E E - 2 7 4 *Introduction to Wavelets & Filter Banks* Spring Semester 2007

HOMEWORK 2

THE CONTINUOUS WAVELET TRANSFORM (CWT).

In this homework we consider the problem of identifying coherent structures using both the DFT and the CWT. Data is atrial fibrillation data. Where normal heart rate is about 1 beat per sec., atrial fibrillation can cause the heart rate to vary from about 5 to 15 beats per sec. We will see the capability of both these characterizations in extracting this particular feature - heart rate- from the data. We first use the fft and then the cwt.

Go to http://www.cems.uvm.edu/~mirchand/classes/EE275/images/ and copy cardio100.mat somewhere in your computer. Go to MATLAB and do "load cardio100". You will see three files: heart100, heart101 and heart102. While typically these may be many minutes of data, you have here only 2048 points. The original continuous signal was sampled at 1 msec (1000 hz), so you have 2.048 seconds of data for each signal. You will be working with heart100, heart101 and heart102.

The goal is to determine as well as possible, the **"period" of the spikes** in the data. The (average) frequency (1/period) of these spikes will be referred to as the "dominant" frequency. We will determine this frequency as best as we can, first using the fft and then using the cwt. Note that for a meaningful characterization of a dominant frequency, the amplitude of a peak at some k should be considerably larger than amplitudes at neighboring values of k. (We will ignore the issue here about what constitutes "considerably larger".) For all three time series heart100, heart101, heart102, do the following: (You may want to invoke "sptool" and import your signals, since the signal browser there allows an easy determination of signal period and other characteristics).

- Determine the period of the spikes in the data. The period between successive pulses is of course not constant. However, obtain an average period for the data of each of the three heart beats series.
- For each of the three (average) periods, determine the value of k in the fft $X[k]$ where you should expect to get a peak, when using a 1000-pt. or a 2048-pt fft.

Note that to use a particular window, say Hamming, you can generate the window using $w = hanning(2048)$ and then multiply the original (2048-pt) signal with this window.

1. ***fft* analysis of cardiogram data - heart100**
See if you are able to get a clear maximum at the desired value of k for each of the following:

(1) 1000 pt. fft without and with a Hamming window.

(2) 2048 pt. fft without and with a Hamming window.

(3) Square the signal. Now repeat [(1)] above.

(4) Square the signal. Now repeat [(2)] above.

(5) Briefly summarize your findings with the fft.

2. ***fft* analysis of cardiogram data - heart 101, 102**

(1) Repeat problem 1 for heart101, heart102.

3. **cwt Scale-to-frequency analysis of wavelets**
Now we will use the *cwt* function. We first need to establish the corresponding "frequencies" of wavelets at various scales. (You can use Help: *doc centfrq*, for example). Do the following in MATLAB:

(1) $>> cfreq = centfrq('mexh', 10, 'plot')$

(2) $>> scal2frq(1, 'mexh', 1)$

(3) $>> scal2frq(1, 'mexh', .001)$

(4) $>> scal2frq(5, 'mexh', .001)$

(5) Repeat for 'db2' and 'db4'.

So now we established the fact that the mexican hat wavelet *mexh* , at scale 5 and sampling frequency 1000, has a "frequency" of 50. In a similar fashion, find the scale for 'db2' and 'db4' that corresponds to a "frequency" of 50. Now we will examine how well this works, first by analyzing a sine function of 50hz frequency and then our cardiograms.

(5) Consider a 50 hz sine wave $x(t) = sin(2 * pt * 50 * t)$ sampled at 1000 hz. Hence

$$x[n] = sin(2 * pi * 50n * 0.001)$$

Generate 100 samples in MATLAB. Do $>>$ *save* x. You now have x.mat saved in your workplace. (Verify this by using 'ls' in the Command Window).

(6) Now find the coefficients of the *cwt* from scales 1 to 32, using the mexican hat wavelet and the plot mode 'glb'.
$>> c = cwt(x, 1:32,' mexh',' glb');$
Note that c is a 32 x 1000 matrix. You can look at c through a contour plot, using 8 levels.
$>> contour(c, 8)$. Also try:
$>> contourf(c, 8)$. Be sure to insert a colorbar from the Insert Colorbar icon above the Figure or by using the Insert drop-down menu. In both cases (cwt and contour), do you detect peaks at the expected scale?

(7) Repeat the experiment above, using the wavelet 'db2'.

(8) Now do a *cwt* on *heart*100 with 'mexh'. You know the scale of the wavelet which corresponds to the dominant frequency of the signal. Are you able to determine the dominant frequency in the signal this way like you did for the sinusoid?

(9) We now turn to a GUI to do the *cwt*. In MATLAB type >> *wavemenu*. Go to Continuous Wavelet 1-D. In the latter window do File → Import signal from Workspace and click on x. Use *mexh* wavelet, sampling frequency 0.001, $step - by - step$ mode, scales 1 → 10 in steps of 1 and push *Analyze*. (Have all 3 Selected Axis checked off. For Coloration Mode, use: init + byscale + abs, Colormap - try gray). In the $C_{a,b}Coefficients$ window, use your mouse and click around scale 5 (Control-Mouse for Mac, right Mouse button for PC). In the Coefficients Line -$C_{a,b}$ observe all the coefficients at that scale. Each time you change scale you press the - New Coefficient Line radio button. By inspection, find the scale a for which you get the largest $|C(a, b)|^2$. Does that correspond to scale 5?

(10) In MATLAB load cardio100. Save variables heart100, heart101, heart102 one-by-one, generating files heart100.mat, etc. Go to the CWT GUI and load heart100. The peaks have an approximate period of 136 and hence a frequency of about 27.8. *mexh* at scale 10 gives a frequency of 25, so a slightly smaller scale will correspond to the desired frequency 27.8 hz. Analyze using scales 1 → 20. Are you able to pick out the period of the pulses?

(11) If that is not satisfactory, are you able to find a scale where you can pick out the high frequency oscillations of the signal. That is the scale (1,2,3?) where the coefficient line looks like the periodic high frequency oscillation? If that is so, perhaps one could just save those coefficients and take their fft which might give the period of these pulses.