# Draft guide to package "lagged"

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#### Abstract

Package lagged provides classes and methods for objects, whose indexing naturally starts from zero. Subsetting, indexing and mathematical operations are defined naturally between lagged objects and lagged and base R objects. Recycling is not used, except for singletons. The single bracket operator doesn't drop dimensions by default.

This vignette is part of package **lagged**, version 0.3-0.

Keywords: lag, autocorrelation, indexing.

## 1. Univariate lagged objects

Create a univariate lagged object<sup>1</sup>:

```
> a1 <- drop(acf(ldeaths)$acf)
> la1 <- Lagged(a1)
> la1
An object of class "Lagged1d"
Slot *data*:
      Lag_0
                                        Lag_3
                 Lag_1
                             Lag_2
                                                    Lag_4
                                                               Lag_5
 1.000000000
            0.755051141
                        0.396956836
                                   0.019395714 -0.355897989 -0.608566374
      Lag_6
                 Lag_7
                             Lag_8
                                        Lag_9
                                                   Lag_10
                                                               Lag_11
-0.681383469 -0.607909875 -0.378212377 -0.012975866 0.383252644
                                                          0.650206704
     Lag_12
                 Lag_13
                            Lag_14
                                       Lag_15
                                                   Lag_16
                                                               Lag_17
0.723167071
            Lag_18
-0.585558984
```

maxLag() returns the maximal lag in the object. Length() returns the number of lags in the object,
i.e. length(la1) == maxLag(la1) + 1. This relation is a definition and holds also for multivariate
lagged objects. In particular, the length is not necessarily the length of the data slot.

```
> maxLag(la1)
[1] 18
> length(la1)
[1] 19
```

<sup>&</sup>lt;sup>1</sup>The datasets ldeaths, fdeaths and mdeaths are in base R. The examples involving them are adapted from the help page of acf().

## 2. Indexing

```
Indexing drops the "laggedness" to allow easy access to the underlying data<sup>2</sup>:
```

```
> la1[0]
[1] 1
> la1[0:4]
    1.00000000 \quad 0.75505114 \quad 0.39695684 \quad 0.01939571 \quad -0.35589799
> la1[c(1,3,5)]
[1] 0.75505114 0.01939571 -0.60856637
> la1[]
    1.000000000 0.755051141 0.396956836 0.019395714 -0.355897989
 [11] 0.383252644 0.650206704 0.723167071 0.638001465 0.371577811
[16] 0.009467461 -0.293699737 -0.496742216 -0.585558984
> la1a <- la1
> la1a[] <- round(la1, 2)
> la1a
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
                                                    -0.38 -0.01
              0.40
                     0.02 -0.36 -0.61 -0.68 -0.61
                                                                  0.38
        0.76
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
              0.64
                     0.37
                           0.01 -0.29 -0.50 -0.59
> la1b <- round(la1, 2)
> all(la1a == la1b)
```

## 3. Unary arithmetic and mathematical functions

Unary arithmetic operations and mathematical functions replace the data part of the object and keep its class.

```
> -la1a
```

[1] TRUE

<sup>&</sup>lt;sup>2</sup>For some indices, such as 0:4, it is possible to keep a Lagged class but it would be confusing if the indexing operation was returning Lagged or non-Lagged objects depending on the values of the index.

```
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
 -1.00 -0.76 -0.40 -0.02 0.36
                                   0.61
                                          0.68
                                                  0.61
                                                        0.38
                                                                0.01 -0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
 -0.65 -0.72 -0.64 -0.37 -0.01
                                    0.29
                                          0.50
> +la1a
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
  1.00 \quad 0.76 \quad 0.40 \quad 0.02 \quad -0.36 \quad -0.61 \quad -0.68 \quad -0.61 \quad -0.38 \quad -0.01 \quad 0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
              0.64 0.37 0.01 -0.29 -0.50 -0.59
        0.72
> ## Math group
> abs(la1a)
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
  1.00 0.76 0.40 0.02 0.36 0.61
                                           0.68
                                                        0.38
                                                                0.01 0.38
                                                  0.61
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
        0.72
              0.64 0.37 0.01
                                    0.29
                                          0.50
> sinpi(la1a)
An object of class "Lagged1d"
Slot *data*:
                             Lag_2
                                         Lag_3
                                                     Lag_4
                 Lag_1
 0.00000000 \quad 0.68454711 \quad 0.95105652 \quad 0.06279052 \quad -0.90482705 \quad -0.94088077
      Lag_6
                 Lag_7
                             Lag_8
                                         Lag_9
                                                    Lag_10
-0.84432793 \ -0.94088077 \ -0.92977649 \ -0.03141076 \ \ 0.92977649 \ \ 0.89100652
                            Lag_14
                                        Lag_15
                Lag_13
                                                    Lag_16
                                                                Lag_17
 Lag_18
-0.96029369
> sqrt(abs(la1a))
An object of class "Lagged1d"
Slot *data*:
   Lag_0
                       Lag_2
                                Lag_3
                                          Lag_4
             Lag_1
                                                     Lag_5
                                                               Lag_6
1.00000000 \ 0.8717798 \ 0.6324555 \ 0.1414214 \ 0.6000000 \ 0.7810250 \ 0.8246211 \ 0.7810250
             Lag_9
                      Lag_10
                                Lag_11
                                          Lag_12
                                                    Lag_13
                                                              Lag_14
0.6164414 \ \ 0.1000000 \ \ 0.6164414 \ \ 0.8062258 \ \ 0.8485281 \ \ 0.8000000 \ \ 0.6082763 \ \ 0.1000000
   Lag_16
            Lag_17
                      Lag_18
0.5385165 0.7071068 0.7681146
```

> ## Math2 group
> round(la1a)

Any

TRUE FALSE

All

```
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
   1 1 0 0 0 -1 -1 -1
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
         1 1 0 0
                                  0
> round(la1a, 2)
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
             0.40
                   0.02 -0.36 -0.61 -0.68 -0.61 -0.38 -0.01 0.38
      0.76
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
 0.65  0.72  0.64  0.37  0.01  -0.29  -0.50  -0.59
> signif(la1a)
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
 1.00 0.76 0.40 0.02 -0.36 -0.61 -0.68 -0.61 -0.38 -0.01 0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
 0.65 0.72 0.64 0.37 0.01 -0.29 -0.50 -0.59
> signif(la1a, 4)
An object of class "Lagged1d"
Slot *data*:
Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
 1.00 0.76 0.40 0.02 -0.36 -0.61 -0.68 -0.61
                                                   -0.38 -0.01 0.38
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
 0.65 0.72 0.64 0.37 0.01 -0.29 -0.50 -0.59
The functions from the summary group work on the data part, as if they were called on it.
> c(Max = max(la1a), Min = min(la1a), Range = range(la1a))
        Min Range1 Range2
  Max
 1.00 -0.68 -0.68 1.00
> c(Prod = prod(la1a), Sum = sum(la1a))
        Prod
-7.582098e-11 9.200000e-01
> c(Any = any(la1a < 0), All = all(la1a >= 0))
```

Binary arithmetic operators are defined between two lagged objects and between a lagged object and a vector. They return a lagged object from one of the "basic" lagged classes, but not necessarilly exactly from the class of the argument(s). The class of the returned value is from a suitable lagged superclass of the argument(s). This concerns operations on objects from classes inheriting from the classes considered here, so is not visible in the examples below, since they use objects from the basic lagged classes.

-0.054872 -0.000001

Lag\_17

-0.024389 -0.125000 -0.205379

Lag\_16

0.054872

Lag\_18

```
> 2*la1a
An object of class "Lagged1d"
Slot *data*:
   Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
                                             0.80
                                                                 0.04 - 0.72 - 1.22 - 1.36 - 1.22
      2.00
                                                                                                                                                                  -0.76 -0.02
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
      1.30
                         1.44
                                             1.28
                                                                 0.74
                                                                                     0.02 -0.58 -1.00 -1.18
> la1a^2
An object of class "Lagged1d"
Slot *data*:
  Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6 Lag_7 Lag_8 Lag_9 Lag_10
1.0000 0.5776 0.1600 0.0004 0.1296 0.3721 0.4624 0.3721 0.1444 0.0001 0.1444
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
0.4225 0.5184 0.4096 0.1369 0.0001 0.0841 0.2500 0.3481
> la1a + la1a^2
An object of class "Lagged1d"
Slot *data*:
     Lag_0
                            Lag_1
                                                   Lag_2
                                                                          Lag_3
                                                                                                 Lag_4
                                                                                                                    Lag_5
                                                                                                                                              Lag_6
                                                                                                                                                                     Lag_7
   2.0000
                         1.3376 0.5600
                                                                     0.0204 -0.2304 -0.2379 -0.2176 -0.2379 -0.2356 -0.0099
   Lag_10 Lag_11
                                             Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
                                                                      1.0496 0.5069 0.0101 -0.2059 -0.2500 -0.2419
   0.5244
                         1.0725
                                               1.2384
> la1a - la1a^2
An object of class "Lagged1d"
Slot *data*:
     Lag_0
                           Lag_1
                                               Lag_2
                                                                      Lag_3
                                                                                             Lag_4
                                                                                                                       Lag_5
                                                                                                                                              Lag_6
                                                                                                                                                                     Lag_7
                                                                                                                                                                                            Lag_8
                                                                                                                                                                                                                   Lag_9
   0.0000 \quad 0.1824 \quad 0.2400 \quad 0.0196 \quad -0.4896 \quad -0.9821 \quad -1.1424 \quad -0.9821 \quad -0.5244 \quad -0.0101 
   Lag_10 Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
   0.2356 0.2275 0.2016
                                                                     0.2304 0.2331 0.0099 -0.3741 -0.7500 -0.9381
> la1a * la1a^2
An object of class "Lagged1d"
Slot *data*:
           Lag_0
                                       Lag_1
                                                                    Lag_2
                                                                                                 Lag_3
                                                                                                                              Lag_4
                                                                                                                                                          Lag_5
                                                                                                                                                                                       Lag_6
   1.000000 0.438976 0.064000 0.000008 -0.046656 -0.226981 -0.314432 -0.226981
           Lag_8
                                       Lag_9
                                                                 Lag_10
                                                                                              Lag_11
                                                                                                                           Lag_12
                                                                                                                                                       Lag_13
                                                                                                                                                                                    Lag_14
```

0.274625 0.373248 0.262144 0.050653 0.000001

```
> la1a / la1a^2
```

12.65

13.72 14.64

```
An object of class "Lagged1d"
Slot *data*:
                               Lag_2
                                                                    Lag_5
      Lag_0
                  Lag_1
                                           Lag_3
                                                       Lag_4
   1.000000
               1.315789
                            2.500000
                                       50.000000
                                                   -2.777778
                                                                -1.639344
      Lag_6
                  Lag_7
                              Lag_8
                                           Lag_9
                                                      Lag_10
                                                                   Lag_11
  -1.470588
              -1.639344
                           -2.631579 -100.000000
                                                    2.631579
                                                                 1.538462
                                                                   Lag_17
     Lag_12
                 Lag_13
                              Lag_14
                                          Lag_15
                                                      Lag_16
                                                                -2.000000
   1.388889
               1.562500
                           2.702703 100.000000
                                                   -3.448276
     Lag_18
  -1.694915
> la1a + 1:length(la1a)
An object of class "Lagged1d"
Slot *data*:
 Lag_0 Lag_1 Lag_2 Lag_3 Lag_4 Lag_5 Lag_6
                                                   Lag_7
                                                          Lag_8 Lag_9 Lag_10
                                                           8.62
  2.00
         2.76
                3.40
                       4.02
                               4.64
                                      5.39
                                             6.32
                                                                   9.99 11.38
                                                    7.39
Lag_11 Lag_12 Lag_13 Lag_14 Lag_15 Lag_16 Lag_17 Lag_18
```

There is a case to argue for keeping the class in some situations, e.g. when the other argument is a scalar but eventually I decided to keep the simple rule of not trying to preserve the class.

17.50

Note however that unary operators and mathematical functions do preserve the class.

15.37 16.01 16.71

### 4. Multivariate lagged objects

Compute the autocorrelations of a multivariate time series and convert it to a lagged object.

```
> acv2 <- acf(ts.union(mdeaths, fdeaths))
> la2 <- Lagged(acv2)

Get the value for lag 1.

> la2[1]

, , 1

        [,1]        [,2]
[1,] 0.7570591 0.7356685
[2,] 0.7443093 0.7295201

> acv2$acf[2, ,] # same

        [,1]        [,2]
[1,] 0.7570591 0.7356685
[2,] 0.7443093 0.7295201
```

Indexing in acf() is somewhat misterious. For some insight, here is a comparison with a DIY calculation of the autocorrelations.

```
> n <- length(mdeaths)</pre>
> tmpcov <- sum((mdeaths - mean(mdeaths)) * (fdeaths - mean(fdeaths)) ) / n
> msd <- sqrt(sum((mdeaths - mean(mdeaths))^2)/n)</pre>
> fsd <- sqrt(sum((fdeaths - mean(fdeaths))^2)/n)</pre>
> tmpcov1 <- sum((mdeaths - mean(mdeaths))[2:n] * (fdeaths - mean(fdeaths))[1:(n-1)] ) / n
> tmpcov1 / (msd * fsd)
[1] 0.7356685
> la2[[1]][1,2] == tmpcov1 / (msd * fsd) # FALSE, but:
[1] FALSE
> la2[[1]][1,2] - tmpcov1 / (msd * fsd) # only numerically different
[1] 2.220446e-16
Some examples for the correspondence between the indices in lagged objects and those from acf().
> la2[[1]][1,2] == acv2$acf[2, 1, 2] # TRUE
[1] TRUE
> la2[0]
, , 1
          [,1]
                   [,2]
[1,] 1.0000000 0.9762413
[2,] 0.9762413 1.0000000
> acv2[0]
Autocorrelations of series Sts.union(mdeaths, fdeaths)S, by lag
, , mdeaths
mdeaths
           fdeaths
 1.000 (0) 0.976 (0)
, , fdeaths
mdeaths
           fdeaths
0.976 (0) 1.000 (0)
> la2[1]
, , 1
          [,1]
                     [,2]
[1,] 0.7570591 0.7356685
```

[2,] 0.7443093 0.7295201

```
> acv2[1]
```

Autocorrelations of series Šts.union(mdeaths, fdeaths)Š, by lag

```
, , mdeaths
```

```
mdeaths fdeaths 0.717 ( 1) 0.708 (-1)
```

, , fdeaths

```
mdeaths fdeaths 0.721 ( 1) 0.716 ( 1)
```

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