Interaction with Matter

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Type of Interactions

- Transmission: we say that there is transmission when the wave (or part of the wave) proceeds on its path in the same direction though the medium
- Absorption: we say that there is absorption when part of the energy of the electromagnetic wave is transferred from the wave to the medium as a consequence of the interaction. This energy can then be given back as a radiation but usually at different wavelengths
- Scattering: we say that there is scattering when the wave is deviated in many different directions by small particles or by small surface elements
- Reflection: we say that there is reflection when the wave is deviated in one predominant direction after hitting a surface
- All related and partially coexisting phenomena

Dielectric Constant

- We used the notation

$$\varepsilon_c = (\varepsilon' - j\varepsilon'')\varepsilon_0$$

for the electric permittivity

- We also had the relations

$$\varepsilon_r = (\varepsilon' - j\varepsilon'')$$
$$n = m - j\kappa$$

where $n^2 = \varepsilon_r$, that is,

$$\varepsilon' = m^2 - \kappa^2$$
$$\varepsilon'' = 2m\kappa$$

Dielectric Constant

- As seen, in a conducting medium the solution of the Maxwell equations can be put in the form

$$E_x = E_0 e^{-\frac{\omega\kappa z}{c}} e^{j\left(\omega t - \frac{\omega m z}{c}\right)}$$

- This showed that the imaginary part of the refractive index determines the attenuation while the real part determines the wavelength

$$\lambda = \frac{2\pi c}{\omega m}$$

- Compared with vacuum

$$\lambda_0 = \frac{2\pi c}{\omega} \quad \Longrightarrow \quad \lambda = \frac{\lambda_0}{m}$$

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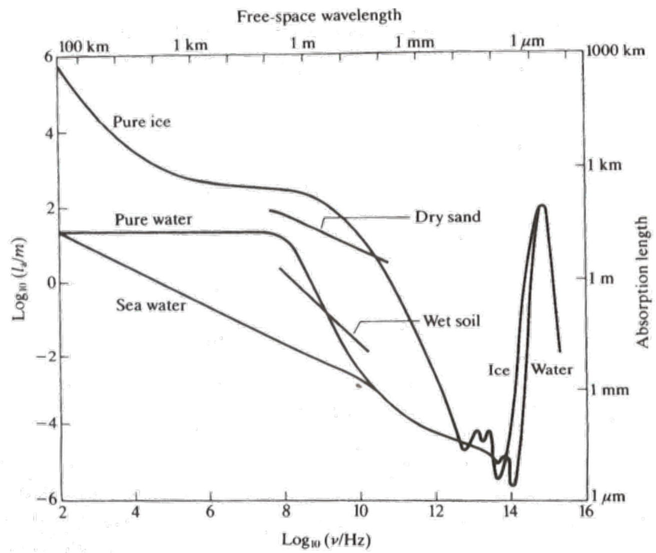
Absorption Lengths



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Dielectric Constant of a Gas

- For a gas, under the assumption that there is no strong absorption, the dielectric constant can be written as

$$\varepsilon_r \approx 1 + \frac{N\alpha}{\varepsilon_0}$$

where N is the density of gas molecules (number of molecules per unit volume) and α is the polarizability of the molecules

- We can compute the refractive index by using the approximation

$$\frac{N\alpha}{\varepsilon_0} \ll 1 \qquad \Longrightarrow \qquad n \approx 1 + \frac{N\alpha}{2\varepsilon_0}$$

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Dielectric Constant of a Gas

- The term α/ε_0 has the dimensions of a volume and it is normally an indication of the actual size of the gas molecules
- Some values

The values are given in units of 10^{-30} m^3

Gas	Optical	Radio
Air	21.7	21.4
Carbon dioxide	33.6	36.8
Hydrogen	9.8	10.1
Oxygen	20.2	19.8
Water vapour	18.9	368

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Solids and Insulating Liquids

- Non polar materials usually have a constant dielectric constant (!), possibly complex
- For polar materials, instead, it is possible to model the phenomena related to resonance and to write the dielectric constant by means of Debye's equations

$$\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_p}{1 + \omega^2\tau^2}$$
$$\varepsilon'' = \frac{\omega\tau\varepsilon_p}{1 + \omega^2\tau^2}$$

where

τ is the relaxation time
ε_∞ is the asymptotic constant at high frequency
ε_p is the polar contribution to the dielectric constant

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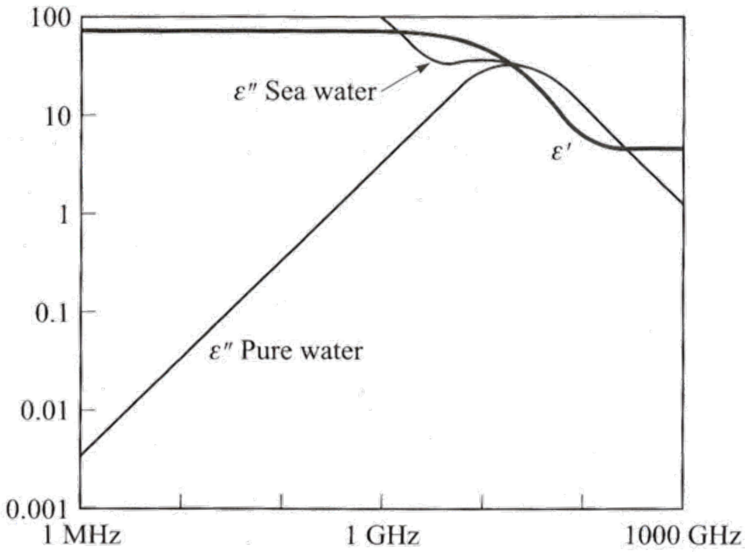
Example: water



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Dielectric Constant of Metals

- The electrical properties of metals are determined prominently by the very high density of delocalized electrons

$$\varepsilon' = 1 - \frac{\sigma\tau}{\varepsilon_0(1+\omega^2\tau^2)}$$
$$\varepsilon'' = \frac{\sigma}{\varepsilon_0\omega(1+\omega^2\tau^2)}$$

where τ is the so called relaxation time

$$\tau = \frac{m_e\sigma}{Ne^2} \qquad \text{with} \qquad \begin{array}{rl} m_e & \text{mass of the electron} \\ e & \text{charge of the electron} \end{array}$$

- For metals, the relaxation time has values of the order of $10^{-15} - 10^{-14}$,
- The behavior can be studied by considering the expressions obtained in the two regions $\omega \gg 1/\tau$ and $\omega \ll 1/\tau$

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Dielectric Constant of Metals

- At low frequecies (radio) we can use the approximation

$$\varepsilon_r \approx -j\frac{\sigma}{\varepsilon_0\omega}$$

that leads to a refractive index n with

$$m = \kappa = \sqrt{\frac{\sigma}{2\varepsilon_0\omega}}$$

and thus to an absorption length

$$l_a = c\sqrt{\frac{\sigma}{2\varepsilon_0\omega}} \quad \left(= \frac{\lambda}{4\pi}\right)$$

- Example: at 5GHz, stainless steel ($\sigma = 1.0 \times 10^6\Omega^{-1}\mathrm{m}^{-1}$) has an absorption length of 3.6μm

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Dielectric Constant of Metals

- At high frequencies ($\gg$optical)

$$\varepsilon_r \approx 1 - \frac{Ne^2}{\varepsilon_0 m_e\omega^2}$$

- Thus, at sufficiently high frequencies the dielectric constant is real and almost equal to one.
- That is, at high frequencies metals are almost transparent
- Alert: the phase velocity can be larger than c, but the group velocity is smaller than c
  - Experiments show that this is not true in other materials in general
  - It seems however that even in those cases still *information* never goes faster than c

Dielectric Constant in Plasma

- Plasma is a state of matter where ionized atoms and electrons can move independently
- The behavior of a plasma is essentially determined by the electrons since they are much lighter than ions
- Hence, for this state of matter the dielectric constant is still as in the previous case

$$\varepsilon_r \approx 1 - \frac{Ne^2}{\varepsilon_0 m_e \omega^2}$$

- We thus have almost complete transparency to high frequencies, those larger than the so called *plasma frequency*

$$\omega_p = \sqrt{\frac{Ne^2}{\varepsilon_0 m_e}}$$

Phase and Group Velocity

- As anticipated, phase and group velocities are not equal in general, and they can be computed from the dispersion relation $\omega = \omega(k)$ as

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}$$

- Often, it is more comfortable to use the law $n = n(\lambda_0)$ from which

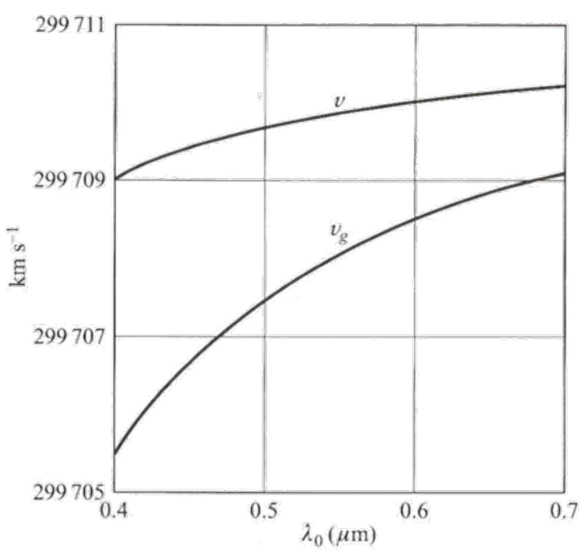
$$\frac{c}{v_g} = n - \lambda_0 \frac{dn}{d\lambda_0}$$

- Example: in air at 15° we have approximately

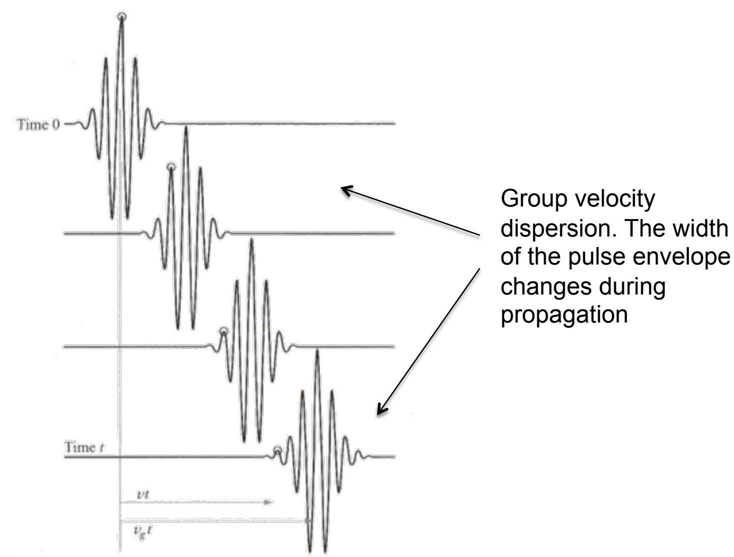
$$n = 1 + \frac{1}{a + b/\lambda_0^2}$$

with $a = 3669$, $b = 12.1173 \times 10^{-11}\mathrm{m}^2$

Phase and Group Velocities



Dispersion of the Group Velocity



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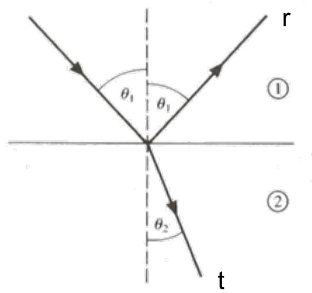
Plane Boundaries

- A plane wave incident on a plane discontinuity between two homogeneous isotropic media is partially reflected and partially transmitted



- The transmission follows the Snell's law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

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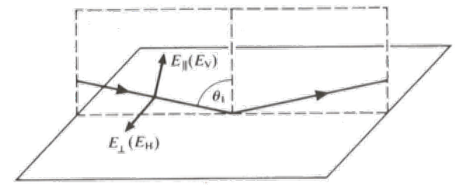
Plane Boundaries

- It is important to study the effect of discontinuities on polarization
- We can decompose the incident ray in two components
  - A component $\parallel$ with parallel polarization (to the plane that contains the rays)
  - A component $\perp$ with orthogonal polarization



- If the media are homogeneous and isotropic each component only generates a component of the same kind

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Plane Boundaries

- Reflection and Transmission coefficients are obtained by solving the Maxwell equations at the surface (Z is the impedance of the medium)

$$r_\perp = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$
$$t_\perp = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_1 +_1\cos\theta_2}$$

$$r_\parallel = \frac{Z_2\cos\theta_2 - Z_1\cos\theta_1}{Z_2\cos\theta_2 + Z_1\cos\theta_1}$$
$$t_\parallel = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_2 + Z_1\cos\theta_1}$$

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Plane Boundaries

- The expansion of those relations in the general case leads to very complicated expressions (for absorbing media)
- In many cases of practical interest the medium 1 is the vacuum or air and we can use $n_1 = 1$ with good approximation
- Under this assumption, we can use *Fresnel's expressions*

$$r_\perp = \frac{\cos\theta_1 - \sqrt{\varepsilon_{r2} - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{\varepsilon_{r2} - \sin^2\theta_1}} \qquad r_\parallel = \frac{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} - \varepsilon_{r2}\cos\theta_1}{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} + \varepsilon_{r2}\cos\theta_1}$$

$$t_\perp = \frac{2\cos\theta_1}{\cos\theta_1 + \sqrt{\varepsilon_{r2} - \sin^2\theta_1}} \qquad t_\parallel = \frac{2\sqrt{\varepsilon_{r2}}\cos\theta_1}{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} + \varepsilon_{r2}\cos\theta_1}$$

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Plane Boundaries

- If medium 2 is non-absorbing

$$r_\perp = \frac{\cos\theta_1 - \sqrt{n_2^2 - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{n_2^2 - \sin^2\theta_1}}$$

$$r_\parallel = \frac{\sqrt{n_2^2 - \sin^2\theta_1} - n_2^2\cos\theta_1}{\sqrt{n_2^2 - \sin^2\theta_1} + n_2^2\cos\theta_1}$$

- There is no reflection in the component with parallel polarization when $\theta_1 = \theta_B$, where θ_B is the Brewster angle, which satisfies

$$\tan\theta_B = n_2$$

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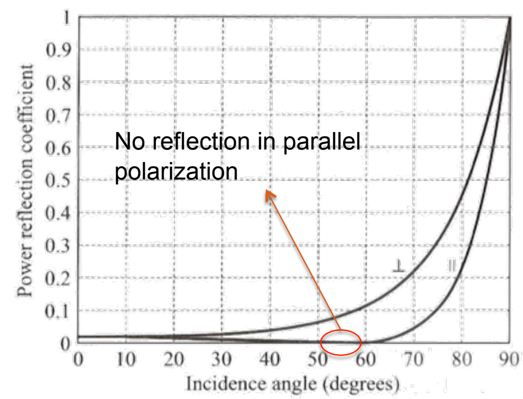
Plane Boundaries

- Example: air-water boundary. We can assume $n_1 = 1$, $n_2 = 1.333$

$$\theta_B = \arctan(1.333) \approx 53.12$$

in fact



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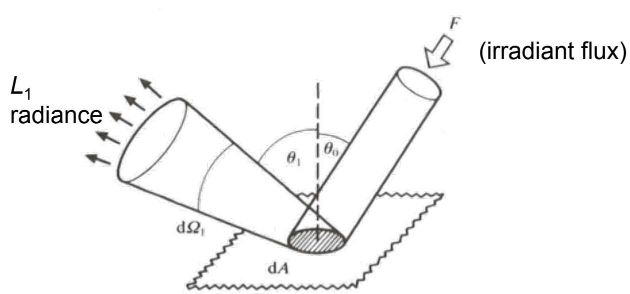
Surface Scattering

- A wave that hits a surface which is not perfectly smooth is not reflected in one direction only but in many different directions with different intensities



- The irradiance E is given by $E = F\cos\theta_0$ and we define the bidirectional reflectance distribution function (BRDF)

$$R = L_1/E$$

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Surface Scattering

- For a radar, we are usually only interested in the *bistatic scattering* coefficient

$$\gamma = 4\pi R\cos\theta_1$$

- Actually, for the radars that we will consider only the backscattering component will be important. In this case $\theta_0 = \theta_1$ and it is useful to define the backscatter coefficient

$$\sigma^0 = \gamma\cos\theta_0 = 4\pi R\cos^2\theta_0$$

- Sometimes it is useful to consider the total radiant flux M in all directions by integrating R over all angles (*albedo*)

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Surface Scattering

- Main surface models

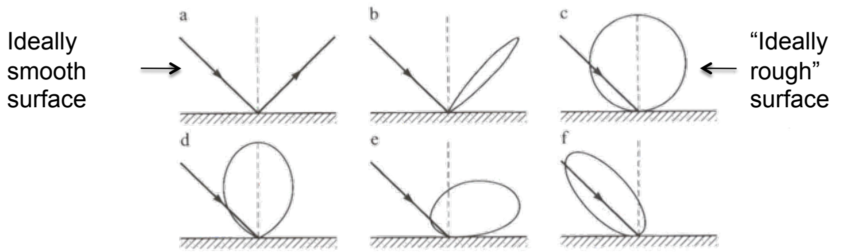

Figure 3.9. Schematic illustration of different types of surface scattering. The lobes are *polar diagrams* of the scattered radiation: the length of a line joining the point where the radiation is incident on the surface to the lobe is proportional to the radiance scattered in the direction of the line. (a) Specular reflection; (b) quasi-specular scattering; (c) Lambertian scattering; (d) Minnaert model ($\kappa = 2$); (e) Henyey–Greenstein model of forward scatter ($\Theta = 0.7$); (f) Henyey–Greenstein model of backscatter ($\Theta = -0.5$).

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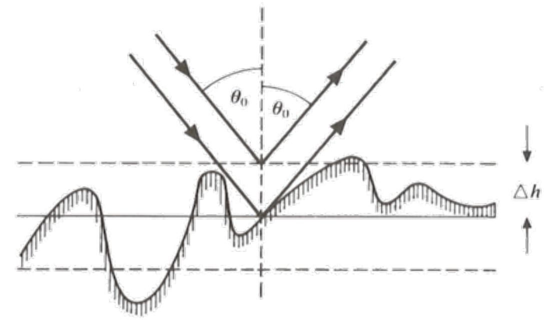
Reflective Surfaces

- We now consider the scattering from a sufficiently smooth surface using the Rayleigh model
- We study the effects of the asperities in the direction of main reflection, that is the specular one to the incident ray



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Reflective Surfaces

- Consider a reference plane and assume asperities are within a range Δh from this plane
- The path difference between a ray reflected on the reference plane and one reflected by an asperity well be nearly $2\Delta h \cos\theta_0$ and the phase difference will then be

$$\Delta\phi = \frac{4\pi\Delta h \cos\theta_0}{\lambda}$$

- If we want the two rays to be in phase we will need to have a small phase difference, say less than $\pi/2$, and thus

$$\Delta h = \frac{\lambda}{2\cos\theta_0}$$

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Reflective Surfaces

- We thus see that the surface will appear more or less reflective depending on the incident angle (as easily observe ordinarily)
- We deduce that for rays incident in the orthogonal direction we have reflection if asperities are in the order of $\lambda/8$ (indicatively).
- Examples
  - At optical frequencies, a surface reflects if $\Delta h \approx 60$nm, that is, in practice only artificially treated or liquid surfaces
  - At radio frequencies (VHF) it suffices to have $\Delta h \approx 40$nm, that is, many surfaces are perfectly reflective (sand, grass etc.)

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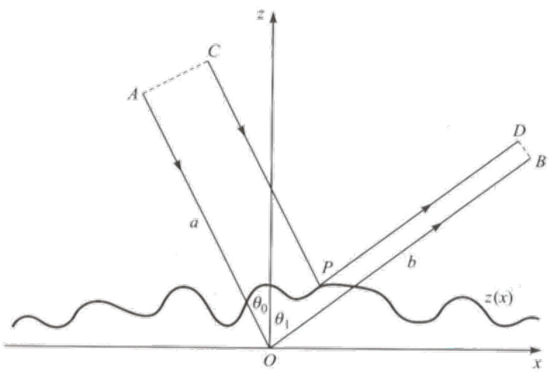
Surface Scattering

- We can study the scattering in a general direction using the model of *small perturbations*
- Essentially, the same model used for the Fraunhofer diffraction integral in the far field



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Surface Scattering

- Consider, as it was done for the case of diffraction, a reference point O, and let us express the contribution of a generic point P by computing the phase difference with respect to the ray though O
- We compute the path difference: with some basic trigonometry

$$\Delta l = x(\sin\theta_0 - \sin\theta_1) - z(x)(\cos\theta_0 + \cos\theta_1)$$

that we can rewrite in terms of phase difference as

$$\Delta\phi = k\alpha x - k\beta z(x)$$

where

$$\alpha = (\sin\theta_0 - \sin\theta_1) \qquad \beta = (\cos\theta_0 + \cos\theta_1)$$

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Surface Scattering

- We obtain the total (complex) amplitude by integrating corresponding exponential function over all P

$$E = \int_{-\infty}^{\infty} e^{-j\phi(x)}dx = \int_{-\infty}^{\infty} e^{-jk\alpha x}e^{jk\beta z(x)}dx$$

- This is the Fourier Transform of the function $e^{jk\beta z(x)}$
- For small $z(x)$ values, we can expand the function in Taylor series to obtain

$$e^{jk\beta z(x)} = 1 + jk\beta z(x) - \frac{(k\beta z(x))^2}{2} + \cdots$$

- Plugging this into the integral we obtain different contributions

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Surface Scattering

- First Term

$$\int_{-\infty}^{\infty} e^{-jk\alpha x}dx \propto \delta(k\alpha)$$

which, by considering that $\alpha = (\sin\theta_0 - \sin\theta_1)$, expresses the fact that in the direction $\theta_1 = \theta_0$ we have the component associated to the ideal reflection

- Second term

$$\int_{-\infty}^{\infty} jk\beta z(x)e^{-jk\alpha x}dx$$

which is proportional to the Fourier Transform of the function $z(x)$

- In other words, the amplitude in the direction α is given by the component of $z(x)$ with spatial frequency $k\alpha$

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Surface Scattering

- If the higher order terms can be neglected, we usually speak of *Bragg scattering*
- The approximation makes sense if

$$k\beta\Delta h \ll 1$$

  that is,

$$\Delta h \ll \frac{\lambda}{2\pi(\cos\theta_0 + \cos\theta_1)}$$

  which implies that the Rayleigh criterion is satisfied
- This model is for example extremely accurate to describe the effect of water wavelets (of the order of 1 cm) on the scattering of microwaves

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Surface Scattering

- The model described with a deterministic $z(x)$ cannot be applied in many cases because there is only a statistical description of $z(x)$...in two dimensions
- We have to describe the behavior of rays and compute a statistical average
- Here the correlation function of $z(x)$ becomes important and it is typically assumed that it is a gaussian function

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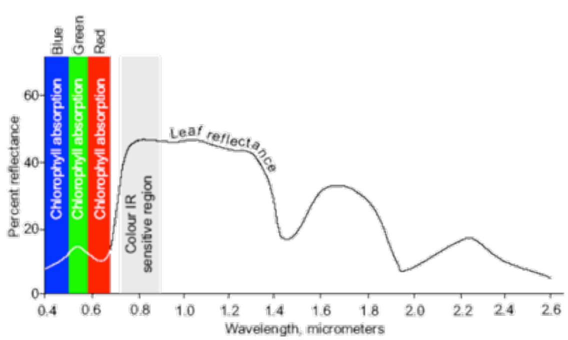
Reflectance Curves

- Different land cover types have different spectral reflectance and this can be used in order to identify the land cover of interest
- For vegetation:



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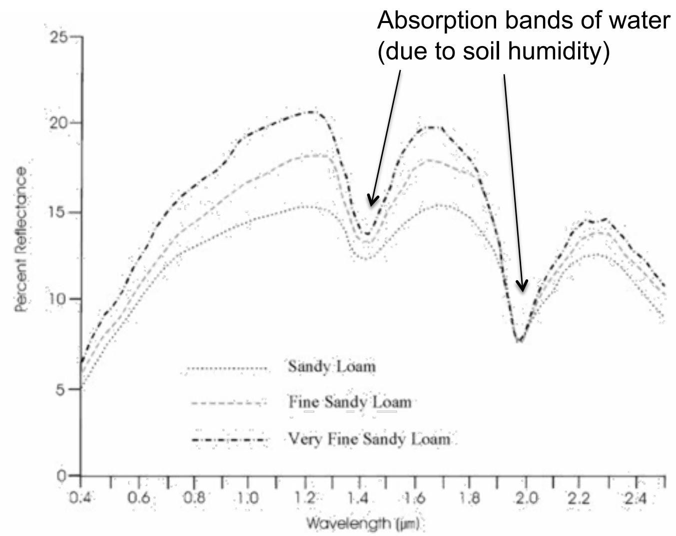
Reflectance Curves

- Soil



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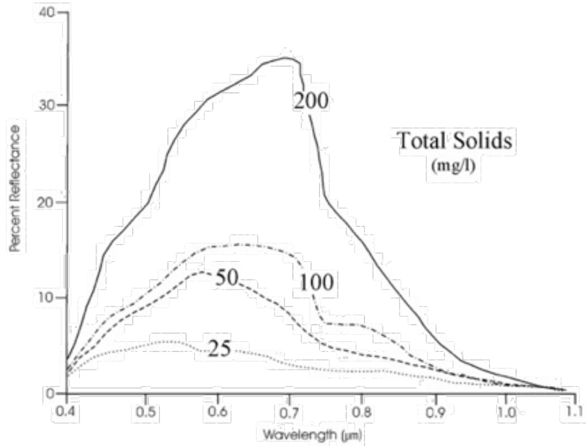
Reflectance Curves

- Clear Water has a very small reflectance
  - Higher values for turbid water
  - Practically no reflectance above 1200nm



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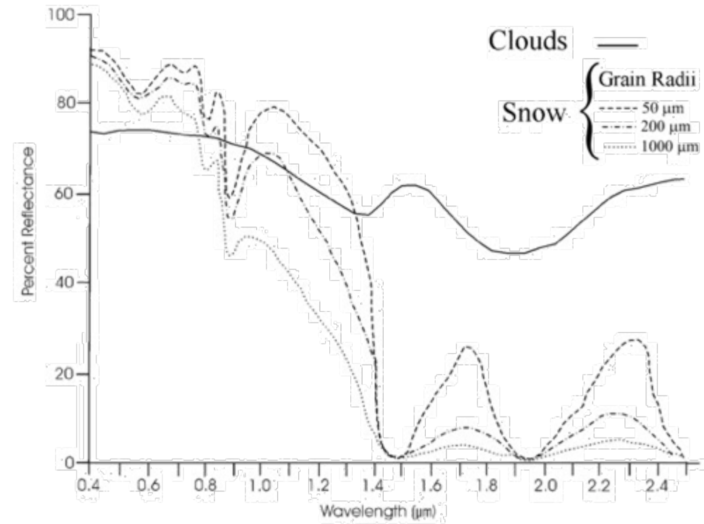
Reflectance Curves

- Clouds and Snow



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Atmosphere of the Earth

Gas	Chemical formula	Volume fraction	Total mass kg m^{-2}
Nitrogen	N_2	0.7808	7797
Oxygen	O_2	0.2095	2389
Argon	Ar	9.34×10^{-3}	133
Carbon dioxide	CO_2	3.5×10^{-4}	5.6
Neon	Ne	1.8×10^{-5}	0.13
Helium	He	5.2×10^{-6}	7.5×10^{-3}
Methane	CH_4	1.8×10^{-6}	1.0×10^{-2}
Krypton	Kr	1.1×10^{-6}	3.4×10^{-2}
Carbon monoxide	CO	$0.06 - 1 \times 10^{-6}$	$0.06 - 1 \times 10^{-2}$
Sulphur dioxide	SO_2	1.0×10^{-6}	2.9×10^{-2}
Hydrogen	H_2	5.0×10^{-7}	4.0×10^{-4}
Ozone	O_3	$0.01 - 1 \times 10^{-6}$	5.4×10^{-3}
Nitrous oxide	N_2O	2.7×10^{-7}	4.0×10^{-3}
Xenon	Xe	9.0×10^{-8}	4.0×10^{-3}
Nitric oxide	NO_2	$0.05 - 2 \times 10^{-8}$	$0.02 - 4 \times 10^{-4}$
Total dry atmosphere		1	1.032×10^{4}
Water vapour	H_2O	0.001 – 0.028	6.5 – 180

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Atmosphere of the Earth

- Layers
  - Troposphere (0-11Km)
  - Stratosphere (11-50 Km)
  - Mesosphere (50-80Km)
  - Thermosphere (above 80 Km)
- The region in the range 60-450 Km is called Ionosphere (partial overlap between mesosphere and thermosphere): composed of highly ionized particles.
- The temperature can achieve 1500 degrees, but thermal exchange is very low

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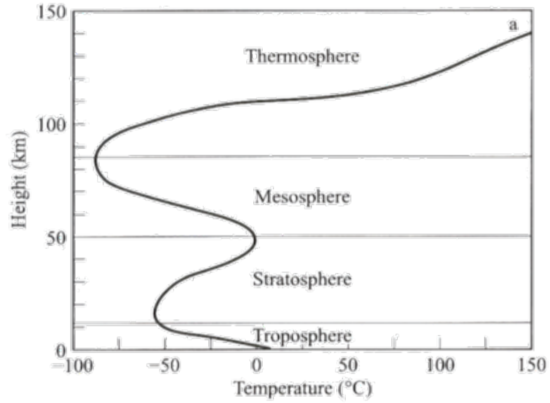
Reflectance Curves

- The temperature has different trends depending on the altitude



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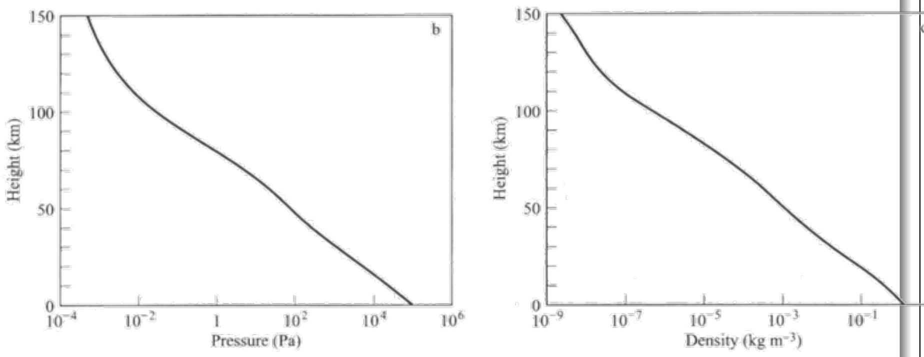
Reflectance Curves

- Density and pressure are instead monotonically decreasing as a function of the altitude



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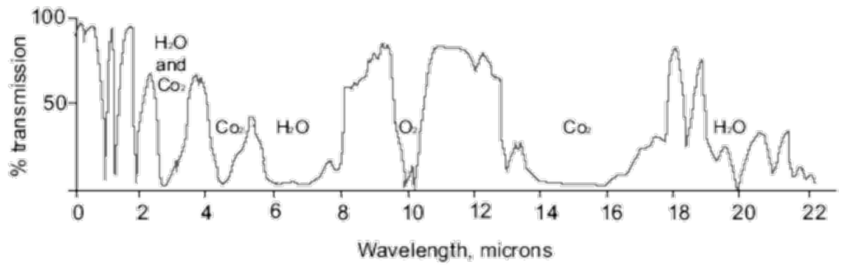
Reflectance Curves

- Electromagnetic waves in the atmosphere are affected by absorption and scattering
- The absorption determines the windows that can be used for remote sensing



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Propagation in the Atmosphere

- As already seen previously, we essentially have the following windows
  - Visible and near infrared (0,4-2μm)
  - Three windows in the thermal infrared, one around 3 μm, one near 5μm and one between 8 and 14μm
  - Microwave window (above 1 mm)

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Atmospheric Scattering

- The gas particles in the atmosphere also cause scattering of the electromagnetic waves. The factors that determines the phenomenon are based on the relation between the wavelength and the particle dimensions, on the quantity of gas particles etc...
- The main particles that contribute to the scattering are the following
  - Gas molecules (0.01 μm)
  - Aerosol (0.1-1 μm)
  - Water particles in the clouds (1-10μm)
  - Suspended ice crystals (1-100μm)
  - Gradine (up to 10 cm...!)

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Atmospheric Scattering

- The ratio between the particle dimension and the wavelength is the critical parameter
- If we define

$$\alpha = \frac{2\pi r}{\lambda}$$

  we have the following cases
  - $\alpha < 0.001$ negligible scattering;
  - $0.001 < \alpha < 0.1$ Rayleigh scattering;
  - $0.1 < \alpha < 50$ Mie scattering
  - $\alpha > 50$ optical geometry

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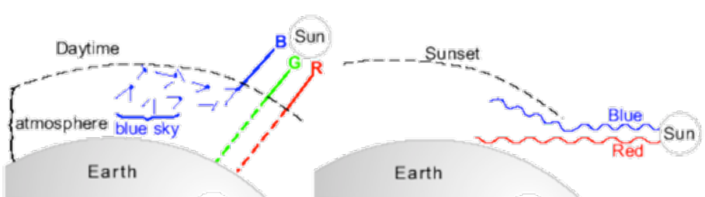
Atmospheric Scattering

- Rayleigh scattering
  - The scattering effect is inversely proportional to the fourth power of the wavelength (shorter wavelength are scattered more)

  - This is why the sky is blue during daytime and the sunset is red



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Atmospheric Scattering

- Mie scattering
  - Mainly caused by aerosol: a mix of gas, water vapor, dust. It is prominent in the lower layers of the atmosphere where dust is more heavily concentrated.
  - It is the dominant scattering in presence of clouds
  - It affects all the spectrum from ultraviolet to infrared