

# *VPI University Program*

Photonics Curriculum Version 7.0

*Lecture Series*



Introduction to Optical Amplifiers

OA1

## Module Prerequisites

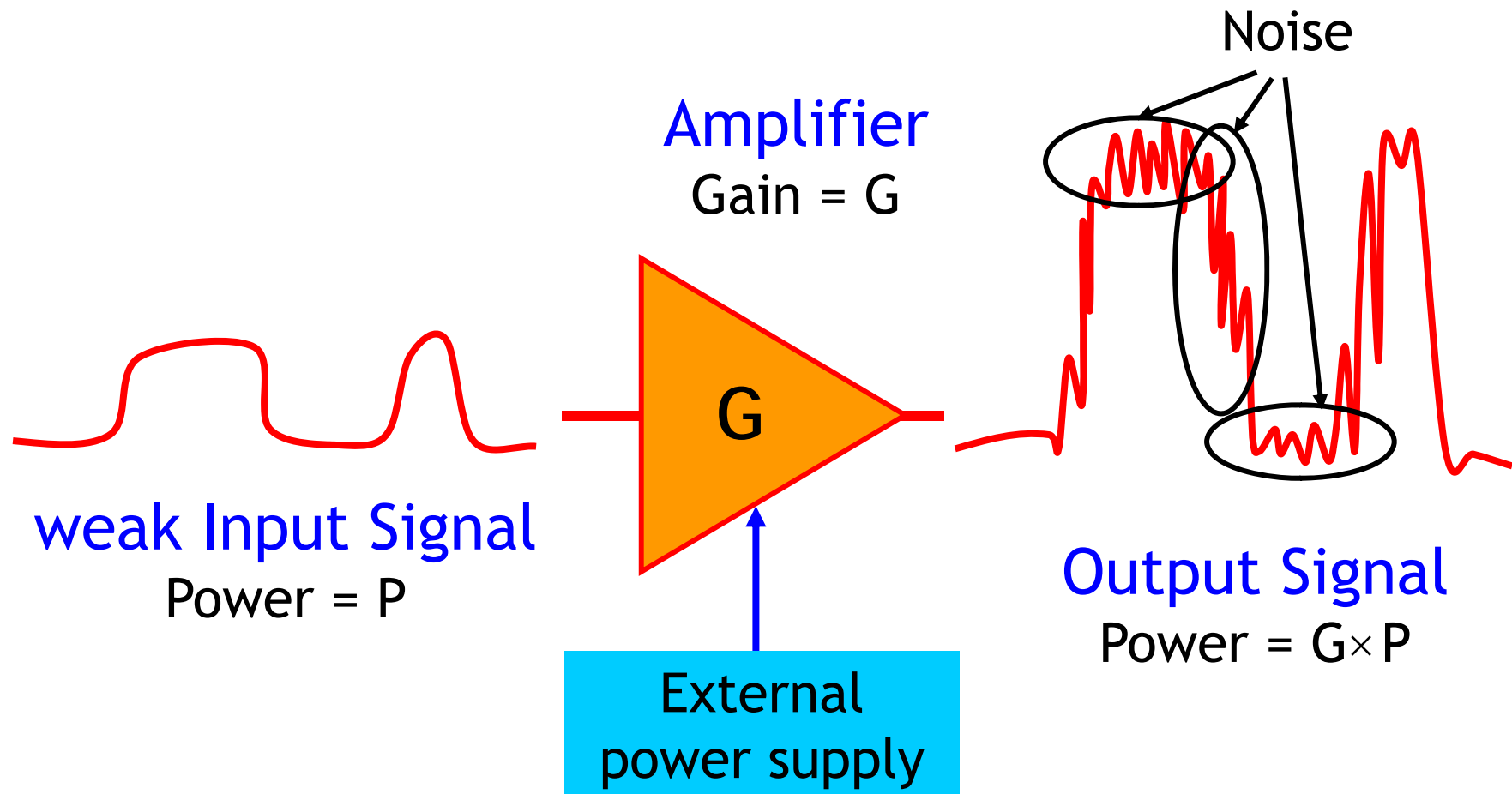
- Introduction to Fiber-Optic Communications I & II
- Recommended - Transmitters I and Receivers I

### *Module Objectives*

- Introduction to optical amplifiers
- How they work:  
Gain, Stimulated and Spontaneous Emission
- Performance Measures:  
Gain, Noise, SNR, Noise Figure
- Performance Limitations and Applications
- Summary

## Introduction

Share some similarities with electrical amplifiers



## Introduction

### Similarities between optical and electrical amplifiers:

- Signal amplification
- Noise added to amplified signal
- Gain and noise can be measured and calculated

### Differences between optical and electrical amplifiers:

- Large gain bandwidth
  - 3 THz – 25 THz (optical)
  - 2 GHz – 50 GHz (electrical)
- Noise spans the same bandwidth

# How Optical Amplifiers work

**LASER** is a good starting point

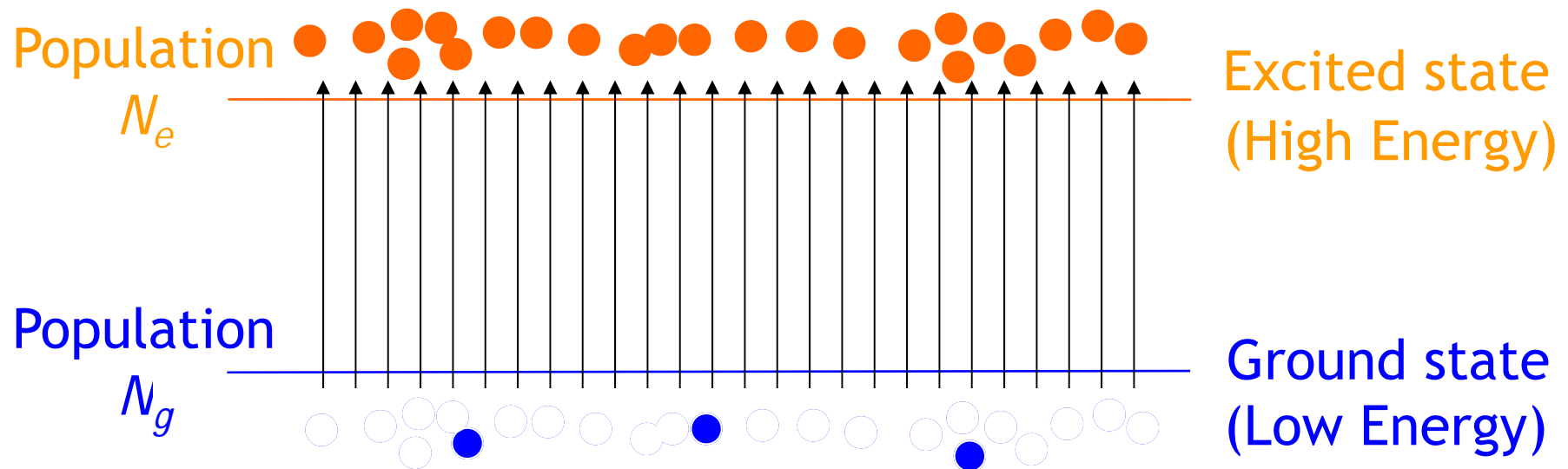
**LIGHT  
AMPLIFICATION by  
STIMULATED  
EMISSION of  
RADIATION**

Energy Level diagram



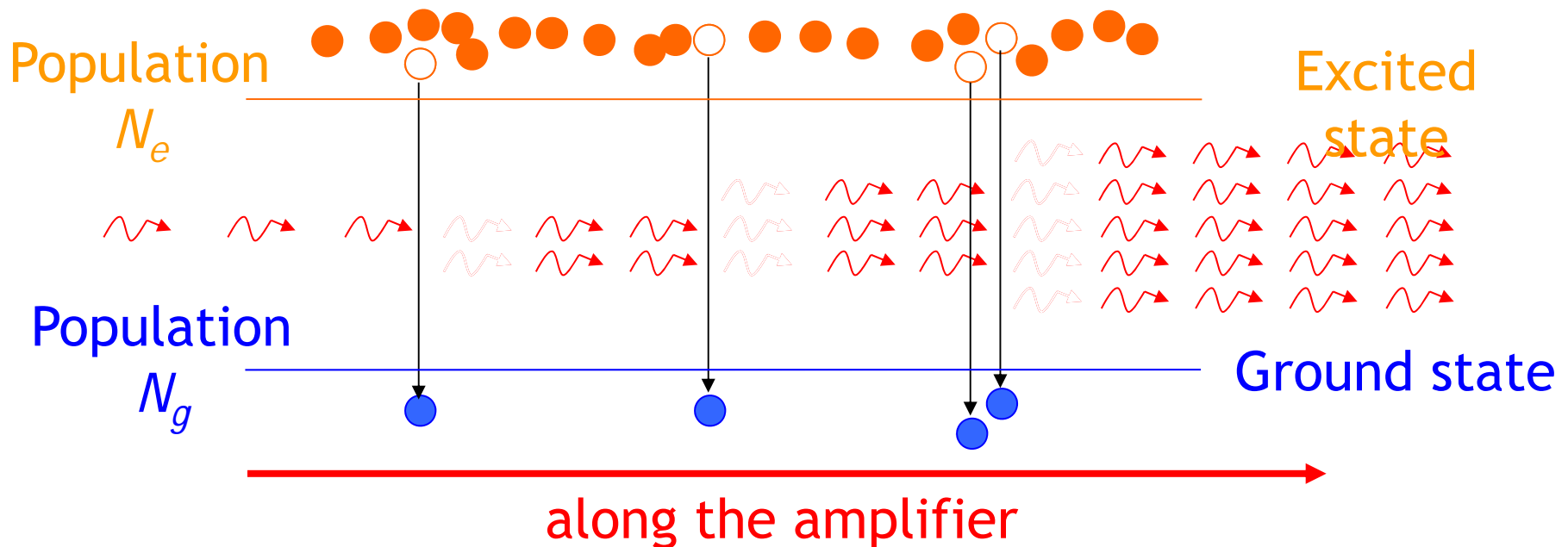
## How Optical Amplifiers work

- First, “Population Inversion” is needed
- “Normal”,  $N_g$  = number in ground state
- “Population Inversion”:  $N_e > N_g$



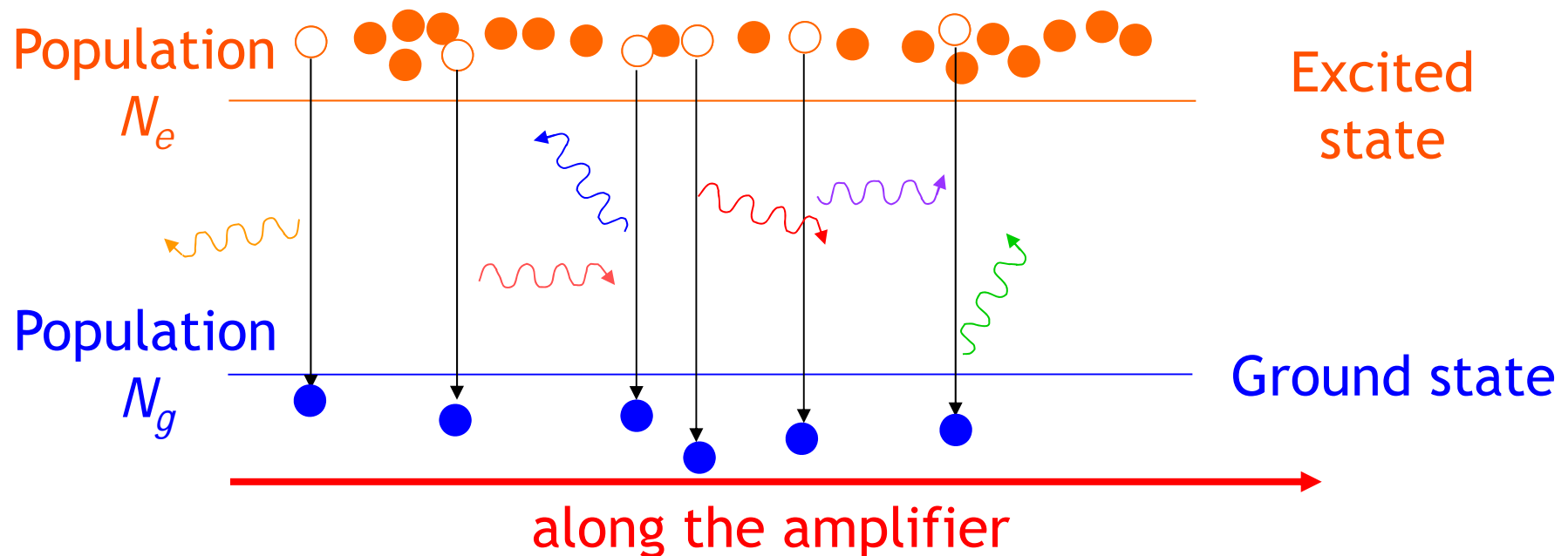
## How Optical Amplifiers work

- Signal photon enters the amplifier
- It **stimulates** an electron to decay to ground state, which **emits** an **identical** photon.
- This process repeats... and the signal is **amplified** (Gain)



## Spontaneous Emission

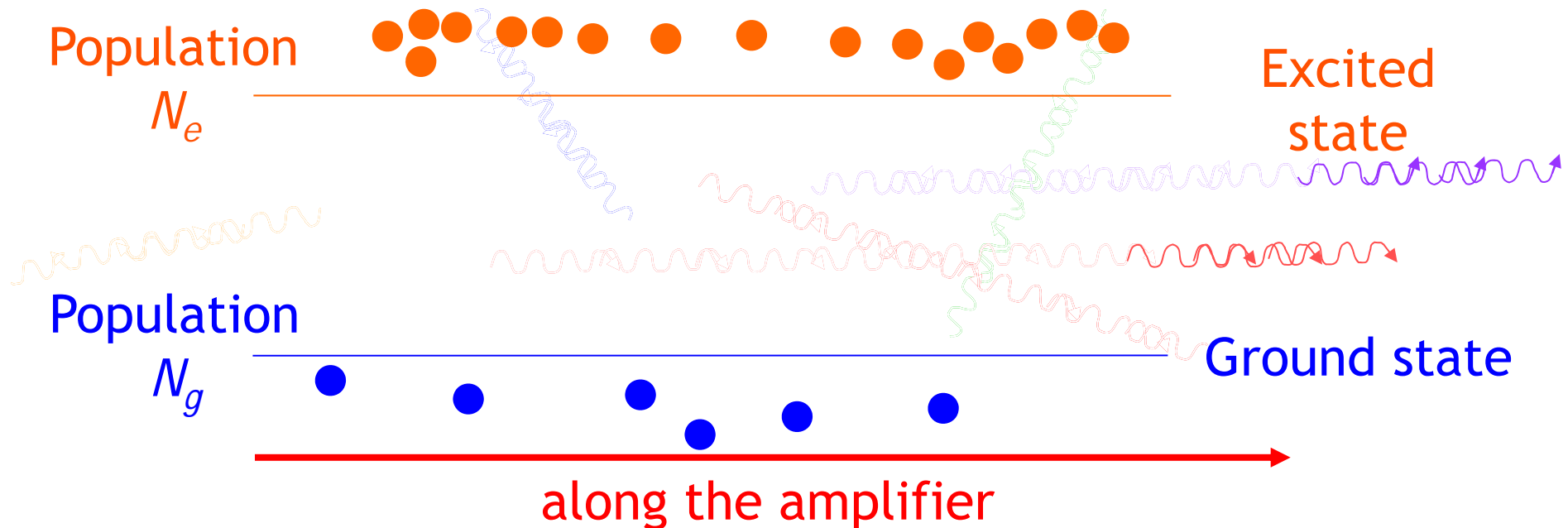
- Electrons can decay to ground state **spontaneously**
- Photons emitted, **random** orientation, phase and  $\lambda$
- “Spontaneous Emission”





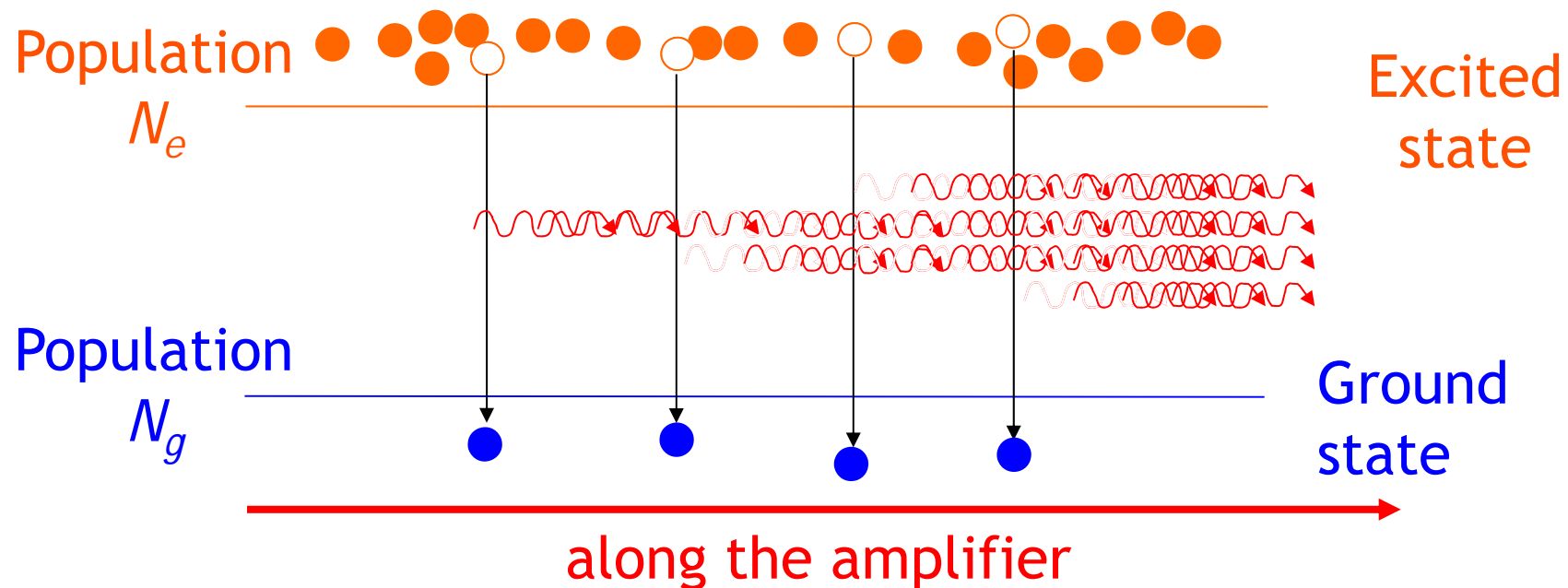
## Spontaneous Emission and Guided Modes

- Most of the spontaneously emitted photons are **lost**
- Only **a portion** is transmitted: those that become **guided modes** of the amplifier's **waveguide** structure



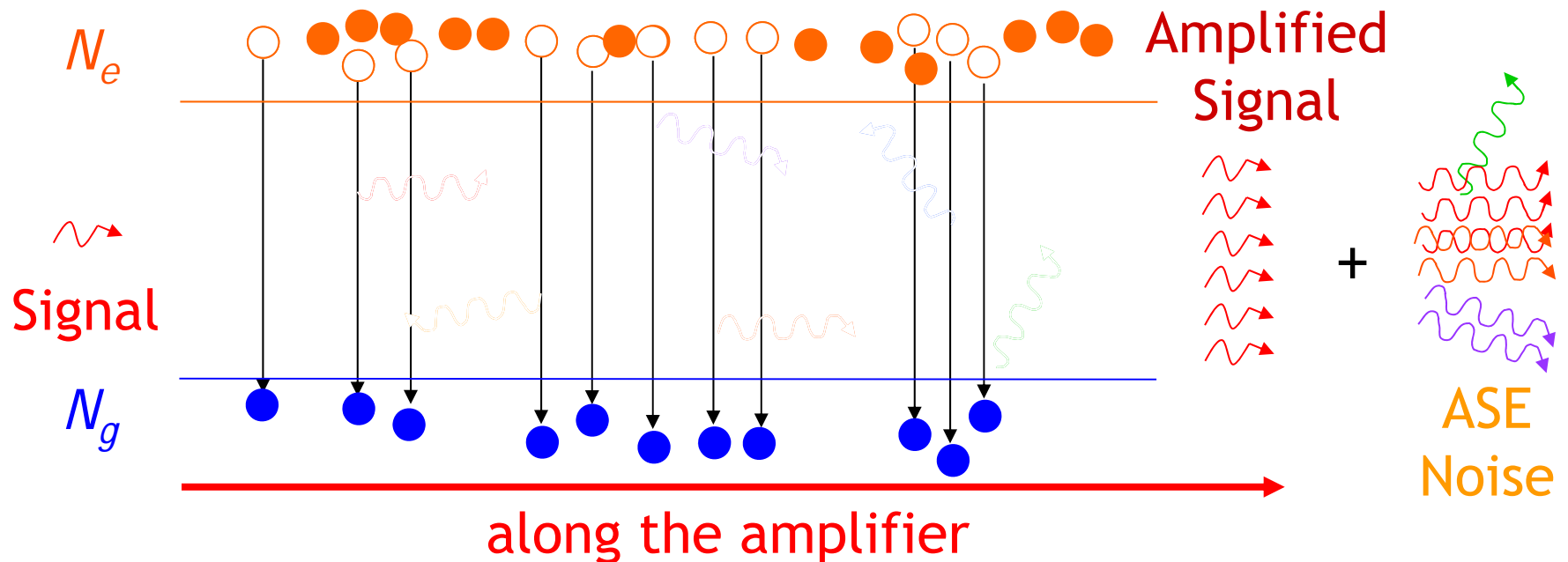
## Amplified Spontaneous Emission

- Spontaneously emitted photons can (and will) get amplified
- “Amplified Spontaneous Emission” (ASE)
- Significant source of Noise



# How (Noisy) Optical Amplifiers work

- A signal entering an optical amplifier will...  
emerge **amplified**...  
and is accompanied by **ASE noise**.



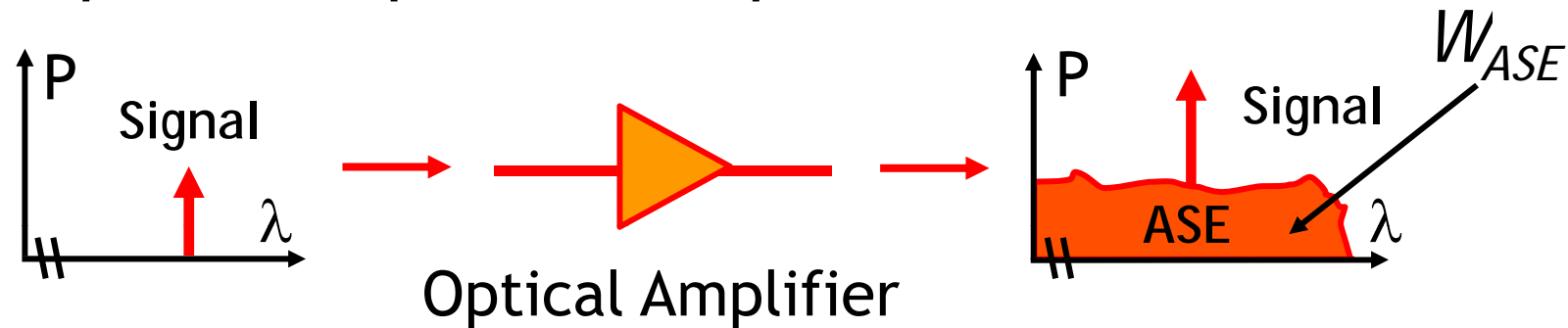
## Optical Amplifier Performance: Gain

$$G \text{ [dB]} = 10 \log_{10} \left[ \frac{P_{\text{signal\_out}}}{P_{\text{signal\_in}}} \right] \text{ dB}$$

- Ratio of signal power at amplifier output to signal power at amplifier input
- Expressed in decibels (dB)

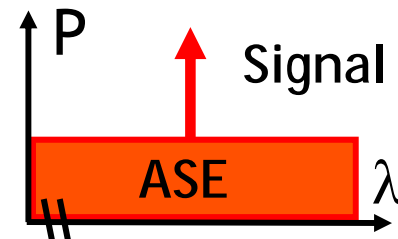
## Optical Amplifier Performance: Noise

- An optical amplifier will produce ASE noise



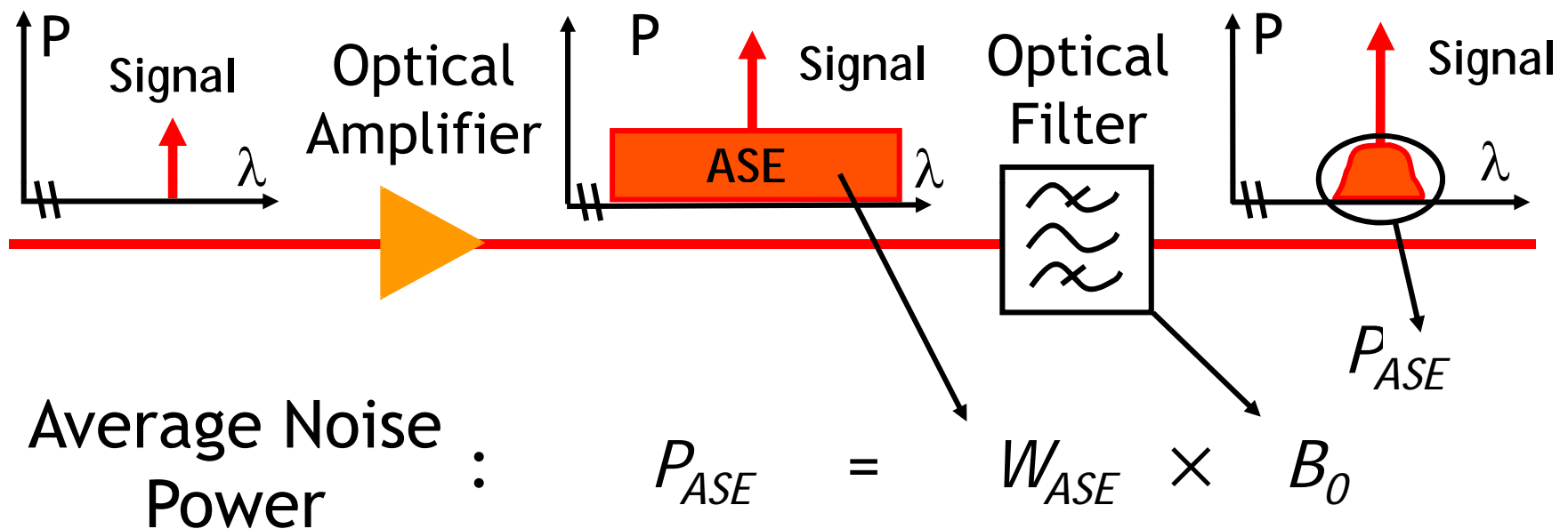
- $W_{ASE}$  = ASE noise Power Spectral Density (PSD)

- $W_{ASE}$  is approximately flat



# Optical Amplifier Performance: Noise

- ASE noise is usually **reduced** by **optical filtering**



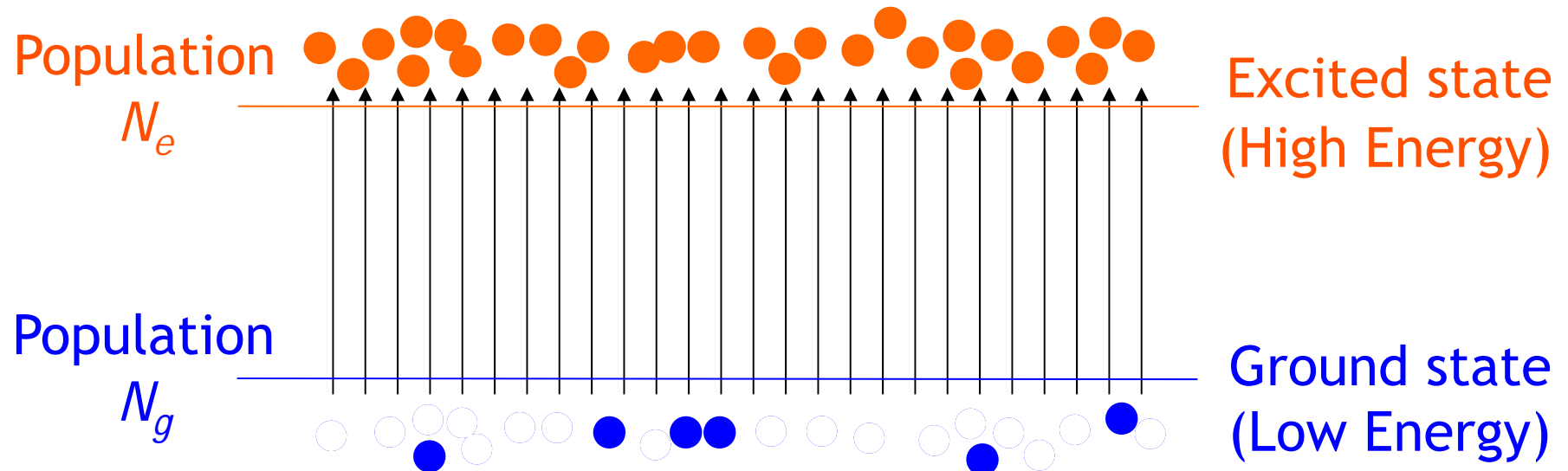
- $B_0$  is the **optical filter bandwidth**
- Expression valid for a brickwall filter and flat  $W_{ASE}$

# Relationship between Gain and Noise

- Gain & Noise depend on Population Inversion

Population Inversion or  
Spontaneous Emission  
Factor

$$n_{sp} = \frac{N_e}{N_e - N_g} > 1$$



## Relationship between Gain and Noise

- $n_{sp}$  is defined only when  $N_e > N_g$  (i.e. inversion)
- Noise  $\propto N_e$ , higher  $N_e \Rightarrow$  more spont. emission
- Absorption  $\propto N_g$ , higher  $N_g \Rightarrow$  greater absorption
- Gain ( $G$ )  $\propto N_e - N_g$ , can only have gain if emission is greater than absorption
- An interpretation of  $n_{sp}$ :  $n_{sp} \approx \frac{\text{Noise}}{\text{Gain}}$
- Low inversion  $\Rightarrow$  low gain  $\Rightarrow$  noise dominates



## Relationship between Gain and Noise

- Noise PSD ( $W_{ASE}$ ) depends on  $n_{sp}$

$$W_{ASE} = hfn_{sp}(G - 1)$$

- High inversion  $\Rightarrow N_g \rightarrow 0 \Rightarrow n_{sp} \approx N_e / N_g \rightarrow 1$

*amplifier less noisy as  $n_{sp} \rightarrow 1$*

- High inversion also desirable because  $G \propto N_e - N_g$ ,

$$N_g \rightarrow 0 \Rightarrow n_{sp} \rightarrow 1 \Rightarrow G \rightarrow \text{maximum value}$$

- For optical amplifiers, **high inversion** and **high gain** is the best.

## Noise Performance Measures

Besides  $n_{sp}$  and Noise Power ...

commonly used performance measures are:

1. Electrical Signal-to-Noise Ratio (SNR)
2. Noise Figure

- Why not  $n_{sp}$ ?

Impossible to count number of excited atoms ...

- Why not Noise Power ?

Not useful without knowing signal power ...

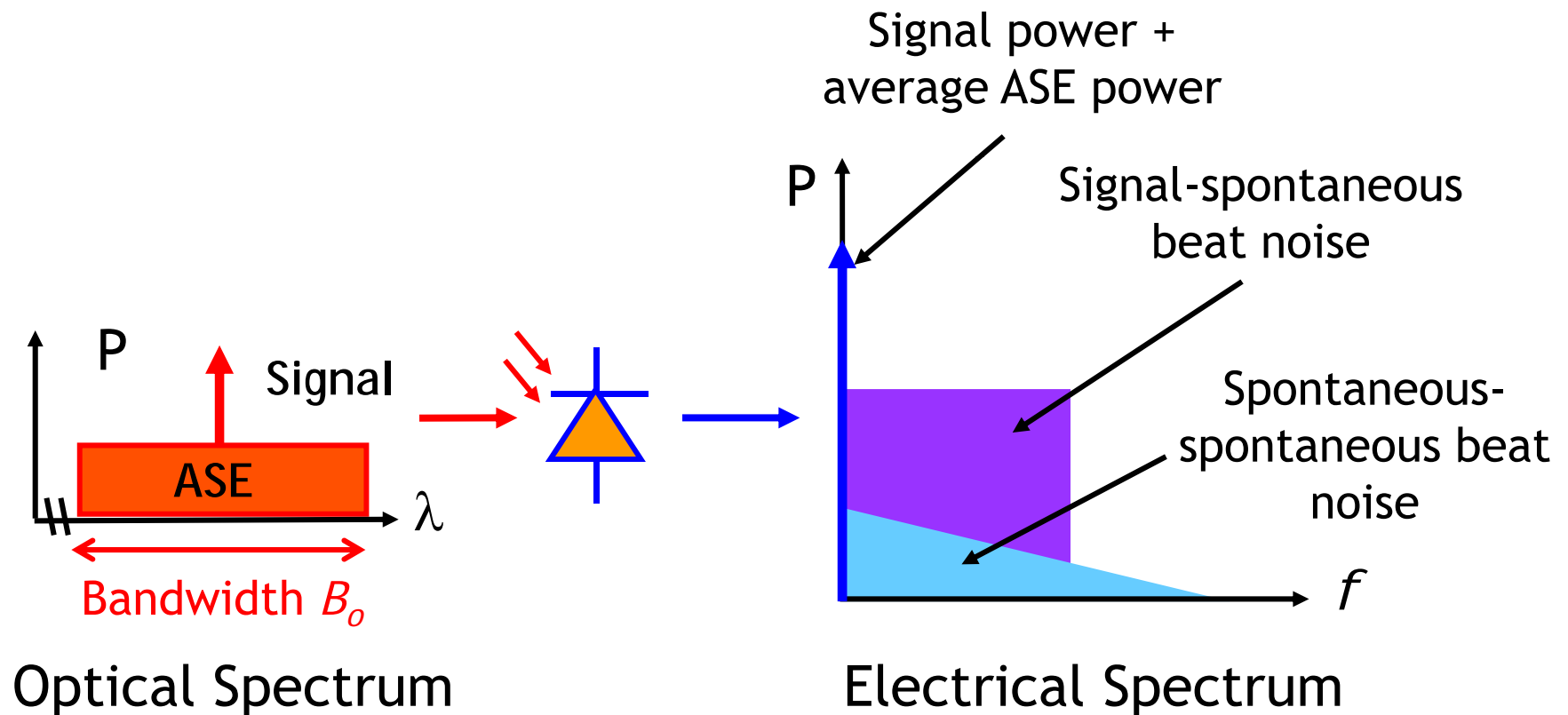
## SNR in Optically Amplified Systems

Two more important differences  
(between optical and electrical amplifiers):

1. SNR meaningful only after the signal has been detected  
(converted to electrical)
2. Optical detectors are square law devices
  - Mixing (or beating) between various frequency components of the optical signal
  - Detected electrical spectrum contains frequency difference components!

# SNR in Optically Amplified Systems

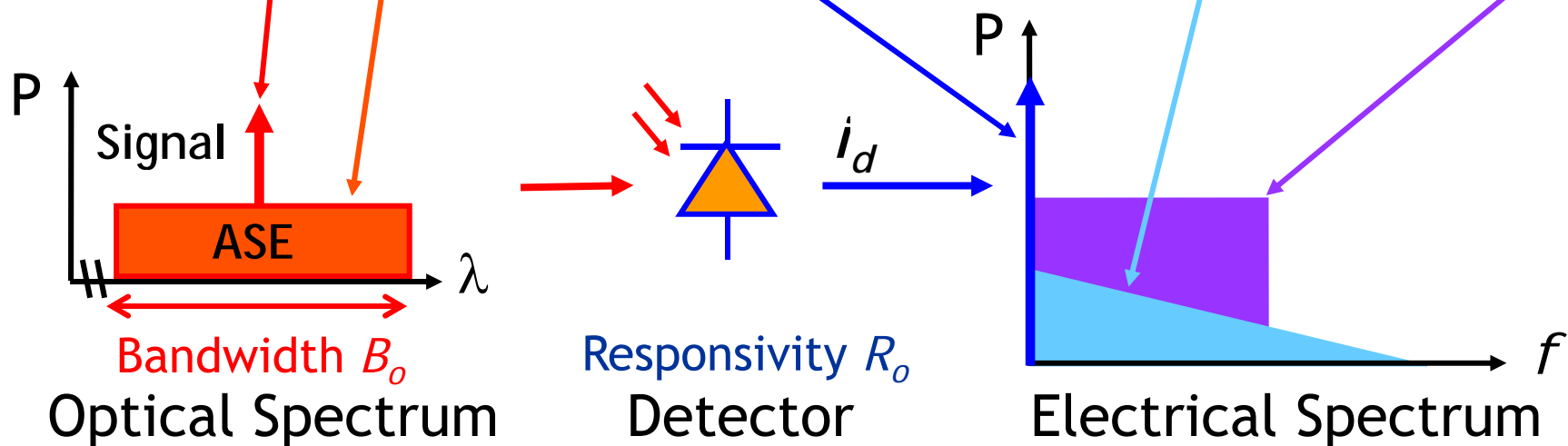
Mixing of various optical spectral components:



# SNR in Optically Amplified Systems

Mixing products in the detected photocurrent:

$$\begin{aligned}
 i_d &\propto R_o / |E_s + E_{ASE}|^2 = R_o ( |E_s|^2 + |E_{ASE}|^2 + 2\text{Re}[E_s E_{ASE}^*] ) \\
 &= R_o P_s + R_o P_{ASE} + R_o P_{ASE-ASE} + R_o P_{s-ASE} \\
 \Rightarrow i_d &= i_s + i_{ASE} + i_{sp-sp} + i_{s-sp}
 \end{aligned}$$



# SNR in Optically Amplified Systems

The SNR at a receiver with responsivity  $R_o$ , and electrical bandwidth  $B_e$ :

$$SNR = \frac{i_s^2}{\left( \overline{i_{s-sp}}^2 + \overline{i_{sp-sp}}^2 + \overline{i_{shot}}^2 + \overline{i_{th}}^2 \right) B_e}$$

signal-spontaneous  
beat noise

spontaneous-  
spontaneous beat  
noise

shot noise

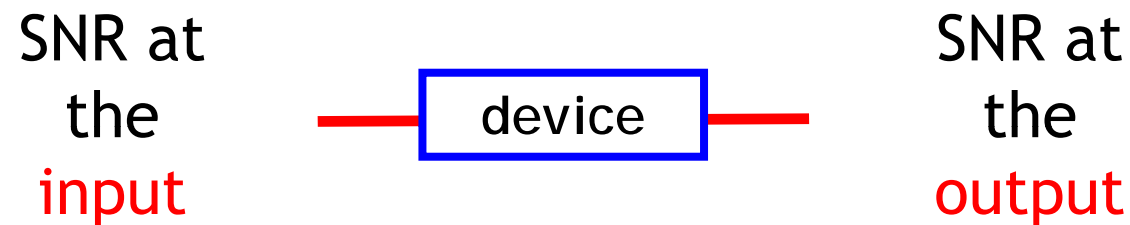
thermal  
noise

contribution from  
optical amplifier

contribution from  
detector

## Noise Figure

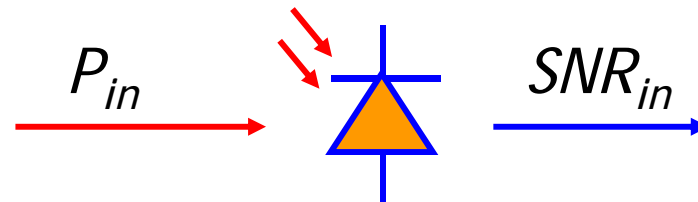
The Noise Figure of a device is defined as follows:



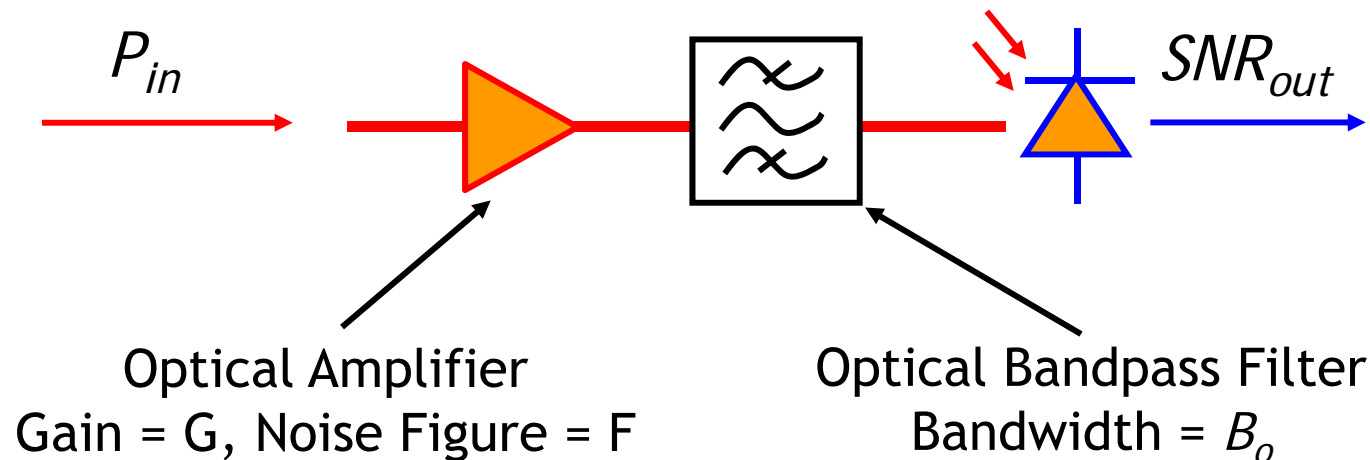
$$\text{Noise Figure, } F = \frac{\text{SNR at the } \mathbf{input}}{\text{SNR at the } \mathbf{output}}$$

# Noise Figure of an Optical Amplifier

SNR at the **input** of the Optical Amplifier,  $SNR_{in}$

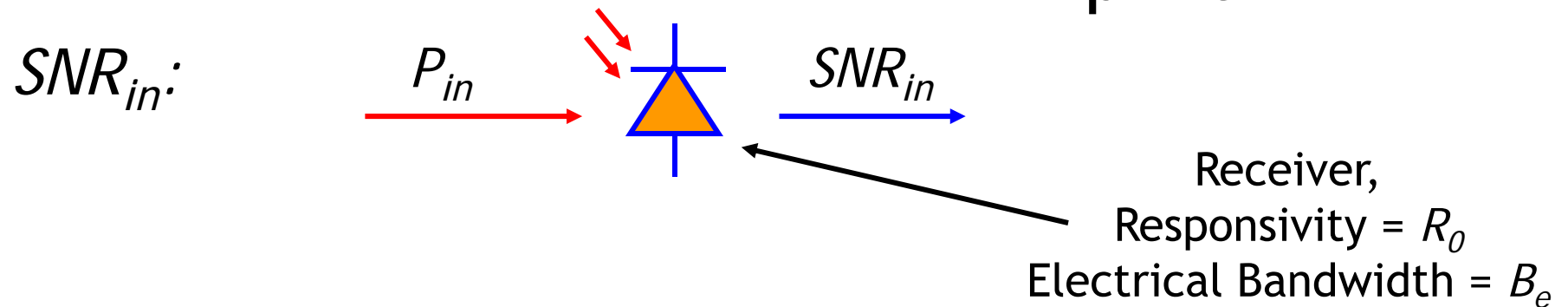


SNR at the **output** of the Optical Amplifier,  $SNR_{out}$





# Noise Figure of an Optical Amplifier



$$SNR_{in} = \frac{(R_0 P_{in})^2}{2eR_0 P_{in} B_e}$$

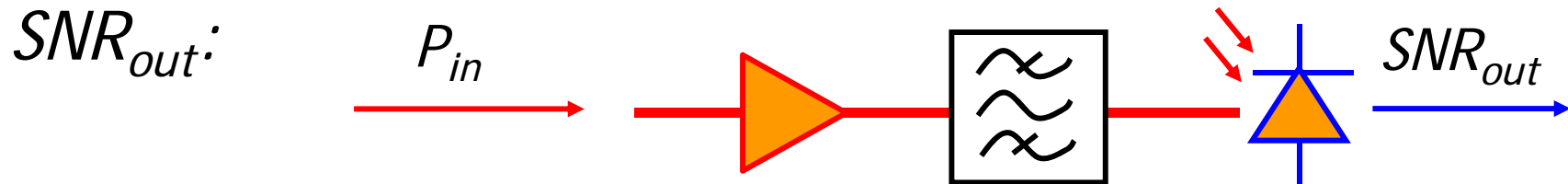
← assuming shot noise limited detector

For a “perfect” detector with 100% Quantum Efficiency,

$$SNR_{in} = \frac{P_{in}}{2hfB_e}$$

$$R_0 = \frac{e}{hf}$$

# Noise Figure of an Optical Amplifier



$$SNR_{out} \approx \frac{(R_o P_{out})^2}{4R_o^2 P_{out} h f n_{sp} (G-1) B_e + 2e R_o P_{in} B_e}$$

$$= \frac{G P_{in}}{4h f n_{sp} (G-1) B_e + 2h f B_e} \quad \leftarrow \text{after simplification}$$

Noise Figure, F:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{4h f n_{sp} (G-1) B_e + 2h f B_e}{2h f B_e G} = \frac{2n_{sp}(G-1) + 1}{G}$$

## Quantum-limited Noise Figure

- Any worthwhile amplifier will have high gain, and

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{2n_{sp}(G-1) + 1}{G}$$

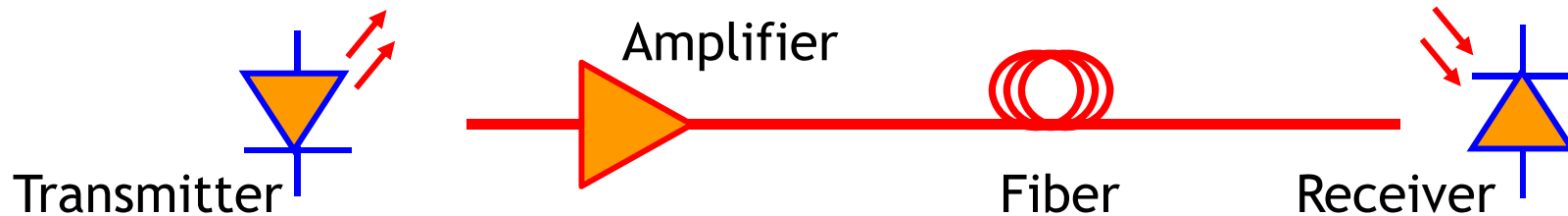
- For an amplifier with high inversion,  $n_{sp} \rightarrow 1$

$$F \approx \frac{2n_{sp}G}{G} = 2 \Rightarrow 3 \text{ dB}$$

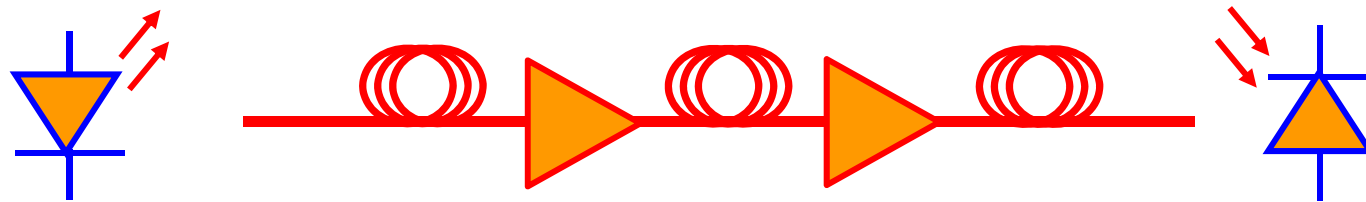
- 3 dB is the quantum limited noise figure

## Applications

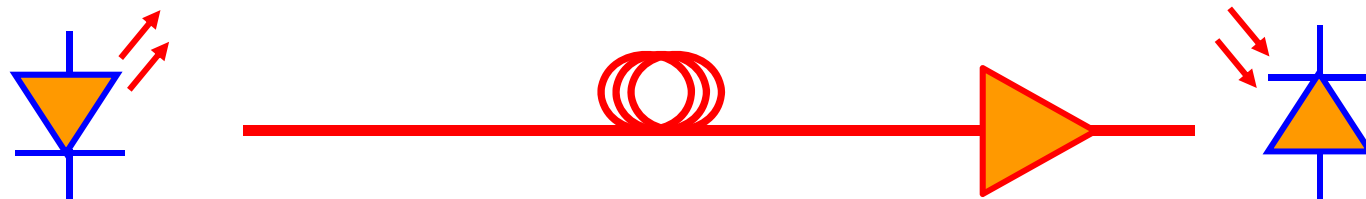
- Power amplifier



- Line amplifier



- Receiver preamplifier



## Summary

- Fundamental characteristics:
  - stimulated emission, amplification, spontaneous emission, ASE
- Performance measures:
  - Inversion Factor, gain, noise power, SNR and noise figure
- Performance Limitations
  - Mixing products, Quantum limit
- Basic mathematical analysis
- Applications

Proceed with the *Interactive Learning Module*