Week #1 - Mesh and projection

1 Projection - estimation of flux at mesh nodes

Use the course notes (Chapter 1) to understand the problem at hand ! Correlate the weak form of the flux equation in the course notes to the code `project_flux` in `Assembly.py` and `project_element_flux` in `Element.py`. Answer the following questions:

1. At which points are evaluated the shape functions and their derivatives? Given your answer, what does the left hand side of the weak form represent?
2. How integration over an element is performed and what is the role of the weights and the jacobian determinant?

In the jupyter notebook, `projection_square_flux.ipynb` (located under `sessions/week_1/`), you will find an example for the creation of an unstructured mesh for the unit square. Moreover, we also give you the analytical solution of a Laplacian problem, i.e.

$$h(x, y) = \sin(\pi x) \sinh(\pi y) / \sinh(\pi)$$

The goal of this exercice is to finish coding up the solution of a projection procedure - notably via the function partially written in the file `MatrixAssembly.py` (function `project_flux` and `assemble_mass_matrix`) in the `src` folder of the base python code. The steps of the procedure are outlined but you need to code up the remaining parts (notably the assembly of the force vector and the global mass matrix out of elemental ones).

After having solved the projection problem, plot the obtained component of the flux q_x and q_y on the mesh using the function plot in the `Mesh` class (`Mesh.py`). Then compute the absolute and relative error with the exact solution for the flux (assuming a permeability coefficient of unity), e.g:

$$q_x = -\pi \cos(\pi x) \sinh(\pi y) / \sinh(\pi)$$
$$q_y = -\pi \sin(\pi x) \cosh(\pi y) / \sinh(\pi)$$

2 Mesh of a sheet-pile wall

In this exercise, we ask you to write a python script to mesh the geometry of a sheet-pile wall as displayed in figure 1 using pygmsh (see example in `projection_square_flux.ipynb`). Write a code where the different dimensions of the problem can be easily changed. We will use that mesh to solve the corresponding steady-state flow problem next week. (Feel free to use another mesher if you want).

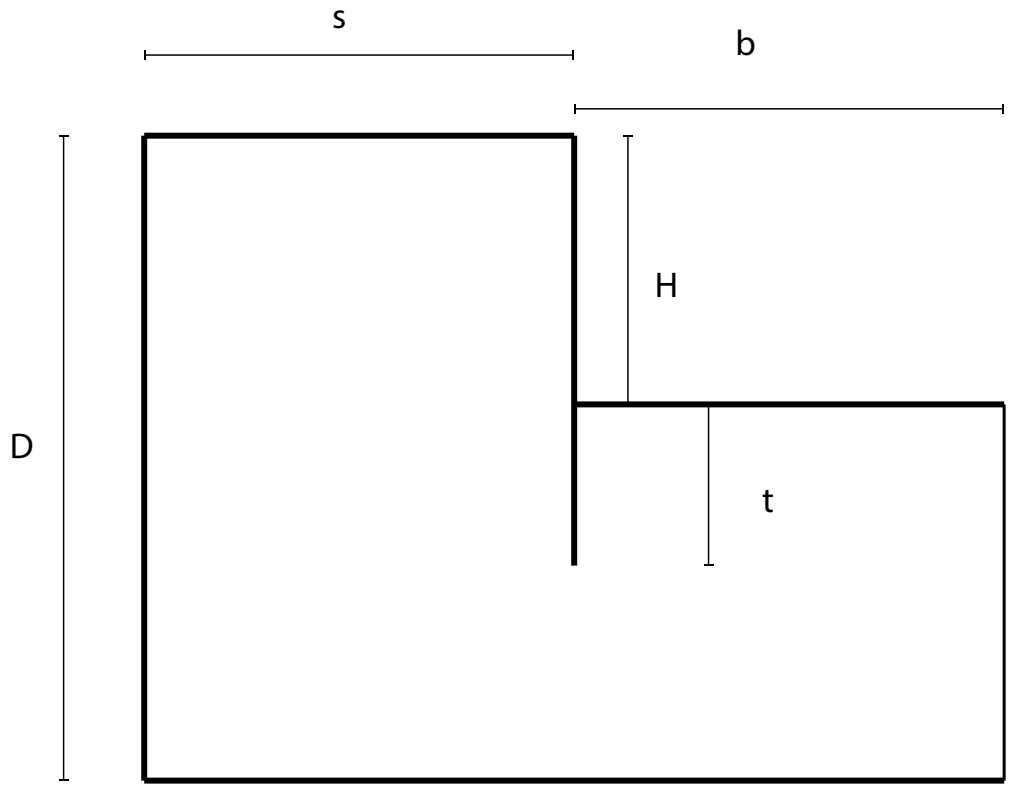

Figure 1: Sketch of a sheet pile wall.