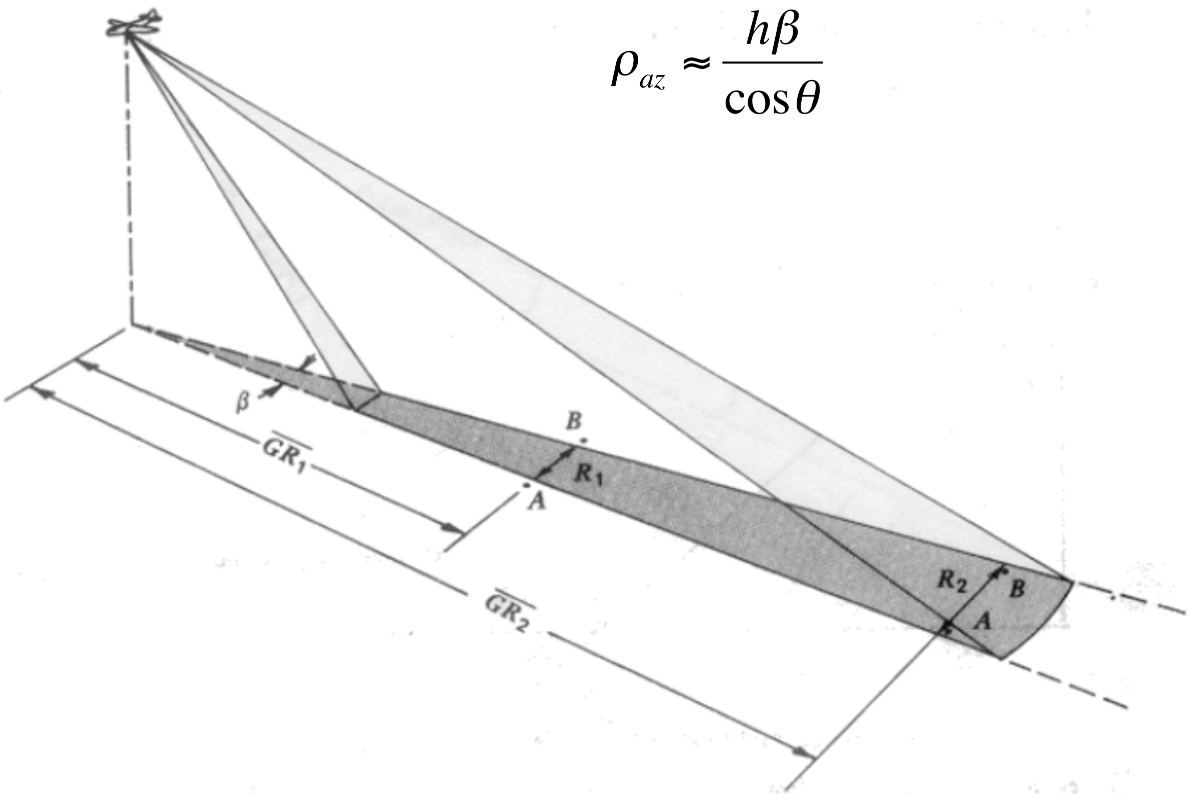
SAR: Synthetic Aperture Radar

Azimuthal resolution

- The minimal detectable distance in the flight direction
- Two points can be distinguished in azimuth only if they are do not reflect the same pulse. Hence

$$\rho_{az} \approx \frac{h\beta}{\cos\theta}$$



Azimuthal resolution

- The azimuthal resolution is a fundamental parameter
- It does not depend in fact on the slant-range resolution and thus not even on the duration of the pulses (or their shape)
- It only depends on the width of the lobe in the flight direction and on the altitude (and whether we are in the near/far range)
- Usually, the width at of the lobe at -3dB can be approximated as

$$\beta \approx \frac{\lambda}{L}$$

where L is the length of the antenna. This implies

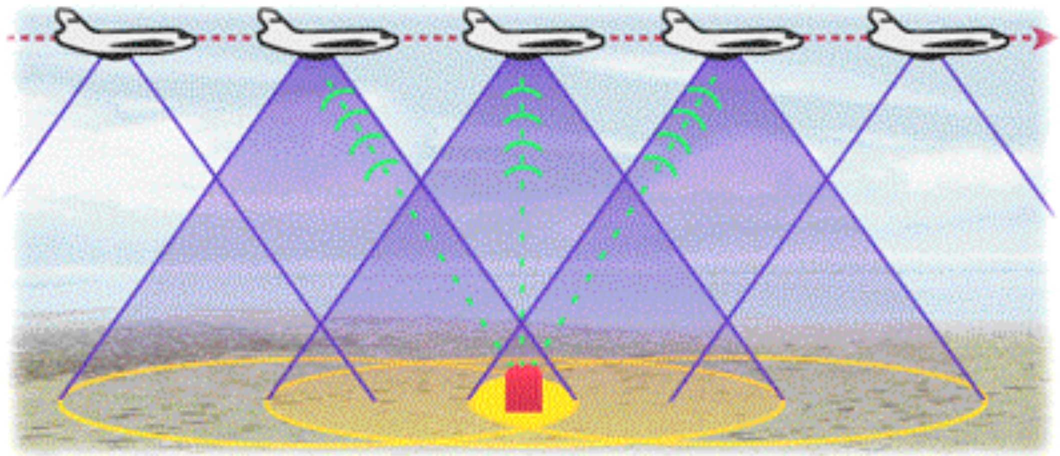
$$\rho_{az} \approx \frac{h\lambda}{L\cos\theta}$$

Synthetic Aperture Radars

- In order to improve the azimuthal resolution a smart solution has been found which introduces a variation on the original idea of how the radar should operate
- Exploit the movement of the platform to emulate a longer antenna in the azimuthal direction.
- In practice, the radar emits a highly coherent wave and analyzes all the echoes from a given point while flying over, so as to simulate what would have been received by a longer antenna than the one physically available

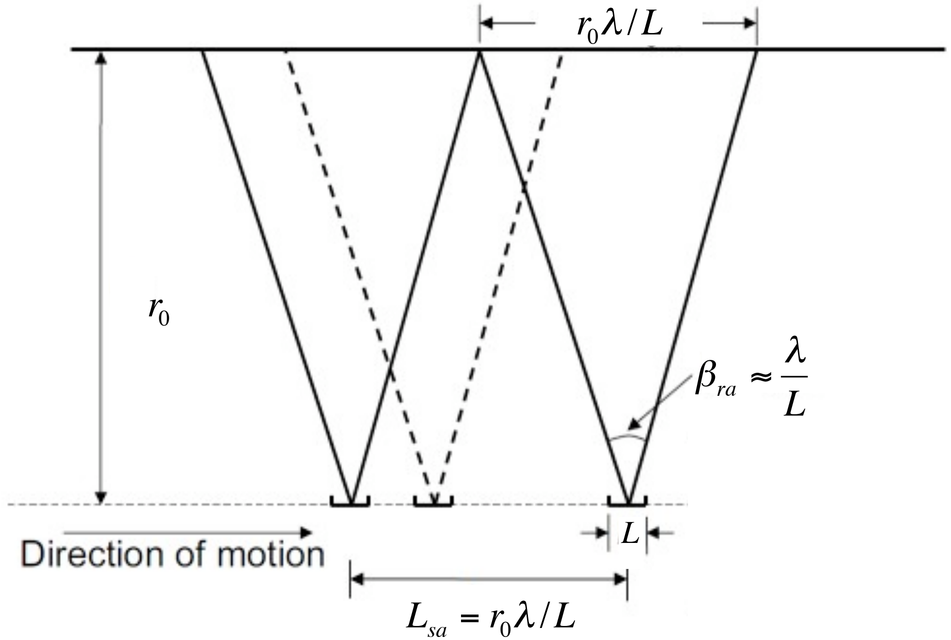
Synthetic Aperture Radar (SAR)

- Principle developed in the ’50s
- If exploits the fact that the platform moves relatively fast
- It can be interpreted as a smart usage of the doppler effect, which acts in a different way depending on whether the target is approaching or had already been passed



Synthetic Aperture

- Definition:
  - The synthetic aperture is defined as the length of the path during which the target is visible
  - It equals the width of lobe at ground in the azimuthal direction



Resolution

- We will see that the *angular resolution* will be related to the synthetic aperture with an expression which is almost the same as that which would be obtained with a real aperture of the same size, with just a penalty factor of 2

$$\alpha_{sa} = \frac{\lambda}{2L_{sa}} = \frac{L}{2r_0}$$

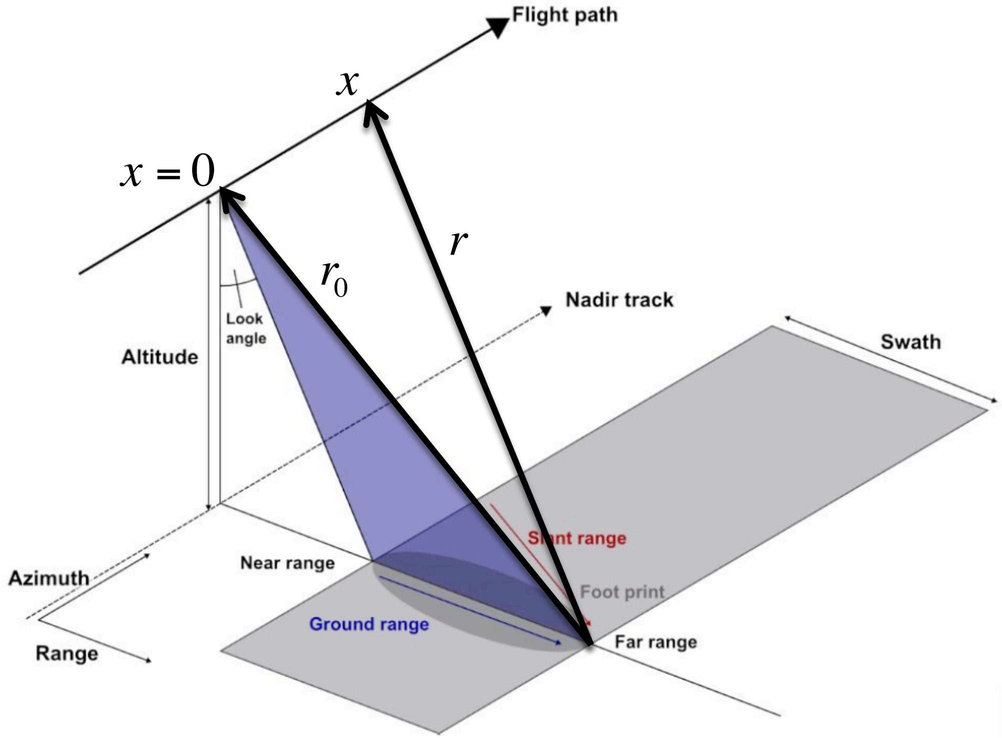
- The angular resolution, thus, depends on the altitude. This is reflected to a ground resolution which is

$$\rho_{sa} = \alpha_{sa} r_0 = \frac{L}{2}$$

- The ground resolution in azimuthal direction is thus half the real aperture of the antenna! (up to a certain point, see later)

Phase History of a target

- Let the antenna be moving in the *x* direction



- With Pitagora, we have

$$r = \sqrt{r_0^{\ 2} + x^2}$$

Phase History of a target

- If we assume that the angular aperture is not too large, that is, $r_0 >> x$, we have

$$r = r_0\sqrt{1+\frac{x^2}{r_0^2}} \approx r_0\left(1+\frac{x^2}{2r_0^2}\right) = r_0 + \frac{x^2}{2r_0}$$

- By studying the phase of the signal received when the sensor is in position *x*, we have

$$\varphi(x) = 2\cdot\frac{2\pi}{\lambda}\left(r_0 + \frac{x^2}{2r_0}\right)$$

$$= \frac{4\pi r_0}{\lambda} + \frac{2\pi x^2}{\lambda r_0}$$

Phase History of a target

- We can rewrite everything as a function of time, using the speed *v*

$$\varphi(t) = \frac{4\pi r_0}{\lambda} + \frac{2\pi v^2 t^2}{\lambda r_0}$$

- Let us now consider the derivative of the phase, that is the instantaneous frequency

$$f(t) = \frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t} = \underbrace{\frac{2v^2}{\lambda r_0}}_{\text{Chirp Rate}} t = kt$$

- Assume we are observing a target placed at x=0. We observe the target for a period of time between the two instants

$$t_{\min} = -\frac{T}{2} = -\frac{L_{sa}}{2v} = -\frac{\beta_{ra} r_0}{2v} \qquad t_{\max} = \frac{T}{2} = \frac{\beta_{ra} r_0}{2v}$$

Phase History of a target

- So, in the interval of observation of the target, the instantaneous frequency moves in a band

$$B_a = f(t_{\max}) - f(t_{\min})$$
$$= \frac{2v^2}{\lambda r_0} \cdot \frac{\beta_{ra} r_0}{v}$$

- Hence, we have a signal with bandwidth

$$B_a = \frac{2\beta_{ra} v}{\lambda}$$

- Since the SAR is based on pulses, we should consider the Shannon theorem and use pulses sufficiently close to allow the reconstruction of a signal with such a band

Phase History of a target

- We thus have a lower bound for the frequency of pulses (PRF, Pulse Repetition Frequency)

$$PRF_{\min} = 2B_a = \frac{4\beta_{ra} v}{\lambda}$$

- Consider the sampling period

$$T_{samp,\max} = \frac{\lambda}{4\beta_{ra} v}$$

and then the (azimuthal) distance between two samples

$$\frac{\lambda}{4\beta_{ra}}$$

…. We see that this is just nearly *L*/4

Focusing

- From what we have seen up to now, each point target is observed on a sequence of pulses and not on in just one pulse
- In general, the return pulses of one target overlap with the return pulses of other targets
- This is usually expressed by saying that the SAR signal is not focused
- It is necessary to process the signal (focusing) so as to concentrate all the energy of a return pulse from a target in one single point

Processing in Azimuth

- We need to process the signal using the phase history. Recall that we had

$$\varphi(t) = \frac{4\pi r_0}{\lambda} + \frac{2\pi v^2 t^2}{\lambda r_0}$$

Constant

- Using a reference phase which is zero at the instant t=0, we can assume we have a signal with phase

$$\varphi(t) = \frac{2\pi v^2 t^2}{\lambda r_0} = \pi k t^2$$

- If we assume that the backscattering is independent from the angle and constant in time, the amplitude will be constant

Processing in Azimuth

- That is, we may assume that we receive a signal of the form

$$S(t) = A_0 \exp\left(j\pi k t^2\right)$$

in the interval [t_{min}, t_{max}]

- The idea is to compute a correlation with a reference pulse so as to compress the signal on a shorter domain.
- Ideally we may use a signal like

$$R(t) = \exp\left(-j\pi k t^2\right)$$

- The intuition is that the correlation between S(t) and R(t) gives a Dirac delta function, that is, it compresses the signal in the origin

Processing in Azimuth

- The fact that the signal S(t) is only available in an interval can be expressed in terms of a windowing operation, that is, we have the signal

$$S(t)W(t)$$

- Let us then compute the correlation

$$V(t) = \int_{-\infty}^{+\infty} S(\alpha)W(\alpha)R(t+\alpha)d\alpha$$

$$= \int_{-\infty}^{+\infty} A_0 \exp(j\pi k\alpha^2)W(\alpha)\exp(-j\pi k(t+\alpha)^2)d\alpha$$

$$= A_0 \exp(-j\pi kt^2)\int_{-\infty}^{+\infty} W(\alpha)\exp(-j2\pi kt\alpha)d\alpha$$

Fourier!

Processing in Azimuth

- So, the result is related to the Fourier trasform of the window

$$V(t) = A_0 \exp(-j\pi k t^2) \int_{-\infty}^{+\infty} W(\alpha) \exp\left(-j2\pi k t \alpha\right) d\alpha$$

$$= A_0 \exp(-j\pi k t^2) \hat{W}\left(kt\right)$$

- For example, if the window is rectangular,

$$W(t) = \mathrm{rect}\left(\frac{\mathrm{t}}{T}\right) \quad \Rightarrow \quad \hat{W}\left(kt\right) = \mathrm{sinc}\left(kTt\right)$$

Processing in Azimuth

- We then see that a point target produces, after the correlation, a pulse that depends on the analysis window
- At a first approximation, we can take as the width of the pulse half the width of the main lobe in the case of rectangular window
- The zeros of the pulse are in positions

$$t = \pm \frac{1}{kT}$$

- And the resolution in azimuth is then

$$\rho_{sa} = \frac{v}{kT} = \frac{v}{B_a} = \frac{L}{2}$$

Processing in Range

- The same description that we have used for the processing in azimuth also helps us to understand the pulse compression, discussed in the previous class, for improving the range resolution
- We said that chirped pulses are used to keep the power low and increase the resolution
- The idea is the same as for the focusing in the processing in azimuth
- For the range, however, the pulses are really generated with a chirp

$$S(t) = A_0 \exp\left(j\pi k t^2\right) W(t)$$

Processing in Range

- The used frequency band depends on the width of the windowing function W(t), that we called τ, and on the chirp rate k

$$B = k\tau$$

- At the receiver a correlation with the pulse

$$R(t) = A_0 \exp\left(-j\pi k t^2\right)$$

is computed, obtaining as seen for the azimuth

$$V(t) = A_0 \exp(-j\pi k t^2)\int_{-\infty}^{+\infty} W(\alpha)\exp\left(-j2\pi k t\alpha\right)d\alpha$$

$$= A_0 \exp(-j\pi k t^2)\hat{W}\left(kt\right)$$

Processing in Range

- Assume again we use a rectangular window

$$W(t) = \mathrm{rect}\left(\frac{\mathrm{t}}{\tau}\right) \quad \Rightarrow \quad \hat{W}(kt) = \mathrm{sinc}(k\tau t)$$

- Considering then the used band we thus have

$$V(t) = A_0 \exp(-j\pi B t^2/\tau)\,\mathrm{sinc}(Bt)$$

- The zeros of the main lob are in $t = \pm 1/B$ and we can then consider the pulse to have length $\tau = 1/B$ which implies that the resolution in range, that we expressed as $\tau c/2$ is, as anticipated

$$\rho_{sr} = \frac{\tau c}{2} = \frac{c}{2B}$$

Processing in Range and Azimuth

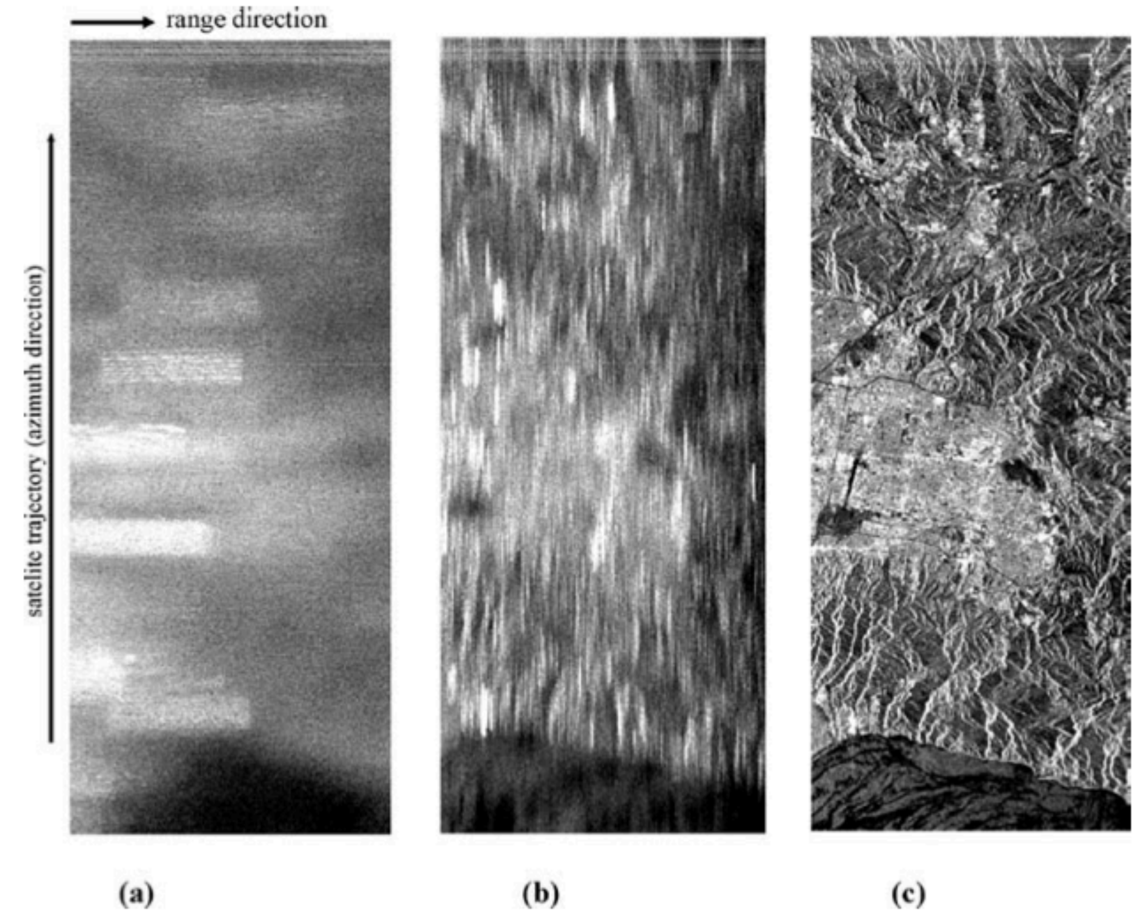


Fig. 2.2 (a) ERS image of raw echoes before SAR processing. Azimuth and range resolution: 5 km; (b) The same data after range processing. (Azimuth resolution: 5 km, range resolution: 20 m); (c) The same data after range processing and SAR synthesis. Azimuth resolution is: 5 m; range resolution is: 20 m. Northridge area, California, 1992.

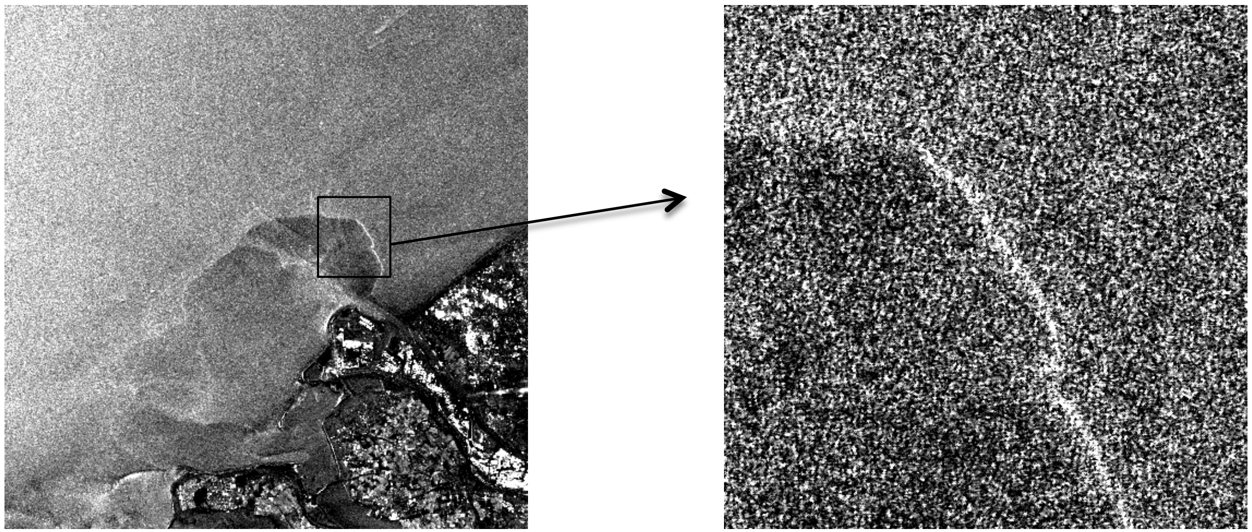
Speckle Noise

- The SAR images present a typical disturbance known as the *speckle noise*
- It is not really a noise, but it appears to our eyes as such
- It is due to the interference between the return waves from different targets which compose each single pixel
- Each pixel, in fact, is obtained by considering the contribution of different targets which, in general, backscatter the pulses with slightly different amplitudes an phases, which exact values cannot be predicted.

Speckle Noise

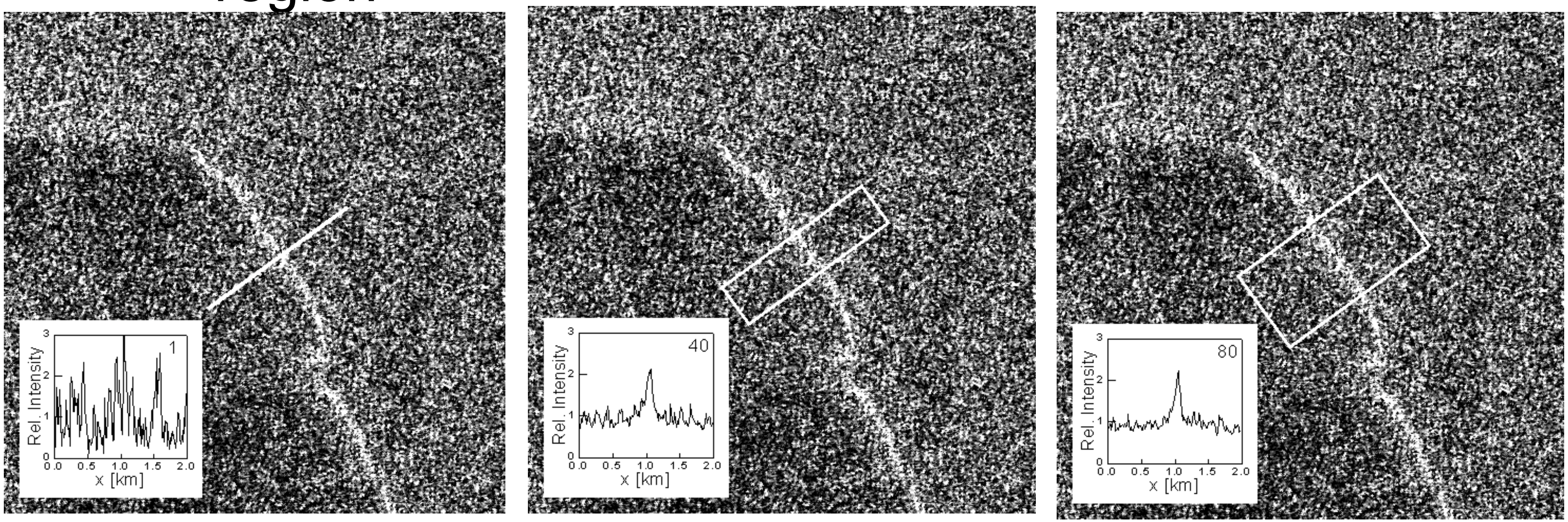
- The received wave is then the vectorial sum of those different contributions that, in general, interfere in a random way

- Example



Speckle Noise

- It is possible to reduce this noise by sacrificing resolution and averaging the values on a given region



Some missions

- Seasat... Already mentioned
- SIRC-C
  - Three Shuttle missions with a radar between 1994 and 2000
  - Operating in band C
  - Ground resolution of 15 m
- ERS
  - ERS-1 and ERS-2, satellites of the ESA (European Space Agency) in the preriod 1991-1995
  - Band C, polarization VV
- Radarsat
  - Radarsat 1 and 2, Canadian satellites, put in orbit in 1995 and in 2007
  - Resolution up to 7m and 3m respectively, different look angles possible
- COSMO/Skymed
- Envisat