Università degli studi di Brescia
Communication Technologies and Multimedia

Multimedia Information Coding and Description

Laboratory Experience No.3

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Chapter 1

Introduction

In this laboratory experience we have analyzed the spectra of bidimensional and then tridimensional signals, in our particular case we observed first images and later videos.

For every signal we have examined the peculiarities of the Fourier transform, verifying them from what we expected basing on the theory seen in class.

The formula for the multidimensional Fourier transform is:

$$U(\underline{f}) = \int_{\Re} u(\underline{x}) \cdot \exp(j2\pi \underline{f^T} \underline{u})$$

Chapter 2

2D cylindric signals

In two dimension the Fourier transform formula is:

$$X(f_x, f_y) = \iint x(x,y) * \exp j2\pi(xf_x + yf_y)dxdy$$

We now analyse the following signal:

$$s_1(x,y) = \frac{rect(\frac{x}{\Delta})}{\Delta}$$

$$s_2(x,y) = rect(\cos(\alpha x) + \sin(\alpha y))$$

$$s_3(x,y) = \exp(j2\pi(ux + vy))$$

$$s_4(x,y) = \cos(2\pi(ux + vy)$$

$$s_5(x,y) = \cos(2\pi(u_1x + v_1y)) * \cos(2\pi(u_2x + v_2y)$$

$$s_6(x,y) = \exp(-\pi x^2) * \cos(2\pi(ux + vy)$$

2.1 2D Rectangular impulse

The Fourier transform of a generic rectangular pulses is:

$$s(x,y) = \frac{rect(\frac{x}{\Delta})}{\Delta} \leftrightarrow S(f_x, f_y) = sinc(\Delta f_x)$$

We have the rectangular pulse that changes support dimension in function of Δ, keeping the volume constant. Decreasing the value of Δ the support shrinks and the signals tends to the Dirac delta. As expected, shown in Figure 2.1 and Figure 2.2, in the Fourier domain decreasing Δ the signal tends to be a constant.

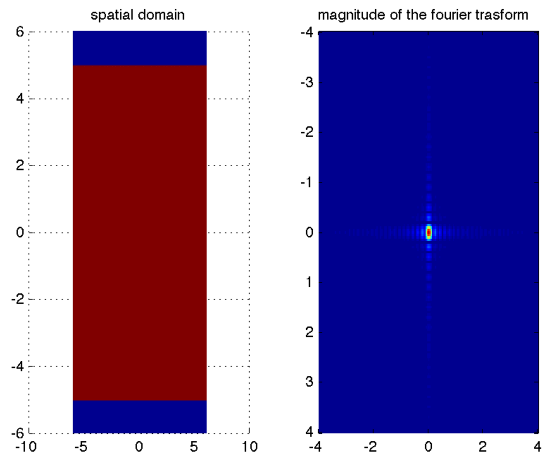

Figure 2.1: Rectangular impulse in spatial and Fourier domain with $\Delta = 10$

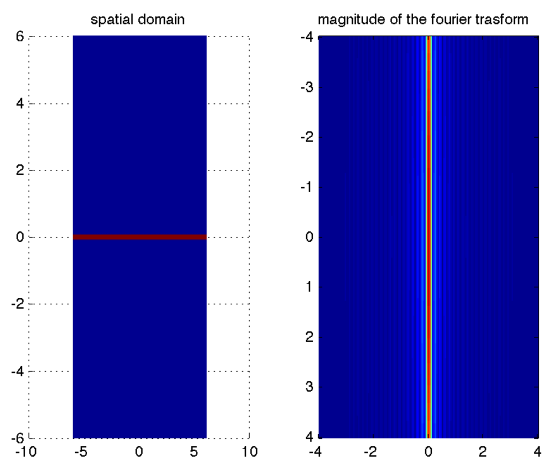

Figure 2.2: Rectangular impulse in spatial and Fourier domain with $\Delta = 0.1$

Afterwards we have put in the arguments of the pulse a sinusoidal function obtaining a periodic one:

$$s_2(x, y) = rect(\cos(\alpha x) + \sin(\alpha y))$$

as a consequence in the frequency domain we obtain a ”line” spectrum, changing the spatial frequency of the sinusoidal function we change the position of the $\delta(f_x, f_y)$. The results are in Figure 2.3 and Figure 2.4.

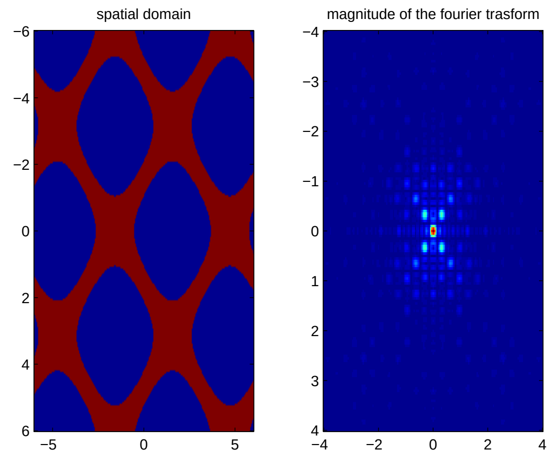

Figure 2.3: rectangular impulse with cosine arguments, $\alpha = 1$

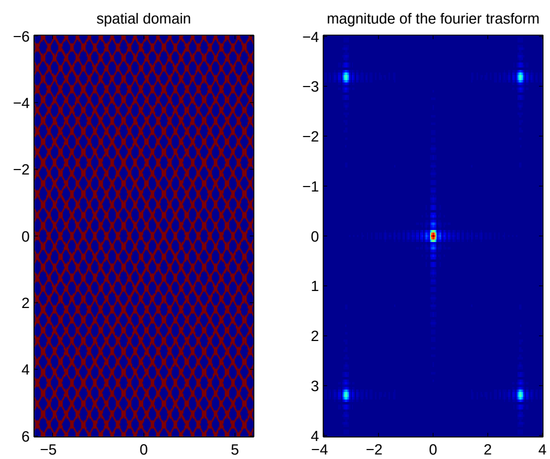

Figure 2.4: rectangular impulse with cosine arguments, $\alpha = 10$

2.2 Sinusoidal signal

In this section we analyse the spectra of two different sinusoidal signal:

$$s_4(x,y) = \cos(2\pi(ux+vy) \leftrightarrow S_4(f_x, fy) = \frac{1}{2}[\delta(f_x-u)\delta(f_y-v)+\delta(f_x+u)\delta(f_y+v)]$$

$$s_5(x,y) = \cos(2\pi(u_1x+v_1y))\cdot\cos(2\pi(u2x+v2y) \leftrightarrow S_5(f_x,f_y) = \frac{1}{2}[\delta(f_x-u)+\delta(f_x+u)]\cdot[\delta(f_y-v)+\delta(f_y+v)]$$

The simulation confirms as expected from the analytical solution. In the first case, Figure 2.5, the spectrum is composed by two Dirac delta at the frequency of the cosine, instead, in the second case we have four Dirac delta that are the multiplication between the δ in f_x and f_y, Figure 2.6.

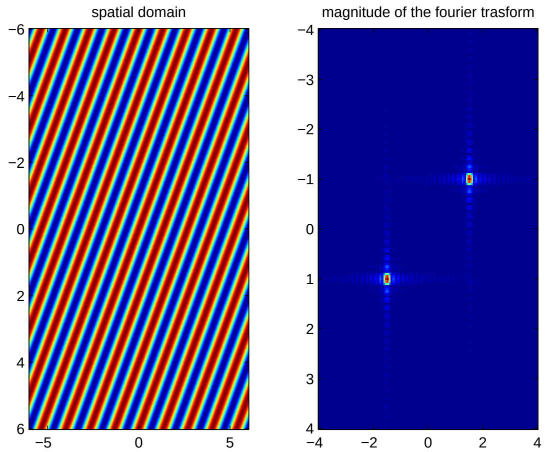

Figure 2.5: Sinusoidal signal in spatial and Fourier signal

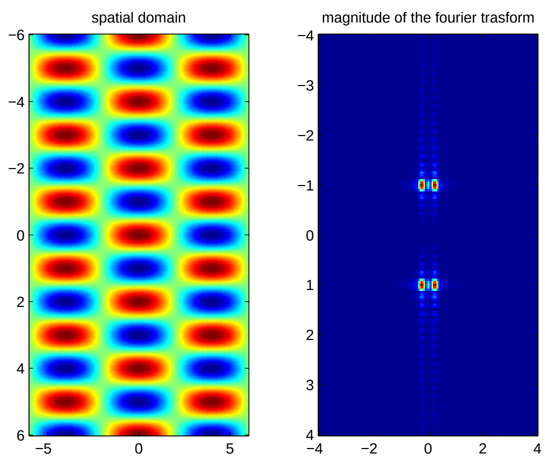

Figure 2.6: Product of two sinusoidal signal in spatial and Fourier signal

2.3 Modulated Gaussian pulse

In the last simulation we have treated the case of modulated Gaussian pulse:

$$s_6(x,y) = \exp(-\pi x^2)\cos(2\pi(ux+vy)) \leftrightarrow X_6(f_x,f_y) = \exp(-f_x^2) * \tfrac{1}{2}[\delta(f_x-u)\delta(f_y-v) + \delta(f_x+u)\delta(f_y+v)]$$

In the spatial domain we have an amplitude modulation of Gaussian pulse so in frequency we obtain a translation of the same pulse, the Gaussian is invariant to the Fourier transform, at the frequency of the cosine, Figure 2.7

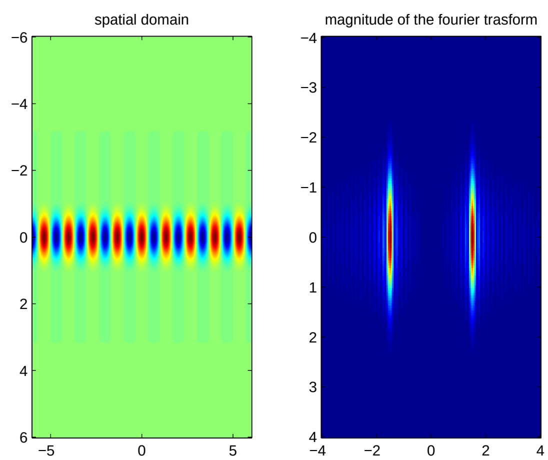

Figure 2.7: Modulated Gaussian pulse

Chapter 3

3D cylindric signals

In this chapter we analyse first the spectrum of a still image video and then a video with motion.
The three dimensional Fourier transform is done thanks to the Matlab function *fftn*.
To observe how varies the spectrum in function of the movement speed we plot on the x axis the temporal frequency f_t and on the y axis the f_y spatial frequency for a value of the the other spatial frequency equal to $f_x = 201$.

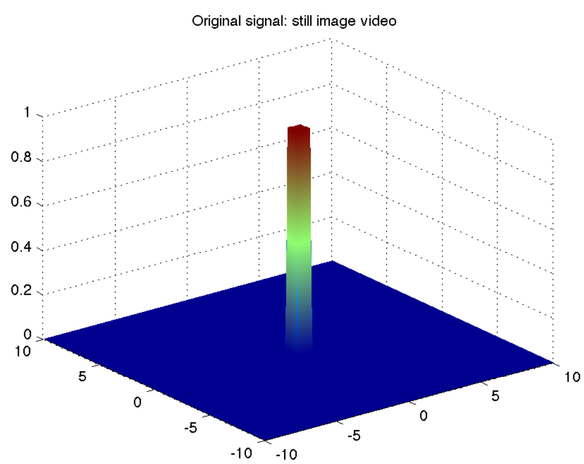

Figure 3.1: Original rectangular pulse used for the simulations

3.1 Still image video

In the case of a still image video, that means $v_x = v_y = 0$, the Fourier transform of the signal is:

$$\tilde{I}(f_x, f_y, f_t) = \tilde{I}_0(f_x, f_y)\delta(f_t)$$

In fact in the $f_t - f_y$ plane we have a ”temporal” delta in the central position that multiplies a sinc in the spatial frequencies, Figure 3.2

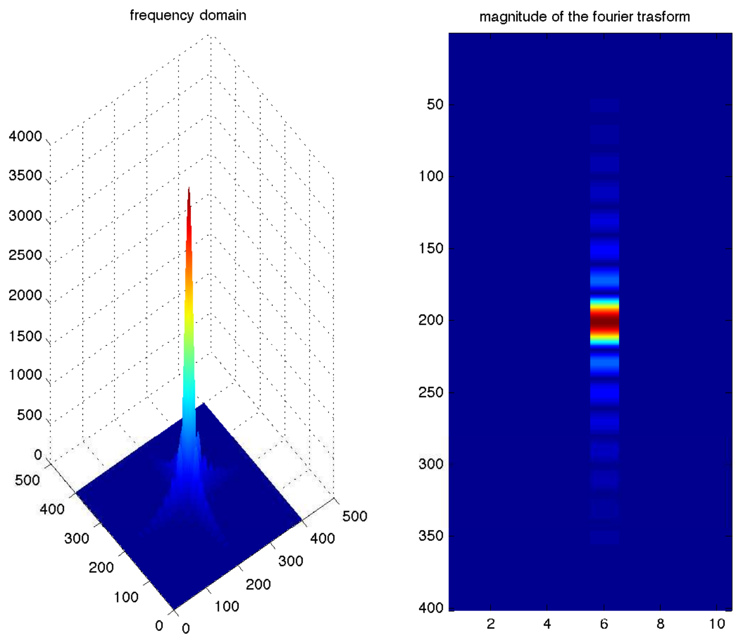

Figure 3.2: Fourier Transform in the $f_x - f_y$ and $f_t - f_y$ plane of the still image video

3.2 Moving Video

Finally we analyse the case of a video with motion, from theory we obtain the following analytical expression for the transform:

$$\tilde{I}(F_x, f_y, f_t) = \tilde{I_0}(f_x, f_y)\delta(f_t - f_x v_x - f_y v_y)$$

the meaning of this is that increasing the moving speed in frequency the spectrum of the original image rotates. Visualizing the spectrum in the $f_t - f_y$ plane as in the previous section we can see how the spectrum translates increasingly with increasing speed.
The results of the simulations are in Figure 3.3 and Figure 3.4

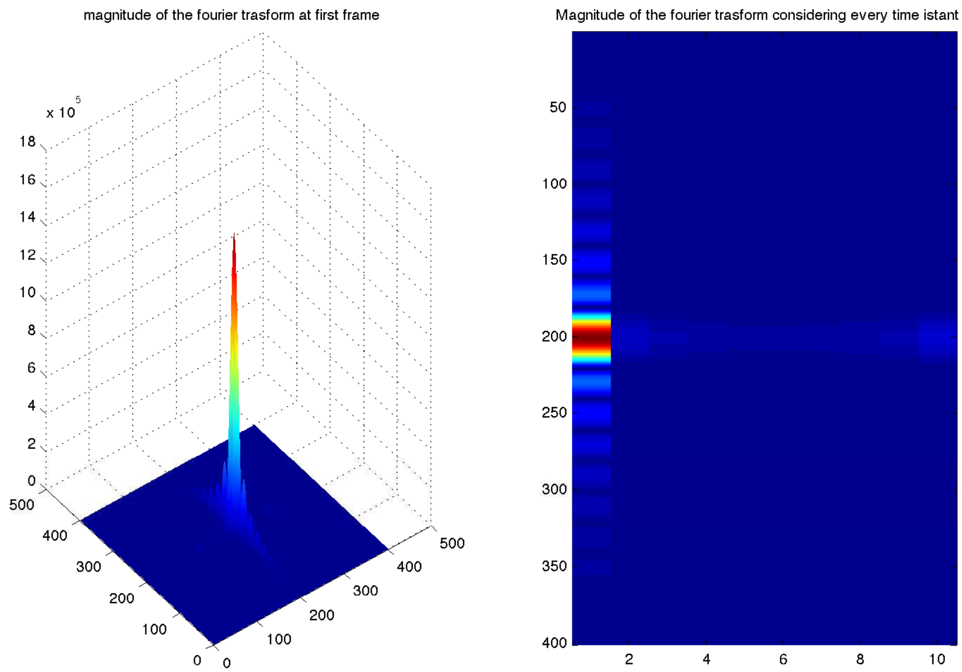

Figure 3.3: Fourier Transform in the $f_t - f_y$ with $v_y = 0.25$

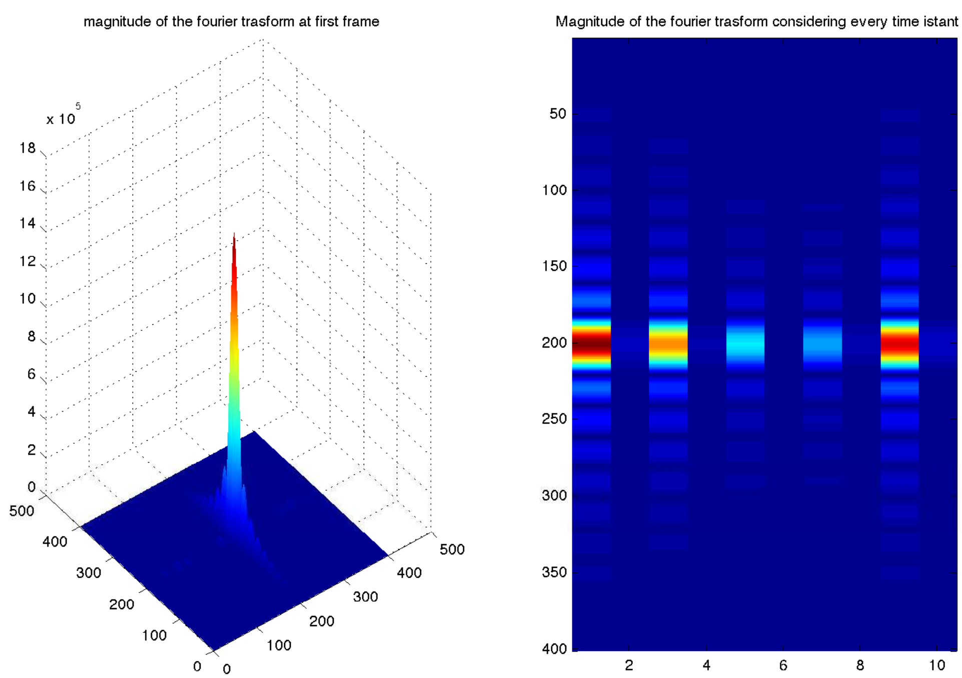

Figure 3.4: Fourier Transform in the $f_t - f_y$ with $v_y = -0.15$