OPTICAL COMMUNICATION COMPONENTS
Lab 4

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Exercise 1

Following is reported our Matlab BPM code that solves the NLSE:

```
function Az = BPM(A,b2,gamma,alpha,Z,T)
%Az returns the propagation of the signal A(T) along the space axis Z,
%in a medium with group-velocity dispersion b2, nonlinearity gamma
%and linear attenuation alpha.

Az=zeros(length(T),length(Z));

N=length(T);
ft=(-N/2:N/2-1)/(T(end)-T(1));

h=Z(2)-Z(1);
E=exp(1i*0.5*b2*(2*pi*ft).^2*h/2);
Az(:,1)=A;

for i=2:length(Z)
    F1=fftshift(fft(A));
    Fdisp1=F1.*E;

    N=ifft(ifftshift(Fdisp1));
    Nl=N.*exp(1i*gamma*h*(abs(N)).^2);

    F2=fftshift(fft(Nl));
    Fdisp2=F2.*E;

    A=ifft(ifftshift(Fdisp2));
    Az(:,i)=A*exp(-alpha*Z(i)/2);
end

end
```

As test we set the following parameters:

- $\alpha = 0$
- $\gamma = 0$
- $\beta_2 = -5 \cdot 10^{-27}$ $[s^2/m]$
- $\lambda = 1.55\mu m$

and we simulate the propagation of the following Gaussian pulse, over a distance of 40 Km:

$$A(T) = A_0 \cdot e^{-\frac{1}{2}\cdot(\frac{T}{T_0})^2} \tag{1}$$

with $A_0 = 1$ and $T_0 = 1.25 \cdot 10^{-11}$ s. In Figure 1 is shown the simulation result.

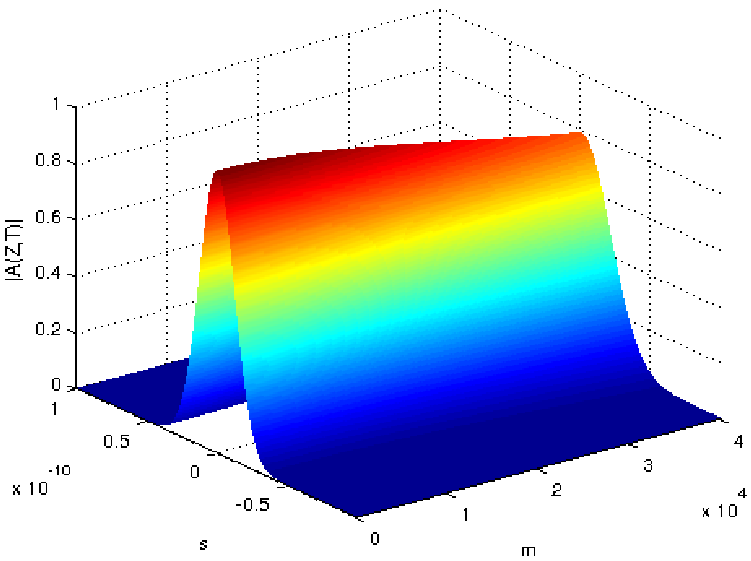

Figure 1: Propagation of the Gaussian pulse $A(T)$.

Exercise 2

Following is reported the procedure to derive the explicit evolution equation for the chirp $C_1(L)$. The pulse at distance L has the following form:

$$A(L,T) = A_0 \cdot \alpha_L \cdot e^{-\frac{1+iC_1}{2}\left(\frac{T}{T_0}\right)^2} \tag{2}$$

that can also be written as:

$$A(L,T) = \frac{A_0}{R(L)} \cdot e^{-\frac{1+iC}{2}\left(\frac{T}{T_0}\right)^2 \frac{1}{R(L)^2}} \tag{3}$$

where

$$R(L) = \sqrt{1 + (C - i)\beta_2 L / {T_0}^2}$$

To compute $C_1(L)$ we have to match the imaginary and the real part of equations (2) and (3). We substitute R(L) in equation (3) and we rearrange the exponent, in order to separate the real and the imaginary parts:

$$\frac{(1+iC)T^2}{2{T_0}^2(1+(C-i)\beta_2 L/{T_0}^2)} = \frac{(1+iC)T^2}{2({T_0}^2+\beta_2 CL - i\beta_2 L)} = \cdots$$

$$\cdots = \frac{(1+iC)T^2({T_0}^2+\beta_2 CL + i\beta_2 L)}{2(({T_0}^2+\beta_2 CL)^2+(\beta_2 L)^2)} = \frac{T^2{T_0}^2 + iT^2(\beta_2 L + C{T_0}^2 + C^2\beta_2 L)}{2(({T_0}^2+\beta_2 CL)^2+(\beta_2 L)^2)}$$

the imaginary part is (with a change of sign):

$$\frac{T^2(\beta_2 L + C{T_0}^2 + C^2\beta_2 L)}{2(({T_0}^2+\beta_2 CL)^2+(\beta_2 L)^2)}$$

and we can equate it to the imaginary part of equation (2), obtaining:

$$\frac{C_1}{2}\left(\frac{T}{T_0}\right)^2 = \frac{T^2(\beta_2 L + C{T_0}^2 + C^2\beta_2 L)}{2(({T_0}^2 + \beta_2 CL)^2 + (\beta_2 L)^2)} \tag{4}$$

from the comparison of the real part of equation (2) with the real part of equation (3), we obtain:

$$\frac{1}{2}\left(\frac{T}{T_0}\right)^2 = \frac{T^2{T_0}^2}{2(({T_0}^2 + \beta_2 CL)^2 + (\beta_2 L)^2)}$$

substituting this value in equation (4) we obtain the final result:

$$C_1 = \frac{T^2(\beta_2 L + C{T_0}^2 + C^2 L\beta_2)}{T^2{T_0}^2}$$

that is:

$$C_1(L) = C + (1 + C^2)\beta_2 L/{T_0}^2 \tag{5}$$

We compute $C_1(L)$ using equation (5) for the following values of length: L/4, L/2, 3/4L, L and for C equal to 0, -2, 2. L is the dispersion distance and it is defined as $L = L_D = {T_0}^2/|\beta_2|$ (in this case $T_0 = 1.25 \cdot 10^{-11}\ s$ and $\beta_2 = -5 \cdot 10^{-27}\ [s^2/m]$).

	$C_1(L/4)$	$C_1(L/2)$	$C_1(3/4L)$	$C_1(L)$
C=0	-0.25	-0.5	-0.75	-1
C=2	0.75	-0.5	-1.75	-3
C=-2	-3.25	-4.5	-5.75	-7

Table 1: Chirp for different lengths.

Now we compute the equation for time duration $T_1(L)$. The equation of the pulse becomes:

$$A(L,T) = A_0 \cdot \alpha_L \cdot e^{-\frac{1+iC}{2}\left(\frac{T}{T_1}\right)^2} \tag{6}$$

it's sufficient to compare the real part of equation (6) with the real part of equation (3), that is:

$$\frac{1}{2}\left(\frac{T}{T_1}\right)^2 = \frac{T^2{T_0}^2}{2(({T_0}^2 + \beta_2 CL)^2 + (\beta_2 L)^2)}$$

rearranging the equation we obtain:

$${T_1}^2 = \frac{(({T_0}^2 + \beta_2 CL)^2 + (\beta_2 L)^2)}{{T_0}^2} = \frac{{T_0}^4 + 2{T_0}^2\beta_2 CL + {\beta_2}^2 C^2 L^2 + {\beta_2}^2 L^2}{{T_0}^2} = \cdots$$

$$\cdots = \frac{{T_0}^4}{{T_0}^2}\left(1 + \frac{2\beta_2 CL}{{T_0}^2} + \frac{{\beta_2}^2 C^2 L^2}{{T_0}^4} + \frac{{\beta_2}^2 L^2}{{T_0}^4}\right)$$

and so:

$$T_1(L) = T_0\left[\left(1 + \frac{\beta_2 CL}{{T_0}^2}\right)^2 + \left(\frac{\beta_2 L}{{T_0}^2}\right)^2\right]^{1/2}$$

The computed values of $T_1(L)$ are:

	$T_1(L/4)$	$T_1(L/2)$	$T_1(3/4L)$	$T_1(L)$
C=0	$1.288 \cdot 10^{-11}$	$1.398 \cdot 10^{-11}$	$1.562 \cdot 10^{-11}$	$1.768 \cdot 10^{-11}$
C=2	$6.99 \cdot 10^{-12}$	$6.25 \cdot 10^{-12}$	$1.127 \cdot 10^{-11}$	$1.768 \cdot 10^{-11}$
C=-2	$1.901 \cdot 10^{-11}$	$2.577 \cdot 10^{-11}$	$3.263 \cdot 10^{-11}$	$3.953 \cdot 10^{-11}$

Table 2: T_1 for different lengths.

Now we verify the correctness of the BPM code by comparing the T_1 computed. In Figure 2, 3, 4, 5, 6 and 7 are reported the measures obtained respectively for C=0, C=2 and C=-2. As we can see, they are almost identical to the ones of Table 2.

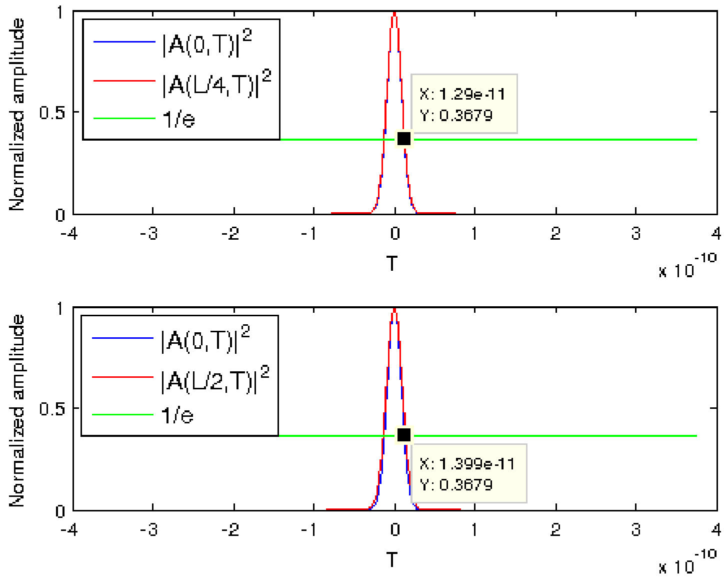

Figure 2: Values of T_1 for C=0.

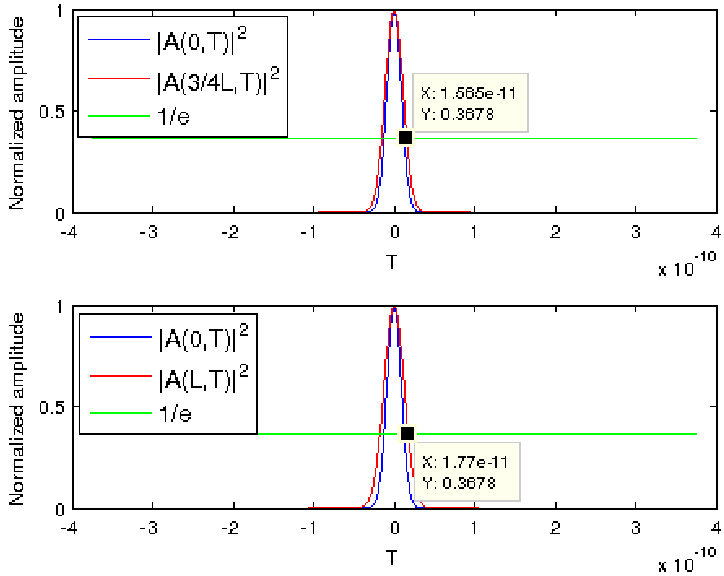

Figure 3: Values of T_1 for C=0.

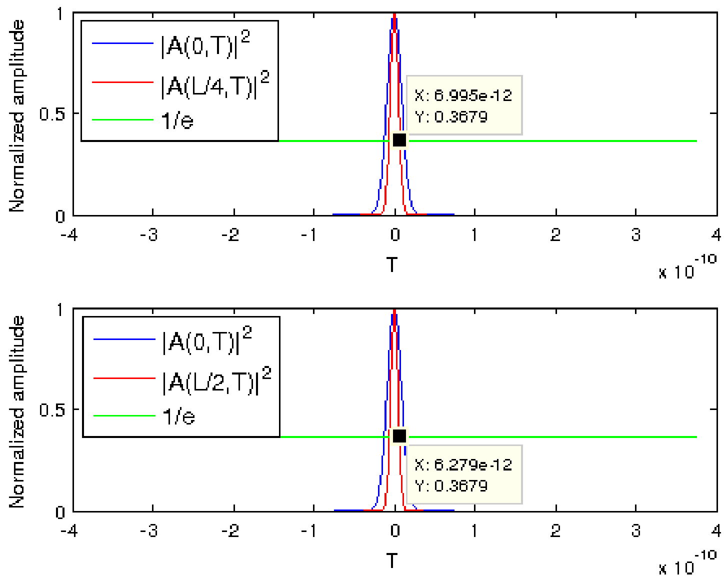

Figure 4: Values of T_1 for C=2.

In Figure 8 are reported the measured values of T_1 versus the length. As we can observe, if the chirp is absent, the pulse broadens slowly and linearly with the distance (red dots). If the initial

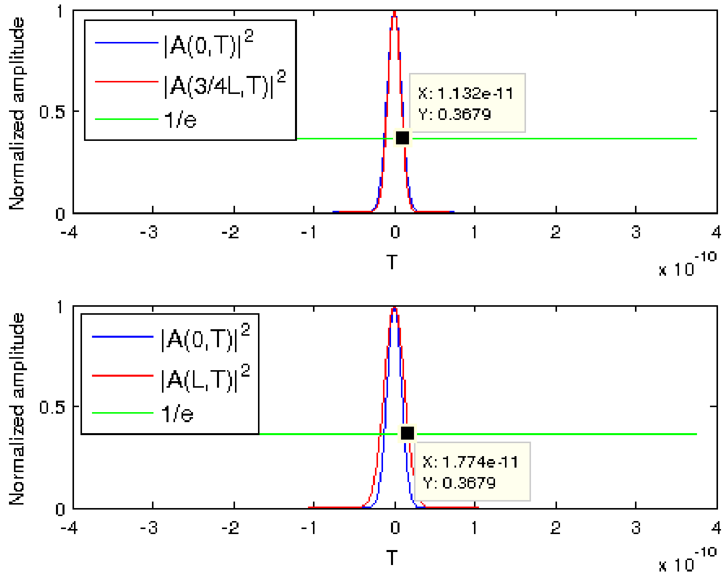

Figure 5: Values of T_1 for C=2.

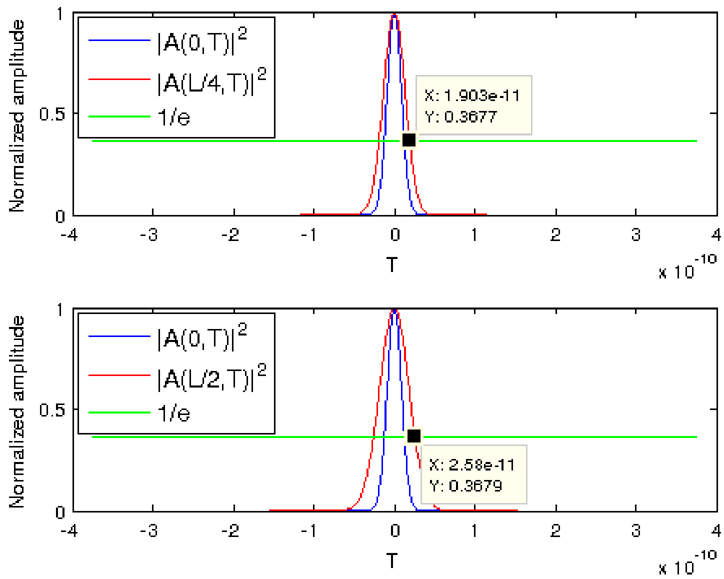

Figure 6: Values of T_1 for C=-2.

chirp is positive (blue dots), the pulse shrinks for the first 16 Km and then starts broadening. At the end of the propagation, the value $T_1(L)$ is identical to the case with C=0. When the initial

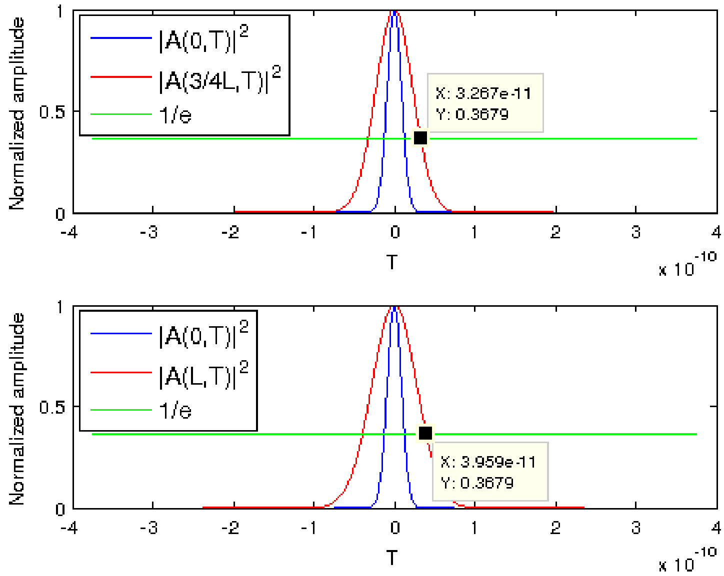

Figure 7: Values of T_1 for C=-2.

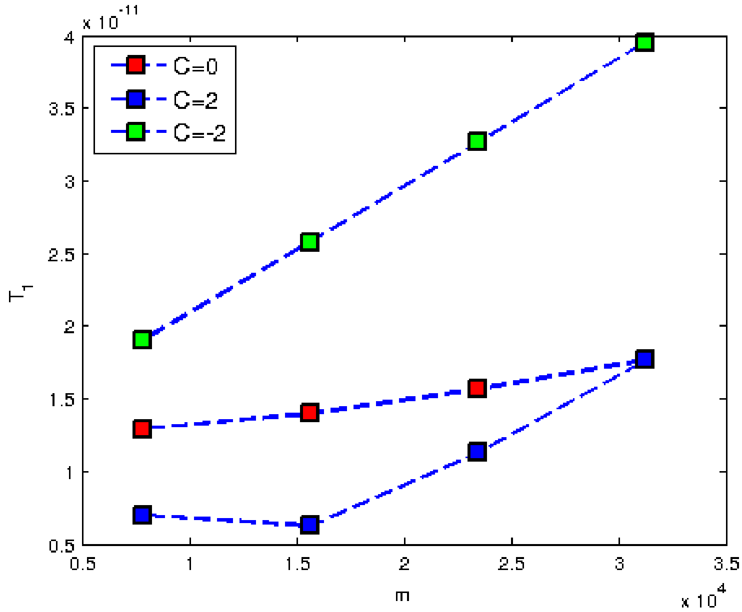

Figure 8: Values of T_1 versus length, for C=0, 2 and -2.

chirp is negative (green dots), the pulse broadens linearly with the distance, but with a higher slope respect to the case with C=0.

Now we repeat the same simulation, changing the sign of the dispersion ($\beta_2 = 5 \cdot 10^{-27}$ $[s^2/m]$). In Figure 9 is shown the result. The curve with C=0 remains unchanged, while the other two curves

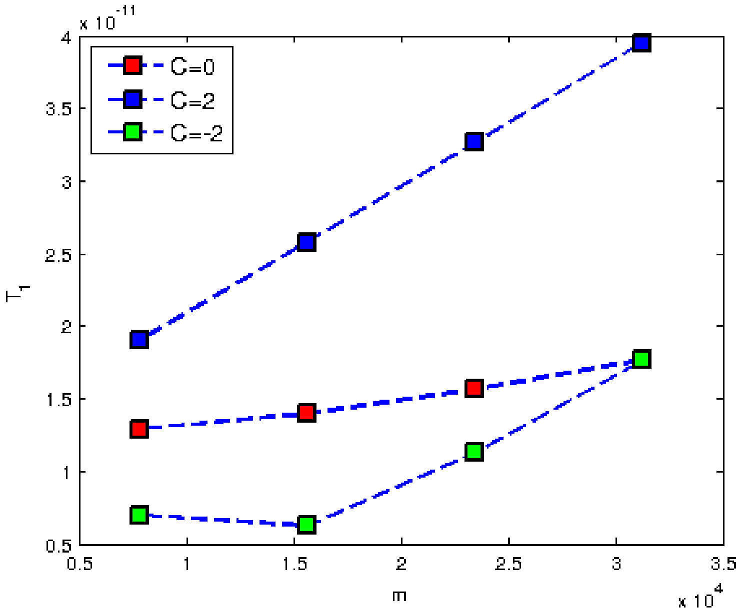

Figure 9: Values of T_1 versus length, for C=0, 2 and -2 and positive dispersion.

are the opposite respect to the case with negative beta.

Exercise 3

We set the dispersion to zero and the attenuation to 0.2 dB/Km. We propagate the pulse of equation (1) for 40 Km and we verify that, the pulse peak power at the end of the simulation, has decreased by 8 dB. The power attenuation relationship is:

$$P(L) = P(0) \cdot e^{-\alpha L}$$

The amplitude of a signal is proportional to the square root of its power, so:

$$A(L) = \sqrt{P(0) \cdot e^{-\alpha L}} = A(0) \cdot e^{-\frac{\alpha L}{2}} \tag{7}$$

But α is expressed in linear units, while our attenuation is given in dB. So we have to convert it:

$$\alpha_{dB} = -10 \log \left(\frac{P_o}{P_i} \right) = -10 \log \left(\frac{P(0)e^{-\alpha}}{P(0)} \right) = -10 \log \left(e^{-\alpha} \right)$$

and we invert the equation to extract α:

$$\alpha = \alpha_{linear} = -\ln \left(10^{-\frac{\alpha_{dB}}{10}} \right)$$

We substitute in the above equation $\alpha_{dB} = 0.2 \cdot 10^{-3}$ $[dB/m]$ and we obtain the equivalent linear coefficient:

$$\alpha = 4.6052 \cdot 10^{-5} \ [1/m]$$

To find the total attenuation we substitute this value and the length into the equation (7) and we obtain:

$$A(L) = A(0) \cdot e^{-\frac{\alpha L}{2}} = A(0) \cdot 0.3981$$

So, after a length of 40000 meters, the pulse is attenuated by a factor 0.3981. This is confirmed by the simulation we have run: in Figure 10 is shown the pulse in time at the beginning of the propagation (in blue) and at the end of the propagation (in red). The respectively maximums are $A(0)_{max} = 0.9995$ and $A(L)_{max} = 0.3979$. The ratio $A(L)_{max}/A(0)_{max} = 0.3981$ corresponds exactly to the theoretical attenuation.

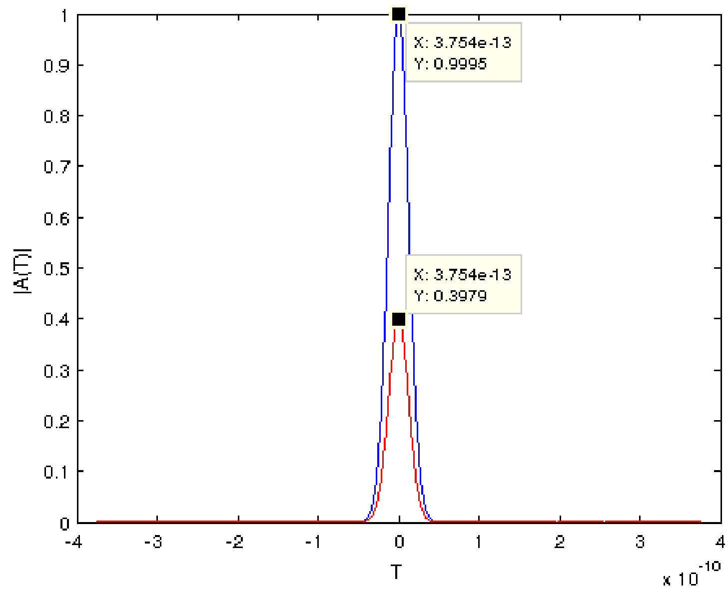

Figure 10: Shape of $|A(T)|$ at the beginning and at the end of propagation.

Exercise 4

Now we introduce in the simulation Kerr non-linearities. We set $\gamma = 2 \cdot 10^{-3}$ $[W^{-1}m^{-1}]$ and $\beta_2 = 0$. We plot the phase profile for three different values of power: $P_1 = 50\ mW$, $P_2 = 100\ mW$ and $P_3 = 150\ mW$. In Figure 11 is shown the phase of the pulse at the end of propagation.

We then run the simulation introducing a dispersion value of $\beta_2 = -5 \cdot 10^{-27}$ $[s^2/m]$ and setting $\gamma = 0$. In Figure 12 are shown the results.

When we have non-linearities also the phase has a non linear behaviour that is proportional to the power of the pulse. Instead, when non-linearities are absent and we introduce dispersion, the phase is quasi linear and it doesn't change for the different powers.

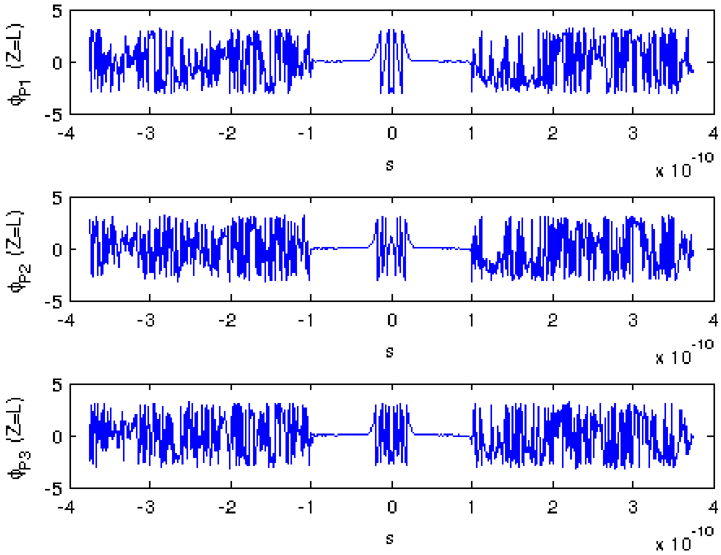

Figure 11: Phase profile for P1, P2, P3 and zero dispersion.

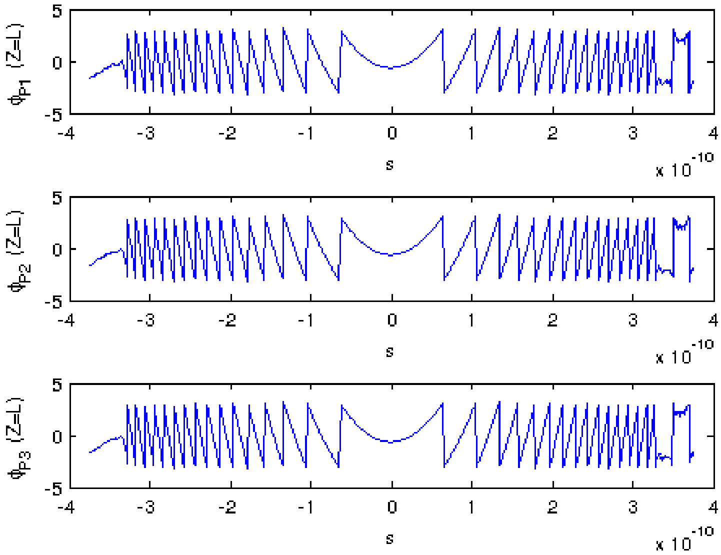

Figure 12: Phase profile for P1, P2, P3 and zero non-linearities.

Now we observe the behaviour of the absolute value of the pulse spectrum, as a function of the propagation distance, for different input power values. We use the same power values of before and we set a length of $L = 10^5\ m$. The non-linear coefficient is $\gamma = 2 \cdot 10^{-3}\ [W^{-1}m^{-1}]$. The spectrum of the propagated signal exhibits a number of peaks M, that depends upon the following relationship:

$$\gamma PL \approx (M - 0.5)\pi$$

and so:

$$M \approx 0.5 + \frac{\gamma PL}{\pi}$$

Substituting the values of the three powers, we obtain:

- $M_1 \approx 4$
- $M_2 \approx 7$
- $M_3 \approx 10$

In Figure 13 is shown the result of the simulation. It is plotted the module of the FFT of the pulse, at the end of propagation. The estimated number of peaks, coincides with the ones obtained in the simulation.

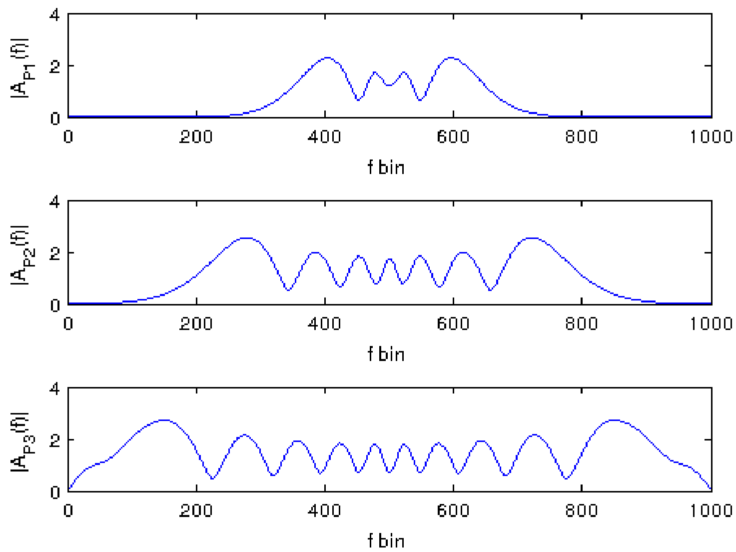

Figure 13: Peaks due to different peak powers.

Exercise 5

We study the propagation of solitons. A soliton is a pulse with the following shape:

$$A(T) = N\sqrt{\frac{|\beta_2|}{\gamma {T_0}^2}} sech\left(\frac{T}{T_0}\right)$$

We set the parameters:

- N=1
- $\beta_2 = -15 \cdot 10^{-27}$ $[s^2/m]$
- $\gamma = 2 \cdot 10^{-3}$ $[W^{-1}m^{-1}]$
- $T_0 = 1.25 \cdot 10^{-11}$ s
- $L = 10^5$ m

In Figure 14 is shown the propagation of a soliton with the above characteristics. We can observe that it propagates unchanged in presence of dispersion and non-linearities. In fact the non linear effect tends to shrink the pulse, while the dispersion tends to enlarge it. The non linear effect compensates the dispersion and the pulse propagates without widening.

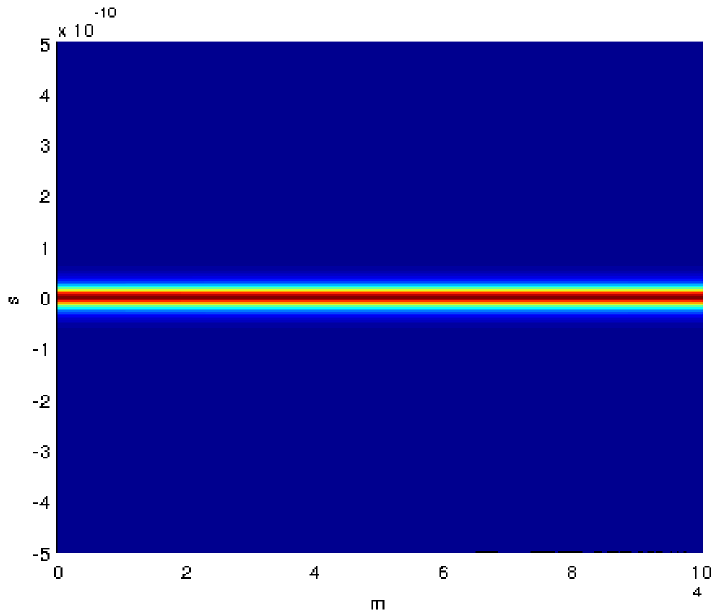

Figure 14: Propagation of a soliton (top view).

Now we increase the power by setting N=3, in Figure 15 is shown the result.

The evolution is periodic, the peaks are in the positions:

$$Z = [0.39, 1.27, 2.05, 2.93, 3.70, 4.57, 5.37, 6.25, 6.87, 7.01, 7.88, 8.62, 9.41] \cdot 10^4 \ m$$

The average of the peak periods is $\Delta T_1 = 8208$ m. Now we reduce the power of 10% and we observe the behaviour of the peaks. In Figure 16 is shown the result. The average period is now $\Delta T_2 = 9529$ m.

We increase the power of 10% and in Figure 17 we can observe the result: the average period is $\Delta T_3 = 3097$ m.

We can conclude that, increasing the power, the period of the peaks decreases and vice versa.

If we change the sign of beta, the propagation is modified: in Figure 18 is shown the propagation for N=1. The effect of dispersion and non-linear effect act in phase and so the pulse is dispersed.

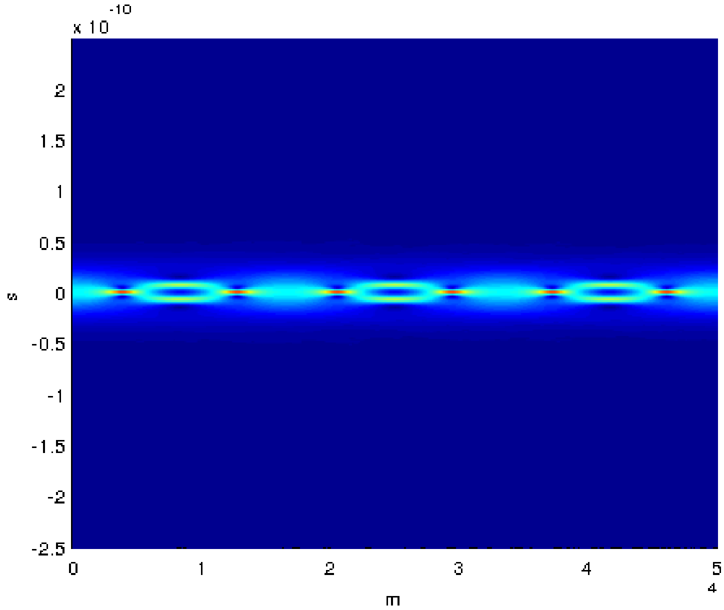

Figure 15: Propagation of a soliton with N=3.

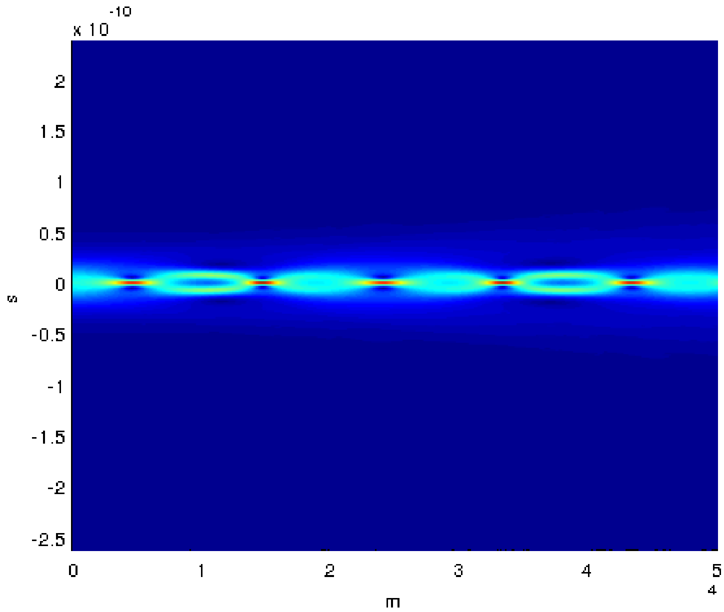

Figure 16: Propagation of a soliton with N=2.7.

Now we propagate a Gaussian pulse with the same characteristics of the soliton (non-linearities and positive dispersion), in Figure 19 is shown the propagation result. The Gaussian pulse is

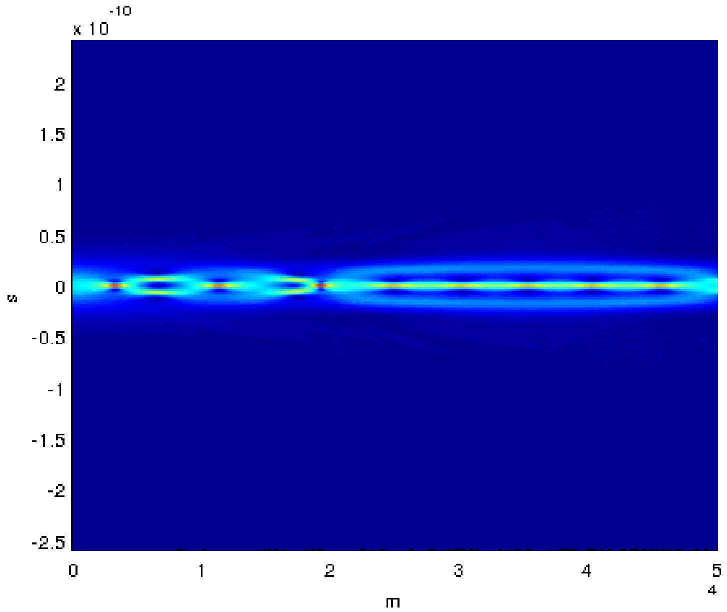

Figure 17: Propagation of a soliton with N=3.3.

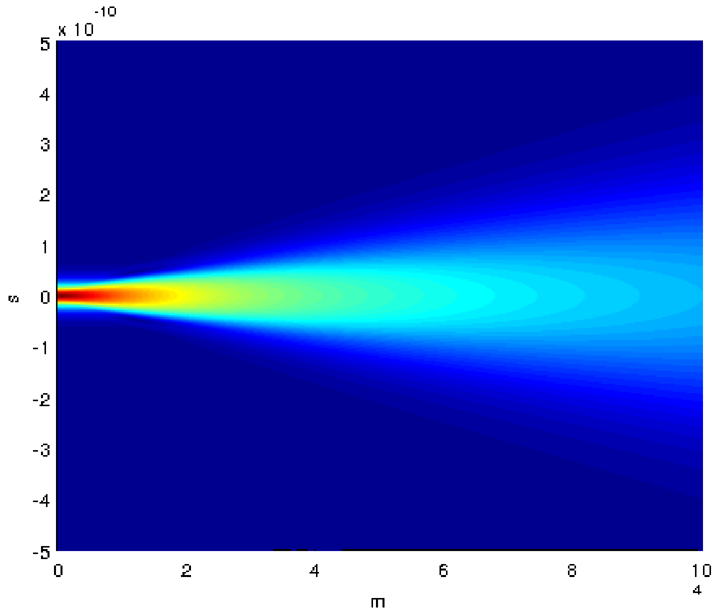

Figure 18: Propagation of a soliton with N=1 and positive beta.

dispersed faster than the soliton.

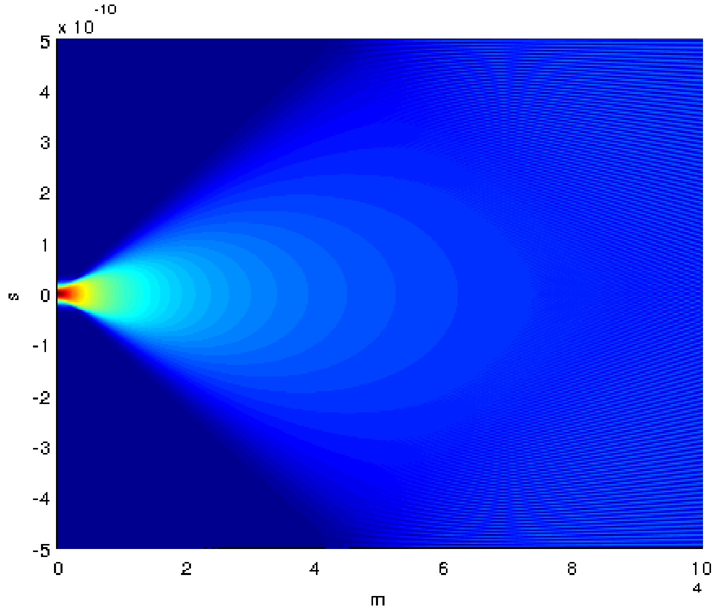

Figure 19: Propagation of a Gaussian pulse with N=1 and positive beta.

Exercise 6

We launch two successive soliton pulses, with the center position shifted in time by KT_0, with K=5 and simulation length $2 \cdot 10^6$ m. The result is shown in Figure 20. The pulses start separately in time but, during propagation, they attract each other and so, there are points in which they hit and create a peak. Measuring the peak distance, we find the average value of the period: $\Delta T = 39319$ m.

We set now K=8 and in Figure 21 we see the result. The average distance between peaks now is increased: in fact it is $\Delta T = 71214$ m.

Now we shift the phase of the second pulse by π, in Figure 22 is shown the result.

The two pulses propagate without hitting one with each other. This is due to the fact that, having opposite phases, they are more distant and so they don't attract each other. Instead they modify their propagation velocity.

These factors are important when transmitting train of pulses: if they are in phase they attract each other causing some hits, which period is proportional to the time shift between pulses. When they are out of phase they don't hit, but they change their velocity.

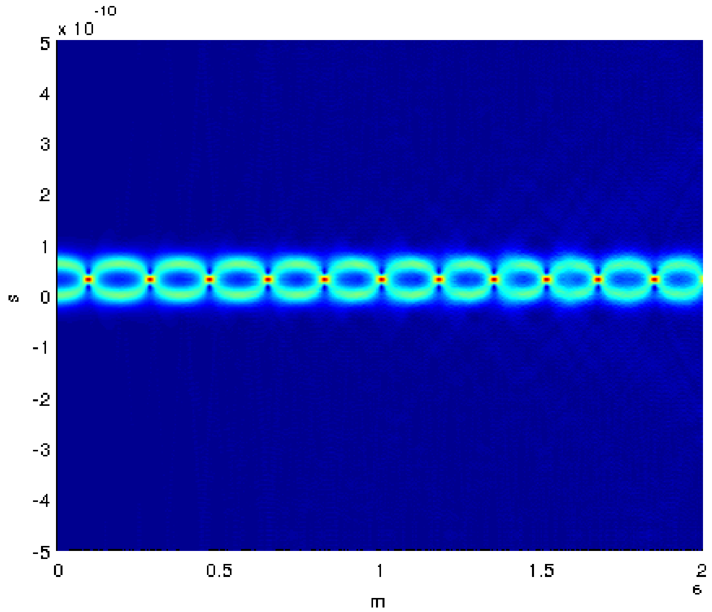

Figure 20: Propagation of two solitons shifted in time (K=5).

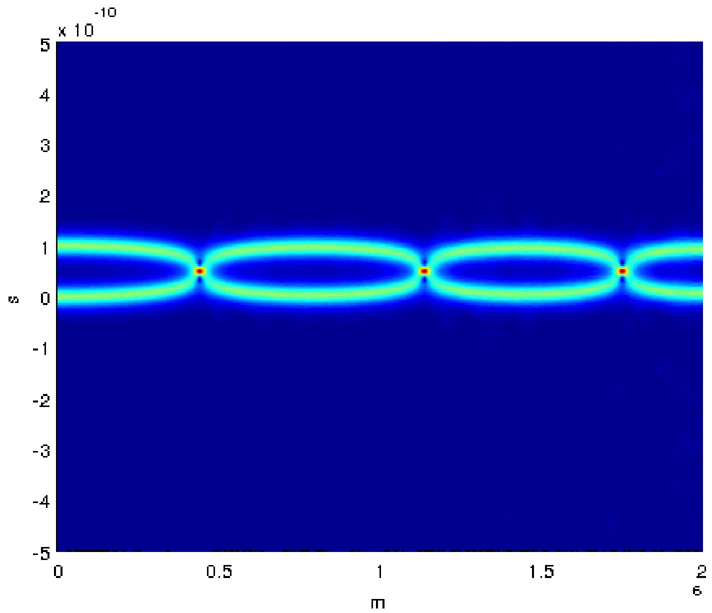

Figure 21: Propagation of two solitons shifted in time (K=8).

Exercise 7

We propagate a Gaussian pulse with the following characteristics:

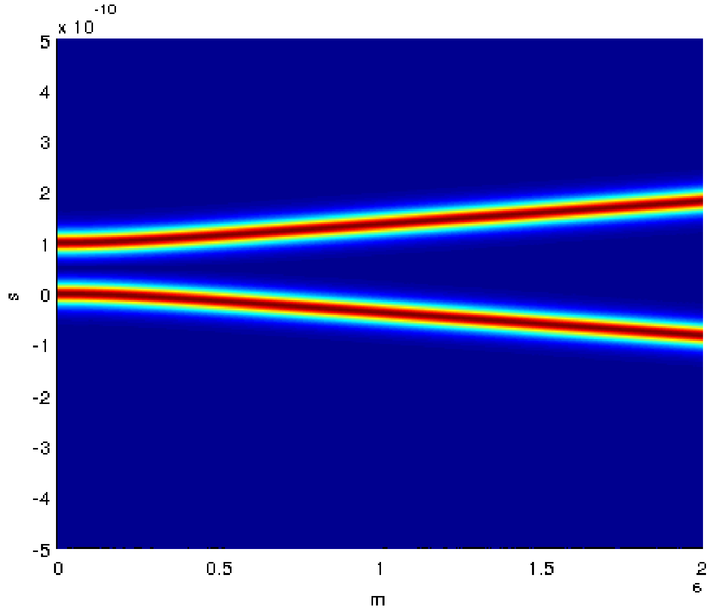

Figure 22: Propagation of two solitons shifted in time (K=5) and with opposite phase.

- Peak power = +6 dBm
- C=0
- $T_0 = 1.25 \cdot 10^{-11}\ s$

the power in linear units is $P = 0.001 \cdot 10^{\frac{dBm}{10}} = 4\ mW$ and so the pulse amplitude is $A = \sqrt{P} = 0.0631$.

The fiber has properties:

- $\beta_2 = -20 \cdot 10^{-27}\ [s^2/m]$
- $\gamma = 2 \cdot 10^{-3}\ [W^{-1}m^{-1}]$
- $L = 100\ Km$

In Figure 23 is shown the simulation result.

Now we propagate the same pulse in a link composed of 4 spans (of length equal to 25 Km each), changing the sign of dispersion at each span. In Figure 24 is shown the result.

When the sign of the dispersion is changed, the effect of the dispersion on the pulse is changed too, and so we have a reconstruction of the pulse. In Figure 25 is shown the pulse at the beginning (blue line) and at the end of the fiber (red line), in time and frequency domain.

We can observe that, the shape of the pulse at the end of the fiber, is almost identical to the shape at the beginning of the fiber (in time and in frequency domain). This is because, at the end of the propagation, the dispersion is fully compensated.

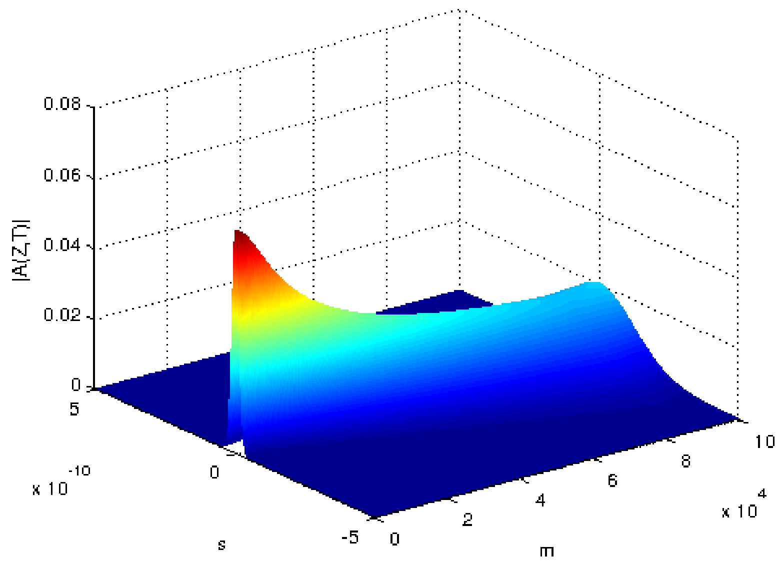

Figure 23: Gaussian pulse.

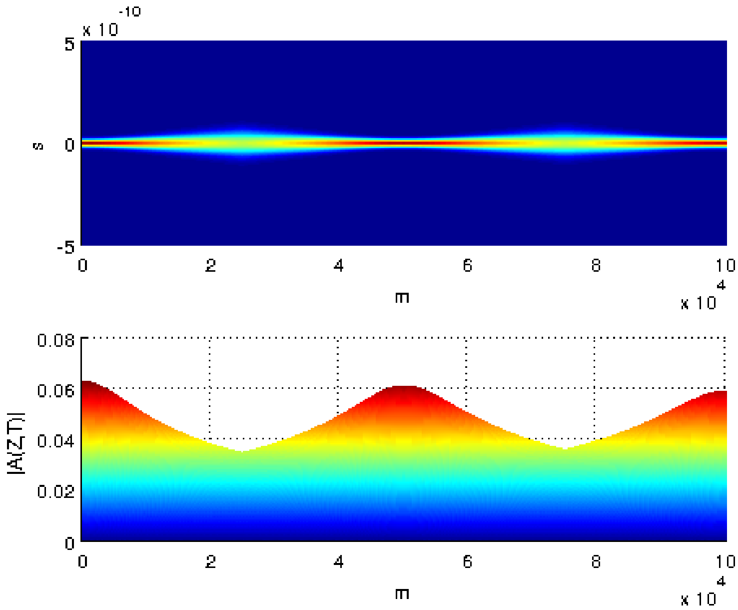

Figure 24: Gaussian pulse with sign of dispersion changed every 25 Km.

Now we simulate the propagation through a 100 Km length link, with the first 90 Km with $\beta_2 = -20{\cdot}10^{-27}$ $[s^2/m]$ and the last 10 Km with a beta that compensates the accumulated dispersion.

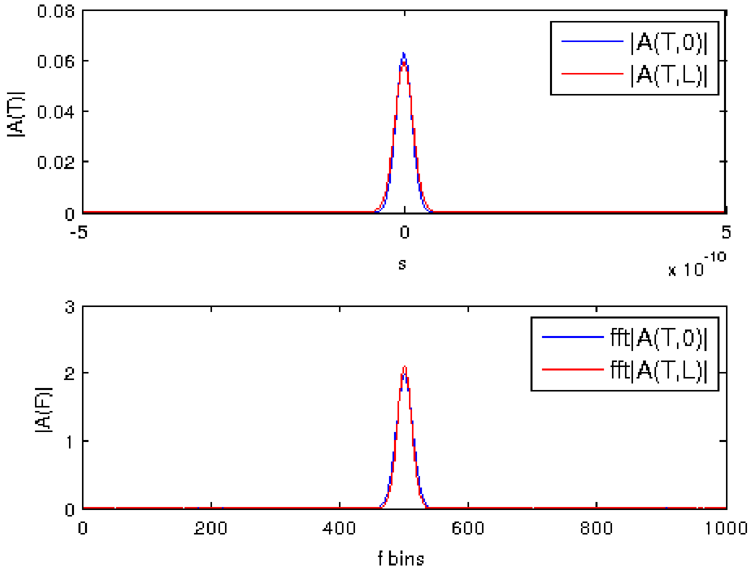

Figure 25: Gaussian pulse with sign of dispersion changed every 25 Km.

The relationship that relates dispersions and lengths is:

$$L_1 D_1 = -L_2 D_2$$

and so:

$$D_2 = -\frac{L_1}{L_2} D_1 = -9D_1$$

In Figure 26 and 27 is shown the simulation result.

We have again a total reconstruction of the pulse, because we compensated the dispersion, even if in an asymmetrical mode respect to the previous case. In the second span the dispersion value is higher, so the desired reconstruction happens in a shorter distance. In this second case, the pulse broadening is higher respect to the previous one and this can create interference problems when transmitting train of pulses.

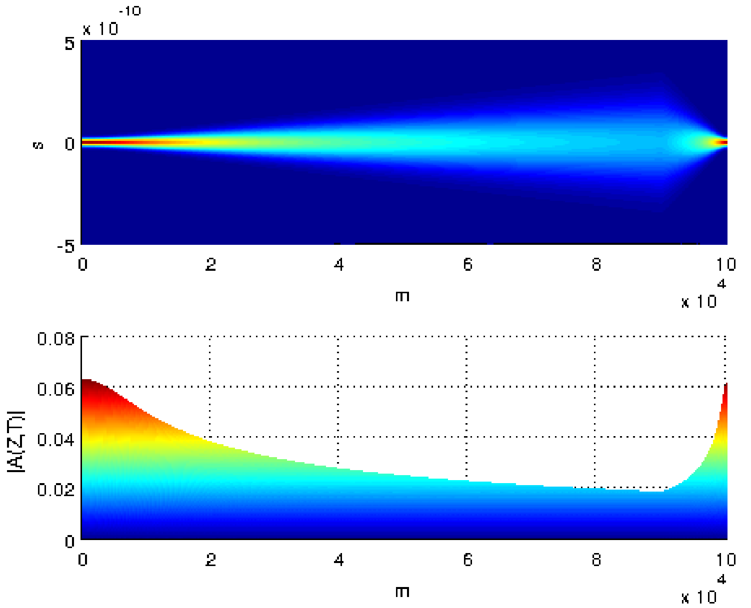

Figure 26: Gaussian pulse with total compensation after 90 Km.

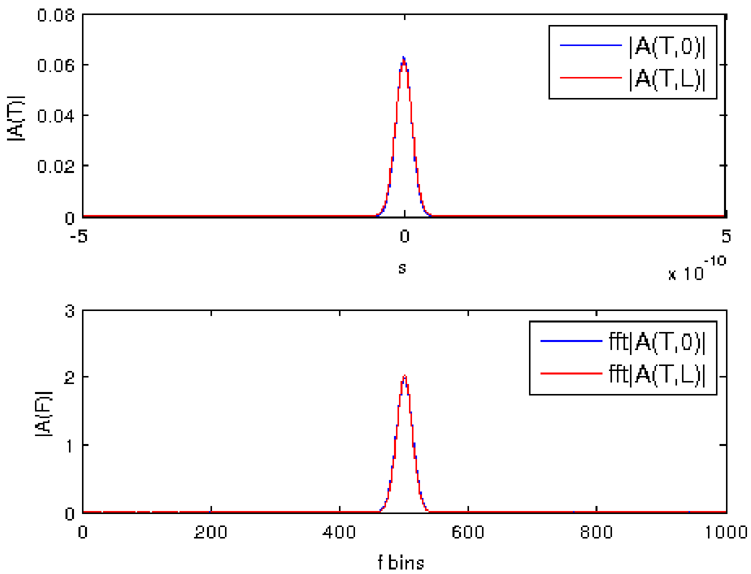

Figure 27: Gaussian pulse with total compensation after 90 Km.

Exercise 8

We propagate a Gaussian pulse in a fiber the following characteristics:

- $\beta_2 = -10 \cdot 10^{-27}$ $[s^2/m]$
- $\gamma = 2 \cdot 10^{-3}$ $[W^{-1}m^{-1}]$
- $L = 7 \cdot 10^4$ m

The pulse has:

- Peak power = +3 dBm
- C=0
- $T_0 = 1.25 \cdot 10^{-11}$ s

along with that, are injected in the fiber other two pulses with the same characteristics, but with frequency shifts of $\Delta f = 100$ GHz and $\Delta f = -100$ GHz. In Figure 28 and 29 is represented the simulation in time domain and in Figure 30 the spectrum at the beginning and at the end of the propagation.

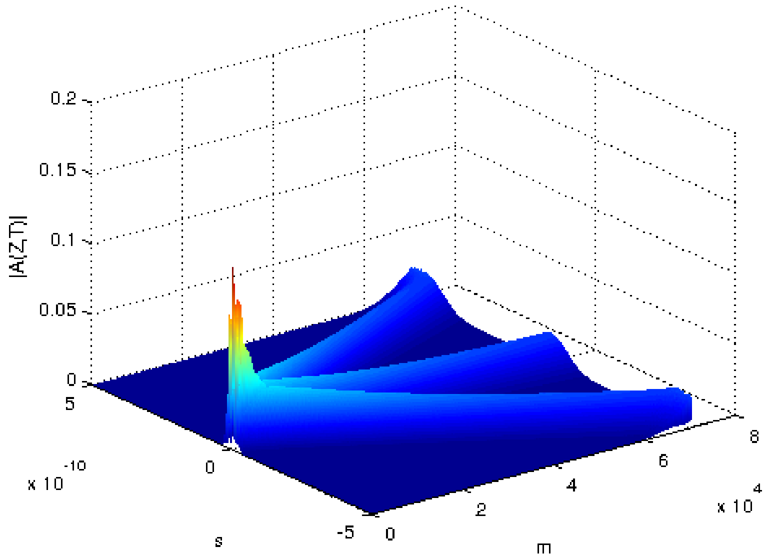

Figure 28: Gaussian pulses with frequency shift.

The pulse with negative frequency shift decreases its velocity, while the one with positive shift increases the velocity. The spectra at the beginning and at the end of the propagation have the same shape.

We now set the value of the dispersion to $\beta_2 = -1 \cdot 10^{-27}$ $[s^2/m]$ and we run the same simulation. In Figure 31, 32 and 33 are shown the results. Again, the velocities of the shifted pulses are changed, but the values are smaller respect to the previous case. The spectra at the beginning and at the end of the simulation remain unchanged.

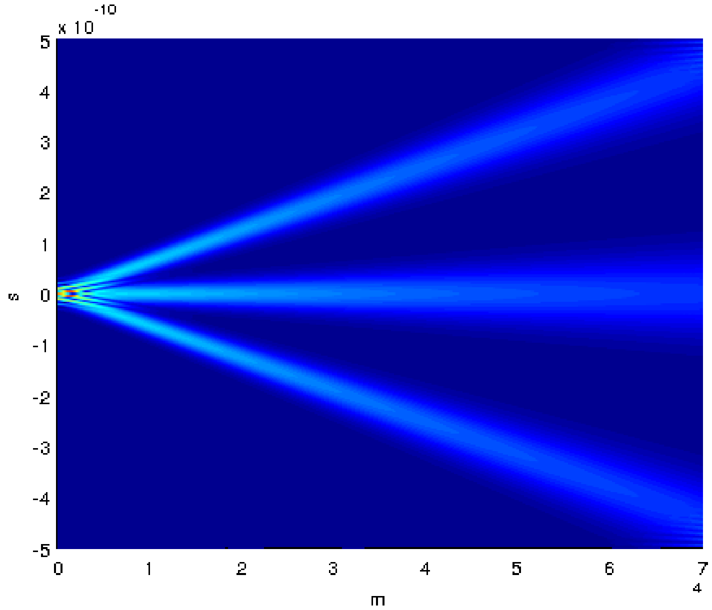

Figure 29: Gaussian pulses with frequency shift (top view).

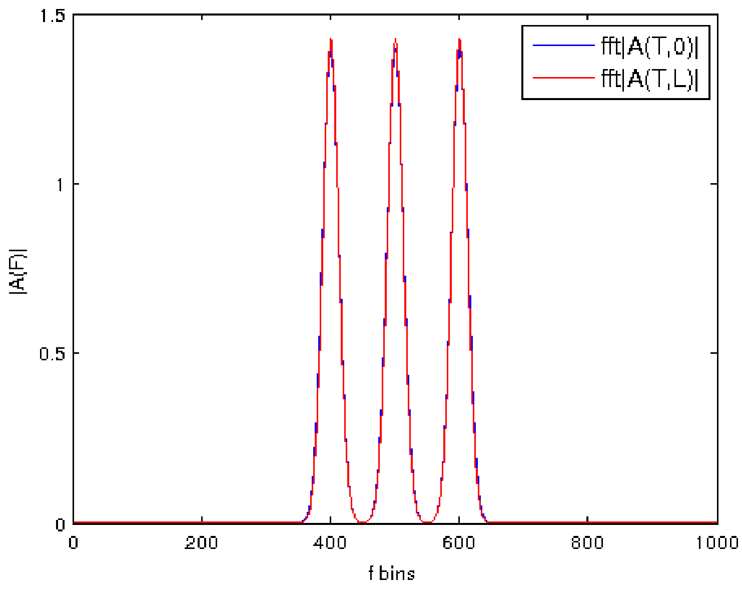

Figure 30: Gaussian pulses with frequency shift in frequency domain.

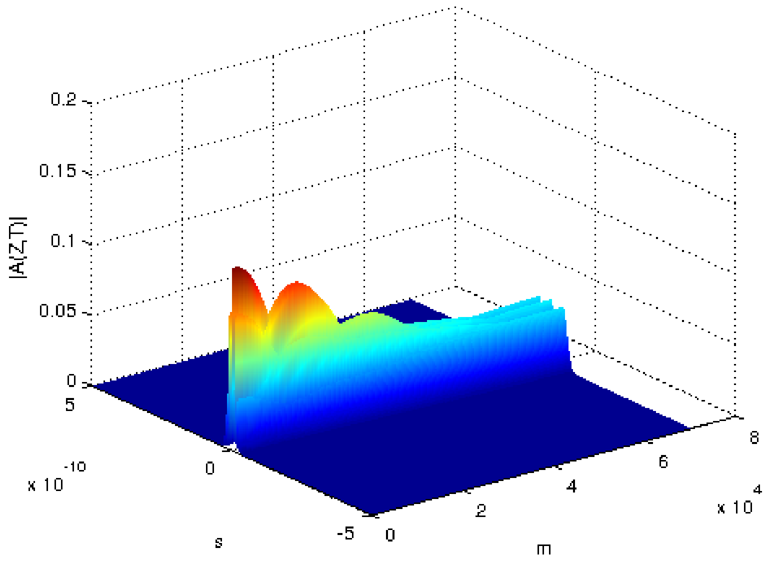

Figure 31: Gaussian pulses with frequency shift.

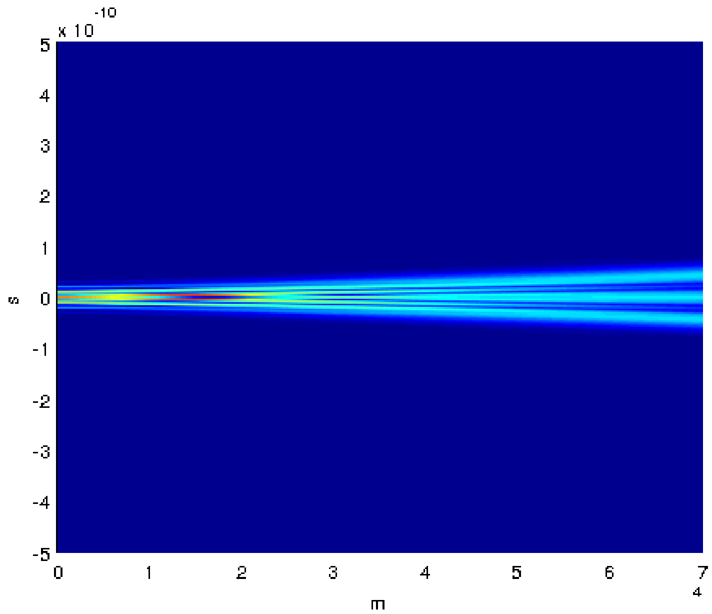

Figure 32: Gaussian pulses with frequency shift (top view).

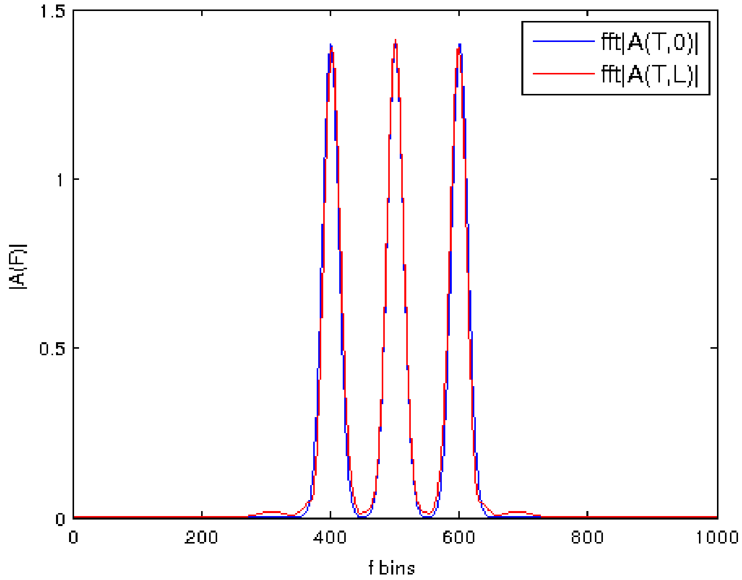

Figure 33: Gaussian pulses with frequency shift in frequency domain.