# Black Body Radiation

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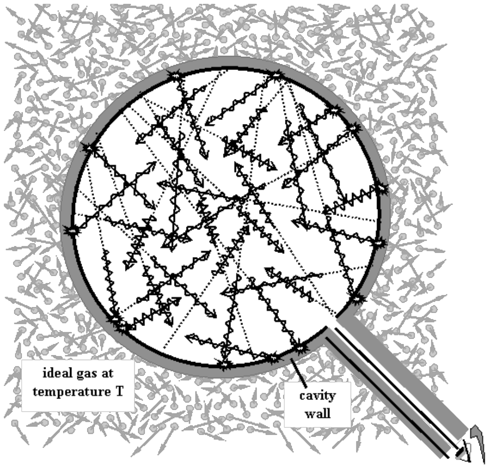
### Cavity

Consider a cavity in thermal equilibrium








### Cavity

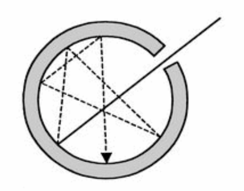
- Any body that is maintained at a temperature $T > 0K$ represents a source of electromagnetic waves
- The body absorbs energy in the form of electromagnetic waves and/or heat and this causes thermal agitation of its molecules
- The thermal motion of these molecules is responsible for the electromagnetic energy radiated (at the cost of the thermal energy)
- If the temperature is kept constant, a dynamic thermal equilibrium is set such that the emitted energy precisely compensates the absorbed one





### Cavity

- The color of the light emitted from a hot body depends both on the temperature and on the characteristics of the body.
- Consider then a cavity with a very small hole



- In this case, the incoming light cannot be reflected out of the body and it is absorbed.
- The energy is instead emitted afterwards as a radiation.
- In this case it is experimentally proved that the “color” of the radiation only depends on the temperature.

## Black Body Radiation

### Radiation Laws

- The emitted radiation in reality is not monochromatic, and it thus contains different "colors" (wavelengths) in different "amounts".
- We can describe this fact using the spectral radiance. **Plank's law** asserts that

$$L(\lambda;T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

where

$h$ is the Plank constant, $h \approx 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$.
$k$ is the Boltzmann constant, $k \approx 1.380 \times 10^{-23}\,\mathrm{J \cdot K^{-1}}$.

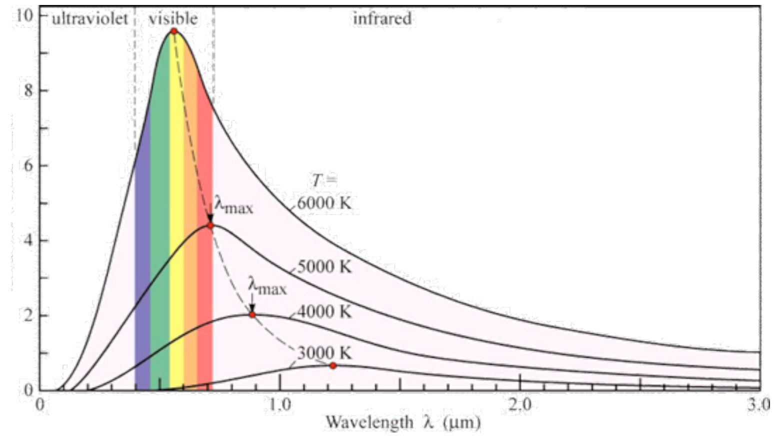
- This law was obtained by Plank in 1900 by assuming that the energy could only be emitted in discrete quantities, multiples of the quantum energy value

$$E = \frac{hc}{\lambda}$$



## Black Body Radiation

### Radiation Laws






## Black Body Radiation

### Radiation Laws

- The exitance is given by the **Stefan law**

$$M = \sigma T^4$$

where $\sigma$ is the Stefan constant and has value $5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$

- The peak in the spectral exitance can be computed according to **Wien's displacement law**

$$\lambda_{\max} = \frac{b}{T}$$

where $b = 2.898 \times 10^{-3}\,\mathrm{mK}$.

- This puts in a clear and simple relation the "color" of light with the temperature.



## Black Body Radiation

### Radiation Laws

- For large wavelengths, that is, if the following condition holds

$$\frac{hc}{\lambda kT} \ll 1$$

then it is possible to use the **Rayleigh-Jeans approximation**

$$L(\lambda;T) \approx \frac{2ckt}{\lambda^4}$$

- Example: if $T \approx 280\mathrm{K}$, the Rayleigh-Jeans approximation holds for $\lambda \gg 50\mu m$. Hence, at room temperature the approximation holds in the microwave and radio region.

## Black Body Radiation

### Real Bodies

- In practice, all bodies are non-ideal and the Planck's model does not describe the radiation exactly
- Some corrections to the model are required: a simple way to describe them is to add a correction coefficient (dimensionless) called **emissivity** $\varepsilon(\lambda)$ such that

$$L(\lambda) = \varepsilon(\lambda)\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

- **Attention:** same symbol as for the permittivity but it has little to do with it.
- In many cases the emissivity $\varepsilon(\lambda)$ can be considered constant over the range of wavelengths of interest, and in these cases one usually speaks of **gray body**.



## Black Body Radiation

### Real Bodies

- A body with an emissivity $\varepsilon \neq 1$ will have a different spectral radiance than the ideal black body *at the same temperature*
- We can define the **brightness temperature** $T_b$ as the temperature of an ideal black body that would have the same spectral radiance as the considered body at the same wavelength, that is,

$$\varepsilon(\lambda)\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT_b}} - 1}$$

- At large wavelengths, using the Rayleigh-Jeans approximations we see that we have

$$T_b \approx \varepsilon(\lambda)T$$



## Black Body Radiation

### Solar Radiation

- The Sun can be considered a gray body with a temperature of around 5800K and an emissivity $\varepsilon \approx 0.99$. It can be approximated with a sphere of radius $R_S = 6.96 \times 10^8$m, at a distance $d = 1.5 \times 10^{11}$m from the Earth.
- The exitance is thus

$$M = \varepsilon\sigma T^4 \approx 6.35 \times 10^7\,\mathrm{Wm^{-2}}$$

- The total radiant flux is thus

$$\Phi = 4\pi R_S^2 \varepsilon\sigma T^4 \approx 3.87 \times 10^{26}\,\mathrm{W}$$

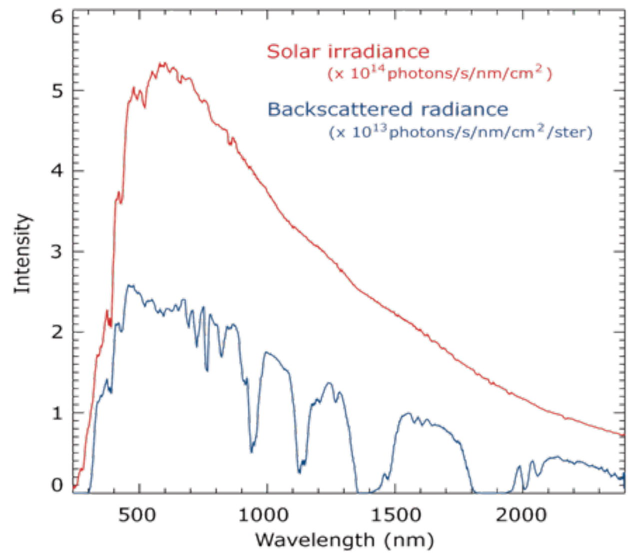
- The irradiance of the Earth is thus

$$E = \frac{\Phi}{4\pi d^2} \approx 1.37 \times 10^3\,\mathrm{Wm^{-2}}$$



## Black Body Radiation

### Solar Radiation



## Terrestrial Radiation

- At a temperature $T = 280\,\mathrm{K}$ a black body radiates with a maximum power density nead the wavelength of $10.3\,\mu$m. Let us consider different sub-bands (as fractions of the total radiant flux)
  - Visible ($0.5 - 0.6\mu$m): $\approx 6 \times 10^{-33}$
  - Near infrared ($1.55 - 1.75\mu$m): $\approx 7 \times 10^{-10}$
  - Thermal infrared ($10.5 - 12.5\mu$m): $\approx 0.12$
  - Microwave ($1.52 - 1.56$cm): $\approx 1 \times 10^{-10}$
- We see a very fast decay at the high frequencies and a slow decay at low frequencies
- In fact, while the usual objects at room temperature essentially do not emit radiation in the visible, they emit small but detectable quantities of radiation in the microwave band (which is actually used for passive systems)





## Terrestrial Radiation