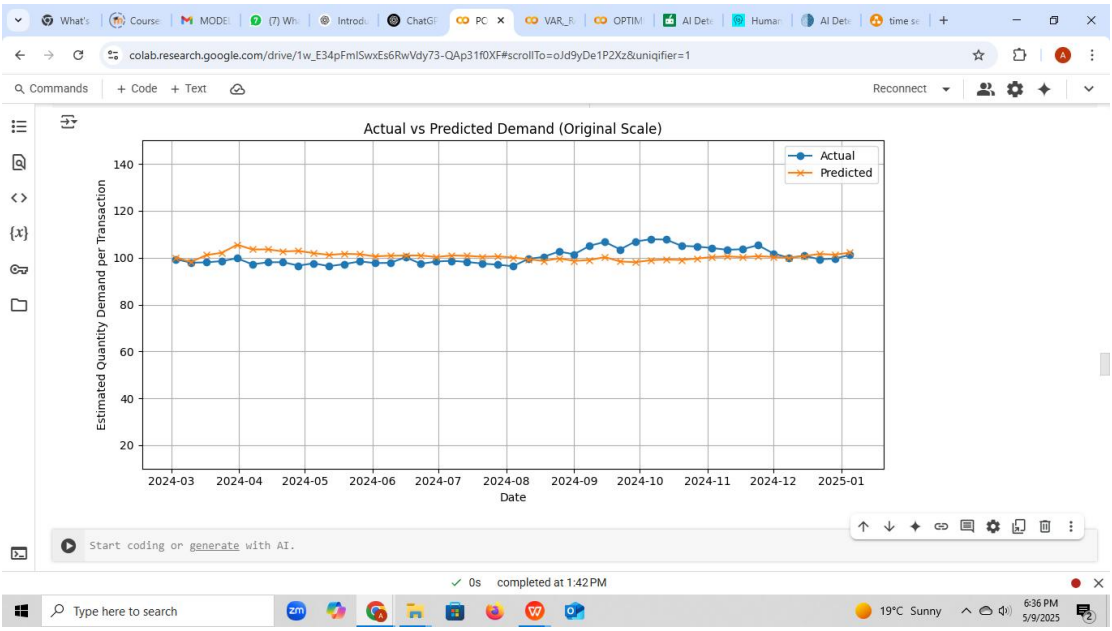



Fig. 23c: TES test actual vs predicted plot

## 4.2 Results

Summarisation of Business Question 1

What forecasting model performs best in capturing the seasonal behavior of Estimated quantity demand per transaction?

SUMMARY OF PREDICTION ERROR METRIC

| MODELS | Mean Absolute Error | Root Mean Squared Error | MAPE |
|---|---|---|---|
| TRIPLE EXPONENTIAL SMOOTHING | 1.242028761 | 1.616442067 | 1.999543581 |
| ARIMA | 1.328542651 | 1.672866358 | 1.014018108 |
| SARIMA | 1.997391161 | 2.465337532 | 2.902488224 |
| SARIMAX | 1.984533419 | 2.486123649 | 2.842760325 |
| VECTOR AUTOREGRESSION | 1.334869801 | 1.684460931 | 0.999655457 |

*From the model plot in fig. 17g, test prediction original scale plot in fig. 23c and prediction error metrics analysis contained in table 1.1 as shown above, the Triple Exponential Smoothing performs best in capturing the seasonal behaviour of estimated quantity demand per transaction having the least prediction errors under MAE and RMSE.*

Summarisation of Business Question 2

What will be the predicted weekly estimated quantity demand per transaction for the next one year?

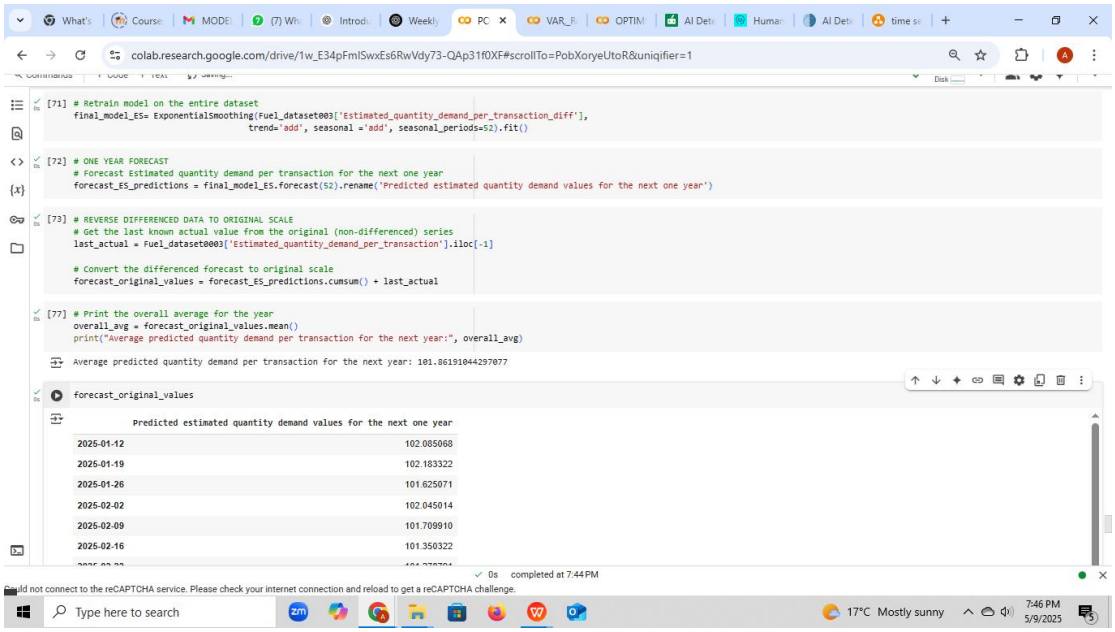



Fig. 24a: one year forecast in original scale

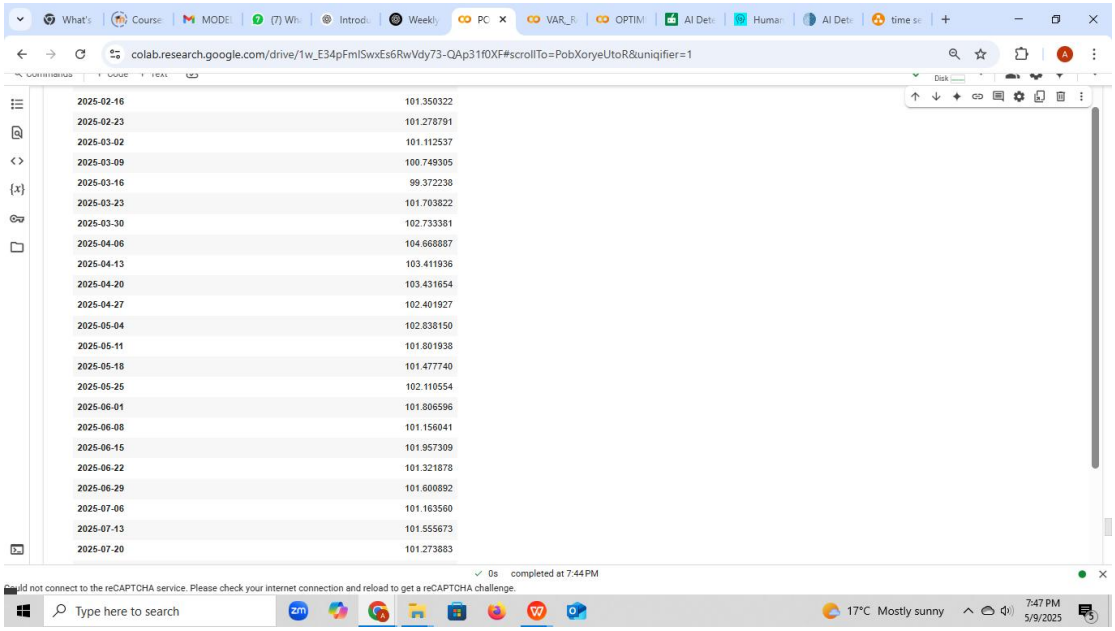



Fig. 24b: one year forecast in original scale

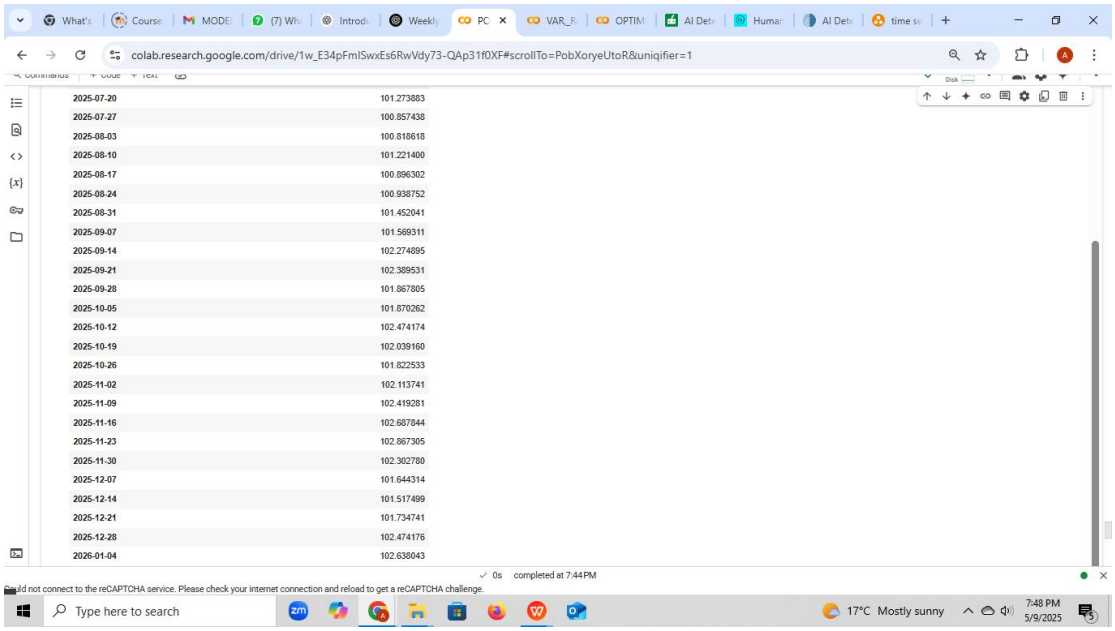



Fig. 24c: one year forecast in original scale

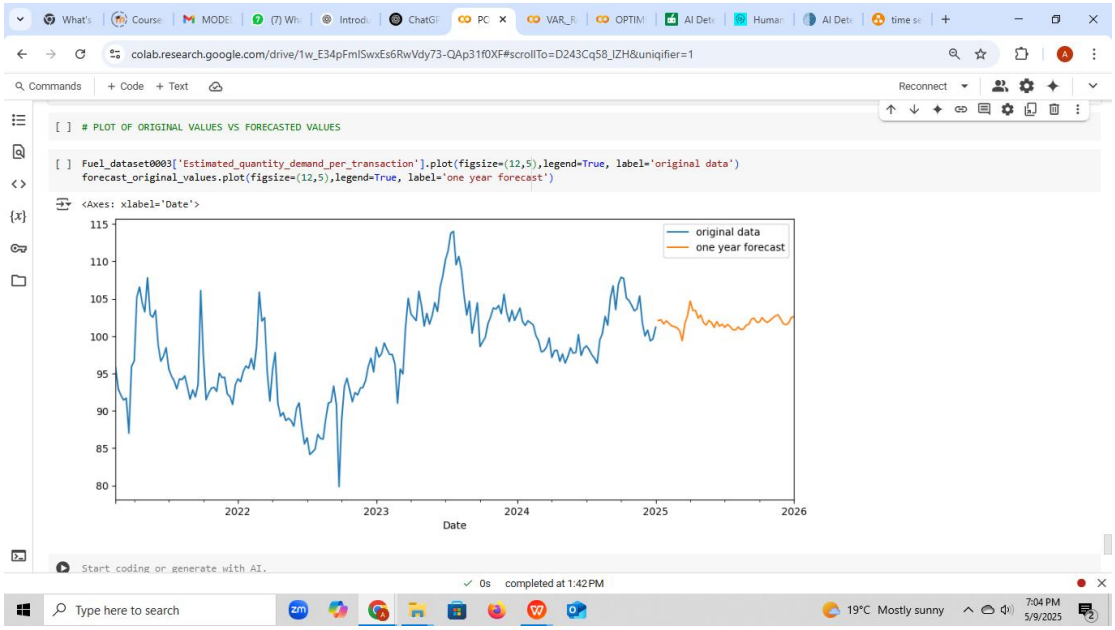



Fig. 24d: one year forecast plot

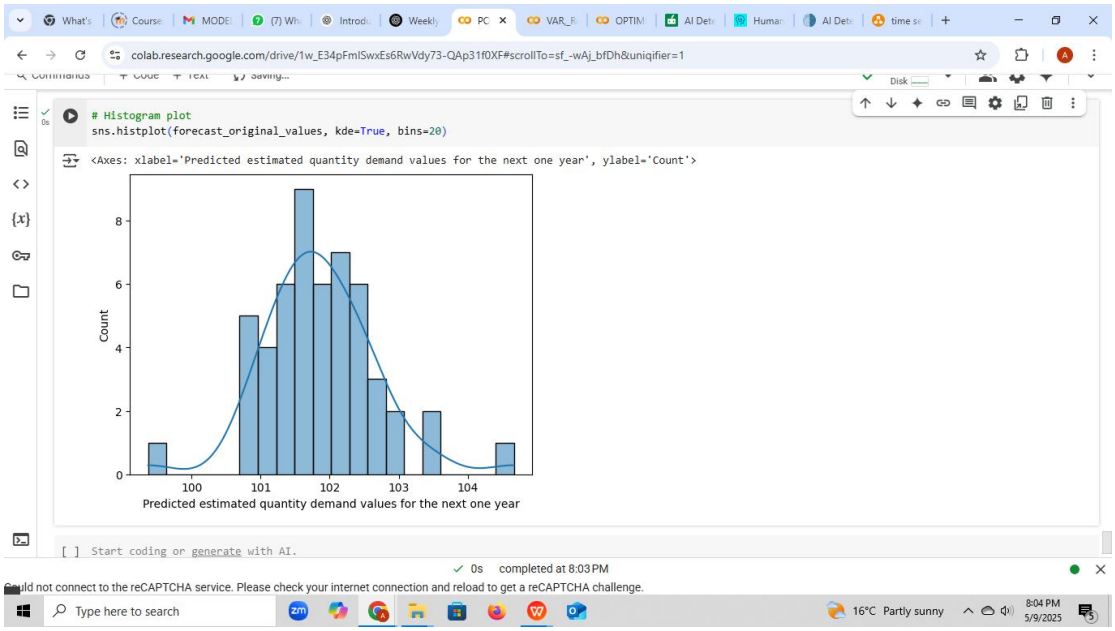



Fig.24e: histogram plot for predicted one year fuel quantity demand per transaction

***Using historical estimated quantity demand per transaction data from the past years and applying Triple Exponential Smoothing model, the forecasted quantity per transaction for the next 52 weeks are shown in fig.24a, fig, 24b and fig. 24c above with a yearly average of 101.86 units per transaction.***

Summarisation of Business Question 3

Which week will record the highest estimated quantity demand per transaction in the next 6 months?

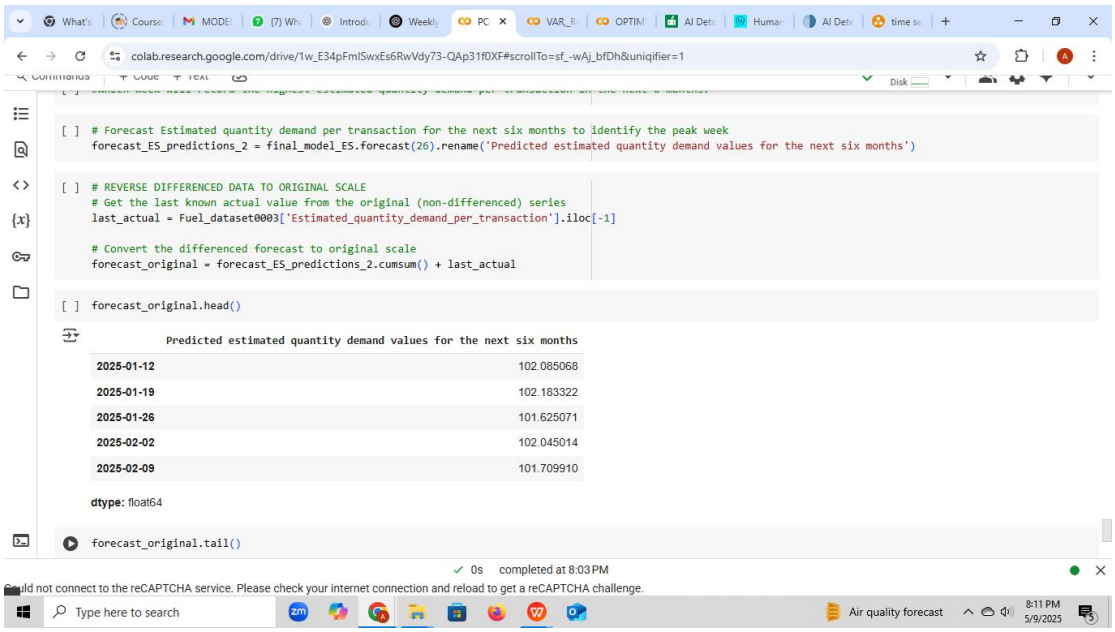


Fig. 25a: peak week quantity demand in the next 6 months

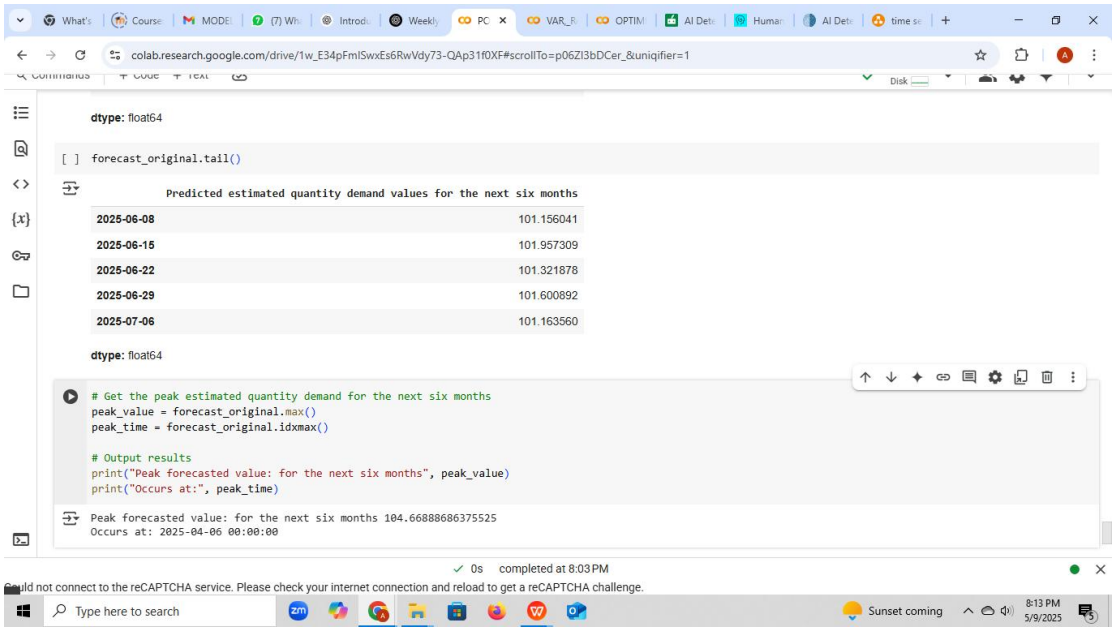



Fig. 25b: peak week quantity demand in the next 6 months

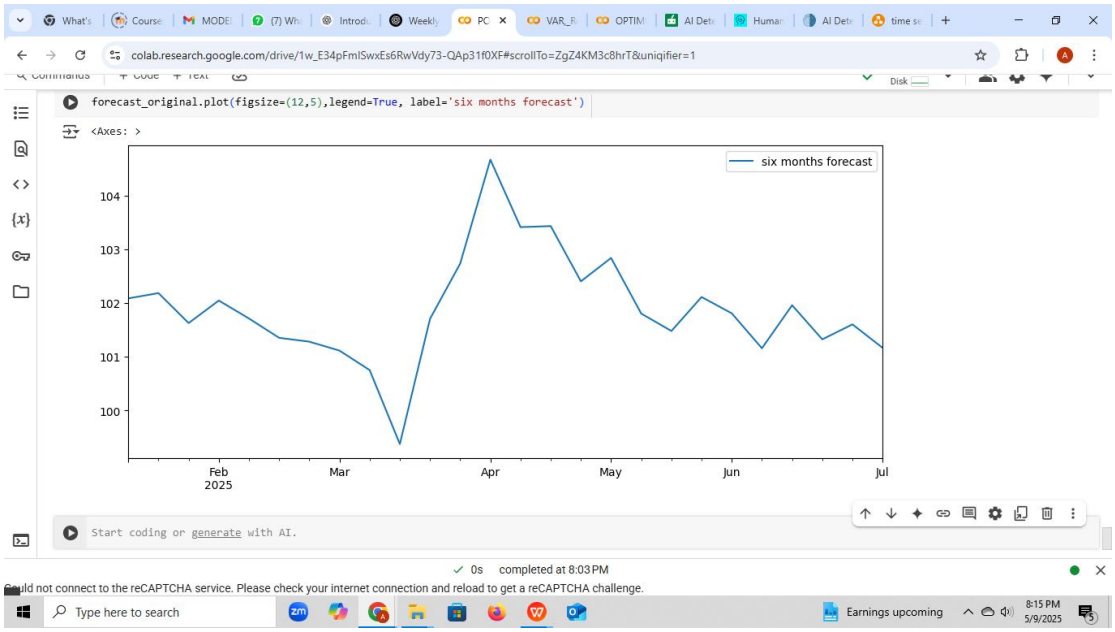



Fig. 25c: peak week quantity demand in the next 6 months plot

*The predicted highest quantity demand per transaction for the next six months is 104.67 and will occur at 06/04/2025*

Summarisation of Business Question 4

What is the relationship between Estimated quantity demand per transaction and motor fuel inflation rate in line plot?

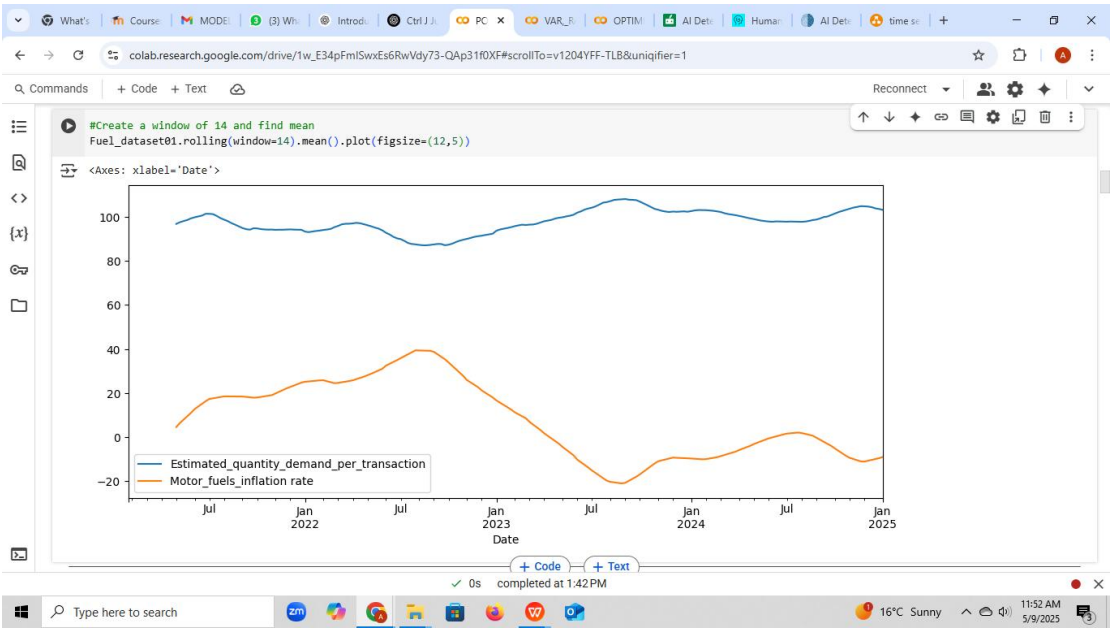



Fig. 9c: fuel data trend plot with rolling window 14

***The line plot shows an inverse relationship between estimated quantity demand per transaction and motor fuel inflation rate. As the inflation rate increases, the quantity per transaction tends to decrease, supporting the economic expectation that higher fuel prices reduce consumer purchasing behavior.***

Summarisation of Business Question 5

Which smoothing technique gives a better trend visualisation?

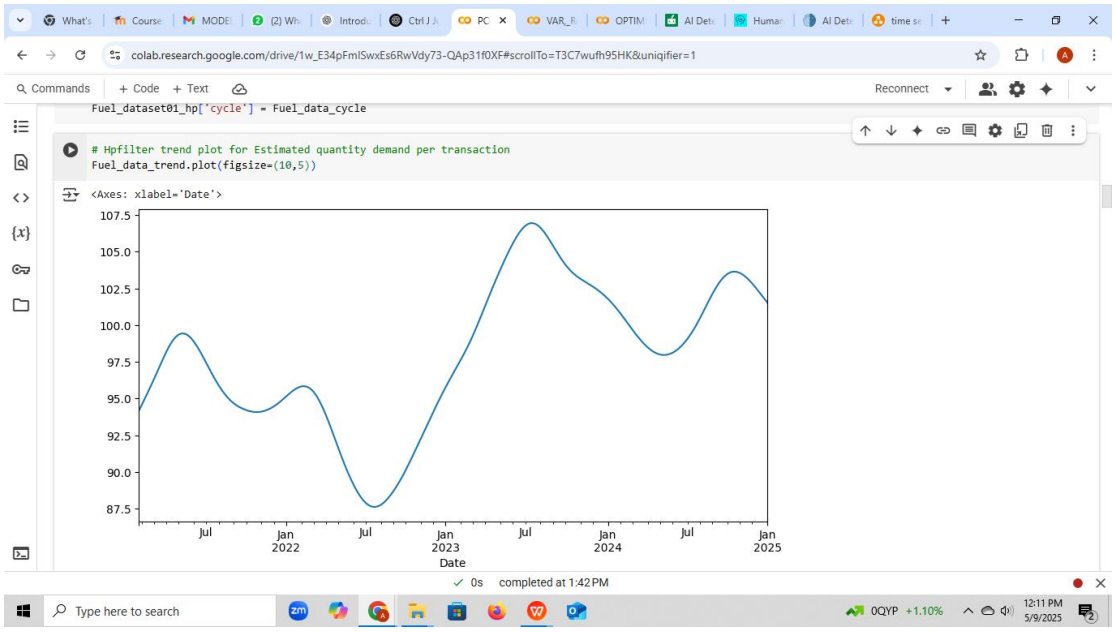



Fig. 9f(2): hpfilter trend plot for estimated quantity demand per transaction

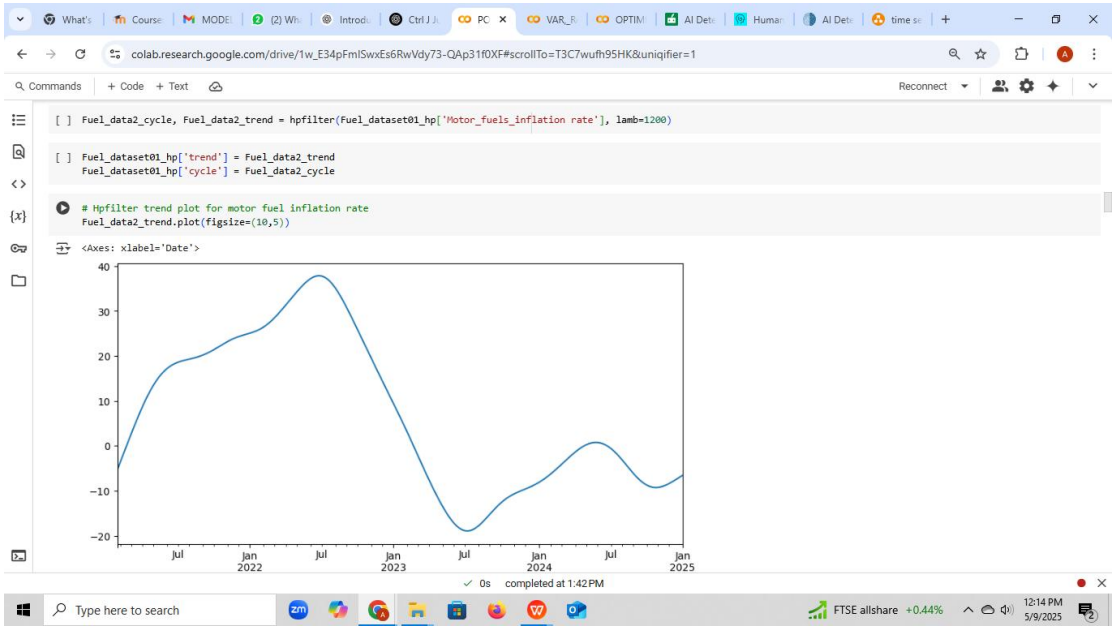



Fig. 9h: hpfilter trend plot for motor fuel inflation rate

*Among the smoothing techniques applied in this study, the Hpfilter provided the most visually interpretable and stable trend line for both the estimated quantity demand per transaction and motor fuel inflation rate. Compared to other methods such as moving average or exponential smoothing and rolling smoothing with window 7 and 14, the Hpfilter effectively captured long term patterns while minimising short term*

*fluctuations. The smoothness and responsiveness of the Hpfilter trend line offered clearer insights into how demand evolved over time.*

# CHAPTER 5

# CONCLUSION AND RECOMMENDATION

## 5.1 Summary of all the results

Below is a summary of all the results from the analysis:

**Best Forecasting Model** Triple Exponential Smoothing outperforms other models in capturing the seasonal behavior of estimated quantity demand per transaction, evidenced by the lowest MAE and RMSE values.

**One Year Forecast**: Using the Triple Exponential Smoothing model, the weekly estimated quantity demand per transaction for the next year averages 101.86 units, with detailed forecasts shown in the corresponding figures.

**Peak quantity demand Week (Next 6 Months)**: The highest predicted demand in the next six months is 104.67 units, occurring on June 4, 2025.

**Estimated quantity demand compared with Motor Fuel Inflation**: An inverse relationship exists; higher motor fuel inflation rates correlate with decreased estimated quantity demanded per transaction.

**Best Trend Visualisation**: The Hpfilter smoothing technique provides the clearest and most stable long-term trend visualisation, outperforming moving average, exponential, and rolling smoothing methods.

## 5.2 Impact of findings on Business and Stakeholder

### Improve Inventory and Supply Chain Planning

With the conclusion that Triple Exponential Smoothing is the optimal model to use for forecasting, the business can now more effectively predict seasonal fluctuations in demand, allowing inventory planners and suppliers to avoid overstocking and stockout. This reduces waste, storage, and customer discontent.

### Informed sales and marketing strategies

With the estimated weekly average (101.86 units) and the estimated high demand week (June 4, 2025), marketing personnel can strategise promotion, campaign, and resource allocation to achieve maximum effect in sales during high demand weeks.

### Proactive pricing and budget adjustments

The reverse relationship of fuel inflation and demand quantity provides directions for financial planning. In higher fuel inflation, the management can anticipate lower

demand and prepare pricing strategies, promotional offers, or cost reduction programs accordingly.

### Enhanced strategic planning with clearness of trends

Enhanced visualisation by the Hpfilter helps managers and analysts to pick up long term shifts in patterns of demand so that annual planning, capital expenditure, and performance measurement can be more precise.

### Stakeholder confidence and data based decision making

High quality evidence of successful forecasting techniques and actionable information inspires stakeholder confidence. It indicates that good data is the foundation for decision making, which enhances investor and partner confidence in business strategy and risk management.

## 5.3 Conclusion

The in-depth analysis revealed that Triple Exponential Smoothing is the most accurate and strongest forecasting model employed to identify seasonal patterns in estimated quantity demand per transaction. This is supported by its best performance in maintaining prediction errors at a minimum, as evidenced through MAE and RMSE. Through this model, the company can comfortably forecast future trend in demand precisely, with weekly average selling forecast standing at 101.86 units per sale for the next year. It is essential in strategic business planning in areas such as logistics, procurement, stock management, and staffing in a bid to balance resources against estimated demand more effectively. Moreover, choosing June 4, 2025, as the high demand date in the coming half a year (at 104.67 units) allows the firm to plan in advance for high demand periods, having sufficient inventory and being ready for marketing. This not only maximises revenue opportunity but also boosts customer satisfaction due to ongoing availability of goods. The analysis also highlights a significant reverse correlation between motor fuel inflation and quantity demand, with a strong need for reactive pricing and promotion reactions in periods of rising inflation. Such economic observation aids the business in forecasting and reacting to market pressure in terms of retaining competitive edge. Lastly, application of the Hpfilter smoothing method has been found to deliver the most stable and uncluttered representation of long run trends among all other smoothing methods. This enhances

the business's ability to track structural shifts in demand and aids in strategic planning with greater certainty. Briefly, such prediction insights make a good foundation for evidence based decision making so that the business and stakeholders can operate more efficiently, evade dangers, and seize market opportunities accurately.

## 5.4 Recommendations Based on the results of Data Analysis

### Embrace Triple Exponential Smoothing for forecasting

Given its increased accuracy in capturing seasonal demand behavior, the firm should formally embrace Triple Exponential Smoothing as the standard forecasting model to calculate quantity demand per transaction. This will improve planning across departments such as inventory, logistics, and finance.

### Align operations with weekly forecast insights

Projected Average 101.86 units weekly operations teams would use this benchmark to provision for procurement, staff planning, and delivery schedules. Implement dynamic inventory restocking systems to align inventories based on expected demand.

### Prepare for Peak demand early June 2025

Since the peak demand (104.67 units) will fall on June 4, 2025, the company has to plan in advance very well. This means having proper inventories, running targeted promotion campaigns, and supply chain preparations in this peak period.

### Develop fuel responsive pricing and promotion plans

The inverse relationship observed between demand and motor fuel inflation suggests that fuel price hikes lower the trend of consumer purchasing. The firm has to employ adaptive pricing strategies or reward-based promotions to keep demand going during times of high inflation.

### Apply Hpfilter trend insights for strategic planning

The Hpfilter smoothing technique gives a clearer picture of long-term trends. Decision-makers should regularly review such filtered trends to guide medium- to long-term plans, e.g., product development, market expansion, and capital investment decisions.

### Enhance cross departmental coordination

Ensure the forecasting results are shared across departments e.g., marketing, finance,

operations, and procurement—through regular reporting dashboards. This ensures a complete and coordinated response to expected demand trends.

### Invest in forecasting and visualisation software

Implement cutting edge analytics platforms that support advanced forecasting models and visualization features (such as used in this study). This will allow internal teams to continuously refresh forecasts and respond quickly to changes in market dynamics.

### Train stakeholders for forecast driven decision making

Conduct workshops or briefings for internal stakeholders and strategic partners to build knowledge and confidence in plan-by-forecast. This encourages buy-in and allows for collaborative implementation of demand-driven initiatives.

# CHAPTER 6

## OPTIMISATION

### a) Problem Statement

In a complex and highly competitive global market, well organised supply chain management is key to improving the profitability and responsiveness of a business. Businesses should make very smart production decisions so they can be meeting demand efficiently with maximum revenue realisation. However, in most companies, production quantities are decided based on past history or static forecasts, without being dynamically linked to real-time revenue performance or product profitability.

The purpose of this study is to increase profitability such as that in a decision-making scenario by employing optimisation methods in enhancing the production planning process for a dataset comprising of 100 SKUs (Stock Keeping Units). All SKUs pertain to various parameters like the product category, cost, supply availabilit y, selling rates, incomes, stocks levels, lead days, shipping as well as manufacture data, and others. With current production volumes, the study aims to optimise them within a capacity constraints so that overall revenue is maximised. The issue revolving around the study is how to adjust production amount for different products within a 20% operating flexibility range so that maximum possible revenue is realised under the condition of feasibility and respecting existing constraints?.

### b) Optimisation Problem

#### Objective

Maximise total revenue gained by production and selling all SKUs.

#### Decision Variables:

production volumes increased by not more than 20% of current production levels

price per unit of SKU is not adjusted.

#### Constraints:

Maintains the production volumes at or higher than current level of production.

Limits production to 20% above current capacity.

These constraints ensure:

No SKU will be produced at a level lower than current level.

No SKU can increase its current level of production by more than 20%.

Production levels are non negative.

### Assumptions

Demand is assumed to be sufficiently large to absorb the increase in production.

Prices remain fixed and are not influenced by the increase in supply.

Supply chain resources like labour, materials and equipment are not strictly constrained in this version.

Revenues are linearly related to production volumes, I.e, everything that is produced is sold.

## c) Implementation

The optimisation model is implemented using linear programming, a common technique for solving resource allocation problems. Python, in combination with PuLP ,libraries are used to formulate and solve the problem.

### Steps

### Data Preparation

The supply chain dataset is uploaded to the google colab environment using the pandas function as shown in fig. . After that, a concise review on the data is carried out to check for missing values, duplicates, data info, description and shape before proceeding to build the optimisation model as shown in fig. 26a, 26b, 26c, 26d, 26e , 26f and 26g;

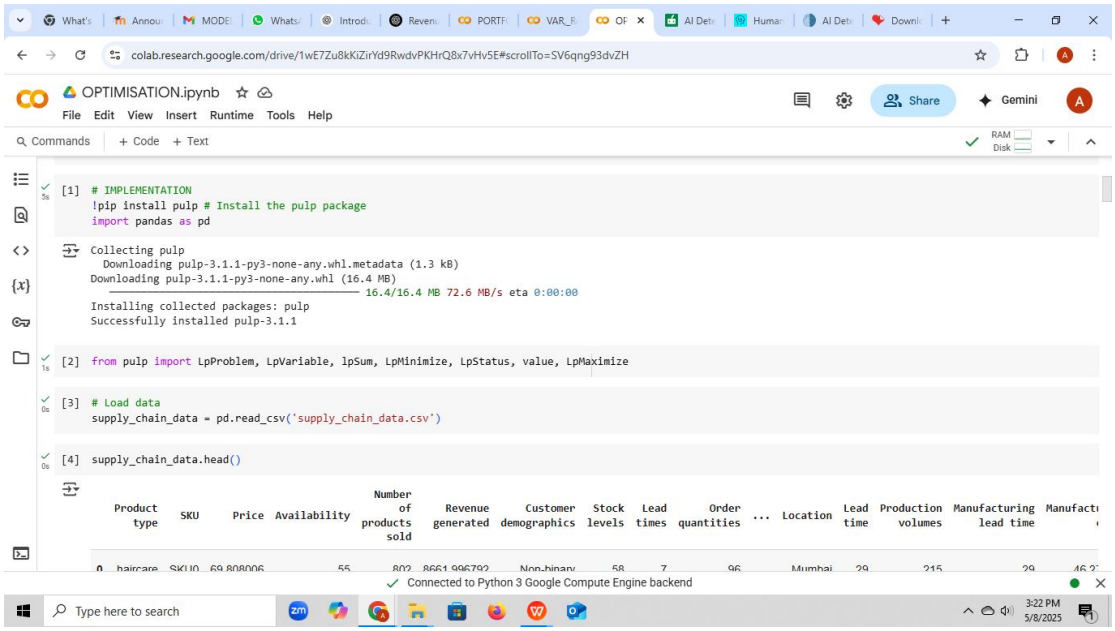



Fig. 26a: Load supply chain data

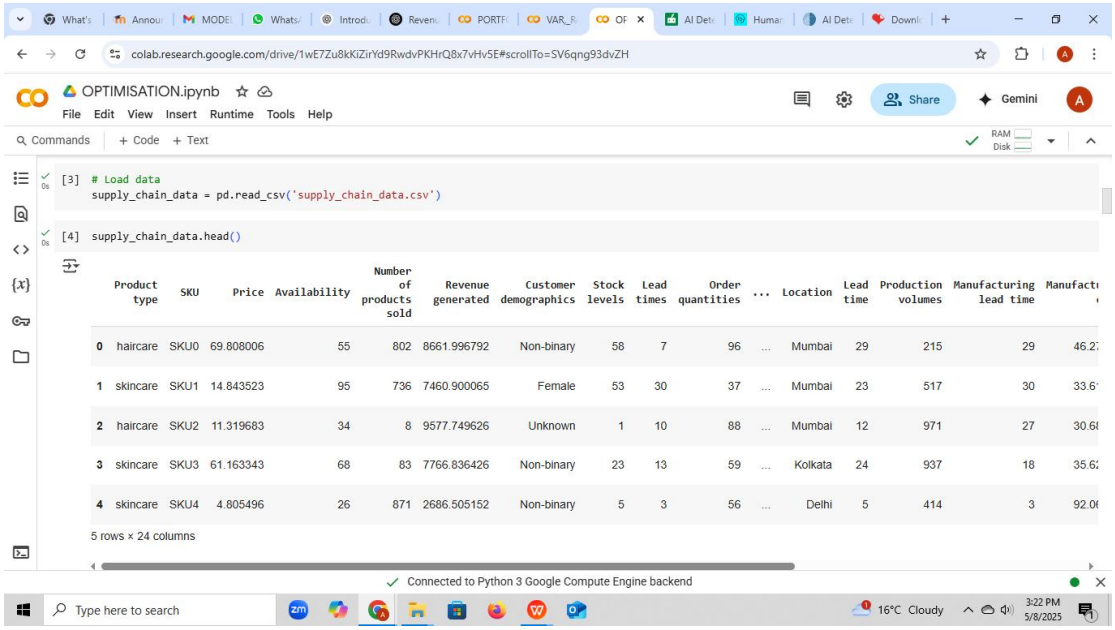



Fig. 26b: Supply chain data head

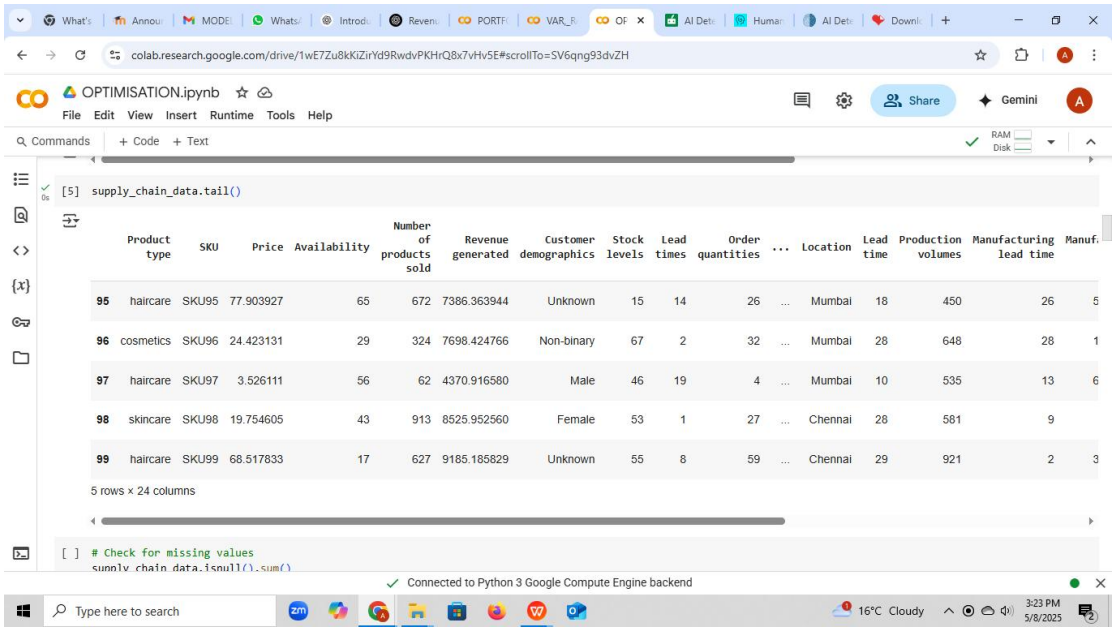



Fig. 26c: Supply chain data tail

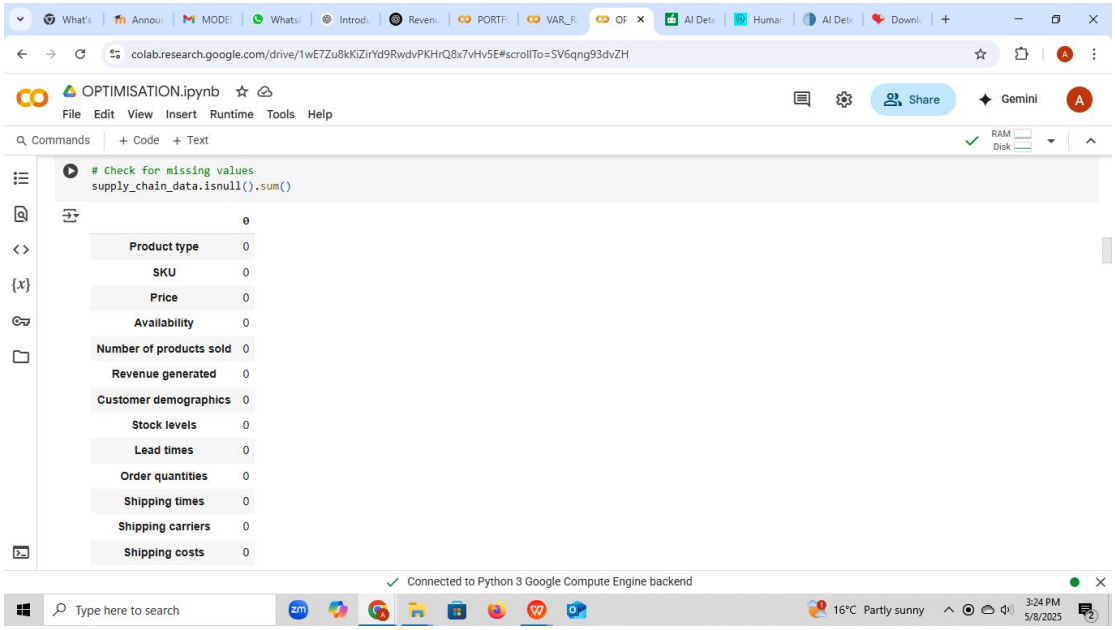



Fig. 26d: Missing values (none)

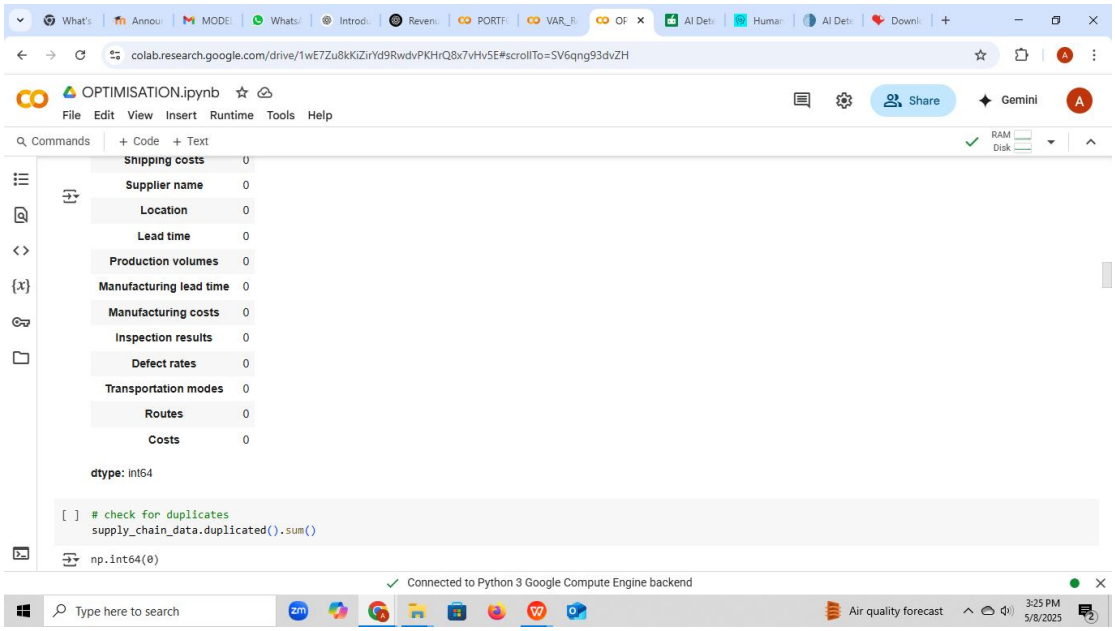



Fig. 26e: Check for duplicates (none)

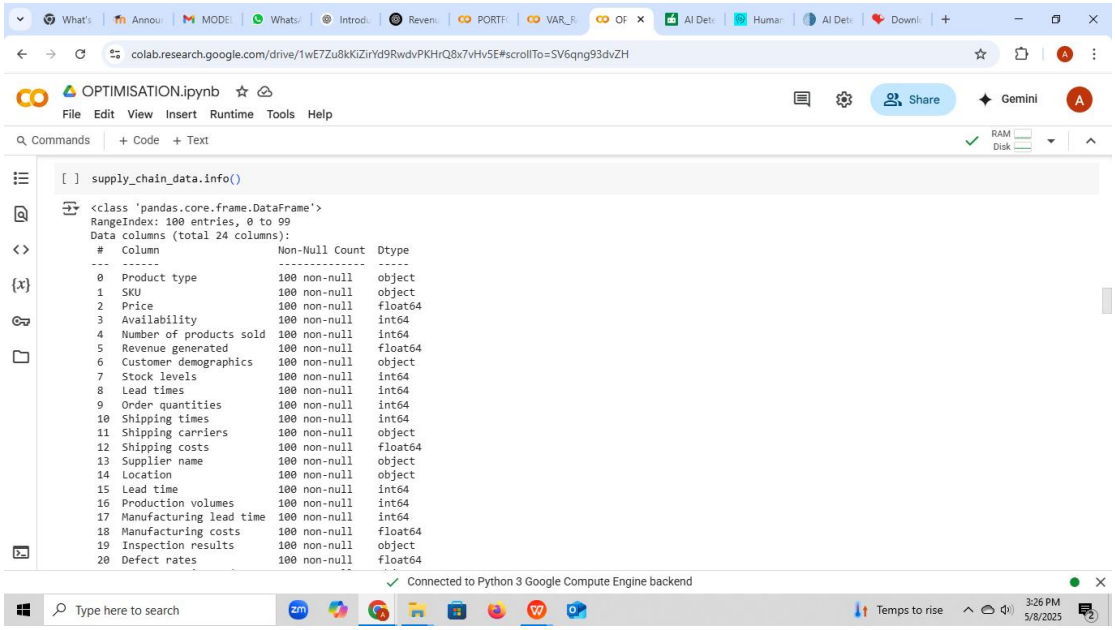
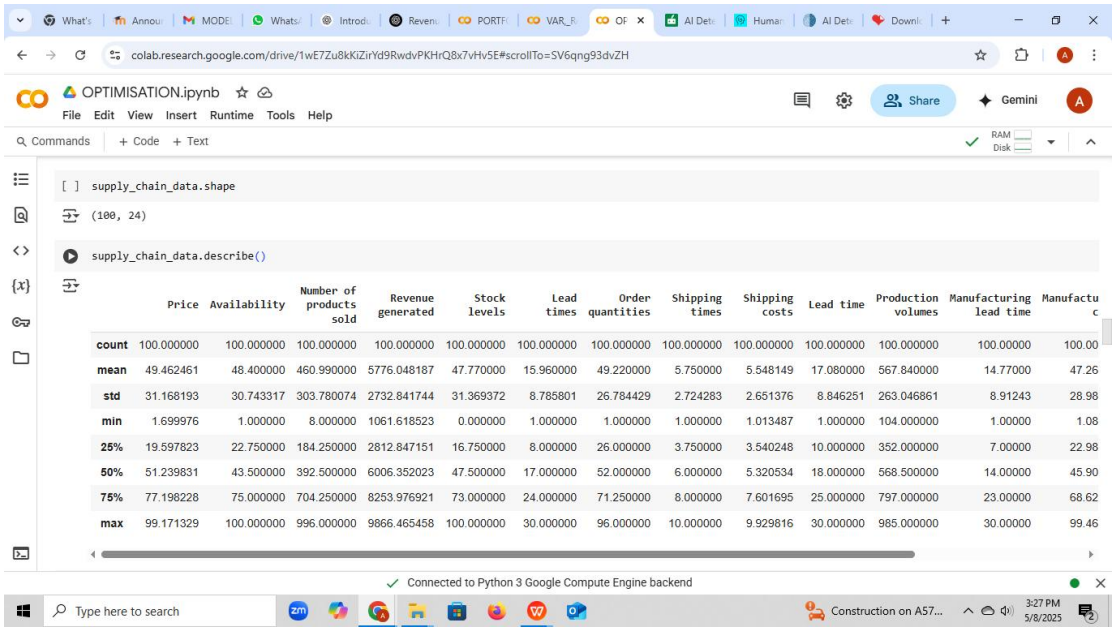



Fig. 26f: Supply chain info



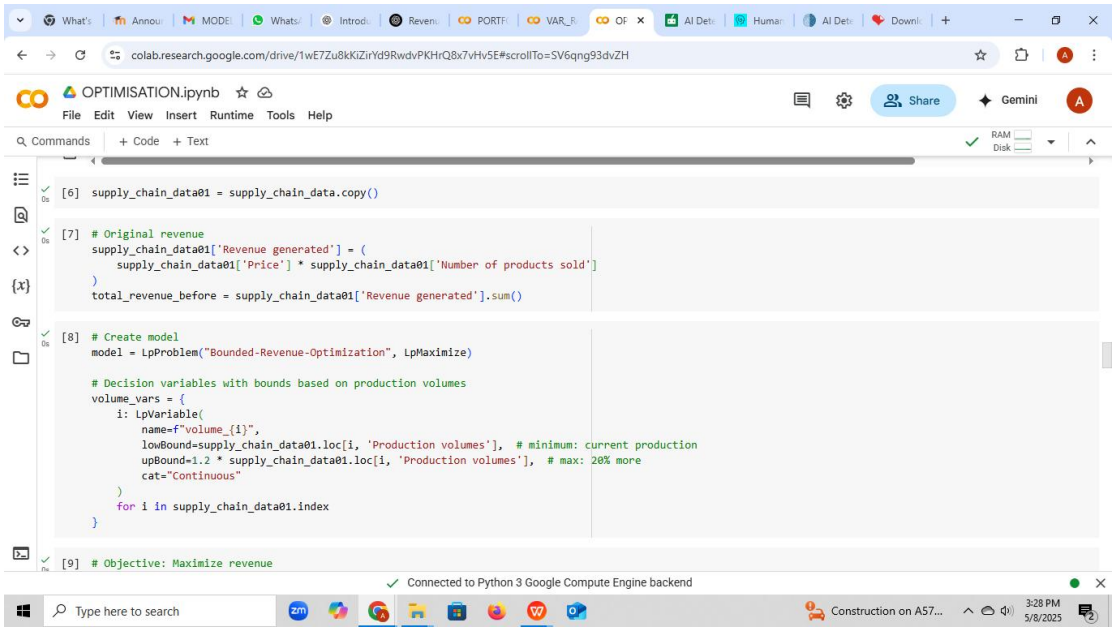
```
[ ] supply_chain_data.shape
(100, 24)
supply_chain_data.describe()
```

| | Price | Availability | Number of products sold | Revenue generated | Stock levels | Lead times | Order quantities | Shipping times | Shipping costs | Lead time | Production volumes | Manufacturing lead time | Manufactu c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| count | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.000000 | 100.00000 | 100.00 |
| mean | 49.462461 | 48.400000 | 460.990000 | 5776.048187 | 47.770000 | 15.960000 | 49.220000 | 5.750000 | 5.548149 | 17.080000 | 567.840000 | 14.77000 | 47.26 |
| std | 31.168193 | 30.743317 | 303.780074 | 2732.841744 | 31.369372 | 8.785801 | 26.784429 | 2.724283 | 2.651376 | 8.846251 | 263.046861 | 8.91243 | 28.98 |
| min | 1.699976 | 1.000000 | 8.000000 | 1061.618523 | 0.000000 | 1.000000 | 1.000000 | 1.000000 | 1.013487 | 1.000000 | 104.000000 | 1.00000 | 1.08 |
| 25% | 19.597823 | 22.750000 | 184.250000 | 2812.847151 | 16.750000 | 8.000000 | 26.000000 | 3.750000 | 3.540248 | 10.000000 | 352.000000 | 7.00000 | 22.98 |
| 50% | 51.239831 | 43.500000 | 392.500000 | 6006.352023 | 47.500000 | 17.000000 | 52.000000 | 6.000000 | 5.320534 | 18.000000 | 568.500000 | 14.00000 | 45.90 |
| 75% | 77.198228 | 75.000000 | 704.250000 | 8253.976921 | 73.000000 | 24.000000 | 71.250000 | 8.000000 | 7.601695 | 25.000000 | 797.000000 | 23.00000 | 68.62 |
| max | 99.171329 | 100.000000 | 996.000000 | 9866.465458 | 100.000000 | 30.000000 | 96.000000 | 10.000000 | 9.929816 | 30.000000 | 985.000000 | 30.00000 | 99.46 |

Fig. 26g: Supply chain shape and description

## Optimisation Model

The objective function is defined where the optimised production volume is bounded between current and 120% of current volume. Use the linear model to find the optimal production volumes that maximise revenue generated. Aggregate all individual revenues to compute total revenue before and after optimisation as shown in fig. 27a and 27b below;



```
[6] supply_chain_data01 = supply_chain_data.copy()

[7] # Original revenue
    supply_chain_data01['Revenue generated'] = (
        supply_chain_data01['Price'] * supply_chain_data01['Number of products sold']
    )
    total_revenue_before = supply_chain_data01['Revenue generated'].sum()

[8] # Create model
    model = LpProblem("Bounded-Revenue-Optimization", LpMaximize)

    # Decision variables with bounds based on production volumes
    volume_vars = {
        i: LpVariable(
            name=f"volume_{i}",
            lowBound=supply_chain_data01.loc[i, 'Production volumes'],  # minimum: current production
            upBound=1.2 * supply_chain_data01.loc[i, 'Production volumes'],  # max: 20% more
            cat="Continuous"
        )
        for i in supply_chain_data01.index
    }

[9] # Objective: Maximize revenue
```

Fig. 27a: optimisation model

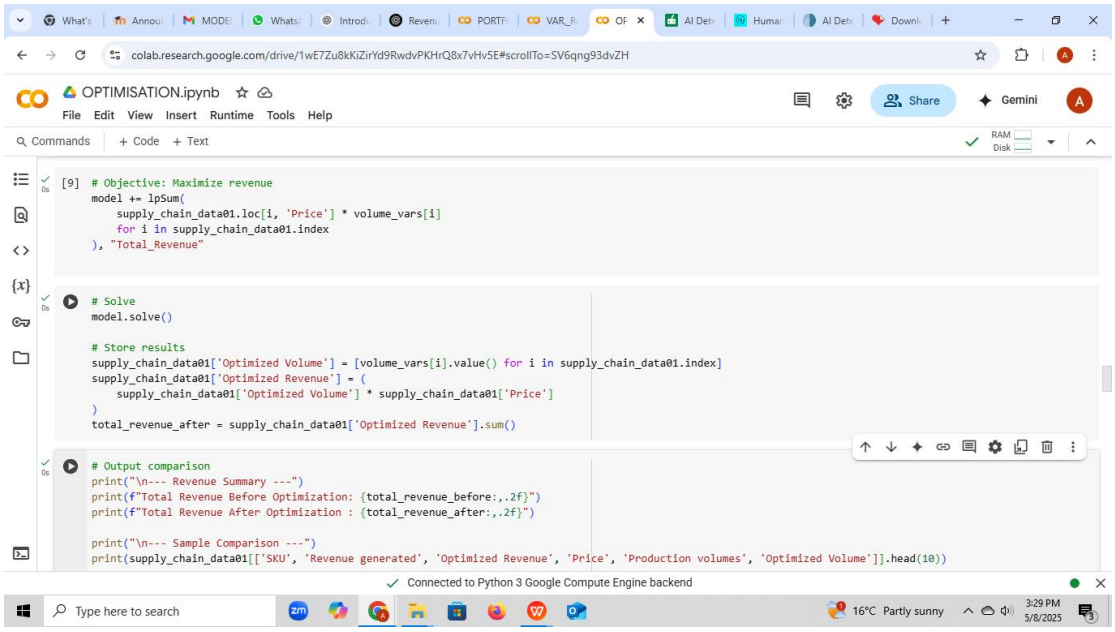



Fig. 27b: optimisation model

## Results storage

Generate a table showing original versus optimised revenue, price, production volumes and optimised volumes. In addition, calculated the percentage change in revenue for the overall SKU as shown in fig.

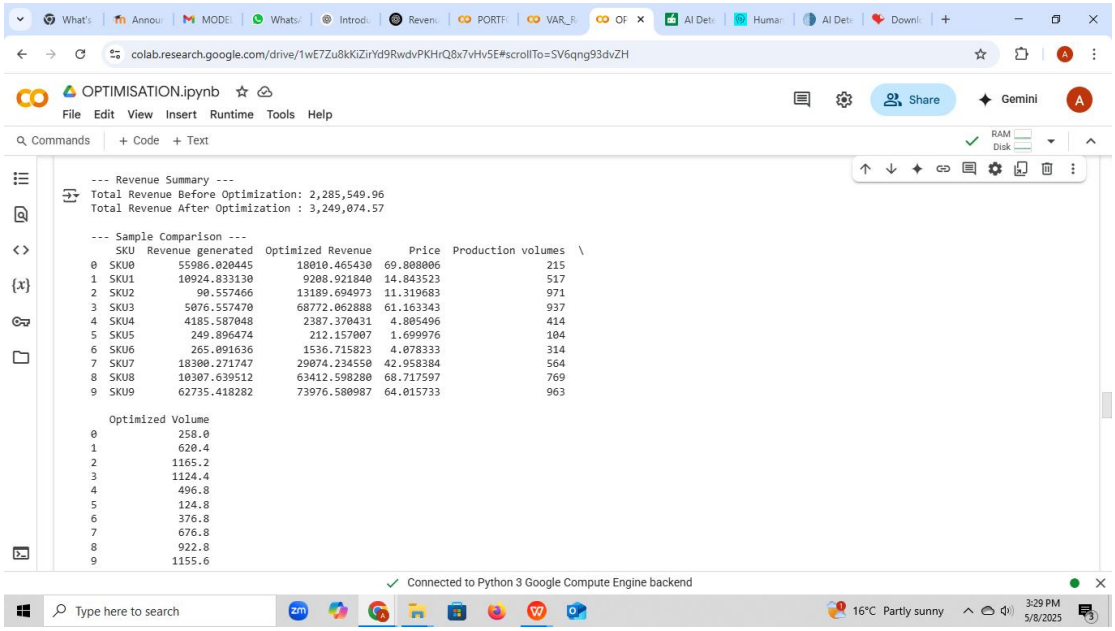



Fig. 28a: Results

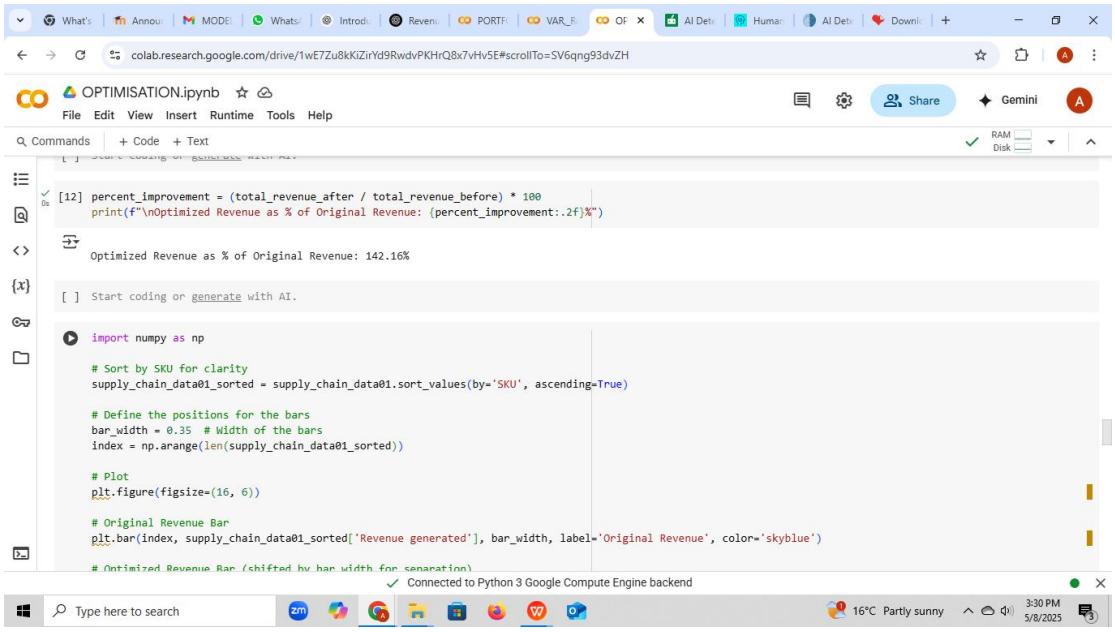



Fig. 28b: Percentage increase for optimised revenue

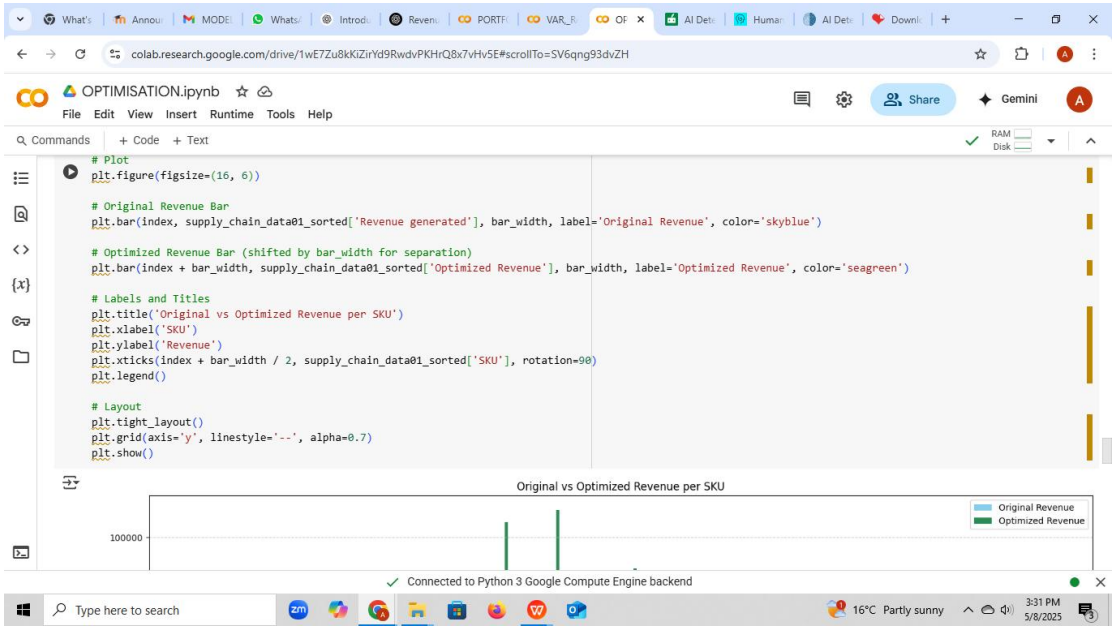



Fig. 28c: Bar plot to show original revenue vs optimised revenue

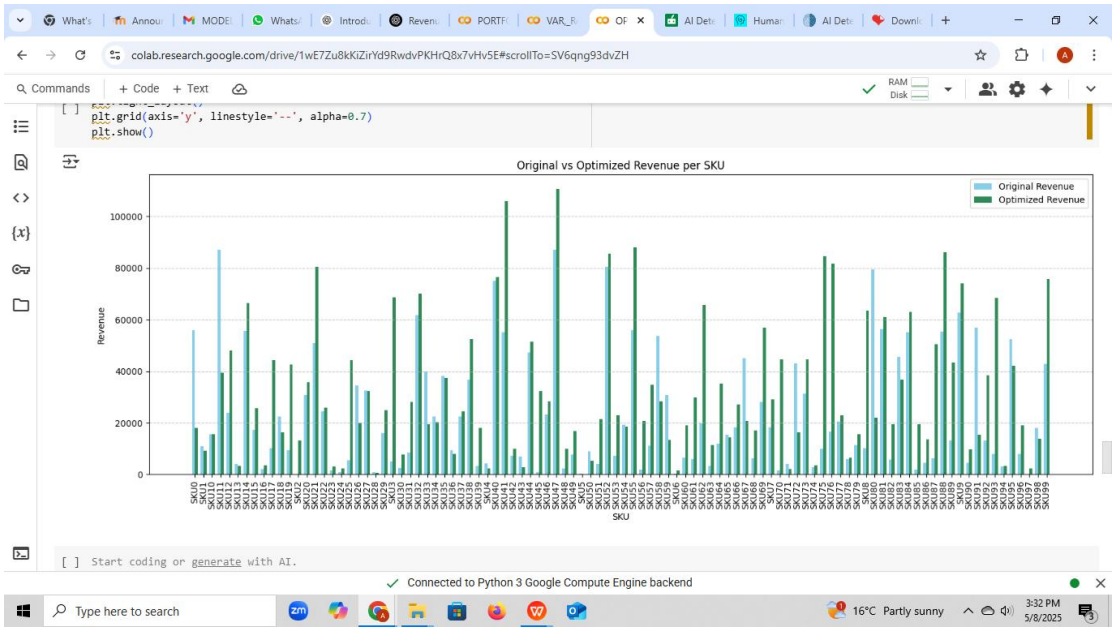



Fig. 28d: Bar plot

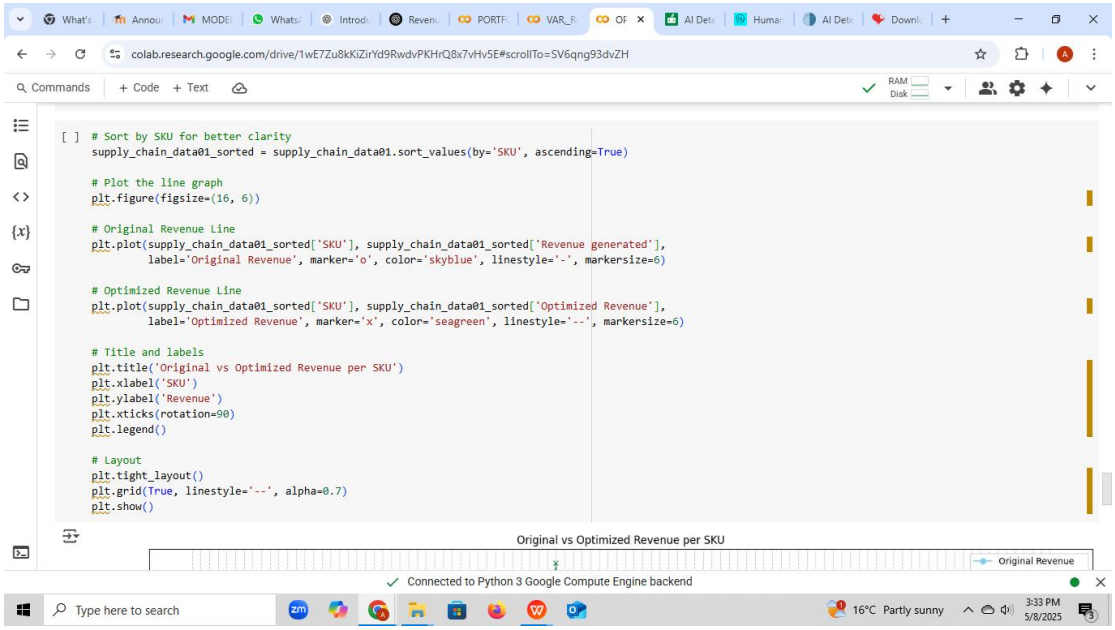



Fig. 28e: Line plot to show original revenue vs optimised revenue

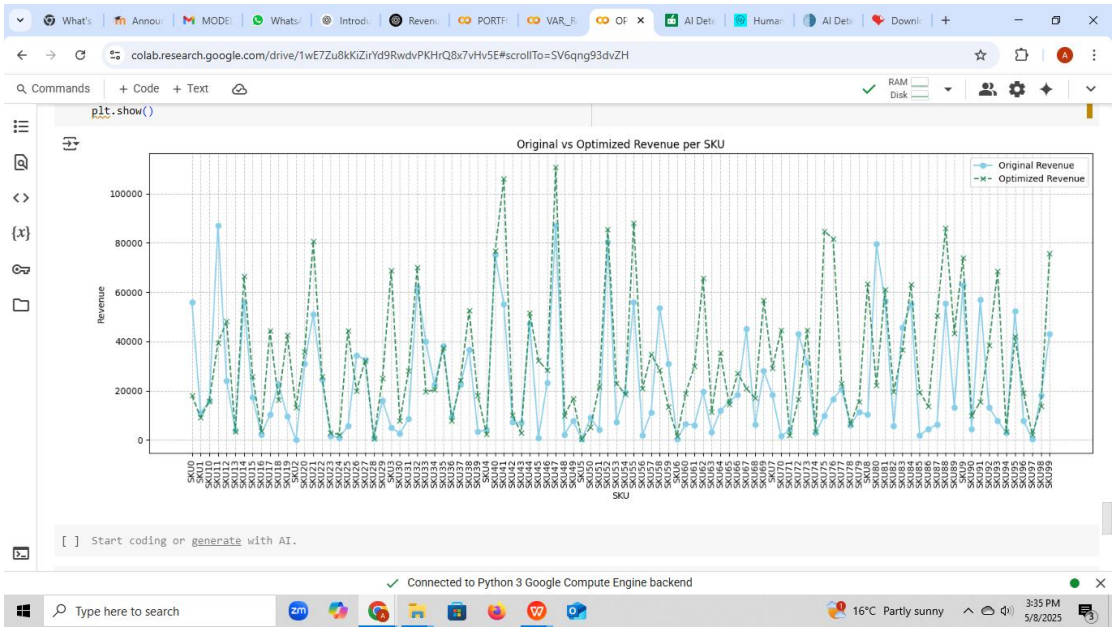



Fig. 28f: Line plot

## Validation

Confirm that all optimised production values lie within the 0% to 20% bounds of their original values.

Ensure that the final model respects all constraints and assumptions.

This linear programming approach, though relatively simple, is highly effective for scalable and constraint-bound production planning problems.

## d) Results and Analysis

Revenue Summary

| Metric | Value |
|---|---|
| Total Revenue (Before) | 2,285,549.96 |
| Total Revenue (After) | 3,249,074.57 |
| Increase in Revenue | 963,524.61 |
| Optimized Revenue % Increase | **42.16%** |

Table 2.1 : Revenue summary

Optimised Revenue % Increased by 42.16%

The optimisation model realised a 42.16% increase in total revenue on all SKUs by

simply adjusting production volumes within a very limited 20% range. That is how even small changes, when well focused, can significantly affect the bottom line.

Sample SKU Analysis

Below is an example of 10 SKUs comparing original and optimised metrics:

| SKU | Original Revenue | Optimized Revenue | Price | Original Volume | Optimised Volume |
|---|---|---|---|---|---|
| SKU0 | 55,986.02 | 18,010.47 | 69.81 | 215 | 258 |
| SKU1 | 10,924.83 | 9,208.92 | 14.84 | 517 | 620.4 |
| SKU2 | 90.56 | 13,189.69 | 11.32 | 971 | 1165.2 |
| SKU3 | 5,076.56 | 68,772.06 | 61.16 | 937 | 1124.4 |
| SKU4 | 4,185.59 | 2,387.37 | 4.81 | 414 | 496.8 |
| SKU5 | 249.90 | 212.16 | 1.7 | 104 | 124.8 |
| SKU6 | 265.09 | 1,536.72 | 4.08 | 314 | 376.8 |
| SKU7 | 18,300.27 | 29,074.23 | 42.96 | 564 | 676.8 |
| SKU8 | 10,307.64 | 63,412.60 | 68.72 | 769 | 922.8 |
| SKU9 | 62,735.42 | 73,976.58 | 64.02 | 963 | 1155.6 |

Table 2.2: sample SKU Analysis

## Observations

High Gain SKUs:

SKU3 and SKU8 showed incredible revenue improvement with their high expense and ability to scale up production.

SKU6 illustrates that underperforming SKUs can be worth it with scaling appropriately.

Revenue Decline

Some SKUs (SKU0 and SKU4) had their revenue fall.

This could occur if revenue has in the past been overestimated (e.g., stockpiling) or when the model puts production into the higher margin SKUs.

### Application of Constraints:

All SKUs were constrained to stay between 100% and 120% of original levels of production, confirming that the model was observing constraints.

### Efficiency of linear programming Model

The results offer proof that linear programming, with or without loose assumptions, can provide very significant outcomes under supply chain scenarios.

## Conclusion

The study highlights how strategic production optimisation has the potential to be a very strong driving force for improving business performance within a supply chain environment. Starting with a simple linear programming model with realistic operating constraints (in this case, a ceiling on a 20% increase in production) and then having the ability to increase total revenue by more than 960,000, or 42.16%. These outcomes are especially note worthy considering fairly modest change necessitated, no larger than a 20% volume boost across each SKU such that the solution was actionable, achievable, and implementable.

The value of this optimisation reaches beyond revenue improvement. This process has validity from a business perspective in general due to the following reasons:

### Planning Agility

The model generates a more dynamic production schedule. Instead of depending on static forecasting, decision makers can now dynamically shift volumes of production in near real time based on which products generate the highest margin of profit per unit of production.

### Data Driven Culture

Plugging these optimisation models into weekly or daily planning cycles creates a culture of fact-driven decision-making. It makes the planning more accurate over time, drives out waste, and aligns operating performance with financial targets.

### SKU Portfolio Analysis

The conclusions of this exercise form a template for SKU rationalisation. Product being negatively impacted by production uplift (or worse) can be reviewed for marketing spend, pricing strategy, or continuous production.

### Scalability of model

Even though this analysis was carried out on 100 SKUs, the process employed to deliver is extremely scalable. It can be delivered on thousands of products by supply chains globally, broken down by geography, or by item category.

### Alignment with digital supply chain initiatives

As businesses enter the fourth industrial revolution which focuses on smart manufacturing, these optimisation models are part of the larger picture of harnessing analytics, automation, and AI for intelligent decision making.

This study shows how relatively modest smart modeling driven changes can positively impact business. Organisations that implement similar quantitative techniques not only create competitive advantage in the short term but also lay the foundation for other advanced analytics applications in supply chain management, including predictive modeling, prescriptive analytics, and AI driven automation.

# CHAPTER 7

## Personal Reflection

Carrying out this study was technically challenging but fulfilling on a personal level. To begin with, I had anticipated that more sophisticated models like SARIMA or VAR would naturally have better forecasts than simple methods, and so it surprised me to see that Triple Exponential Smoothing had the best and least biased predictions. This outcome disappointed me in what I expected regarding model complexity and reminded me of the power simplicity can have if it is proportional to the properties of the data. Walking through the process of data preprocessing, especially stationarity and residual diagnosis, educated me on the merits of rigorous validation over believing what theoretical expectations do. I learned better also that external economic markers like fuel inflation can influence the behavior of demand. Overall, this study not only improved me as a technical practitioner of time series forecasting but also prompted me to be more critical and objective in evaluating model performance. It was an enriching and enlightening experience that reinforced the need to let the data speak for itself, rather than letting preconceived notions guide decisions on which models should work best. In addition, maximising revenue generated in supply chain using linear optimisation programming has deepened my understanding of how data driven decisions can significantly enhance operational efficiency and maximise revenue, by aligning supply more closely with demand and reducing losses from overstocking or shortages.

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