Optical Communication Systems-2013/2014

Simulation Lab on: **Beam Propagation Method**

Consider the nonlinear Schrödinger equation (NLSE) for pulse propagation in optical fibers:

$$j\frac{\partial A}{\partial Z}-\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2}+j\frac{\alpha}{2}A+\gamma\left|A\right|^2 A=0$$

where α is the power loss coefficient.

1. Write a BPM code to solve the NLSE. Suppose at first that the loss α as well as the nonlinearity γ are zero, and set the group-velocity dispersion (GVD) β_2= -5 ps^2/km (at the wavelength λ=1.55 μm).

2. Whenever at the fiber input is present a chirped Gaussian pulse with amplitude

$$A(T)=A_0\exp\left(-\frac{1+iC}{2}\left(\frac{T}{T_0}\right)^2\right)$$

   where C is the chirp parameter, then the absolute value squared of the output pulse after propagation over a fiber length *L* is still of Gaussian shape as given above, but with A_0 and T_0 replaced by the expressions:

$$A_0(L)=A_0/R(L);\;\; T_0(L)=T_0R(L);\;\; R(L)=\sqrt{1+(C-i)\beta_2L/T_0^2}$$

   Obtain the explicit evolution equations for the chirp $C_1(L)$ and for the time duration $T_1(L)$ of the Gaussian pulse. Verify the correctness of the BPM code by comparing T_1 as obtained from the above formula, with the result that is obtained after propagating a Gaussian pulse with C=0, C=+2 and C=-2, over the distances L/4, L/2, 3L/4 and L, where L is equal to the dispersion distance, i.e., L= $L_D=T_0^2/|\beta_2|$ (with T_0=12.5 ps and β_2= -5 ps^2/km). Plot the numerical values of the time duration T_1 as a function of distance for the three different values of C. Discuss the results that are obtained: what is the effect of the initial chirp C on dispersive pulse broadening in the fiber? Repeat the above simulations with the GVD β_2=+5 ps^2/km: what is the effect of changing the sign of β_2?

3. Set now the dispersion to zero, and the attenuation to 0.2 dB/km: verify by simulation that, after propagation over 40 km, the pulse peak power has indeed decreased by 8 dB.

4. Let us introduce now in the BPM code the Kerr nonlinearity (with γ=2 W^{-1} km^{-1}). Neglecting at first the effect of GVD (β_2=0), observe the phase profile (for peak powers of the order of a few hundreds of mW), and qualitatively compare it with the phase profiles that were obtained under the action of GVD only. Observe the behaviour of the absolute value of the pulse spectrum as a function of the propagation distance for different input power values. Verify that the spectrum exhibits a number of peaks *M*, which may be approximately determined by the expression:

$$\gamma PL \approx (M - 0.5)\pi$$

where P in the peak power of the input Gaussian pulse, and L is the propagation distance.

5. Verify that the input pulse which is described by the following expression:

$$A(T) = N\sqrt{\frac{|\beta_2|}{\gamma T_0^2}}\,\mathrm{sech}\left(\frac{T}{T_0}\right)$$

propagates unchanged along the fiber whenever N=1 and β_2 is negative (set for example β_2=-15 ps^2/km, γ=2 W^{-1} km^{-1}). A pulse which propagates unchanged in the presence of both dispersion and nonlinearity is called "soliton". Next, verify the accuracy of your numerical solver by setting N=3: a periodic evolution of the pulse with distance Z should be observed. What is the spatial period? If the pulse does not reproduce itself periodically, decrease your spatial step size h. Observe what happens when the pulse amplitude is slightly increased or decreased (for example, by 10%) with respect to the soliton value. How is the pulse propagation modified the sign of β_2 is changed (i.e., β_2 is positive) ? Finally, propagate a Gaussian pulse of same peak power and time width equal to that of the soliton, and describe its evolution. Note: the full width at half maximum (of the intensity) of the soliton is $T^S_{fwhm}=1.763T_0$, whereas for the Gaussian pulse $T^G_{fwhm}=2(\ln 2)^{1/2}T_0$.

6. Verify that, when launching at the fiber input two successive pulses equal to the soliton of (8), but with their center position shifted in time by KT_0 (set for example K=5), one obtains a periodic evolution of the two pulses along the fiber: describe the observed behaviour. From the simulations, estimate the value of the period. What happens when N is varied, for example set K=8? And when the two input pulses are phase shifted by π (or the two amplitudes have opposite signs)? What are the practical implications of the above results for the transmission of a stream of consecutive soliton pulses?

7. Numerically propagate an initially chirp-free, +6 dBm peak power Gaussian pulse (i.e., with C=0), with T_0=12.5 ps, in a lossless but nonlinear and dispersive fiber span (γ=2 W^{-1} km^{-1}, β_2=-20 ps^2/km) of length equal to 100 km. Next, propagate the same pulse in a link composed by 4 fiber spans, of length equal to 25 each, with the same absolute value of GVD as before, but where the sign of dispersion alternates from one span to the next. Observe the evolution of the pulse shape and of its spectrum, and compare the pulse profile after a total of 100 km with the input pulse. Finally, simulate the pulse propagation through a link with total length of 100 km, composed of a first 90 km span with β_2=-20 ps^2/km and a second 10 km span whose GVD is chosen so that the average dispersion (or the accumulated dispersion) over the whole link is equal to zero. Discuss the obtained results.

8. Let us consider a nonlinear fiber span with negligible loss, with γ=2 W^{-1} km^{-1} and β_2= -10 ps^2/km. Inject at the fiber input a +3 dBm peak power chirp-free Gaussian pulse with T_0=12.5 ps, along with two additional pulses with the same time position, amplitude and duration, but one shifted in frequency by Δf=+100 GHz, and the other shifted by Δf=-100 GHz. Observe, as a function of the propagation distance z, the evolution of the total field and of its spectrum. Repeat the simulation with β_2= -1 ps^2/km. What changes in the second case?