Optical Fibers: the Split-Step Algorithm

In a fiber, the Kerr nonlinearity is represented by separating the effects of dispersion and nonlinearity within a series of sections dividing the fiber's length. This allows the dispersion to be calculated efficiently in the frequency domain, while the nonlinearity requires calculation in the time domain.

The number of sections that a fiber is divided into greatly affects the calculation accuracy: infinitesimally-short sections would give the least approximation, but would require large computational effort as both the dispersion and nonlinear operators would have to be implemented a large number of times.

Part of the reason that short sections are required is that the FWM accumulates over the fiber's length from the contributions of the nonlinearity in all parts of the fiber. If the nonlinearity is calculated only at discrete points along the fiber (once per section), then it is important that the phases of the contributions at these points accurately represent the spread of phases and amplitudes of the distributed contributions in the real fiber.

Dispersion

Given a dispersion-less fiber, all contributions to FWM will contribute with the same phase to the accumulated FWM tone. Thus, the phase will not matter, and so inaccuracy will only be a result of misrepresenting the amplitude of the FWM tone, as the contributions will reduce along the fiber length due to fiber attenuation. However, fiber attenuation is weak, and the power decreases by $1/e$ over 20 kilometer or more. Thus, a section length (entered as **MaxStepWidth**) of 1 kilometer is accurate enough to represent the changing amplitude of the FWM contributions in this case.

However, if the fiber has dispersion, then phase of the contributions to FWM will rotate along the fiber. The resultant power is the square of the vector sum of the contributions, which trace a circle on which the origin lies. This leads to a sine^2-like fluctuation in the power in a FWM tone along the fiber. The contributions to FWM in this case is shown in Figure 6-1 A.

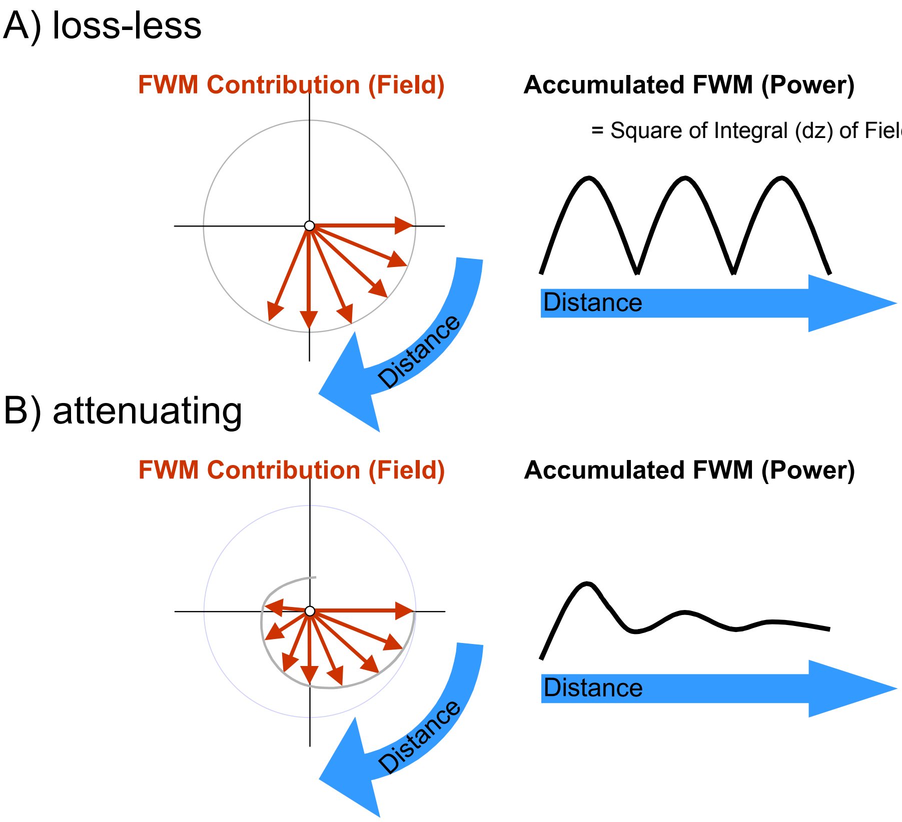

Figure 6-1. *Representation of the contributions to the FWM tone from each section of the fiber model. (A) loss-less, (B) attenuation*

Because of attenuation, the contribution from each section reduces along the fiber (see Figure 6-1 B), which because of their diminishing power when added, trace a spiral path around a point away from the origin. Thus the FWM power reaches a non-zero steady state. That is, for a fiber longer than approximately 60 kilometers, the oscillations in accumulated FWM power will die away.

See the demo *WDM* → *Tutorials* → *Component Behavior* → *Kerr Nonlinearity* → *Four Wave Mixing* to see the growth of FWM along a fiber.

This effect can be seen in Figure 6-2, which shows the simulated power in a carrier and FWM tones 100 GHz apart, for zero-dispersion, near-zero-dispersion and standard fiber at 1550 nm. Two carriers of 1-mW each were transmitted along the fibers. The loss was 0.25 dB/km and the third-order dispersion 0.08 ps/nm^2/km. The reference wavelength of the fiber was placed on the upper carrier, and the power in the lower FWM tone was monitored. Note the rapid oscillation in power in the high-dispersion power due to the rapid rotation in phase of the FWM contribution with respect to the FWM tone.

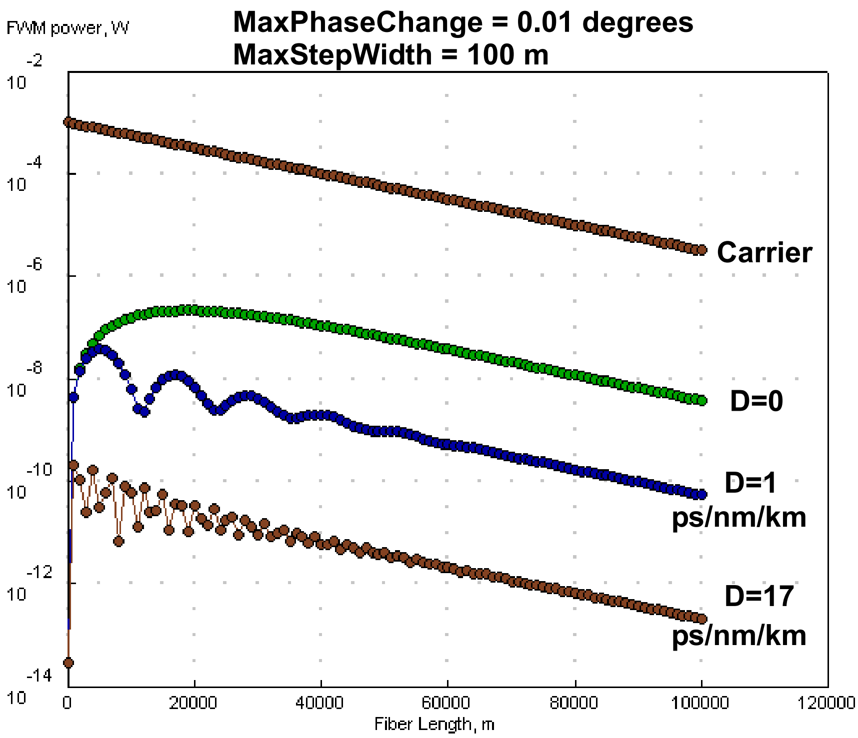

Figure 6-2. *Power in one carrier of a two-carrier system, and the simulated powers in the lower FWM product for three types of fibers*

The rapid oscillation in FWM power suggests that very short step lengths should be used (at least several steps per period of the oscillation, and preferably an integer number of steps), particularly for high-dispersion fibers. This can be shown by plotting the FWM power at 100 km versus the step length for the three fibers (see Figure 6-3). Here the predicted FWM power in the standard fiber oscillates with step length for lengths greater than 600 meters, and only converges to a small value for very short step lengths (<100 meters). This oscillation is because of the small number of sampling points, there are step lengths at which the sample points always sample FWM contributions with the same phase, so the accumulated FWM is a large overestimate.

This error may not matter to system performance, because the predicted FWM power is small compared with the output power of the fiber (-72 dB), and the FWM power is always an overestimate; however it is of course preferable that the results of a numerical simulation do not depend on numerical (non-physical) parameters.

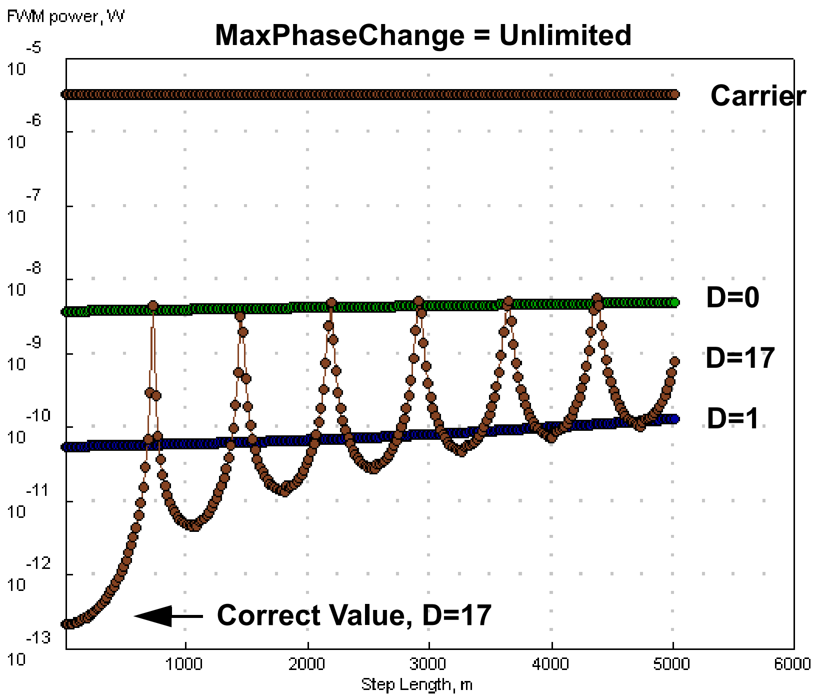

Figure 6-3. *Variation of predicted FWM power with step length for three fiber types*

Improving the Accuracy

The algorithm can be greatly improved if the step length is varied dynamically through a range of values. This means that the phase of the FWM contribution covers more phase space. This can be achieved by setting the step length to limit phase shift (due to nonlinearities) per section. Naturally, limiting the phase shift improves the accuracy of the solution of the nonlinear equation.

Figure 6-4 shows the variation of FWM power with step length for a maximum phase shift per step of 0.01 degrees. The prediction of FWM power for the standard fiber is now much more stable. However, there are discrepancies at certain phase values. Importantly, however, below 600 meter step length gives a reliable prediction, unlike the previous simulation.

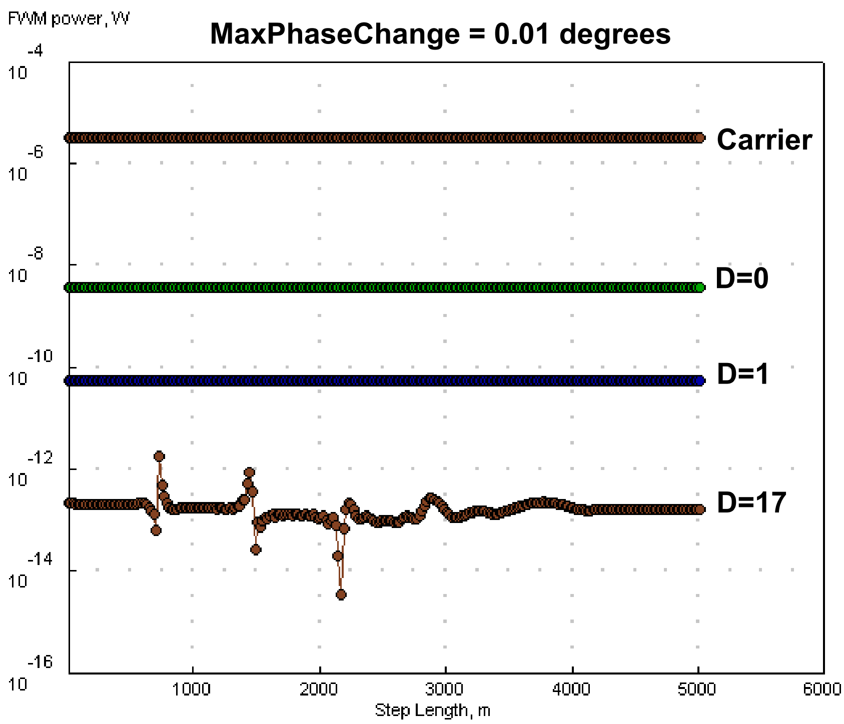

Figure 6-4. *Variation of predicted FWM power with maximum step length (the step width is also limited to give a maximum phase shift) for three fiber types*

From these simulations, a combination of maximum step length and maximum phase change is recommended for standard fiber. This is set in the Numerical Tab-pane of the fiber model by setting:

MaxStepWidth = `100` m

MaxPhaseChange = `0.01` degrees

Note that for low-dispersion fibers, the step size can be scaled upwards.

These tests can be repeated using the schematic shown in Figure 6-5. A multidimensional-sweep is used to run through fiber or step length, for the three fiber types.

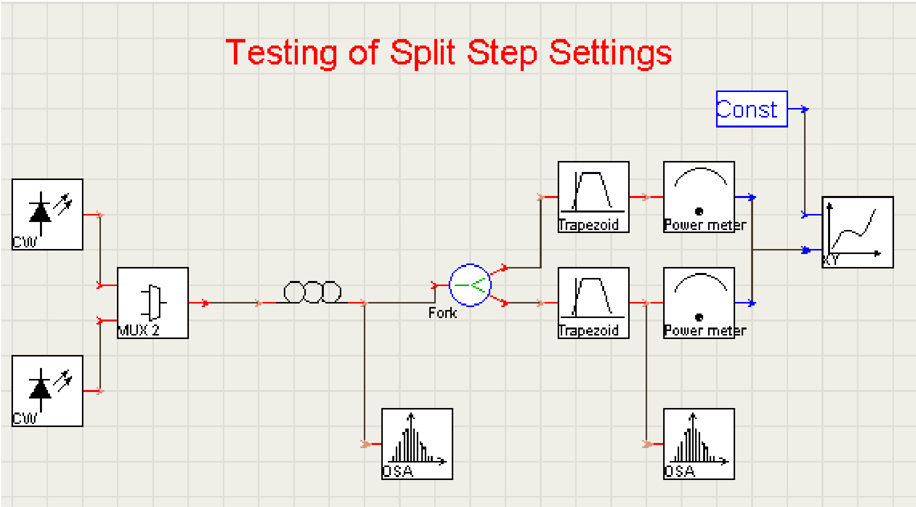

Figure 6-5. *Schematic for examining the performance of the split-step algorithm*

Dithering Optical Channel Frequencies

The results of a simulation of Four-Wave Mixing will depend on the relative phases of the optical carriers in a system, as the phase of a FWM product (relative to the carrier of the channel it is polluting) will determine whether constructive of destructive interference occurs. One method of ensuring that all phase space is covered is to detune the carriers from a regular channel spacing. For example, for three carriers,

- space two channels at 100 GHz
- space the third channel at 100 GHz + 1/**TimeWindow** from the first two.

This will ensure that the FWM product from the 100-GHz spaced channels will fall at one frequency resolution point from the third carrier. Thus is will rotate in phase relative to the third carrier over 360-degrees during the simulated block.

The waveform of Channel 2 of a 4-channel WDM system with zero-linewidth sources with and without detuning is shown in Figure 6-6. The amplitude variation with detuning is far higher than without as the phase of the FWM produces from adjacent channels rotates relative to the channel 2 carriers.

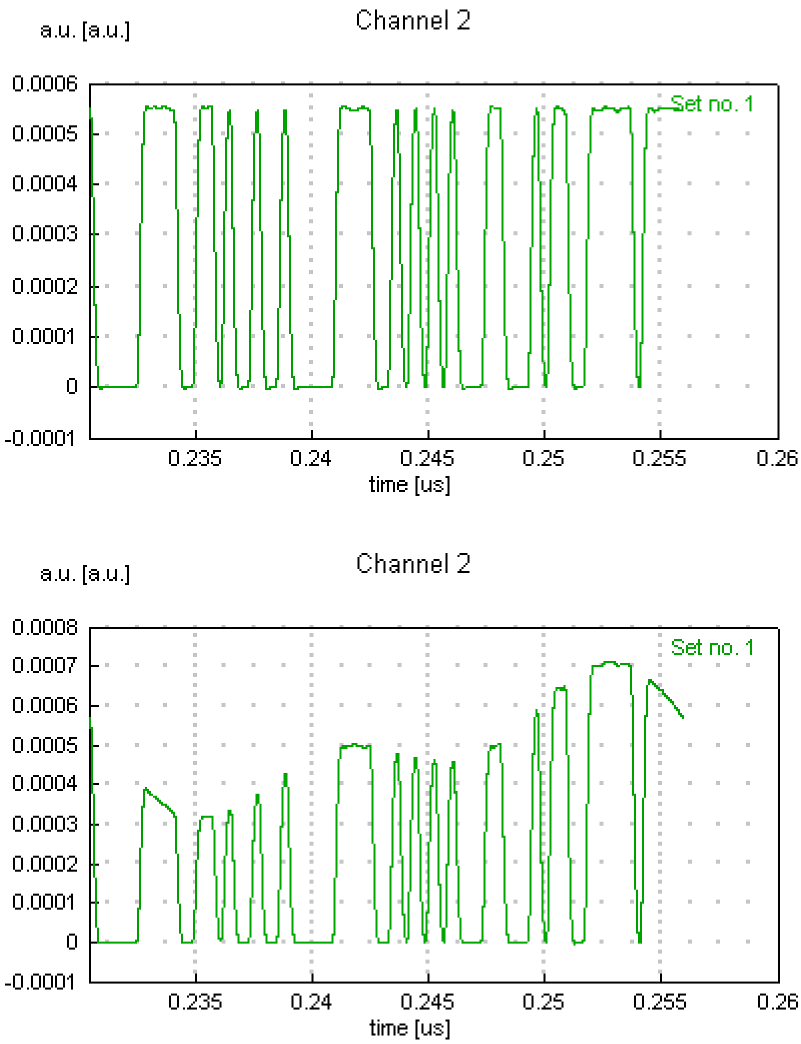

Figure 6-6. *Simulated waveform of Channel 2 (zero linewidth sources) subject to interference generated by FWM. Top: regularly spaced channels. Bottom: channel 2 carrier detuned by 1/***TimeWindow**

Alternatively, non-zero linewidth sources can be used and the simulation should be performed over at least a time equal to the inverse of the linewidth of the sources. This will ensure that the laser's phases walk over a reasonable range of phases during a simulation. This is similar to having several random walkers, who all set off from the same point: they will tend to remain near to that point during a short simulation. All sources will initially start at zero phase (unless set otherwise), and so it is unlikely that all phase combinations will be covered. A method of ensuring that phase space is reasonably covered is to set the **InitialPhase** of each transmitter randomly from block to block.

The waveform of Channel 2 of a 4-channel WDM system with 12-MHz linewidth sources is shown in Figure 6-7. Note the apparently random amplitudes of the bits that have been affected by FWM.

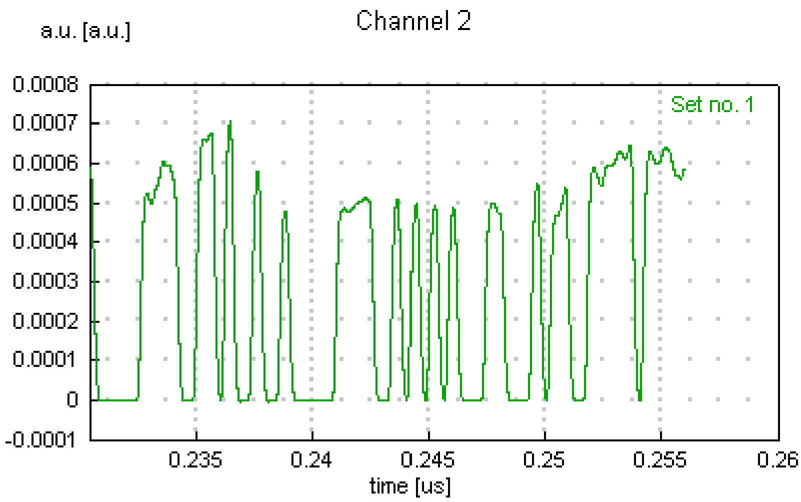

Figure 6-7. *Simulated waveform subject to interference generated by FWM with 12-MHz linewidth sources*

TDM Demos

Comparison of TDM-Systems

Tutorials → Component Behavior → Linear Effects → NRZ pre-post compensation

This demo introduces simulations of time-division-multiplexing (TDM) scenarios, where we mainly focus on the optical transmission layer. Just like Wavelength Division Multiplexing (WDM), TDM is another way to increase the transmission capacity of a given fiber link. Multiplexing and demultiplexing schemes will be considered in detail in the next release. Ten Gbit/s NRZ and 40 Gbit/s RZ transmission using different dispersion compensating schemes and standard fiber links are taken into account.

Simulation Layout and Parameters

Demo NRZ pre-post compensation describes a comparison of three TDM transmission networks, which introduce different dispersion compensation schemes (see Figure 8-1). The system is as follows:

- The transmitter is a conventional NRZ source, realized by the module *Externally Modulated Laser Transmitter*, which transmits at a bit rate of 10 Gbit/s. Multiplexing schemes are not considered in our simulation.
- The optical signal is propagated through a fiber link, which is realized by a ring setup and various fiber dispersion management modules.
- The receiver uses an optical filter for suppression of ASE noise, a PIN photodiode (including shot and thermal noise) and an electrical low pass filter.

The eye diagram of the received signal and the transmitted as well as received time signal are also shown. A sequence of 32 bits is transmitted at a **BitRate** of 10 GHz. Each sequence contains two preceding and two succeeding zero bits.

The **SampleRate** of the system is set to 320 GHz. Generally, the goal in medium distance fiber transmission is large amplifier spacing. In our example, 120 km is assumed.

To guarantee a high OSNR at the receiver, a large signal level must be generated by the transmitter (with a **PeakPower** of 13 dBm). This causes distortion of the signal due to fiber non-linearity.

Dispersion compensation is necessary for 10 Gbit/s transmission over a single-mode fiber link of several-hundred kilometers in total length. In our simulation, three examples are introduced for fiber dispersion management using dispersion compensated fibers (DCF). Precompensation, postcompensation and symmetrical compensation using two amplifiers are regarded. Two amplifiers in the schemes are necessary in order to compensate for the high total fiber losses.

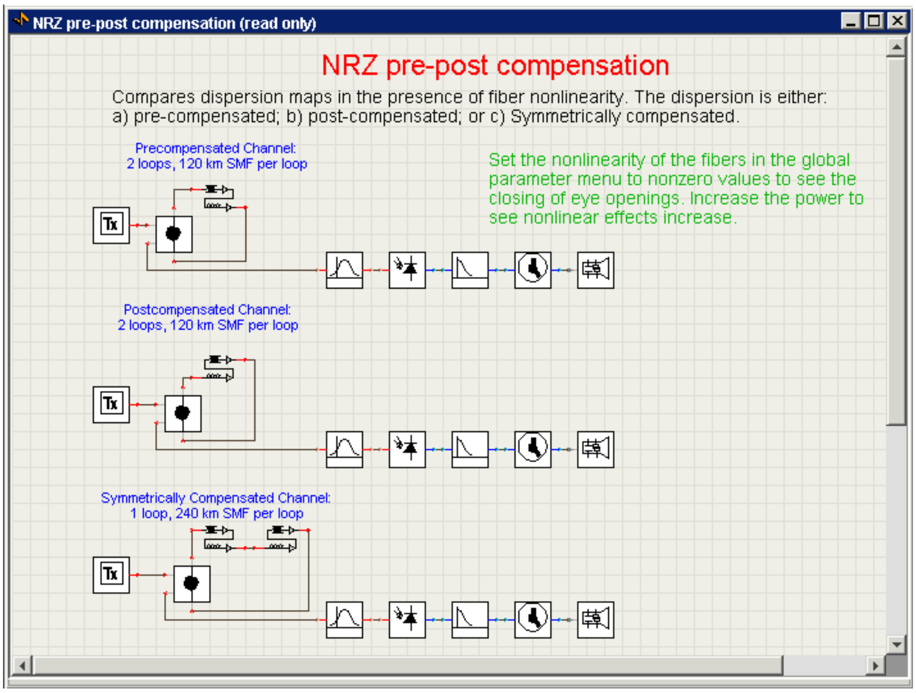

Figure 8-1. *Schematic for NRZ pre-/post-compensation*

For the SMF link and the succeeding amplifier, the following parameters are chosen:

- The length of the transmission fiber **SMF_Length** is set to 120 km.
- The gain of its succeeding amplifier **SMF_Gain** is chosen to be 24 dB to compensate for the fiber loss of 0.2 dB/km.
- The dispersion **SMF_Dispersion** is 16×10^{6} s/m^2 and the nonlinear index **SMF_NonLinearIndex** is 2.6×10^{-20}m^2/W.

For the DCF and its amplifier, they are set to:

- The dispersion **DCF_Dispersion** is -90×10^{6} s/m^{2}
- Therefore, a fiber length **DCF_Length** of 24 km is necessary to compensate for the SMF dispersion
- The EDFA gain **DCF_Gain** is 12.8 dB, to compensate for the fiber loss of 0.6 dB/km.
- The non-linearity index **DCF_NonLinearIndex** is chosen to be 3.0×10^{-20} m^{2}/W

Typical Results

Running the simulation without non-linearity (**DCF_NonLinearIndex** and **SMF_NonLinearIndex** nullified by multiplying them with zero) produces the same result for all three schemes, as shown in the following examples.

Now remove the zero multipliers from the parameters. **DCF_NonLinearIndex** and **SMF_NonLinearIndex** to re-instate the effect of nonlinear indices. Re-running the simulation shows the eye diagrams of the three schemes are identical because the dispersion of the SMF is completely compensated by the DCF.

In the case of *precompensation,* an overshoot at longer "1" trains is visible. Single "1" pulses show a pulse compression, which induces a higher 1-level in these pulses. Due to these effects, the eye will close in the horizontal direction (see Figures 8-2 and 8-3).

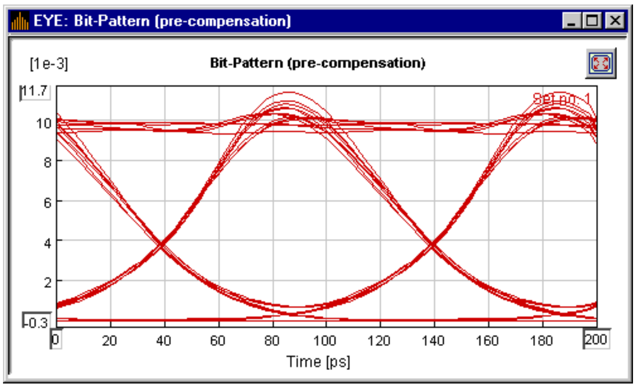

Figure 8-2. *Eye diagram (pre-compensation)*

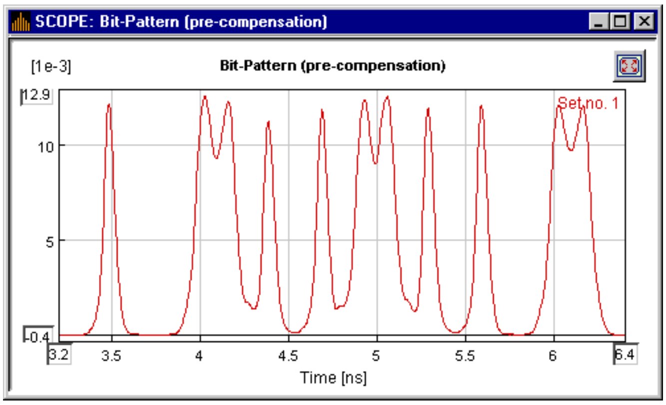

Figure 8-3. *Waveform (pre-compensation)*

If *postcompensation* is applied (see Figure 8-4), the received peak power is reduced (especially at single pulses). Therefore, the effect of non-linearity on the pulses changes. Pulse broadening induces a lower 1-level in single pulses. The resulting eye will close in the vertical direction and is asymmetrically deformed. No overshoot appears, in contrast to the precompensation scheme.

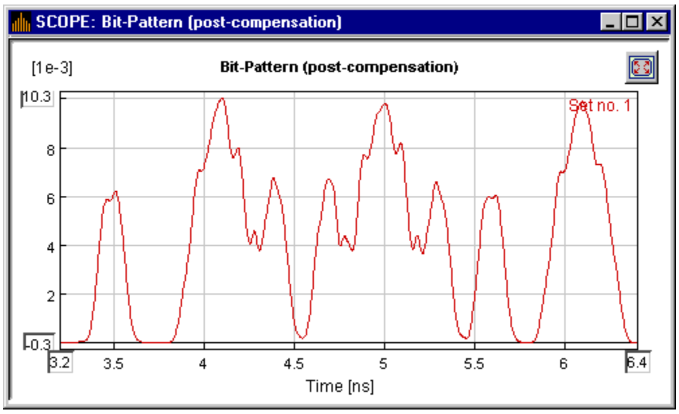

Figure 8-4. *Waveform (post-compensation)*

In case of *symmetrical compensation* (see Figure 8-5), both pre- and post-compensation are applied. Therefore, the effects of pulse compression and pulse broadening nullify each other. Overcompensation lowers overshoot significantly in comparison to a precompensation scheme. However, some overshoot still appears for longer sequences of 1's. Single pulses show a pulse shape almost similar to the linear case. The horizontal opening of the eye is larger then in pre- or postcompensation schemes. The vertical eye opening is better than in a precompensation scheme, and comparable to the eye in the postcompensation scheme. Unlike the postcompensation scheme, the eye in the symmetrical compensation scheme will have a symmetrical shape. Eye closure in this case results from an increased 0-level.

If a further enhancement of performance is desired, this can be achieved by a reduction of the power level in the DCF (e.g., by changing the gain of the EDFAs or using a lower transmitter **PeakPower**). Decreasing the power level in the DCF decreases the influence of the non-linearity. To see the influence of the DCF non-linearity on the performance, just set the parameter **DCF_NonLinearIndex** in the global parameter menu to zero. This causes a larger eye opening, especially at symmetrical components and less overshoot at longer sequences of 1's.

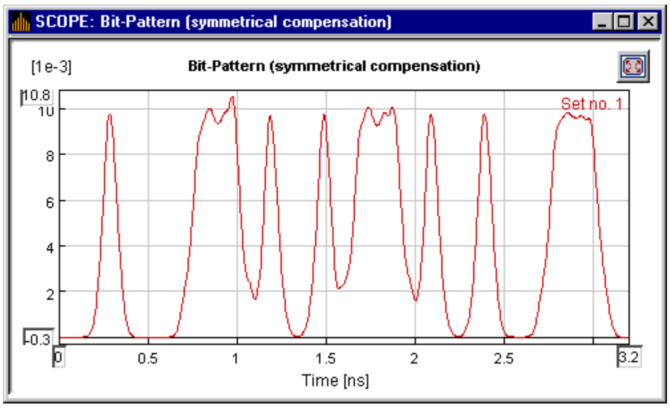

Figure 8-5. *Waveform (symmetrical compensation)*