REMOTE SENSING

MODULE OF

REMOTE SENSING DATA ANALYSIS

(6 CFU)

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MASTER OF SCIENCE IN COMMUNICATION TECHNOLOGIES AND MULTIMEDIA

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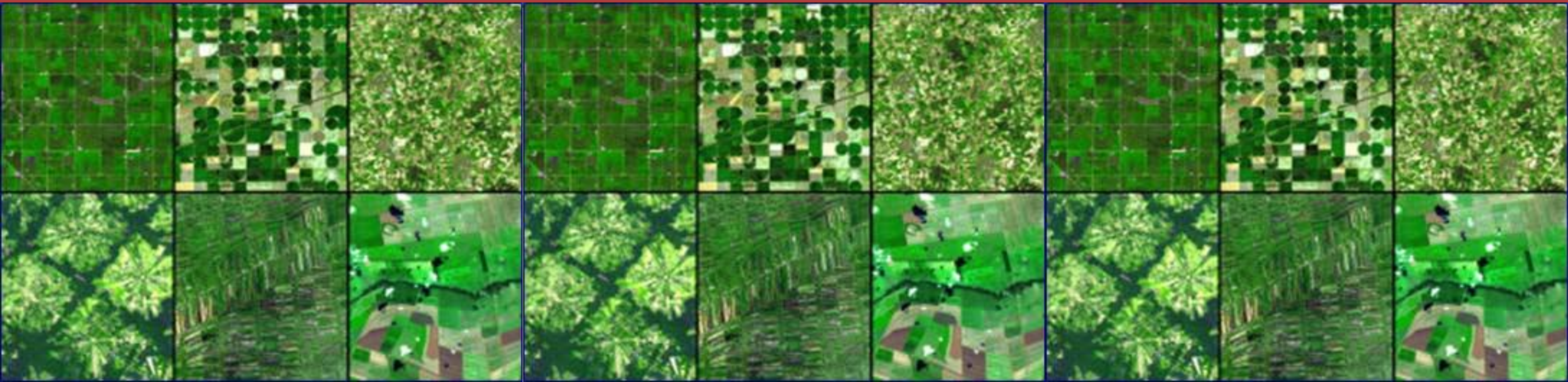
MULTISPECTRAL TRANSFORMATIONS OF IMAGE DATA



Introduction

- The multispectral or vector character of most remote sensing image data renders it amenable to **spectral transformations** that **generate new sets of image components** or bands.
- These components then represent an **alternative description of the data**, in which the new components of a pixel vector are related to its old brightness values in the original set of spectral bands via a linear operation.
- The transformed image
  - may make evident features not discernable in the original data
    - this has significance for **image enhancement**
    - or as **preconditioning** of image data **prior to classification** techniques
  - or alternatively it might be possible to preserve the essential information content of the image (for a given application) with a reduced number of the transformed dimensions.
    - this has significance for **displaying data** in the three dimensions available on a **colour** monitor or in colour hardcopy,
    - and for **transmission and storage** of data.
- The techniques covered, which appeal directly to the vector nature of the image, include
  - the principal components transformation
  - the so-called band arithmetic (which includes the creation of ratio images)
  - and some specialised transformations, such as the Kauth-Thomas tasseled cap transform

THE PRINCIPAL COMPONENTS TRANSFORMATION

The principal components transform defined in the following is also known as the Principal Component Analysis (PCA), the Karhunen-Loève transform (KLT) or the Hotelling transform.

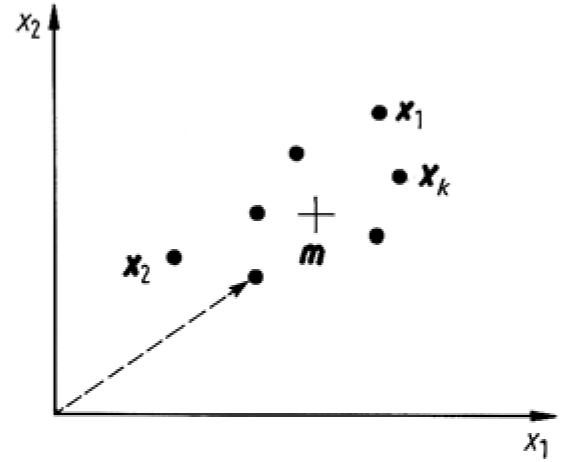
The Principal Components Transformation

- The multispectral or multidimensional nature of remote sensing image data can be accommodated by constructing a vector space with as many axes or dimensions as there are spectral components associated with each pixel.
  - In the case of **Landsat Thematic Mapper** data it will have **seven dimensions**
  - for **SPOT HRV** data it will be **three dimensional**.
  - For **hyperspectral** data there may be **several hundred axes**.

- A particular *pixel in an image is plotted as a point in such a space* with *coordinates* that correspond to the *brightness values* of the pixels in the appropriate spectral components.
  - For simplicity the treatment to be developed in this topic will be based upon a two dimensional multispectral space (say visible red and infrared) since the diagrams are then easily understood and the mathematical detail is readily assimilated.
  - The results derived however are perfectly general and apply to data of any dimensionality.

The Mean Vector and Covariance Matrix

- The positions of pixel points in multispectral space can be described by vectors, whose components are the individual spectral responses in each band.
  - Consider a multispectral space with a large number of pixels plotted (see Figure), with each pixel described by its appropriate vector ***x*.**
  - The **mean position** of the pixels in the space is defined by the expected value of the pixel vector ***x***, according to



$$m = \mathscr{E}(x\} = \frac{1}{K}\sum_{k=1}^{K} x_k$$

where ***m*** is the mean pixel vector and the x_k are the individual pixel vectors of total number *K;* $\mathscr{E}$ is the expectation operator.

  - While the mean vector is useful to define the average or expected position of the pixels in multispectral vector space, it is of value to have available a means by which their scatter or spread is described. This is the role of the **covariance matrix** which is defined as

$$\Sigma_x = \mathscr{E}\{(x - m)(x - m)^t\}$$

in which the superscript '*t*' denotes vector transpose.

The Mean Vector and Covariance Matrix

- An *unbiased estimate* of the covariance matrix is given by

$$\Sigma_x = \frac{1}{K-1}\sum_{k=1}^{K}(x_k - m)(x_k - m)^t$$

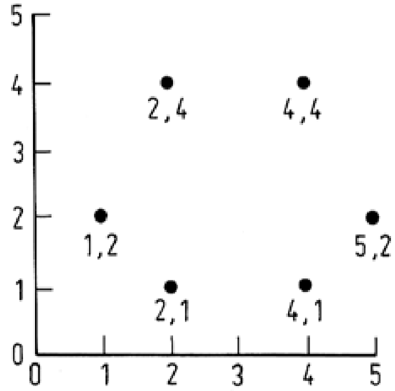
- The covariance matrix is one of the most important mathematical concepts in the analysis of multispectral remote sensing data.
  - If there is correlation between the responses in a pair of spectral bands the corresponding *off-diagonal element* in the covariance matrix will be large by comparison to the diagonal terms.
  - On the other hand, if there is a little correlation, the off-diagonal terms will be close to zero.
- This behaviour can also be described in terms of the correlation matrix *R* whose elements are related to those of the covariance matrix by

$$\varrho_{ij} = v_{ij} / \sqrt{v_{ii} v_{jj}}$$

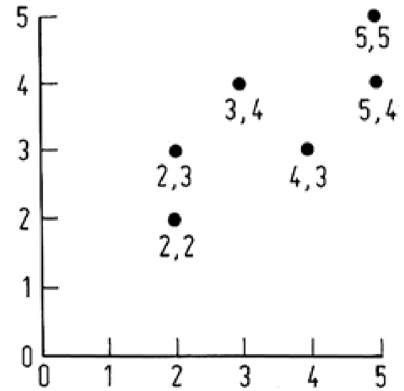
where ϱ_{ij} *is an element of the correlation matrix and* v_{ij} *etc. are elements of the* covariance matrix; v_{ii} *and* v_{ij} *are the variances of the i*th and *j*th bands of data. The ϱ_{ij} describe the correlation between band *i* and band *j*.

An example

- **Figure a** shows *little correlation* between the two components: in other words, both components are necessary to describe where a pixel lies in the space.
- The data shown in **Figure b** however exhibits a *high degree of correlation* between its two components, evident in the elongated spread of the data at an angle to the axes.



a



b

x	$x - m$	$[x-m]\,[x-m]^t$
$\begin{bmatrix}1\\2\end{bmatrix}$	$\begin{bmatrix}-2.00\\-0.33\end{bmatrix}$	$\begin{bmatrix}4.00 & 0.66\\0.66 & 0.11\end{bmatrix}$
$\begin{bmatrix}2\\1\end{bmatrix}$	$\begin{bmatrix}-1.00\\-1.33\end{bmatrix}$	$\begin{bmatrix}1.00 & 1.33\\1.33 & 1.77\end{bmatrix}$
$\begin{bmatrix}4\\1\end{bmatrix}$	$\begin{bmatrix}1.00\\-1.33\end{bmatrix}$	$\begin{bmatrix}1.00 & -1.33\\-1.33 & 1.77\end{bmatrix}$
$\begin{bmatrix}5\\2\end{bmatrix}$	$\begin{bmatrix}2.00\\-0.33\end{bmatrix}$	$\begin{bmatrix}4.00 & -0.66\\-0.66 & 0.11\end{bmatrix}$
$\begin{bmatrix}4\\4\end{bmatrix}$	$\begin{bmatrix}1.00\\1.67\end{bmatrix}$	$\begin{bmatrix}1.00 & 1.67\\1.67 & 2.79\end{bmatrix}$
$\begin{bmatrix}2\\4\end{bmatrix}$	$\begin{bmatrix}-1.00\\1.67\end{bmatrix}$	$\begin{bmatrix}1.00 & -1.67\\-1.67 & 2.79\end{bmatrix}$

whereupon $m = \begin{bmatrix}3.00\\2.33\end{bmatrix}$ $\Sigma_x = \begin{bmatrix}2.40 & 0\\0 & 1.87\end{bmatrix}$

and $R = \begin{bmatrix}1.00 & 0\\0 & 1.00\end{bmatrix}$

where R is the correlation matrix.

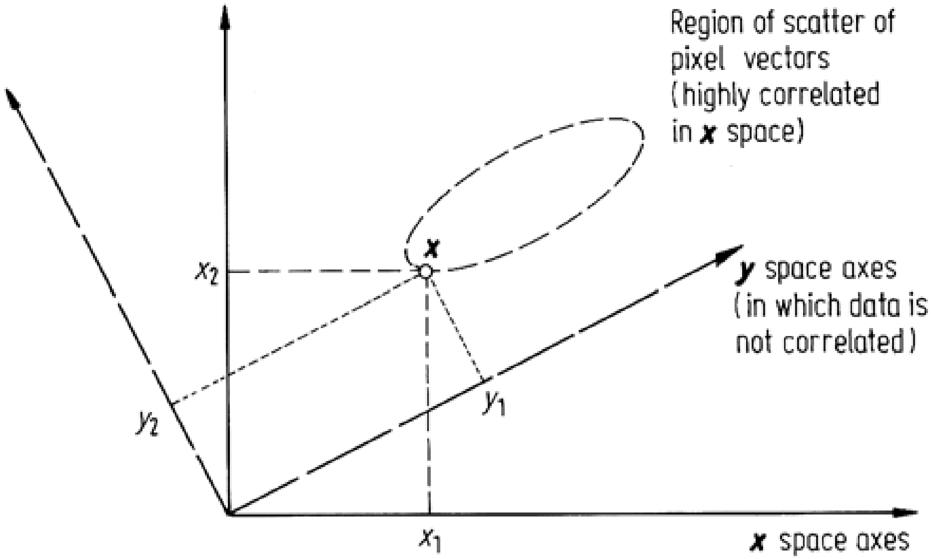
$$m = \begin{bmatrix}3.50\\3.50\end{bmatrix} \quad \Sigma_x = \begin{bmatrix}1.900 & 1.100\\1.100 & 1.100\end{bmatrix}$$

$$R = \begin{bmatrix}1.000 & 0.761\\0.761 & 1.000\end{bmatrix}$$

Thus components 1 and 2 of the data in Figure b are 76% correlated.

A Zero Correlation, Rotational Transform

- It is fundamental to the development of the *principal components transformation* to ask whether there is a *new co-ordinate system in the multispectral vector space in which the data can be represented without correlation* (see Figure); in other words, such that the covariance matrix in the new co-ordinate system is *diagonal.*



If the vectors describing the pixel points are represented as $\boldsymbol{y}$ in the new co-ordinate system then it is desired to find a linear transformation G of the original co-ordinates, such that

$$\boldsymbol{y} = G\boldsymbol{x} = D^t\boldsymbol{x}$$

subject to the constraint that the covariance matrix of the pixel data in $\boldsymbol{y}$ space is diagonal. Expressing G as D^t will make the comparison of principal components with other transformation operations, treated later, much simpler.

In $\boldsymbol{y}$ space the covariance matrix is, by definition,

$$\Sigma_y = \mathscr{E}\{(\boldsymbol{y} - \boldsymbol{m}_y)(\boldsymbol{y} - \boldsymbol{m}_y)^t\}$$

where $\boldsymbol{m}_y$ is the mean vector expressed in terms of the $\boldsymbol{y}$ co-ordinates. It is shown readily that

$$\boldsymbol{m}_y = \mathscr{E}\{\boldsymbol{y}\} = \mathscr{E}\{D^t\boldsymbol{x}\} = D^t\mathscr{E}\{\boldsymbol{x}\} = D^t\boldsymbol{m}_x \text{ [1]}$$

[1] $\mathscr{E}\{D^t\boldsymbol{x}\} = \frac{1}{K}\sum_{k=1}^{K} D^t\boldsymbol{x}_k = D^t\frac{1}{K}\sum_{k=1}^{K}\boldsymbol{x}_k = D^t\boldsymbol{m}_x$

i.e. D^t, being a matrix of constants, can be taken outside

A Zero Correlation, Rotational Transform

where $\boldsymbol{m}_x$ is the data mean in $\boldsymbol{x}$ space. Therefore

$$\Sigma_y = \mathscr{E}\{(D^t\boldsymbol{x} - D^t\boldsymbol{m}_x)(D^t\boldsymbol{x} - D^t\boldsymbol{m}_x)^t\}$$

which can be written as

$$\Sigma_y = D^t\mathscr{E}\{(\boldsymbol{x} - \boldsymbol{m}_x)(\boldsymbol{x} - \boldsymbol{m}_x)^t\}D \text{ [2]}$$

i.e. $$\Sigma_y = D^t\,\Sigma_x\,D$$

where Σ_x is the covariance of the pixel data in $\boldsymbol{x}$ space. Since Σ_y must, by demand, be diagonal, D can be recognised as the matrix of eigenvectors of Σ_x, provided D is an orthogonal matrix. This can be seen from the material presented in Appendix D dealing with the diagonalization of a matrix. As a result, Σ_y can then be identified as the diagonal matrix of eigenvalues of Σ_x,

$$\Sigma_y = \begin{bmatrix} \lambda_1 & 0 & & & \\ 0 & \lambda_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \lambda_N \end{bmatrix}$$

where N is the dimensionality of the data. Since Σ_y is, by definition, a covariance matrix and is diagonal, its elements will be the variances of the pixel data in the respective transformed co-ordinates. It is arranged such that $\lambda_1 > \lambda_2 > \ldots\lambda_N$ so that the data exhibits maximum variance in y_1, the next largest variance in y_2 and so on, with minimum variance in y_N.

[2] Since $[A\zeta]^t = \zeta^t A^t$ (reversed law of matrices). Note also $[A\zeta]^{-1} = \zeta^{-1}A^{-1}$.

An example (contd)

Before proceeding it is of value at this stage to pursue further the examples of Fig. 6.2, to demonstrate the computational aspects of principal components analysis. Recall that the original $\boldsymbol{x}$ space covariance matrix for the highly correlated image data of Fig. 6.2b is

$$\Sigma_x = \begin{bmatrix} 1.90 & 1.10 \\ 1.10 & 1.10 \end{bmatrix}$$

To determine the principal components transformation it is necessary to find the eigenvalues and eigenvectors of this matrix. The eigenvalues are given by the solution to the characteristic equation

$$|\Sigma_x - \lambda \boldsymbol{I}| = 0, \quad \boldsymbol{I} \text{ being the identity matrix.}$$

i.e.

$$\begin{vmatrix} 1.90 - \lambda & 1.10 \\ 1.10 & 1.10 - \lambda \end{vmatrix} = 0$$

or $\lambda^2 - 3.0\lambda + 0.88 = 0$

which yields $\lambda = 2.67$ and 0.33

As a check on the analysis it may be noted that the sum of the eigenvalues is equal to the trace of the covariance matrix, which is the sum of its diagonal elements.

The covariance matrix in the appropriate $\boldsymbol{y}$ co-ordinate system (with principal components as axes) is therefore

$$\Sigma_y = \begin{bmatrix} 2.67 & 0 \\ 0 & 0.33 \end{bmatrix}$$

Note that the first principal component, as it is called, accounts for $2.67/(2.67 + 0.33) \equiv 89\%$ of the total variance of the data in this particular example. It is now of interest to find the actual principal components transformation matrix $\boldsymbol{G} = \boldsymbol{D}^t$.

An example (contd)

Note that this is the *transposed* matrix of eigenvectors of Σ_x. Consider first, the eigenvector corresponding to $\lambda_1 = 2.67$. This is the vector solution to the equation

$$[\Sigma_x - \lambda_1 \boldsymbol{I}]\, \boldsymbol{g}_1 = 0$$

with $\boldsymbol{g}_1 = \begin{bmatrix} g_{11} \\ g_{21} \end{bmatrix} \equiv \boldsymbol{d}_1^t$ for the two dimensional example at hand.

Substituting for $\Sigma_x$and λ_1 gives the pair of equations

$$-0.77 g_{11} + 1.10 g_{21} = 0$$
$$1.10 g_{11} - 1.57 g_{21} = 0$$

which are not independent, since the set is homogeneous. It does have a non-trivial solution however because the coefficient matrix has a zero determinant. From either equation it can be seen that

$$g_{11} = 1.43 g_{21} \tag{6.6}$$

At this stage either g_{11} or g_{21} would normally be chosen arbitrarily, and then a value would be computed for the other. However the resulting matrix G has to be orthogonal so that $G^{-1} \equiv G^t$. This requires the eigenvectors to be normalised, so that

$$g_{11}^2 + g_{21}^2 = 1 \tag{6.7}$$

This is a second equation that can be solved simultaneously with (6.6) to give

$$g_1 = \begin{bmatrix} 0.82 \\ 0.57 \end{bmatrix}$$

An example (contd)

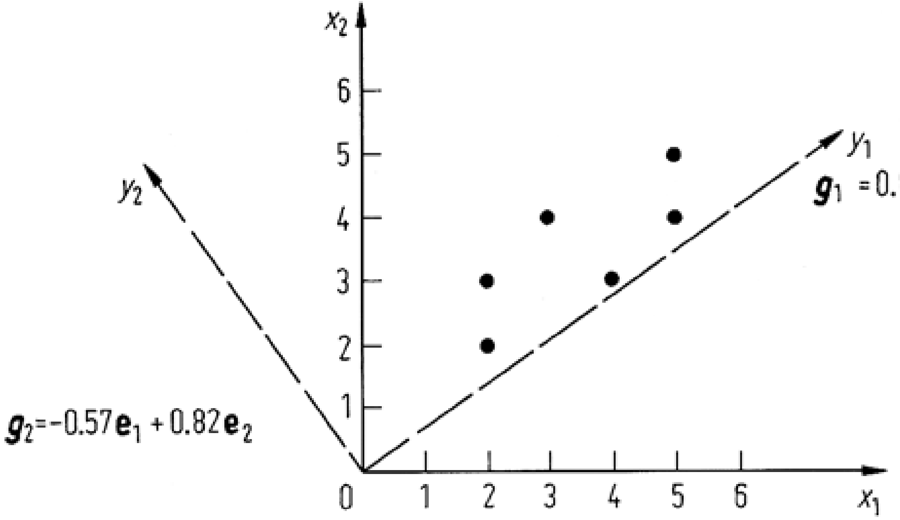
In a similar manner it can be shown that the eigenvector corresponding to $\lambda_2 = 0.33$ is

$$g_2 = \begin{bmatrix} -0.57 \\ 0.82 \end{bmatrix}$$

The required principal components transformation matrix therefore is

$$G = D^t = \begin{bmatrix} 0.82 & -0.57 \\ 0.57 & 0.82 \end{bmatrix}^t = \begin{bmatrix} 0.82 & 0.57 \\ -0.57 & 0.82 \end{bmatrix}$$

Now consider how these results can be interpreted. First of all, the individual eigenvectors $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ are vectors which define the principal component axes in terms of the original co-ordinate space. These are shown in Fig. 6.4: it is evident that the data is uncorrelated in the new axes and that the new axes are a rotation of the original set. For this reason (even in more than two dimensions) the principal components transform is classed as a rotational transform.



An example (contd)

Secondly, consider the application of the transformation matrix G to find the positions (i.e., the brightness values) of the pixels in the new uncorrelated co-ordinate system. Since $\boldsymbol{y} = G\boldsymbol{x}$ this example gives

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.82 & 0.57 \\ -0.57 & 0.82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{6.8}$$

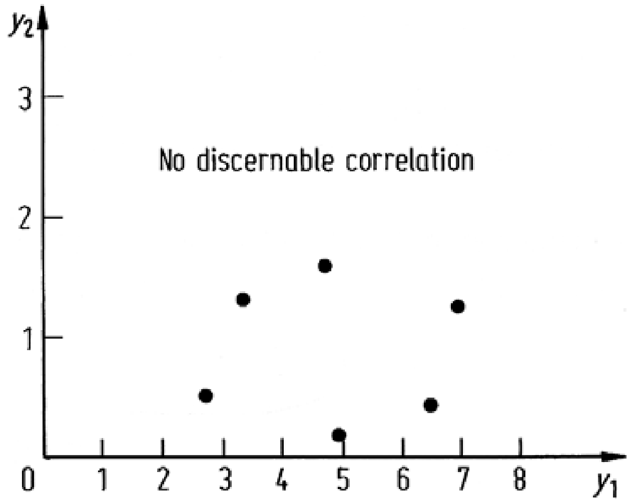
which is the actual principal components transformation to be applied to the image data. Thus, for

$$\boldsymbol{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

we find

$$\boldsymbol{y} = \begin{bmatrix} 2.78 \\ 0.50 \end{bmatrix}, \begin{bmatrix} 4.99 \\ 0.18 \end{bmatrix}, \begin{bmatrix} 6.38 \\ 0.43 \end{bmatrix}, \begin{bmatrix} 6.95 \\ 1.25 \end{bmatrix}, \begin{bmatrix} 4.74 \\ 1.57 \end{bmatrix}, \begin{bmatrix} 3.35 \\ 1.32 \end{bmatrix}.$$

The pixels plotted in $\boldsymbol{y}$ space are shown in Fig. 6.5. Several points are noteworthy.



Practical Considerations

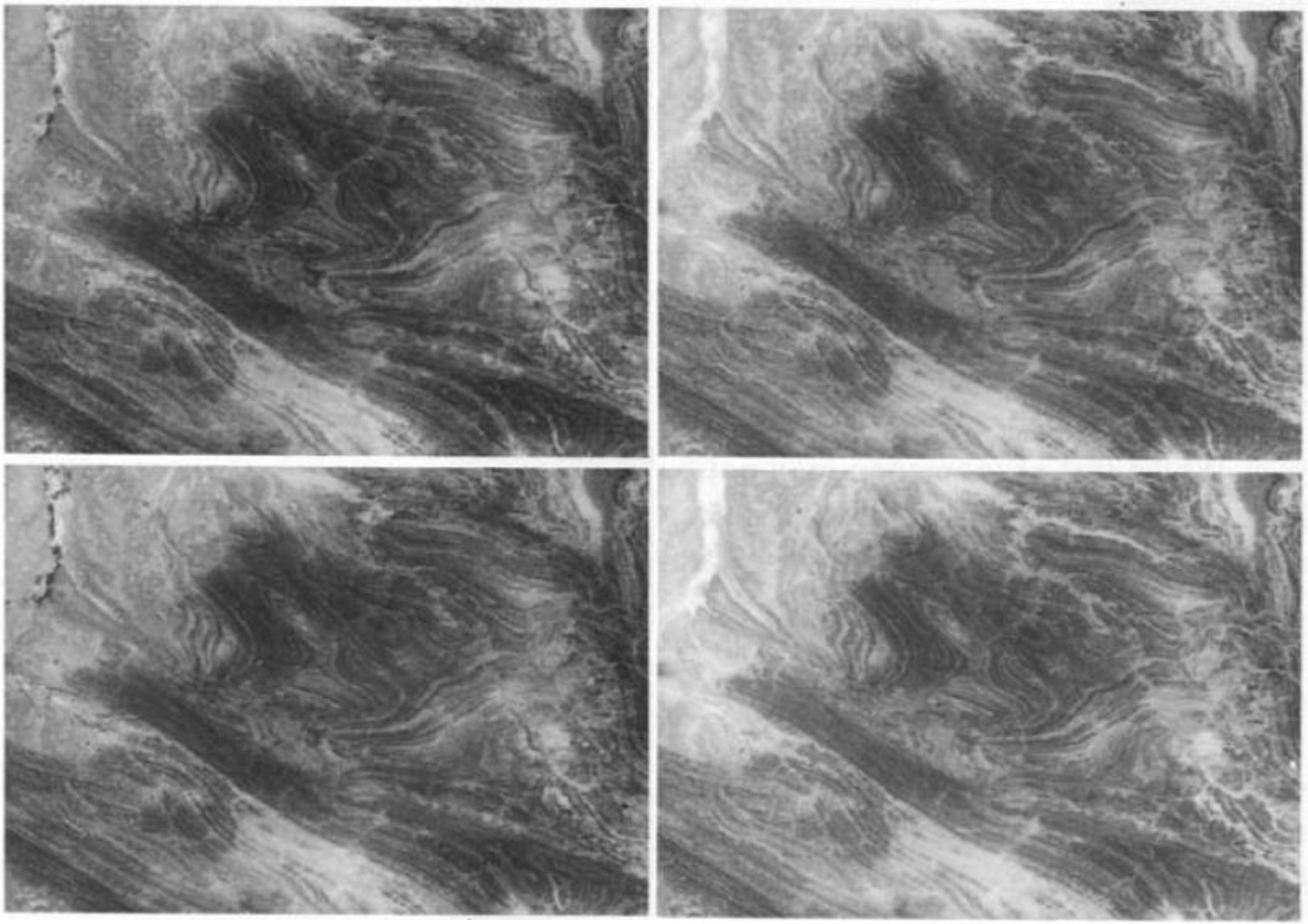
- From the above example some considerations can be drawn:

1. First, the data exhibits <u>no discernable correlation between the pair of new axes</u> (i.e., the principal components).
2. Secondly, <u>most of the data spread is in the direction of the first principal component</u>. It could be interpreted that this component contains most of the information in the image.
3. Finally, if the pair of principal component images are produced by using the $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$ component brightness values for the pixels, the first principal component image will show a *high degree of contrast* whereas the second will *have limited contrast*.

   - By comparison to the first component, the second will make use of only a few available brightness levels. It will be seen, therefore, to lack the detail of the former.
   - While this phenomenon may not be particularly evident for a simple two dimensional example, it is especially noticeable in the fourth component of a principal component transformed Landsat multispectral scanner image as can be assessed in the next example.

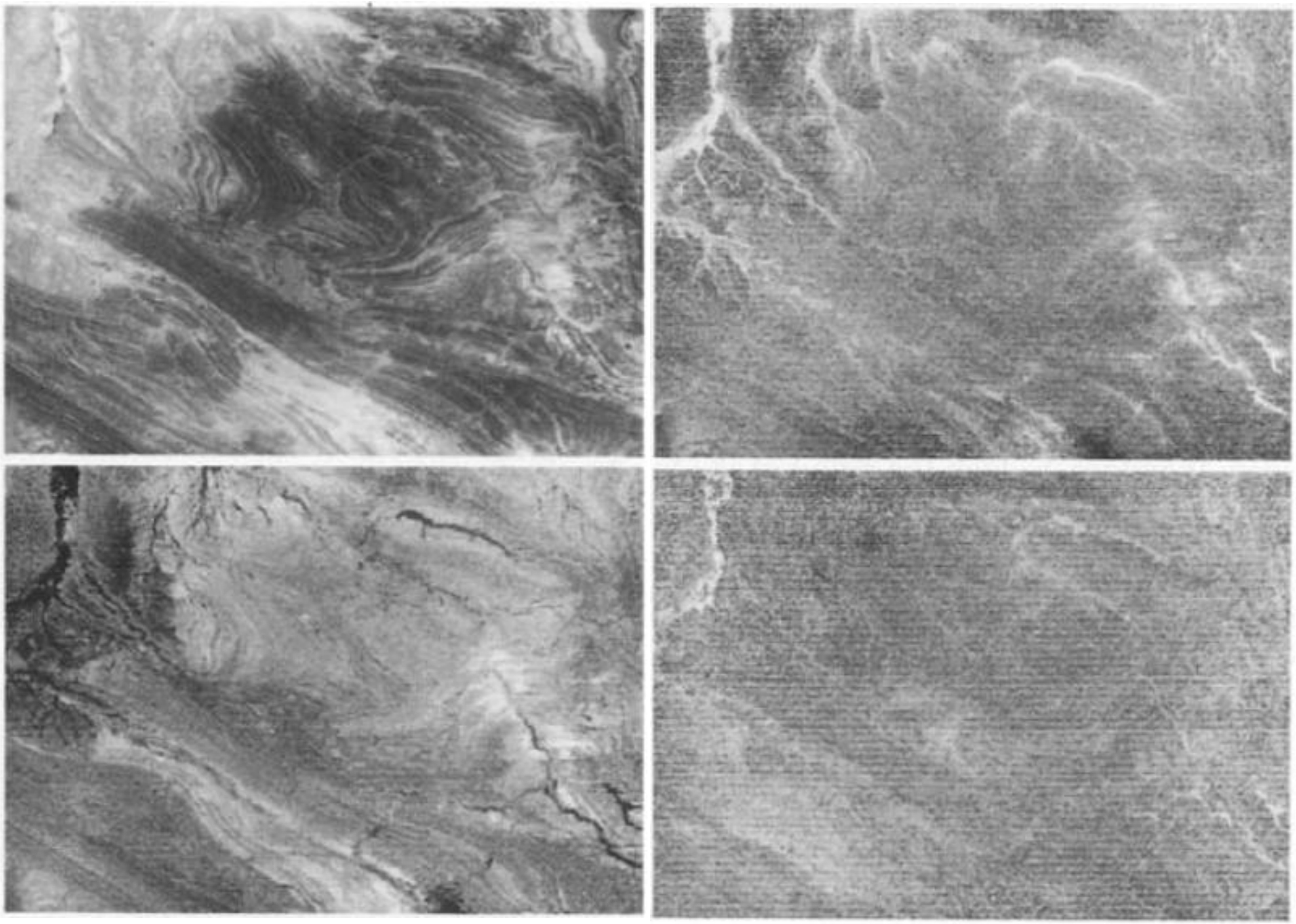
Real Example

- Four original bands acquired by the Landsat MS scanner (Andamooka, central Australia)



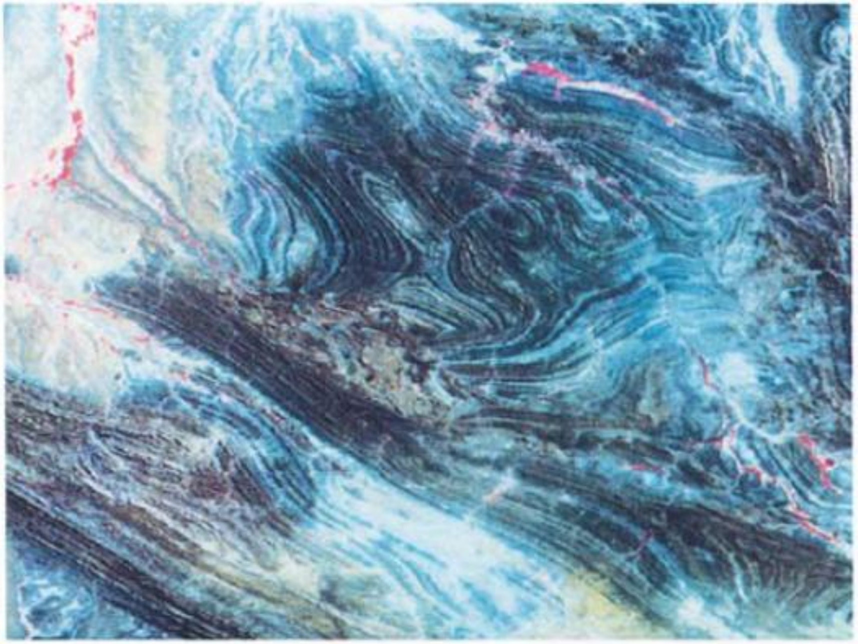
Real Example (contd)

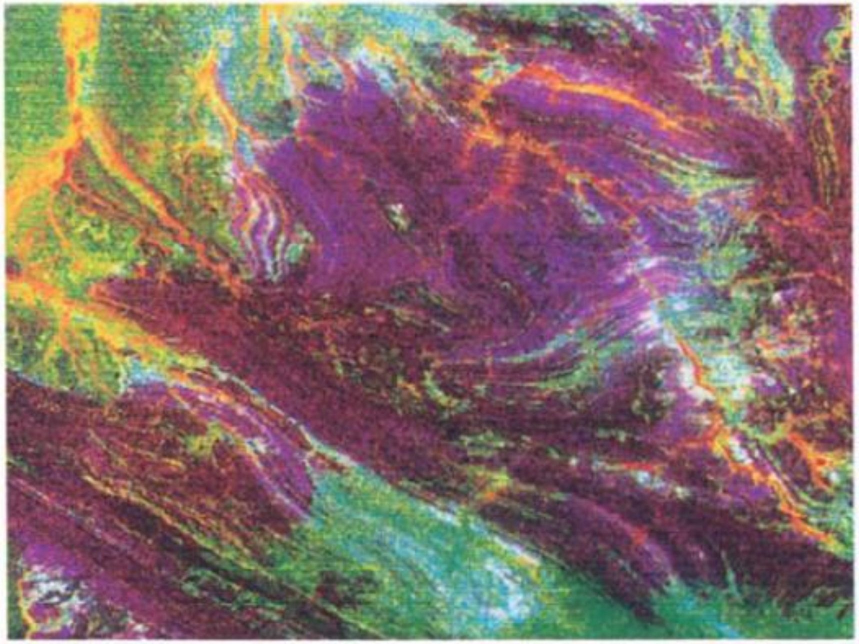
- Four principal components of the same image segment



Real Example (contd)

- Comparison of standard false colour composite (band 7 to red, band 5 to green and band 4 to blue)
- with a principal component composite (first component to red, second to green and third to blue)
- … below some pictures from the neighbor





Real Example (contd)

- The covariance matrix for this image is

$$\Sigma_x = \begin{bmatrix} 34.89 & 55.62 & 52.87 & 22.71 \\ 55.62 & 105.95 & 99.58 & 43.33 \\ 52.87 & 99.58 & 104.02 & 45.80 \\ 22.71 & 43.33 & 45.80 & 21.35 \end{bmatrix}$$

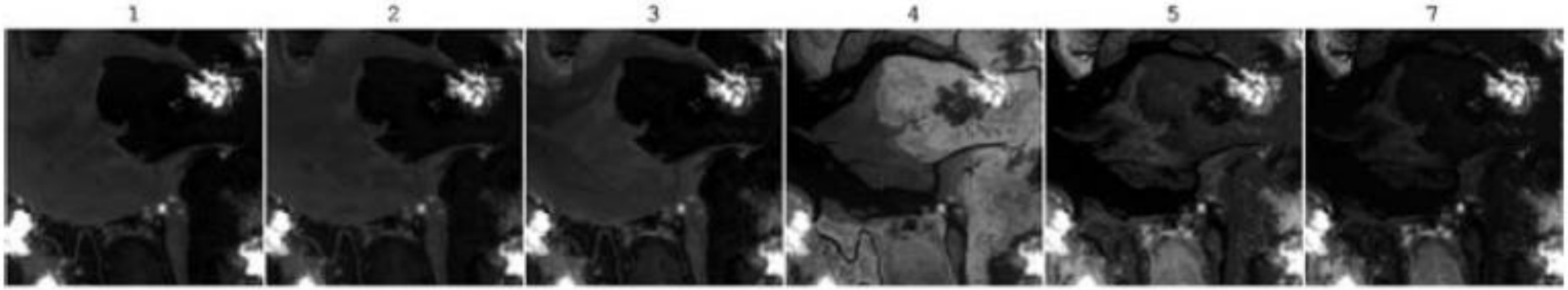
and its eigenvalues and eigenvectors are:

eigenvalues	253.44	7.91	3.96	0.89
eigenvector	0.34	−0.61	0.71	−0.06
components	0.64	−0.40	−0.65	−0.06
(vertically)	0.63	0.57	0.22	0.48
	0.28	0.38	0.11	−0.88

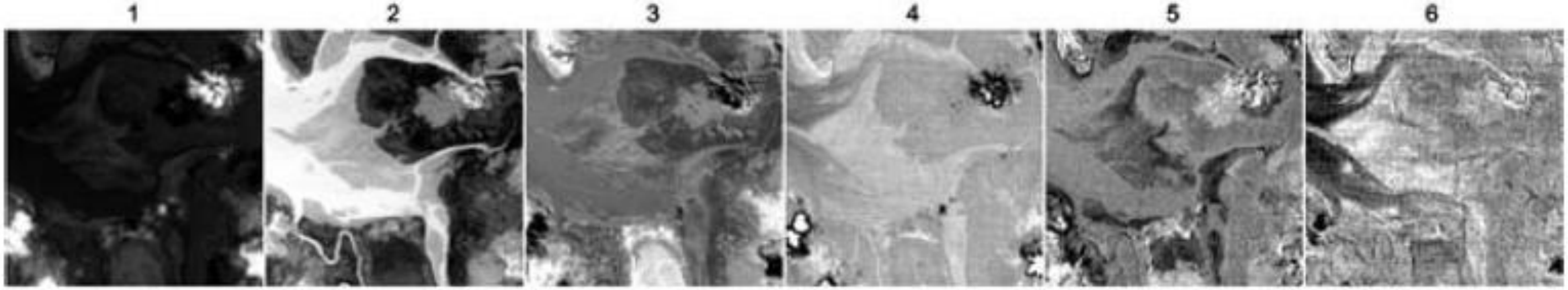
- The first principal component image will be expected therefore to contain 95% of the data variance. By comparison, the variance in the last component is seen to be negligible. It is to be expected that this component will appear almost totally as noise of low amplitude.
- The four principal component images show the information redistribution and compression properties of the transformation are illustrated. By association with the numerical example it would be anticipated that the later components should appear dull and poor in contrast. The high contrasts displayed are a result of a contrast enhancement applied to the components for the purpose of display. This serves to highlight the poor signal to noise ratio.

A second Real Example

- A second example of the principal components transformation is shown in Figure, this time based on the 6 reflective TM bands for a region in the Northern Territory of Australia.
- The original TM bands: visible (1,2,3), near IR, NIR (4), shortwave IR, SWIR (5,7)



- The full set of principal components



A second Real Example (contd)

- The covariance and correlation matrices for the image are:

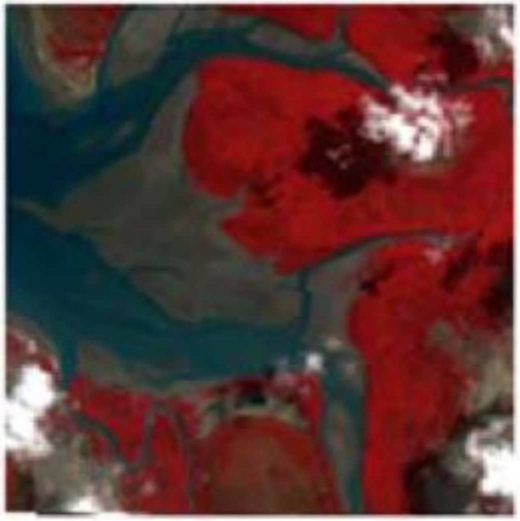
$$\Sigma_x = \begin{bmatrix} 874.98 & 550.56 & 698.00 & 335.54 & 858.15 & 551.21 \\ 550.56 & 363.82 & 454.79 & 230.30 & 558.88 & 358.38 \\ 689.00 & 454.79 & 580.63 & 288.11 & 747.97 & 471.72 \\ 335.54 & 230.30 & 288.11 & 722.46 & 742.35 & 387.61 \\ 858.15 & 558.88 & 747.97 & 742.35 & 1544.70 & 871.29 \\ 551.21 & 358.38 & 471.72 & 387.61 & 871.29 & 514.18 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1.00 & 0.98 & 0.97 & 0.42 & 0.74 & 0.82 \\ 0.98 & 1.00 & 0.99 & 0.45 & 0.75 & 0.83 \\ 0.97 & 0.99 & 1.00 & 0.44 & 0.79 & 0.86 \\ 0.42 & 0.45 & 0.44 & 1.00 & 0.70 & 0.64 \\ 0.74 & 0.75 & 0.79 & 0.70 & 1.00 & 0.98 \\ 0.82 & 0.83 & 0.86 & 0.64 & 0.98 & 1.00 \end{bmatrix}$$

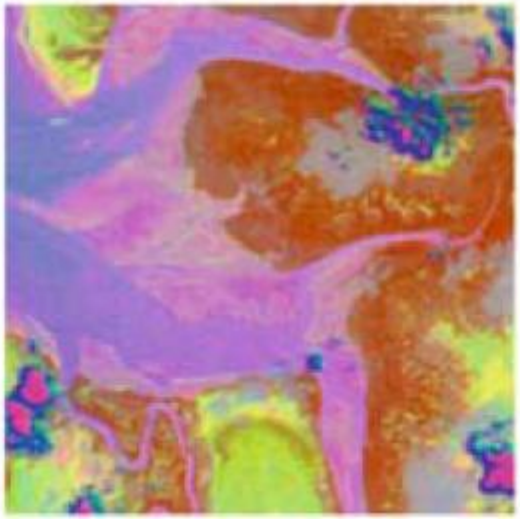
  - From the correlation matrix we directly see that the correlation among bands is high
  - Thus we can expect a significant effect of the principal component transform
  - The eigenvalues and eigenvectors are:

eigenvalues	3727.35	613.34	226.14	23.52	8.16	2.25
eigenvectors	first	second	third	fourth	fifth	sixth
	0.433	0.485	−0.307	−0.684	−0.089	0.088
	0.282	0.294	−0.218	0.369	0.094	−0.801
	0.364	0.347	−0.127	0.627	−0.153	0.561
	0.303	−0.673	-0.671	0.018	0.042	0.056
	0.615	−0.322	0.562	−0.052	−0.429	−0.129
	0.362	−0.047	0.275	−0.026	0.880	0.127

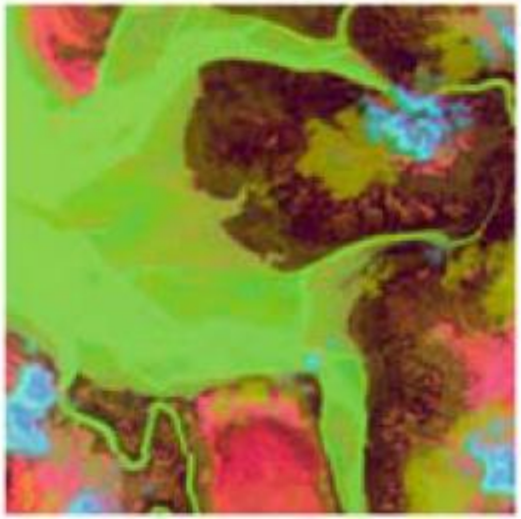
A second Real Example (contd)



a



b



c

- Pseudocolor composites comparison
  - Figure a shows a colour composite formed by mapping the original bands 4, 3, and 2 to red, green and blue respectively.
  - Figure b shows PC3, PC2 and PC1 mapped to red, green and blue,
  - while Figure c shows PC4, PC3 and PC2 mapped to red, green and blue.
- Interestingly, the last **PC4, PC3, PC2** colour composite shows more detail for those ground covers whose **spectral** responses are dominant in the visible to near infrared regions, since PC4 (determined by the fourth eigenvector) is largely a difference image in the visible region (this can be seen from the linear combination coefficients in the fourth eigenvector).
- In contrast **PC1** is essentially just a total brightness image (**topographic**), as can be seen from the first eigenvector, so that it does little to enhance spectral differences.

Remarks about the Principal Components Transform

- The covariance matrix used to generate the principal component transformation matrix is a *global measure of the variability of the original image segment*.

- Princpal Component images are useful
  - for reducing data dimensionality,
  - condensing topographic and spectral information,
  - improving image colour presentation,
  - and enhancing some spectral features.

- Notwithstanding the anticipated negligible information content of the last, or last few, image components resulting from a principal components analysis it is important to examine all components since often local detail may appear in a later component.
- Abnormal local detail therefore may not necessarily be mapped into one of the earlier components but could just as easily appear later. This is often the case with geological structure.
  - Some variants of the principal component transform have been proposed to enforce the possibility of a better separation between topographic and specific spectral information

The Effect of an Origin Shift

- It is evident that some principal component pixel brightnesses could be negative owing to the fact that the transformation is a simple axis rotation.
  - Clearly a combination of positive and negative brightnesses cannot be displayed.
  - Nor can negative brightness pixels be ignored since their appearance relative to the other pixels in a component serve to define detail.
  - In practice, the problem with negative values is accommodated by shifting the origin of the principal components space to yield all components with positive and thus displayable brightnesses.
  - This has no effect on the properties of the transformation as can be seen by inserting an origin shift term in the definition of the covariance matrix in the principal components axes.
  - Define $\boldsymbol{y}' = \boldsymbol{y} - \boldsymbol{y}_0$ where $\boldsymbol{y}_0$ is the position of a new origin. In the new $\boldsymbol{y}'$ co-ordinates

$$\Sigma_{y'} = \mathcal{E}\{(\boldsymbol{y}' - \boldsymbol{m}_{y'})(\boldsymbol{y}' - \boldsymbol{m}_{y'})^t\}$$

  - Now $\boldsymbol{m}_y = \boldsymbol{m}_{y'} - \boldsymbol{y}_0$ so that $\boldsymbol{y}' - \boldsymbol{m}_{y'} = \boldsymbol{y} - \boldsymbol{y}_0 - \boldsymbol{m}_y + \boldsymbol{y}_0 = \boldsymbol{y} - \boldsymbol{m}_y$.
  - Thus $\Sigma_{y'} = \Sigma_y$ – i.e. the origin shift has no influence on the covariance of the data in the principal components axes, and can be used for convenience in displaying principal component images.

Application of Principal Components in Image Enhancement and Display

- In constructing a colour display of remotely sensed data only three dimensions of information can be mapped to the three colour primaries of the display device.
  - For imagery with more than three bands that means the user must choose the most appropriate subset of three to use.
  - A less ad hoc means for colour assignment rests upon performing a principal components transform and assigning the first three components to the red, green and blue colour primaries.
  - Examination of a typical set of principal component images for Landsat data, such as those seen in the first example, reveals that there is very little detail in the fourth component so that, in general, it could be ignored without prejudicing the ability to extract meaningful information from the scene.
- A difficulty with principal components colour display, however, is that there is no longer a one to one mapping between sensor wavelength bands and colours.
  - Rather each colour now represents a linear combination of spectral components, making photointerpretation difficult for many applications.
  - An exception would be in exploration geology where structural differences may be enhanced in principal components imagery, there often being little interest in the meanings of the actual colours.

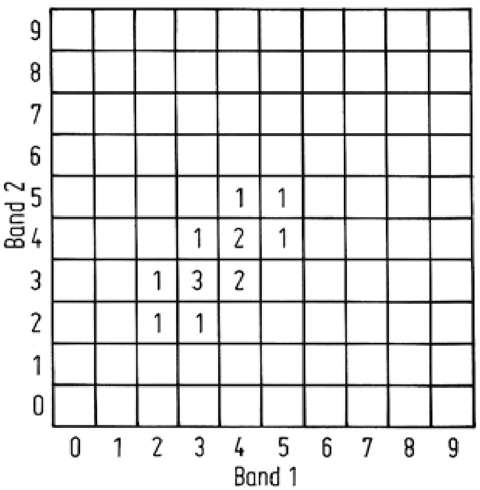
The Taylor Method of Contrast Enhancement (Decorrelation Stretch)

- Application of common contrast modification techniques (e.g. linear stretch, histogram equalization) to each of the individual components of a highly correlated vector image yields an **enhanced image in which certain highly saturated hues are missing.**
  - It is a direct result of the correlation in the image that the highly saturated colour primaries are not displayed.
  - In the display of three dimensional correlated image data, simple contrast enhancement of each component independently will yield an image *without* highly saturated reds, blues and greens *but also without* saturated yellows, cyans and magentas (color space extremes).

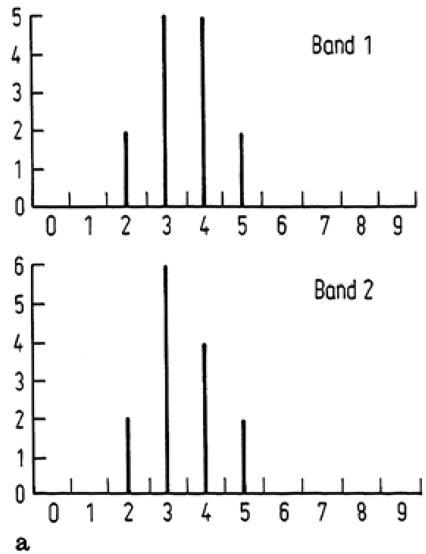
- An interesting **contrast stretching procedure** which can be used to create a modified image with *good utilisation of the range of available hues* **rests upon the use of the principal components transformation**.
  - It was developed by Taylor (1973) and has also been presented by Soha and Schwartz (1978). A more recent and general treatment has been given by Campbell (1996).
  - Also called (on other books) Principal Component Analysis **Decorrelation Stretch**, PCA-DS

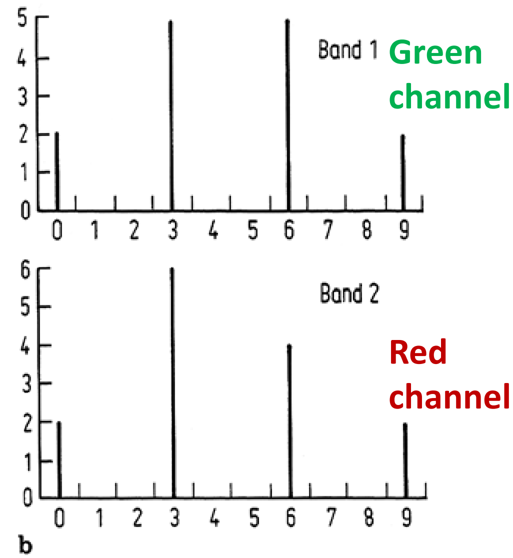
The Taylor Method of Contrast Enhancement (Decorrelation Stretch)

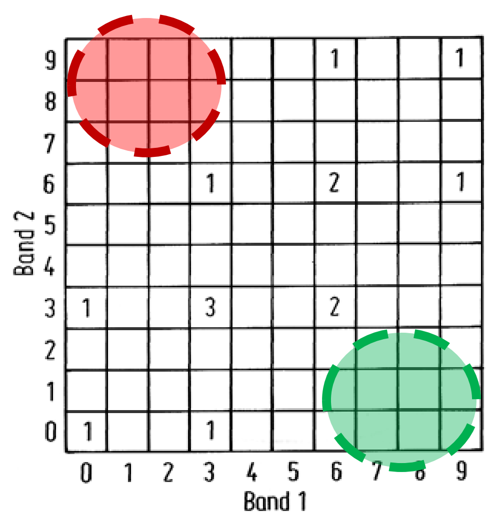
- The problem of a separate band (linear) contrast enhancement



Histogram for a hypothetical two dimensional image showing correlation in its components







- The components' correlation still remains
- Suboptimal exploitation of color channels (hue saturation not reached)

The Taylor Method of Contrast Enhancement (Decorrelation Stretch)

- The solution based on the use of the Principal Component Transformation:

1. Apply PCT to the data to transform the original bands into Principal Components
2. Apply contrast enhancement of each of the PCs (by linear stretching)
3. Apply Inverse PCT to convert back into image bands

- The procedure recommended by Taylor overcomes the lack of hue saturation because it fills the available colour space on the display more fully.

$$\Sigma_x = \mathscr{E}\{(\boldsymbol{x}-\boldsymbol{m})(\boldsymbol{x}-\boldsymbol{m})^t\}$$

Presenting non-zero off-diagonal elements (band correlation)

1.

$$\Sigma_y = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

2.

$$\Sigma'_y = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix} = \sigma^2 \boldsymbol{I}$$

Possible noise amplification problem can be reduced by denoising in the PC domain

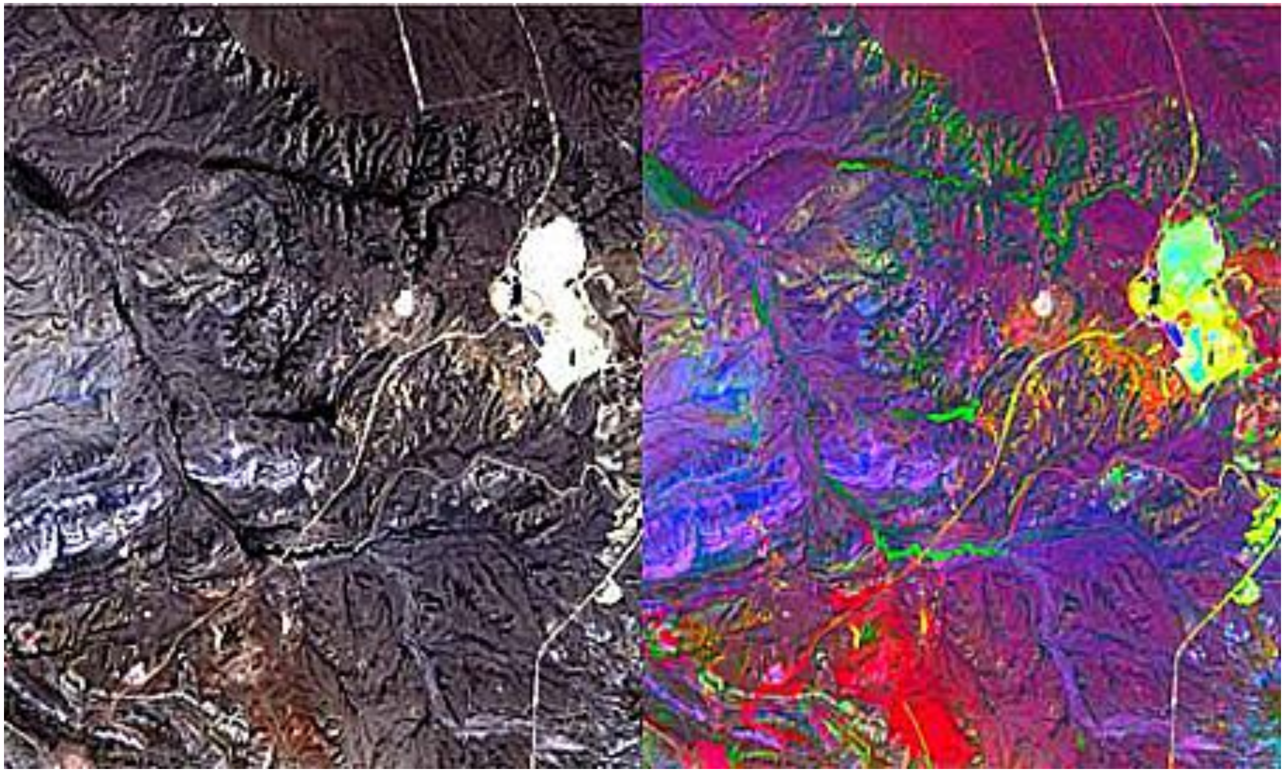
3.

$$\begin{aligned} \Sigma'_x &= G^t \mathscr{E}\{(y' - \mathscr{E}(y'))(y' - \mathscr{E}(y'))\}^t G \\ &= G^t \Sigma'_y G \\ &= G^t \sigma^2 I G \end{aligned}$$

$$\Sigma'_x = \sigma^2 I \ .$$

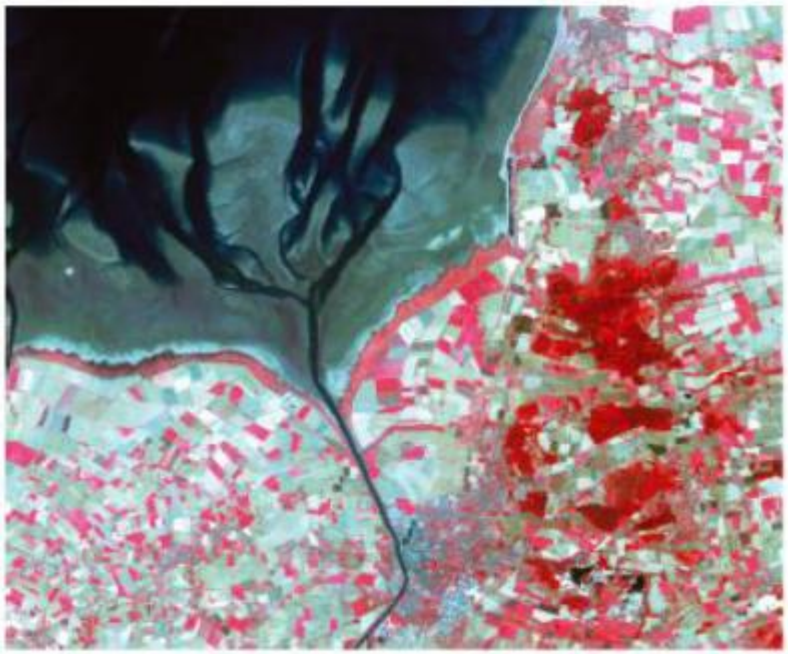
Decorrelation Stretch Example 1

- On the left is a standard false color composite; on the right a DS image - this illustrates the ability to extract and emphasize the tonal differences not apparent in the left image (from http://rst.gsfc.nasa.gov/Sect1/Sect1_14.html)

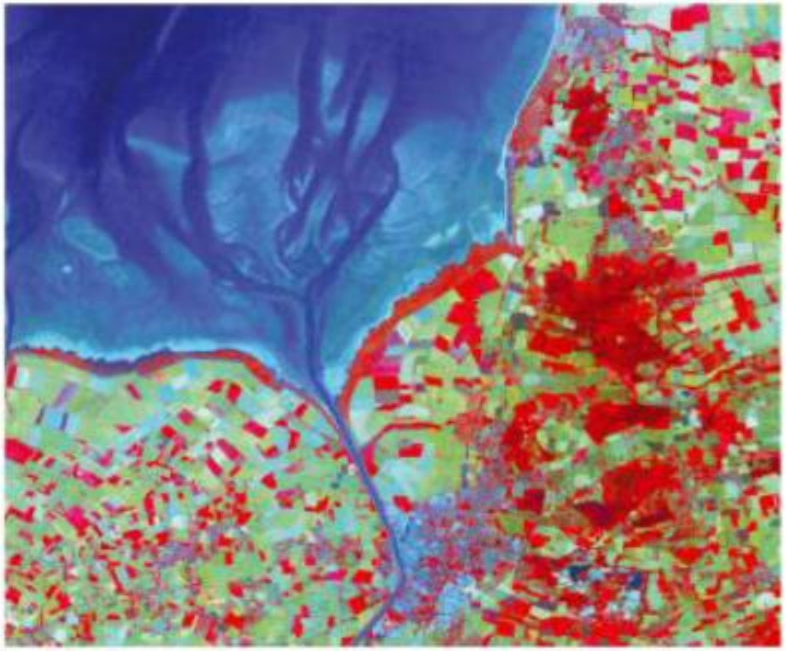


Decorrelation Stretch Example 2

a) Landsat TM bands 4,3,2 false colour composite of the coastline of The Wash, eastern England, after a 5-95% linear contrast stretch
b) The same Wash image after a decorrelation stretch based on the covariance matrix



a)



b)

Computer Processing of Remotely-Sensed Images, An Introduction, 4th edition: Paul M. Mather and Magaly Koch

Other Applications of Principal Components Analysis

- Owing to the information compression properties of the principal components transformation it lends itself to reduced representation of image data for storage or transmission.
  - In such a situation only **the uppermost significant components are retained** as a representation of an image, with the information content so lost being indicated by the sum of the eigenvalues corresponding to the components ignored.
  - Thereafter if the original image is to be restored, either on reception through a communications channel or on retrieval from memory, then *the inverse of the transformation matrix is used to reconstruct the image from the reduced set of components*.
  - Since the matrix is orthogonal its inverse is simply its transpose.
- This technique is known as **bandwidth compression** in the field of telecommunications.
  - Until recently it had not found great application in satellite remote sensing image processing, because hitherto image transmission has not been a consideration and available memory has not placed stringent limits on image storage.
  - With increasing use of imaging spectrometry (hyperspectral) data however, *bandwidth compression has become more important*.

- Another interesting application of principal components analysis is in the *detection of features that change with time* between images of the same region. This will be described when dealing change detection.

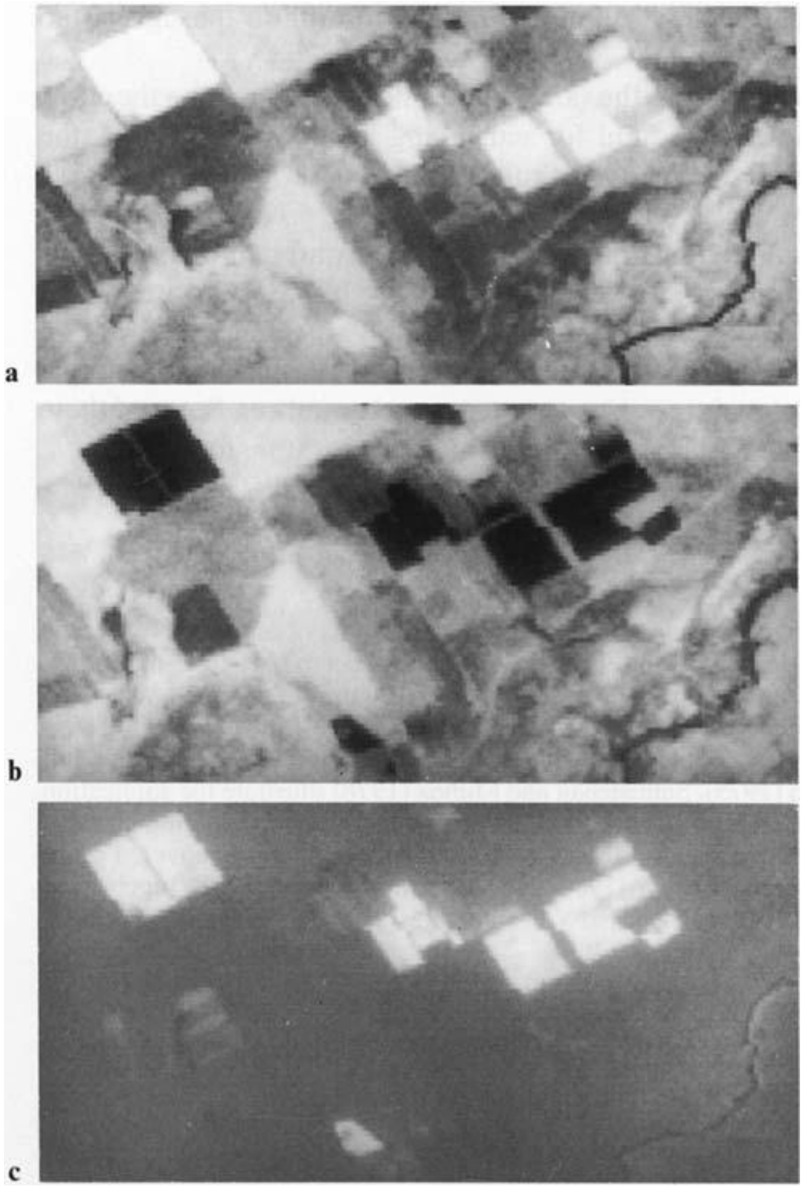
IMAGE ARITHMETIC, BAND RATIOS AND VEGETATION INDICES

Image Arithmetic, Band Ratios and Vegetation Indices

- Addition, subtraction, multiplication and division of the pixel brightnesses from two bands of image data to form a new image are particularly simple transformations to apply.
  - Multiplication seems not to be as useful as the others, band differences and ratios being most common.
- Differences can be used to highlight regions of change between two images of the same area. This requires that the images be registered beforehand.
  - The resultant difference image must be scaled to remove negative brightness values.
  - Normally this is done so that regions of no change appear mid-grey, with changes shown as brighter or duller than mid-grey according to the sign of the difference.

- Ratios of different spectral bands from the same image find use in reducing the effect of topography, as a *vegetation index*, and for *enhancing subtle differences in the spectral reflectance characteristics for rocks and soils*.
  - As an illustration of the value of band ratios for providing a single vegetation index image, next Figure shows Landsat multispectral scanner band 5 and band 7 images of an agricultural region along with the band 7/band 5 ratio.
  - As seen,
    - healthy vegetated areas are bright,
    - soils are mid to dark grey,
    - and water is black.
  - These shades are readily understood from an examination of the corresponding spectral reflectance curves.
- Note that band ratioing is not a linear transformation.

Image Arithmetic, Band Ratios and Vegetation Indices



Landsat multispectral scanner band 7, **a** and band 5, **b** images of an arid region containing irrigated crop fields.
The ratio of these two images **c** shows vegetated regions as bright, soils as mid to dark grey and water as black

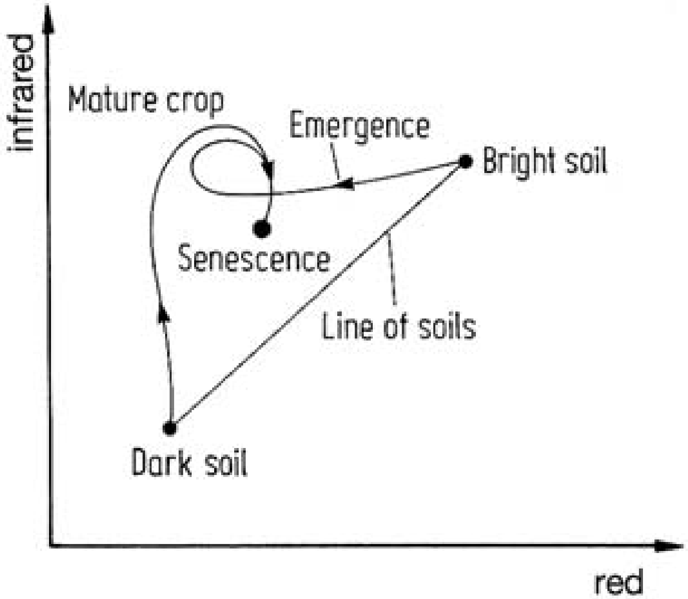
THE KAUTH-THOMAS TASSELED CAP TRANSFORMATION

The Kauth-Thomas Tasseled Cap Transformation

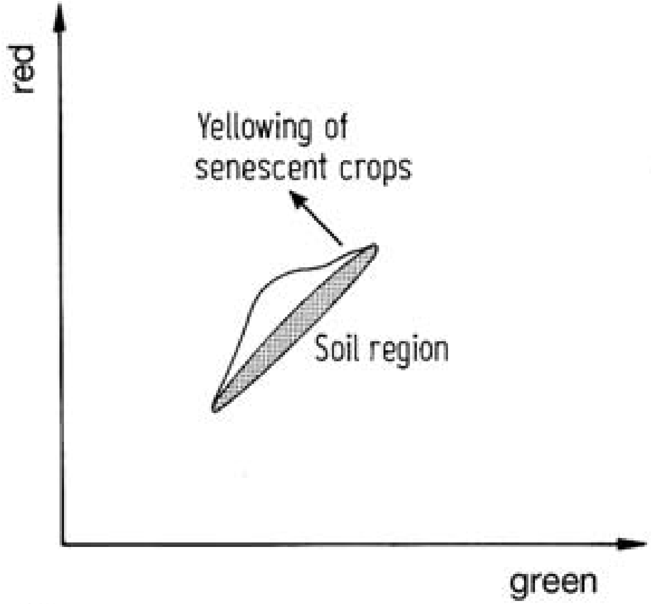
- The principal components transformation yields a new co-ordinate description of multispectral remote sensing image data by establishing a diagonal form of the global covariance matrix.
  - The new co-ordinates (components) are linear combinations of the original spectral bands.
  - Other linear transformations are of course possible, for example to define new axes in which data are described have been devised to maximise information of importance to agriculture.
  - Other application-specific special transformations would also be possible.
- The so-called “tasseled cap” transformation (Crist and Kauth, 1986) developed by Kauth and Thomas (1976) is a means for highlighting the most important (spectrally observable) phenomena of crop development in a way that allows discrimination of specific crops, and crops from other vegetative cover, in Landsat multitemporal, multispectral imagery.
  - Its basis originally lies in an observation of crop trajectories in band 6 versus band 5, and band 5 versus band 4 subspaces.

The Kauth-Thomas Tasseled Cap Transformation

- **a** Infrared versus red subspace showing trajectories of crop development;
- **b** Red versus green subspace also depicting crop development



a



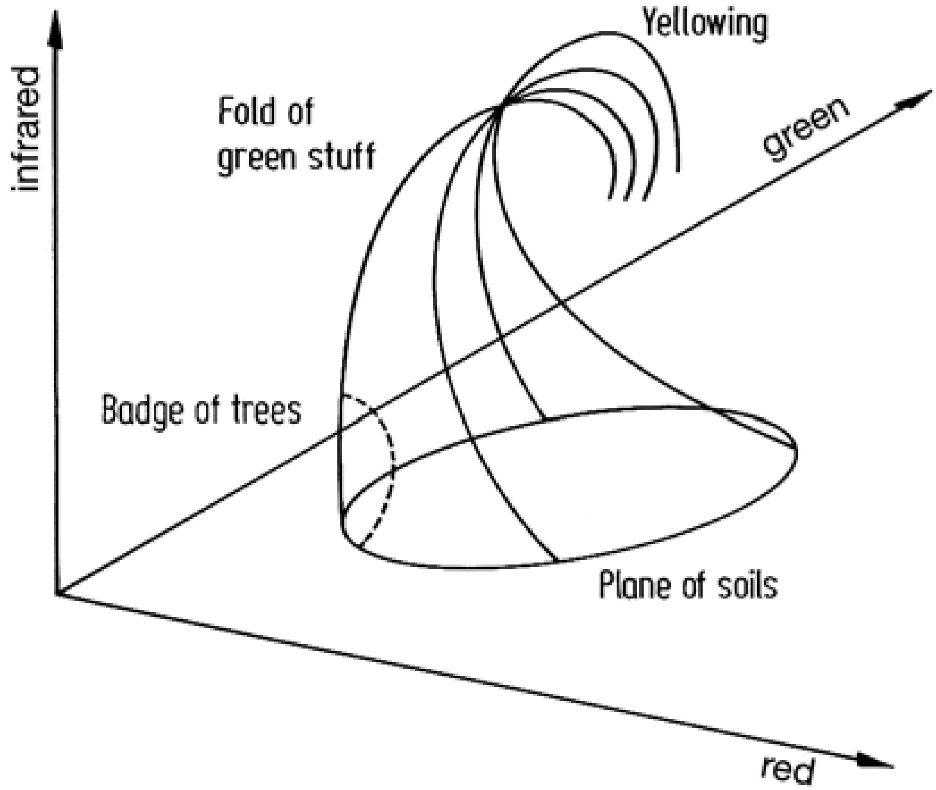
b

The Kauth-Thomas Tasseled Cap Transformation

- A first observation that can be made is that the variety of soil types on which specific crops might be planted appear as points along a diagonal in an infrared, red space as shown.
- This is well-known and can be assessed from an observation of the spectral reflectance characteristics for soils.
  - Darker soils lie nearer the origin and lighter soils at higher values in both bands. The actual slope of this line of soils will depend upon global external variables such as atmospheric haze and soil moisture effects.
  - If the transformation to be derived is to be used quantitatively these effects need to be modelled and the data calibrated or corrected beforehand.
- Consider now the trajectories followed in infrared versus red subspace for crop pixels corresponding to growth on different soils – in this case take the extreme light and dark soils as depicted in Figure a.
  - For both regions at planting the multispectral response is dominated by soil types, as expected.
  - As the crops emerge the shadows cast over the soil dominate any green matter response.
  - As a result there is considerable darkening of the response of the lighter soil crop field and only a slight darkening of that on dark soil.
  - When both crops reach maturity their trajectories come together implying closure of the crop canopy over the soil.
  - The response is then dominated by the green biomass, being in a high infrared and low red region, as is well known.
- When the crops senesce and turn yellow their trajectories remain together and move away from the green biomass point in the manner depicted in the diagram.
  - However, whereas the development to maturity takes place almost totally in the same plane, the yellowing development in fact moves out of this plane, as can be assessed by how the trajectories develop in the red versus green subspace during senescence as illustrated in Figure b.
- Should the crops then be harvested, the trajectories beyond senescence move, in principle, back towards their original soil positions.

The Kauth-Thomas Tasseled Cap Transformation

- Crop trajectories in a green, red, infrared space, having the appearance of a tasseled cap



The Kauth-Thomas Tasseled Cap Transformation

- The first point to note is that the line of soils used in the previous Figure a is shown now as a plane of soils.
  - Its maximum spread is along the three dimensional diagonal as indicated;
  - however it has a scatter about this line consistent with the spread in red versus green as shown in Fig. 6.11b.
- Kauth and Thomas note that this plane of soils forms the brim and base of the cap.
  - As crops develop on any soil type their trajectories converge essentially towards the crown of the cap at maturity whereupon they fold over and continue to yellowing as indicated. Thereafter they break up to return ultimately to various soil positions, forming tassels on the cap as shown.
- The behaviour observable in the above Figure led Kauth and Thomas to consider the development of a linear transformation that would be useful in crop discrimination.
  - As with the principal components transform, this transformation will yield four orthogonal axes.
  - However the axis directions are chosen according to the behaviour seen in Figure.
- Three major orthogonal directions of significance in agriculture can be identified.
  - The first is the principal diagonal along which soils are distributed. This was chosen by Kauth and Thomas as the first axis in the tasseled cap transformation.
  - The development of green biomass as crops move towards maturity appears to occur orthogonal to the soil major axis. This direction was then chosen as the second axis, with the intention of providing a greeness indicator.
  - Crop yellowing takes place in a different plane to maturity. Consequently choosing a third axis orthogonal to the soil line and greeness axis will give a yellowness measure.
  - Finally a fourth axis is required to account for data variance not substantially associated with differences in soil brightness or vegetative greeness or yellowness.
    - Again this needs to be orthogonal to the previous three.
    - It was called “non-such” by Kauth and Thomas in contrast to the names “soil brightness”, “green-stuff” and “yellow-stuff” they applied to the previous three.

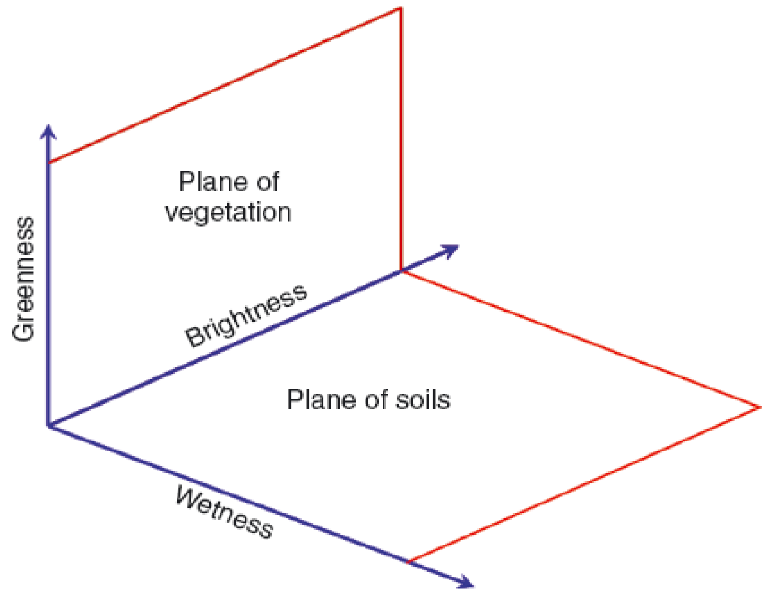
The Kauth-Thomas Tasseled Cap Transformation

- The transformation that produces the new description of the data may be expressed as ***u = Rx + c*** where ***x*** is the original Landsat vector, and ***u*** is the vector of transformed brightness values.
  - This has soil brightness as its first component, greeness as its second and yellowness as its third. T
  - hese can therefore be used as indices, respectively.
  - ***R*** is the transformation matrix and ***c*** is a constant vector chosen (arbitrarily) to avoid negative values in ***u.***
- The transformation matrix ***R*** is the transposed matrix of column unit vectors along each of the transformed axes (compare with the principal components transformation matrix).
  - For a particular agricultural region Kauth and Thomas chose the first unit vector as a line of best fit through a set of soil classes.
  - The subsequent unit vectors were generated by using a Gram-Schmidt orthogonalization procedure in the directions required.
  - The transformation matrix generated for Landsat MSS data was

$$R = \begin{bmatrix} 0.433 & 0.632 & 0.586 & 0.264 \\ -0.290 & -0.562 & 0.600 & 0.491 \\ -0.829 & 0.522 & -0.039 & 0.194 \\ 0.223 & 0.012 & -0.543 & 0.810 \end{bmatrix}$$

  - From this it can be seen, at least for the region investigated by Kauth and Thomas, that the soil brightness is a weighted sum of the original four Landsat bands with approximately equal emphasis.
  - The greeness measure is the difference between the infrared and visible responses. In a sense therefore this is more a biomass index.
  - The yellowness measure can be seen to be substantially the difference between the Landsat visible red and green bands.
  - Just as new images can be synthesised to correspond to various principal components so can the actual transformed images be created for this approach.
  - By applying the transform to every pixel in a Landsat multispectral scanner image, soil brightness, greeness, yellowness and non-such images can be produced.
  - These can then be used to assess stages in crop development.
  - The method can also be applied to other sensors.

The Kauth-Thomas Tasseled Cap Transformation



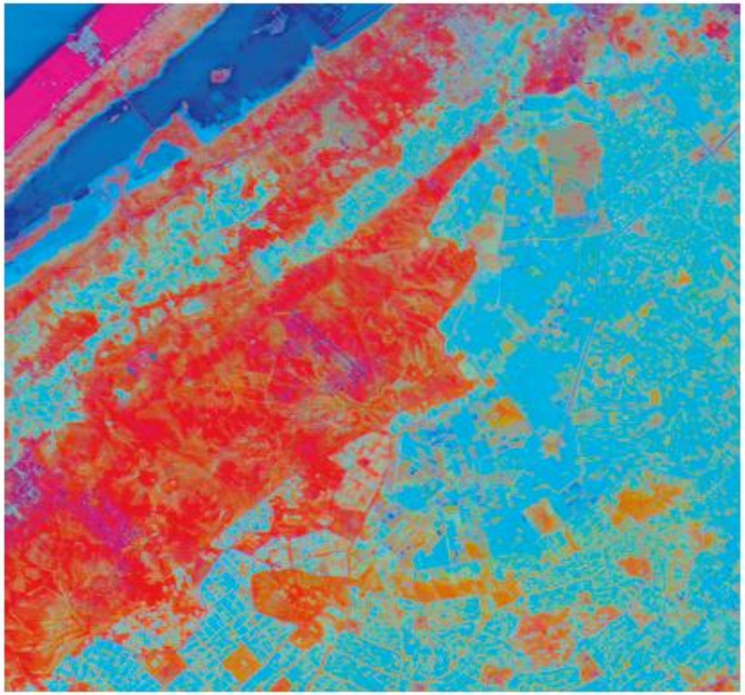


Figure 6.8 The Tasselled Cap transformation defines three fixed axes. Image pixel data are transformed to plot on these three axes (greenness, brightness and wetness) which jointly define the Plane of Vegetation and the Plane of Soils. See text for discussion. Based on Crist, E.P. and Cicone, R.C., 1986, Figure 3. Reproduced with permission from American Society for Photogrammetry and Remote Sensing, Manual of Remote Sensing.

Figure 6.9 Tasselled Cap image derived from 1993 ETM+ image of Alexandria, Egypt, shown in Figure 6.2b. Brightness is shown in red – with the sandy desert area being clearly delineated. Greenness is allocated to the green band, and wetness to the blue band. The water areas are clearly identified in the top left corner of the image, while the agricultural areas (shown in red on Figure 6.2b) are shown in shades of cyan, a mixture of green (greenness) and blue (wetness). More detail of field boundaries and roads and tracks can be seen on this image, compared to Figure 6.2b. Landsat data courtesy NASA/USGS.