University of Brescia
Faculty of Engineering

Communication Technologies and Multimedia

Multimedia Information Coding and Description

Laboratory Experience, No.4

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Abstract

In this laboratory experience, we talk about sampling: before in two dimensions, i.e. images, and after in three dimensions, i.e. videos. In the first part of this experience we use a simple synthetic image, created in Matlab ambient, and we *subsample* this image with some type of sampling, like *quincunx grid*. After that we compute its Fourier Transform comparing the obtained results. Then repeat the experiments with different signals. In the second part of experience, we create a synthetic video, compute its Fourier Transform and display it, comparing the effect of different speeds in both space and frequency domains. After that we sampling the video with the different strategy, like *interlaced* strategy, and compare the results.

1 Image Sampling

As anticipated above, in the first part of experience we implement some exercise in order to understand the operation of sampling in two dimensions. So, we create a synthetic image with a rectangle in the middle, as shown in Figure 1.

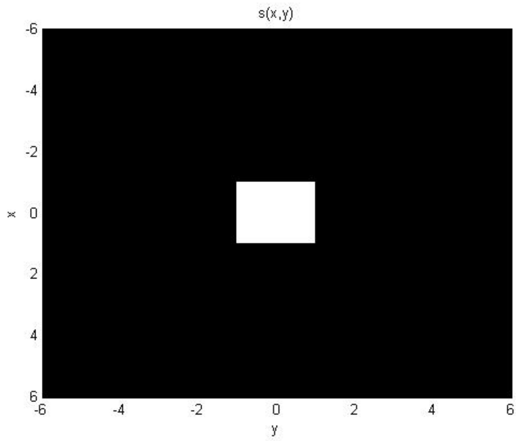

Figure 1: The synthetic image $s(x, y)$.

Using the $fft2$ and $fftshift$ functions of Matlab, we compute its Fourier Transform as is shown in Figure 2.

Note that the magnitude is for us important, while the phase does not yield much new information about the structure of image into the spatial domain; so, in this and the following examples, we restrict ourselves to displaying only the magnitude of the Fourier Transform.

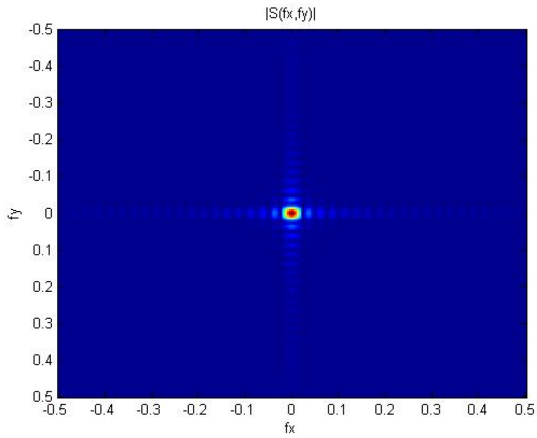

Figure 2: The magnitude of Fourier Transform $|S(x,y)|$.

As we expect the spectrum of $s(x,y)$ signal is a 2D-$sinc$ (see laboratory experience, no. 2).

After these preliminary step, we start to apply the subsampling operation to the test image.

We subsample the test image by a factor of 4 in the vertical direction (x) by multiplying it by a sampling matrix made of zeros and ones. Obviously one means "take the sample", vice versa the zero means. The obtained result is shown in Figure 3.

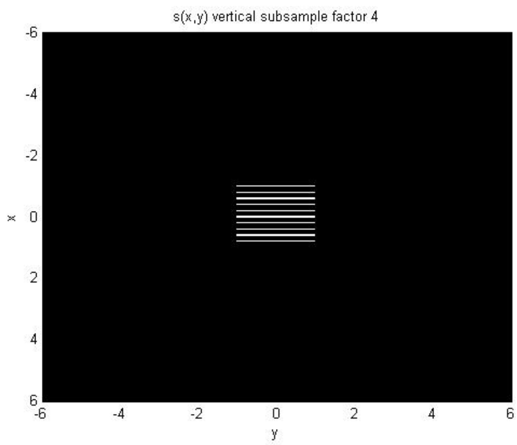

Figure 3: The vertical subsample of image $s(x,y)$, $s_2(x,y)$.

Like before, we also compute its Fourier Transform and display it in Figure 4.

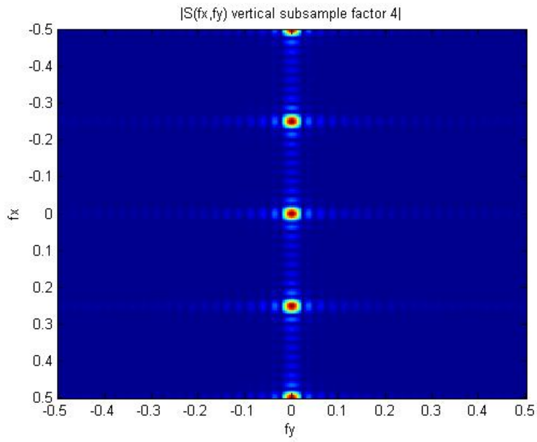

Figure 4: The module of Fourier Transform $|S_2(x, y)|$.

As it is possible see in the Figure 4 there is a periodic repetition along f_x direction of the spectrum of original image. Note that the locations of repetitions depend on the sampling factor, in our case the distance between two repetitions is equal to $\frac{1}{4}$ because the sampling factor is 4.

To better understand this concept we make a new example. We subsample the original image by a factor of 4 in both horizontal and vertical directions (Figure 5).

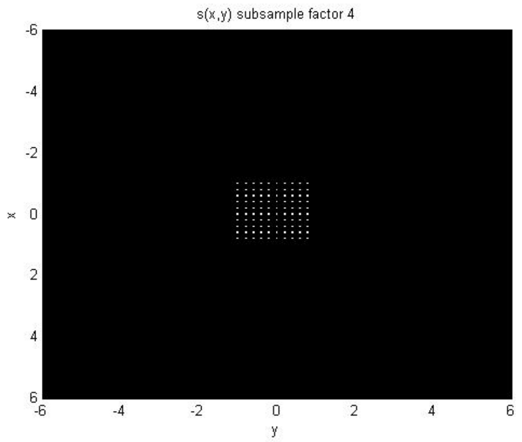

Figure 5: The subsample of image $s(x, y)$, $s_3(x, y)$.

The Fourier Transform of this signal is reported in Figure 6.

Again we have the repetition of the spectrum of the original image but, in this case, the replicas are in the both f_x and f_y frequency directions.

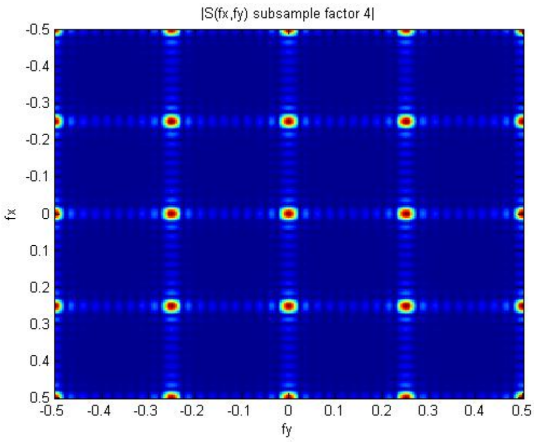

Figure 6: The module of Fourier Transform $|S_3(x, y)|$.

Note that the distance between two different replicas is, along both vertical and horizontal direction, the same of the previous case.

Then we repeat the experiment using a *quincunx grid*. This pattern is produced as the lattice in Figure 7.

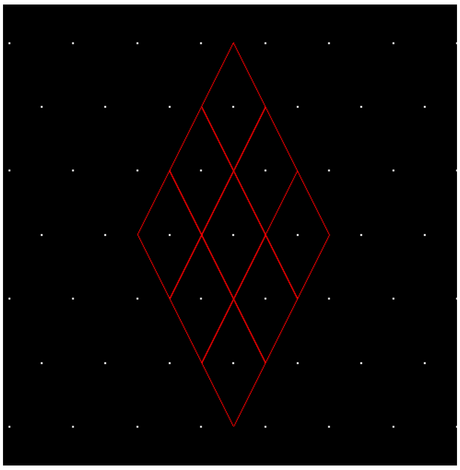

Figure 7: The *quincux* grid.

You can think of it this way, or you can think of it as a rotated square grid that has been squeezed and stretched. The red lines show the latter interpretation. There are two nearest neighbors at a distance of 2, and four more neighbors at a distance of $\sqrt{5}$.

We apply the *quincunx* grid at our test image, the result is shown in Figure 8.

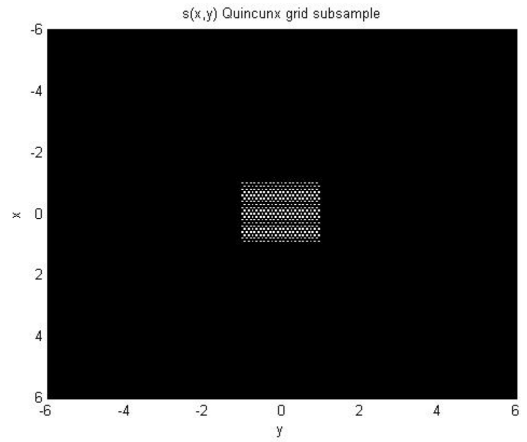

Figure 8: The *quincunx* subsample of image $s(x,y)$, $s_4(x,y)$.

Like before, we also compute its Fourier Transform and display it in Figure 9.

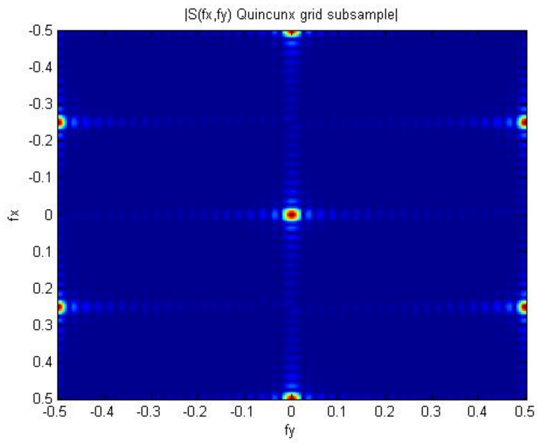

Figure 9: The module of Fourier Transform $|S_4(x,y)|$.

In above Figure 9, it is possible to see in the spectrum that there are some replicas, which are located following the *quinqunx* pattern. The distances between different replicas are proportional at the inverse of the sampling step.

Now follow other examples of image sampling of the signals: $i(x,y) = sin(2 \cdot \pi \cdot x)$. Using the previous sampling techniques on this test image, we compare the results with the previous ones.

2 Video Sampling

After having talk about the image sampling in the above section, in this second part of experience we implement some exercise in order to understand the operation of sampling in three dimensions.

So, we create a synthetic video $sin(2 \cdot \pi \cdot x + t \cdot v_x)$ moving in the vertical direction (x) considering 15 fps (frames per second) of 15 seconds long, as shown in Figure 10.

(Click to play video)

Figure 10: The synthetic video $v(x, y, t)$.

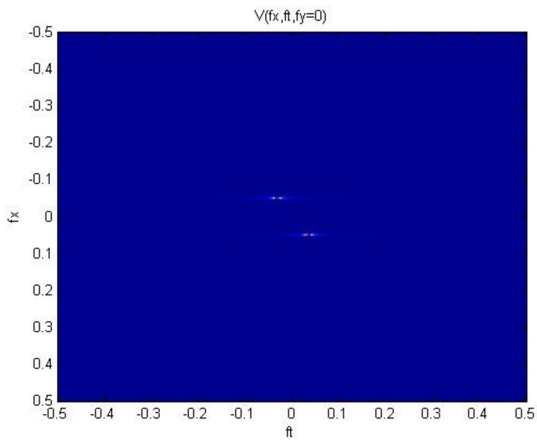

Figure 11: The spectrum of the video $v(x, y, t)$: $V(f_x, f_t, f_y = 0)$.

We can note that we have simplified the situation, in fact we have a video in which only one spatial-dimension is moving (x), while the other spatial-direction (y) remain constant.

That means the energy of the spectrum of our video in located only in spatial-frequencies f_y equal to 0 (constant along an direction means to have a wall of delta in that frequency-direction).

So, we can visualize the spectrum of video by extracting the only components in $f_y = 0$, as shown in Figure 11.

After this notation, we have observed the effect of different speeds in both space and frequency domains. In particular, as a result of spatial-velocity change (v_x), the frequency response gets a rotation with some degree. By increasing the velocity, we note as the spectrum is moved to the boundary of our frequency window. This until to critic velocity that give rise the aliasing effect. In fact, the near spectrum replicas appears into our window domain, so the observer assumes the object is moving in opposite direction. As it is possible see this concept in Figure 12.

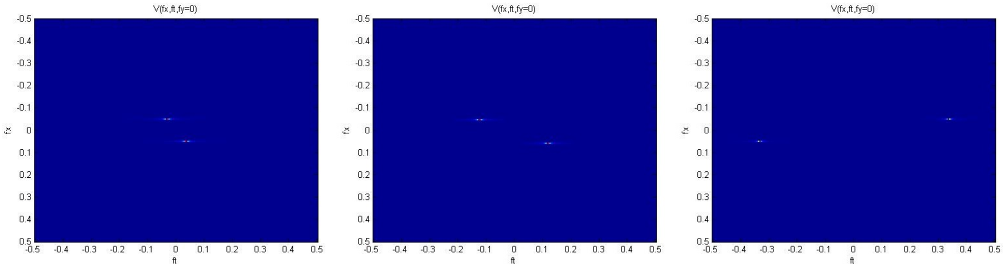

Figure 12: The spectrum of the video $v(x, y, t)$ with a different spatial speeds (v_x): π, 4π and 20π.

When the video is static, the energy of its Fourier Transform lie at $f_t = 0$, as in Equation 1.

$$v(x, y, t) = i_0(x, y) \iff V(f_x, f_y, f_t) = I_0(f_x, f_y) \cdot \delta(f_t) \tag{1}$$

When instead the video moves with constant velocity $[v_x, v_y]$, its Fourier Transform become to Equation 2.

$$\begin{gathered} v(x, y, t) = i_0(x + v_x \cdot t, y + v_y \cdot t, 0) \\ \Updownarrow \\ V(f_x, f_y, f_t) = I_0(f_x, f_y) \cdot \delta(f_t - f_x \cdot v_x - f_y \cdot v_y) \end{gathered} \tag{2}$$

This means that there is a change of reference system, like in the previous Figure 12.

Instead, changing the frequency of the sinusoid we note that the $delta$s are different spaced.

After have seen this, we sample our video by a factor of 4 in the time direction, as shown in Figure 13. Then we compute, and display, its Fourier Transform in Figure 14.

(Click to play video)

Figure 13: Video test.

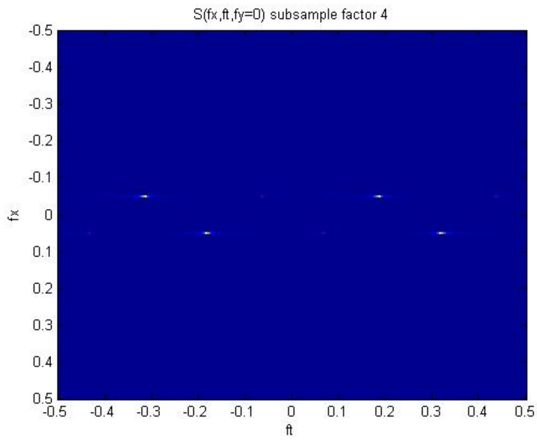

Figure 14: The spectrum of the video $v(x, y, t)$: sampled version.

When the video is sampled at a factor of n in time direction (t), in the frequency domain the Fourier Transform of the original video is repeated along f_t direction at the period of $1/n$. Like Equation 3

$$\begin{gathered} v_s(x,y,t) = v(x,y,t) \cdot \delta_{\underline{\Delta}}^{(3)}(x,y,t) \\ \Updownarrow \\ V_s(f_x,f_y,f_t) = V(f_x,f_y,f_t) * \mathfrak{F}\left\{\delta_{\underline{\Delta}}^{(3)}(x,y,t)\right\} \end{gathered} \tag{3}$$

In the last point of our laboratory experience, we have downsampled the video with an interlaced strategy, and like before we computed its transform.

The interlaced video is made by by sampling of the original video not only in time direction, like in previous case, but also in space domain. In particular along x direction, while along y direction there isn't sampling.

We take the odd frame and maintain its odd raws; the next even frame, we maintain its even raws, and so on.

The result of this sampling technique is shown in Figure 15.

(Click to play video)

Figure 15: Video test.

Like the previous case, we are talking about sampling and then we can use the previous Equation 3. So we expect to see in the spectrum a replica of the original one.

But the interlaced pattern of sampling taking into account both the spacial (x) and temporal (t) dimension. So, the replica lie in both f_x and f_t.

Then we compute the Fourier Transform and display it in Figure 16.

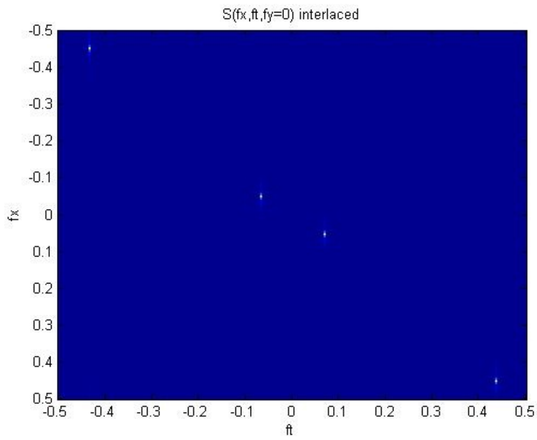

Figure 16: The spectrum of the video $v(x, y, t)$: interlaced version.