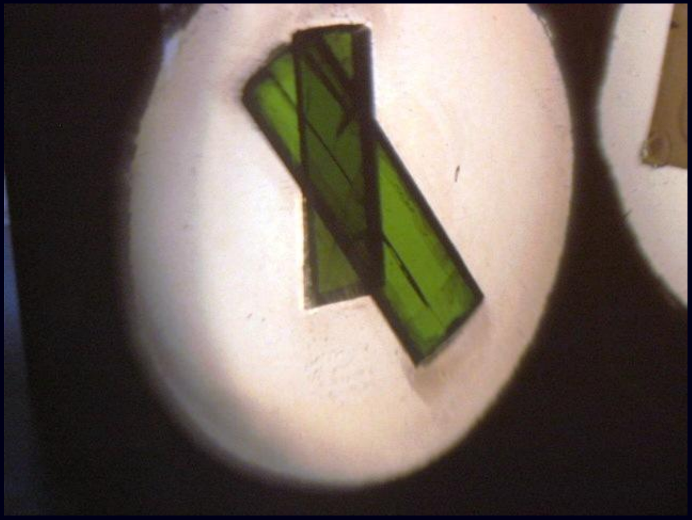
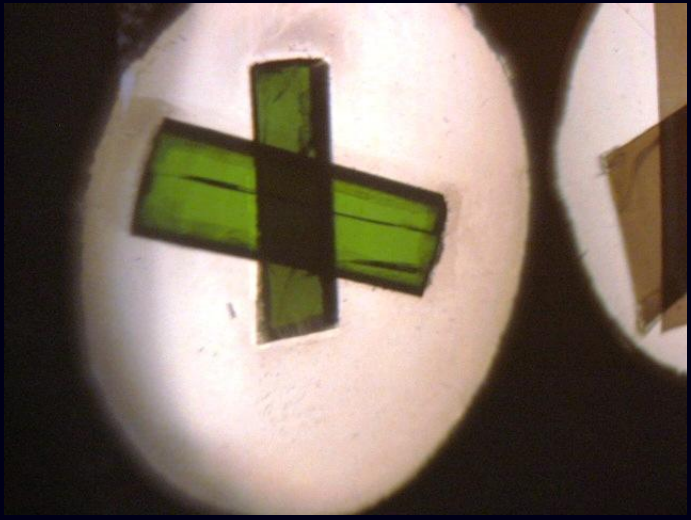
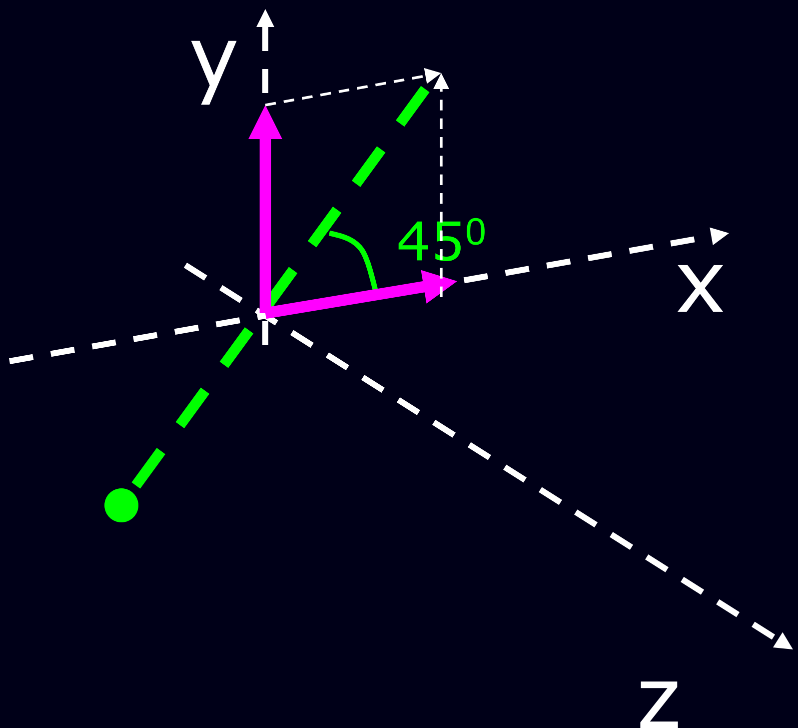
Polarization of light PART 2

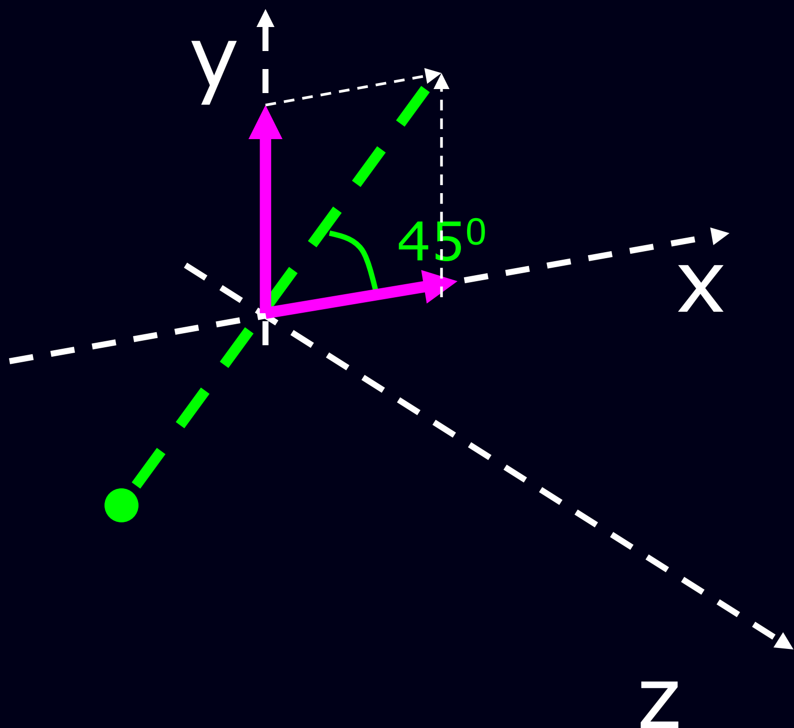






$$|s>_P = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$|s>_P = \frac{1}{\sqrt{2}}|s>_V + \frac{1}{\sqrt{2}}|s>_H$$



$$ {}_V\langle s | s \rangle_P = \frac{1}{\sqrt{2}} $$

$$ {}_H\langle s | s \rangle_P = \frac{1}{\sqrt{2}} $$

$$ | s \rangle_P = \frac{1}{\sqrt{2}} | s \rangle_V + \frac{1}{\sqrt{2}} | s \rangle_H $$

More generally.....

Orthogonal set in Jones Space

$$|p> = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \qquad |q> = \begin{pmatrix} q_x \\ q_y \end{pmatrix}$$

$$<p|p> = 1$$

$$<q|q> = 1$$

$$<p|q> = 0$$

$$<p|q> = \left(p_x^*, p_y^*\right)\begin{pmatrix} q_x \\ q_y \end{pmatrix} = p_x^* q_x + p_y^* q_y$$

More generally.....
Orthogonal set in Jones Space

$$|p> = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \qquad |q> = \begin{pmatrix} q_x \\ q_y \end{pmatrix}$$

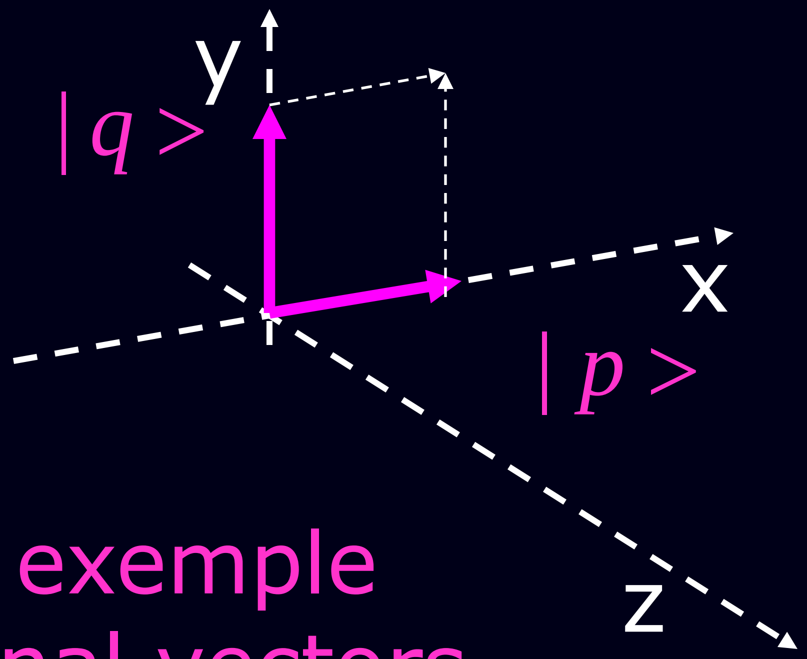
$$<p|p> = 1$$

$$<q|q> = 1$$

$$<p|q> = 0$$

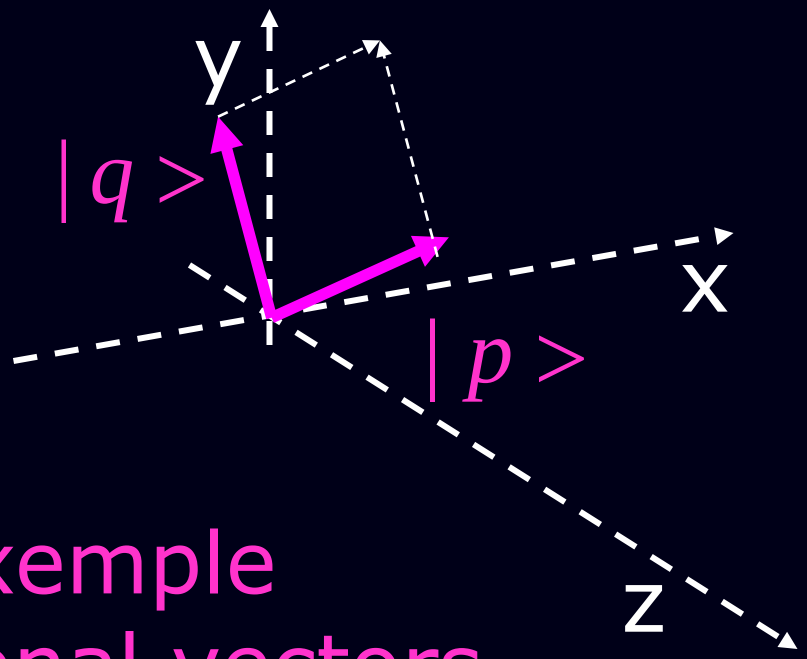
$$|s> = a|p> + b|q>$$

Jones vectors are complex vectors on a vector space so we cannot sketch all possible cases in a 2D real diagram



A possible exemple
of orthogonal vectors

Jones vectors are complex vectors on a vector space so we cannot sketch all possible cases in a 2D real diagram



another exemple
Of orthogonal vectors

Orthonormal Jones vectors:
How do they look like in the Poincaré Sphere?

$$|p> = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \qquad |q> = \begin{pmatrix} q_x \\ q_y \end{pmatrix}$$

$$\downarrow \qquad \downarrow$$

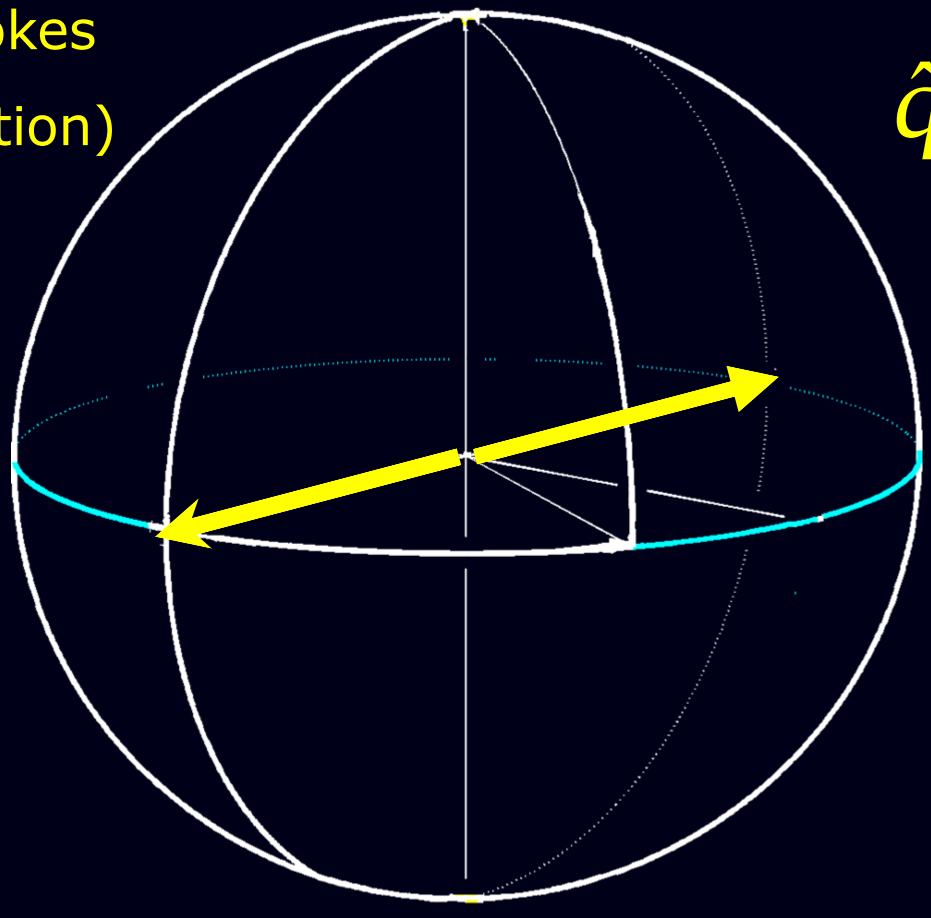
$$\hat{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \qquad \hat{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

Simple Case: $|p\rangle = |s\rangle_H$ and $|q\rangle = |s\rangle_V$

The corresponding Stokes Vectors
are antiparallel
(Think at the Stokes

Parameter definition)

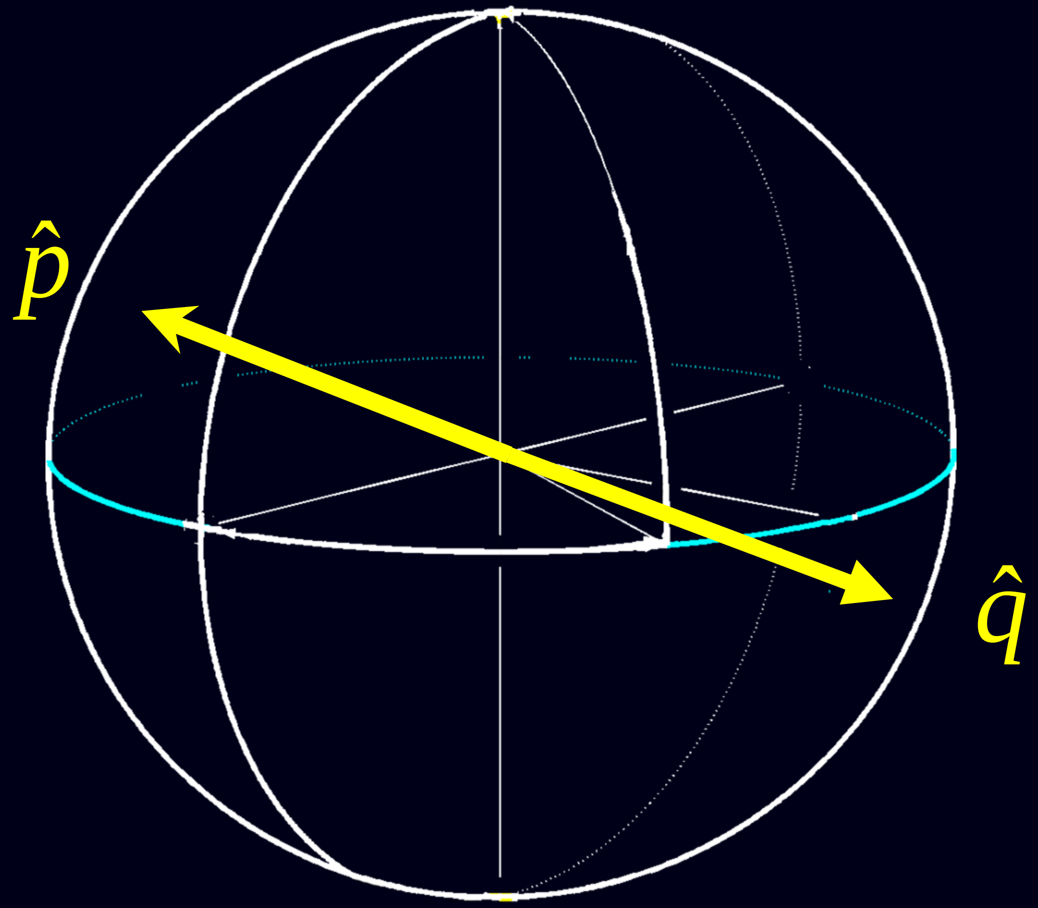
$$\hat{q} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$



$$\hat{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

General Rule for <q|p>=0

$$\hat{p} \cdot \hat{q} = -1$$



These few following slides are not necessary for a basic undertanding of the problem.

However they will show some intriguing relations between Stokes and Jones space.

Formal relation between Stokes and Jones formalism

$$|s> = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$\hat{S} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

$$\boxed{S_i = < s | \sigma_i | s >}$$

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \; \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \sigma_3 = \begin{pmatrix} 0 & -j \\ j & 0 \end{pmatrix}$$

Pauli Spin Matrices

Marginal subject

$$<p|q> = (p_x^*, p_y^*)\begin{pmatrix} q_x \\ q_y \end{pmatrix} = p_x^* q_x + p_y^* q_y$$

$$|p><q| = ? = \begin{pmatrix} p_x \\ p_y \end{pmatrix}(q_x^*, q_y^*) = \begin{pmatrix} p_x q_x^* & p_x q_y^* \\ p_y q_x^* & p_y q_y^* \end{pmatrix}$$

Dyadic operator: Note that <q|p> is simply Tr[|p><q|]

Marginal subject

Important property of the diadic operator

$$|p\rangle\langle q| = \begin{pmatrix} p_x q_x^* & p_x q_y^* \\ p_y q_x^* & p_y q_y^* \end{pmatrix}$$

$$|s\rangle\langle s| = \frac{1}{2}(I + \hat{s}\cdot\vec{\sigma})$$

Pauli Spin Vector

Jones vectors
"nose to nose"

Corresponding
Stokes Vector

Marginal subject

Let use this property for the case of orthogonal |p> and |q>

$$|p><p| = \frac{1}{2}(I + \hat{p}\cdot\vec{\sigma})$$

$$<q|p><p|q> = \frac{1}{2} <q|(I + \hat{p}\cdot\vec{\sigma})q>$$

Orthogonality

It follows from the relation

$$q_i = <q|\sigma_i|q>$$

And hence

$$0 = \frac{1}{2}(1 + \hat{p}\cdot\hat{q})$$

$$\hat{p}\cdot\hat{q} = -1$$

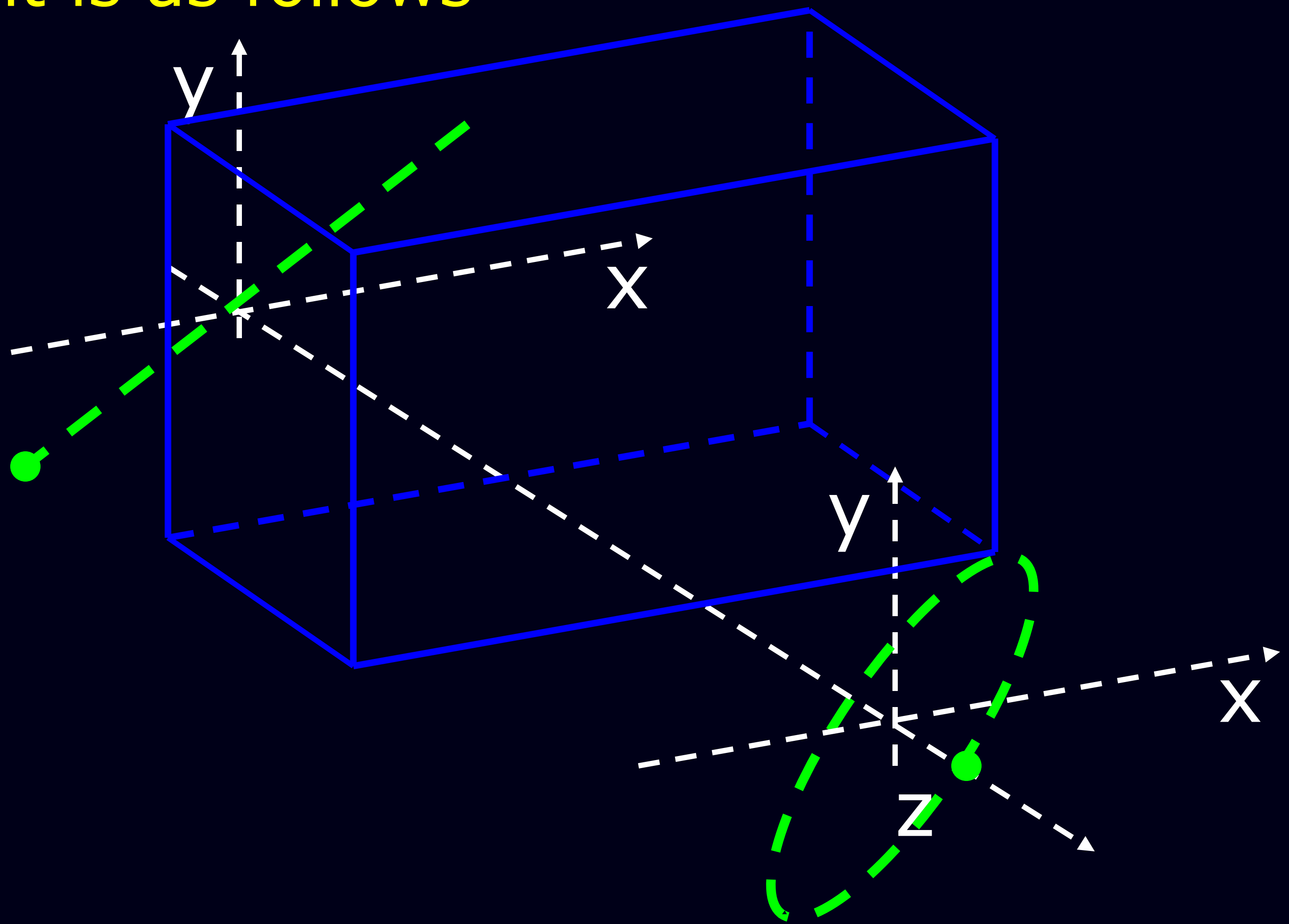
The corresponding Stokes vectors
are always antiparallel

Marginal subject

Not to get lost in our digression,
we take to the main road again

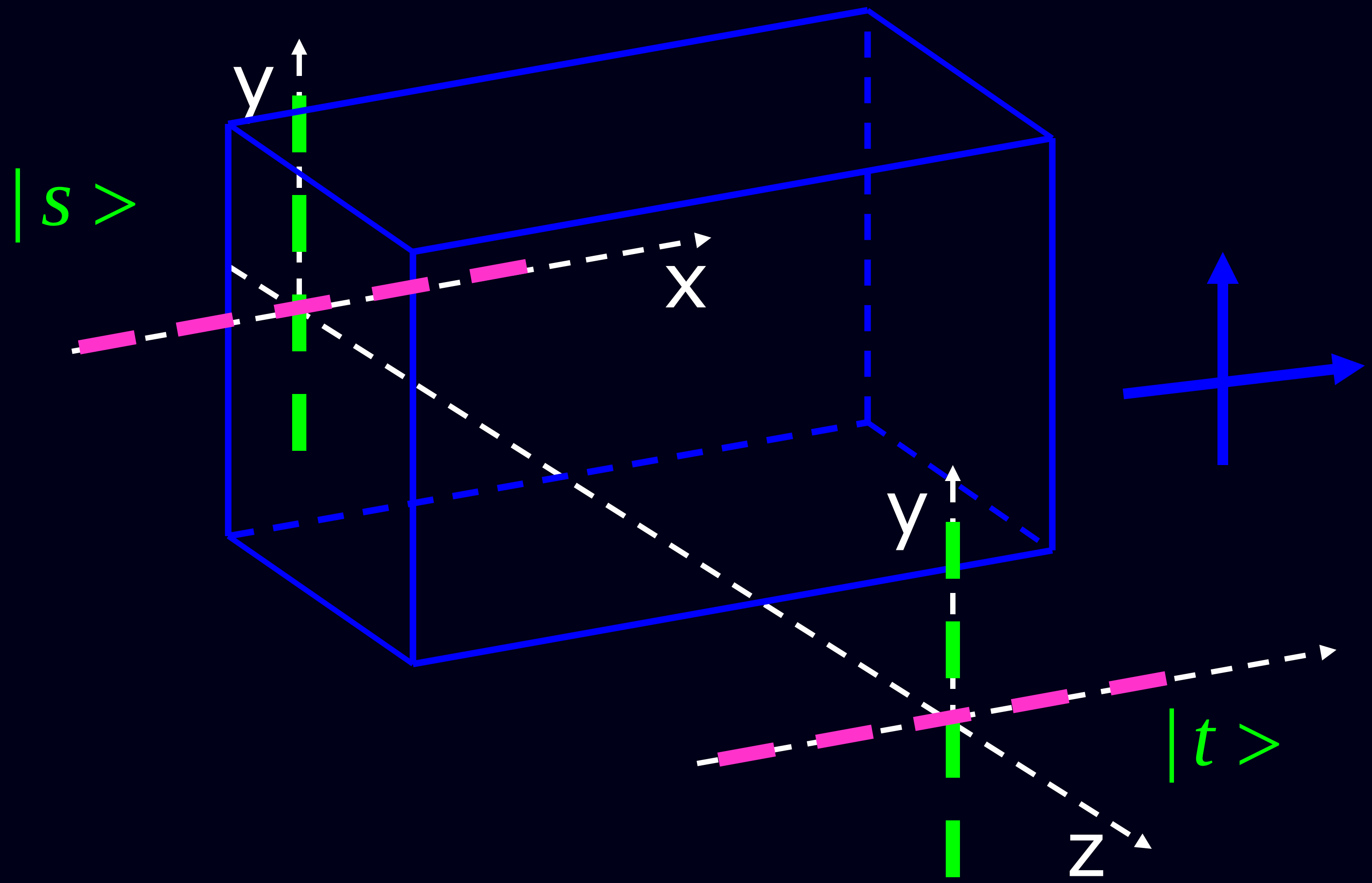
Let us consider the polarization evolution when the light passes though and anysotropic dielectric medium (like in a birefringent fiber)

One possible result with input monochromatic light is as follows



Input and output polarization states are different

Case of linear birefringence (absence of circular birefringence or optical activity)



The output polarization state is different

$$|t> = e^{-j\beta_0 z} U \, |s>$$

Exemple of Jones Matrix

$$U(\omega, z) = \begin{pmatrix} e^{-j\varphi/2} & 0 \\ 0 & e^{+j\varphi/2} \end{pmatrix}$$

$$\varphi = \frac{2\pi}{\lambda} \Delta n \, z = \frac{\omega}{c} \Delta n \, z$$

$$\left.\begin{aligned} |t> &= U\,|s>_H = e^{-j\beta_H z}\,|s>_H \\ |t> &= U\,|s>_V = e^{+j\beta_V z}\,|s>_V \end{aligned}\right\}$$

For these eigenstates the polarization state does not evolve (However there is a phase tag)

$$\beta_H = \frac{\omega}{c} n_H = \frac{\omega}{c}\left(n_0 + \frac{\Delta n}{2} \right)$$

$$\beta_V = \frac{\omega}{c} n_V = \frac{\omega}{c}\left(n_0 - \frac{\Delta n}{2} \right)$$

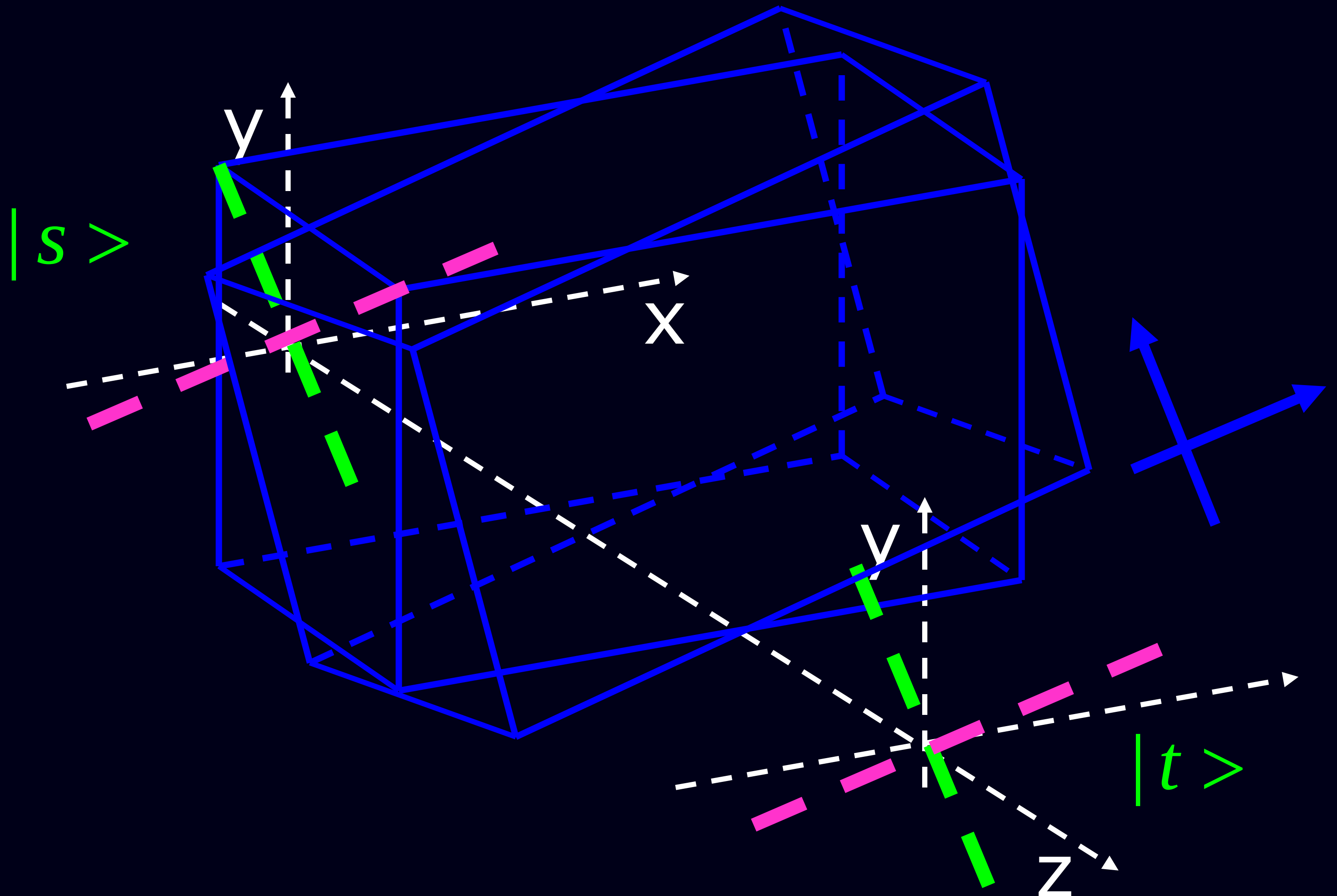
$$U = \begin{pmatrix} e^{-j\varphi/2} & 0 \\ 0 & e^{+j\varphi/2} \end{pmatrix}$$

$$UU^* = I$$

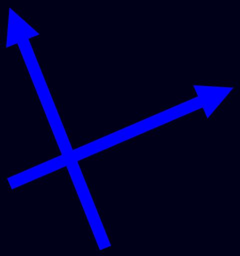
$$\left.\begin{aligned} |t> &= U\,|s>_H = e^{-j\beta_H z}\,|s>_H \\ |t> &= U\,|s>_V = e^{+j\beta_V z}\,|s>_V \end{aligned}\right\}$$

$$U = \begin{pmatrix} e^{-j\varphi/2} & 0 \\ 0 & e^{+j\varphi/2} \end{pmatrix}$$

Such special axes are peculiarities of the birefringent element, not of the specific reference frame we used to describe these phenomena


y
x
|s >
y
z
|t >

$$\left.\begin{array}{l} |t> = U|s>_{Slow} = e^{-j\beta_{Slow}z}\,|s>_{Slow} \\ \\ |t> = U|s>_{Fast} = e^{+j\beta_{Fast}z}\,|s>_{Fast} \end{array}\right\}$$



$$\beta_{Slow} = \frac{\omega}{c} n_{Slow} = \frac{\omega}{c}\left(n_0 + \frac{\Delta n}{2}\right)$$

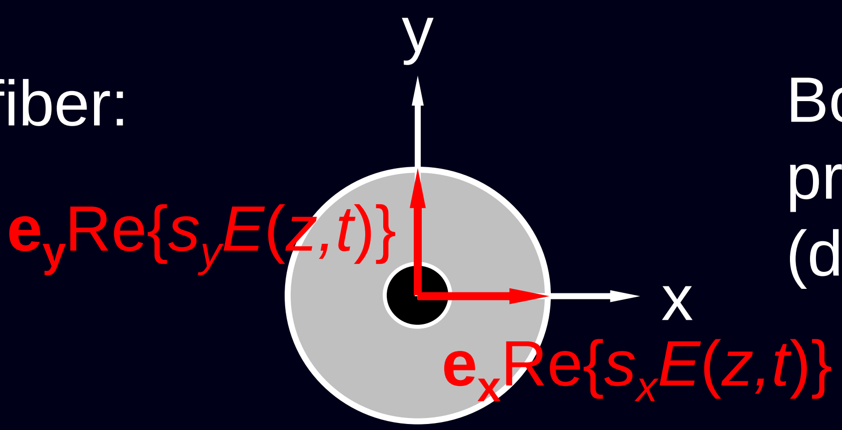
$$\beta_{Fast} = \frac{\omega}{c} n_{Fast} = \frac{\omega}{c}\left(n_0 - \frac{\Delta n}{2}\right)$$

Referring to the birefringence axes frame

Birefringent Fibers

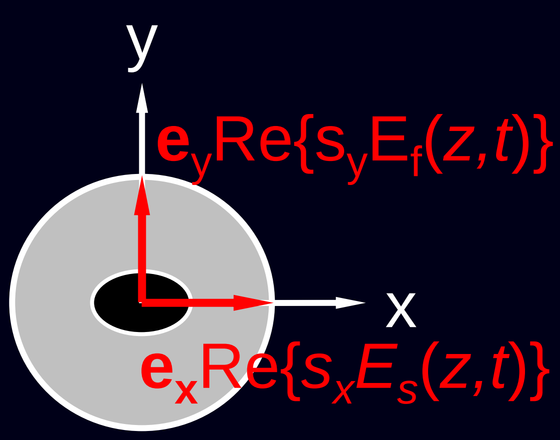
A single-mode fiber supports two orthogonally polarized HE_{11}-modes.

Ideal fiber:



Both modes have the same propagation constant β (degenerate modes).

Short birefringent fiber: (uniform birefringence over the fiber length)



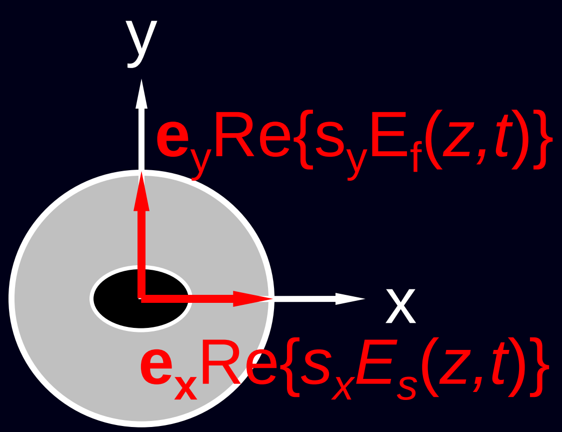
Different propagation constants:

$$\Delta\beta = \beta_s - \beta_f = \frac{\omega_0 n_s}{c} - \frac{\omega_0 n_f}{c} = \frac{\omega_0 \Delta n}{c}$$

β_s, β_f :	propagation constant of slow and fast mode
n_s, n_f :	respective effective refractive indices
ω_0:	angular carrier frequency
c :	speed of light

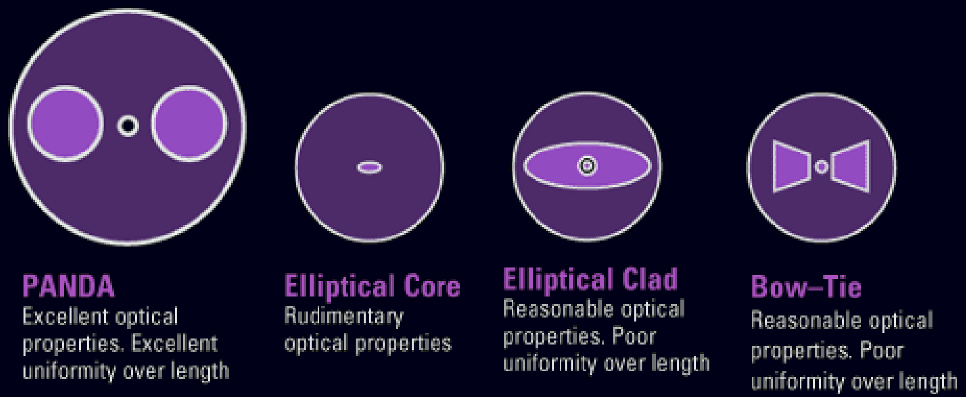
Birefringent Fibers

Short birefringent fiber: (uniform birefringence over the fiber length)



Different propagation constants:

$$\Delta\beta = \beta_s - \beta_f = \frac{\omega_0 n_s}{c} - \frac{\omega_0 n_f}{c} = \frac{\omega_0 \Delta n}{c}$$



From: www.nufern.com

Jones Matrix U preserves the scalar product

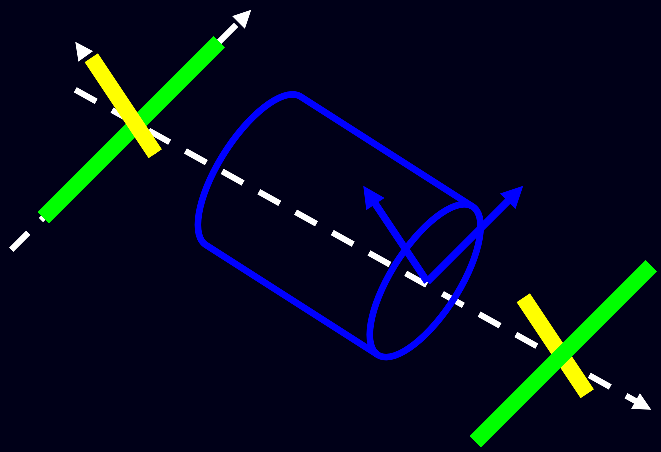
$$|t> = e^{-j\beta_0 z} U |s>$$

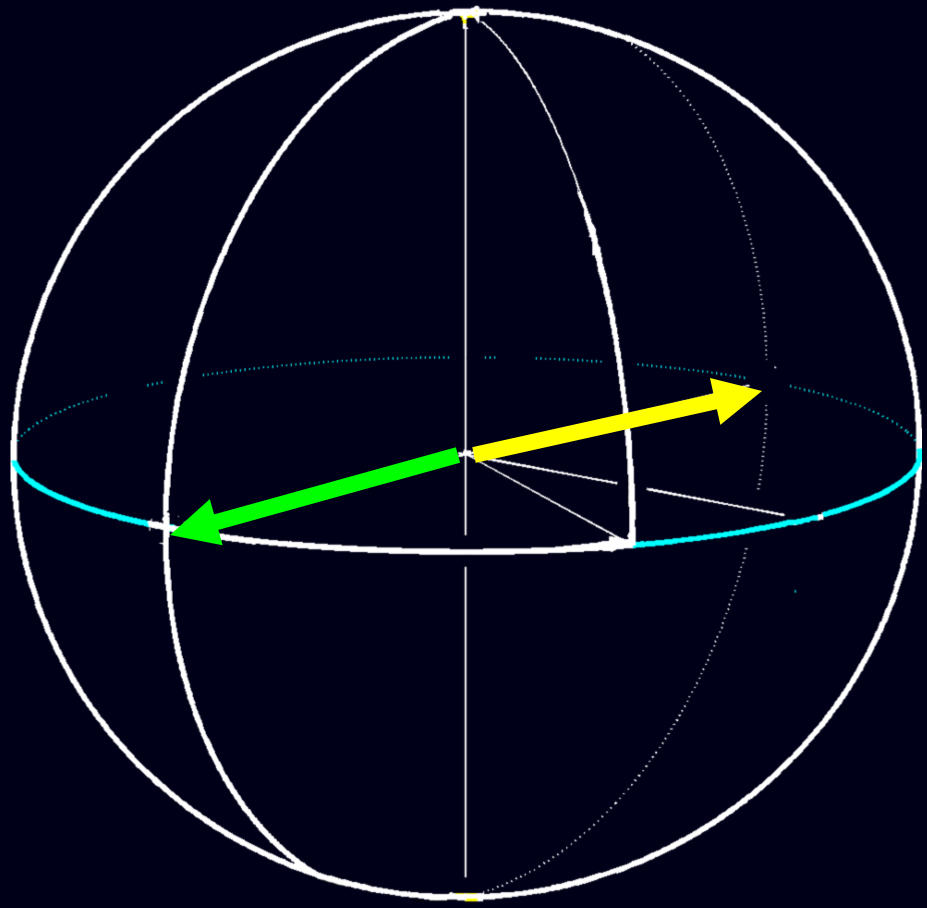
$$|w> = e^{-j\beta_0 z} U |q>$$

$$<t|w> = <s|U^* U|q> = <s|q>$$

Hence input orthogonal SOP give output orthogonal SOP

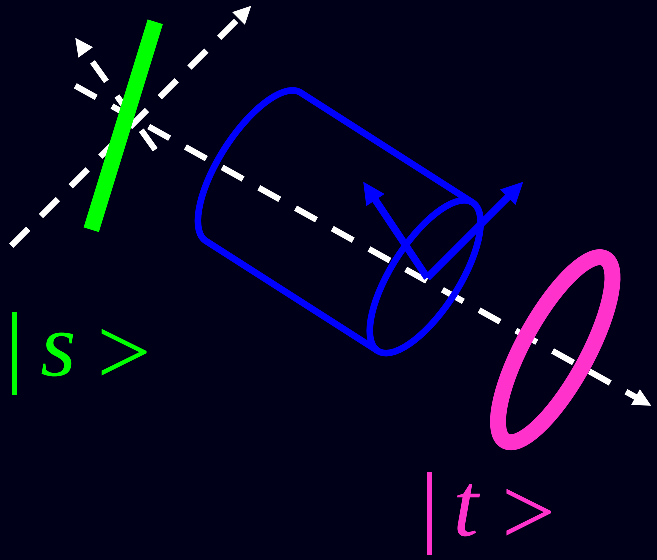
Stokes vectors dynamics in birefringent fiber For monochromatic light

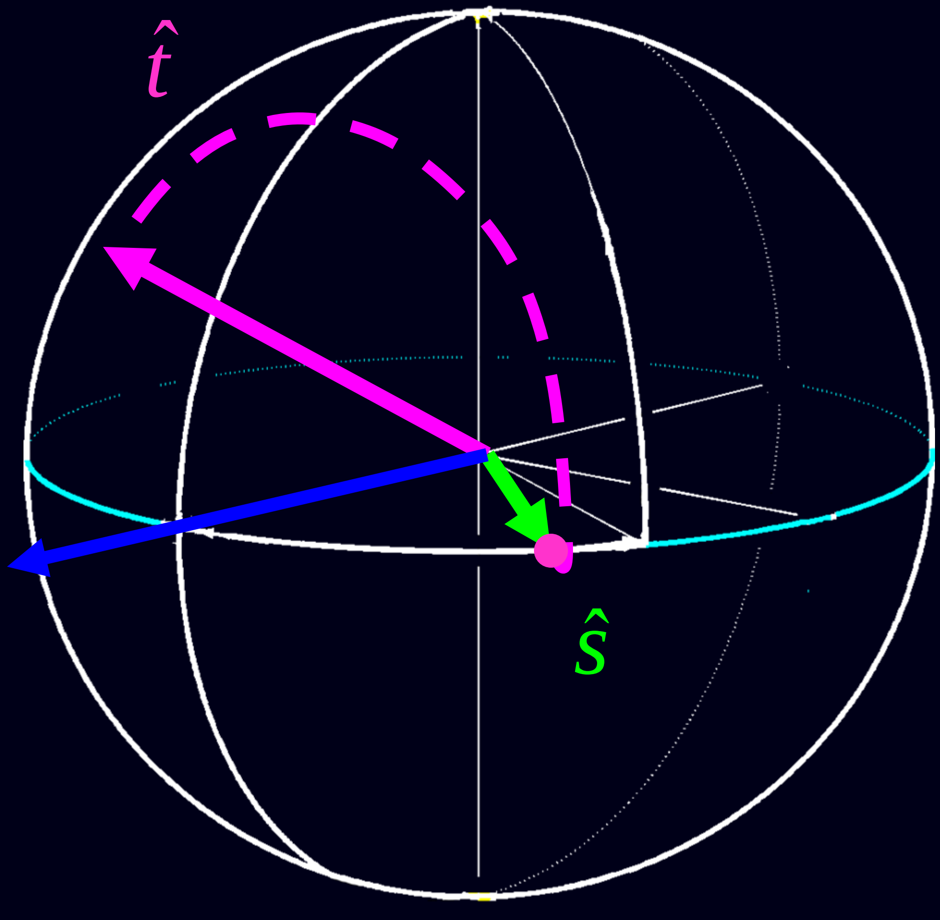




This is the reason why birefringent fibers are also known as polarization preserving fibers (obviously using their principal axes)

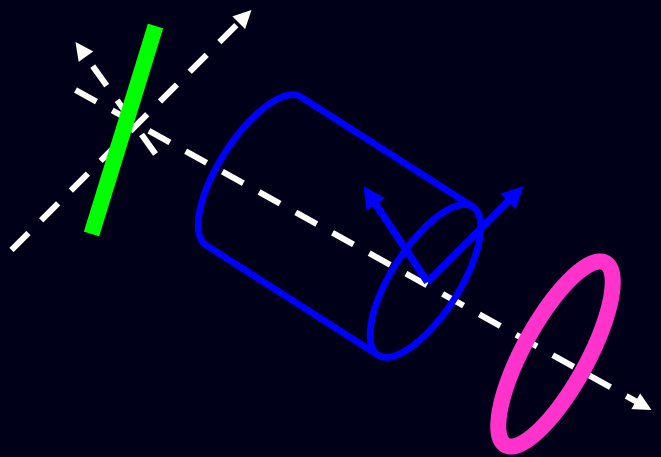
Stokes vectors dynamics in birefringent fiber
For monochromatic light

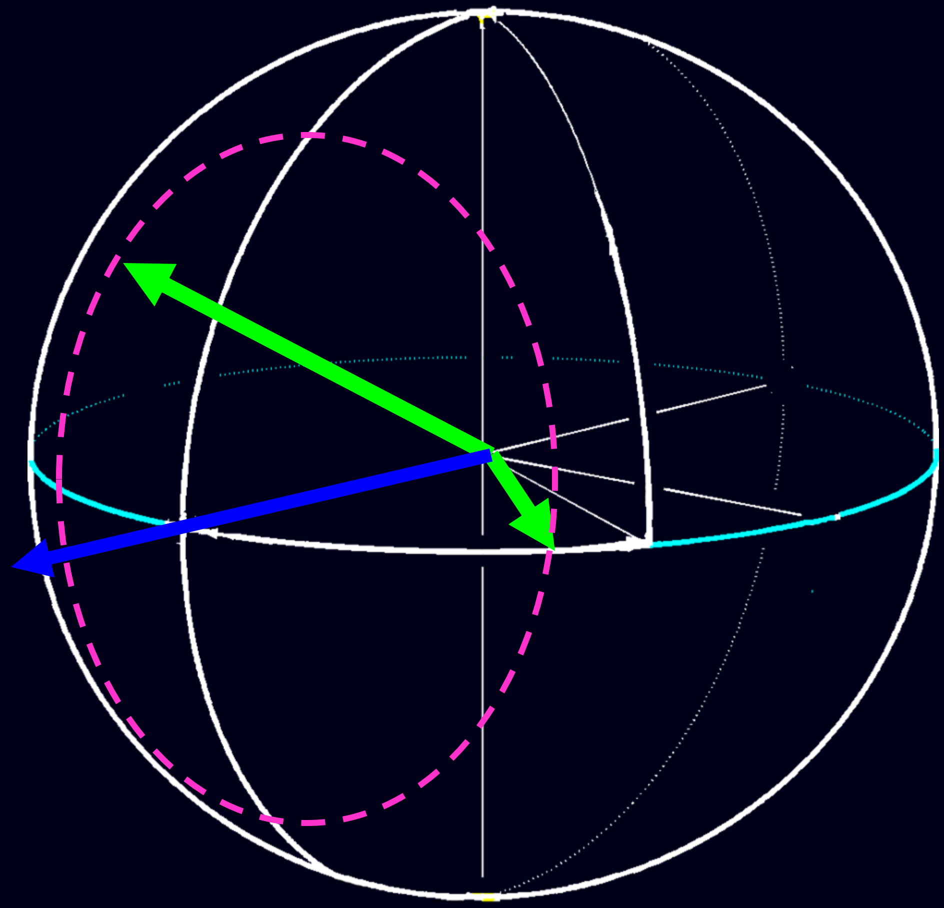




$$\frac{d\hat{s}}{dz} = \beta \times \hat{s}$$

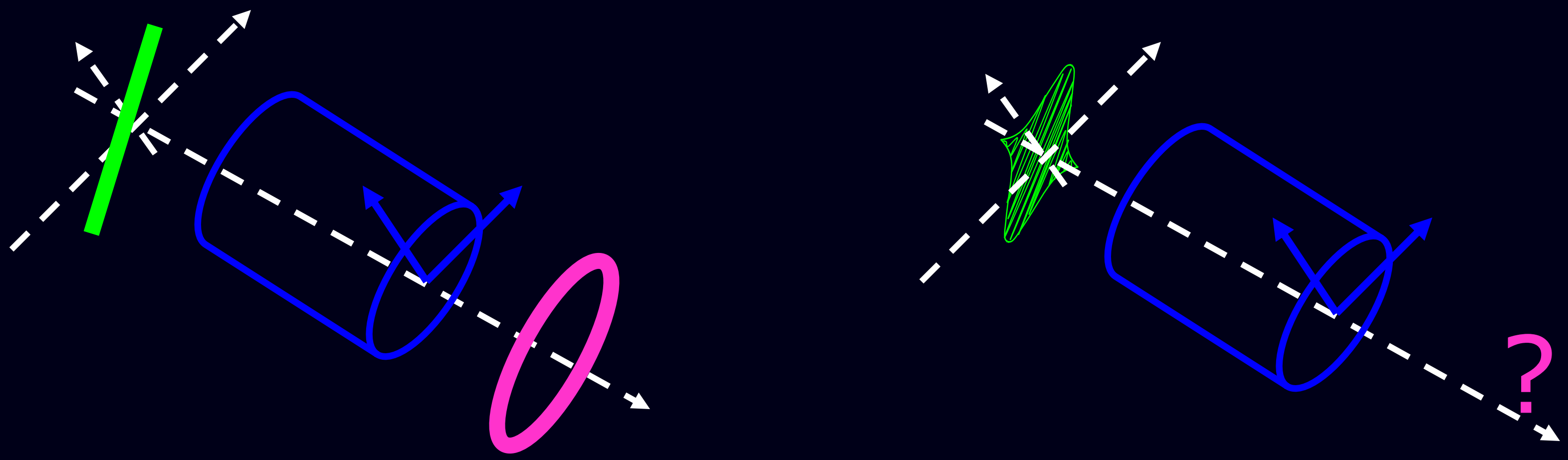
Stokes vectors dynamics in birefringent fiber
For monochromatic light





$$\frac{d\hat{s}}{dz} = \beta \times \hat{s}$$

Monochromatic light does not carry information. What happens with pulses?



We should evolve our diagrams to consider The polarization state of each frequency or temporal mode

$$|t>=e^{-j\beta_0 z}U|s>$$

$$\frac{d}{d\omega}|t>=-jz\frac{d\beta_0}{d\omega}e^{-j\beta_0 z}U|s>+e^{-j\beta_0 z}\frac{dU}{d\omega}|s>$$

$$\frac{d}{d\omega}|t>=-jz\frac{d\beta_0}{d\omega}|t>+\frac{dU}{d\omega}U^*|t>$$

$$\frac{d}{d\omega}|t>=-j\left(\tau_0+j\frac{dU}{d\omega}U^*\right)|t>$$

$$|t> = e^{-j\beta_0 z} U\,|s>$$

$$\frac{d}{d\omega}|t> = -j\left(\tau_0 + j\frac{dU}{d\omega}U^*\right)|t>$$

$$\tau_0 = z\frac{d\beta_0}{d\omega} = \frac{d\varphi_0}{d\omega} = z\left(\frac{n(\omega)}{c} + \frac{\omega}{c}\frac{dn}{d\omega}\right)$$

Mean group delay

$$|t> = e^{-j\beta_0 z} U |s>$$

$$\frac{d}{d\omega} |t> = -j\left(\tau_0 + j\frac{dU}{d\omega}U^*\right)|t>$$

Hermitian operator: something I can measure in practice. Real Eigenvalues

zero trace: (sum of real eigenvalues is zero)

Physical dimensions of time

$$|t> = e^{-j\beta_0 z} U \, |s>$$

$$\frac{d}{d\omega} |t> = -j\left(\tau_0 + j \frac{dU}{d\omega} U^* \right) |t>$$

$$\tau_g = \tau_0 \pm \frac{\tau}{2}$$

Two times delay for the two eigenstates of $jU_\omega U^*$

$$|t> = e^{-j\beta_0 z} U\,|s>$$

$$\frac{d}{d\omega}|t> = -j\left(\tau_0 + j\frac{dU}{d\omega}U^*\right)|t>$$

$$\left(j\frac{dU}{d\omega}U^*\right)|p>_+ = \frac{1}{2}\tau\,|p>_+$$ Slow PSP (largest group delay)

$$\left(j\frac{dU}{d\omega}U^*\right)|p>_- = -\frac{1}{2}\tau\,|p>_-$$ Fast PSP (smallest group delay)

$$\left(j\frac{dU}{d\omega}U^* \right) | p >_+ = \frac{1}{2}\tau \, | p >_+$$

Slow PSP (largest group delay)

$$\left(j\frac{dU}{d\omega}U^* \right) | p >_- = -\frac{1}{2}\tau \, | p >_-$$

Fast PSP (smallest group delay)

τ is called the differential group delay DGD

$$\hat{\tau} = \tau \, \hat{p}_+$$

This is the PMD vector In Stokes space: it is the slow PSP multiplied by the DGD

$$\frac{d}{d\omega}|t> = -j\left(\tau_0 + j\frac{dU}{d\omega}U^*\right)|t>$$