Geodesi 2008-08-01

Gauss Conformal Projection (Transverse Mercator)

Krüger's Formulas

Symbols and Definitions

a	semi-major axis of the ellipsoid
f	flattening of the ellipsoid
e^2	first eccentricity squared
φ	geodetic latitude, positive north
λ	geodetic longitude, positive east
x	grid coordinate, positive north
y	grid coordinate, positive east
λ_0	longitude of the central meridian
k_0	scale factor along the central meridian
$\delta\lambda$	difference $\lambda - \lambda_0$
FN	false northing
FE	false easting

All angles are expressed in radians. Please note that the x-axis is directed to the north and the y-axis to the east.

The following variables are defined out of the ellipsoidal parameters a and f:

$$e^2 = f(2-f)$$

$$n = \frac{f}{(2-f)}$$

$$\hat{a} = \frac{a}{(1+n)}\left(1+\frac{1}{4}n^2+\frac{1}{64}n^4+\ldots\right)$$

Conversion from geodetic coordinates (φ,λ) to grid coordinates (x,y).

Compute the conformal[1] latitude $\varphi*$

$$\varphi* = \varphi - \sin\varphi\cos\varphi\left(A + B\sin^2\varphi + C\sin^4\varphi + D\ \sin^6\varphi + \ldots\right)$$

$$q = \operatorname{artanh}\left(\sin\varphi\right) - e\operatorname{artanh}(e\sin\varphi); \quad \tan\varphi^* = \sinh q; \quad \cos\varphi^* = \operatorname{sech} q$$

The coefficients *A*, *B*, *C*, and *D* are computed using the following formulas:

$$A = e^2$$

$$B = \frac{1}{6}\left(5e^4 - e^6\right)$$

$$C = \frac{1}{120}\left(104e^6 - 45e^8 + \ldots\right)$$

$$D = \frac{1}{1260}\left(1237e^8 + \ldots\right)$$

Let $\delta\lambda = \lambda - \lambda_0$ and

$$\xi' = \arctan(\tan\varphi * / \cos\delta\lambda)$$

$$\eta' = \operatorname{arctanh}(\cos\varphi * \sin\delta\lambda)$$

then

$$x = k_0\hat{a}\begin{pmatrix}\xi' + \beta_1\sin 2\xi'\cosh 2\eta' + \beta_2\sin 4\xi'\cosh 4\eta' + \beta_3\sin 6\xi'\cosh 6\eta' + \\ + \beta_4\sin 8\xi'\cosh 8\eta' + \ldots\end{pmatrix} + FN$$

$$y = k_0\hat{a}\begin{pmatrix}\eta' + \beta_1\cos 2\xi'\sinh 2\eta' + \beta_2\cos 4\xi'\sinh 4\eta' + \beta_3\cos 6\xi'\sinh 6\eta' + \\ + \beta_4\cos 8\xi'\sinh 8\eta' + \ldots\end{pmatrix} + FE$$

[1] Older Swedish literature refers to this quantity as the isometric latitude. Today the term isometric latitude is applied to the quantity $\psi = \ln\{\tan(\pi/4 + \varphi/2)[(1 - e\sin\varphi)/(1 + e\sin\varphi)]^{e/2}\}$. The isometric latitude is related to the conformal latitude by $\psi = \ln\tan(\pi/4 + \varphi*/2)$. Cf. John P. Snyder: Map Projections - A Working Manual, U.S. Geological Survey Professional Paper 1395.

where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are computed by

$$\beta_1 = \frac{1}{2}n - \frac{2}{3}n^2 + \frac{5}{16}n^3 + \frac{41}{180}n^4 + \ldots$$

$$\beta_2 = \frac{13}{48}n^2 - \frac{3}{5}n^3 + \frac{557}{1440}n^4 + \ldots$$

$$\beta_3 = \frac{61}{240}n^3 - \frac{103}{140}n^4 + \ldots$$

$$\beta_4 = \frac{49561}{161280}n^4 + \ldots$$

Conversion from grid coordinates (*x*,*y*) to geodetic coordinates (φ,λ)

Introduce the variables ξ and η as

$$\xi = \frac{x - FN}{k_0 \cdot \hat{a}}$$

$$\eta = \frac{y - FE}{k_0 \cdot \hat{a}}$$

Let

$$\xi' = \xi - \delta_1 \sin 2\xi \cosh 2\eta - \delta_2 \sin 4\xi \cosh 4\eta - \delta_3 \sin 6\xi \cosh 6\eta$$
$$- \delta_4 \sin 8\xi \cosh 8\eta - \ldots$$

$$\eta' = \eta - \delta_1 \cos 2\xi \sinh 2\eta - \delta_2 \cos 4\xi \sinh 4\eta - \delta_3 \cos 6\xi \sinh 6\eta -$$
$$- \delta_4 \cos 8\xi \sinh 8\eta - \ldots$$

where

$$\delta_1 = \frac{1}{2}n - \frac{2}{3}n^2 + \frac{37}{96}n^3 - \frac{1}{360}n^4 + \ldots$$

$$\delta_2 = \frac{1}{48}n^2 + \frac{1}{15}n^3 - \frac{437}{1440}n^4 + \ldots$$

$$\delta_3 = \frac{17}{480}n^3 - \frac{37}{840}n^4 + \ldots$$

$$\delta_4 = \frac{4397}{161280}n^4 + \ldots$$

The conformal latitude φ^* and the difference in longitude $\delta\lambda$ are obtained by the formulas

$$\varphi^* = \arcsin(\sin\xi' / \cosh\eta')$$

$$\delta\lambda = \arctan(\sinh\eta' / \cos\xi')$$

Finally, the latitude φ and the longitude λ are obtained by the formulas

$$\lambda = \lambda_0 + \delta\lambda$$

$$\varphi = \varphi^* + \sin\varphi^* \cos\varphi^* (A^* + B^* \sin^2\varphi^* + C^* \sin^4\varphi^* + D^* \sin^6\varphi^* + \ldots)$$

where

$$A^* = (e^2 + e^4 + e^6 + e^8 + \ldots)$$

$$B^* = -\frac{1}{6}(7e^4 + 17e^6 + 30e^8 + \ldots)$$

$$C^* = \frac{1}{120}(224e^6 + 889e^8 + \ldots)$$

$$D^* = -\frac{1}{1260}(4279e^8 + \ldots)$$

Worked example:

ELLIPSOID GRS 1980	
Semi-major axis (a)	6378137.0000 m.
Flattening (f)	1/298.257222101

TRANSVERSE MERCATOR PARAMETERS	
Longitude of the central meridian	13 35 7.692000 degr. min. sec.
Scale factor on the central merdian	1.000002540000
False northing	-6226307.8640 m.
False easting	84182.8790 m.

Latitude and longitude	66 0 0.0000	24 0 0.0000 degr. min. sec.
Grid coordinates	1135809.413803	555304.016555 m