Università degli studi di Brescia
Communication Technologies and Multimedia

Multimedia Information Coding and Description

Laboratory Experience No. 7

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Chapter 1

Abstract

This laboratory is about block transforms. In particular we focus on the Karhunen-Loève transform (KLT), on the 2-dimensional Discrete Fourier Transform (DFT) and then on the 2-dimensional DCT (Discrete Cosine Transform). Block transforms are related to image compression. An image in raw form contains a high degree of redundant data, due to the correlation that exists between pixels. The aim of block transforms is to remove such redundancy, allowing to store an image in a more compact way. In the implementation of the various functions we use as test image the picture of Lena of dimensions 512 x 512.

Chapter 2

Karhunen Loève Transform (KLT)

We start with the study of the Karhunen Loève transform (KLT).
The KLT is a reversible transformation that removes the redundancy by decorrelating the data. The main steps to compute it are the followings:

- Partition the image into blocks of 8 x 8 size
- Stack the rows of the blocks into column vectors
- Estimate the covariance matrix
- Compute the eigenvalues and the eigenvectors of the covariance matrix
- The eigenvectors obtained are the basis functions of the KLT
- Apply the transformation to the column vectors

2.1 Covariance matrix

We partition the image into blocks of equal size (8 x 8), then we stack the rows of the blocks into columns and we put them together into a matrix $\underline{\underline{U}}$. To estimate the covariance matrix we can use two different approaches. The first one is to assume ergodicity, compute the autocorrelation of every block and average them as shown in the following equations:

$$\varphi_{U_k}[m,n] = U_k^*[-n] * U_k[n] = \sum_{m=0}^{N-1} U_k[m] \cdot U_k[m+n]$$

$$\underline{\underline{R_U}}_{ij} = \frac{1}{B}\sum_{k=1}^{B} \varphi_{U_k}[i-j]$$

The second method, that is the one that we choose, is to use a stochastic approach. So we estimate the covariance matrix in the following way:

$$\underline{\underline{R_U}} = E\left[UU^T\right] = \begin{pmatrix} E\left[{U_1}^2\right] & E\left[U_1U_2\right] & \cdots & E\left[U_1U_N\right] \\ E\left[U_2U_1\right] & E\left[{U_2}^2\right] & \cdots & E\left[U_2U_N\right] \\ \vdots & \vdots & \ddots & \vdots \\ E\left[U_NU_1\right] & E\left[U_NU_2\right] & \cdots & E\left[{U_N}^2\right] \end{pmatrix}$$

2.2 Basis functions

The basis functions of the KLT are the eigenvectors of the covariance matrix. We arrange them in decreasing order of importance according to the associated eigenvalues and we store them in the matrix $\underline{\underline{T}}$. In Figure 2.1 are shown the basis functions displayed as an 8 x 8 block each. The matrix $\underline{\underline{T}}$ is the one that diagonalizes $\underline{\underline{R_U}}$, that is:

$$\underline{\underline{R_V}} = \underline{\underline{T}}^T \underline{\underline{R_U}}\, \underline{\underline{T}}$$

Where $\underline{\underline{R_V}}$ is a diagonal matrix that contains the eigenvalues of $\underline{\underline{R_U}}$.

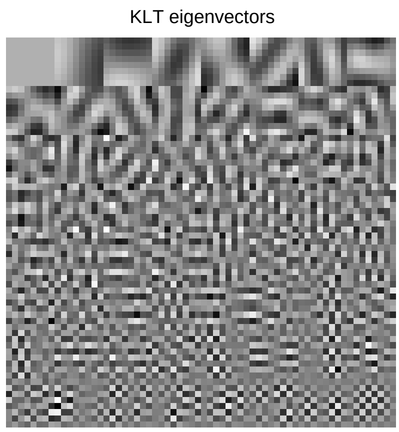

Figure 2.1: KLT basis functions

2.3 Transform

Now we apply the KLT transform to the $\underline{\underline{U_i}}$ column vectors in the following way:

$$\underline{V} = \underline{\underline{T}}^T \cdot \underline{U}$$

In Figure 2.2 we can see the KLT coefficients rearranged in 8 x 8 blocks and in Figure 2.3 the same coefficients rearranged by rows.

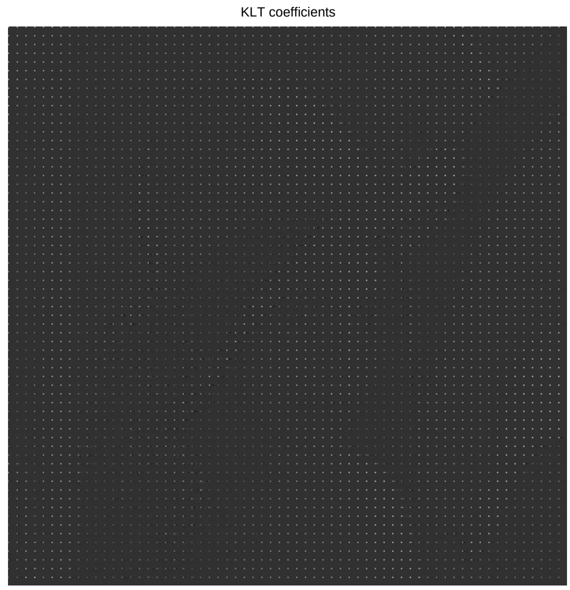

Figure 2.2: KLT coefficients

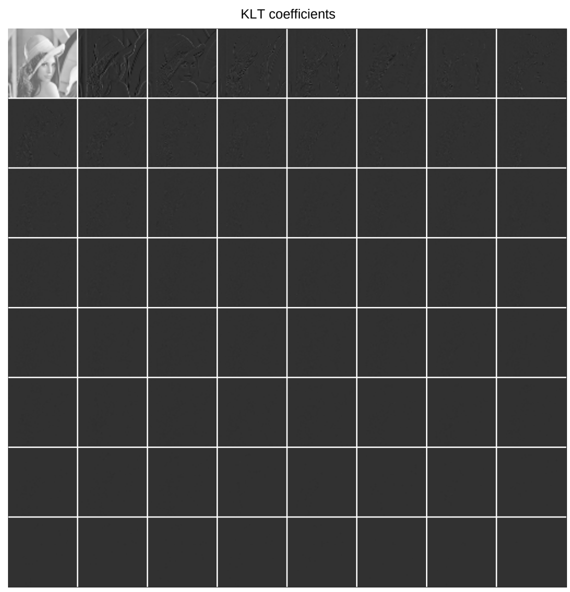

Figure 2.3: KLT coefficients rearranged by rows

We compute the energy for each block of the Figure 2.3 and we obtain the following values:

$$E_{KLT} = 10^6 \cdot \begin{pmatrix} 1.0558 & 0.0056 & 0.0021 & 0.0013 & 0.0009 & 0.0005 & 0.0004 & 0.0003 \\ 0.0003 & 0.0003 & 0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

We can observe that the energies obtained correspond to the eigenvalues of the matrix $\underline{\underline{R_U}}$. Moreover the top-left block is the one that contains the most energy, then energies monotonically rapidly decrease. This is due to the energy compaction property of the KLT.

Chapter 3

2-D Discrete Fourier Transform (DFT)

Now we apply a Discrete Fourier Transform to the image. In particular we:

- Partition the image into blocks of 8 x 8 size
- Apply the two dimensional DFT to each block

In Figure 3.1 and 3.2 we can see the obtained results.

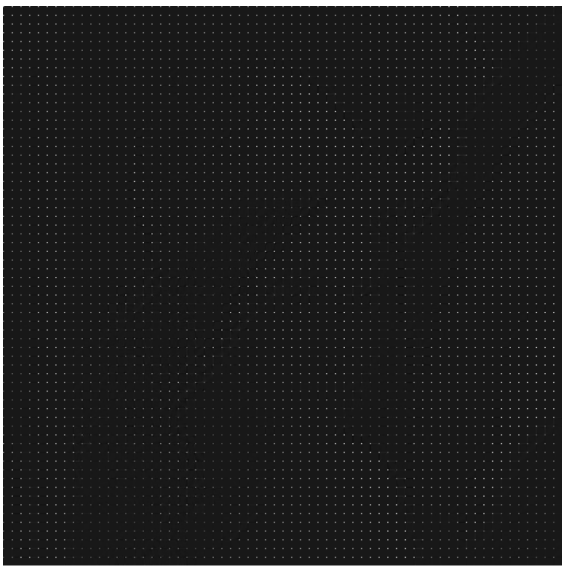

Figure 3.1: DFT coefficients

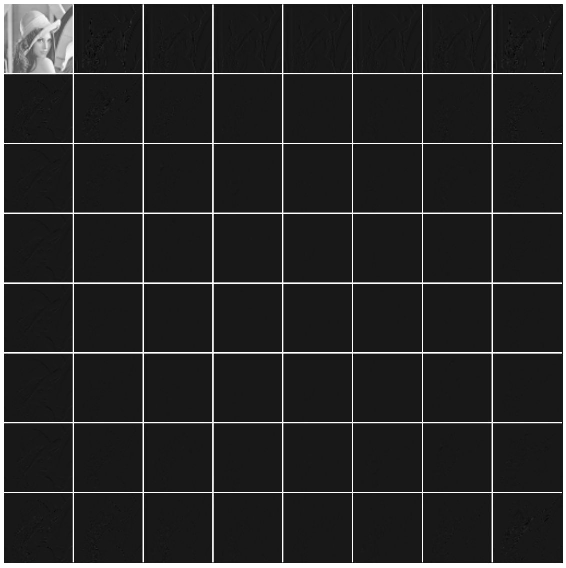

Figure 3.2: DFT coefficients rearranged by rows

The energies computed for the blocks in the Figure 3.2 are the followings:

$$E_{DFT} = 10^7 \cdot \begin{pmatrix} 6.7568 & 0.0168 & 0.0038 & 0.0019 & 0.0015 & 0.0019 & 0.0038 & 0.0168 \\ 0.0061 & 0.0036 & 0.0008 & 0.0003 & 0.0002 & 0.0003 & 0.0007 & 0.0022 \\ 0.0013 & 0.0006 & 0.0003 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0004 \\ 0.0007 & 0.0002 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0002 \\ 0.0006 & 0.0002 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0002 \\ 0.0007 & 0.0002 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0002 \\ 0.0013 & 0.0004 & 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0003 & 0.0006 \\ 0.0061 & 0.0022 & 0.0007 & 0.0003 & 0.0002 & 0.0003 & 0.0008 & 0.0036 \end{pmatrix}$$

We can observe that the energies obtained are higher and more sparse respecto to the ones obtained with the KLT transform.

Chapter 4

2-D Discrete Cosine Transform (DCT)

Now we apply a Discrete Fourier Transform to the image. In particular we:

- Partition the image into blocks of 8 x 8 size
- Apply the two dimensional DCT to each block

In Figure 4.1 and 4.2 we can see the results obtained.

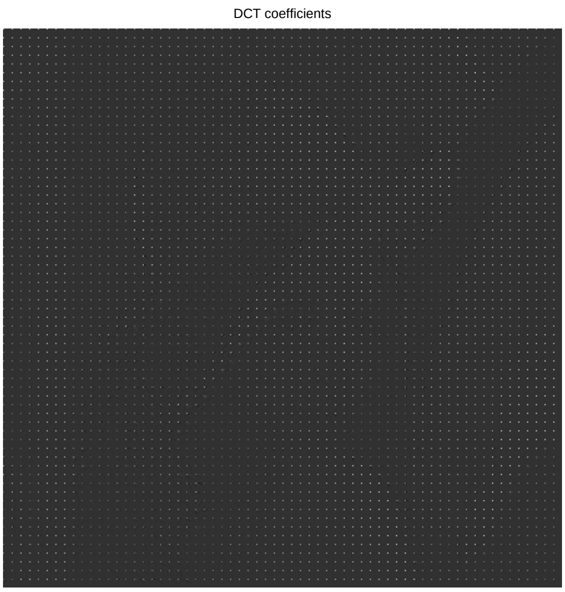

Figure 4.1: DCT coefficients

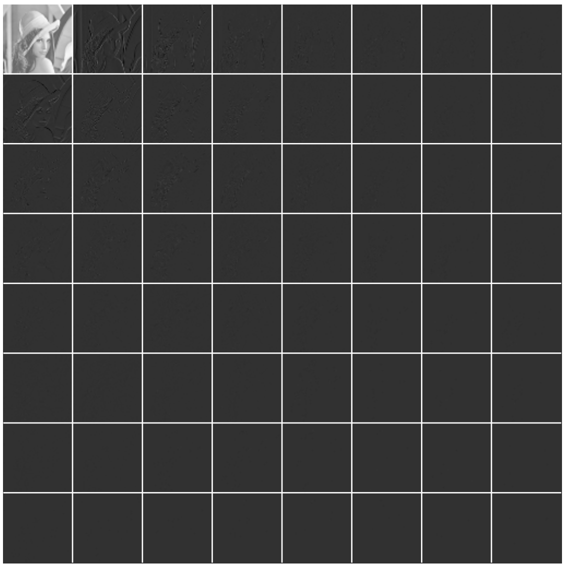

Figure 4.2: DCT coefficients rearranged by rows

The energies computed for the blocks in the Figure 4.2 are the followings:

$$E_{DCT} = 10^6 \cdot \begin{pmatrix} 1.0558 & 0.0054 & 0.0012 & 0.0003 & 0.0002 & 0.0001 & 0.0000 & 0.0000 \\ 0.0021 & 0.0010 & 0.0005 & 0.0002 & 0.0001 & 0.0000 & 0.0000 & 0.0000 \\ 0.0003 & 0.0003 & 0.0003 & 0.0001 & 0.0001 & 0.0000 & 0.0000 & 0.0000 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

We can observe that the energies obtained are similar to the ones obtained with the KLT. In fact the DCT is the best approximation of the KLT for images: one of the differences between the two transforms is that DCT bases are fixed while the KLT ones are data dependent.

Chapter 5

Inversion

Now we apply compression to the data. We invert the transforms by only using the most significant coefficients of each block. We adopt the same model for the three transforms according to the following matrix:

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Where the matrix above represents a block of the transform and the symbol $\bullet$ means that we keep the value corresponding to that position, while all the others are set to zero. This pattern has been chosen by looking at the blocks and trying to keep the most significant coefficients. Of course the importance of coefficients is different for each transform, but we use this model with all the transformed data, in order to compare the results.

5.1 KLT inversion

To invert the KLT transform we use the equation:

$$\underline{U} = \underline{\underline{T}}^{T^{-1}} \underline{V}$$

Then we have to rearrange the vectors $\underline{U_i}$ by 8 x 8 blocks. In Figure 5.1 is shown the result.

In Figure 5.2 is shown the reconstruction error obtained as the difference between the original image and the reconstructed one.

Figure 5.1: KLT reconstructed

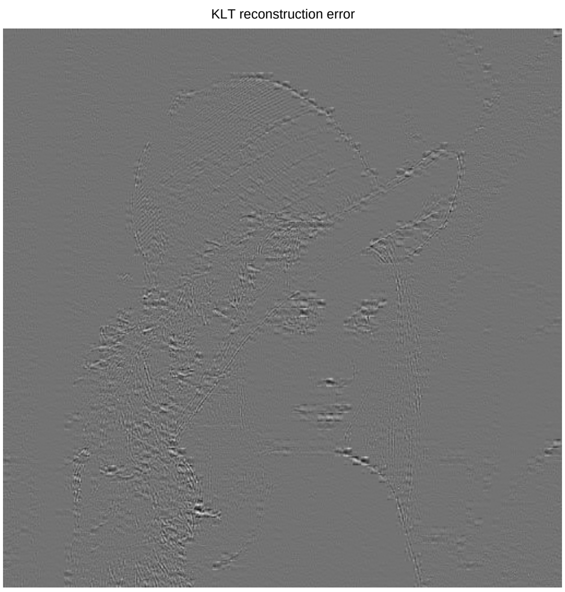

Figure 5.2: KLT reconstruction error

The energy of the reconstruction error of the blocks is the following:

$$Error_{KLT} = \begin{pmatrix} 13.5772 & 16.4966 & 21.8808 & 25.8373 & 33.3612 & 32.8382 & 23.0529 & 17.3925 \\ 8.9302 & 10.2524 & 10.1697 & 12.2760 & 18.5415 & 14.1270 & 13.1716 & 12.1192 \\ 10.6839 & 15.6651 & 16.2328 & 12.7195 & 14.4944 & 11.0995 & 16.1080 & 11.2022 \\ 13.8595 & 23.7678 & 23.0076 & 20.7697 & 15.9561 & 15.9057 & 15.1229 & 8.5274 \\ 11.3131 & 23.7756 & 22.5471 & 21.9871 & 16.7357 & 16.0369 & 14.8965 & 10.9152 \\ 9.3163 & 18.9596 & 14.0216 & 15.6337 & 12.4422 & 13.7388 & 14.6613 & 10.2669 \\ 15.7819 & 16.2185 & 13.3479 & 13.0209 & 12.6380 & 9.8621 & 12.5418 & 8.0034 \\ 30.6511 & 36.2730 & 32.5418 & 26.3883 & 21.2544 & 17.1969 & 11.9813 & 16.3340 \end{pmatrix}$$

5.2 DFT inversion

To invert the DFT transform we apply the 2-D inverse DFT to each block. In Figure 5.3 is shown the result.

Figure 5.3: DFT reconstructed

In Figure 5.4 is shown the reconstruction error obtained as the difference between the original image and the reconstructed one.

Figure 5.4: DFT reconstruction error

The energy of the reconstruction error of the blocks is the following:

$$Error_{DFT} = \begin{pmatrix} 189.5680 & 170.8719 & 161.7842 & 148.4984 & 133.6950 & 132.0245 & 127.2323 & 123.6461 \\ 92.0558 & 86.4259 & 84.6947 & 81.4182 & 79.5152 & 78.6084 & 75.3613 & 74.1843 \\ 87.1627 & 91.6959 & 94.0764 & 93.3339 & 91.5959 & 87.8302 & 85.2974 & 84.8858 \\ 82.8435 & 85.0899 & 85.0884 & 84.8998 & 82.5285 & 81.1612 & 80.7900 & 80.0250 \\ 77.5154 & 81.4289 & 80.6678 & 82.1235 & 85.2892 & 85.8296 & 83.7545 & 83.4368 \\ 83.4316 & 86.3796 & 88.7496 & 87.3197 & 93.9485 & 96.9676 & 92.5422 & 86.6929 \\ 76.4922 & 75.2371 & 80.2354 & 78.5210 & 82.4269 & 88.1667 & 89.4179 & 92.9767 \\ 120.0566 & 131.1545 & 139.7054 & 146.3248 & 145.3549 & 154.6473 & 162.8657 & 192.0169 \end{pmatrix}$$

We can notice that the energy of the reconstruction error is much higher than the one obtained for the KLT.

5.3 DCT inversion

To invert the DCT transform we apply the 2-D inverse DCT to each block. In Figure 5.5 is shown the result.

In Figure 5.6 is shown the reconstruction error obtained as the difference between the original image and the reconstructed one.

Figure 5.5: DCT reconstructed

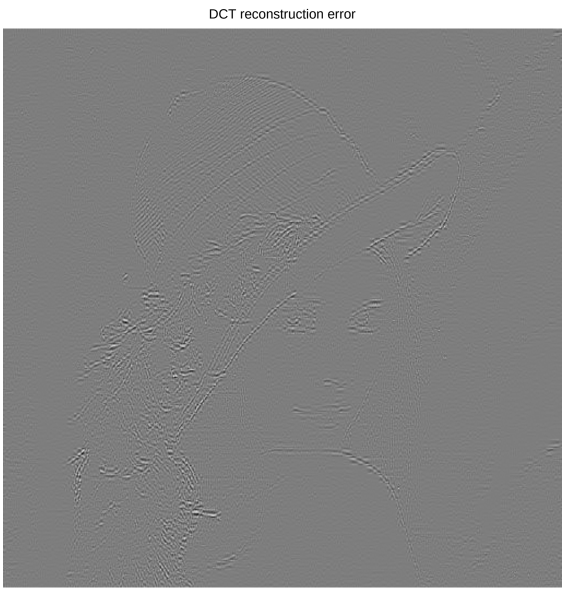

Figure 5.6: DCT reconstruction error

The energy of the reconstruction error of the blocks is the following:

$$Error_{DCT} = \begin{pmatrix} 17.3006 & 17.3256 & 20.5964 & 17.2879 & 19.9219 & 18.9882 & 19.2074 & 16.1439 \\ 8.6888 & 9.0964 & 9.6601 & 9.9178 & 9.4307 & 10.8705 & 8.6814 & 9.1857 \\ 17.9322 & 17.6561 & 19.7070 & 18.4806 & 19.9330 & 18.9524 & 18.3172 & 17.5406 \\ 12.4268 & 13.5312 & 12.9107 & 14.1594 & 14.6032 & 13.0346 & 14.6835 & 12.2510 \\ 11.8596 & 13.2591 & 13.6820 & 14.0783 & 15.0800 & 14.3996 & 13.5928 & 13.2973 \\ 18.1089 & 18.3477 & 19.1057 & 18.0166 & 19.1817 & 19.9738 & 17.7690 & 18.4498 \\ 9.6640 & 9.0135 & 10.1120 & 9.1829 & 10.5148 & 10.3495 & 9.8450 & 8.6196 \\ 15.5027 & 19.1465 & 17.4172 & 19.2939 & 18.3106 & 19.1652 & 17.9235 & 18.3649 \end{pmatrix}$$

We can notice that the energy of the reconstruction error is not very different to the one obtained for the KLT, but it is anyway higher.