

Inserting figures and evaluated examples

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Abstract

Package **lagged** provides classes and methods for objects, whose indexing naturally starts from zero.

This vignette is part of package **lagged**, version 0.1-2.

Keywords: lag, autocorrelation, indexing.

1. Univariate lagged objects

Create a univariate lagged object¹:

```
> a1 <- drop(acf(ldeaths)$acf)
> la1 <- Lagged(a1)
> la1
```

An object of class "LaggedId"

Slot *data*:

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5
1.000000000	0.755051141	0.396956836	0.019395714	-0.355897989	-0.608566374
Lag_6	Lag_7	Lag_8	Lag_9	Lag_10	Lag_11
-0.681383469	-0.607909875	-0.378212377	-0.012975866	0.383252644	0.650206704
Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17
0.723167071	0.638001465	0.371577811	0.009467461	-0.293699737	-0.496742216
Lag_18					
-0.585558984					

`maxLag()` returns the maximal lag in the object. `length()` returns the number of lags in the object, i.e. `length(la1) == maxLag(la1) + 1`. This relation is a definition and holds also for multivariate lagged objects. In particular, the length is not necessarily the length of the data slot.

```
> maxLag(la1)
```

```
[1] 18
```

```
> length(la1)
```

```
[1] 19
```

2. Indexing

¹The datasets `ldeaths`, `fdeaths` and `mdeaths` are in base R. The examples involving them are adapted from the help page of `acf()`.

Indexing drops the "laggedness" to allow easy access to the underlying data²:

```
> la1[0]

[1] 1

> la1[0:4]

[1] 1.00000000 0.75505114 0.39695684 0.01939571 -0.35589799

> la1[c(1,3,5)]

[1] 0.75505114 0.01939571 -0.60856637

> la1[]

[1] 1.000000000 0.755051141 0.396956836 0.019395714 -0.355897989
[6] -0.608566374 -0.681383469 -0.607909875 -0.378212377 -0.012975866
[11] 0.383252644 0.650206704 0.723167071 0.638001465 0.371577811
[16] 0.009467461 -0.293699737 -0.496742216 -0.585558984

> la1a <- la1
> la1a[] <- round(la1, 2)
> la1a

An object of class "Lagged1d"
Slot *data*:
  Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
    1.00  0.76  0.40  0.02 -0.36 -0.61 -0.68 -0.61 -0.38 -0.01  0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
    0.65  0.72  0.64  0.37  0.01 -0.29 -0.50 -0.59

> la1b <- round(la1, 2)
> all(la1a == la1b)

[1] TRUE
```

3. Unary arithmetic and mathematical functions

Unary arithmetic operations and mathematical functions replace the data part of the object and keep its class.

```
> -la1a

An object of class "Lagged1d"
Slot *data*:
  Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
 -1.00 -0.76 -0.40 -0.02  0.36  0.61  0.68  0.61  0.38  0.01 -0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
 -0.65 -0.72 -0.64 -0.37 -0.01  0.29  0.50  0.59
```

²For some indices, such as 0:4, it is possible to keep a Lagged class but it would be confusing if the indexing operation was returning Lagged or non-Lagged objects depending on the values of the index.

```
> +la1a
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.00	0.76	0.40	0.02	-0.36	-0.61	-0.68	-0.61	-0.38	-0.01	0.38

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.65	0.72	0.64	0.37	0.01	-0.29	-0.50	-0.59

```
> ## Math group
```

```
> abs(la1a)
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.00	0.76	0.40	0.02	0.36	0.61	0.68	0.61	0.38	0.01	0.38

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.65	0.72	0.64	0.37	0.01	0.29	0.50	0.59

```
> sinpi(la1a)
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5
0.00000000	0.68454711	0.95105652	0.06279052	-0.90482705	-0.94088077

Lag_6	Lag_7	Lag_8	Lag_9	Lag_10	Lag_11
-0.84432793	-0.94088077	-0.92977649	-0.03141076	0.92977649	0.89100652

Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17
0.77051324	0.90482705	0.91775463	0.03141076	-0.79015501	-1.00000000

Lag_18
-0.96029369

```
> sqrt(abs(la1a))
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7
1.0000000	0.8717798	0.6324555	0.1414214	0.6000000	0.7810250	0.8246211	0.7810250

Lag_8	Lag_9	Lag_10	Lag_11	Lag_12	Lag_13	Lag_14	Lag_15
0.6164414	0.1000000	0.6164414	0.8062258	0.8485281	0.8000000	0.6082763	0.1000000

Lag_16	Lag_17	Lag_18
0.5385165	0.7071068	0.7681146

```
> ## Math2 group
```

```
> round(la1a)
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1	1	0	0	0	-1	-1	-1	0	0	0

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
1	1	1	0	0	0	0	-1

```
> round(la1a, 2)
```

```
An object of class "LaggedId"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.00	0.76	0.40	0.02	-0.36	-0.61	-0.68	-0.61	-0.38	-0.01	0.38

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.65	0.72	0.64	0.37	0.01	-0.29	-0.50	-0.59

```
> signif(la1a)
```

```
An object of class "LaggedId"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.00	0.76	0.40	0.02	-0.36	-0.61	-0.68	-0.61	-0.38	-0.01	0.38

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.65	0.72	0.64	0.37	0.01	-0.29	-0.50	-0.59

```
> signif(la1a, 4)
```

```
An object of class "LaggedId"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.00	0.76	0.40	0.02	-0.36	-0.61	-0.68	-0.61	-0.38	-0.01	0.38

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.65	0.72	0.64	0.37	0.01	-0.29	-0.50	-0.59

The functions from the summary group work on the data part, as if they were called on it.

```
> c(Max = max(la1a), Min = min(la1a), Range = range(la1a))
```

Max	Min	Range1	Range2
1.00	-0.68	-0.68	1.00

```
> c(Prod = prod(la1a), Sum = sum(la1a))
```

Prod	Sum
-7.582098e-11	9.200000e-01

```
> c(Any = any(la1a < 0), All = all(la1a >= 0))
```

Any	All
TRUE	FALSE

Binary arithmetic operators are defined between two lagged objects and between a lagged object and a vector. They return a lagged object from one of the "basic" lagged classes, but not necessarily exactly from the class of the argument(s). The class of the returned value is from a suitable lagged superclass of the argument(s). This concerns operations on objects from classes inheriting from the classes considered here, so is not visible in the examples below, since they use objects from the basic lagged classes.

```
> 2*la1a
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
2.00	1.52	0.80	0.04	-0.72	-1.22	-1.36	-1.22	-0.76	-0.02	0.76

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
1.30	1.44	1.28	0.74	0.02	-0.58	-1.00	-1.18

```
> la1a^2
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
1.0000	0.5776	0.1600	0.0004	0.1296	0.3721	0.4624	0.3721	0.1444	0.0001	0.1444

Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.4225	0.5184	0.4096	0.1369	0.0001	0.0841	0.2500	0.3481

```
> la1a + la1a^2
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9
2.0000	1.3376	0.5600	0.0204	-0.2304	-0.2379	-0.2176	-0.2379	-0.2356	-0.0099

Lag_10	Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.5244	1.0725	1.2384	1.0496	0.5069	0.0101	-0.2059	-0.2500	-0.2419

```
> la1a - la1a^2
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9
0.0000	0.1824	0.2400	0.0196	-0.4896	-0.9821	-1.1424	-0.9821	-0.5244	-0.0101

Lag_10	Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18
0.2356	0.2275	0.2016	0.2304	0.2331	0.0099	-0.3741	-0.7500	-0.9381

```
> la1a * la1a^2
```

```
An object of class "Lagged1d"
```

```
Slot *data*:
```

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7
1.000000	0.438976	0.064000	0.000008	-0.046656	-0.226981	-0.314432	-0.226981

Lag_8	Lag_9	Lag_10	Lag_11	Lag_12	Lag_13	Lag_14	Lag_15
-0.054872	-0.000001	0.054872	0.274625	0.373248	0.262144	0.050653	0.000001

Lag_16	Lag_17	Lag_18
-0.024389	-0.125000	-0.205379

```
> la1a / la1a^2
```

An object of class "Lagged1d"

Slot *data*:

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5
1.000000	1.315789	2.500000	50.000000	-2.777778	-1.639344
Lag_6	Lag_7	Lag_8	Lag_9	Lag_10	Lag_11
-1.470588	-1.639344	-2.631579	-100.000000	2.631579	1.538462
Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17
1.388889	1.562500	2.702703	100.000000	-3.448276	-2.000000
Lag_18	-1.694915				

```
> la1a + 1:length(la1a)
```

An object of class "Lagged1d"

Slot *data*:

Lag_0	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_10
2.00	2.76	3.40	4.02	4.64	5.39	6.32	7.39	8.62	9.99	11.38
Lag_11	Lag_12	Lag_13	Lag_14	Lag_15	Lag_16	Lag_17	Lag_18			
12.65	13.72	14.64	15.37	16.01	16.71	17.50	18.41			

There is a case to argue for keeping the class in some situations, e.g. when the other argument is a scalar but eventually I decided to keep the simple rule of not trying to preserve the class.

Note however that unary operators and mathematical functions do preserve the class.

4. Multivariate lagged objects

Compute the autocorrelations of a multivariate time series and convert it to a lagged object.

```
> acv2 <- acf(ts.union(mdeaths, fdeaths))
> la2 <- Lagged(acv2)
```

Get the value for lag 1.

```
> la2[1]
```

```
, , 1
```

	[,1]	[,2]
[1,]	0.7570591	0.7356685
[2,]	0.7443093	0.7295201

```
> acv2$acf[2, ,] # same
```

	[,1]	[,2]
[1,]	0.7570591	0.7356685
[2,]	0.7443093	0.7295201

Indexing in `acf()` is somewhat misterious. For some insight, here is a comparison with a DIY calculation of the autocorrelations.

```
> n <- length(mdeaths)
> tmpcov <- sum((mdeaths - mean(mdeaths)) * (fdeaths - mean(fdeaths)) ) / n
> msd <- sqrt(sum((mdeaths - mean(mdeaths))^2)/n)
> fsd <- sqrt(sum((fdeaths - mean(fdeaths))^2)/n)
> tmpcov1 <- sum((mdeaths - mean(mdeaths))[2:n] * (fdeaths - mean(fdeaths))[1:(n-1)] ) / n
> tmpcov1 / (msd * fsd)
```

```
[1] 0.7356685
```

```
> la2[[1]][1,2] == tmpcov1 / (msd * fsd) # FALSE, but:
```

```
[1] FALSE
```

```
> la2[[1]][1,2] - tmpcov1 / (msd * fsd) # only numerically different
```

```
[1] 2.220446e-16
```

Some examples for the correspondence between the indices in lagged objects and those from `acf()`.

```
> la2[[1]][1,2] == acv2$acf[2, 1, 2] # TRUE
```

```
[1] TRUE
```

```
> la2[0]
```

```
, , 1
```

```
      [,1]      [,2]
[1,] 1.0000000 0.9762413
[2,] 0.9762413 1.0000000
```

```
> acv2[0]
```

Autocorrelations of series 'ts.union(mdeaths, fdeaths)', by lag

```
, , mdeaths
```

```
mdeaths  fdeaths
1.000 (0) 0.976 (0)
```

```
, , fdeaths
```

```
mdeaths  fdeaths
0.976 (0) 1.000 (0)
```

```
> la2[1]
```

```
, , 1
```

```
      [,1]      [,2]
[1,] 0.7570591 0.7356685
[2,] 0.7443093 0.7295201
```

```
> acv2[1]
```

```
Autocorrelations of series 'ts.union(mdeaths, fdeaths)', by lag
```

```
, , mdeaths
```

mdeaths	fdeaths
0.717 (1)	0.708 (-1)

```
, , fdeaths
```

mdeaths	fdeaths
0.721 (1)	0.716 (1)

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