VPI University Program

Photonics Curriculum Version 7.0

Lecture Series

Chromatic Dispersion and Kerr Nonlinearities

Fiber 2

Developed in cooperation with
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Module Prerequisites

- **Fiber 1: Basics of Fiber Propagation**

Module Objectives

- **Dispersion, dispersion slope**
- **Dispersion compensation and management**
- **Kerr nonlinearities**
  - **Self-phase modulation (SPM)**
  - **Cross-phase modulation (XPM)**
  - **Four-wave mixing (FWM)**
- **Nonlinear transmission**

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Propagation of pulses

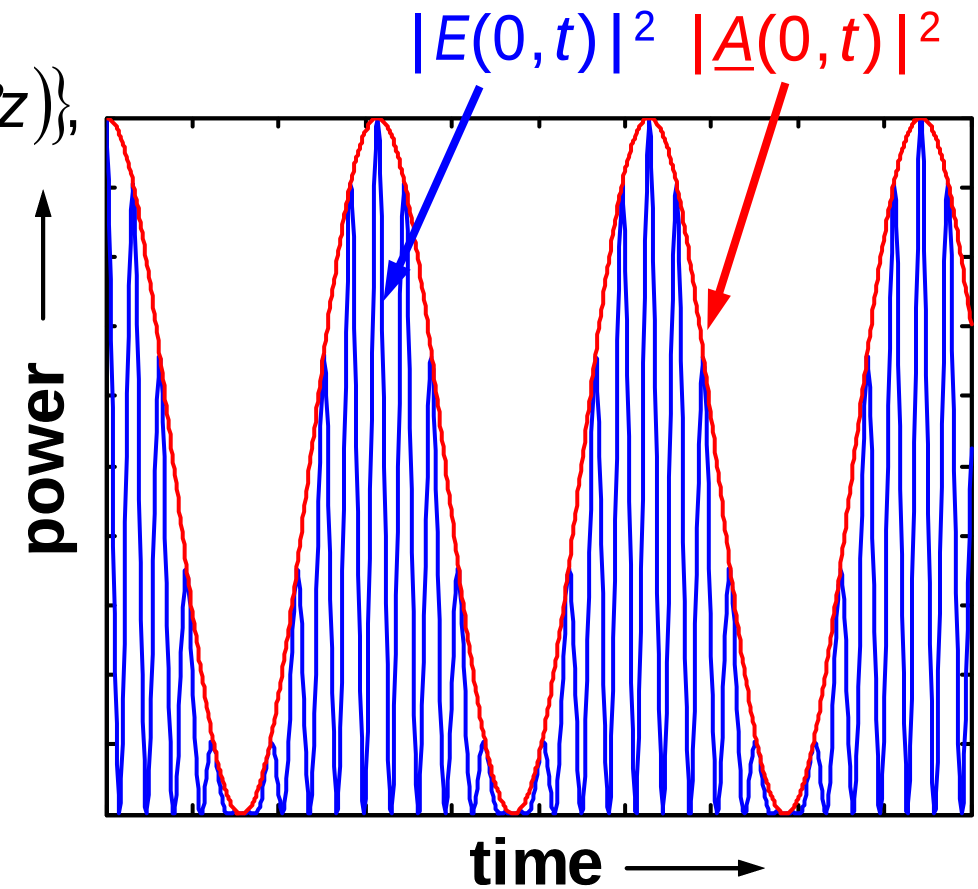
Electrical field:

$$E(z,t) = \mathrm{Re}\{\underline{A}(z,t)\exp(j\omega t - j\beta z)\},$$

with propagation constant

$$\beta = \frac{2\pi \cdot n}{\lambda},$$

where n is the effective refractive index.



$\underline{A}(z,t)$: slowly varying complex field envelope

$|\underline{A}(z,t)|^2$: pulse shape in time domain

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Nonlinear Schrödinger Equation (NLSE)

Pulse evolution along a fiber is governed by the NLSE.

$$\frac{\partial \underline{A}(z,t)}{\partial z} = \underbrace{-\frac{\alpha}{2}\underline{A}(z,t)}_{\text{attenuation}} + \underbrace{i\frac{\beta_2}{2}\frac{\partial^2 \underline{A}(z,t)}{\partial t^2}}_{\text{1st order GVD}} + \underbrace{\frac{\beta_3}{6}\frac{\partial^3 \underline{A}(z,t)}{\partial t^3}}_{\text{2nd order GVD}} \underbrace{- i\gamma\left|\underline{A}(z,t)\right|^2 \underline{A}(z,t)}_{\text{Kerr nonlinearities}}$$

1st order GVD: Characterized by the *dispersion parameter* *D* [ps/(km.nm)]

$$D = -\frac{2\pi \cdot c}{\lambda^2}\beta_2$$

2nd order GVD: Characterized by the *differential-dispersion parameter* (dispersion slope) *S* [ps/(km.nm²)]

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi \cdot c}{\lambda^2}\right)^2 \beta_3 - \frac{2}{\lambda}D$$

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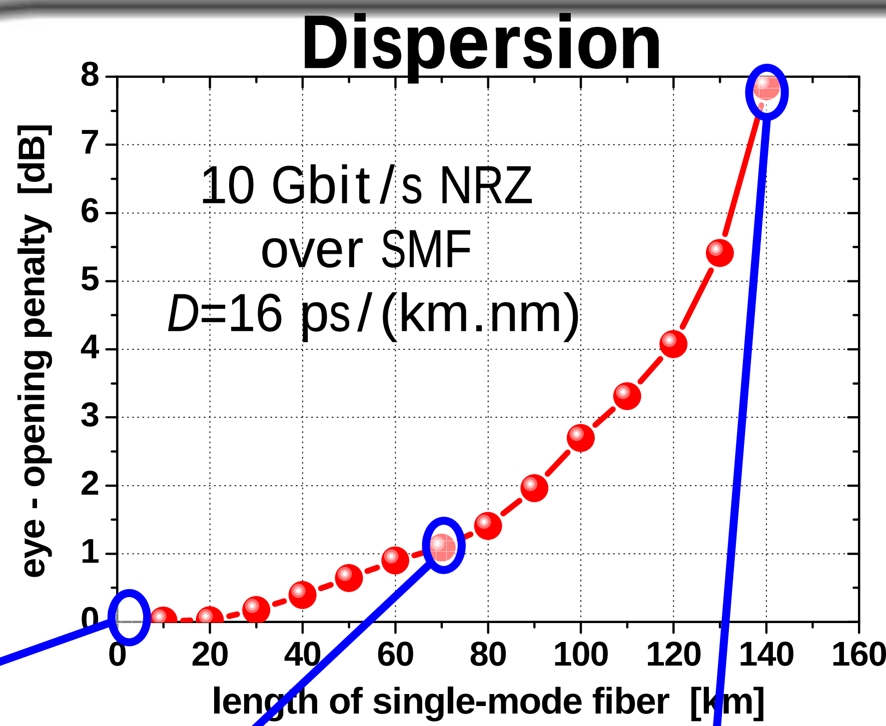
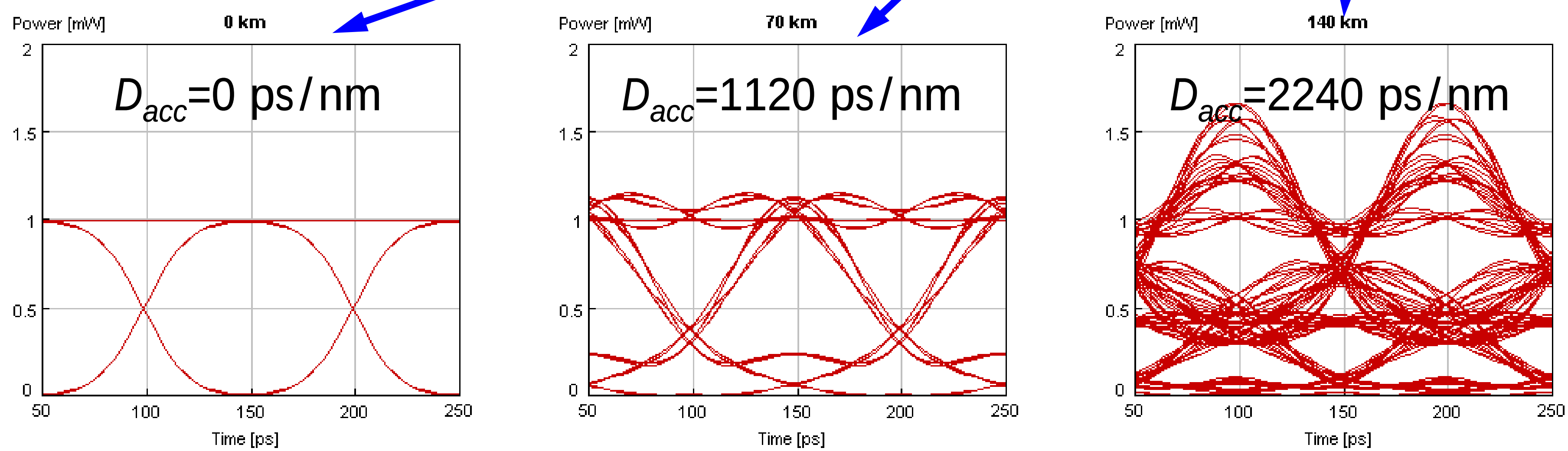
Dispersion



Pulse broadening in the time-domain due to dispersion leads to an increased eye-closure.

Characterized by accumulated dispersion D_{acc} [ps/nm].

$$D_{acc} = D \cdot L$$



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Dispersion length

Consider a Gaussian shaped pulse with $\underline{A}(0,t) = \underline{A}_0 \exp\left(-\frac{1}{2}\frac{t^2}{T_0^2}\right)$

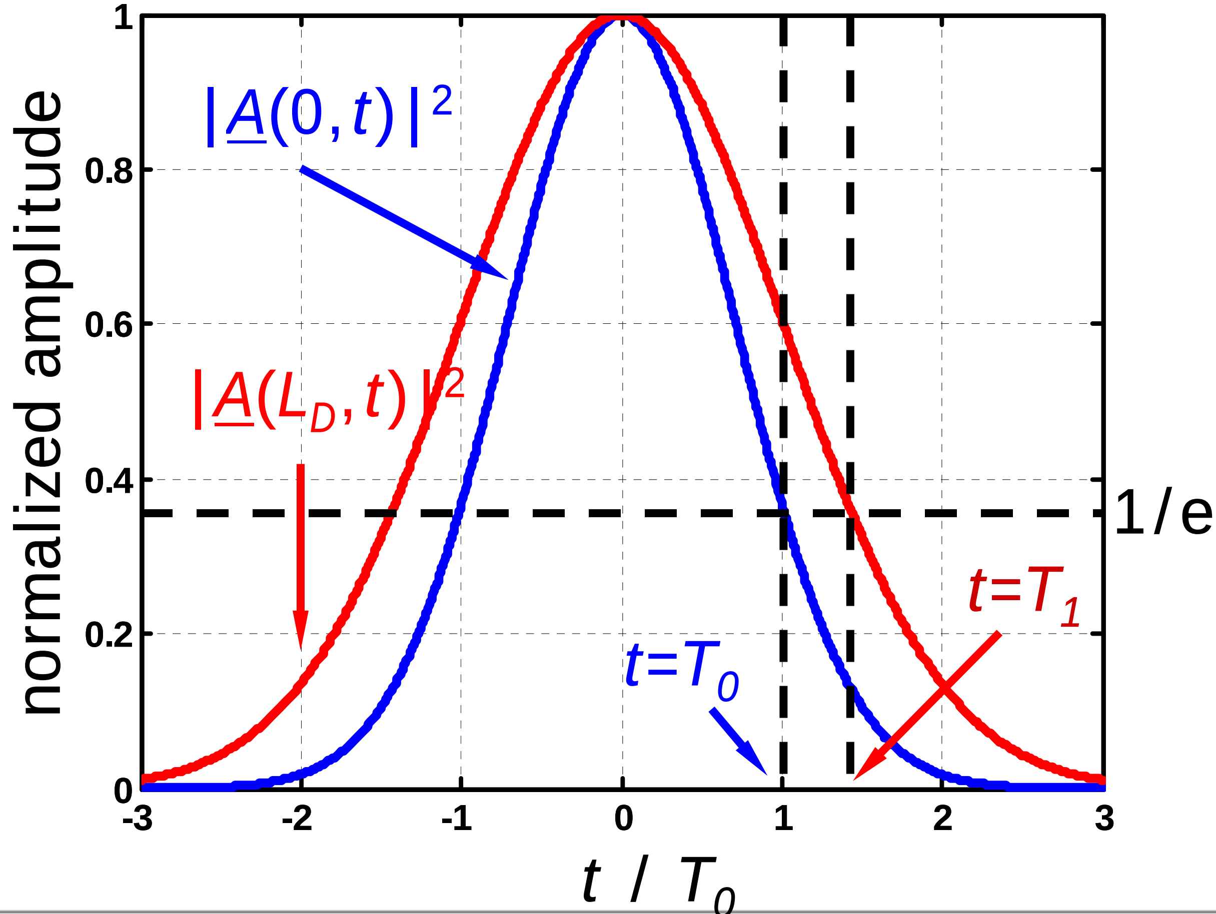
Dispersion length is defined as:

$$\boxed{L_D = \frac{T_0^2}{|\beta_2|}}$$

Pulse-shape at $z=L_D$?

Broadening factor at $z=L_D$:

$$\frac{T_1}{T_0} = \sqrt{2}$$



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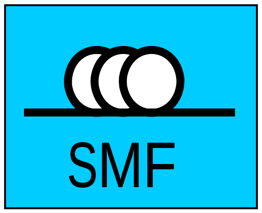
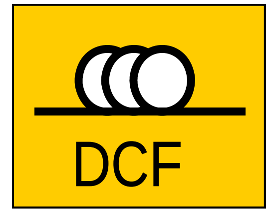
Dispersion compensation

Dispersion is a linear effect. → It can be compensated.

Commonly used: *dispersion-compensating fiber* (DCF)



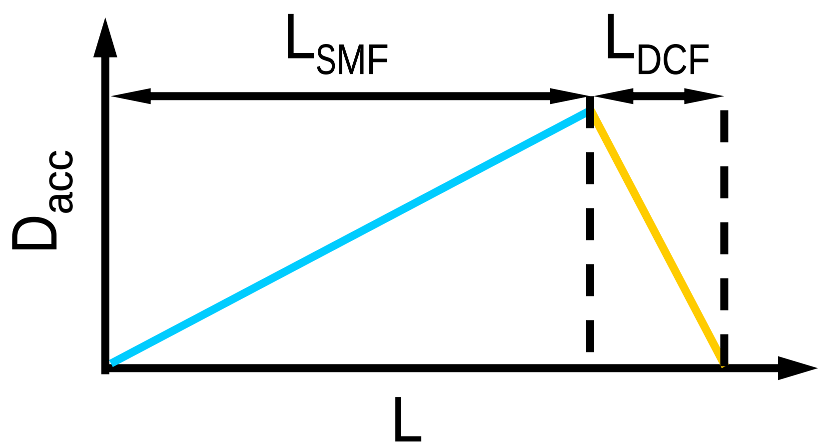


Positive dispersion parameter:

$$D_{SMF} \approx 17 \frac{ps}{km \cdot nm}$$

Negative dispersion parameter:

$$D_{DCF} \approx -100 \frac{ps}{km \cdot nm}$$



Requirement for complete compensation of 1st order GVD at a single wavelength:

$$\boxed{L_{SMF} D_{SMF} = -L_{DCF} D_{DCF}}$$

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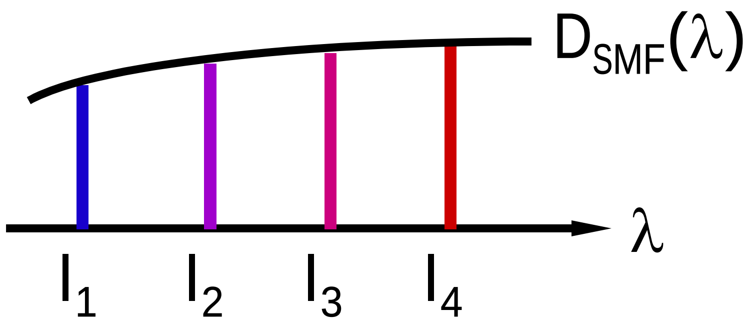
Dispersion slope

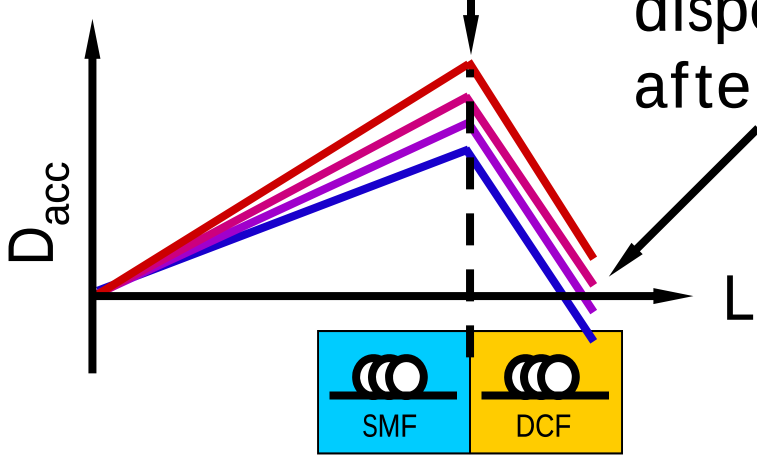
Dispersion parameter D is a function of λ.



Consider transmission of four wavelength channels.

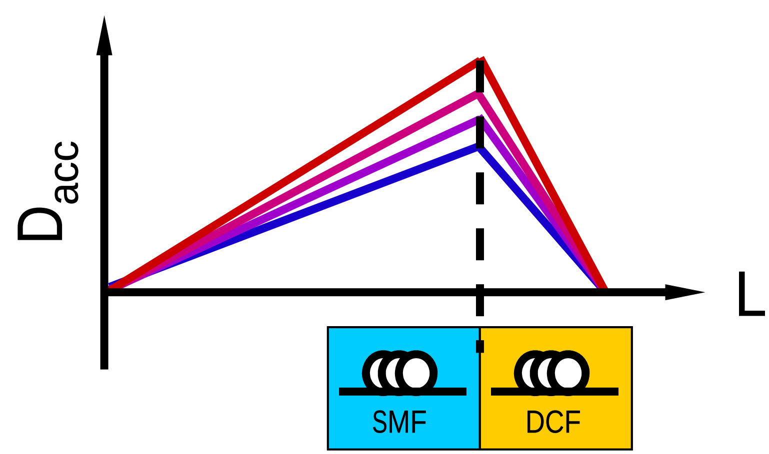
Different accumulated dispersion.

Residual dispersion after DCF.



Requirement for ideal 2nd order GVD compensation

$$\boxed{\frac{S_{SMF}}{S_{DCF}} = \frac{D_{SMF}}{D_{DCF}}}$$ (for all λ)



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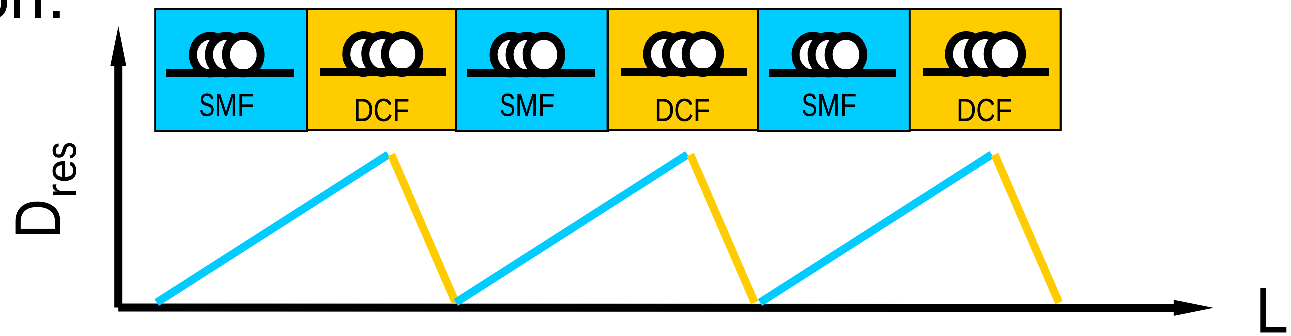
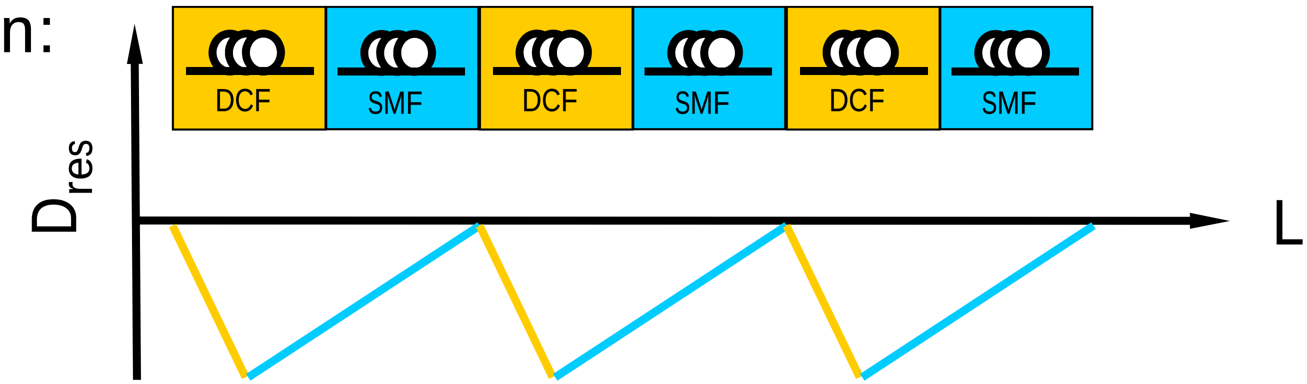
Dispersion-compensation schemes

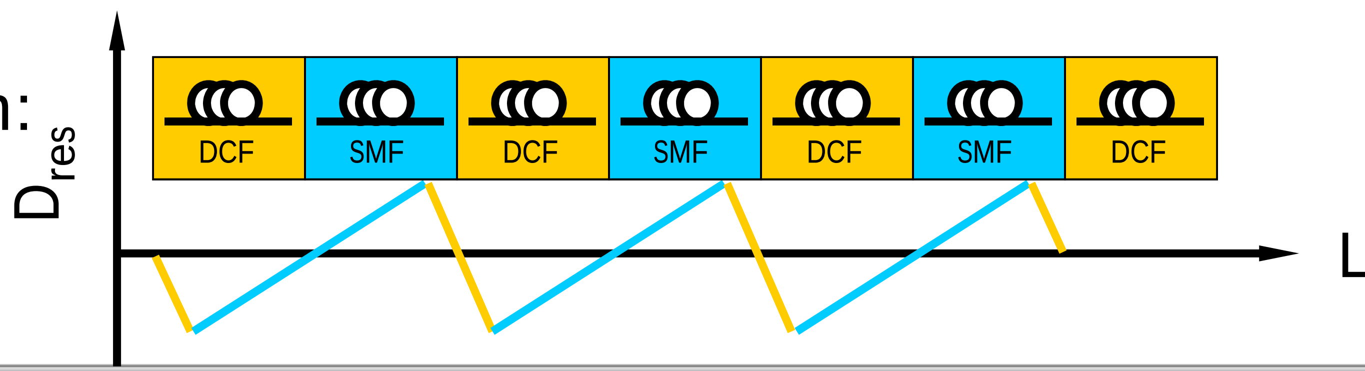
Postcompensation:



Precompensation:



Hybrid-compensation:



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Kerr nonlinearities

In general, the refractive index varies with the power of the optical field.

$$n' = n + \underbrace{\boxed{n_2 \frac{P}{A_{eff}}}}_{\text{nonlinear contribution}}$$

n_2: *nonlinear-index coefficient*

A_{eff}: *effective core area*

→ Propagation constant becomes power dependent.

$$\beta' = \beta + \boxed{\gamma \cdot P}$$

Nonlinearity coefficient: $\gamma = \frac{k_0 n_2}{A_{eff}}$

NLSE:

$$\frac{\partial \underline{A}(z,t)}{\partial z} = -\frac{\alpha}{2}\underline{A}(z,t) + i\frac{\beta_2}{2}\frac{\partial^2 \underline{A}(z,t)}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 \underline{A}(z,t)}{\partial t^3} - \boxed{i\gamma\left|\underline{A}(z,t)\right|^2}\underline{A}(z,t)$$

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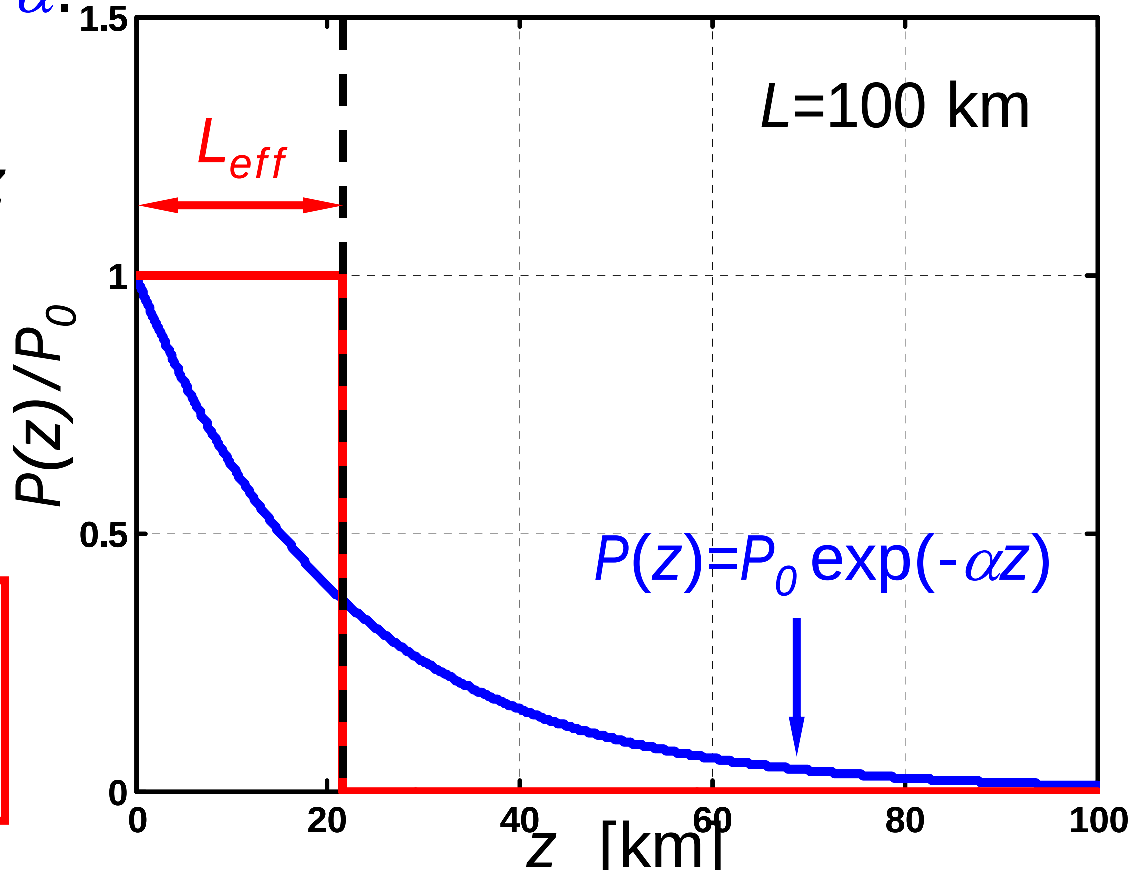
Effective interaction length L_{eff}

Signal power decreases exponentially with distance *z*.

Effective interaction length L_{eff} is the length of a fiber with zero attenuation, which has the same nonlinear impact as a fiber with attenuation α.

$$P_0 L_{eff} = \int_0^L P_0 \exp(-\alpha z) dz$$

$$\boxed{L_{eff} = \frac{1-\exp(-\alpha \cdot L)}{\alpha}}$$



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Self-phase modulation (SPM)

- Phase of one wavelength channel is modulated by its power.
- Phase shift is time dependent, since signal power varies with time.

Example: Gaussian and super Gaussian pulses

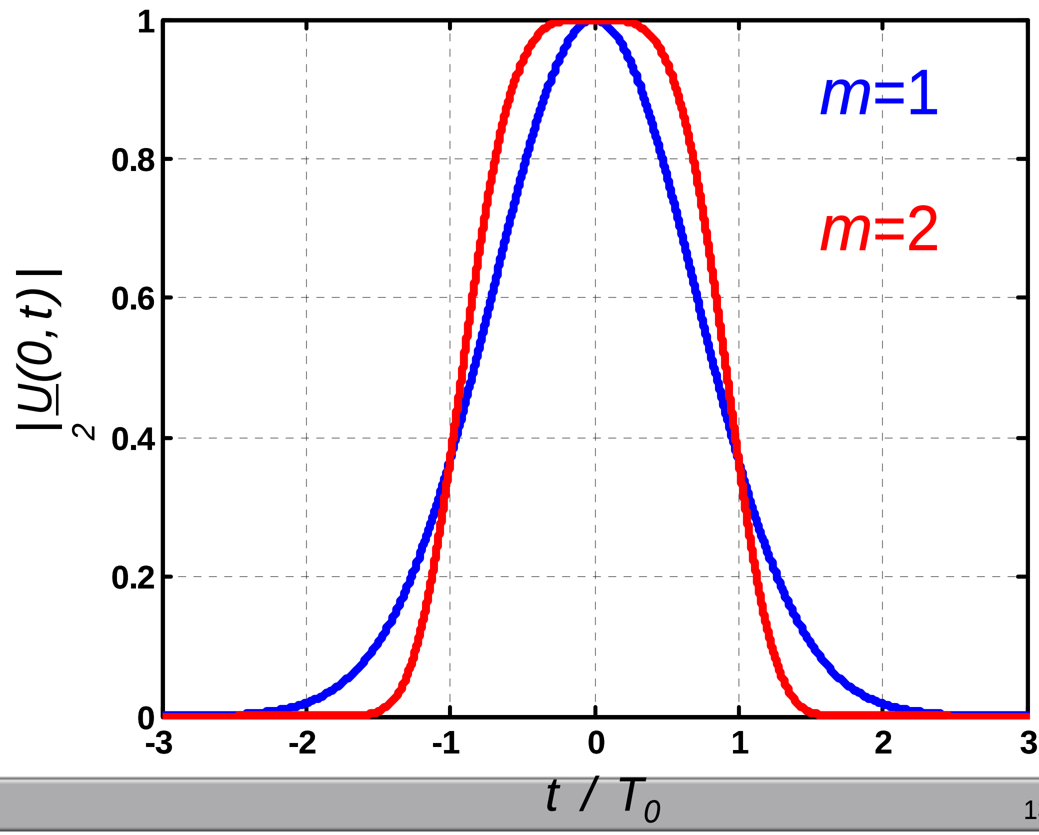
Normalized pulse shape:

$$\left|\underline{U}(z,t)\right|^2 = \frac{\left|\underline{A}(z,t)\right|^2}{P_0 \exp(-\alpha z)}$$

P_0: pulse peak power

Gaussian shape:

$$\left|\underline{U}(0,t)\right|^2 = \exp\left(-\frac{t^{2m}}{T_0^{2m}}\right)$$



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Self-phase modulation (SPM)

Nonlinear phase shift after a fiber of length L is given by:

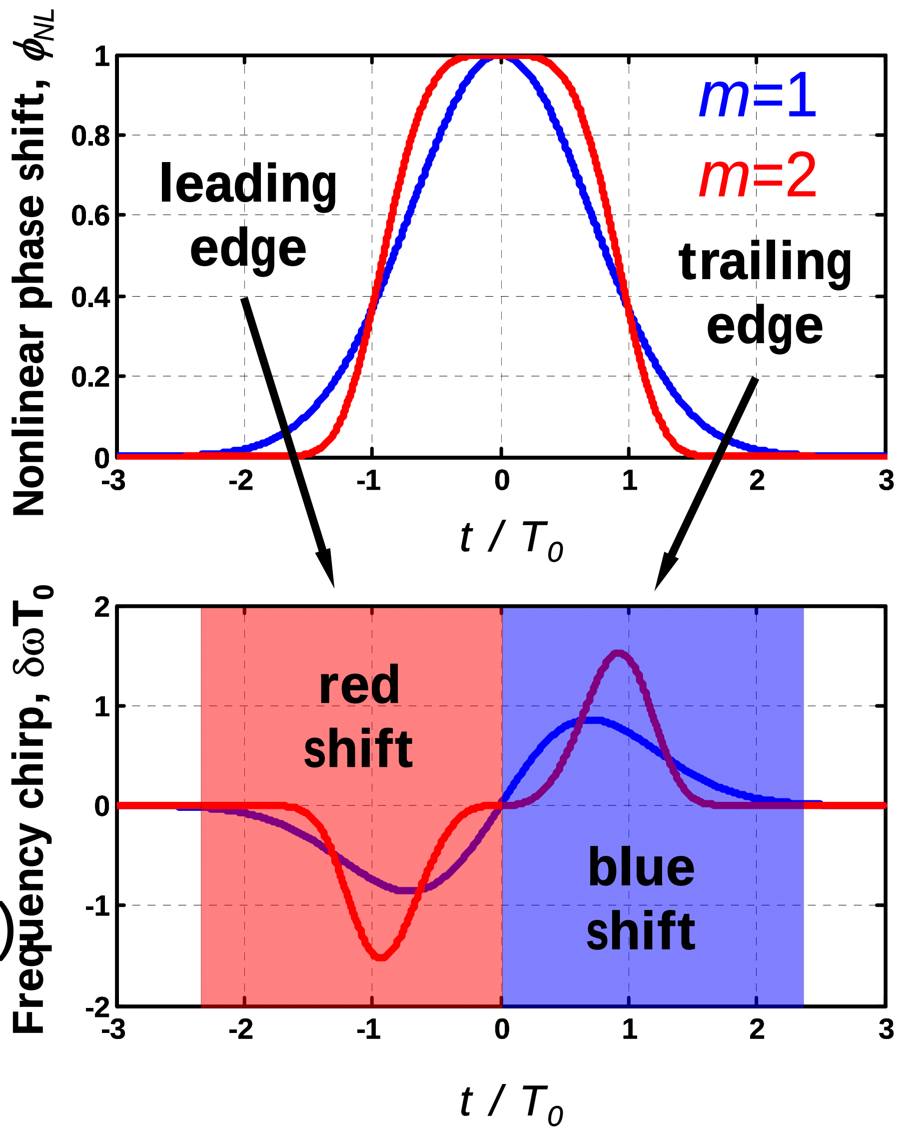
$$\phi_{NL}(L,t) = |U(0,t)|^2 \frac{L_{eff}}{L_{NL}}$$

Where the *nonlinear length* L_{NL} is defined as:

$$L_{NL} = \frac{1}{\gamma \cdot P_0}$$

A time dependent phase shift leads to a variation of frequency with time (chirp *dw*)

$$\delta\omega = -\frac{\partial \phi_{NL}}{\partial t}$$



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Self-phase modulation (SPM)

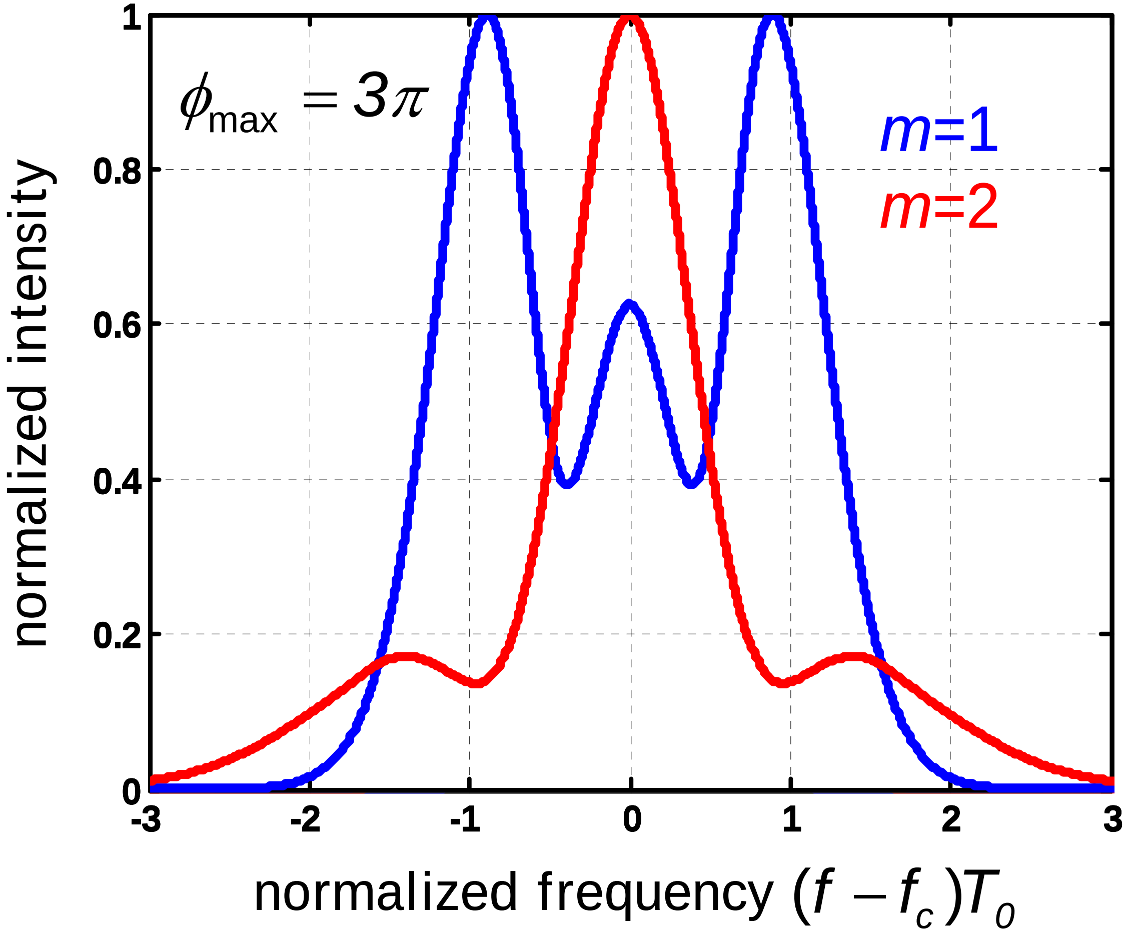
Through the frequency chirp $\delta\omega$, SPM generates new frequency components when a pulse propagates along a fiber. This leads to spectral broadening.

Example:
Gaussian and super Gaussian pulse with pulse shape

$$|\underline{U}(0,t)|^2 = \exp\left(-\frac{t^{2m}}{T_0^{2m}}\right)$$

f_c : carrier frequency

Spectral evolution?



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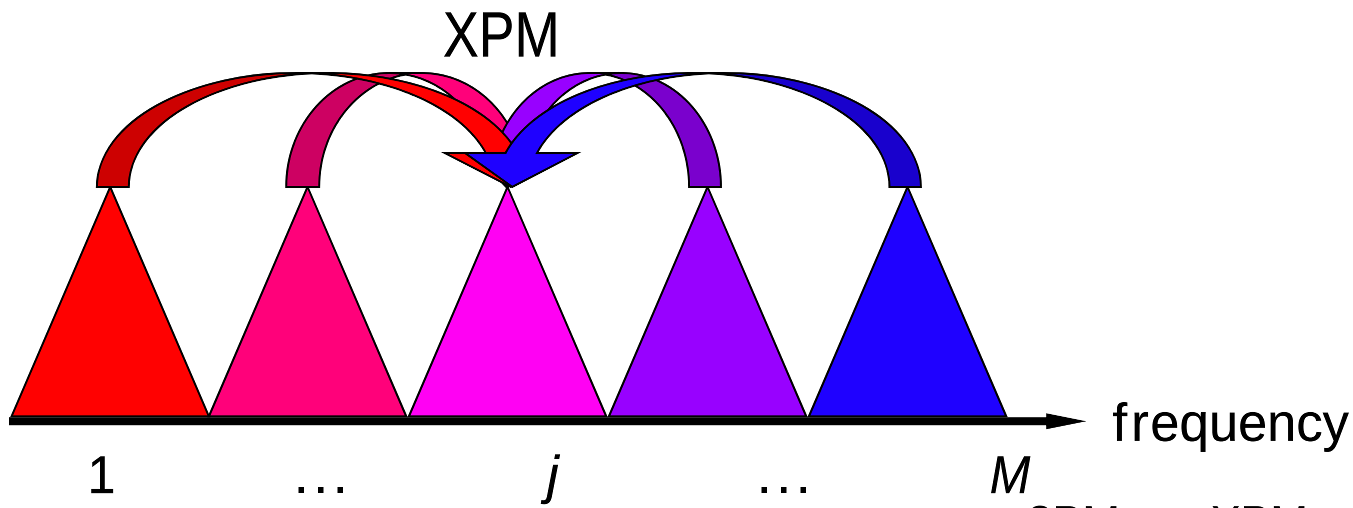
Cross-phase modulation (XPM)

When several waves co-propagate inside a fiber, the nonlinear contribution to the refractive index depends on the power of all co-propagating waves.

⇨ Phase in one wavelength channel is modulated by the power in all other wavelength channels.



Nonlinear phase shift in channel j : $\phi_{NL,j} = \gamma \cdot L_{eff}\left(\overbrace{\boxed{P_j}}^{\text{SPM}} + \overbrace{\boxed{2\sum_{m\neq j}^{M} P_m}}^{\text{XPM}}\right)$

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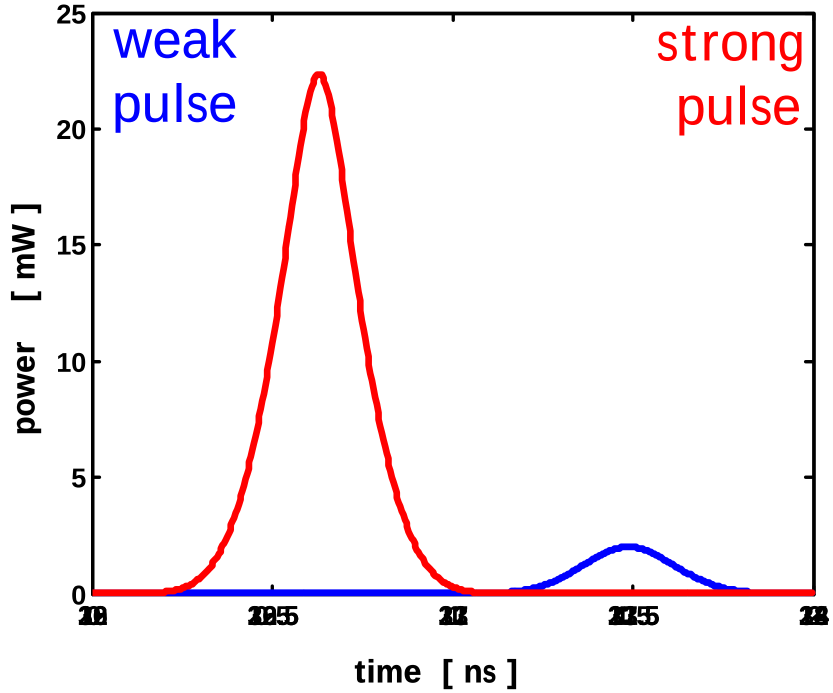
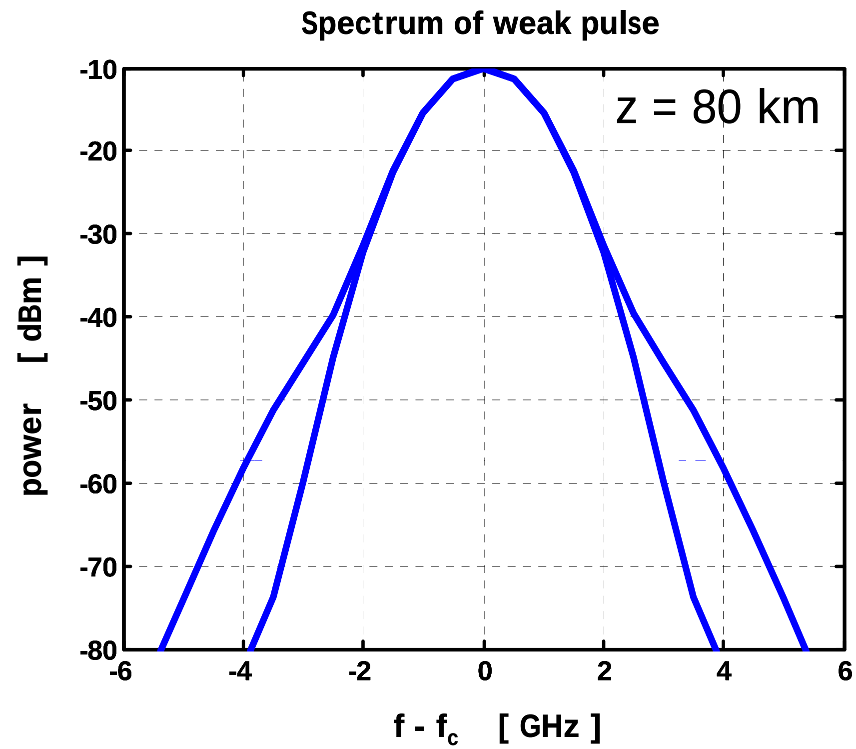
Cross-phase modulation (XPM)

XPM arises when two pulses of different WDM channels cross each other in the time domain.





Complete crossing of pulses leaves timing jitter due to fiber attenuation.

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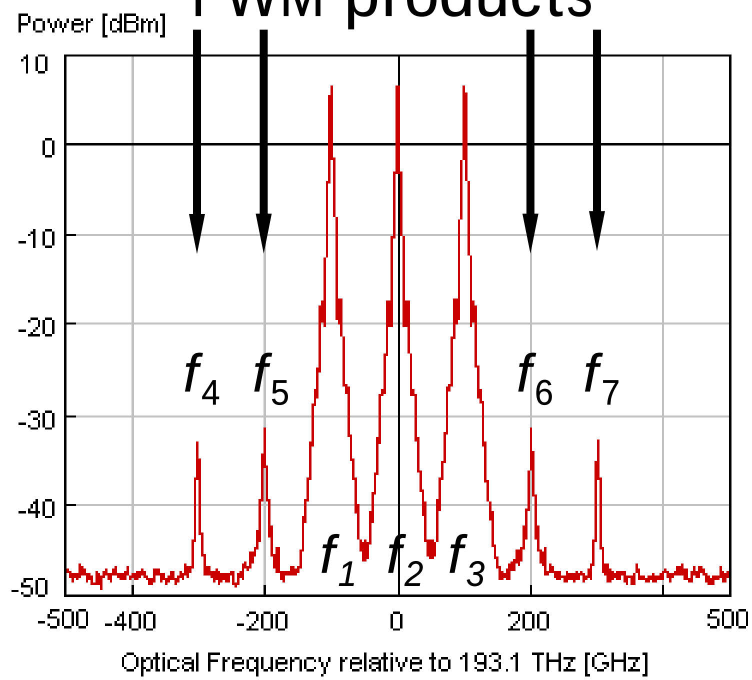
Four-wave mixing (FWM)

- The intensity dependence of the refractive index also leads to frequency mixing of optical waves.
- Consider three copropagating optical fields with carrier frequencies f_1, f_2, and f_3.
- Four-wave mixing generates new mixing products at frequencies:

$$\boxed{f_n = f_i + f_j - f_k}$$

with $f_i, f_j \neq f_k$.

out-of-band FWM products



Example:

FWM products at f_5:

$$f_5 = f_1 + f_2 - f_3 \quad \text{and} \quad f_5 = 2f_1 - f_2$$

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Effects of FWM

- Loss of signal power in all wavelength channels
- Coherent interchannel crosstalk between wavelength channels in systems employing equidistant channel spacing

⇒ In WDM systems with many channels, FWM effects can be considered as a degradation of signal-to-noise ratio.

How can the effects of FWM be minimized?

- Large channel spacing → Lower FWM-efficiency
- Unequal channel spacings → Incoherent out-of-band crosstalk
- Large fiber dispersion → Lower FWM-efficiency

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Dispersion management

Dispersion compensation has to accommodate different needs depending on system design.

Linear transmission systems:

- Zero residual dispersion at receiver
- Optimization of signal-to-noise ratio at receiver

Additionally in nonlinear transmission systems:

- Minimization of nonlinear effects

Optimization of the GVD profile along a transmission link with respect to the impact of fiber nonlinearities is commonly referred to as *dispersion management*.

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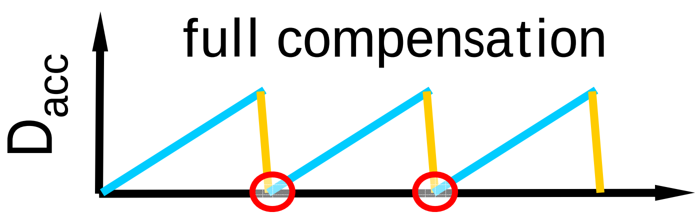
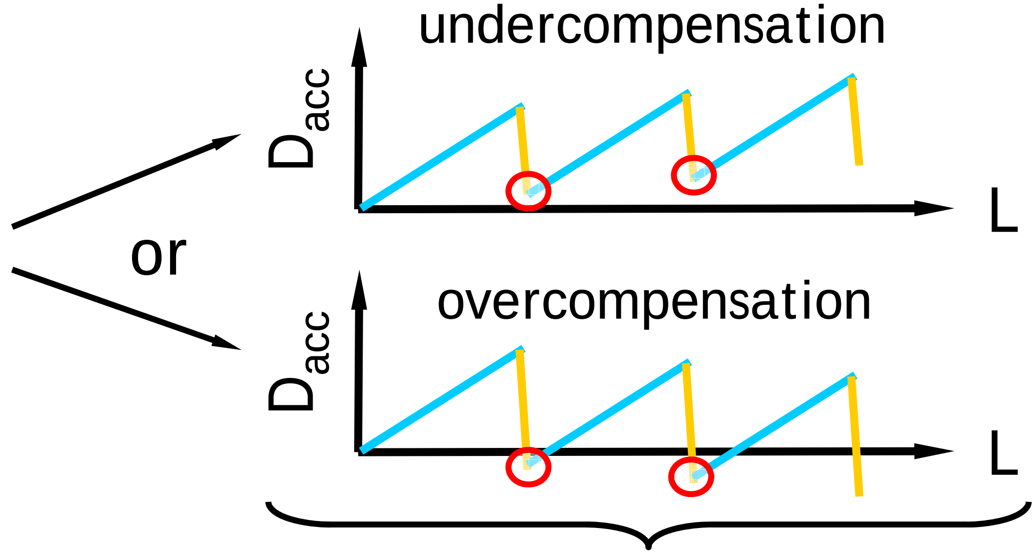
Dispersion management

Optimization of GVD profile through:

- choice of compensation scheme
- amount of residual dispersion per span





D_{acc} at start of each span:	equal (zero)	unequal
Nonlinear perturbations accumulate:	coherently	incoherently

⇨ Slight under- or overcompensation can reduce the accumulation of nonlinear perturbations.

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Summary

- **Nonlinear Schrödinger Equation**
- **Dispersion and dispersion-compensation schemes**
- **Kerr nonlinearities**
  - **Self-phase modulation (SPM)**
  - **Cross-phase modulation (XPM)**
  - **Four-wave mixing (FWM)**
- **Dispersion-management**

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