University of Brescia
Faculty of Engineering

Communication Technologies and Multimedia

Multimedia Information Coding and Description

Laboratory Experience, No.8

Students:

Arici Giorgio
Arrigoni Simone
Migliorati Andrea
Mutti Andrea
Yonas Etansa
Wasyhun Asefa
Weise Eduardo

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Abstract

In this laboratory experience we talk about *sub-band coding* using the perfect reconstruction *Haar* analysis and synthesis filters. In the first part of this experience we decompose a simple image (called X) using the analysis bank filter, first filtering by columns and then filtering by rows, obtaining four components X_{LL}, X_{LH}, X_{HL}, X_{HH}. After that we reconstruct the image using the synthesis bank filter. Then we evaluate the quality of the image obtained using only using the LL sub-band in the synthesis phase. Iterate the sub-band analysis and synthesis process on the LL sub-band, and repeat the experiment of the previous two points. In the end we apply a uniform (non-zero) quantization to different sub-bands and study its effect on the quality of the reconstructed image.

1 Sub–band filtering

Consider now the separable application of these filter bank to an image according to the following schemes (Figure 1 and Figure 2) for horizontal and vertical filtering and sub-sampling.

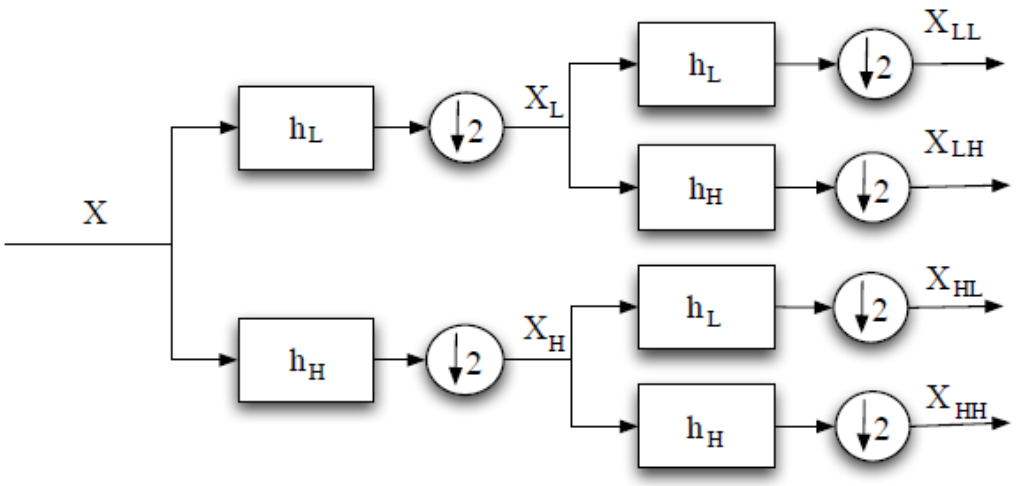

Figure 1: The analysis bank: filters the signal X first by columns and then filters by rows.

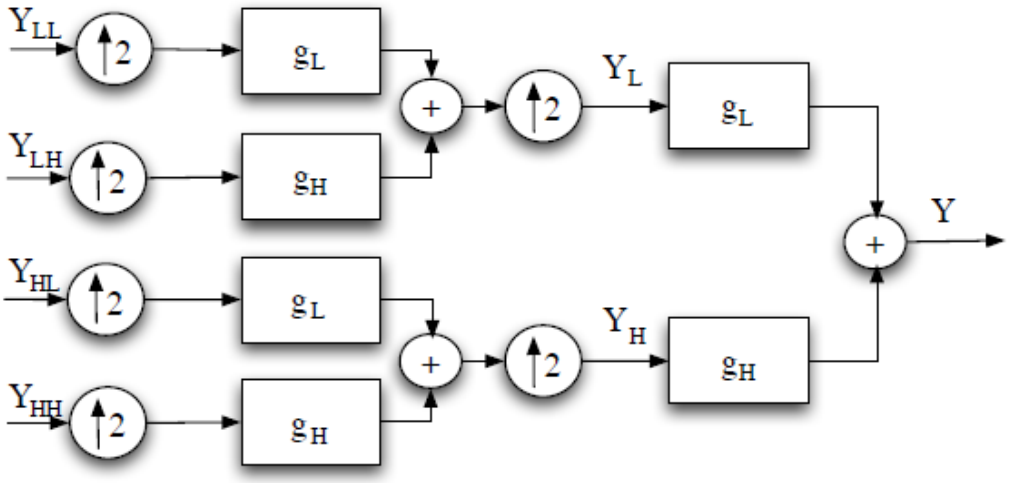

Figure 2: The synthesis bank: filters the signal obtained by the analysis part first by rows and then filters by columns.

1.1 Analysis Phase

As anticipated above, in the first part of experience we implement the *analysis* bank by filtering the image X using the following perfect *Haar* decomposition analysis; shown in Figure 3 and Equation 1.

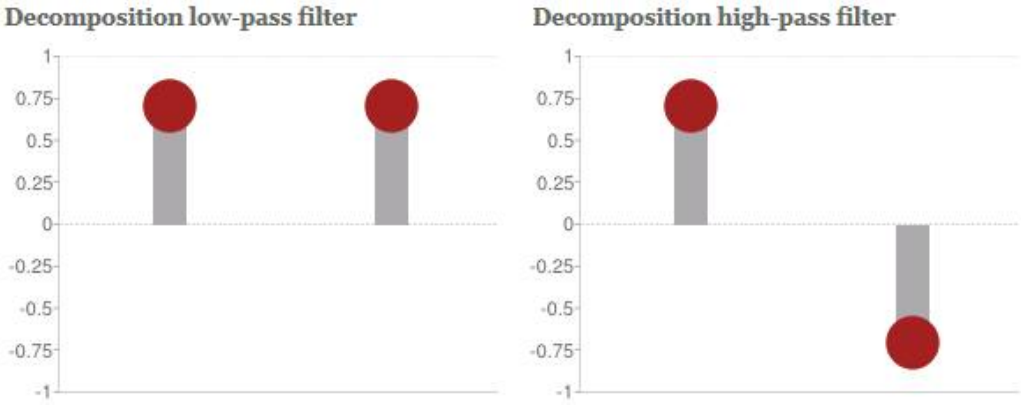

Figure 3: Wavelet families: Haar. Coefficients of low-pass filter h_L and high-pass filter h_H.

$$h_L = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \qquad h_H = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \tag{1}$$

As it is possible to see in Figure 1, first we filter the image by columns, obtaining X_L and X_H. This intermediate results are used in the second step, in which X_L and X_H are filtered by rows.

The results of the analysis phase are shown in Figure 4, in which it is possible to see the obtained components X_{LL}, X_{LH}, X_{HL}, X_{HH}.

Figure 4: The results of the analysis bank: X_{LL} top-left, X_{LH} top-right, X_{HL}down-left and X_{HH} down-right.

It is important to notice that almost all the energy is in the first component X_{LL}, which is very near to a small replica of the original image, while in the other components (expecially in X_{HH}) we can observe some features, for example in the eye area, and some high-frequency noise com-

ponents.

1.2 Synthesis Phase

After the *analysis* phase implementation we proceed in *synthesis* phase by filtering the components obtained in the previous part using the following perfect reconstruction synthesis; shown in Figure 5 and Equation 2.

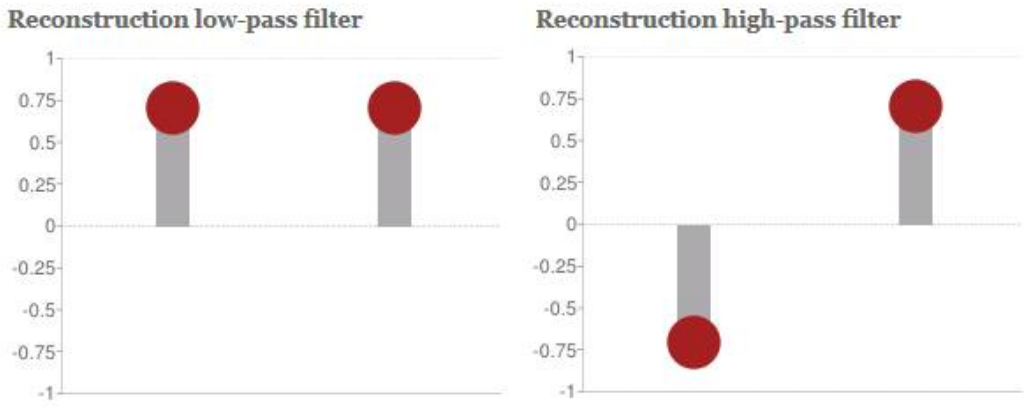

Figure 5: Wavelet families: Haar. Coefficients of low-pass filter g_L and high-pass filter g_H.

$$g_L = \left[\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}\right] \qquad g_H = \left[-\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}\right] \tag{2}$$

The result of the synthesis phase is shown in Figure 6, in which it is possible to see how the reconstructed image is an exact replica of the original one, up to a quantization error shown in the third image. The error is in the order of 10^{-16}.

2 Not perfect reconstruction

In this part of our experience we try to evaluate the quality of the image obtained by only using the *LL sub-band* in the synthesis phase , in order to reduce the amount of information to transmit. The remaining sub-bands are quantized to zero (we can think of them as not trasmitted sub-bands).

The result of the reconstruction is shown in the Figure 7 below:

We've also computed the error, which is in the order of 0.01. We have then obtained a much worse reconstruction of the original image than the

Figure 6: The results of perfect reconstruction of the image. From left: original, reconstructed and the error.

Figure 7: The results of not-perfect reconstruction of the image. From left: original, reconstructed and the error.

case in which all the bands are used. Yet it is still a reasonably acceptable result, in terms of quality/definition of the image.

3 Iterative experiment

In this section we show the result of the *iteration* of the sub-band coding process on the LL sub-band, repeating the experiment of the previous two points. We use the LL sub-band component obtained at the previous step as input for the next. We chose 5 as the number of iteration.

In the Figure 8 we show the decomposition of the original image. Each interation produces a new decomposition in the LL sub-band component (i.e. the top left square), leading to a sequence of smaller squares inside it.

At the fifth iteration the process is over, so we can proceed to reconstruct the image. We used the two different types of reconstruction

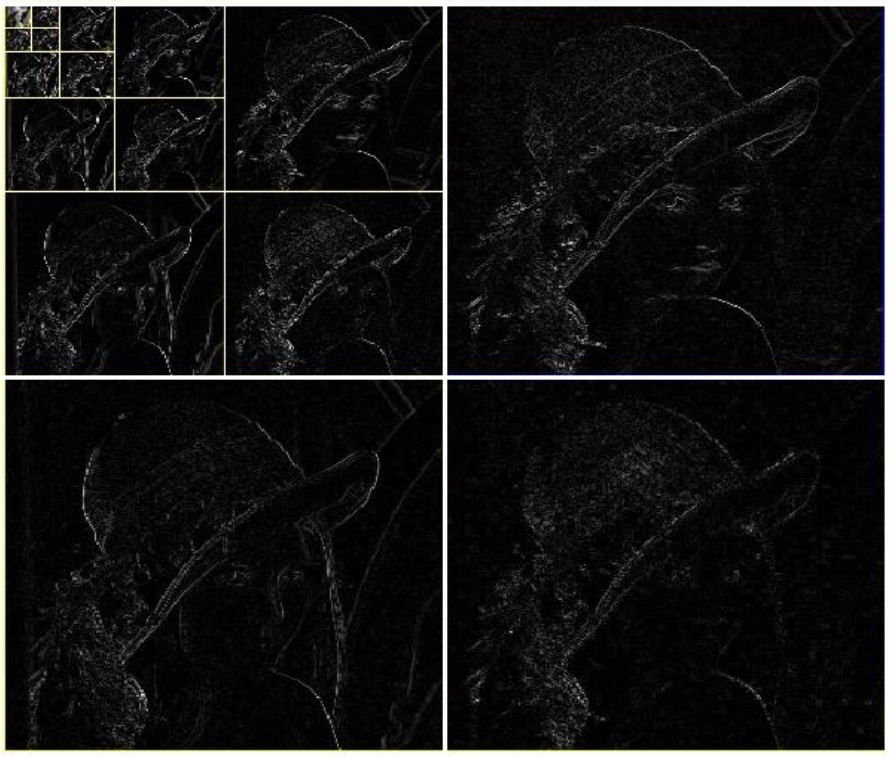

Figure 8: The results of iterative decomposition of the image.

(perfect and not-perfect) also in this case.

In the Figure 9 it is possible to see the single step of the reconstruction in case of perfect reconstruction. Note that the dimension of the images are double step by step.

Figure 9: The partial results of iterative perfect reconstruction of the image. The proportion between the component and original image are reported.

The result of the perfect and not-perfect reconstruction are shown in the Figure 10 and Figure 11.

Figure 10: The final results of iterative perfect reconstruction of the image.

Figure 11: The final results of iterative not-perfect reconstruction of the image.

As we can see, the perfect reconstruction in this case lead almost to the same result obtained before.

For the second case (non-perfect reconstruction) we observe an error that is in the same range of values of the error obtained in the other non-perfect reconstruction operated before.

4 Bonus: Uniform Quantization

In the last section of our experience we are proposed to apply a uniform (non-zero) quantization to the different sub-bands observing its effect on the quality of the reconstructed image. We have considered two different case:

- 128 levels-quantization for just LH, HL and HH sub-bands;
- 128 levels-quantization for all sub-bands;
- 16 levels-quantization for just LH, HL and HH sub-bands and
- 16 levels-quantization for all sub-bands.

In the following images (Figure 12, Figure 13, Figure 14, Figure 15) are shown the results of our experiment: in each triplet it's possible to compare the original image with the quantized one and then it's represented the error.

Figure 12: Image quantized with 128 levels for the LH, HL and HH sub-bands.

Figure 13: Image quantized with 128 levels for all sub-bands.

In these two cases it's used just one bit/pixel less than the original image (7 bits/pixel instead of 8) and therefore the resulting error is very small (there are just few pixels where it's evident).

In the last two cases the amount of information in each quantized sub-band is halfed with respect to the original image since we use 4 bits/pixel

Figure 14: Image quantized with 16 levels for the LH, HL and HH sub-bands.

Figure 15: Image quantized with 16 levels for all sub-bands.

instead of 8 bits/pixel. In these images the error is more prominent, expecially where all sub-bands are quantized.