VPI University Program

Photonics Curriculum Version 7.0

Lecture Series

VPIphotonics™

Polarization Effects

Fiber 3

Developed in cooperation with
Prof. Klaus Petermann
and his group at
Technische Universität Berlin

Author: J.K. Fischer

Module Prerequisites

- **Fiber 1: Basics of Fiber Propagation**

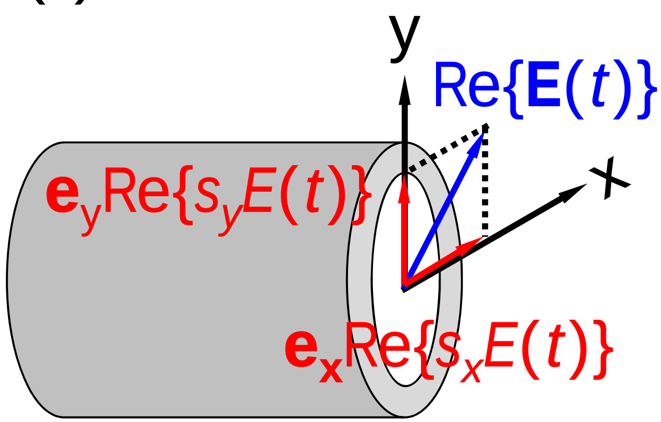
Module Objectives

- Polarization
- Polarization-mode dispersion (PMD)
- Polarization-dependent loss (PDL)

Polarization: Jones-Formalism

Complex transverse electric field vector $\mathbf{E}(t)$:

$$\mathbf{E}(t) = |s\rangle E(t)$$



$E(t)$: Complex electric field amplitude

$|s\rangle = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$: Unit Jones-vector (ket-vector)

$\langle s| = \begin{pmatrix} s_x^* & s_y^* \end{pmatrix}$: Corresponding complex conjugate vector (bra-vector)

$|s\rangle$ is normalized, such that (bra-ket notation):

$$\langle s|s\rangle = s_x^* s_x + s_y^* s_y = 1$$

Notation adopted from:
J.P. Gordon and H. Kogelnik,
„PMD Fundamentals: Polarization mode dispersion in optical fibers,“
Proc. of the National Academy of Sciences USA, vol. 97, no. 9, pp. 4541-4550

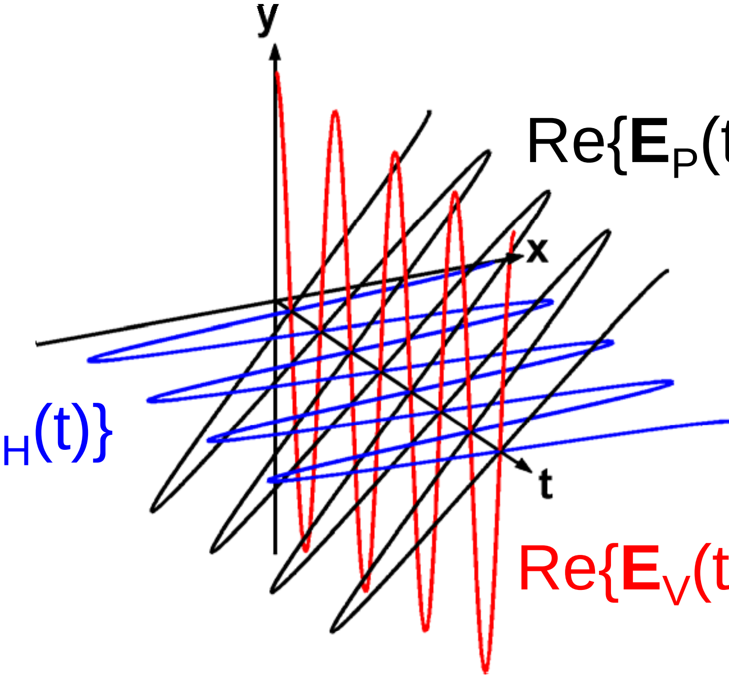
Examples of Jones-Vectors

Linear Polarization:

$$|s\rangle_H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ horizontal

$$|s\rangle_V = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ vertical

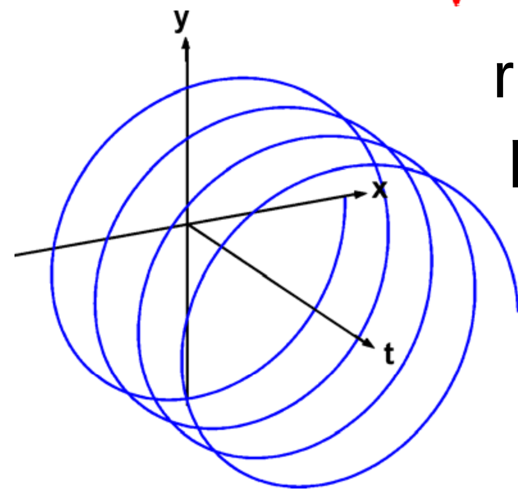
$$|s\rangle_P = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ 45° azimuth



Circular Polarization:

$$|s\rangle_R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix}$$ right-circular

$$|s\rangle_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix}$$ left-circular

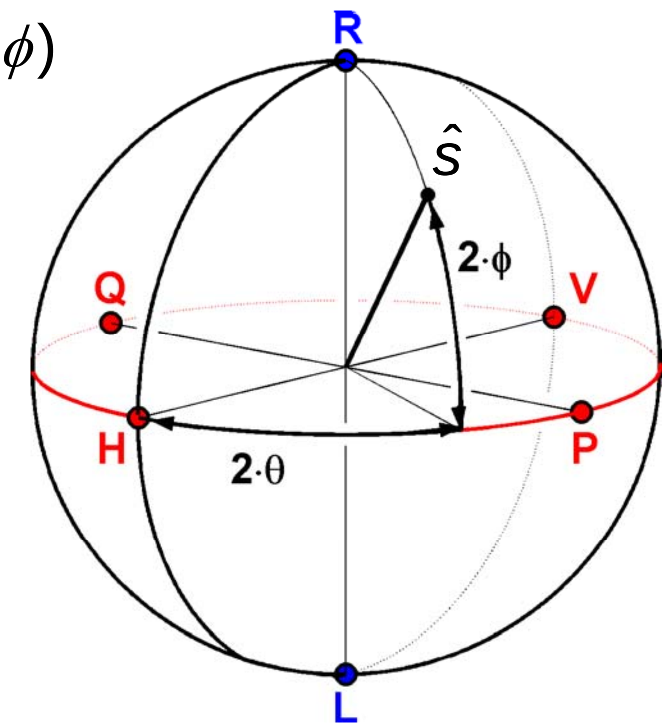


Polarization - Poincaré-Sphere

Representation of the unit Jones-Vector in Stokes space:

$$\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} \quad \text{with} \quad \begin{aligned} s_1 &= s_x s_x^* - s_y s_y^* = \cos(2\theta)\sin(2\phi) \\ s_2 &= s_x s_y^* + s_x^* s_y = \sin(2\theta)\sin(2\phi) \\ s_3 &= j\left(s_x s_y^* - s_x^* s_y\right) = \cos(2\phi) \end{aligned}$$

θ : azimuth, ϕ : ellipticity



Linear polarization: $\phi = 0$
(equator)

H: horizontal (θ=0), **V**: vertical ($\theta=\pi/2$)

P: 45° azimuth ($\theta=\pi/4$), **Q**: -45° azimuth ($\theta=3\pi/4$)

Circular polarization: $2\phi = \pm\frac{\pi}{2}$
(poles)

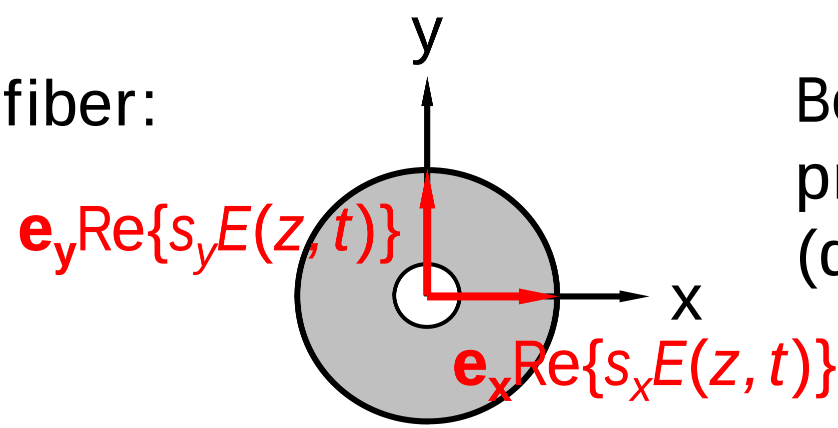
R: right circular ($2\phi=\pi/2$), **L**: left circular ($2\phi=-\pi/2$)

Elliptical polarization: $-\frac{\pi}{2} < 2\phi < 0$ or $0 < 2\phi < \frac{\pi}{2}$

Birefringence - Short Fiber

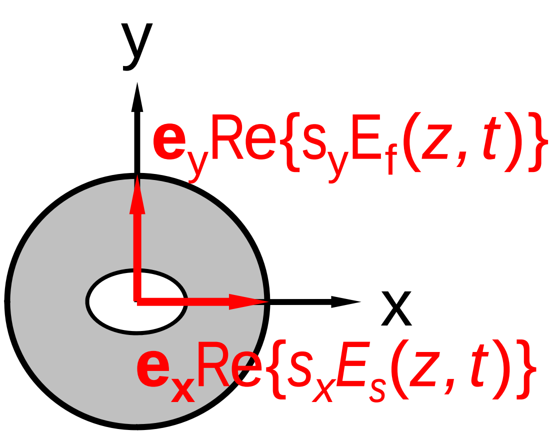
A single-mode fiber supports two orthogonally polarized HE_{11}-modes.

Ideal fiber:



Both modes have the same propagation constant β (degenerate modes).

Short birefringent fiber: (uniform birefringence over the fiber length)



Different propagation constants:

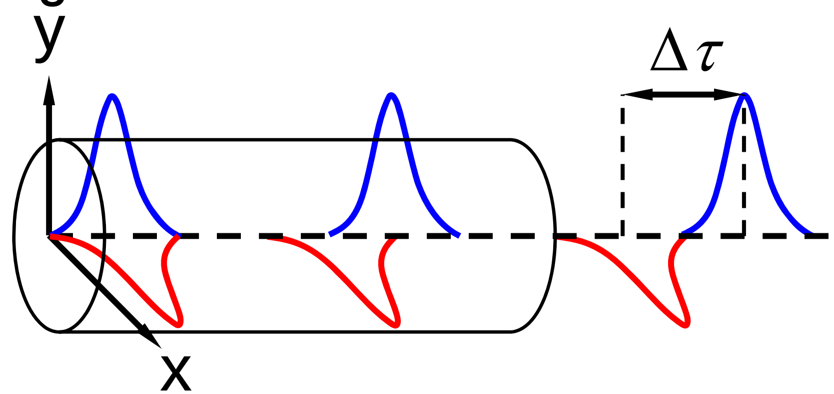
$$\Delta\beta = \beta_s - \beta_f = \frac{\omega_0 n_s}{c} - \frac{\omega_0 n_f}{c} = \frac{\omega_0 \Delta n}{c}$$

$\beta_s,\ \beta_f$: propagation constant of slow and fast mode
$n_s,\ n_f$: respective effective refractive indices
ω_0: angular carrier frequency
c : speed of light

Differential Group Delay (DGD)

Consider a wave that is linearly polarized at an azimuth of $\theta \neq 0$ with respect to the fiber birefringent axes.

⇒ Signal power propagates along both fiber axes.



Different propagation constants

$$\Delta\beta = \beta_s - \beta_f$$

⇓

Different group delays

$$\Delta\tau = \tau_s - \tau_f$$

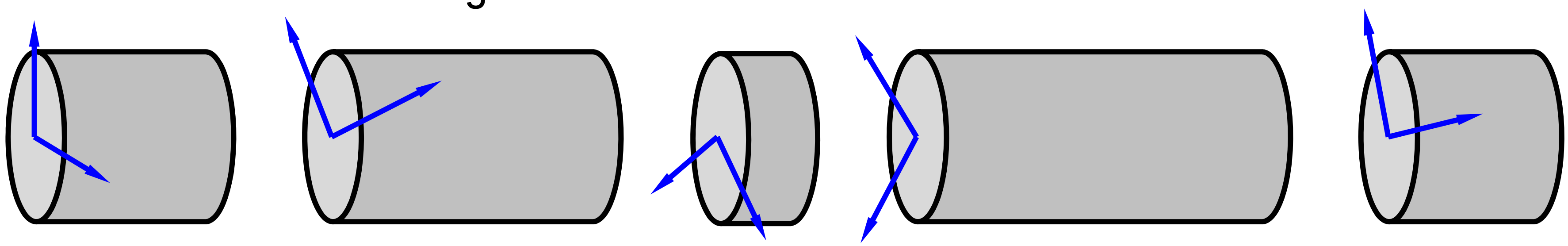
DGD of a short fiber: $$\frac{\Delta\tau}{L} = \frac{d\Delta\beta}{d\omega} = \frac{1}{c}\frac{d\left(\omega\Delta n\right)}{d\omega} = \frac{\Delta n}{c} + \frac{\omega}{c}\frac{d\Delta n}{d\omega}$$

⇒ Linear length dependence of DGD $\Delta\tau$ in short fibers, where uniform birefringence can be assumed.

Birefringence - Long Fiber

In long fibers, the orientation of the axes on birefringence varies randomly along the fiber length.

Model: Concatenation of short fibers with randomly oriented axes of birefringence and random DGD $\Delta\tau$.



Fast and slow polarization modes of one segment are not aligned with the birefringent axes of the next segment.

⇨ The power in each mode is distributed between the two birefringent axes of the next segment.

This is called polarization-mode coupling.

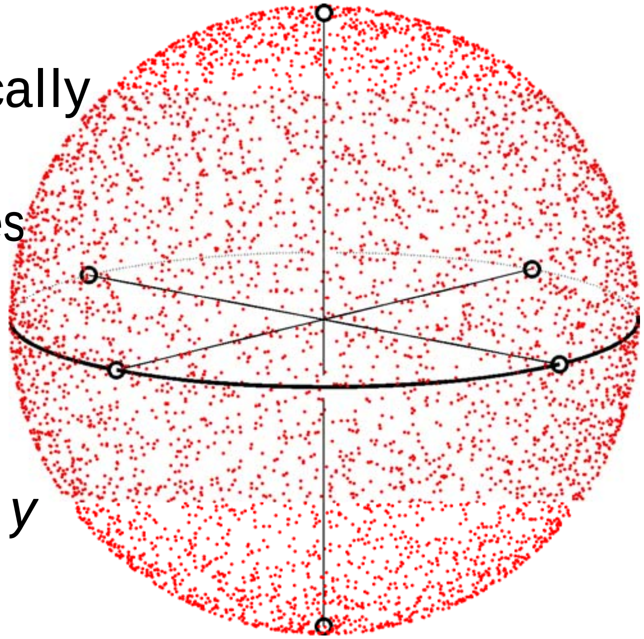
It has been shown that in long fibers the mean DGD evolves proportional to the square-root of the fiber length. Usually fibers are characterized with the PMD parameter D_{PMD} in units ps/$\sqrt{}$km.

Correlation Length

In order to distinguish between a short and a long fiber, the parameter correlation length L_C is used.

Consider an ensemble of long fibers with statistically equivalent properties.

For a fixed input polarization, the observed states of polarization at the output are uniformly distributed on the Poincaré-sphere.



Output states of polarization of an ensemble of 5000 fibers.

The correlation length can be characterized with the ensemble averages of the power in the *x* and *y* polarizations ($\langle P_x \rangle$, $\langle P_y \rangle$).

For fixed values $\langle P_x \rangle=1$, $\langle P_y \rangle=0$ at the input, the difference $\langle P_x \rangle - \langle P_y \rangle$ will approach zero at long lengths, due to polarization-mode coupling.

L_C is defined as the length, where $\langle P_x \rangle - \langle P_y \rangle = \dfrac{1}{e^2}$

Short fiber: $L \ll L_C$

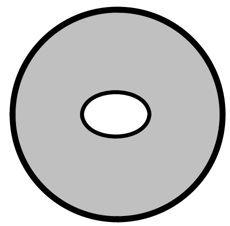
Long fiber: $L \gg L_C$

L_C~1 m to 1 km

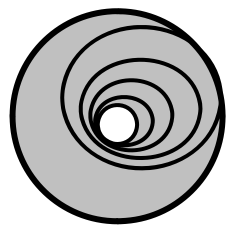
Origins of Fiber Birefringence

Intrinsic origins of birefringence:

Noncircular core:



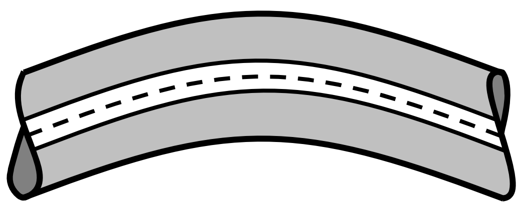
Mechanical stress:



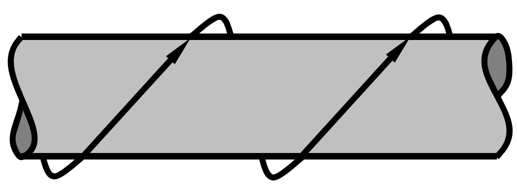
Induced by manufacturing process

Extrinsic origins of birefringence:

Bending:



Torsion:



Induced by embedding the fiber into the ground, spooling or cabling

Transmission Matrix

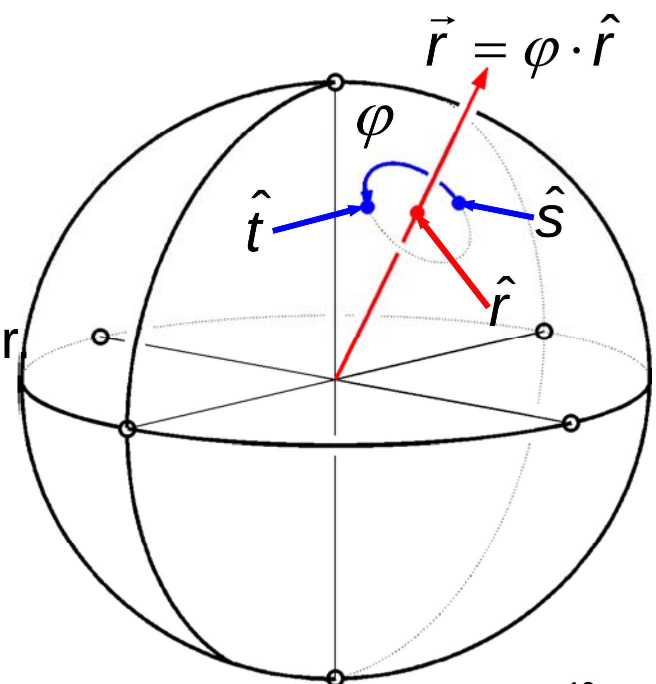
Consider transmission over a fiber without polarization-dependent loss. The output Jones-vector is related to the input Jones-vector by a 2×2 unitary (i.e. $\underbrace{\mathbf{T}^\dagger}_{\text{conjugate transpose}} = \underbrace{\mathbf{T}^{-1}}_{\text{inverse matrix}}$) transmission matrix $\mathbf{T}(\omega)$.

$$\Rightarrow \quad \underbrace{|t\rangle}_{\text{output}} = \mathbf{T}(\omega)\underbrace{|s\rangle}_{\text{input}}$$ or, equivalently: $$|t\rangle = e^{-j\phi_0}\mathbf{U}(\omega)|s\rangle$$

ϕ_0: common phase

$\mathbf{U}(\omega)$: normalized transmission matrix

On the Poincaré-sphere, these relations can be represented by a rotation around a rotation vector.



$\vec{r}$: rotation vector φ : rotation angle

$\hat{r}$: unit rotation vector

$\hat{s}, \hat{t}$: Poincaré respresentations of $|s\rangle, |t\rangle$

Principal States of Polarization

In the absence of polarization-dependent loss, there exist two orthogonal input polarization states, where the output polarization does not change with frequency to first order.

They are called *Principal States of Polarization* (PSP) and correspond to the states with the longest and shortest group delay, respectively.

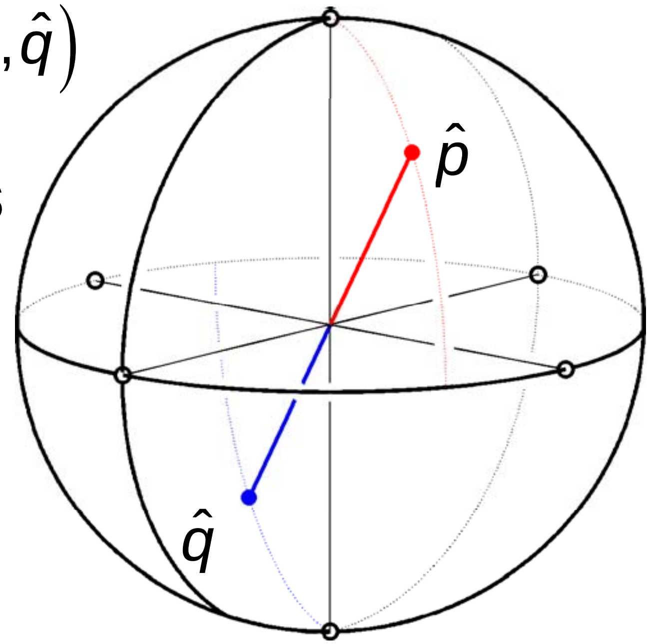
⇨ Slow PSP $\left(|p\rangle, \hat{p}\right)$ and fast PSP $\left(|q\rangle, \hat{q}\right)$

Mathematically, they can be expressed as eigenvectors of the operator $j\mathbf{U}_\omega\mathbf{U}^\dagger$.

$$\underbrace{\frac{1}{2}\Delta\tau}_{\text{Eigenvalue of } j\mathbf{U}_\omega\mathbf{U}^\dagger}|p\rangle = j\mathbf{U}_\omega\mathbf{U}^\dagger \underbrace{|p\rangle}_{\text{Eigenvector of } j\mathbf{U}_\omega\mathbf{U}^\dagger}$$

$\Delta\tau$: DGD

$\mathbf{U}_\omega = \mathrm{d}\mathbf{U}/\mathrm{d}\omega$



Polarization-Mode Dispersion (PMD)

Fiber birefringence and random variations of the birefringent axes along a fiber are the origins of PMD.

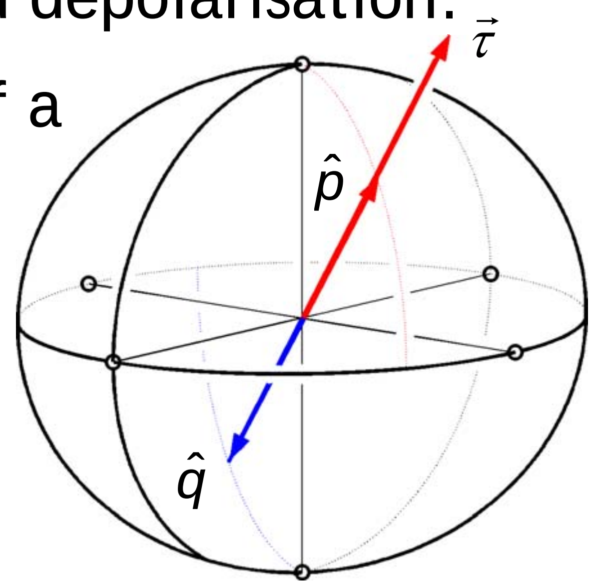
In a first order approximation, PMD leads to different group delays for different polarizations. The difference in group delays of slow and fast PSP is called differential group delay (DGD).
In a second order approximation, PMD also leads to polarization-dependent chromatic dispersion (PCD) and depolarisation.

In the frame of the PSP-model, the PMD of a fiber is conveniently described by a PMD vector:

$$\vec{\tau} = \Delta\tau \cdot \hat{p}$$

$\Delta\tau$: DGD

$\hat{p}$: unit Stokes vector of the slow PSP



PMD Vector

The PMD vector is frequency dependent. For frequencies around the angular center frequency ω_0, the PMD vector can be expanded into a Taylor-series expansion.

$$\vec{\tau}\left(\omega\right) = \underbrace{\vec{\tau}\left(\omega_0\right)}_{\text{1st order term}} + \underbrace{\vec{\tau}_{\omega}\left(\omega_0\right)\cdot\left(\omega - \omega_0\right)}_{\text{2nd order term}} + \ldots$$

Small signal bandwidth ⇒ First order PMD is the dominating impairment.

Large signal bandwidth ⇒ Second order PMD has to be taken into account.

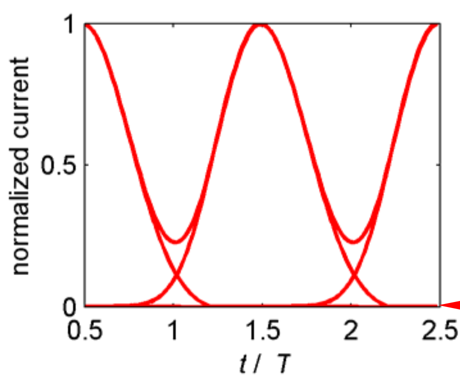
First Order PMD

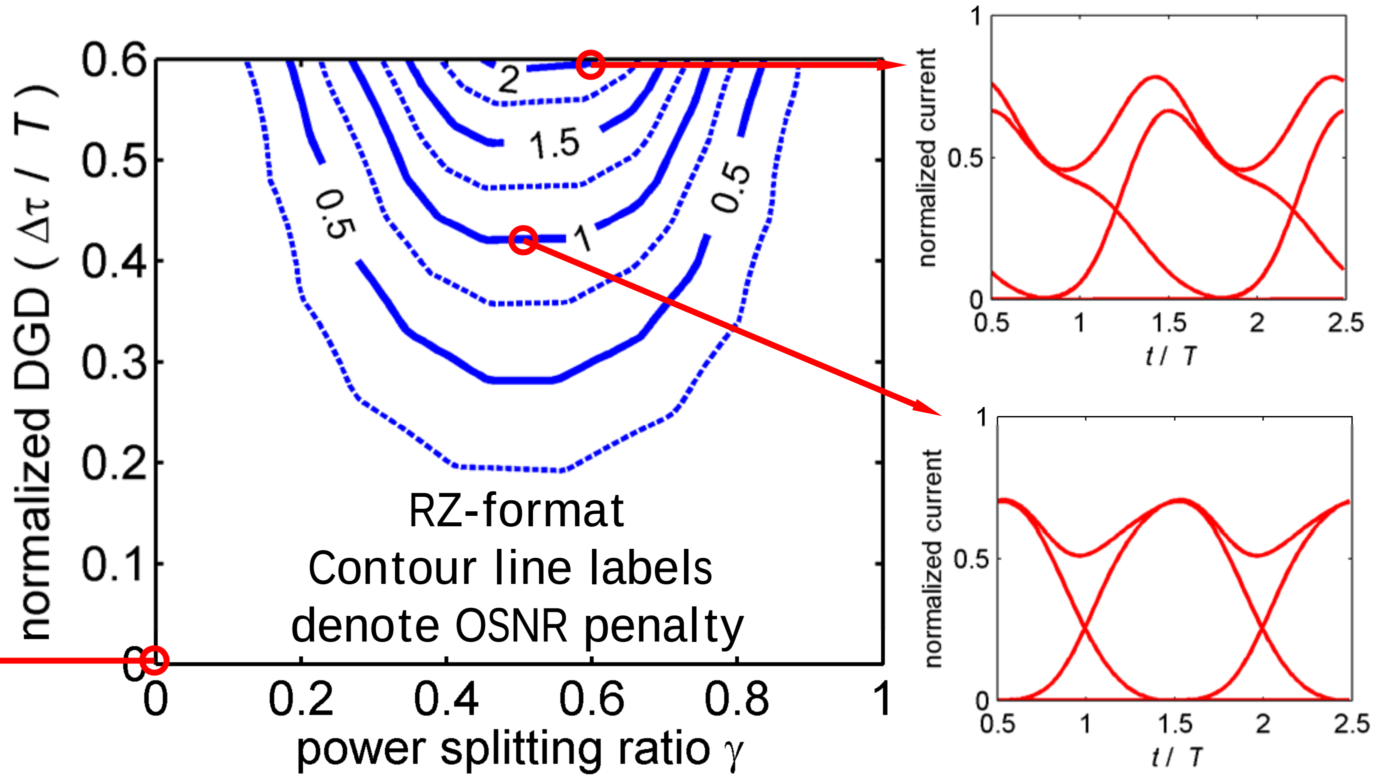
Pure first order PMD can be characterized by two parameters:

Differential group delay: $\Delta\tau$

Fraction of signal power aligned with the slow PSP: γ
(Power splitting ratio between the PSPs)

T: bit period





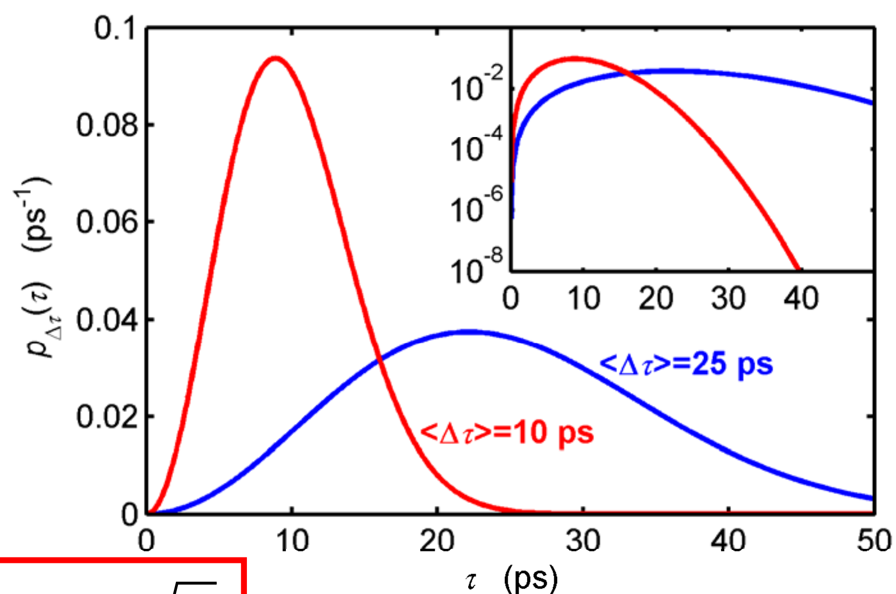
Statistical Properties of 1st order PMD

Unfortunately, due to temperature and environmental changes, the PMD properties of a fiber link change stochastically with wavelength and time.

For the design of transmission links impaired by 1st order PMD, it is important to have knowledge about the distribution of DGD.

It has been established that the probability distribution function of the DGD in long fiber links can be well approximated by a Maxwell distribution of the form:

$$p_{\Delta\tau}(\tau) = \sqrt{\frac{2}{\pi}}\frac{\tau^2}{\alpha^3} e^{-\frac{\tau^2}{2\alpha^2}}$$



The factor α is connected to the mean DGD of the link:

$$\alpha^2 = \frac{\pi}{8}\langle\Delta\tau\rangle^2$$

$\langle\Delta\tau\rangle$: mean DGD

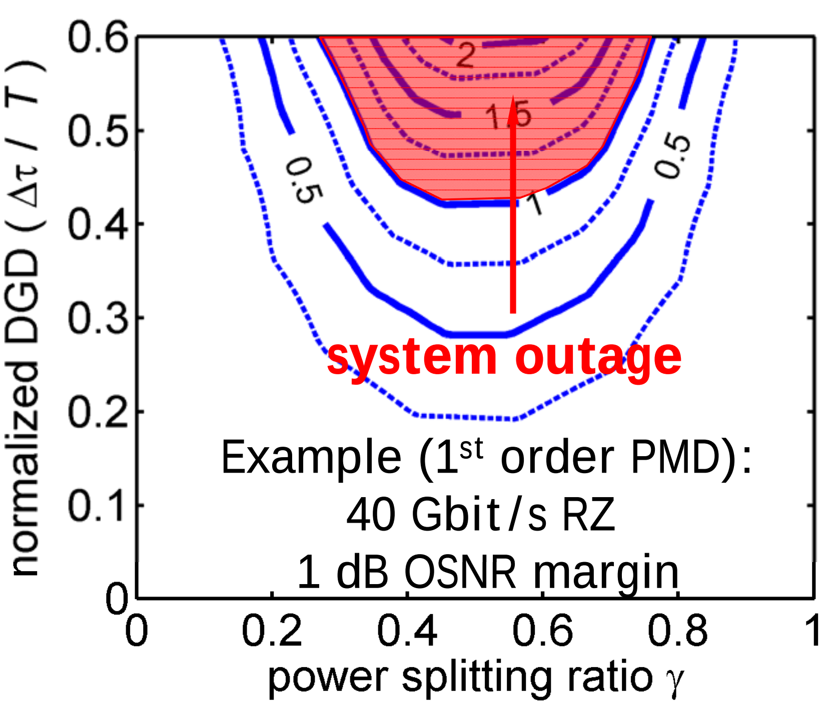
Mean DGD of a long fiber: $\boxed{\langle\Delta\tau\rangle = D_{PMD}\sqrt{L}}$

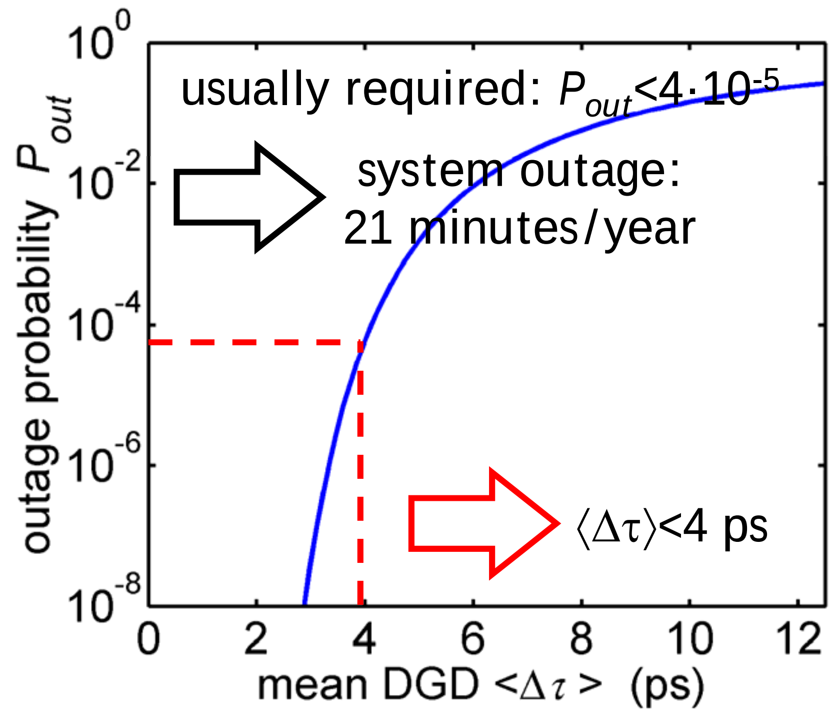
Outage Probability

Due to the stochastic nature of PMD it is not possible to find a „worst case“ scenario and design a system accordingly.

Instead, a safety margin, usually an OSNR margin is allocated to account for fluctuations of received signal quality.

The outage probability of a system is defined as the probability, that the allocated margin is exceeded.





Second Order PMD

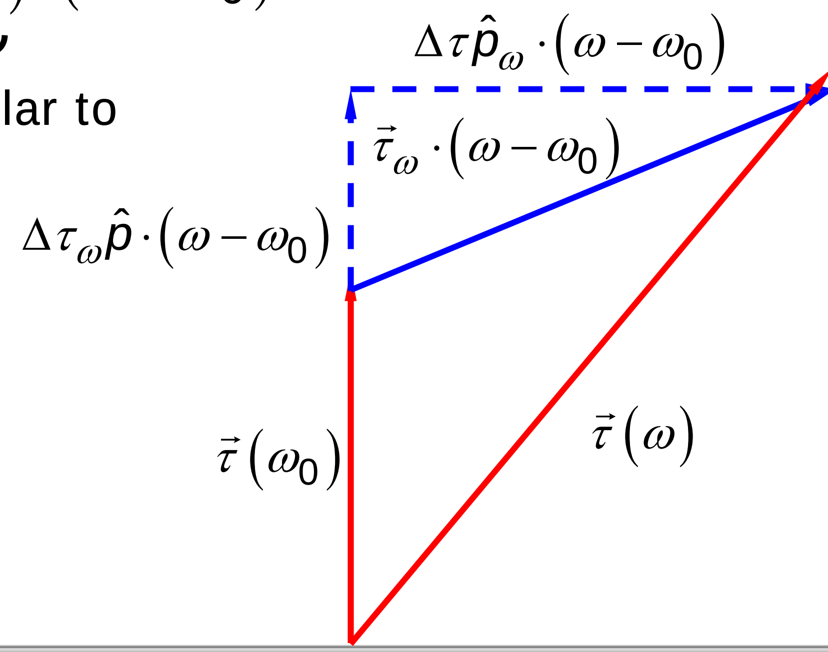
Second order PMD arises from the frequency dependence of the PMD vector.

It is characterized by the second order term of the Taylor-series expansion of the PMD vector.

$$\vec{\tau}_\omega(\omega_0)\cdot(\omega-\omega_0)=\Big(\overbrace{\Delta\tau_\omega\hat{p}}^{\text{parallel to } \vec{\tau}(\omega_0)}+\underbrace{\Delta\tau\hat{p}_\omega}_{\substack{\text{perpendicular to} \\ \vec{\tau}(\omega_0)}}\Big)\cdot(\omega-\omega_0)$$

$\Delta\tau_\omega\hat{p}$ polarization-dependent chromatic dispersion (parallel to $\vec{\tau}(\omega_0)$)

$\Delta\tau\hat{p}_\omega$ PSP depolarization (perpendicular to $\vec{\tau}(\omega_0)$)



Second Order PMD

Polarization-dependent chromatic dispersion:
Frequency components of a signal experience different DGDs.

⇒ Different effective chromatic dispersion for slow and fast PSP.

$$\boxed{\beta_{2,eff} = \underbrace{\beta_2}_{\text{CD}} \underbrace{\pm \frac{1}{2}\frac{\Delta\tau_\omega}{L}}_{\text{PCD}}}$$

„+“: slow PSP
„-“ : fast PSP

PSP depolarization:

Orientation of the PSPs rotates with frequency.

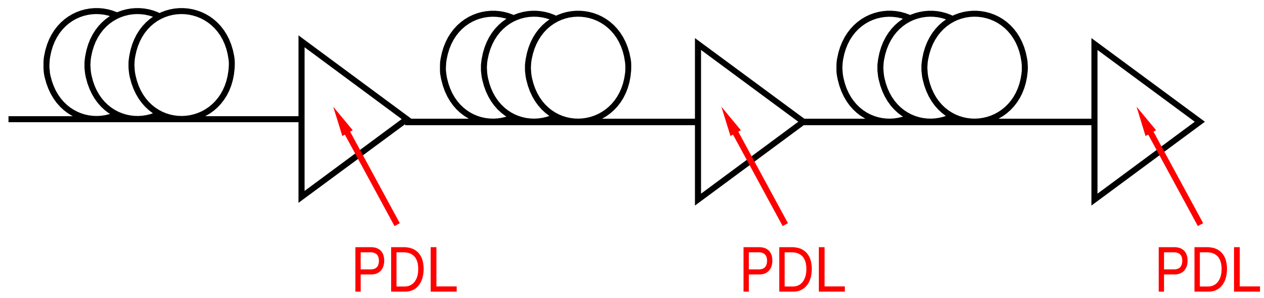
⇓

Compensation of first order PMD for large bandwidth signals can become very difficult.

Polarization-Dependent Loss (PDL)

Many optical components (e.g. filters, amplifiers) used in todays transmission systems have a polarization dependent insertion loss.

Consider transmission over some amplified spans.



The output OSNR depends on the orientation of signal polarization and PDL axes of the amplifiers.

Since both signal polarization and PDL axes change randomly, PDL (like PMD) is a stochastic impairment.

To combat PDL induced impairments, all components have to be carefully designed to exhibit only very small PDL.

Summary

- Polarization
  - Jones Formalism
  - Stokes Formalism
  - Poincaré-Sphere
- Fiber Birefringence
- Polarization-Mode Dispersion
  - First Order PMD
  - Second Order PMD
    - Polarization-Dependent Chromatic Dispersion
    - PSP Depolarization
  - Outage Probabilty
- Polarization-Dependent Loss

Proceed with the ***Interactive Learning Module***