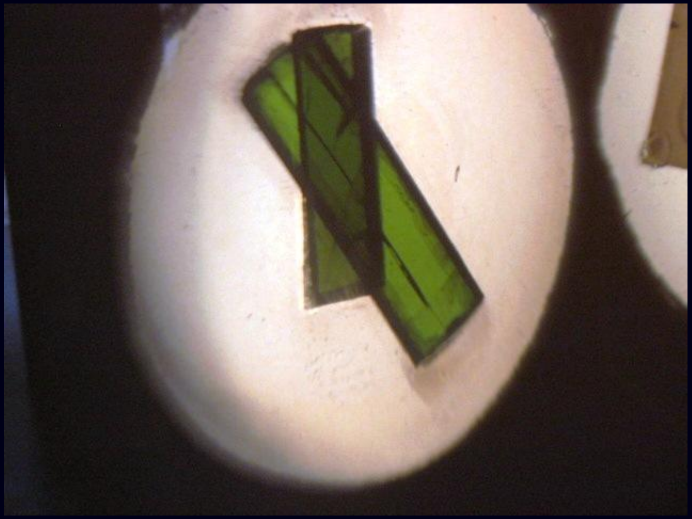
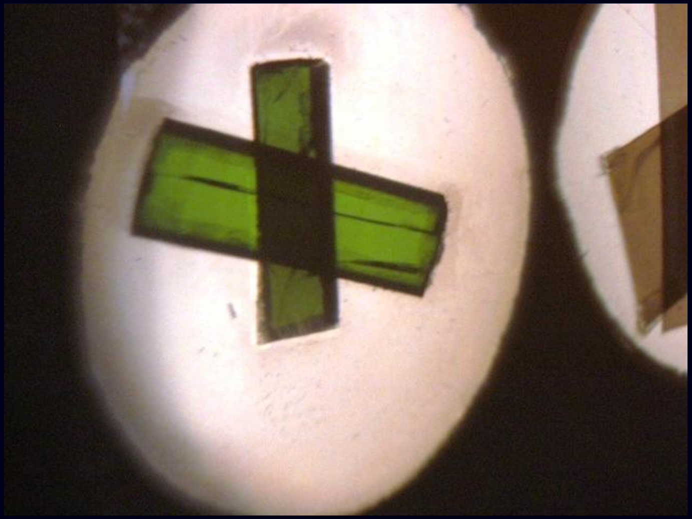
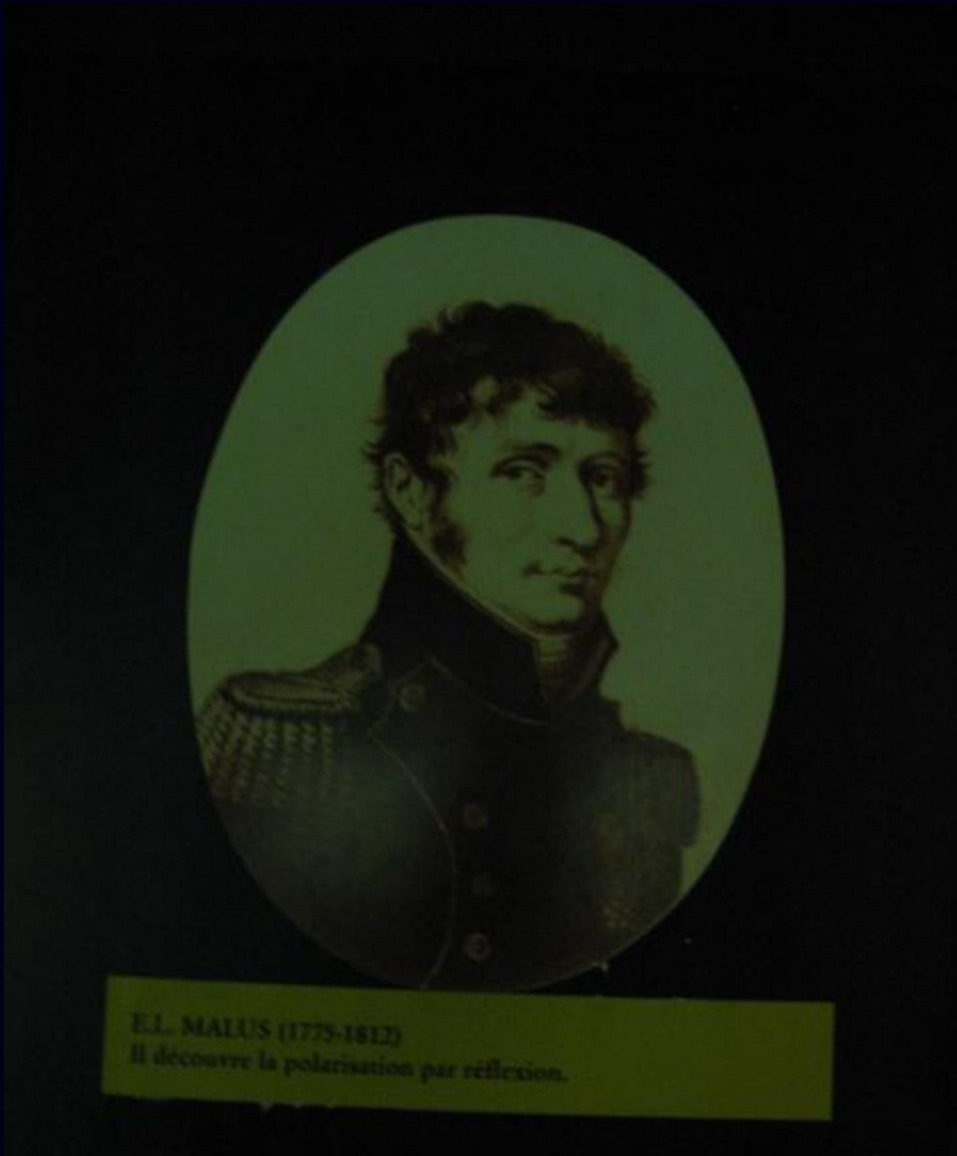
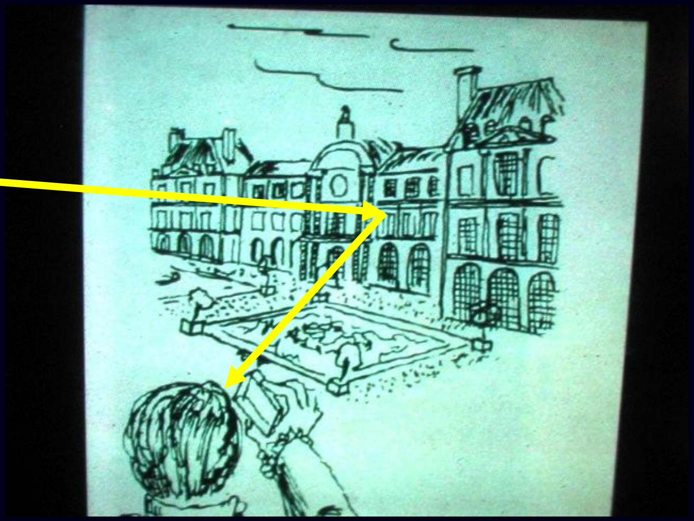
Polarization of light PART 1

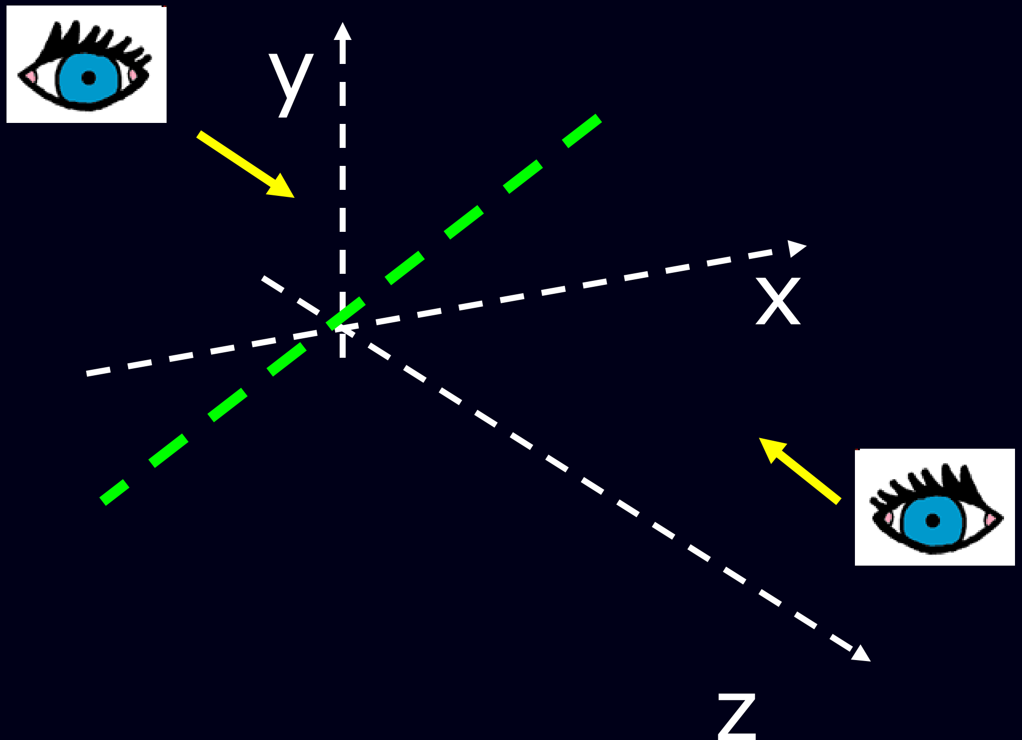
Malus (1808): the discovery of polarization by reflection



Malus observation of sunset reflected by the Windows of the palais du Luxebourg



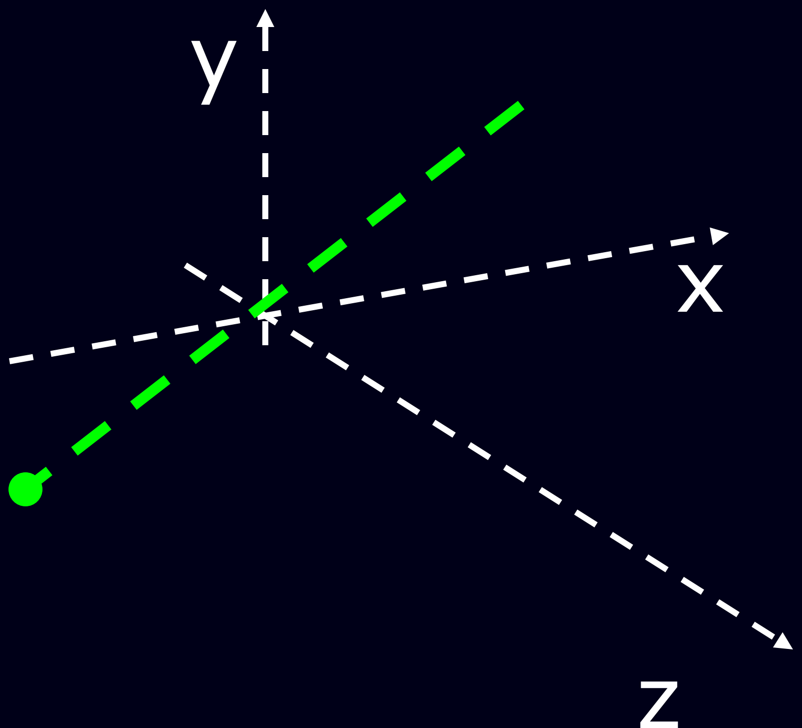
Polarization of light



Two possible descriptions:

- from the perspective of the source
- from the perspective of the receiver

Polarization of light



$$E_x = s_x E(t) e^{j(\omega_0 t - \beta_0 z)}$$

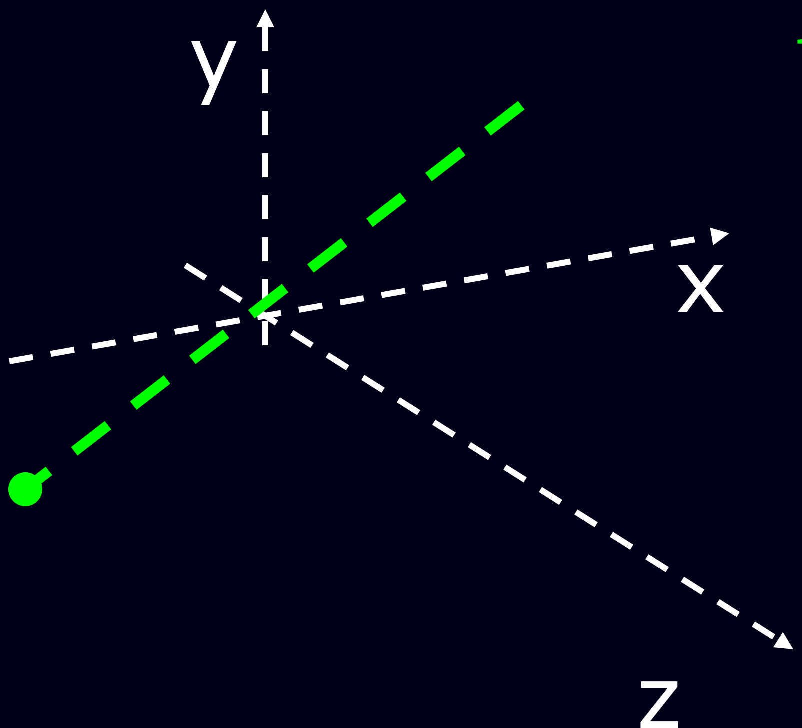
$$E_y = s_y E(t) e^{j(\omega_0 t - \beta_0 z)}$$

Propagation constant

Angular carrier frequency

Complex electric field amplitude

Polarization of light

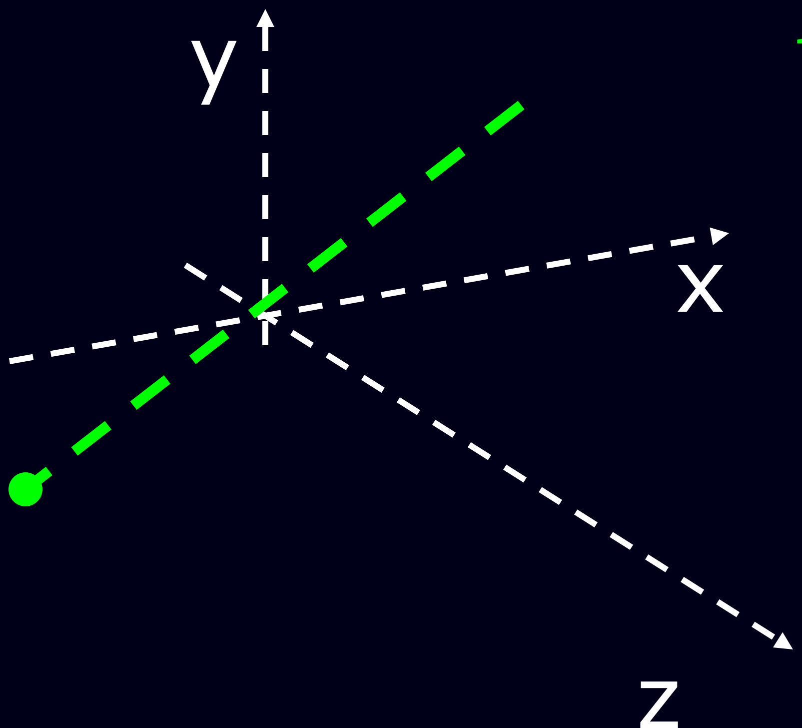


$$\overline{\mathbf{E}} = E(t)e^{j(\omega_0 t - \beta_0 z)} \,|\, s >$$

$$|\, s >= \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

2D complex (column) Jones ket vector

Polarization of light



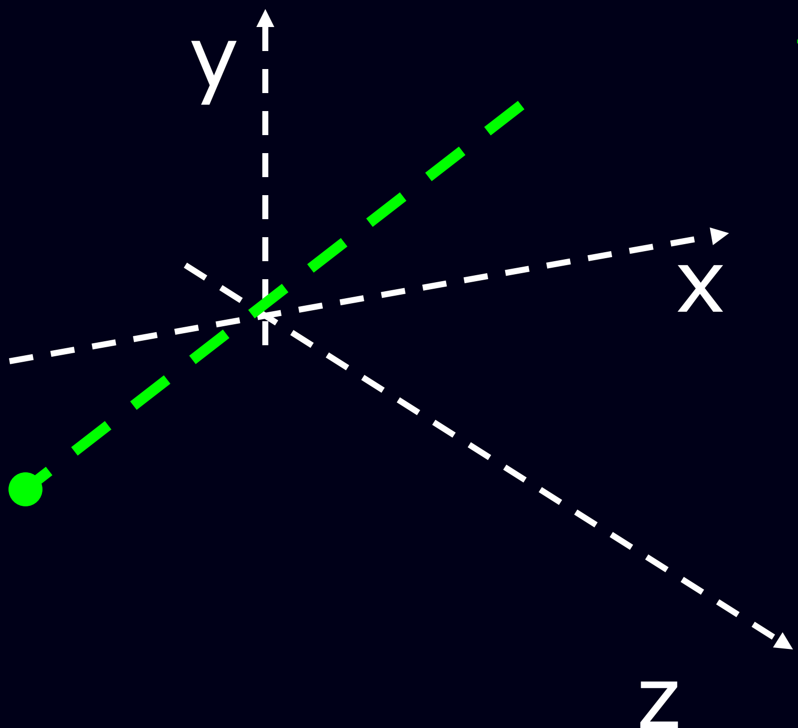
$$\overline{\mathbf{E}} = E(t)e^{j(\omega_0 t - \beta_0 z)}\,|\,s>$$

$$|\,s> = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$<s\,| = \left(s_x^*, s_y^*\right)$$

Corresponding complex conjugate (row) bra vector

Polarization of light



$$\overline{\mathbf{E}} = E(t)e^{j(\omega_0 t - \beta_0 z)} \,|\, s >$$

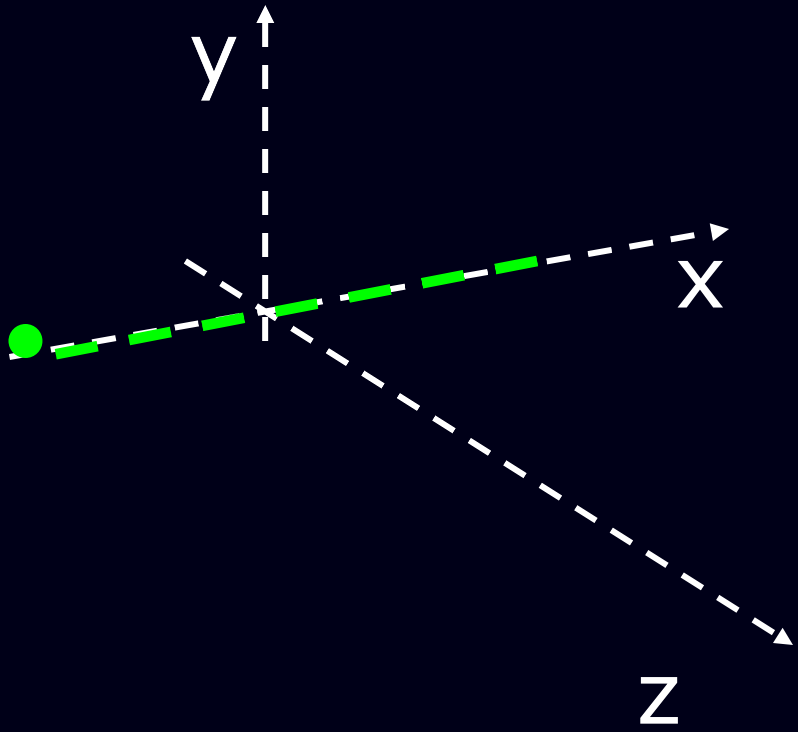
$$|\, s >= \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$< s \,| = \left(s_x^*, s_y^*\right)$$

We normalise |s> such that

$$< s \,|\, s >= \left(s_x^*, s_y^*\right) \begin{pmatrix} s_x \\ s_y \end{pmatrix} = s_x^* s_x + s_y^* s_y = 1$$

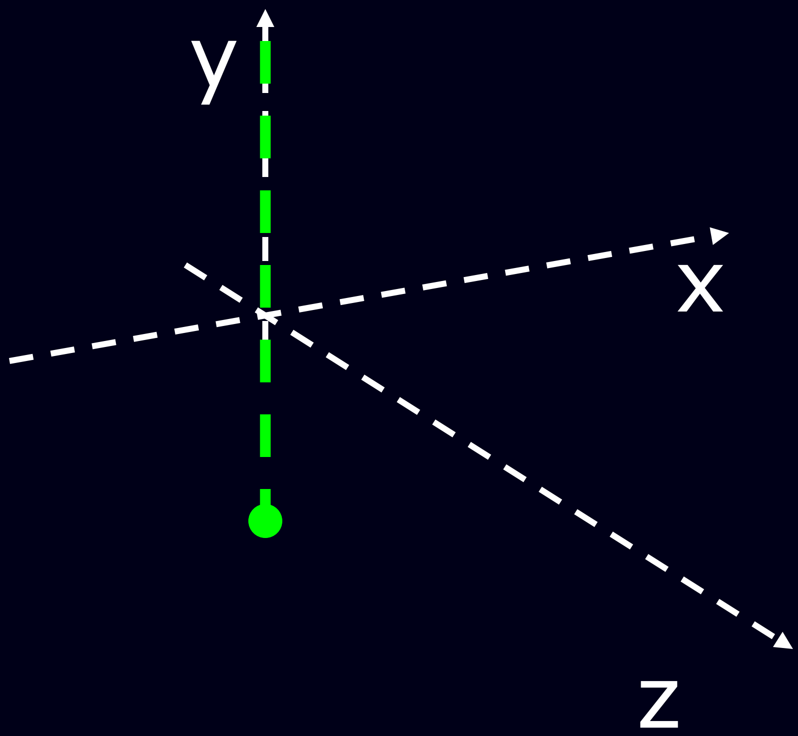
Examples of Jones vectors



$$|s>_H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Linear Polarization
Horizontal

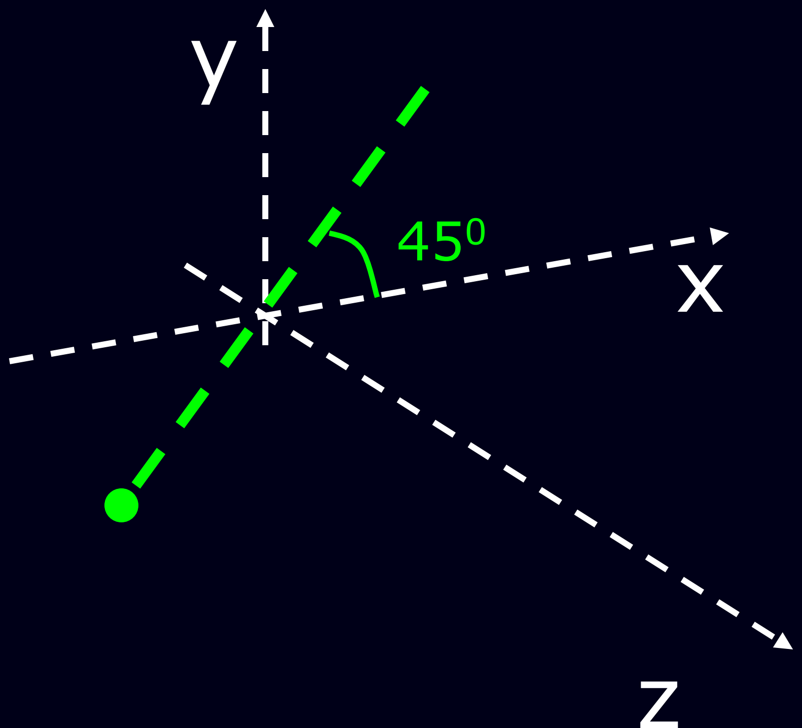
Examples of Jones vectors



$$|s>_V = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Linear Polarization
Vertical

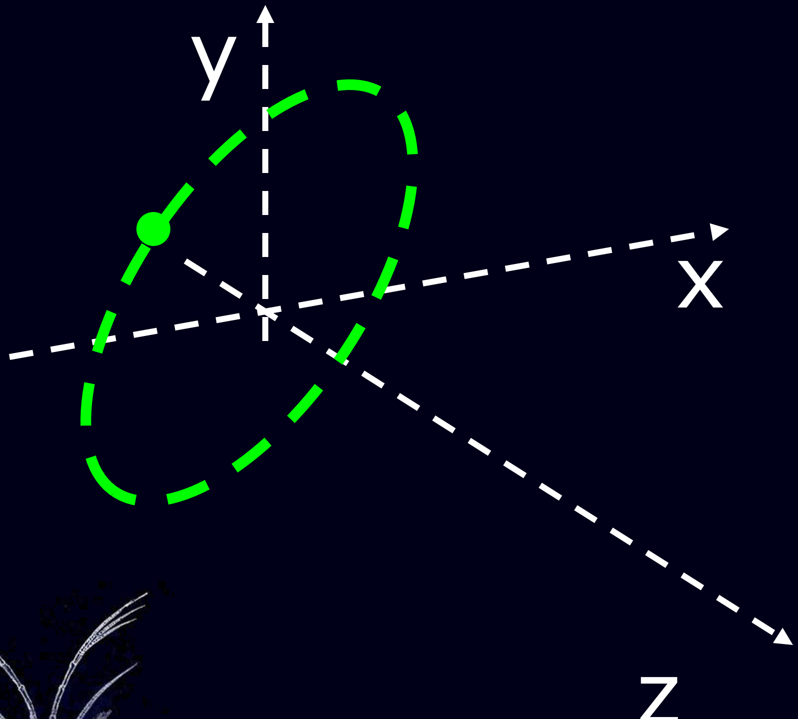
Examples of Jones vectors



$$|s>_P = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

Linear Polarization
Azimuth 45^0

Examples of Jones vectors

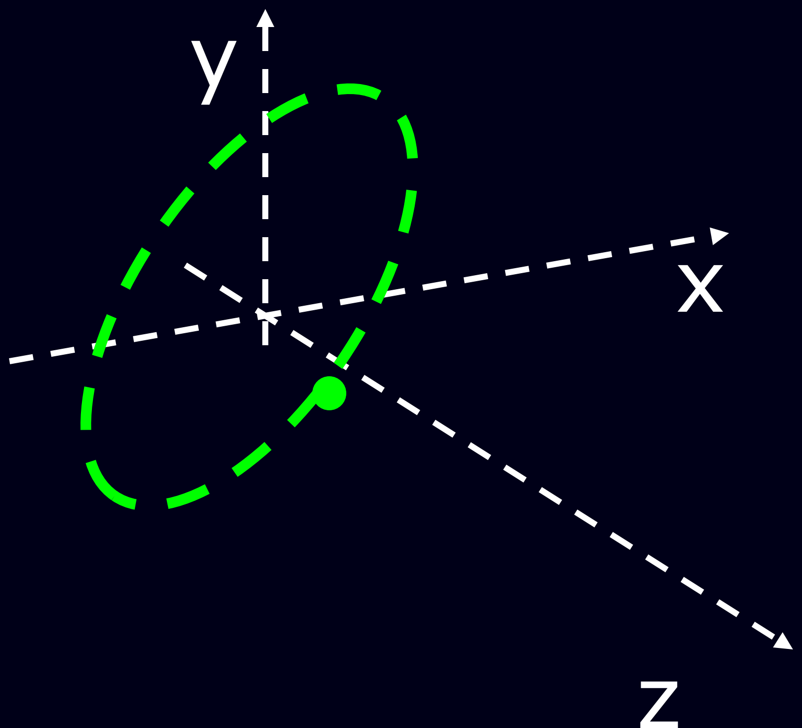


$$|s>_R = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\end{pmatrix}$$

Right Circular polarization

An animal capable to distinguish circular polarization: the mantis shrimp

Examples of Jones vectors



$$|s>_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix}$$

Left Circular polarization

Jones vectors are complex vectors on a vector space

$$| s >= \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$< s | s >= \left(s_x^*, s_y^* \right) \begin{pmatrix} s_x \\ s_y \end{pmatrix} = s_x^* s_x + s_y^* s_y = 1$$

- Intuitive understanding of changes in polarization (lab-space coordinates)
- Jones vectors are of unit magnitude as we assume, for the moment, coherent light
- Optical phase is included in complex Jones vectors
-

Jones vectors are complex vectors on a vector space

$$| s > = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$< s | s > = \left(s_x^*, s_y^*\right)\begin{pmatrix} s_x \\ s_y \end{pmatrix} = s_x^* s_x + s_y^* s_y = 1$$

- Only fully polarized waves can be described by Jones vectors

- Complex values can be hardly measured in real word physics

Another formalism can be used to describe the polarization of light:

George Gabriel Stokes

Stokes formalism (1852)

• The Stokes parameters are function only of observables of the light waves.

• They can describe any polarization state of a light beam (totally, partially, not polarized)

Stokes parameters for fully coherent light

$$|s> = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^*$$

$$s_2 = s_x s_y^* + s_x^* s_y$$

$$s_3 = j(s_x s_y^* - s_x^* s_y)$$

$$s_0 = s_x s_x^* + s_y s_y^*$$

$$\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

Stokes parameters for fully coherent light

$$|s> = e^{j\vartheta}\begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^*$$

$$s_2 = s_x s_y^* + s_x^* s_y$$

$$s_3 = j(s_x s_y^* - s_x^* s_y)$$

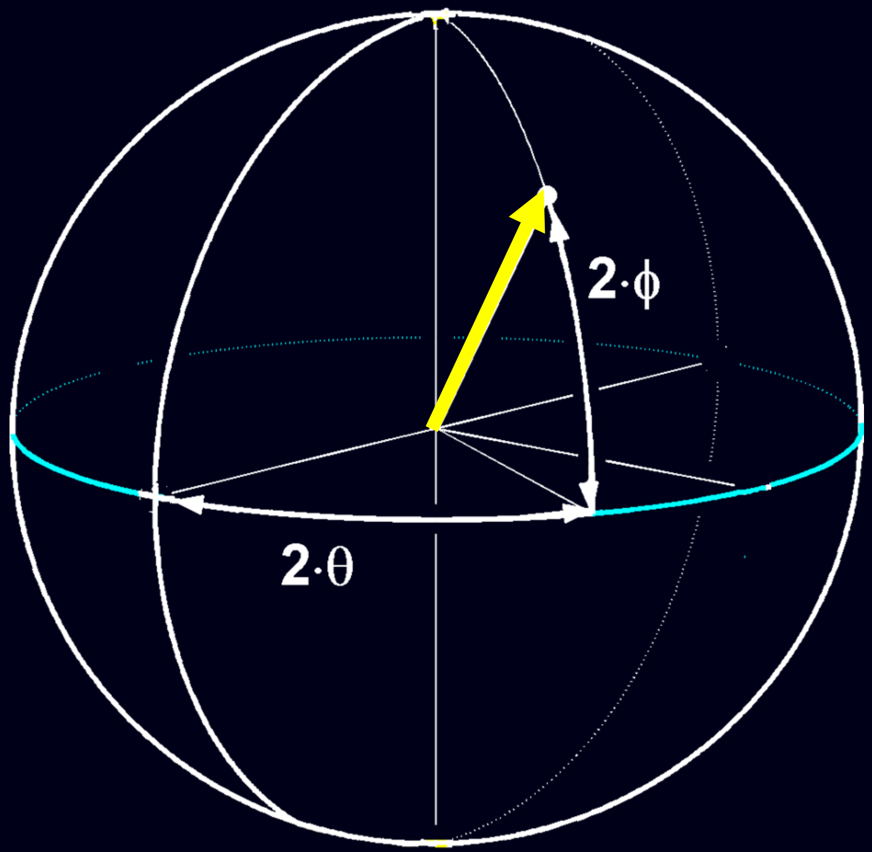
$$\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

A common phase shift of both components of |s> does not change $\hat{s}$

The locus of Stokes vectors representing all possible states of polarization of coherent light forms a unit sphere in Stokes space: the Poincaré sphere

$$\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

$$s_0 = 1$$

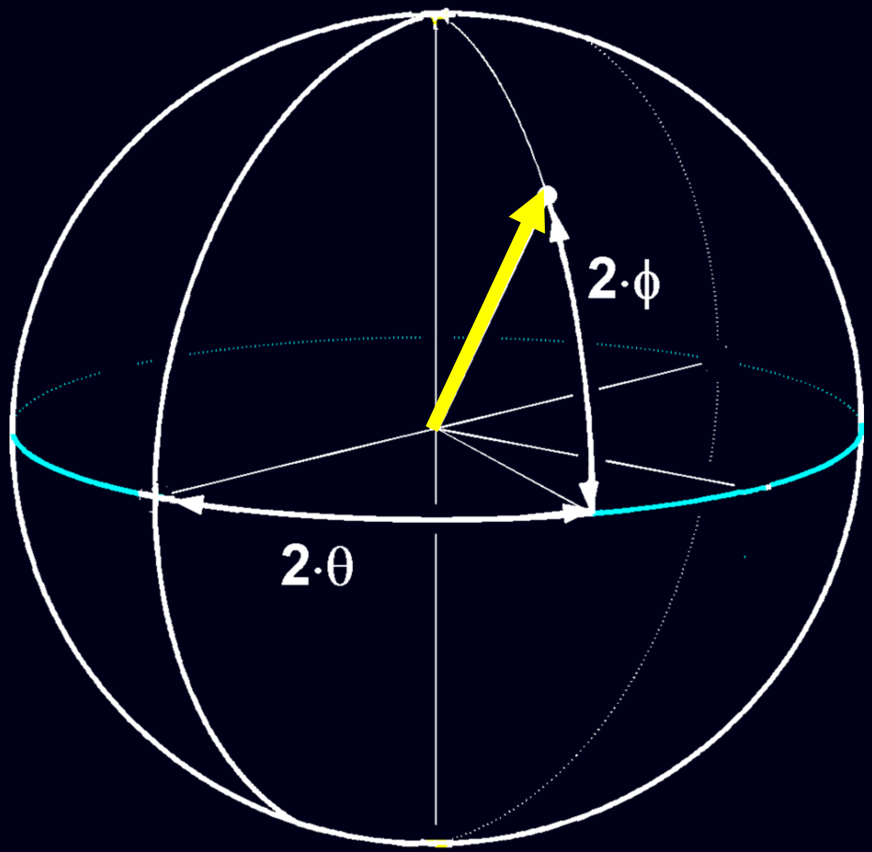


The locus of Stokes vectors representing all possible states of polarization of coherent light forms a unit sphere in Stokes space: the Poincaré sphere

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

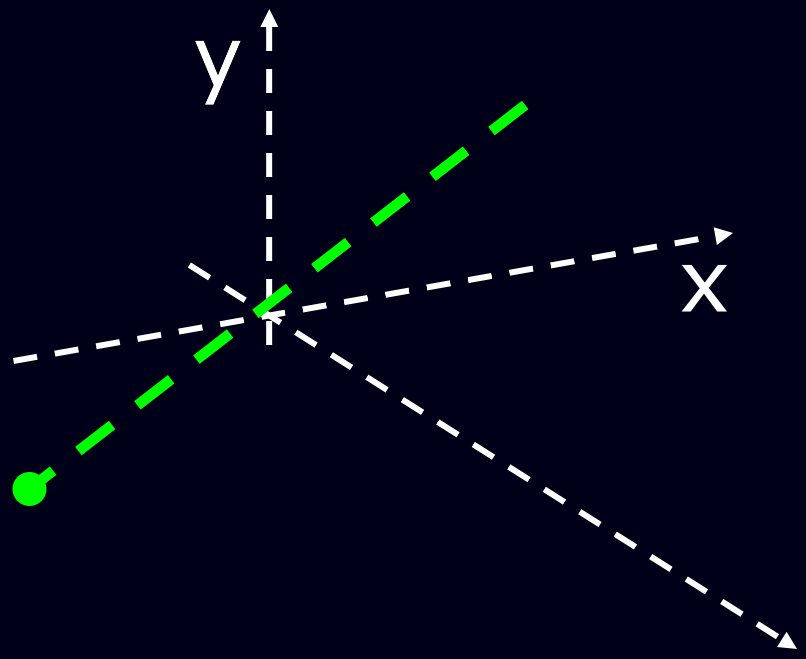
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

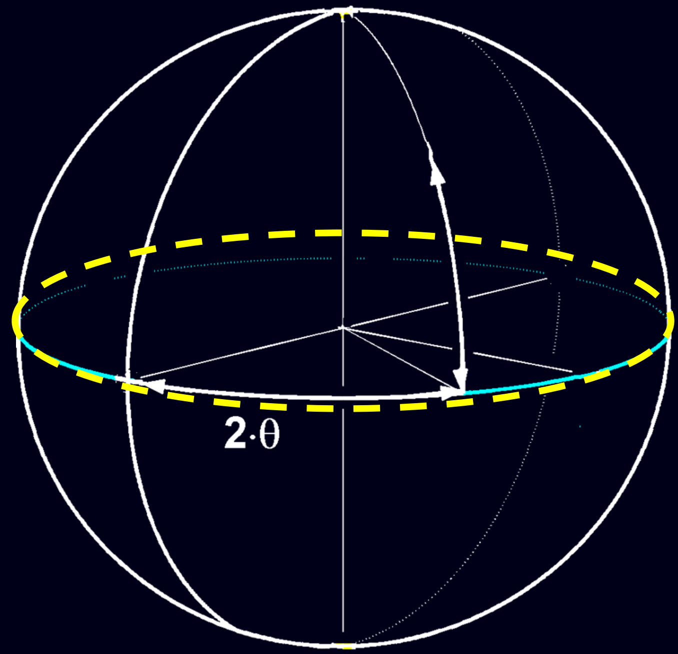


θ Azimuth

Φ Ellipticity

Stokes vector representation





Linear Polarizations are plotted along the equator

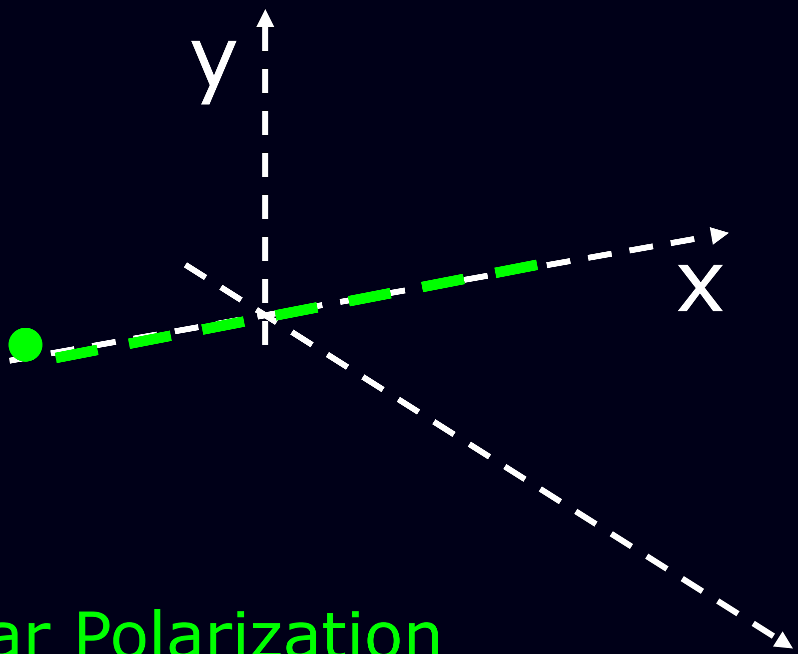
$\Phi = 0$ Ellipticity

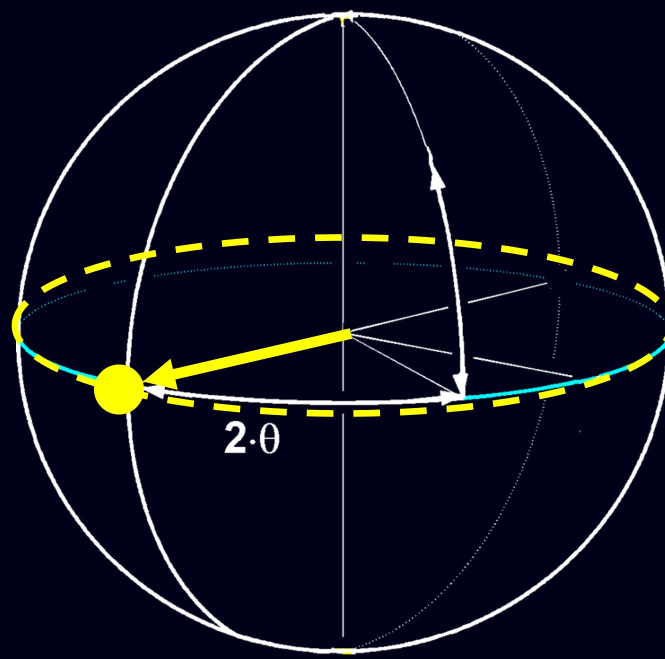
$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

Stokes vector representation





Linear Polarization
Horizontal

$$|s>_H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

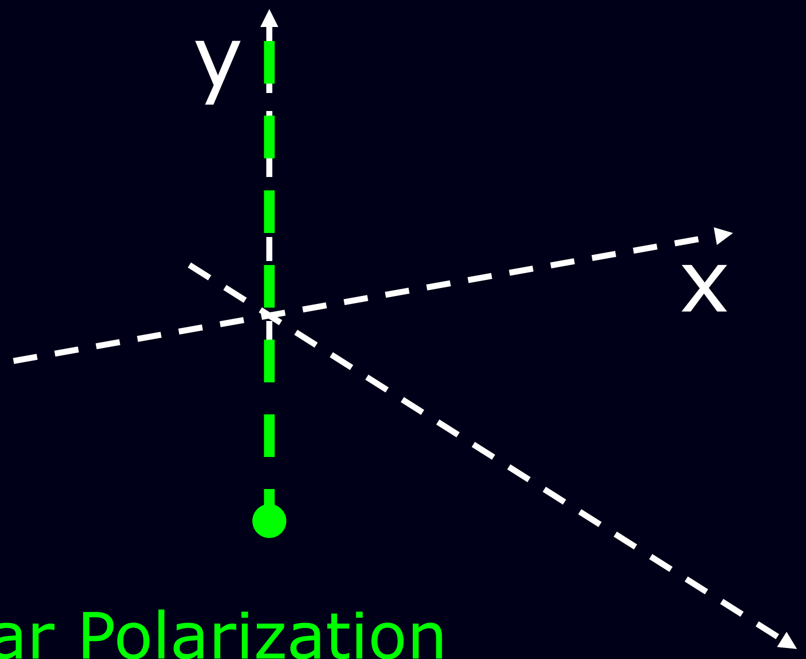
$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

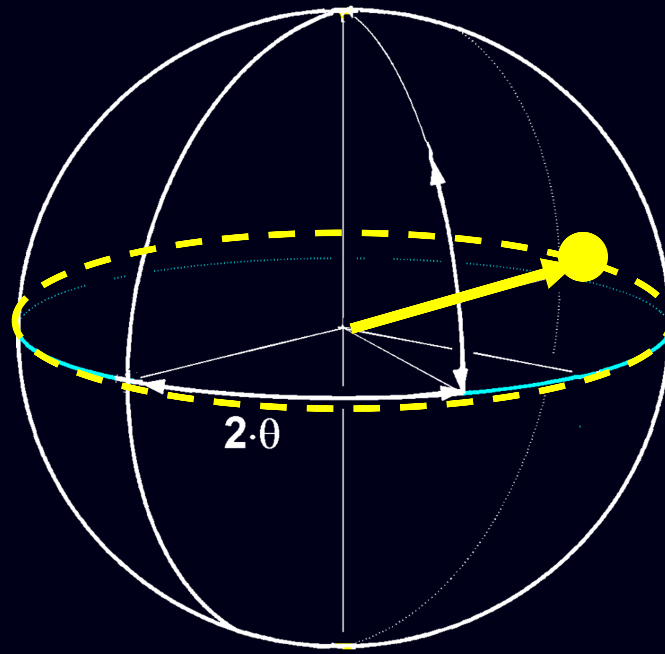
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{s} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\theta = 0, \phi = 0$$

Stokes vector representation





Linear Polarization
Vertical

$$|s>_V = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

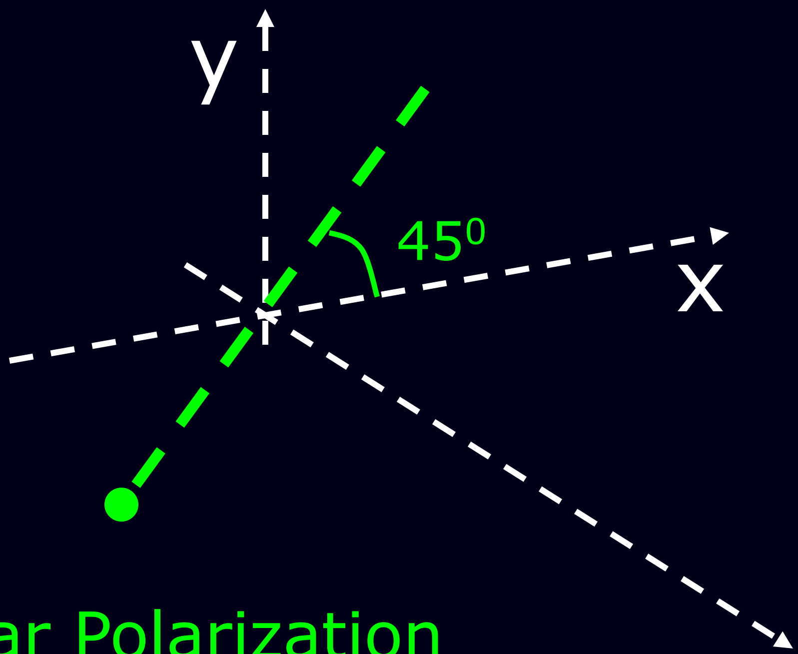
$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

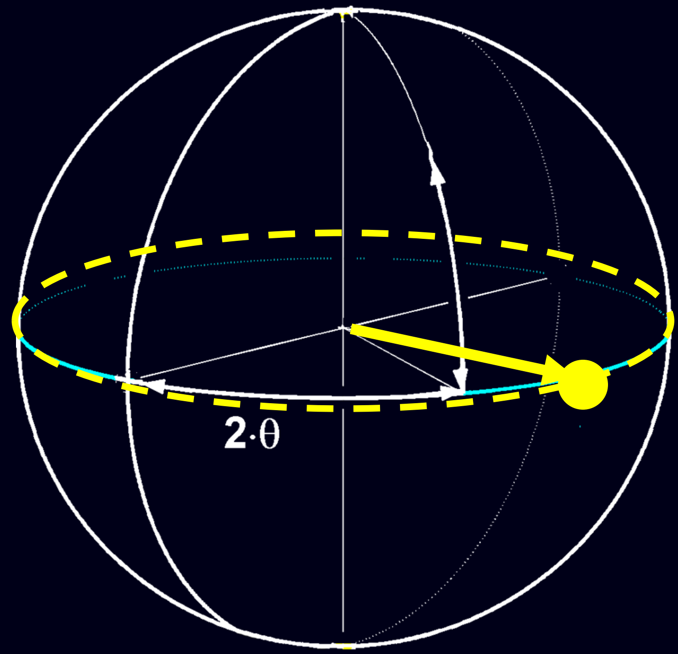
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{s} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$\theta = \pi/2, \phi = 0$$

Stokes vector representation





Linear Polarization
45^0 Azimuth

$$| s >_P = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

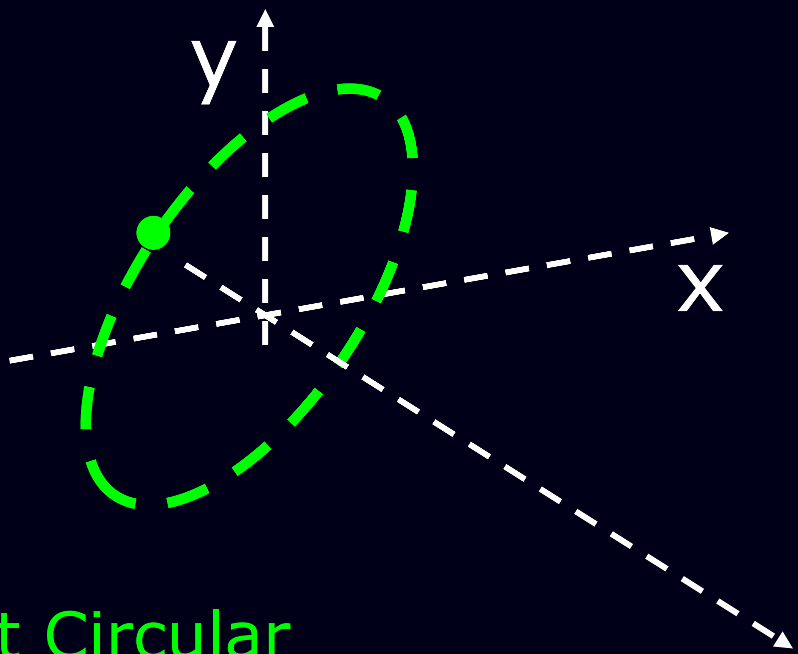
$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

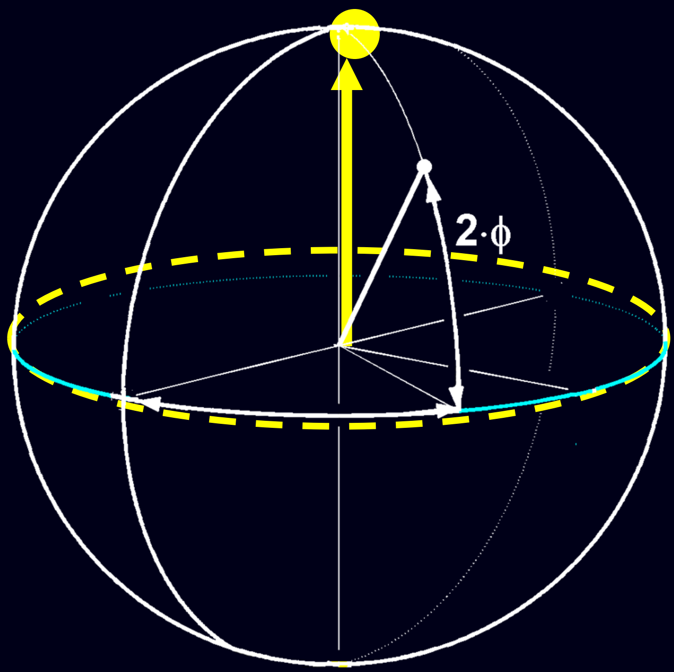
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{s} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\theta = \pi/4, \phi = 0$$

Stokes vector representation





Right Circular Polarization

$$| s >_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

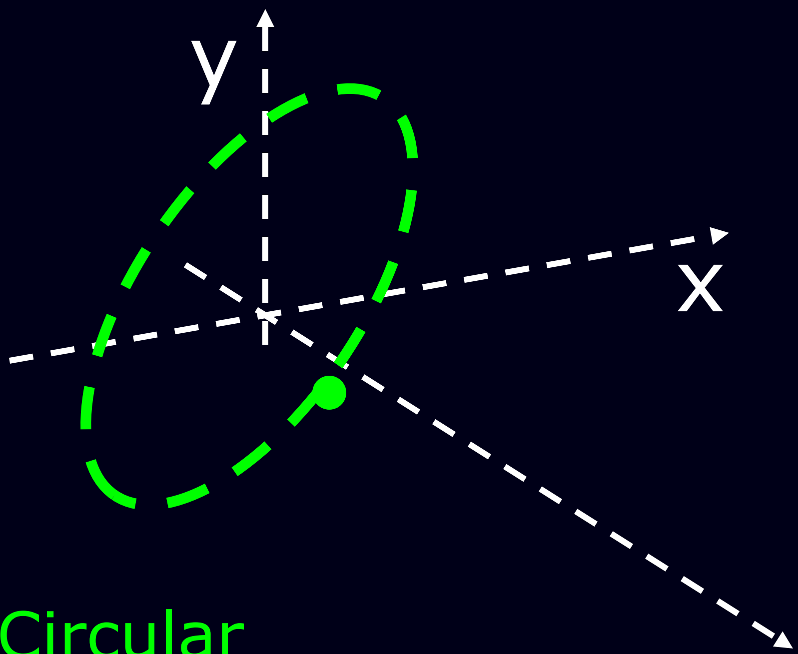
$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

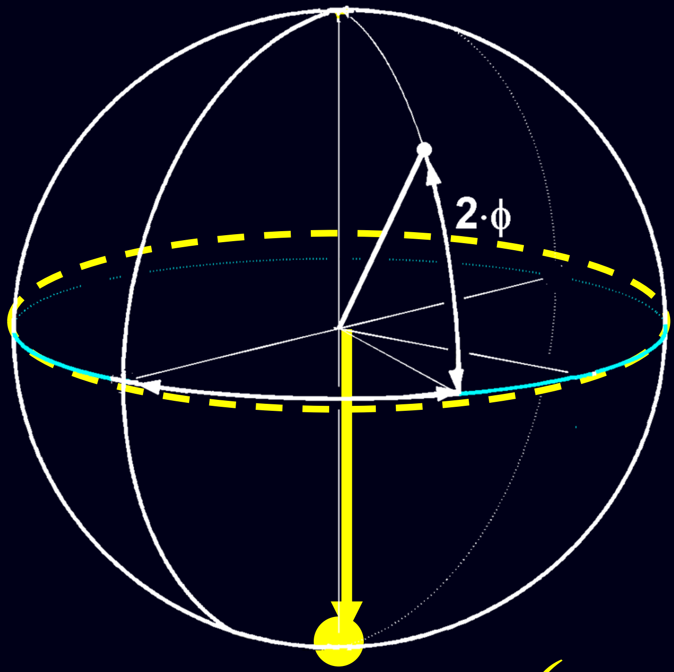
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$2\phi = \pi / 2$$

Stokes vector representation





Left Circular Polarization

$$|s>_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

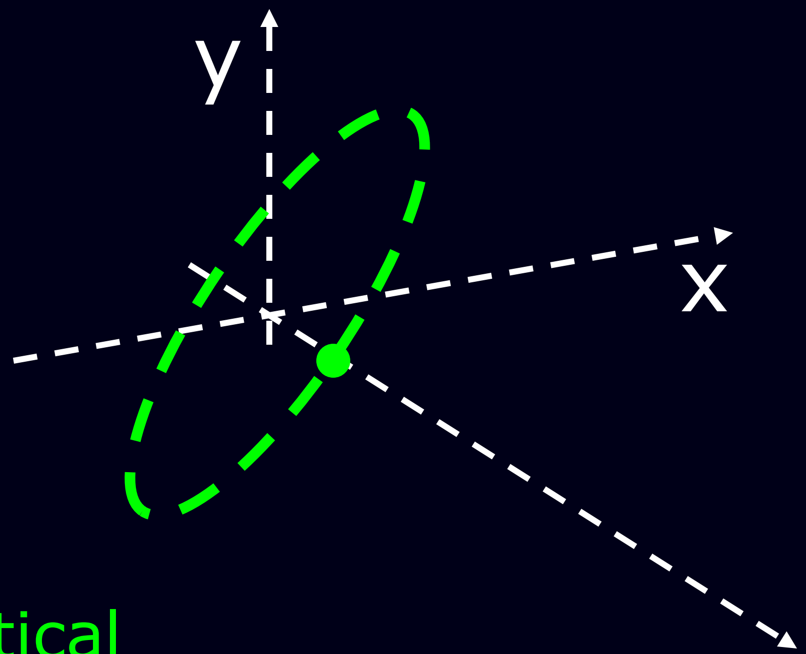
$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

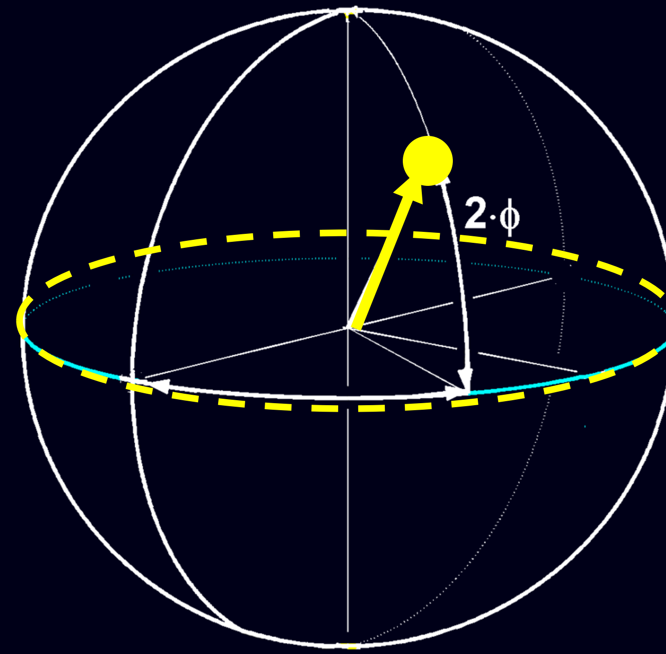
$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{s} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$2\phi = -\pi/2$$

Stokes vector representation





Elliptical Polarization

$$| s \rangle = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$s_1 = s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi)$$

$$s_2 = s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi)$$

$$s_3 = j(s_x s_y^* - s_x^* s_y) = \sin(2\phi)$$

$$\hat{S} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

$$S_0 = \left\| \hat{S} \right\| = \langle s | s \rangle = 1$$

Stokes vector representation

For fully polarized light the total beam intensity s_0 coincides with the norm of the Stokes vector

$$s_0 = \left\| \hat{s} \right\| = \sqrt{s_1^2 + s_2^2 + s_3^2} = \langle s | s \rangle = 1$$

$$| s \rangle = \begin{pmatrix} s_x \\ s_y \end{pmatrix} \qquad \hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

Stokes vector representation

Stokes parameters are not merely an alternative way of representing Jones vectors.

The one by one correspondence with the Jones vectors |s> is valid only for fully polarized light

Stokes parameters imply power measurements i.e. measurements on observables of light.

Stokes vector representation

Gabriel Stokes was the first one to understand that partially polarized light or unpolarized light cannot be described with complex amplitudes.

He abndoned the complex amplitudes to develop a new approach based on observable intensities.

In the interactive learning module the Stokes parameter measurement will be presented

Stokes vector representation

Partially polarized light or unpolarized light was a hot topic of discussion in the middle of the 19^{th} century.

An optical beam composed by several temporal or frequecy modes either can exhibit a common state of polarization, or its polarization state may varies extemely rapidly in time.

Hence partially polarized light is also a matter of interest in optics for communications

Stokes vector representation

The key element of Stokes' approach is that the beam intensity may be different from the norm of its corresponding average Stokes vector

Fully polarized light

$$s_0 = \left\|\hat{s}\right\| = \sqrt{s_1^2 + s_2^2 + s_3^2} = \; < s \,|\, s >= 1$$

Unpolarized light

$$\left\|\hat{s}\right\| = \sqrt{s_1^2 + s_2^2 + s_3^2} = 0$$

Stokes vector representation

With the Stokes parameters and the beam intensity s_0 one can define the degree of polarization

$$DOP = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0}$$

Fully polarized light: DOP=1

Unpolarized light: DOP=0

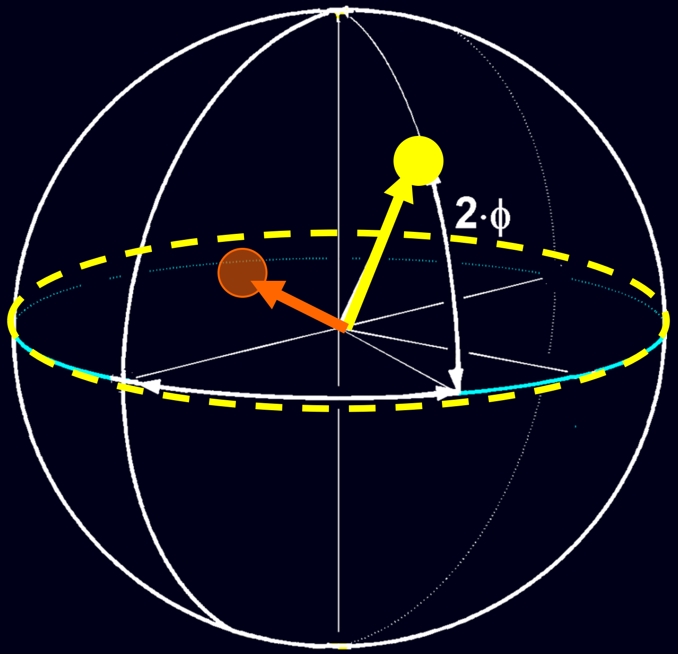
Partially polarized light: 0<DOP<1

Stokes vector for partially coherent light

Fully polarized light

$$s_0 = \left\| \hat{s} \right\| = < s \,|\, s > = 1$$

at the surface



Partially polarized light
Inside the Sphere